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1) 1001

0110

multiply (1001, 0110)

$X = 1001 \quad Y = 0110$

$n = 4$

$X_L = 10 \quad X_R = 01$

$Y_L = 01 \quad Y_R = 10$

$P_1 =$ multiply (10, 01)

$X = 10 \quad Y = 01$

$n = 2$

$X_L = 1 \quad X_R = 0$

$Y_L = 0 \quad Y_R = 1$

$P_1 =$ multiply (1, 0)

$X = 1 \quad Y = 0$

$n = 1$

return $1 \cdot 0 = 0$

$P_1 = 0$

$P_2 =$ multiply (0, 1)

$X = 0 \quad Y = 1$

$n = 1$

return $0 \cdot 1 = 0$

$P_2 = 0$

$P_3 =$ multiply (1+0, 0+1)

$X = 1 \quad Y = 1$

$n = 1$

return $1 \cdot 1 = 1$

$P_3 = 1$

return $0 \times 2^2 + (1 - 0 - 0) \times 2^{2/2} + 0$

return $000 + 10 + 0$

return 010

$P_1 = 010$

$P_2 =$ multiply$(01, 10)$

$X = 01 \quad Y = 10$

$n = 2$

$X_L = 0 \quad X_R = 1$

$Y_L = 1 \quad Y_R = 0$

$P_1 =$ multiply$(0, 1)$

$X = 0 \quad Y = 1$

$n = 1$

return $0 \cdot 1 = 0$

$P_1 = 0$

$P_2 =$ multiply$(1, 0)$

$x = 1 \quad y = 0$

$n = 1$

return $1 \cdot 0 = 0$

$P_2 = 0$

$P_3 =$ multiply$(0+1, 1+0)$

$x = 1 \quad y = 1$

$n = 1$

return $1 \cdot 1 = 1$

$P_3 = 1$

return $0 \times 2^2 + (1 - 0 - 0) \times 2^{2/2} + 0$

return $000 + 10 + 0$

return 010

$P_2 = 010$

$P_3 =$ multiply$(10 + 01, 01 + 10)$

$X = 11 \quad Y = 11$

$n = 2$

$X_L = 1 \quad X_R = 1$

$Y_L = 1 \quad Y_R = 1$

$P_1 = \text{multiply}(1, 1)$

$X = 1 \quad Y = 1$

$n = 1$

return $1 \cdot 1 = 1$

$P_1 = 1$

$P_2 = \text{multiply}(1, 1)$

$X = 1 \quad Y = 1$

$n = 1$

return $1 \cdot 1 = 1$

$P_2 = 1$

$P_3 = \text{multiply}(1+1, 1+1)$

$X = 10 \quad Y = 10$

$n = 2$

$X_L = 1 \quad X_R = 0$

$Y_L = 1 \quad Y_R = 0$

$P_1 = \text{multiply}(1, 1)$

$X = 1 \quad Y = 1$

$n = 1$

return $1 \cdot 1 = 1$

$P_1 = 1$

$P_2 = \text{multiply}(0, 0)$

$X = 0 \quad Y = 0$

$n = 1$

return $0 \cdot 0 = 0$

$P_2 = 0$

$P_3 = \text{multiply}(1+0, 1+0)$

$X = 1 \quad Y = 1$

$n = 1$

return $1 \cdot 1 = 1$

$P_3 = 1$

return $1 \times 2^2 + (1 - 1 - 0) \times 2^{2/2} + 0$

return $100 + 00 + 0$

return 100

$P_3 = 100$

return $1 \times 2^2 + (100 - 1 - 1) \times 2^{2/2} + 1$

return $100 + 10 \times 2^1 + 1$

return $100 + 100 + 1$

return 1001

$P_3 = 1001$

return $010 \times 2^4 + (1001 - 010 - 010) \times 2^{4/2} + 010$

return $0100000 + 0101 \times 2^2 + 010$

return $0100000 + 010100 + 010$

return 0110110

multiply $(1001, 0110) = 0110110$

2) a) $T(n) = 5\,T\left(\frac{n}{2}\right) + O(n)$

$a = 5$ $\quad \log_2 5 \quad 1$ $\qquad 5 = 2^x$

$b = 2$ $\quad \log_2 5 > 1$ $\qquad$ entre $]2, 3[$

$d = 1$ $\quad O(n^{\log_2 5})$

b) $T(n) = 2T(n-1) + O(1)$

$= 2[2T(n-2) + O(1)] + O(1) = 2^2 T(n-2) + 2O(1) + O(1)$

$= 2^2[2T(n-3) + O(1)] + 2O(1) + O(1) = 2^3 T(n-3) + 2^2 O(1) + 2O(1) + O(1)$

$\vdots$

$= 2^k T(n-k) + 2^{k-1} O(1) + \dots + 2O(1) + O(1)$

$= 2^k T(n-k) + \sum_{i=0}^{k-1} 2^i O(1)$

$n - k = 0 \to n = k$

$$= 2^n T(0) + \frac{1(2^n - 1)}{2 - 1} \cdot O(1)$$

$$= 2^n O(1) + (2^n - 1) O(1)$$

$$= O(2^n) + O(2^n - 1)$$

$$T(n) = O(2^n)$$

c) $T(n) = 9\left(\frac{n}{3}\right) + O(n^2)$

$a = 9$ $\qquad \log_3 9 \quad 2$

$b = 3$ $\qquad 2 = 2$

$d = 2$ $\qquad O(n^2 \log n)$

a : $T(n) = O(n^{\log_2 5}) \approx O(n^{2,32})$

b : $T(n) = O(2^n)$

c : $T(n) = O(n^2 \log n)$

$$\lim_{n \to \infty} \frac{n^{2,32}}{n^2 \log n} = \lim_{n \to \infty} \frac{n^2 \cdot n^{0,32}}{n^2 \log n} = \lim_{n \to \infty} \frac{n^{0,32}}{\log n} = \infty$$

Dessa forma, $n^{\log_2 5} = \omega(n^2 \log n)$, logo, a função c é o limite inferior para a função a.

Assim, o algoritmo c deve ser escolhido já que ele possui a menor complexidade dentre os três