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Lista 3 de Matemática Discreta II

1) $a_n = a_{n-1} + 2a_{n-2}$

$a_0 = 2 \qquad a_1 = 7$

a) $a_2 = a_1 + 2a_0$ $\qquad a_3 = a_2 + 2a_1$ $\qquad a_4 = a_3 + 2 \cdot a_2$

$a_2 = 7 + 2 \cdot 2$ $\qquad a_3 = 11 + 2 \cdot 7$ $\qquad a_4 = 25 + 2 \cdot 11$

$a_2 = 11$ $\qquad a_3 = 25$ $\qquad a_4 = 47$

b) $\Delta(x) = x^2 - x - 2$ $\qquad a_n = c_1(2)^n + c_2(-1)^n$

$\Delta = 1 - 4 \cdot 1(-2) = 9$

$x = \frac{1 \pm 3}{2}$

$x' = 2 \qquad x'' = -1$

c) $n = 0$ $\qquad n = 1$ $\qquad a_n = 3(2)^n + (-1)(-1)^n$

$2 = c_1 + c_2$ $\qquad 7 = 2c_1 - c_2$

$$\begin{cases} c_1 + c_2 = 2 \\ 2c_1 - c_2 = 7 \end{cases}$$

$3c_1 = 9$

$\boxed{c_1 = 3 \qquad c_2 = -1}$

2) $\Delta(x) = x^3 - 6x^2 + 12x - 8$ $\quad$ ∴ Por tentativa, uma raiz é 2

$$\begin{array}{l|l} x^3 - 6x^2 + 12x - 8 & x - 2 \\ \underline{-x^3 + 2x^2} & x^2 - 4x + 4 \\ \quad -4x^2 + 12x & \\ \quad \underline{+4x^2 - 8x} & \\ \qquad 4x - 8 & \\ \qquad \underline{-4x + 8} & \\ \qquad\quad 0 & \end{array}$$

$\Delta = 16 - 4 \cdot 1 \cdot 4 = 0$

$x' = 2$

raiz = 2

multiplicidade = 3

$(x-2)^3$

$a_n = (\alpha_1 + \alpha_2 n + \alpha_3 n^2) 2^n$

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01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

b) $a_0 = 3 \quad a_1 = 4 \quad a_2 = 12$

$n=0$ $\quad$ $n=1$ $\quad$ $n=2$

$(x+0y+0z)2^0 = 3 \qquad (x+y+z)2 = 4 \qquad (x+2y+4z)4 = 12$

$\begin{cases} x = 3 \\ x+y+z = 2 \\ x+2y+4z = 3 \end{cases}$

$\begin{cases} 3+y+z = 2 \\ 3+2y+4z = 3 \end{cases}$

$\begin{cases} y+z = -1 \\ 2y+4z = 0 \end{cases}$

$\begin{cases} -2y-2z = 2 \\ 2y+4z = 0 \end{cases}$

$2z = 2$

$z = 1$

$3+y+1 = 2$

$y = -2$

$c_1 = 3$

$c_2 = -2$

$c_3 = 1$

$(3-2n+n^2)2^n$

3a) $a_n = 3a_{n-1} + 10a_{n-2} \qquad a_0 = 5 \quad a_1 = 11$

$x^2 - 3x - 10$

$\Delta = 9 - 4 \cdot 1(-10) = 49$

$x = \frac{3 \pm 7}{2} \qquad x' = 5 \quad x'' = -2$

$c_1 \cdot 5^n + c_2(-2)^n$: geral

$n=0 \rightarrow c_1 + c_2 = 5 \qquad n=1 \Rightarrow 5c_1 - 2c_2$

$\begin{cases} c_1 + c_2 = 5 \quad (\cdot 2) \\ 5c_1 - 2c_2 = 11 \end{cases}$

$\begin{cases} 2c_1 + 2c_2 = 10 \\ 5c_1 - 2c_2 = 11 \end{cases}$

$7c_1 = 21$

$c_1 = 3 \quad c_2 = 2$

$3 \cdot 5^n + 2(-2)^n$: específico

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b) $a_n = 4a_{n-1} + 21 a_{n-2}$ $\quad a_0 = 9 \quad a_1 = 13$

$x^2 - 4x - 21$

$\Delta = 16 - 4(1)(-21) = 100$

$x = \dfrac{4 \pm 10}{2}$

$x' = 7$ $\quad C_1 \cdot 7^n + C_2(-3)^n$: geral

$x'' = -3$

$n = 0$ $\quad\quad n = 1$

$9 = C_1 + C_2$ $\quad\quad 7C_1 + 3C_2 = 13$

$\begin{cases} C_1 + C_2 = 9 \quad (3) \\ 7C_1 - 3C_2 = 13 \end{cases}$

$\begin{cases} 3C_1 + 3C_2 = 27 \\ 7C_1 - 3C_2 = 13 \end{cases}$

$10C_1 = 40$

$C_1 = 4 \quad C_2 = 5$ $\quad\quad 4 \cdot 7^n + 5(-3)^n$: explícita

$C_1 = 9$

$C_2 = 2$

c) $a_n = 3a_{n-1} - 2a_{n-2}$ $\quad a_0 = 5 \quad a_1 = 8$

$x^2 - 3x + 2$

$\Delta = 9 - 4 \cdot 1 \cdot 2 = 1$ $\quad x = \dfrac{3 \pm 1}{2}$ $\quad x' = 2 \quad x'' = 1$

$C_1 \cdot 2^n + C_2 \cdot 1^n$: geral

$\begin{cases} C_1 + C_2 = 5 \quad (-2) \\ 2C_1 + C_2 = 8 \end{cases}$

$\begin{cases} -2C_1 - 2C_2 = -10 \\ 2C_1 + C_2 = 8 \end{cases}$

$-C_2 = -2$

$C_2 = 2 \quad C_1 = 3$ $\quad\quad 3 \cdot 2^n + 2 \cdot 1^n$: explícita

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d) $a_n = 5a_{n-1} - 6a_{n-2}$ $a_0 = 2$ $a_1 = 8$

$x^2 - 5x + 6$

$\Delta = 25 - 4 \cdot 1 \cdot 6 = 1$ $x = \frac{5 \pm 1}{2}$ $x' = 3$ $x'' = 2$ geral: $C_1 \cdot 3^n + C_2 \cdot 2^n$

$\begin{cases} C_1 + C_2 = 2 \quad (-2) \\ 3C_1 + 2C_2 = 8 \end{cases}$ específico: $4 \cdot 3^n - 2 \cdot 2^n$

$\begin{cases} -2C_1 - 2C_2 = -4 \\ 3C_1 + 2C_2 = 8 \end{cases}$

$C_1 = 4$ $C_2 = -2$

e) $a_n = 3a_{n-1} - a_{n-2}$ $a_0 = 0$ $a_1 = 1$

$x^2 - 3x + 1$

$\Delta = 9 - 4 \cdot 1 \cdot 1 = 5$ $x' = \frac{3 + \sqrt{5}}{2}$, $x'' = \frac{3 - \sqrt{5}}{2}$

Geral: $\left(\frac{3 + \sqrt{5}}{2}\right)^n C_1 + \left(\frac{3 - \sqrt{5}}{2}\right)^n C_2$

$\begin{cases} C_1 + C_2 = 0 \\ \frac{3+\sqrt{5}}{2} C_1 + \frac{3-\sqrt{5}}{2} C_2 = 1 \end{cases}$ $C_1 = -C_2$

$-\frac{3 - \sqrt{5}}{2} C_2 + \frac{3 - \sqrt{5}}{2} C_2 = 1$

$\frac{-2\sqrt{5}}{2} C_2 = 1$

$C_2 = -\frac{1}{\sqrt{5}}$

$C_2 = \frac{\sqrt{5}}{5}$

$C_1 = \frac{-\sqrt{5}}{5}$

específico: $-\frac{\sqrt{5}}{5}\left(\frac{3 + \sqrt{5}}{2}\right)^n + \frac{\sqrt{5}}{5}\left(\frac{3 - \sqrt{5}}{2}\right)^n$

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b) $a_n = 5a_{n-1} - 3a_{n-2}$ $\quad a_0 = 0 \quad a_1 = 1$

$x^2 - 5x + 3$

$\Delta = 25 - 4 \cdot 1 \cdot 3 = 13$

$x' = \frac{5+\sqrt{13}}{2} \quad x'' = \frac{5-\sqrt{13}}{2}$ geral $C_1 \cdot \left(\frac{5+\sqrt{13}}{2}\right)^n + C_2\left(\frac{5-\sqrt{13}}{2}\right)^n$

$C_1 = -C_2$

$-C_2\left(\frac{5+\sqrt{13}}{2}\right) + C_2\left(\frac{5-\sqrt{13}}{2}\right) = 1$

$C_2\left(\frac{-5-\sqrt{13}}{2} + \frac{5-\sqrt{13}}{2}\right) = 1$

$C_2(-\sqrt{13}) = 1$

$C_2 = -\frac{\sqrt{13}}{13} \quad C_1 = \frac{\sqrt{13}}{13}$ específica: $\frac{\sqrt{13}}{13}\left(\frac{5+\sqrt{13}}{2}\right)^n + \left(\frac{-\sqrt{13}}{13}\right)\left(\frac{5-\sqrt{13}}{2}\right)^n$

4a) $a_n = 6a_{n-1}$ $\quad a_0 = 5$

$\Delta(x) = x - 6$

$x = 6 \quad 5 = C_1 \cdot 6^0$

$\boxed{a_n = C_1 \cdot 6^n} \quad C_1 = 5 \quad \boxed{a_n = 5 \cdot 6^n}$

b) $a_n = 7a_{n-1}$ $\quad a_0 = 5$

$x - 7 = 0$

$x = 7$

$a_n = C_1 \cdot 7^n \quad C_1 = 5 \quad a_n = 5 \cdot 7^n$

$n = 0$

$1 = C_1 \cdot 1 + C_2 \cdot 0 \cdot 1$

$1 = C_1$

$n = 1$

$8 = C_1 \cdot 2 + C_2 \cdot 1 \cdot 2^2$

c) $a_n = 4a_{n-1} - 4a_{n-2}$ $\quad a_0 = 1 \quad a_1 = 8$

$8 = C_1 \cdot 2 + C_2 \cdot 1 \cdot 2^1$

$x^2 - 4x + 4$

$\Delta = 16 - 4 \cdot 1 \cdot 4 = 0$

$x = 2$

geral: $C_1 \cdot 2^n + C_2 \cdot n \cdot 2^n$

$\begin{cases} C_1 = 1 \\ 2C_1 + 2C_2 = 8 \end{cases}$

$2 + 2C_2 = 8$

$C_2 = 3$

$2^n + 3n2^n$

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d) $a_n = 10a_{n-1} - 25a_{n-2}$ $a_0 = 2$ $a_1 = 15$

$x^2 - 10x + 25$

$\Delta = 100 - 4 \cdot 1 \cdot 25 = 0$

$x = \frac{10}{2} = 5$

multiplicidade = 2

$$\begin{cases} C_1 = 2 \\ 5C_1 + 5C_2 = 15 \end{cases}$$

$C_1 + C_2 = 3$

$2 + C_2 = 3$

$C_2 = 1$

geral: $C_1 \cdot 5^n + C_2 \cdot n \cdot 5^n$ $2 \cdot 5^n + n \cdot 5^n$

5 a) $a_n = 10a_{n-1} - 32a_{n-2} + 32a_{n-3}$ $a_0 = 5$ $a_1 = 18$ $a_2 = 76$

$x^3 - 10x^2 + 32x - 32$

Por tentativa, uma das raízes é o número 2

$$\begin{array}{l|l} x^3 - 10x^2 + 32x - 32 & x - 2 \\ \underline{x^3 + 2x^2} & x^2 + 8x + 16 \\ \quad -8x^2 + 32x & \\ \quad \underline{8x^2 - 16x} & \\ \qquad 16x - 32 & \\ \qquad \underline{16x - 32} & \\ \qquad\quad 0 & \end{array}$$

$\Delta = 64 - 4 \cdot 1 \cdot 16 = 0$

$x = \frac{8}{2} = 4$

2 com m = 1

4 com m = 2

$(x-2)(x-4)^2$

$a_n = C_1 \cdot 2^n + (C_2 + C_3 n) 4^n$

$a_n = 2^n C_1 + 4^n C_2 + 4^n n C_3$

$n = 0$

$C_1 + C_2 = 5$

$n = 1$

$2C_1 + 4C_2 + 4C_3 = 18 \rightarrow C_1 + 2C_2 + 2C_3 = 9$

$n = 2$

$4C_1 + 16C_2 + 32C_3 = 76 \rightarrow C_1 + 4C_2 + 8C_3 = 19$

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$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 5 \\ 1 & 2 & 2 & 9 \\ 1 & 4 & 8 & 19 \end{array}\right] \begin{array}{l} \\ L_2-L_1 \\ L_3-L_1 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 5 \\ 0 & 1 & 2 & 4 \\ 0 & 3 & 8 & 14 \end{array}\right] \begin{array}{l} L_1-L_2 \\ \\ L_3-L_2 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 2 & 2 \end{array}\right] \cdot \tfrac{1}{2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} L_1+2L_3 \\ L_2-2L_3 \\ \\ \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$C_1 = 3$

$C_2 = 2$ $\quad a_n = 3 \cdot 2^n + 2 \cdot 4^n + 4^n$

$C_3 = 1$

b) $a_n = 9a_{n-1} - 27a_{n-2} + 27a_{n-3}$ $\quad a_0 = 5 \quad a_1 = 24 \quad a_2 = 117$

$x^3 - 9x^2 + 27x - 27$

Por tentativa, uma das raízes é o 3

$$\begin{array}{l|l} x^3 - 9x^2 + 27x - 27 & x-3 \\ \hline -x^3 + 3x^2 & x^2 - 6x + 9 \\ \quad -6x^2 + 27x & \\ \quad +6x^2 - 18x & \\ \qquad 9x - 27 & \\ \qquad -9x + 27 & \\ \qquad\quad 0 & \end{array}$$

$\Delta = 36 - 4 \cdot 1 \cdot 9 = 0$

$x = \frac{6}{2} = 3$

Raiz do polinômio característico $= 3$

multiplicidade $= 3$

$(x-3)^3$

$a_n = C_1 3^n + C_2 \cdot n \cdot 3^n + C_3 \cdot n^2 \cdot 3^n$

$n = 0 \rightarrow C_1 = 5$

$n = 1 \rightarrow 3C_1 + 3C_2 + 3C_3 = 24 \rightarrow C_1 + C_2 + C_3 = 8 \rightarrow C_2 + C_3 = 3$

$n = 2 \rightarrow 9C_1 + 18C_2 + 36C_3 = 117 \rightarrow C_1 + 2C_2 + 4C_3 = 13 \rightarrow 2C_2 + 4C_3 = 8 \rightarrow C_2 + 2C_3 = 4$

$$\begin{cases} C_2 + C_3 = 3 \quad (-1) \\ C_2 + 2C_3 = 4 \end{cases}$$

$$\begin{cases} -C_2 - C_3 = -3 \\ C_2 + 2C_3 = 4 \end{cases}$$

$C_3 = 1$

$C_2 = 2$

$a_n = 5 \cdot 3^n + 2 \cdot 3^n \cdot n + 3^n n^2$

$a_n = (5 + 2n + n^2) 3^n$

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6) a) $x^m = s x^{m-1} + t x^{m-2} \quad (x^2)$

$x^{m+2} = s x^{m+1} + t x^m \quad (\cdot \frac{1}{x^m})$

$\frac{x^{m+2}}{x^m} = \frac{s x^{m+1}}{x^m} + \frac{t x^m}{x^m}$

$x^2 = s x + t$

Como $x^m = a_m$, logo $m = 2$, logo:

$a_m = s a_{m-1} + t a_{m-2}$

$a_2 = s a_1 + t a_0 = s x + t$

b) i) $s = 2x$ $\quad a_m = 2 x a_{m-1} - x^2 a_{m-2}$

$t = -x^2$ $\quad$ Polinômio característico: $\Delta(x) = x^2 - 2x x + x^2$

Como x é uma raiz multipla da equação de 2° grau, logo, $m = 2$

$(x - x)^2$

$x^2 - 2 x x + x^2$

ii) $a_m = m x^m$

$m x^m = s(m-1) x^{(m-1)} + t(m-2) x^{m-2} \quad \cdot \frac{1}{x^{m-2}}$

$m x^2 = s(m-1) x + t(m-2)$

$m x^2 = 2x(m-1) x - x^2 (m-2)$

$m x^2 = 2 m x^2 - 2 x^2$