Mecânica Lagrangeana

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Princípio da Mínima Ação

$$S = \int_{t_1}^{t_2} dt\ L(q, \dot{q}, t)$$

$$q \to q + \delta q$$

$$\delta q(t_1) = \delta q(t_2) = 0$$

$$\delta S = \int_{t_1}^{t_2} dt\ L(q + \delta q, \dot{q} + \delta \dot{q}, t) - \int_{t_1}^{t_2} dt\ L(q, \dot{q}, t)$$

$$\delta S = \int_{t_1}^{t_2} dt\ \left(\frac{\partial L}{\partial q} \delta q + \frac{\partial L}{\partial \dot{q}} \delta \dot{q} + L(q, \dot{q}, t) \right) - \int_{t_1}^{t_2} dt\ L(q, \dot{q}, t)$$

$$\delta S = \int_{t_1}^{t_2} dt\ \left(\frac{\partial L}{\partial q} \delta q + \frac{\partial L}{\partial \dot{q}} \delta \dot{q} \right)$$

$$\delta \dot{q} = \frac{d}{dt} \delta q$$

$$\delta S = \int_{t_1}^{t_2} dt\ \left(\frac{\partial L}{\partial q} \delta q + \frac{\partial L}{\partial \dot{q}} \frac{d}{dt} \delta q \right)$$

$$\frac{d}{dt} \left[\frac{\partial L}{\partial \dot{q}} \delta q \right] = \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} \delta q + \frac{\partial L}{\partial \dot{q}} \frac{d}{dt} \delta q$$

$$\delta S = \int_{t_1}^{t_2} dt\ \left(\frac{\partial L}{\partial q} \delta q - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} \delta q \right) + \int_{t_1}^{t_2} dt\ \frac{d}{dt} \left[\frac{\partial L}{\partial \dot{q}} \delta q \right]$$

$$\delta S = \int_{t_1}^{t_2} dt\ \left(\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} \right) \delta q$$

$$\delta S = 0$$

$$\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} = 0$$

$$S = \int_{t_1}^{t_2} dt\ L(q, \dot{q}, t)$$

$$L = \int d^3x\ \mathcal{L}(\phi, \partial_\mu \phi)$$

$$S = \int d^4x\ \mathcal{L}(\phi, \partial_\mu \phi)$$

$$\delta S = \int d^4x\ \left(\frac{\partial \mathcal{L}}{\partial \phi} \delta\phi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta(\partial_\mu \phi) \right)$$

$$\delta S = \int d^4x\ \left(\frac{\partial \mathcal{L}}{\partial \phi} \delta\phi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial_\mu \delta\phi \right)$$

$$\partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta\phi \right] = \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta\phi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial_\mu \delta\phi$$

$$\delta S = \int d^4x\ \left(\frac{\partial \mathcal{L}}{\partial \phi} \delta\phi - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta\phi \right) + \int d^4x\ \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta\phi \right]$$

$$\delta S = \int d^4x\ \left(\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) \delta\phi$$

$$\delta S = 0$$

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = 0$$