
Algorithm 1 CI-TGO($f(\cdot)$, $v(\cdot)$, \mathbf{l} , \mathbf{u} , \mathbf{PS} , \mathbf{K} , Max_LS , LS_1 , LS_2 , ϕ , α , β)

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1: best  $\leftarrow$  random( $\mathbf{l}$ ,  $\mathbf{u}$ );
2: iteration  $\leftarrow$  0;
3: while Converged(best) == False do
4:   Populations  $\leftarrow$  CreatePop( $\mathbf{l}$ ,  $\mathbf{u}$ ,  $\mathbf{PS}(1)$ , 1.0);
5:   for  $p \in \{1, \dots, |\mathbf{PS}|\}$  do
6:     NewPops  $\leftarrow$   $\{\}$ ;
7:     for  $pop \in$  Populations do
8:       Topop  $\leftarrow$  TopographicalHeuristic( $f(\cdot)$ ,  $v(\cdot)$ ,  $pop$ ,  $\mathbf{K}(p)$ ,  $\alpha$ );
9:       if  $p < |\mathbf{PS}|$  then
10:        for  $\mathbf{x} \in$  Topop do
11:          NewPops  $\leftarrow$  NewPops  $\cup$ 
              {CreatePop( $\mathbf{x}$ ,  $\mathbf{l}$ ,  $\mathbf{u}$ ,  $\mathbf{PS}(p+1)$ ,  $\phi^p$ )};
12:        end for
13:      else
14:        TopoBest  $\leftarrow$  TopoBest  $\cup$  Topop;
15:      end if
16:    end for
17:    if  $p < |\mathbf{PS}|$  then
18:      Populations  $\leftarrow$  NewPops;
19:    end if
20:  end for
21:  TopoBest  $\leftarrow$   $\{\mathbf{x} = \mathbf{x}^1, \dots, \mathbf{x}^{\min(|TopoBest|, Max\_LS)} : \mathbf{x} \in sort(TopoBest)\}$ ;
22:  for  $\mathbf{x} \in$  TopoBest do
23:     $\mathbf{x} \leftarrow$  LocalSearch( $f(\cdot)$ ,  $v(\cdot)$ ,  $\mathbf{x}$ ,  $LS_1$ );
24:    if Compare( $f(\cdot)$ ,  $v(\cdot)$ ,  $\mathbf{x}$ , best) or  $f(\mathbf{x}) < f(\mathbf{best})$  then
25:       $\mathbf{x} \leftarrow$  LocalSearch( $f(\cdot)$ ,  $v(\cdot)$ ,  $\mathbf{x}$ ,  $LS_2$ );
26:      if Compare( $f(\cdot)$ ,  $v(\cdot)$ ,  $\mathbf{x}$ , best) then
27:        best  $\leftarrow$   $\mathbf{x}$ ;
28:      end if
29:    end if
30:  end for
31:  iteration  $\leftarrow$  iteration + 1;
32: end while
33: return best;

```

Pseudocode for Constrained I-TGO.

Algorithm 2 TopographicalHeuristic($f(\cdot)$, $v(\cdot)$, $Population$, K , α)

```
1:  $M \leftarrow |Population|$ ;  
2:  $best \leftarrow Population(0)$ ;  
3:  $KNN_K \leftarrow Build\_KNN(Population, K)$ ;  
4:  $R \leftarrow random([0, 1]^{M \times M})$ ;  
5:  $TopoBest \leftarrow \{\}$ ;  
6: for  $i \in \{1, \dots, |Population|\}$  do  
7:    $insert = True$ ;  
8:   for  $j \in \{1, \dots, |PS|\} \cap \{x_j \in KNN_K(x_i)\}$  do  
9:     if  $R_{i,j} < \alpha$  then  
10:       $insert \leftarrow insert \ \& \ Compare(f(\cdot), v(\cdot), x_i, x_j)$ ;  
11:     else  
12:       $insert \leftarrow insert \ \& \ f(x) < f(y)$ ;  
13:     end if  
14:   end for  
15:   if  $insert = True$  then  
16:      $TopoBest \leftarrow TopoBest \cup \{x\}$ ;  
17:   end if  
18:   if  $Compare(f(\cdot), v(\cdot), x, best)$  then  
19:      $best \leftarrow x$ ;  
20:   end if  
21: end for  
22: if  $|TopoBest| = 0$  then  
23:    $TopoBest \leftarrow TopoBest \cup \{best\}$ ;  
24: end if  
25: return  $TopoBest$ ;
```

Creates topographical ordering and returns the best elements.

Algorithm 3 CreatePop($\mathbf{x}, \mathbf{l}, \mathbf{u}, popSize, \phi$)

- 1: $\mathbf{l}' \leftarrow \max(\mathbf{l}, \mathbf{x} - (0.5 * \phi) * (\mathbf{u} - \mathbf{l}))$;
 - 2: $\mathbf{u}' \leftarrow \min(\mathbf{u}, \mathbf{x} + (0.5 * \phi) * (\mathbf{u} - \mathbf{l}))$;
 - 3: $Population \leftarrow \{\mathbf{x} = \mathbf{x}^1, \dots, \mathbf{x}^{popSize} : \mathbf{x} \in \text{random}(\mathbf{l}', \mathbf{u}')\}$;
 - 4: **return** $Population$;
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Create a new population shrunk by ϕ around the point \mathbf{x} .
