Teoria Clássica de Campos

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Abstract: Trabalho em Progresso

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Sumário

1 Aula 1

1.1 Formalismo Lagrangeano

$$S = \int_{t_1}^{t_2} L(q, \dot{q}, t)dt$$

$$q(t) \to q'(t) = q(t) + \delta q(t)$$

$$\delta q(t_1) = \delta q(t_2) = 0$$

$$\delta S = S[q + \delta q, \dot{q} + \delta \dot{q}] - S[q, \dot{q}] = 0$$

$$\delta S = \int L(q+\delta q, \dot{q}+\delta\dot{q})dt - L(q,\dot{q})dt$$

$$L(q+\delta q) = \frac{\partial L}{\partial q}\delta q + L(q)$$

$$L(\dot{q}+\delta\dot{q}) = \frac{\partial L}{\partial \dot{q}}\delta\dot{q} + L(\dot{q})$$

$$L(q+\delta q, \dot{q}+\delta\dot{q}) = \frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot{q}}\delta\dot{q} + L(q,\dot{q})$$

$$\delta S = \int \left(\frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot{q}}\delta\dot{q}\right)\mathrm{d}t = \left(\frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot{q}}\frac{\mathrm{d}}{\mathrm{d}t}\delta\dot{q}\right)\mathrm{d}t$$

Podemos integrar o segundo termo por partes

$$\delta S = \int \left[\frac{\partial L}{\partial q}\delta q - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right)\delta q + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\delta q\right)\right]$$

A integral da derivada total fornece um termo proporcional às variações δq mas como elas são nulas nos extremos ficamos simplesmente com

$$\delta S = \int \left(\frac{\partial L}{\partial q} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}}\right)\delta q.dt = 0$$

$$\boxed{\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = 0}$$

Onde $L = T - U$

1.2 Oscilador Harmônico

$$U(x) = \frac{1}{2}k.x^2$$

$$T = \frac{1}{2}m.\dot{x}^2$$

$$L = \frac{1}{2}m.\dot{x}^2 - \frac{1}{2}k.x^2$$

$$\frac{\partial L}{\partial x} = -k.x \quad , \quad \frac{\partial L}{\partial \dot{x}} = m.\dot{x}$$

$$m.\ddot{x} = -k.x$$

1.3 Formalismo Hamiltoniano

$$p_i \equiv \frac{\partial L}{\partial \dot{q}_i}$$

$$L(q,\dot{q},t) \rightarrow H(q,p,t)$$

Transformada de Legendre

$$H(q,p,t) = \sum_i p_i.\dot{q}_i - L(q,\dot{q},t)$$

Equações de Movimento

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

1.4 Leis de Conservação

1.4.1 L não não depende de t - Conservação da Energia

$$\frac{\mathrm{d}L}{\mathrm{d}t} = \sum_i \left(\frac{\partial L}{\partial q_i}\frac{\mathrm{d}q_i}{\mathrm{d}t} + \frac{\partial L}{\partial \dot{q}_i}\frac{\mathrm{d}\dot{q}_i}{\mathrm{d}t} \right)$$

$$\frac{\mathrm{d}L}{\mathrm{d}t} = \sum_i \left(\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_i}\dot{q}_i + \frac{\partial L}{\partial \dot{q}_i}\ddot{q} \right)$$

$$\frac{\mathrm{d}L}{\mathrm{d}t} = \sum_i \frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\partial L}{\partial \dot{q}_i}\dot{q}_i \right)$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \left(\sum_i \frac{\partial L}{\partial \dot{q}_i}\dot{q}_i - L \right) = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \left(\sum_i p_i.\dot{q}_i - L \right) = 0$$

$$\frac{\mathrm{d}H}{\mathrm{d}t} = 0$$

1.4.2 L não depende de q - Conservação de Momento

$$\frac{\partial L}{\partial q_i} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_i} = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_i} = 0$$

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = 0$$

1.4.3 L não depende de ϕ - Conservação do Momento Angular

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$$

Em coordenadas polares

$$T = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2)$$

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) - U(r)$$

$$p_\phi = \frac{\partial L}{\partial \phi} = mr^2\dot{\phi} =$$

momento angular

2 Aula 2

2.1 Variação diretamente na ação

$$S = \int \mathrm{d}t \left(\frac{1}{2} m \dot{q}_i^2 - U(q) \right)$$

$$\delta S = \int \mathrm{d}t \left(m \dot{q}_i . \delta \dot{q} - \frac{\mathrm{d}U}{\mathrm{d}q} \delta q \right) = 0$$

$$\delta S = \int \mathrm{d}t \, . \delta q \left(-m . \ddot{q} - \frac{\mathrm{d}U}{\mathrm{d}q} \right) = 0$$

$$m . \frac{\mathrm{d}^2 q}{\mathrm{d}t^2} = - \frac{\mathrm{d}U}{\mathrm{d}q}$$

2.2 Rotações Infinitesimais

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} . \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & \theta \\ -\theta & 1 \end{pmatrix} . \begin{pmatrix} x \\ y \end{pmatrix}$$

$$x' = x + \theta . y$$

$$y' = y - \theta . x$$

$$\delta x = x' - x = \theta . y$$

$$\delta y = y' - y = -\theta . x$$

Generalização

$$\delta x_i = \epsilon_{ij} . x_j$$

Invariância da norma

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{x_1^2 + x_2^2 + x_3^2} = \sqrt{x_i x_i}$$

$$\delta r = \frac{1}{2} \frac{2 x_i \delta x_i}{\sqrt{x_i x_i}} = \frac{1}{r} x_i \delta x_i = \frac{1}{r} x_i \epsilon_{ij} x_j$$

ϵ_{ij} Tem de ser anti-simétrico para que $\delta r = 0$ e a norma seje invariante.

2.3 Invariância da ação por rotação infinitesimal e simetria associada

$$U = U(r)$$

$$\delta U = \frac{\mathrm{d}U}{\mathrm{d}r}\delta r = 0$$

$$\delta S \int \mathrm{d}t\,(\delta T - \delta U)$$

$$\delta S \int \mathrm{d}t\,\delta T$$

$$\delta T = \frac{\mathrm{d}T}{\mathrm{d}\dot{x}}\delta\dot{x}_i = m.\dot{x}_i\delta\dot{x}_i$$

$$\delta S = \int \mathrm{d}t\,(m.\dot{x}_i\delta\dot{x}_i)$$

$$\delta S = \int \mathrm{d}t\left[m\frac{\mathrm{d}}{\mathrm{d}t}(\dot{x}_i\delta x_i) - m\ddot{x}_i\delta x_i\right]$$

Equação de movimento

$$m.\ddot{x}_i = -\frac{\mathrm{d}V}{\mathrm{d}x_i} = \frac{\mathrm{d}V}{\mathrm{d}r}\frac{\mathrm{d}r}{\mathrm{d}x_i}$$

$$\frac{\mathrm{d}r}{\mathrm{d}x_i} = \frac{\mathrm{d}x_i}{\mathrm{d}r}$$

$$m.\ddot{x}_i = -\frac{\mathrm{d}V}{\mathrm{d}r}\frac{\mathrm{d}x_i}{\mathrm{d}r}$$

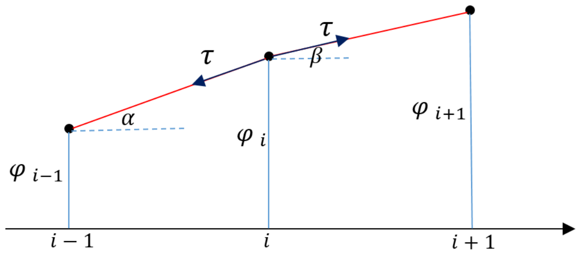
$$\delta S = \int \mathrm{d}t\left(m\frac{d}{dt}(\dot{x}_i\delta x_i) + \frac{dV}{dr}\frac{x_i}{r}\delta x_i\right)$$

$$\delta S = \int \mathrm{d}t\left[m\frac{\mathrm{d}}{\mathrm{d}t}(\dot{x}_i\delta x_i)\right]$$

$$\delta S = m.\dot{x}_i.\delta x_i|_{t_1}^{t_2} = 0$$

$$m.\dot{x}_i.\epsilon_{ij}x_j \rightarrow$$

Momento angular conservado



2.4 Passagem do discreto para o Contínuo

Consideremos um sistema de massa acopladas e que oscilam verticalmente Lei de Newton

$$m.\ddot{\varphi}_i = \tau \sin\beta - \tau \sin\alpha$$

Como os ângulos são pequenos:

$$\sin\beta = \tan\beta$$
$$\sin\alpha = \tan\alpha$$

$$m.\ddot{\varphi}_i = \frac{\tau}{a}[(\varphi_{i+1} - \varphi_i) - (\varphi_i - \varphi_{i-1})]$$

$$\frac{\tau}{a}[(\varphi_{i+1} - \varphi_i) - (\varphi_i - \varphi_{i-1})] = F_R$$

$$U(\varphi_i) = -\int F.\,\mathrm{d}\varphi_i$$

$$U = \frac{\tau}{2a}(\varphi_{i+1} - \varphi_i)^2 + \frac{\tau}{2a}(\varphi_i - \varphi_{i-1})^2$$

$$U = \sum_k \frac{\tau}{2a}(\varphi_{k+1} - \varphi_k)^2$$

$$T = \sum_k \frac{1}{2}m.\dot{\varphi}_k^2$$

$$L = \sum_k \frac{1}{2}m.\dot{\varphi}_k^2 - \sum_k \frac{\tau}{2a}(\varphi_{k+1} - \varphi_k)^2$$

$$a = \Delta x \to 0, \quad \varphi_i(t) \to \varphi(x,t)$$

$$L = \sum \Delta x \frac{1}{2}\frac{m}{\Delta x}\left(\frac{\partial\varphi}{\partial t}\right)^2 - \sum \Delta x^2 \frac{\tau}{2\Delta x}\left(\frac{\varphi(x+\Delta x,t) - \varphi(x,t)}{\Delta x}\right)^2$$

$$L = \lim_{\Delta x\to 0} \sum \Delta x \frac{1}{2}\frac{m}{\Delta x}\left(\frac{\partial\varphi}{\partial t}\right)^2 - \sum \Delta x \frac{\tau}{2}\left(\frac{\varphi(x+\Delta x,t) - \varphi(x,t)}{\Delta x}\right)^2$$

$$L = \mathrm{d}x \left[\frac{\sigma}{2} \left(\frac{\partial \varphi}{\partial t} \right)^2 - \frac{\tau}{2} \left(\frac{\partial \varphi}{\partial t} \right)^2 \right]$$

3 Aula 3

3.1 Equação de Euler-Lagrange para Campos

De forma geral

$$\mathcal{L} = \mathcal{L}\left(\varphi, \frac{\partial \varphi}{\partial t}, \frac{\partial \varphi}{\partial x}\right)$$

$$S = \int\int \mathrm{d}t\,\mathrm{d}x\,\mathcal{L}\left(\varphi, \frac{\partial \varphi}{\partial t}, \frac{\partial \varphi}{\partial x}\right)$$

$$\delta S = \int\int \mathrm{d}t\,\mathrm{d}x\left(\frac{\partial \mathcal{L}}{\partial \varphi}\delta\varphi + \frac{\partial \mathcal{L}}{\partial \dot{\varphi}}\delta\dot{\varphi} + \frac{\partial \mathcal{L}}{\partial \varphi'}\delta\varphi'\right)$$

$$\delta S = \int\int \mathrm{d}t\,\mathrm{d}x\left(\frac{\partial \mathcal{L}}{\partial \varphi} - \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \dot{\varphi}} - \frac{\partial}{\partial x}\frac{\partial \mathcal{L}}{\partial \varphi'}\right)\delta\varphi$$

$$\delta S = 0$$

$$\frac{\partial \mathcal{L}}{\partial \varphi} - \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \dot{\varphi}} - \frac{\partial}{\partial x}\frac{\partial \mathcal{L}}{\partial \varphi'} = 0$$

3.2 Relatividade Especial

$$\mathrm{d}s = \mathrm{d}s'$$

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2$$

Quadri-vetor posição

$$x^\mu = (x^0, x^1, x^2, x^3)$$

$$\mathrm{d}s^2 = \eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$$

Referencial $S \to x^\mu$
Referencial $S' \to x'^\mu$

$$x'^\mu = \Lambda^\mu_\nu . x^\nu$$

$$\Lambda^\mu_\nu = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Lambda^{\mu}_{\nu} = \begin{pmatrix} \cosh\alpha & -\sinh\alpha & 0 & 0 \\ -\sinh\alpha & \cosh\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$x'^{0} = \cosh\alpha . x^{0} - \sinh\alpha . x^{1}$$

$$x'^{1} = -\sinh\alpha . x^{0} + \cosh\alpha . x^{1}$$

$$x'^{2} = x^{2}$$

$$x'^{3} = x^{3}$$

$$\mathrm{d}s'^{2} = \eta_{\mu\nu}\,\mathrm{d}x'^{\mu}\,\mathrm{d}x'^{\nu} = \eta_{\mu\nu}\Lambda^{\mu}_{\alpha}\,\mathrm{d}x^{\alpha}\,\Lambda^{\nu}_{\beta}\,\mathrm{d}x^{\beta} = \eta_{\alpha\beta}\,\mathrm{d}x^{\alpha}\,\mathrm{d}x^{\beta}$$

$$\eta_{\mu\nu}\Lambda^{\mu}_{\alpha}\Lambda^{\nu}_{\beta} = \eta_{\alpha\beta}$$

3.3 Transformações de Lorentz infinitesimais

$$x'^{\mu} = x^{\mu} + \epsilon^{\mu}_{\nu}.x^{\nu}$$

$$\Lambda^{\mu}_{\nu} = \delta^{\mu}_{\nu} + \epsilon^{\mu}_{\nu}$$

$$\eta_{\mu\nu} = (\delta^{\mu}_{\alpha} + \epsilon^{\mu}_{\alpha})(\delta^{\nu}_{\beta} + \epsilon^{\nu}_{\beta}) = \eta_{\alpha\beta}$$

$$\eta_{\mu\nu} = (\delta^{\mu}_{\alpha}\delta^{\nu}_{\beta} + \delta^{\mu}_{\alpha}\epsilon^{\nu}_{\beta} + \delta^{\nu}_{\beta}\epsilon^{\mu}_{\alpha} + \epsilon^{\mu}_{\alpha}\epsilon^{\nu}_{\beta}) = \eta_{\alpha\beta}$$

$$\eta_{\alpha\beta} + \eta_{\alpha\mu}\epsilon^{\nu}_{\beta} + \eta_{\mu\beta}\epsilon^{\mu}_{\alpha} = \eta_{\alpha\beta}$$

$$V^{\mu} \to V_{\mu} = \eta_{\mu\nu}V^{\nu}$$

$$\eta_{\alpha\nu}\epsilon^{\nu}_{\beta} \equiv \epsilon_{\alpha\beta}$$

$$\eta_{\mu\beta}\epsilon^{\mu}_{\alpha} = \epsilon_{\mu\alpha}$$

$$\epsilon_{\alpha\beta} + \epsilon_{\beta\alpha} = 0$$

$\epsilon_{\alpha\beta}$ é anti-simétrica

3.4 rotações

3.5 Translações

Translações

$$x'^{\mu} = x^{\mu} + a^{\mu}$$

Translações infintesimais

$$x'^{\mu} = x^{\mu} + \epsilon^{\mu}$$

$$\delta x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu} + \epsilon^{\mu}$$

$$\frac{\partial}{\partial x^{\mu}} = \left(\frac{\partial}{\partial x^0}, \frac{\partial}{\partial x^1} . \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3} \right) = \left(\frac{\partial}{\partial x^0, \vec{\nabla}} \right)$$

$$\partial_{\mu} = \frac{\partial}{\partial x^{\mu}}$$

3.6 Equação de Movimento para Campos Relativísticos

$$\mathcal{L} = \mathcal{L}(\phi, \partial_{\mu}\phi)$$

$$S = \int \mathrm{d}^4 x \mathcal{L}(\phi, \partial_{\mu}\phi)$$

$$\delta S = \int \mathrm{d}^4 x \left[\frac{\partial \mathcal{L}}{\partial \phi} \delta\phi + \frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\phi)} \delta(\partial_{\mu}\phi) \right]$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\phi)} \delta(\partial_{\mu}\phi) = \frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\phi)} \partial_{\mu}(\delta\phi) = \partial_{\mu} \left[\frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\phi)} \delta\phi \right] - \partial_{\mu} \left[\frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\phi)} \right] \delta\phi$$

$$\delta S = \int \mathrm{d}^4 x \left\{ \frac{\partial \mathcal{L}}{\partial \phi} \delta\phi + \partial_{\mu} \left[\frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\phi)} \delta\phi \right] - \partial_{\mu} \left[\frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\phi)} \right] \delta\phi \right\}$$

$$\delta S = \int \mathrm{d}^4 x \left\{ \left[\frac{\partial \mathcal{L}}{\partial \phi} - \partial_{\mu} \left(\frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\phi)} \right) \right] \delta\phi + \partial_{\mu} \left[\frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\phi)} \delta\phi \right] \right\}$$

$$\boxed{\frac{\partial \mathcal{L}}{\partial \phi} - \partial_{\mu} \left(\frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\phi)} \right) = 0}$$

4 Aula 4

4.1 Teorema de Noether

Simetrias na ação:

Variação nas coordenadas

$$x^\mu \to x'^\mu = x^\mu + \delta x^\mu$$

Variação no campo

$$\phi \to \phi' = \phi + \delta\phi$$

É importante tomar o cuidado de considerar que uma variação nas coordenadas vai induzir uma mudança no elemento de volume quadrimensional $\mathrm{d}^4x \to \mathrm{d}^4x'$, essa diferença é fornecida pelo determinante do Jacobiano

$$\mathrm{d}^4x' = \det J \ \mathrm{d}^4x$$

Jacobiano da transformação de coordenadas

$$J^\mu_\nu = \frac{\partial x'^\mu}{\partial x^\nu}$$

$$J^\mu_\nu = \delta^\mu_\nu + \partial_\nu \delta x^\mu$$

$$\det J = \det(\mathbb{1} + \partial\delta) = 1 + \operatorname{tr} \partial\delta$$

$$\det J = 1 + \partial_\mu \delta x^\mu$$

$$\mathrm{d}^4x' = \det J \ \mathrm{d}^4x = (1 + \partial_\mu \delta x^\mu)\mathrm{d}^4x$$

$$\mathrm{d}^4x' = \mathrm{d}^4x + \mathrm{d}^4x \partial_\mu \delta x^\mu$$

$$\delta(\mathrm{d}^4x) = \mathrm{d}^4x' - \mathrm{d}^4x = \mathrm{d}^4x \ \partial_\mu \delta x^\mu$$

$$S = \int \mathrm{d}^4x \ \mathcal{L}(\phi, \partial_\mu\phi)$$

$$\delta S = \int \left[\delta(\mathrm{d}^4x) \ \mathcal{L} + \mathrm{d}^4x \ \delta\mathcal{L}\right]$$

$$\delta S = \int \left(\mathrm{d}^4x\ \partial_\mu \delta x^\mu \mathcal{L} + \mathrm{d}^4x\ \delta\mathcal{L}\right)$$

$$\delta S = \int \mathrm{d}^4x \left(\partial_\mu \delta x^\mu \mathcal{L} + \delta\mathcal{L}\right)$$

$$\delta\mathcal{L} = \mathcal{L}'(x') - \mathcal{L}(x)$$

Como δx é uma variação infinitesimal, posso fazer uma expansão da função $\mathcal{L}(x+\delta x)$ em série de Taylor até primeira ordem

$$\delta\mathcal{L} = \mathcal{L}'(x+\delta x) - \mathcal{L}(x) = \mathcal{L}'(x) + \partial_\mu \mathcal{L}' \delta x^\mu - \mathcal{L}(x)$$

Definimos a diferença $\mathcal{L}'(x) - \mathcal{L}(x)$ como $\delta_0\mathcal{L}$, denominada de variação funcional de $\mathcal{L}$, enquanto que $\delta x^\mu \partial_\mu \mathcal{L}'$ é chamado de termo de transporte

$$\delta_0\mathcal{L} \equiv \mathcal{L}'(x) - \mathcal{L}(x)$$

$$\delta\mathcal{L} = \delta_0\mathcal{L} + \delta x^\mu \partial_\mu \mathcal{L}'$$

Como estamos fazendo variações infinitesimais, a diferença entre $\mathcal{L}'$ e $\mathcal{L}$ é da ordem de δx, então $\mathcal{L}'$ e $\mathcal{L}$ só diferem por um termo quadrático adicional, que pode ser ignorado.

$$\delta\mathcal{L} = \delta_0\mathcal{L} + \delta x^\mu \partial_\mu \mathcal{L}$$

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\phi}\delta_0\phi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta_0\partial_\mu\phi + \delta x^\mu \partial_\mu \mathcal{L}$$

Integrando por partes

$$\frac{\partial\mathcal{L}}{\partial\phi}\delta_0\phi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\partial_\mu\delta_o\phi = \frac{\partial\mathcal{L}}{\partial\phi}\delta_0\phi + \partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta_0\phi\right] - \partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right]\delta_0\phi$$

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\phi}\delta_0\phi + \partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta_0\phi\right] - \partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right]\delta_0\phi + \delta x^\mu \partial_\mu \mathcal{L}$$

Fatoramos $\delta_0\phi$

$$\delta\mathcal{L} = \left[\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right)\right]\delta_0\phi + \partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta_0\phi\right] + \delta x^\mu \partial_\mu \mathcal{L}$$

O primeiro termo entre parênteses é a equação de movimento do campo ϕ

$$\delta S = \int \mathrm{d}^4x \; \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta_0 \phi + \delta x^\mu \mathcal{L} \right]$$

$$\delta \phi = \phi'(x') - \phi(x)$$

$$\delta \phi = \phi'(x + \delta x) - \phi(x)$$

$$\delta \phi = \phi'(x) + \delta x^\nu \partial_\nu \phi' - \phi(x) = \delta_0 \phi + \delta x^\nu \partial_\nu \phi$$

$$\delta \phi = \delta_0 \phi + \delta x^\nu \partial_\nu \phi$$

$$\delta S = \int \mathrm{d}^4x \; \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} (\delta \phi - \delta x^\nu \partial_\nu \phi) + \delta x^\mu \mathcal{L} \right]$$

$$\delta S = \int \mathrm{d}^4x \; \partial_\mu \left[\delta x^\mu \mathcal{L} - \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta x^\nu \partial_\nu \phi + \delta \phi \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right]$$

$$\delta S = \int \mathrm{d}^4x \; \partial_\mu \left[\delta x^\nu \left(\delta^\mu_\nu \mathcal{L} - \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial_\nu \phi \right) + \delta \phi \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right]$$

Lorentz

$$\delta x^\mu = \epsilon^\mu_\nu x^\nu$$

$$\delta \omega^a \to \epsilon^{\mu\nu}, \frac{\delta x^\mu}{\delta \epsilon^{\alpha\beta}} = x_\alpha \delta^\mu_\nu - x_\nu \delta^\mu_\alpha$$

Translação

$$\delta x^\mu = \epsilon^\mu$$

$$\delta \omega^a \to \epsilon^\mu, \frac{\delta x^\mu}{\delta \epsilon^\nu} = \delta^\mu_\nu$$

Parâmetro genérico de transformação ω_a

$$\delta x^\mu = \frac{\delta x^\mu}{\delta \omega^a} \delta \omega^a$$

$$\delta \phi = \frac{\delta \phi}{\delta \omega^a} \delta \omega^a$$

Caso genérico

$$\delta S = \int \mathrm{d}^4x \partial_\mu \left[\left(\delta^\mu_\nu \mathcal{L} - \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) \frac{\delta x^\nu}{\delta \omega^a} + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \frac{\delta \phi}{\delta \omega^a} \right] \delta \omega^a = 0$$

$$\frac{\delta x^\nu}{\delta \omega^a} \left(\delta^\mu_\nu \mathcal{L} - \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) + \frac{\delta \phi}{\delta \omega^a} \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = J^\mu_a$$

$$\int \mathrm{d}^4x \partial_\mu J^\mu_a \delta\omega^a = 0$$

Corrente de Noether

$$\partial_\mu J^\mu_a = 0$$

Carga associada

$$Q = \int \mathrm{d}^3x J^0$$

Conservação

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = 0$$

Acknowledgments

This is the most common positions for acknowledgments. A macro is available to maintain the same layout and spelling of the heading.

Note added. This is also a good position for notes added after the paper has been written.

Referências

[1] Author, *Title*, *J. Abbrev.* **vol** (year) pg.

[2] Author, *Title*, arxiv:1234.5678.

[3] Author, *Title*, Publisher (year).