Data

17 e) $\sum_{n=1}^{\infty} (-1)^{n-1} \cdot \frac{n!}{2^{n+1}}$

$$\left| (-1)^{n-1} \cdot \frac{n!}{2^{n+1}} \right| = \frac{n!}{2^{n+1}}$$

$$L = \lim_{n\to+\infty} \frac{(n+1)!}{2^{n+2}} \cdot \frac{2^{n+1}}{n!} = \lim_{n\to+\infty} \frac{(n+1) \cdot \cancel{n!} \cdot \cancel{2^{n+1}}}{2 \cdot \cancel{2^{n+1}} \cdot \cancel{n!}} = +\infty$$

∴ O módulo da série diverge, pelo critério da razão

Para n ímpar:

$$\frac{n!}{2^{n+2}} = \frac{1}{2}\left[\frac{n!}{2^n}\right] = \frac{1}{2}\left(\frac{n\,(n-1)\,(n-2)\ldots 4.3.2.1}{2.\ 2\ .\ 2\ \ldots\ 2.2.2.2}\right) \geq \frac{1}{4}\left(\frac{3}{2}\right)^{n-2}$$

$$\lim_{n\to+\infty} \frac{n!}{2^{n+1}} \geq \lim_{n\to+\infty} \frac{1}{4}\left(\frac{3}{2}\right)^{n-2} = +\infty$$

Como diverge para n ímpar, a série diverge pelo critério do Termo geral.

f) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n^2+2n}$

$$\left| (-1)^{n-1} \cdot \frac{1}{n^2+2n} \right| = \frac{1}{n^2+2n}$$

$$L = \lim_{n\to+\infty} \frac{1}{(n+1)^2+2(n+1)} \cdot n^2+2n = \lim_{n\to+\infty} \frac{n^2+2n}{n^2+4n+3} = 1$$

Como L=1, nada se pode afirmar

Data

Mas $\frac{1}{n^2+2n} \leq \frac{1}{n^2}$ (série-p com $p=2>1 \Rightarrow$ convergente)

Então o módulo da série converge, por comparação, e, pelo teorema, a série dada converge.

∴ absolutamente convergente

g) $\sum_{n=1} (-1)^{n-1} \frac{3^n}{n!}$

$\left|(-1)^{n-1} \frac{3^n}{n!}\right| = \frac{3^n}{n!}$

$L = \lim_{n\to+\infty} \frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n} = \lim_{n\to+\infty} \frac{3 \cdot \cancel{3^n} \cdot \cancel{n!}}{(n+1) \cdot \cancel{n!} \cdot \cancel{3^n}} = 0$

Como $L=0<1$, o módulo da série converge, pelo critério da razão, e, pelo teorema, a série dada converge.

∴ absolutamente convergente

h) $\sum_{n=1} (-1)^{n-1} \frac{n^2+1}{n^3}$

$\left|(-1)^{n-1} \frac{n^2+1}{n^3}\right| = \frac{n^2+1}{n^3} = \frac{1}{n} + \frac{1}{n^3}$

convergente (série-p com $p=3>1$)

divergente (série harmônica)

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Como divergente + convergente = divergente, o módulo da série diverge.

Por Leibnitz

(i) $\lim_{n\to+\infty} \frac{n^2+1}{n^3} = \lim_{n\to+\infty} \frac{1}{n} + \frac{1}{n^3} = 0$

(ii) $\frac{(n+1)^2+1}{(n+1)^3} \leq \frac{n^2+1}{n^3}$

$\frac{n^2+2n+2}{n^3+3n^2+3n+1} \leq \frac{n^2+1}{n^3}$

$\cancel{n^5}+\cancel{2n^4}+\cancel{2n^3} \leq \cancel{n^5}+\cancel{3}n^4+\cancel{3}n^3+n^2+n^3+3n^2+3n+1$

$0 \leq n^4+2n^3+4n^2+3n+1$

A série dada converge, por Leibnitz.

$\therefore$ Condicionalmente convergente

i) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^n}{n!}$

$\left| (-1)^{n-1} \cdot \frac{n^n}{n!} \right| = \frac{n^n}{n!}$

$L = \lim_{n\to+\infty} \frac{(n+1)^{n+1}}{(n+1)!} \cdot \frac{n!}{n^n} = \lim_{n\to+\infty} \frac{\cancel{(n+1)} \cdot (n+1)^n \cdot \cancel{n!}}{\cancel{(n+1)} \cdot \cancel{n!} \cdot n^n}$

$= \lim_{n\to+\infty} \left(\frac{n+1}{n}\right)^n = \lim_{n\to+\infty} \left(1+\frac{1}{n}\right)^n = e$

Como $L = e > 1$, o módulo da série diverge, pelo critério da razão.

Data

Para n ímpar

$n^n \geq n!$ ∴ Pelo critério do Termo Geral, a série diverge quando n é ímpar, assim, a série dada diverge.

$\frac{n^n}{n!} \geq 1$

$\lim_{n\to+\infty} \frac{n^n}{n!} \geq 1$

j) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n^{2/3}+n}$

$\left|(-1)^{n-1} \cdot \frac{1}{n^{2/3}+n}\right| = \frac{1}{n^{2/3}+n} \geq \frac{1}{2n}$ (série harmônica => divergente)

O módulo da série diverge, por comparação

Por Leibnitz:

(i) $\lim_{n\to+\infty} \frac{1}{n^{2/3}+n} = 0$

(ii) $\frac{1}{(n+1)^{2/3}+n+1} \leq \frac{1}{n^{2/3}+n}$

$n^{2/3} + \cancel{n} \leq (n+1)^{2/3} + \cancel{n}+1$

A série dada converge, por Leibnitz

∴ Condicionalmente convergente.

K) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^n 2^n}{(2n-5)^n}$

Data

$$\left|(-1)^{n-1}\cdot\frac{n^n 2^n}{(2n-5)^n}\right| = \frac{(2n)^n}{|2n-5|^n} \geq \frac{\cancel{2^n}.n^n}{\cancel{2^n}.n^n+2^n} \geq \frac{n^n}{n^n+1}$$

$$\geq \frac{n^n}{n^n+n^{n+1}} = \frac{1}{1+n} \geq \frac{1}{2n} \text{ (série harmônica)}$$

O módulo da série diverge, por comparação

Para n ímpar $\begin{cases} 2n-5=m \\ m=5K \end{cases}$

$$\lim_{n\to+\infty}\left(\frac{2n}{2n-5}\right)^n = \lim_{m\to+\infty}\left(\frac{m+5}{m}\right)^{\frac{m+5}{2}}$$

$$\lim_{K\to+\infty}\left(1+\frac{1}{K}\right)^{\frac{5}{2}(K+1)} = \lim_{K\to+\infty}\left(1+\frac{1}{K}\right)^{\frac{5}{2}}\cdot\left(1+\frac{1}{K}\right)^{\frac{5}{2}K} = e^{5/2} \neq 0$$

∴ A série dada diverge pelo critério do Termo Geral

l) $\sum_{n=1}^{\infty} (-1)^{n-1}\cdot\frac{n^4}{e^n}$

$$\left|(-1)^{n-1}\frac{n^4}{e^n}\right| = \frac{n^4}{e^n}$$

$$L = \lim_{n\to+\infty}\left(\frac{n^4}{e^n}\right)^{1/n} = \frac{1}{e}\lim_{n\to+\infty} n^{4/n}$$

$$\ln L = \frac{1}{e}\lim_{n\to+\infty}\frac{4}{n}\ln n = \frac{4}{e}\lim_{n\to+\infty}\frac{1/n}{1} = 0$$

$L = 1$

Como $L=1$ nada pode-se afirmar.

$$\lim_{b\to+\infty}\int_1^b x^4 e^{-x}$$

Data

	u	dv
$+$	x^4	e^{-x}
$-$	$4x^3$	$-e^{-x}$
$+$	$12x^2$	e^{-x}
$-$	$24x$	$-e^{-x}$
$+$	24	e^{-x}
		$-e^{-x}$

$$= \lim_{b \to +\infty} -\frac{x^4}{e^x} - \frac{4x^3}{e^x} - \frac{12x^2}{e^x} - \frac{24x}{e^x} - \frac{24}{e^x} \Big|_1^b$$

$$= \lim_{b \to +\infty} -\frac{b^4}{e^b} - \frac{4b^3}{e^b} - \frac{12b^2}{e^b} - \frac{24b}{e^b} - \cancel{\frac{24}{e^b}} + \frac{1}{e} + \frac{4}{e} + \frac{12}{e} + \frac{24}{e} + \frac{24}{e}$$

$$= \frac{65}{e}$$

O módulo da série converge, pelo critério da integral, e pelo Teorema, a série dada converge.

$\therefore$ Absolutamente convergente

m) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2+1}$

$$\left| (-1)^{n-1} \frac{n}{n^2+2} \right| = \frac{n}{n^2+1}$$

$$L = \lim_{n \to +\infty} \frac{n+1}{(n+1)^2+2} \cdot \frac{n^2+1}{n} = \lim_{n \to +\infty} \frac{n^3+n+n^2+1}{n^3+2n^2+2n} = 1$$

Como $L = 1$ nada pode-se concluir.

Data

$$\lim_{b\to+\infty}\int_1^b \frac{x}{(x^2+1)}\,dx\ \frac{2}{2} = \lim_{b\to+\infty} \frac{\ln(x^2+1)}{2}\Big|_1^b =$$

$$= \frac{\ln(b^2+1)}{2} - \frac{\ln 2}{2} = +\infty$$

O módulo da série diverge, pelo critério da integral

Por Leibnitz

(i) $\lim_{n\to+\infty} \frac{n}{n^2+1} = \lim_{n\to+\infty} \frac{1/n}{1+1/n^2} = 0$

(ii) $\frac{n+1}{(n+1)^2+1} \leq \frac{n}{n^2+1}$

$(n+1)(n^2+1) \leq n(n^2+2n+2)$

$\cancel{n^3}+n^2+n+1 \leq \cancel{n^3}+2n^2+2n$

$1 \leq n^2+n$

A série converge, por Leibnitz

$\therefore$ Condicionalmente convergente

n) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^3+3}$

$\left|(-1)^{n-1}\frac{n}{n^3+3}\right| = \frac{n}{n^3+3} \leq \frac{n}{n^2+3} \leq \frac{n}{n^2+1}$ (questão anterior)

O módulo da série converge por comparação, pelo critério da integral e, pelo teorema, a série dada também converge.

$\therefore$ Absolutamente convergente.

Data $n!^{n!} \overset{10^{19}}{\geq} n^{n!} \overset{10^{23}}{\geq} n!^{n} \overset{10^{22}}{\geq} n^{n} \overset{10^{23}}{\geq} n!$

θ) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{2n^2-n}}$

$\left|\frac{(-1)^n}{\sqrt{2n^2-n}}\right| = \frac{1}{\sqrt{2n^2-n}} \geq \frac{1}{\sqrt{2n^2}+\sqrt{n}} \geq \frac{1}{(\sqrt{2}+1)n}$ (série harmônica)

O módulo da série diverge, por comparação.

Por Leibnitz

(i) $\lim_{n\to+\infty} \frac{1}{\sqrt{n(2n-1)}} = 0$

(ii) $\frac{1}{\sqrt{2(n+1)^2-(n+1)}} \leq \frac{1}{\sqrt{2n^2-n}}$

$\sqrt{2n^2-n} \leq \sqrt{2(n^2+2n+1)-n-1}$

$\cancel{2n^2}-\cancel{n} \leq \cancel{2n^2}+4n+2-\cancel{n}-1$

$0 \leq 4n+1$

A série dada converge, por Leibnitz

∴ Condicionalmente convergente

(18) a) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(2^{3n+4}-n)}{e^n n^{3n}}$

$\left|\frac{(-1)^{n-1}(2^{3n+4}-n)}{e^n n^{3n}}\right| = \frac{2^{3n+4}-n}{e^n n^{3n}} \leq \frac{2^4 \cdot 2^{3n}}{e^n \cdot n^{3n}} \leq 2^4\left(\frac{2}{n}\right)^{3n}$

$L = \lim_{n\to+\infty} \left(2^4 \cdot \left(\frac{2}{n}\right)^{3n}\right)^{1/n} = \lim_{n\to+\infty} 2^{4/n} \cdot \frac{8}{n^3} = 0$

Data

O módulo da série converge por comparação, pelo critério da raiz e, pelo teorema, a série dada converge.

∴ Absolutamente convergente.

b) $\sum_{n=1}^{\infty} \frac{n \cos(n\pi)}{n^2+n+1}$

$\left|\frac{n \cos(n\pi)}{n^2+n+1}\right| = \frac{n}{n^2+n+1} \geq \frac{n}{n^2+n} = \frac{1}{n+1} \geq \frac{1}{2n}$ (série harmônica)

O módulo da série diverge, por comparação.

Por Leibnitz

(i) $\lim_{n\to+\infty} \frac{n}{n^2+n+1} = \lim_{n\to+\infty} \frac{1}{2n+1} = 0$

(ii) $\frac{n+1}{(n+1)^2+(n+1)+1} \leq \frac{n}{n^2+n+1}$

$(n+1)(n^2+n+1) \leq n(n^2+2n+1+n+2)$

$\cancel{n^3}+\cancel{n^2}+\cancel{n}+\cancel{n^2}+\cancel{n}+1 \leq \cancel{n^3}+\cancel{3}n^2+\cancel{3}n$

$1 \leq n^2+n$

A série dada converge, por Leibnitz

∴ Condicionalmente convergente.

Data

c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+\sqrt{n}}}$

$\left|\frac{(-1)^n}{\sqrt{n+\sqrt{n}}}\right| = \frac{1}{\sqrt{n+\sqrt{n}}} \geq \frac{1}{n+\sqrt{n}} \geq \frac{1}{2n}$ (série harmônica)

O módulo da série diverge

Por Leibnitz

(i) $\lim_{n\to+\infty} \frac{1}{\sqrt{n+\sqrt{n}}} = 0$

(ii) $\frac{1}{\sqrt{n+1+\sqrt{n+1}}} \leq \frac{1}{\sqrt{n+\sqrt{n}}}$

$\sqrt{n+\sqrt{n}} \leq \sqrt{n+1+\sqrt{n+1}}$

$\cancel{n}+\sqrt{n} \leq \cancel{n}+1+\sqrt{n+1}$

$\sqrt{n} \leq \sqrt{n+1}+1$

120

$2^n = 120 \cdot 3!$

$2^n = 30 \cdot 2^2$

$2^n = 15 \cdot 2^3$

$2^n = 2^3 \cdot 3^1 \cdot 5^1$

$3 | 3!$

A série dada converge, por Leibnitz

∴ Condicionalmente convergente.

d) $\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)!}{(2n)!}$

$\left|\frac{(-1)^n (n+1)!}{(2n)!}\right| = \frac{(n+1)!}{(2n)!}$

$L = \lim_{n\to+\infty} \frac{(n+2)!}{(2n+2)!} \cdot \frac{(2n)!}{(n+1)!} = \lim_{n\to+\infty} \frac{(n+2)\cdot\cancel{(n+1)!} \cdot \cancel{(2n)!}}{(2n+2)(2n+1)\cdot\cancel{(2n)!}\cdot\cancel{(n+1)!}}$

Data

$$L = \lim_{n \to +\infty} \frac{n+2}{4n^2+6n+2} = \lim_{n \to +\infty} \frac{1}{8n+6} = 0$$

Como $L = 0 < 1$, o módulo da série converge, pelo critério da razão e, pelo teorema, a série dada converge.

$\therefore$ Absolutamente convergente.

e) $\sum_{n=1}^{\infty} (-1)^n \frac{5^{4n+1}}{n^{3n}}$

$$\left| (-1)^n \frac{5^{4n+1}}{n^{3n}} \right| = \frac{5 \cdot 5^{4n}}{n^{3n}}$$

$$L = \lim_{n \to +\infty} \left(\frac{5 \cdot 5^{4n}}{n^{3n}} \right)^{1/n} = \lim_{n \to +\infty} 5^{1/n} \cdot \frac{5^4}{n^3} = 0$$

Como $L = 0 < 1$, o módulo da série converge, pelo critério da raiz e, pelo teorema, a série dada converge

$\therefore$ Absolutamente convergente.

f) $\sum_{n=1}^{\infty} (-1)^n \frac{7^{3n+1}}{(\ln n)^n}$

$$\left| (-1)^n \frac{7^{3n+1}}{(\ln n)^n} \right| = \frac{7 \cdot 7^{3n}}{(\ln n)^n}$$

$$L = \lim_{n \to +\infty} \left(\frac{7 \cdot 7^{3n}}{(\ln n)^n} \right)^{1/n} = \lim_{n \to +\infty} 7^{1/n} \cdot \frac{7^3}{\ln n} = 0$$

Data

Como $L = 0 < 1$, o módulo da série converge, pelo critério da raiz, e, pelo teorema, a série dada converge

$\therefore$ Absolutamente convergente.

g) $\sum_{n=1}^{\infty} \frac{n\,\mathrm{sen}(n\pi)+n}{n^2+5}$

$$\left|\frac{n\,\mathrm{sen}(n\pi)+n}{n^2+5}\right| = \frac{n}{n^2+5} \geq \frac{n}{n^2+5n^2} = \frac{1}{6n}$$

O módulo da série diverge, por comparação

Por Leibnitz

(i) $\lim\limits_{n\to+\infty} \frac{n}{n^2+5} = \lim\limits_{n\to+\infty} \frac{1}{2n+5} = 0$

(ii) $\frac{n+1}{(n+1)^2+5} \leq \frac{n}{n^2+5}$

$(n+1)(n^2+5) \leq n(n^2+2n+6)$

$\cancel{n^3} + \cancel{5n} + n^2 + 5 \leq \cancel{n^3} + 2n^2 + \cancel{6}n$

$n^2 + n \geq 5$

Falso para $n = 1$, então não se pode afirmar nada.

$$\lim_{n\to+\infty} \frac{n\,\mathrm{sen}(n\pi)+n}{n^2+5} = \lim_{n\to+\infty} \frac{n\,(\mathrm{sen}(n\pi)+1)}{n^2+5} =$$

$$= \lim_{n\to+\infty} \frac{1/n\,(\mathrm{sen}(n\pi)+1)}{1+5/n^2} = \nexists$$

A série dada diverge, pelo critério do termo geral.

Data

h) $\sum_{n=1}^{\infty} \frac{\cos(n)+\operatorname{sen}(n)}{n^3+\sqrt{n}}$

$\left|\frac{\cos(n)+\operatorname{sen}(n)}{n^3+\sqrt{n}}\right| \leq \frac{2}{n^3+\sqrt{n}} \leq \frac{2}{n^3}$ (série-p com $p=3>1$ $\Rightarrow$ convergente)

O módulo da série converge, por comparação e, pelo teorema, a série dada converge.

$\therefore$ Absolutamente convergente.

i) $\sum_{n=1}^{\infty} \frac{n e^{2n}}{n^2 e^n - 1}$

$\frac{n e^{2n}}{n^2 e^n - 1} = \frac{n e^{2n}}{n^2 e^n - 1} \geq \frac{n e^{2n}}{n^2 e^n} = \frac{e^n}{n}$

$\lim_{n\to+\infty} \frac{e^n}{n} = \lim_{n\to+\infty} e^n = +\infty$

A série dada diverge por comparação e pelo critério do termo geral.

19 a) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

$L = \lim_{n\to+\infty} \left|\frac{x^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{x^n}\right| = |x| \underbrace{\lim_{n\to+\infty} \sqrt{\frac{n}{n+1}}}_{1} = |x|$

$\therefore$ A série converge se $L<1 \Rightarrow |x|<1 \Rightarrow -1<x<1$

Data

Para $x=-1$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

(i) $\lim_{n\to+\infty} \frac{1}{\sqrt{n}} = 0$

(ii) $\frac{1}{\sqrt{n+1}} \leq \frac{1}{\sqrt{n}}$

$\sqrt{n} \leq \sqrt{n+1}$

$n \leq n+1$

$\therefore$ a série converge, por Leibnitz

Para $x=1$

$$\sum_{n=1} \frac{1^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

$\therefore$ a série diverge, série-p com $p=1/2<1$.

Assim, $R=1$, $I=[-1,1)$

b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n^3}$

$$L = \lim_{n\to\infty} \left| \frac{(-1)^n \cdot \cancel{x}^{n+1}}{(n+1)^3} \cdot \frac{n^3}{(-1)^{n-1} \cdot \cancel{x}^n} \right| = |x| \lim_{n\to+\infty} \frac{n^3}{n^3+3n^2+3n+1} = |x|$$

$\therefore$ a série converge se $L<1 \Rightarrow |x|<1 \Rightarrow -1<x<1$

Data

Para $x = -1$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(-1)^n}{n^3} = \sum_{n=1}^{\infty} \frac{(-1)^{2n-1}}{n^3} = \sum_{n=1}^{\infty} \frac{-1}{n^3}$$

$\therefore$ a série converge, série-p com $p = 3 > 1$

Para $x = 1$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cdot 1^n}{n^3} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$$

$$\left| \frac{(-1)^{n-1}}{n^3} \right| = \frac{1}{n^3}$$

$\therefore$ a série converge, pelo teorema com relação a série-p com $p = 3 > 1$

Assim, $R = 1$, $I = [-1, 1]$

c) $\sum_{n=0}^{\infty} \frac{(3x-2)^n}{n!}$

$$L = \lim_{n \to +\infty} \left| \frac{\cancel{(3x-2)}^{n+1}}{(n+1)!} \cdot \frac{n!}{\cancel{(3x-2)^n}} \right| = |3x-2| \cdot \lim_{n \to +\infty} \frac{1}{n+1} = 0 < 1$$

$\therefore$ a série converge $\forall x \in \mathbb{R} \Rightarrow R = +\infty$, $I = (-\infty, +\infty)$

d) $\sum_{n=1}^{\infty} (-1)^n \, n \, 4^n x^n$

Data

$$L = \lim_{n\to+\infty} \left| \frac{(-1)^{n+1}(n+1) \cdot 4^{n+1} \cdot \cancel{x^{n+1}}}{(-1)^n \, n \cdot 4^n \cdot \cancel{x^n}} \right| = 4|x| \lim_{n\to+\infty} 1 + \frac{1}{n} = 4|x|$$

$\therefore$ a série converge se $L < 1 \Rightarrow 4|x| < 1 \Rightarrow -\frac{1}{4} < x < \frac{1}{4}$

Para $x = -1/4$

$$\sum_{n=1}^{\infty} (-1)^n n 4^n \left(\frac{-1}{4}\right)^n = \sum_{n=1}^{\infty} (-1)^n n \cancel{4^n} \frac{(-1)^n}{\cancel{4^n}} = \sum_{n=1}^{\infty} n$$

$$\lim_{n\to+\infty} n = +\infty$$

$\therefore$ a série diverge, pelo critério do termo geral.

Para $x = 1/4$

$$\sum_{n=1}^{\infty} (-1)^n n 4^n \left(\frac{1}{4}\right)^n = \sum_{n=1}^{\infty} (-1)^n n$$

Para n par:

$$\sum_{n=1}^{\infty} n$$

$$\lim_{n\to+\infty} n = +\infty$$

$\therefore$ a série diverge pelo critério do termo geral quando n é par, assim, a série dada diverge.

$\therefore R = \frac{1}{4}$, $I = \left(-\frac{1}{4}, \frac{1}{4}\right)$

Data

e) $\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{\sqrt[4]{n}} = \sum_{n=1}^{\infty} (-1)^n \frac{2^n x^n}{\sqrt[4]{n}}$

$L = \lim_{n\to+\infty} \left| \frac{\cancel{(-1)^{n+1}} \cdot \cancel{2^{n+1}} \cancel{x^{n+1}}}{\sqrt[4]{n+1}} \cdot \frac{\sqrt[4]{n}}{\cancel{2^n} \cdot \cancel{x^n} \cdot \cancel{(-1)^n}} \right| = 2|x| \lim_{n\to+\infty} \sqrt{\frac{n}{n+1}} = 2|x|$

$\therefore$ a série converge se $L<1 \Rightarrow 2|x|<1 \Rightarrow -\frac{1}{2} < x < \frac{1}{2}$

Para $x = -1/2$

$\sum_{n=1} (-1)^n \cdot \frac{2^n}{n^{1/4}} \left(\frac{-1}{2}\right)^n = \sum_{n=1} (-1)^n \cdot \frac{\cancel{2^n}}{n^{1/4}} \frac{(-1)^n}{\cancel{2^n}} = \sum_{n=1} \frac{1}{n^{1/4}}$

$\therefore$ a série diverge, série-p com $p = 1/4 < 1$

Para $x = 1/2$

$\sum_{n=1} (-1)^n \cdot \frac{2^n}{n^{1/4}} \cdot \left(\frac{1}{2}\right)^n = \sum_{n=1} (-1)^n \cdot \frac{\cancel{2^n}}{n^{1/4}} \frac{1^n}{\cancel{2^n}} = \sum_{n=1} \frac{(-1)^n}{n^{1/4}}$

(i) $\lim_{n\to+\infty} \frac{1}{n^{1/4}} = 0$

(ii) $\frac{1}{\sqrt[4]{n+1}} \leq \frac{1}{\sqrt[4]{n}}$

$\sqrt[4]{n} \leq \sqrt[4]{n+1}$

$n \leq n+1$

$\therefore$ a série converge, por Leibnitz

$\therefore R = \frac{1}{2}, \ I = \left[-\frac{1}{2}, \frac{1}{2}\right]$

Data

f) $\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{4^n \ln n}$

$L = \lim_{n\to+\infty} \left| \frac{(-1)^{n+1} x^{n+1}}{4^{n+1} \cdot \ln(n+1)} \cdot \frac{4^n \ln n}{x^n (-1)^n} \right| = \frac{|x|}{4} \lim_{n\to+\infty} \frac{\ln n}{\ln(n+1)}$

$= \frac{|x|}{4} \lim_{n\to+\infty} \frac{1}{n} \cdot \frac{n+1}{1} = \frac{|x|}{4}$

$\therefore$ a série converge se $L < 1 \Rightarrow \frac{|x|}{4} < 1 \Rightarrow -4 < x < 4$

Para $x = -4$

$\sum_{n=2}^{\infty} \frac{(-1)^n (-1)^n \cdot \cancel{4^n}}{\cancel{4^n} \ln n} = \sum_{n=2} \frac{1}{\ln n}$

$\ln n \leq n$

$\frac{1}{\ln n} \geq \frac{1}{n}$

$\therefore$ a série diverge, pelo critério da comparação com a série harmônica.

Para $x = 4$

$\sum_{n=1}^{\infty} \frac{(-1)^n \cancel{4^n}}{\cancel{4^n} \ln n} = \sum_{n=1} \frac{(-1)^n}{\ln n}$

(i) $\lim_{n\to+\infty} \frac{1}{\ln n} = 0$

Data

(ii) $\frac{1}{\ln(n+1)} \leq \frac{1}{\ln n}$

$\ln n \leq \ln(n+1)$

$e^{\ln n} \leq e^{\ln(n+1)}$

$n \leq n+1$

∴ a série converge, por Leibnitz

Assim, $R = 4$, $I = (-4, 4]$

g) $\sum_{n=0}^{\infty} \frac{n(x+2)^n}{3^{n+1}}$

$L = \lim_{n \to +\infty} \left| \frac{(n+1)\cancel{(x+2)}^{n+1}}{3^{n+2}} \cdot \frac{\cancel{3^{n+1}}}{n \cdot \cancel{(x+2)^n}} \right| = \frac{|x+2|}{3} \lim_{n \to +\infty} \frac{n+1}{n} = \frac{|x+2|}{3}$

a série converge se $L < 1 \Rightarrow \frac{|x+2|}{3} < 1 \Rightarrow -3 < x+2 < 3$

$-5 < x < 1$

Para $x = -5$

$\sum_{n=0}^{\infty} \frac{n(-5+2)^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{n \cdot (-1)^n \cdot \cancel{3^n}}{3 \cdot \cancel{3^n}} = \sum_{n=0}^{\infty} (-1)^n \frac{n}{3}$

Para n par

$\sum_{n=0}^{\infty} \frac{n}{3}$

∴ a série diverge, pelo critério do termo geral quando n é par, assim, a série dada é divergente

$\lim_{n \to +\infty} \frac{n}{3} = +\infty$

Data

Para $x = 1$

$$\sum_{n=0}^{\infty} \frac{n(1+2)^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{n}{3}$$

$$\lim_{n\to+\infty} \frac{n}{3} = +\infty$$

$\therefore$ a série diverge, pelo critério do termo geral.

Assim, $R = 3$, $I = (-5, 1)$

h) $\sum_{n=0}^{\infty} \sqrt{n}(x-4)^n$

$$L = \lim_{n\to+\infty} \left| \frac{\sqrt{n+1} \cdot \cancel{(x-4)}^{n+1}}{\sqrt{n} \cancel{(x-4)^n}} \right| = |x-4| \lim_{n\to+\infty} \sqrt{\frac{n+1}{n}} = |x-4|$$

$\therefore$ a série converge se $L < 1 \Rightarrow |x-4| < 1 \Rightarrow -1 < x-4 < 1$

$3 < x < 5$.

Para $x = 3$

$$\sum_{n=0}^{\infty} \sqrt{n}(3-4)^n = \sum_{n=0}^{\infty} (-1)^n \sqrt{n}$$

Para n par

$$\sum_{n=0}^{\infty} \sqrt{n}$$

$$\lim_{n\to+\infty} \sqrt{n} = +\infty$$

$\therefore$ a série diverge, pelo critério do termo geral quando n é par, assim, a série dada diverge.

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Data

Para $x=5$

$$\sum_{n=0}^{\infty} \sqrt{n}\,(5-4)^n = \sum_{n=0}^{\infty} \sqrt{n}$$

$$\lim_{n\to+\infty} \sqrt{n} = +\infty$$

$\therefore$ a série diverge, pelo critério do termo geral.

Assim, $R=1$, $I=(3,5)$

i) $\sum_{n=1}^{\infty} \frac{(-1)^n (x+2)^n}{n 2^n}$

$$L = \lim_{n\to+\infty} \left| \frac{\cancel{(-1)^{n+1}}\,\cancel{(x+2)^{n+1}}}{(n+1)\,\cancel{2^{n+1}}} \cdot \frac{n\,\cancel{2^n}}{\cancel{(-1)^n}\,\cancel{(x+2)^n}} \right| = \frac{|x+2|}{2} \lim_{n\to+\infty} \frac{n}{n+1} = \frac{|x+2|}{2}$$

$\therefore$ a série converge se $L<1 \Rightarrow \frac{|x+2|}{2} < 1 \Rightarrow -2 < x+2 < 2$

$-4 < x < 0$

Para: $x=-4$

$$\sum_{n=0}^{\infty} \frac{(-1)^n(-4+2)^n}{n 2^n} = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot (-1)^n \cdot 2^n}{n \cdot 2^n} = \sum_{n=0}^{\infty} \frac{1}{n}$$

$\therefore$ a série diverge, série harmônica

Para $x=0$

$$\sum_{n=0}^{\infty} \frac{(-1)^n \cdot (0+2)^n}{n \cdot 2^n} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n}$$

Data

(i) $\lim_{n\to+\infty} \frac{1}{n} = 0$

(ii) $\frac{1}{n+1} \leq \frac{1}{n}$

$n \leq n+1$

$\therefore$ a série converge, por Leibnitz

Assim, $R = 2$, $I = (-4, 0]$

j) $\sum_{n=1}^{\infty} n!(2x-1)^n$

$$L = \lim_{n\to+\infty} \left| \frac{(n+1)!\,\cancel{(2x-1)^{n+1}}}{n!\,\cancel{(2x-1)^n}} \right| = |2x-1| \lim_{n\to+\infty} n+1 = +\infty > 1$$

$\therefore$ a série diverge $\forall x \in \mathbb{R} \Rightarrow R = 0$, $I = \{1/2\}$

k) $\sum_{n=1}^{\infty} \frac{x^n}{n\sqrt{n}\,3^n}$

$$L = \lim_{n\to+\infty} \left| \frac{\cancel{x^{n+1}}}{\sqrt{(n+1)^3}\,.\,\cancel{3^{n+1}}} \cdot \frac{\sqrt{n^3}\,\cancel{3^n}}{\cancel{x^n}} \right| = \frac{|x|}{3} \lim_{n\to+\infty} \underbrace{\sqrt{\frac{n^3}{n^3+3n^2+3n+1}}}_{1} = \frac{|x|}{3}$$

$\therefore$ a série converge se $L < 1 \Rightarrow \frac{|x|}{3} < 1 \Rightarrow -3 < x < 3$

Para $x = -3$

$$\sum_{n=1}^{\infty} \frac{(-3)^n}{n\sqrt{n}\,3^n} = \sum_{n=1}^{\infty} \frac{(-1)^n\,\cancel{3^n}}{n\sqrt{n}\,.\,\cancel{3^n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$$

Data

(i) $\lim_{n\to+\infty} \frac{1}{n\sqrt{n}} = 0$

(ii) $\frac{1}{(n+1)\sqrt{n+1}} \leq \frac{1}{n\sqrt{n}}$

$\sqrt{n^3} \leq \sqrt{n^3+3n^2+3n+1}$

$\cancel{n^3} \leq \cancel{n^3} + 3n^2 + 3n+1$

$0 \leq 3n^2+3n+1$

∴ a série converge por Leibnitz

Para $x=3$

$\sum_{n=1}^{\infty} \frac{\cancel{3^n}}{n\sqrt{n}\,\cancel{3^n}} = \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

∴ a série converge, série-p com $p = 3/2 > 1$

Assim, $R=3$, $I=[-3,3]$

l) $\sum_{n=1}^{\infty} \frac{(4x-5)^{2n+1}}{n^{3/2}}$

$L = \lim_{n\to+\infty} \left| \frac{(4x-5)^{2n+3}}{n^{3/2}} \cdot \frac{(n+1)^{3/2}}{(4x-5)^{2n+1}} \right| = (4x-5)^2 \cdot \underbrace{\lim_{n\to+\infty} \left(1+\frac{1}{n}\right)^{3/2}}_{1}$

$L = 16x^2 - 40x + 25$

∴ a série converge se $L<1 \Rightarrow |16x^2-40x+25|<1$

$-1 < 16x^2-40x+25 < 1$

$-26 < 16x^2 - 40x < -24$

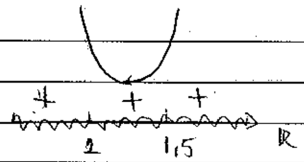
Data

(I) $16x^2-40x+26>0$

$\Delta = 1600-4.16.26$

$\Delta = -64$

I)



(II) $16x^2-40+24<0$

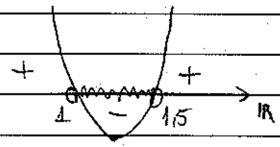
$\Delta = 1600-4.16.24$

$\Delta = 64$

$x = \frac{40 \pm 8}{32}$

$x_1 = \frac{3}{2}$ $\quad x_2 = 1$

II)



$I \cap II = \left(1, \frac{3}{2}\right)$

Para $x=1$

$$\sum_{n=1}^{\infty} \frac{(4-5)^{2n+1}}{n^{3/2}} = \sum_{n=1}^{\infty} \frac{-1}{n^{3/2}}$$

∴ a série converge, série-p com $p=3/2>1$

Para $x=3/2$

$$\sum_{n=1}^{\infty} \frac{(6-5)^{2n+1}}{n^{3/2}} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$

∴ a série converge, série-p com $p=3/2>1$

Assim, $R=\frac{1}{4}$, $I=\left[1, \frac{3}{2}\right]$.

m) $\sum_{n=0}^{\infty} \frac{n(x-5)^n}{n^2+1}$

Data

$$L = \lim_{n\to+\infty} \left| \frac{(n+1)\,\cancel{(x-5)}^{n+1}}{(n+1)^2+1} \cdot \frac{n^2+1}{n\,\cancel{(x-5)^n}} \right| = |x-5| \underbrace{\lim_{n\to+\infty} \frac{n^3+n^2+n+1}{n^3+2n^2+2n}}_{1}$$

$L = |x-5|$

$\therefore$ a série converge se $L<1 \Rightarrow |x-5|<1 \Rightarrow 4<x<6$

Para $x=4$

$$\sum_{n=0}^{\infty} \frac{n\,(4-5)^n}{n^2+1} = \sum_{n=0}^{\infty} (-1)^n \frac{n}{n^2+1}$$

(i) $\lim_{n\to+\infty} \frac{n}{n^2+1} = 0$

(ii) $\frac{(n+1)}{(n+1)^2+1} \leq \frac{n}{n^2+1}$

$(n+1)(n^2+1) \leq n(n^2+2n+2)$

$\cancel{n^3}+\cancel{n^2}+\cancel{n}+1 \leq \cancel{n^3}+\cancel{2}n^2+\cancel{2}n$

$1 \leq n^2+n \quad \forall n \geq 1$

$\therefore$ a série converge, por Leibnitz

Para $x=6$

$$\sum_{n=0}^{\infty} \frac{n\,(6-5)^n}{n^2+1} = \sum_{n=0}^{\infty} \frac{n}{n^2+1}$$

$n^2+1 \leq n^2+n^2 \quad \forall n \geq 1$ $\therefore$ a série diverge, por comparação

$\frac{n}{n^2+1} \geq \frac{\cancel{n}}{2n^{\cancel{2}}} = \frac{1}{2n}$

Assim, $R=1$, $I=[4,6)$

Data

n) $\sum_{n=0}^{\infty} \frac{n^n (x+2)^n}{(2n-5)^n}$

$L = \lim_{n\to+\infty} \left| \left(\frac{n^n (x+2)^n}{(2n-5)^n} \right)^{1/n} \right| = |x+2| \lim_{n\to+\infty} \frac{n}{|2n-5|} = \frac{|x+2|}{2}$

$\therefore$ A série converge se $L < 1 \Rightarrow \frac{|x+2|}{2} < 1 \Rightarrow -4 < x < 0$

Para $x = -4$

$\sum_{n=0}^{\infty} \frac{n^n (-4+2)^n}{(2n-5)^n} = \sum_{n=0}^{\infty} \frac{(-1)^n (2n)^n}{(2n-5)^n}$

Para n par

$\lim_{n\to+\infty} \frac{(2n)^n}{(2n-5)^n} = \lim_{m\to+\infty} \left(\frac{m+5}{m}\right)^{\frac{m+5}{2}} = \lim_{K\to+\infty} \left(1+\frac{1}{K}\right)^{\frac{5}{2}(1+K)}$

$\lim_{K\to+\infty} \left(1+\frac{1}{K}\right)^{\frac{5}{2}} \cdot \lim_{n\to+\infty} \left(1+\frac{1}{K}\right)^{\frac{5}{2}K} = e^{5/2} \neq 0$

$\therefore$ A série diverge, pelo critério do termo geral

Para $x = 0$

$\sum_{n=0}^{\infty} \frac{n^n (0+2)^n}{(2n-5)^n} = \sum_{n=0}^{\infty} \frac{(2n)^n}{(2n-5)^n}$

$\therefore$ A série diverge, pelo critério do termo geral.

Assim, $R = 2$, $I = (-4, 0)$

Data

$\theta)\ \sum_{n=0}^{\infty} \frac{n^4(x-1)^n}{e^n}$

$L = \lim_{n\to+\infty} \left| \frac{(n+1)^4 (x-1)^{n+1}}{e^{n+1}} \cdot \frac{e^n}{n^4 (x-1)^n} \right|$

$= \frac{|x-1|}{e} \cdot \lim_{n\to+\infty} \frac{n^4+4n^3+6n^2+4n+1}{n^4} = \frac{|x-1|}{e}$

$\therefore$ a série converge se $L<1 \Rightarrow \frac{|x-1|}{e} < 1 \Rightarrow 1-e < x < 1+e$

Para $x = 1-e$

$\sum_{n=0}^{\infty} \frac{n^4(\cancel{1}-e-\cancel{1})^n}{e^n} = \sum_{n=0}^{\infty} \frac{(-1)^n n^4 \cancel{e^n}}{\cancel{e^n}} = \sum_{n=0}^{\infty} (-1)^n n^4$

Para n par

$\lim_{n\to+\infty} n^4 = +\infty$

$\therefore$ a série diverge, pelo critério do termo geral

Para $x = 1+e$

$\sum_{n=0}^{\infty} \frac{n^4(\cancel{1}+e-\cancel{1})^n}{e^n} = \sum_{n=0}^{\infty} n^4 \frac{\cancel{e^n}}{\cancel{e^n}} = \sum_{n=0}^{\infty} n^4$

$\therefore$ a série diverge, pelo teste anterior

assim, $R = e$, $I = (1-e, 1+e)$

Data

p) $\sum_{n=0}^{\infty} \frac{2^n (x+1)^n}{n^2+1}$

$L = \lim_{n\to+\infty} \left| \frac{\cancel{2^{n+1}}\cancel{(x+1)}^{n+1}}{(n+1)^2+1} \cdot \frac{n^2+1}{\cancel{2^n}\cancel{(x+1)}^n} \right| = 2|x+1| \lim_{n\to+\infty} \frac{n^2+1}{n^2+2n+2} = 2|x+1|$

$\therefore$ a série converge se $L<1 \Rightarrow 2|x+1|<1 \Rightarrow -\frac{3}{2} < x < -\frac{1}{2}$

Para $x = -3/2$

$\sum_{n=0}^{\infty} \frac{2^n (-3/2+1)^n}{n^2+1} = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot \cancel{2^n}}{n^2+1} \cdot \frac{1}{\cancel{2^n}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$

(i) $\lim_{n\to+\infty} \frac{1}{n^2+1} = 0$

(ii) $\frac{1}{(n+1)^2+1} \leq \frac{1}{n^2+1}$

$\cancel{n^2} + \cancel{1} \leq \cancel{n^2} + 2n + 1 + \cancel{1}$

$0 \leq 2n+1$

$\therefore$ a série converge, por Leibnitz

Para $x = -1/2$

$\sum_{n=0}^{\infty} \frac{2^n (-1/2+1)^n}{n^2+1} = \sum_{n=0}^{\infty} \frac{\cancel{2^n}}{n^2+1} \cdot \frac{1}{\cancel{2^n}} = \sum_{n=0}^{\infty} \frac{1}{n^2+1}$

$n^2+1 \geq n^2$

$\frac{1}{n^2+1} \leq \frac{1}{n^2}$

Data

$\therefore$ a série converge, por comparação com série-p com $p = 2 > 1$.

Assim, $R = \frac{1}{2}$, $I = \left[-\frac{3}{2}, -\frac{1}{2}\right]$

q) $\sum_{n \geq 0}^{\infty} \frac{n(x-1)^{2n}}{n^3+3}$

$$L = \lim_{n \to +\infty} \left| \frac{(n+1)\cancel{(x-1)^{2n+2}}}{(n+1)^3+3} \cdot \frac{n^3+3}{n\cancel{(x-1)^{2n}}} \right|$$

$$= |x-1|^2 \lim_{n \to +\infty} \frac{n^4+n^3+3n+3}{n^4+3n^3+3n^2+4n} = x^2-2x+1$$

$\therefore$ a série converge se $L < 1 \Rightarrow |x^2-2x+1| < 1$

$-1 < x^2-2x+1 < 1$

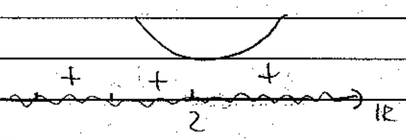
$-2 < x^2-2x < 0$

I) $x^2-2x+2 > 0$

$\Delta = 4 - 4.1.2$

$\Delta = -4$

I)



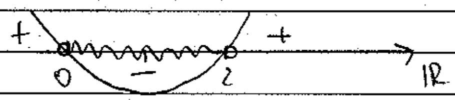
II) $x^2-2x < 0$

$x(x-2) < 0$

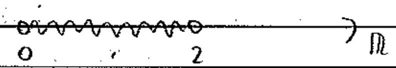
$x_1 = 0$

$x_2 = 2$

II)



I∩II)



Para $x = 0$

Data

$$\sum_{n=0}^{\infty} \frac{n(0-1)^{2n}}{n^3+3} = \sum_{n=0}^{\infty} \frac{n}{n^3+3}$$

$n^3+3 \geq n^3$

$$\frac{n}{n^3+3} \leq \frac{n}{n^3} = \frac{1}{n^2}$$

$\therefore$ a série converge, por comparação com série-p com $p=2>1$.

Para $x=2$

$$\sum_{n=0}^{\infty} \frac{n(2-1)^{2n}}{n^3+3} = \sum_{n=0}^{\infty} \frac{n}{n^3+3}$$

$\therefore$ a série converge, pelo teste anterior

Assim, $R=1$, $I=[0,2]$

r) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1.3.5.7\cdots(2n-1)x^n}{3.6.9\cdots 3n}$

$$L = \lim_{n\to+\infty} \left| \frac{\cancel{1.3.5.7\cdots(2n-1)}(2n+1)\cancel{x^{n+1}}}{\cancel{3.6.9\cdots 3n}\,.3(n+1)} \cdot \frac{\cancel{3.6.9\cdots 3n}}{\cancel{1.3.5.7\cdots(2n-1)}\cancel{x^n}} \right| = \frac{2}{3}|x|$$

$\therefore$ a série converge se $L<1 \Rightarrow \frac{2}{3}|x|<1 \Rightarrow -\frac{3}{2}<x<\frac{3}{2}$

Assim, $R=\frac{3}{2}$, $I=\left(-\frac{3}{2},\frac{3}{2}\right)$

Data

(20) $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^2}$

$L = \lim_{n\to+\infty} \left| \frac{x^{n+1}}{(n+1)^2} \cdot \frac{n^2}{x^n} \right| = |x| \lim_{n\to+\infty} \frac{n^2}{n^2+2n+1} = |x|$

$\therefore$ a série converge se $L < 1 \Rightarrow |x| < 1 \Rightarrow -1 < x < 1$

Para $x = 1$

$f(x) = \sum_{n=1}^{\infty} \frac{(1)^n}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^2}$

$\therefore$ a série converge, série-p com $p = 2 > 1$

Para $x = -1$.

$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

(i) $\lim_{n\to+\infty} \frac{1}{n^2} = 0$

(ii) $\frac{1}{(n+1)^2} \leq \frac{1}{n^2}$

$\cancel{n^2} \leq \cancel{n^2} + 2n + 1$

$0 \leq 2n + 1$

$\therefore$ a série converge, por Leibnitz

Assim, $R = 1$, $I = [-1, 1]$

Data

$$f'(x) = \sum_{n=1}^{\infty} \frac{\cancel{n} X^{n-1}}{n^2} = \sum_{n=1}^{\infty} \frac{X^{n-1}}{n}$$

$$L = \lim_{n\to+\infty} \left| \frac{x^n}{n+1} \cdot \frac{n}{x^{n-1}} \right| = |x| \lim_{n\to+\infty} \frac{n}{n+1} = |x|$$

$\therefore$ a série converge se $L < 1 \Rightarrow |x| < 1 \Rightarrow -1 < x < 1$

Para $x = 1$

$$f'(x) = \sum_{n=1}^{\infty} \frac{(1)^{n-1}}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$$

$\therefore$ a série diverge, série harmônica

Para $x = -1$

$$f'(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$$

(i) $\lim_{n\to+\infty} \frac{1}{n} = 0$

(ii) $\frac{1}{n+1} \leq \frac{1}{n}$

$n \leq n+1$

$0 \leq 1$

$\therefore$ a série converge, por Leibnitz

Assim, $R = 1$, $I = [-1, 1)$

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Data

$$f''(x) = \sum_{n=1}^{\infty} \frac{(n-1)}{n} x^{n-2}$$

$$L = \lim_{n\to+\infty} \left| \frac{n \cdot \cancel{x^{n-1}}}{(n+1)} \cdot \frac{n}{(n-1)\cdot \cancel{x^{n-2}}} \right| = |x| \lim_{n\to+\infty} \frac{n^2}{n^2-1} = |x|$$

$\therefore$ a série converge se $L<1, \Rightarrow |x|<1 \Rightarrow -1<x<1$

Para $x=1$

$$f''(x) = \sum_{n=1}^{\infty} \frac{(n-1) \cdot 1^{n-2}}{n} = \sum_{n=1}^{\infty} \frac{n-1}{n} = \sum_{n=1}^{\infty} 1 - \frac{1}{n}$$

$$\lim_{n\to+\infty} 1 - \frac{1}{n} = 1 \neq 0$$

$\therefore$ a série diverge, pelo critério do termo geral

Para $x=-1$

$$f''(x) = \sum_{n=1}^{\infty} (-1)^{n-2} \left[1 - \frac{1}{n}\right]$$

Para n par

$$f''(x) = \sum_{n=1}^{\infty} 1 - \frac{1}{n}$$

$\therefore$ a série diverge, pelo teste anterior.

Assim, $R=1$, $I=(-1,1)$

21) $\sum_{n=1}^{\infty} x^n = \frac{1}{1-x}, \forall x \in (-1,1)$

a) $\sum_{n=1}^{\infty} n x^{n-1} = \left(\sum_{n=1}^{\infty} x^n\right)' = \left(\frac{1}{1-x}\right)' = \frac{1}{(1-x)^2}$

b) $\sum_{n=1}^{\infty} n x^n = \sum_{n=1}^{\infty} n x^{n-1} . x = \frac{x}{(1-x)^2}$

c) $\sum_{n=1}^{\infty} \frac{n}{2^n} = \sum_{n=1}^{\infty} n . \left(\frac{1}{2}\right)^n = \frac{(1/2)}{(1-1/2)^2} = \frac{\cancel{(1/2)}}{(1-1/2)^{\cancel{2}}} = 2$

d) $\sum_{n=1}^{\infty} n(n-1) . x^n = \sum_{n=1}^{\infty} n(n-1) x^{n-2} . x^2 = x^2 \left(\frac{1}{(1-x)^2}\right)'$

$= x^2 . \frac{2}{(1-x)^3} = \frac{2x^2}{(1-x)^3}$

e) $\sum_{n=2}^{\infty} \frac{n^2-n}{2^n} = \sum_{n=2}^{\infty} n(n-1)\left(\frac{1}{2}\right)^n = \frac{2(1/2)^2}{(1-1/2)^3} = \frac{2\cancel{(1/2)^2}}{(1/2)^{\cancel{3}}} = 4$

f) $\sum_{n=1}^{\infty} \frac{n^2}{2^n} = \sum_{n=1}^{\infty} \frac{n^2}{2^n} - \frac{n}{2^n} + \frac{n}{2^n} = \sum_{n=1}^{\infty} (n^2-n)\left(\frac{1}{2}\right)^n + n\left(\frac{1}{2}\right)^n$

$= \sum_{n=1}^{\infty} n(n-1)\left(\frac{1}{2}\right)^n + \sum_{n=1}^{\infty} n\left(\frac{1}{2}\right)^n = 4 + 2 = 6$

g) $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{n+1}}{n+1} = -\int (-1)^n x^n = -\int \frac{dx}{1+x}$

$= -\ln(1+x) + K$

Para $x=0$

$$\sum_{n=0}^{\infty} \frac{(-0)^{n+2}}{n+2} = -\ln(1+0)+K$$

$$0 = 0 + K \therefore K = 0$$

Assim, $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n} = -\ln(x+1)$

h) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n(n+1)} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} \cdot (1/2)^n \frac{(1/2)}{(1/2)} = \sum_{n=0}^{\infty} \frac{2(-1)^n (1/2)^{n+1}}{n+1}$

$= 2\ln(1+1/2) = 2\ln\left(\frac{3}{2}\right)$

(22) a) $f(x) = \frac{1}{1+x^3} = \sum_{n=0}^{\infty} (-x^3)^n = (-1)^n \cdot x^{3n}$

b) $f(x) = \frac{1}{4+x^3} = \frac{1}{4} \cdot \frac{1}{1+x^3/4} = \frac{1}{4} \sum_{n=0}^{\infty} \left(\frac{-x^3}{4}\right)^n =$

$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{3n}}{4^{n+1}}$

c) $f(x) = \frac{x}{9+4x^2} = \frac{x}{9} \cdot \frac{1}{1+4x^2/9} = \frac{x}{9} \sum_{n=0}^{\infty} \left(\frac{-4x^2}{9}\right)^n$

$= \sum_{n=0}^{\infty} \frac{(-1)^n 4^n x^{2n} \cdot x}{9^n \cdot 9} = \sum_{n=0}^{\infty} \frac{(-1)^n 4^n \cdot x^{2n+1}}{9^{n+1}}$

d) $f(x) = \frac{x^2}{(1-2x)^2} = \sum_{n=0}^{\infty} n(2x)^{n-2} \cdot x^2 = \sum_{n=0}^{\infty} n \cdot 2^{n-1} \cdot x^{n+1}$

e) $f(x) = \frac{x^3}{(x-2)^2} = \frac{x^3}{\left[-2\left(1-\frac{x}{2}\right)\right]^2} = \frac{1}{4} \cdot \frac{x^3}{\left(1-\frac{x}{2}\right)^2}$

Data

$$= \frac{x^3}{4}\sum_{n=0}^{\infty} n\left(\frac{x}{2}\right)^{n-1} = \sum_{n=0}^{\infty} \frac{n x^{n-1}\cdot x^3}{2^2\cdot 2^{n-1}} = \sum_{n=0}^{\infty} \frac{n x^{n+2}}{2^{n+1}}$$

f) $$f(x) = \ln(5-x) = \ln\left[\frac{1}{5}\left(1-\frac{x}{5}\right)\right] = \ln\frac{1}{5} + \ln\left(1-\frac{x}{5}\right)$$

$$= -\ln 5 + \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}\cdot\left(\frac{-x}{5}\right)^{n+1}$$

$$= -\ln 5 + \sum_{n=0}^{\infty} \frac{(-1)^n\cdot(-1)^{n+1}\cdot x^{n+1}}{5^{n+1}\cdot n+1}$$

$$= -\ln 5 - \sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)\cdot 5^{n+1}}$$

g) $$f(x) = x\ln(x^2+1) = x\sum_{n=0}^{\infty} \frac{(-1)^n\cdot(x^2)^{n+1}}{n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n\cdot x^{2n+3}}{n+1}$$

23 a) $$\int \frac{x}{1-x^8}\,dx = \int x\sum_{n=0}^{\infty}(x^8)^n\,dx = \int\sum_{n=0}^{\infty} x^{8n+1}$$

$$= \sum_{n=0}^{\infty} \frac{x^{8n+2}}{8n+2} + K$$

b) $$\int \frac{\ln(1-x^2)}{x^2}\,dx = \int \frac{1}{x^2}\sum_{n=0}^{\infty} \frac{(-1)^n(-x^2)^{n+1}}{n+1}\,dx$$

$$= \int \sum_{n=0}^{\infty} \frac{(-1)^n(-1)^{n+2}\cdot x^{2n+2}}{(n+1)\cdot x^2}\,dx$$

$$= -\int\sum_{n=0}^{\infty} \frac{x^{2n}}{n+1}\,dx$$

$$= -\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)(n+1)} + K$$

$$= -\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n^2+3n+1} + K$$

$$= -\sum_{n=1}^{\infty} \frac{x^{2n-1}}{n(2n-1)} + K$$

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Data

c) $\int \frac{x - \operatorname{arctg} x}{x^3}\, dx =$

$$\frac{x - \operatorname{arctg} x}{x^3} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\, x^{2n-2}}{2n+1}$$

$$\int \frac{x - \operatorname{arctg} x}{x^3} = \int \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\, x^{2n-2}}{2n+1}\, dx$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\, x^{2n-1}}{4n^2-1} + K$$

d) $\int \operatorname{arctg} x^2\, dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n (x^2)^{2n+1}}{2n+1}\, dx$

$$= \int \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{4n+2}}{2n+1}\, dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{4n+3}}{(4n+3)(2n+1)} + K$$

(24) $f(x) = \operatorname{arctg} x = \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+1}}{2n+1}$

$$\frac{\pi}{6} = \sum_{n=0}^{\infty} (-1)^n \cdot \left(\frac{1}{\sqrt{3}}\right)^{2n+1} \cdot \frac{1}{2n+1}$$

$$\frac{\pi}{6} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} \cdot \frac{1}{\sqrt{3}} \cdot \frac{1}{3^n}$$

$$\pi = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} \cdot \frac{1}{3^n} \cdot \frac{1}{\sqrt{3}} \cdot 6 \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\pi = 2\sqrt{3} \sum_{n=0}^{\infty} \frac{(-1)^n}{3^n(n+1)}$$

Data

(25) $f(x)=\sum_{n=0}^{\infty} \frac{x^n}{n!}$

$f'(x)=\sum_{n=1}^{\infty} n\frac{x^{n-1}}{n!}=\sum_{n=0}^{\infty} \frac{(n+1).x^n}{(n+1)!}=\sum_{n=0}^{\infty} \frac{x^n}{n!}$

$\therefore f'(x)=f(x)$

(26) $f_1(x)=\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

$f_1'(x)=\sum_{n=1}^{\infty} \frac{(-1)^n.2n\,x^{2n-1}}{(2n)!}=\sum_{n=1}^{\infty} \frac{(-1)^n.x^{2n-1}}{(2n-1)!}$

$=\sum_{n=0}^{\infty} \frac{(-1)^{n+1}.x^{2n+1}}{(2n+1)!}$

$f_1''(x)=\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n+1).x^{2n}}{(2n+1)!}=\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^{2n}}{(2n)!}$

$f_1(x)+f_1''(x)=\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}-\sum_{n=1}^{\infty} \frac{(-1)^n.x^{2n}}{(2n)!}=0$

$f_2(x)=\sum_{n=0}^{\infty} \frac{(-1)^n.x^{2n+1}}{(2n+1)!}$

$f_2'(x)=\sum_{n=1}^{\infty} \frac{(-1)^n.x^{2n}}{(2n)!}=\sum_{n=0}^{\infty} \frac{(-1)^{n+1}.x^{2n+2}}{(2n+2)!}$

$f_2''(x)=\sum_{n=1}^{\infty} \frac{(-1)^{n+1}.2n+2.x^{2n+1}}{(2n+2)!}=\sum_{n=1}^{\infty} \frac{(-1)^{n+1}.x^{2n+1}}{(2n+1)!}$

$f_2(x)+f_2''(x)=\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}-\sum_{n=1}^{\infty} \frac{(-1)^n.x^{2n+1}}{(2n+1)!}=0$

Data

(27) a) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{4^{2n+1}(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \cdot \left(\frac{\pi}{4}\right)^{2n+1}$

$= \operatorname{arctg}\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

b) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot \left(\frac{\pi}{6}\right)^{2n} = \operatorname{arctg}\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$

c) $\sum_{n=1}^{\infty} \frac{3^n}{n!} = \sum_{n=0}^{\infty} \frac{3^n}{n!} - 1 = e^3 - 1$

d) $\sum_{n=0}^{\infty} \frac{3^n}{5^n \cdot n!} = \sum_{n=0}^{\infty} \left(\frac{3}{5}\right)^n \cdot \frac{1}{n!} = e^{3/5}$

(28) $\sum_{n=0}^{\infty} \frac{2^n}{5^n} \frac{(x-2)^n}{(1+n^2)}$

$L = \lim_{n \to +\infty} \left| \frac{\cancel{2^{n+1}} \cancel{(x-2)^{n+1}}}{\cancel{5^{n+1}}(1+(n+1)^2)} \cdot \frac{\cancel{5^n} \cdot (1+n^2)}{\cancel{2^n} \cdot \cancel{(x-2)^n}} \right| = \frac{2}{5} |x-2| \lim_{n \to +\infty} \frac{n^2+1}{n^2+2n+2}$

$= \frac{2}{5}|x-2|$

$\therefore$ a série converge se $L < 1 \Rightarrow \frac{2}{5}|x-2| < 1 \Rightarrow -\frac{1}{2} < x < \frac{9}{2}$

Para $x = 9/2$

$\sum_{n=0}^{\infty} \left(\frac{2}{5}\right)^n \cdot \frac{(9/2-2)^n}{(1+n^2)} = \sum_{n=0}^{\infty} \frac{\cancel{2^n}}{\cancel{5^n}} \cdot \frac{\cancel{5^n}}{\cancel{2^n}} \cdot \frac{1}{1+n^2} = \sum_{n=0}^{\infty} \frac{1}{1+n^2}$

Data

$n^2+2 \geq n^2$

$\frac{1}{n^2+1} \leq \frac{1}{n^2}$

$\therefore$ a série converge, por comparação com série-p com $p=2>1$

Para $x=-1/2$

$$\sum_{n=0}^{\infty} \frac{2^n}{5^n} \cdot \frac{(-1/2-2)^n}{(1+n^2)} = \sum_{n=0}^{\infty} \frac{2^n}{5^n} \cdot \left(\frac{-5}{2}\right)^n \cdot \frac{1}{1+n^2} = \sum_{n=0}^{\infty} \frac{(-1)^n}{1+n^2}$$

(i) $\lim_{n\to+\infty} \frac{1}{1+n^2} = 0$

(ii) $\frac{1}{1+(n+1)^2} \leq \frac{1}{1+n^2}$

$\cancel{1}+\cancel{n^2} \leq \cancel{1}+\cancel{n^2}+2n+1$

$0 \leq 2n+1$

$\therefore$ a série converge, por Leibnitz

Assim, $R=\frac{5}{2}$, $I=\left[-\frac{1}{2}, \frac{9}{2}\right]$

(29) $\sum_{n=1}^{\infty} \frac{(3x-5)^n}{7^n n}$

$$L=\lim_{n\to+\infty}\left|\frac{(\cancel{3x-5})^{n+1}}{\cancel{7}^{n+1}\cdot(n+1)} \cdot \frac{\cancel{7^n}\, n}{\cancel{(3x-5)^n}}\right| = \frac{|3x-5|}{7} \lim_{n\to+\infty} \frac{n}{n+1} = \frac{|3x-5|}{7}$$

$\therefore$ a série converge se $L<1 \Rightarrow \frac{|3x-5|}{7}<1 \Rightarrow -\frac{2}{3}<x<4$

Data

Para $x = 4$

$$\sum_{n=1}^{\infty} \frac{(3.4-5)^n}{7^n . n} = \sum_{n=1}^{\infty} \frac{\cancel{7^n}}{\cancel{7^n} . n} = \sum_{n=1}^{\infty} \frac{1}{n}$$

∴ a série diverge, série harmônica

Para $x = -2/3$

$$\sum_{n=1}^{\infty} \frac{(-2-5)^n}{7^n . n} = \sum_{n=1}^{\infty} \frac{(-1)^n \cancel{7^n}}{\cancel{7^n} . n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

(i) $\lim\limits_{n \to +\infty} \frac{1}{n} = 0$

(ii) $\frac{1}{n+1} \leq \frac{1}{n}$

$\cancel{n} \leq \cancel{n}+1$

$0 \leq 1$

∴ a série converge, por Leibnitz

Assim, $R = \frac{7}{3}$, $I = \left[\frac{-2}{3}, 4\right)$

30) $$\sum_{n=0}^{\infty} \frac{(-1)^n . x^{2n}}{3^{2n}} = \sum_{n=0}^{\infty} (-1)^n \left(\frac{x}{3}\right)^{2n}$$

$$L = \lim_{n \to +\infty} \left| \frac{(-1)^{n+1} . \cancel{x^{2n+2}}}{3^{2n+2}} . \frac{3^{2n}}{\cancel{x^{2n}} . (-1)^n} \right| = \frac{x^2}{9}$$

Data

$\therefore$ a série converge se $L < 1 \Rightarrow \frac{x^2}{9} < 1 \Rightarrow -3 < x < 3$

Para $x = 3$:

$\sum_{n=0}^{\infty} (-1)^n \cdot \frac{3^{2n}}{3^{2n}} = \sum_{n=0}^{\infty} (-1)^n$

Para n par

$\sum_{n=0}^{\infty} 1$

$\lim_{n \to +\infty} 1 = 1 \neq 0$

$\therefore$ a série diverge, pelo critério do termo geral

Para $x = -3$

$\sum_{n=0}^{\infty} (-1)^n \cdot \frac{(-3)^{2n}}{3^{2n}} = \sum_{n=0}^{\infty} (-1)^n$

$\therefore$ a série diverge, pelo teste anterior

Assim, $R = 3$, $I = (-3, 3)$

$S = \frac{1}{1 + \left(\frac{x^2}{9}\right)} = \frac{9}{9 + x^2}$

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Data

(31) $f(x) = \frac{4}{x^2}$

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$

$\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} n x^{n-1}$

$\frac{1}{(1-x)^2} = \sum_{n=0}^{\infty} (n+1) x^n$

$\frac{4}{(1-x)^2} = \sum_{n=0}^{\infty} 4(n+1) x^n$

$\frac{4}{x^2} = \sum_{n=0}^{\infty} 4(n+1)(1-x)^n$

$\frac{4}{x^2} = \sum_{n=0}^{\infty} (-1)^n (4n+4)(x-1)^n$

$L = \lim_{n \to +\infty} \left| \frac{(-1)^{n+2} (4(n+1)+4) \cancel{(x-1)}^{n+2}}{(-1)^n . (4n+4) \cancel{(x-1)}^n} \right| = |x-1|$

$\therefore$ a série converge se $L < 1 \Rightarrow |x-1| < 1 \Rightarrow 0 < x < 2$

Para $x = 2$

$\sum_{n=0}^{\infty} (4n+4)(-1)^n (2-1)^n = \sum_{n=0}^{\infty} (4n+4)(-1)^n$

Para n par

$\lim_{n \to +\infty} 4n+4 = +\infty$

$\therefore$ a série diverge, pelo critério do termo geral

Data

Para $x=0$

$$\sum_{n=0}^{\infty} (-1)^n \cdot (4n+4) \cdot (-1)^n = \sum_{n=0}^{\infty} 4n+4$$

$$\lim_{n \to +\infty} 4n+4 = +\infty$$

$\therefore$ a série diverge, pelo critério do termo geral

Assim, $R=1$, $I=(0,2)$

32) $f(x) = \cosh(x^3) =$

$$\cosh(x) = \cosh(0) + \operatorname{senh}(0) + \frac{\cosh(0)}{2!} + \frac{\operatorname{sen}(0)}{3!} + \frac{\cosh(0)}{4!} + \ldots$$
$$= 1 + \frac{1}{2!} + \frac{1}{4!} + \ldots +$$
$$= \frac{1}{(2n)!}$$

$$\cosh(x) = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$$

$$\cosh(x^3) = \sum_{n=0}^{\infty} \frac{x^{6n}}{(2n)!}$$

$$L = \lim_{n \to +\infty} \left| \frac{x^{6n+6}}{(2n+2)!} \cdot \frac{(2n)!}{x^{6n}} \right| = x^6 \lim_{n \to +\infty} \frac{1}{(2n+2)(2n+1)} = 0$$

$\therefore$ a série converge $\forall x \in \mathbb{R} \Rightarrow R = +\infty$, $I = (-\infty, +\infty)$

Data

(33) $f(x) = \frac{e^{x^2}-1}{x}$

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \dots$

$e^{x^2} = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!} = 1 + x^2 + \frac{x^4}{2!} + \dots$

$e^{x^2} - 1 = \sum_{n=1}^{\infty} \frac{x^{2n}}{n!} = x^2 + \frac{x^4}{2!} + \dots$

$\frac{e^{x^2}-1}{x} = \sum_{n=1}^{\infty} \frac{x^{2n}}{n!} \cdot \frac{1}{x} = x + \frac{x^3}{2!} + \dots$

$\frac{e^{x^2}-1}{x} = \sum_{n=1}^{\infty} \frac{x^{2n-1}}{n!}$

$L = \lim_{n \to +\infty} \left| \frac{x^{2n+1}}{(n+1)!} \cdot \frac{n!}{x^{2n-1}} \right| = x^2 \lim_{n \to +\infty} \frac{1}{n+1} = 0$

$\therefore$ a série converge $\forall x \in \mathbb{R} \Rightarrow R = +\infty$, $I = (-\infty, +\infty)$

(34) $\int \cos x \, dx = \operatorname{sen} x + K$

$\cos x = \cos(0) - \operatorname{sen}(0) - \frac{\cos(0)}{2!} + \frac{\operatorname{sen}(0)}{3!} + \frac{\cos(0)}{4!} - \frac{\operatorname{sen}(0)}{5!} + \dots$

$= \cos(0) - \frac{\cos(0)}{2!} + \frac{\cos(0)}{4!} + \dots$

$= \frac{(-1)^n}{2!}$

$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n}}{(2n)!}$

Data

$$\int \cos x\, dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+1}}{(2n+1)(2n)!} + K$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} + K$$

$$= \operatorname{sen} x + K$$

(35) $f(x) = \int_0^x t^2 \ln(1+4t^2)\, dt$

$$\ln(1+t) = \sum_{n=0}^{\infty} \frac{(-1)^n\, t^{n+1}}{n+1}$$

$$\ln(1+4t^2) = \sum_{n=0}^{\infty} \frac{(-1)^n (4t^2)^{n+1}}{n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 4^{n+1} \cdot t^{2n+2}}{n+1}$$

$$t^2 \ln(1+4t^2) = \sum_{n=0}^{\infty} \frac{(-1)^n\, 4^{n+1}\, t^{2n+4}}{n+1}$$

$$\int_0^x t^2 \ln(1+4t^2)\, dt = \int_0^x \sum_{n=0}^{\infty} \frac{(-1)^n\, 4^{n+1}\, t^{2n+4}}{n+1}\, dt$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n\, 4^{n+1}\, t^{2n+5}}{(2n+5)(n+1)} \Big|_0^x$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n\, 4^{n+1}\, x^{2n+5}}{(2n+5)(n+1)}$$

$$L = \lim_{n\to+\infty} \left| \frac{(-1)^{n+1}\, 4^{n+2}\, x^{2n+7}}{(2n+7)(n+2)} \cdot \frac{(2n+5)(n+1)}{(-1)^n\, 4^{n+1}\, x^{2n+5}} \right| = 4x^2$$

$\therefore$ a série converge se $L<1 \Rightarrow 4x^2 < 1 \Rightarrow -\frac{1}{2} < x < \frac{1}{2}$

Para $x = \frac{1}{2}$

Data

$$\sum_{n=0}^{\infty} \frac{(-1)^n \cdot 2^{2n+2}}{(2n+5)(n+1)\cdot 2^{2n+5}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{8(2n+5)(n+1)}$$

(i) $\lim_{n\to+\infty} \frac{1}{8(2n+5)(n+1)} = 0$

(ii) $\frac{1}{\cancel{8}(2n+7)(n+2)} \leq \frac{1}{\cancel{8}(2n+5)(n+1)}$

$\cancel{2n^2} + 7n + 5 \leq \cancel{2n^2} + 11n + 14$

$0 \leq 4n + 9$

∴ a série converge, por Leibnitz

Para $x = -\frac{1}{2}$

$$\sum_{n=0}^{\infty} \frac{(-1)^n \cdot 2^{2n+2}}{(2n^2+7n+5)} \cdot \frac{(-1)^{2n+5}}{2^{2n+5}} = \sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{8(2n^2+7n+5)}$$

∴ a série converge, pelo teste anterior

Assim, $R = \frac{1}{2}$, $I = \left[-\frac{1}{2}, \frac{1}{2}\right]$

36 a) $f(x) = \operatorname{sen}^2 x = \frac{1}{2}(1 - \cos(2x))$

$$\cos(2x) = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot (2x)^{2n}}{(2n)!} = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \dots$$

$$1 - \cos(2x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}\, 2^{2n} \cdot x^{2n}}{(2n)!} = \frac{(2x)^2}{2!} - \frac{(2x)^4}{4!} + \frac{(2x)^6}{6!}$$

Data

$$\frac{1-\cos(2x)}{2} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}\, 2^{2n-1}\, x^{2n}}{(2n)!} = \frac{2x^2}{2!} - \frac{2^3 x^4}{4!} + \frac{2^5 x^6}{6!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n\, 2^{2n+1}\, x^{2n+2}}{(2n+2)!}$$

b) $f(x) = x^2 \operatorname{sen}(2x)$

$$\operatorname{sen}(2x) = \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n+1}}{(2n+1)!}$$

$$x^2 \operatorname{sen}(2x) = \sum_{n=0}^{\infty} \frac{(-1)^n\, 2^{2n+1} \cdot x^{2n+3}}{(2n+1)!}$$

c) $f(x) = e^{3x}$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{3x} = \sum_{n=0}^{\infty} \frac{3^n \cdot x^n}{n!}$$

d) $f(x) = e^{-x^2}$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n}}{n!}$$

e) $f(x) = \cos(2x)$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\cos(2x) = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 4^n \cdot x^{2n}}{(2n)!}$$

f) $f(x) = \frac{\operatorname{sen}(x^3)}{x^3}$

$$\operatorname{sen} x = \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+1}}{(2n+1)!}$$

Data

$$\operatorname{sen}(x^5) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{10n+5}}{(2n+1)!}$$

$$\frac{\operatorname{sen}(x^5)}{x^3} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{10n+2}}{(2n+1)!}$$

g) $f(x) = \dfrac{\cos x - 1}{x^2}$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\cos x - 1 = \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\frac{\cos x - 1}{x^2} = \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-2}}{(2n)!}$$

h) $f(x) = x^3 e^{x^2}$

$$e^{x^2} = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$$

$$x^3 e^{x^2} = \sum_{n=0}^{\infty} \frac{x^{2n+3}}{n!}$$

37 a) $\displaystyle\lim_{x \to 0} \frac{\cos 2x + 2x^2 - 1}{x^4}$

$$\cos 2x - 1 = \sum_{n=1}^{\infty} \frac{(-1)^n \cdot 2^{2n} \cdot x^{2n}}{(2n)!} = -\frac{2^2 x^2}{2!} + \frac{2^4 x^4}{4!} - \frac{2^6 x^6}{6!} + \ldots$$

$$\cos 2x - 1 + 2x^2 = \sum_{n=2}^{\infty} \frac{(-1)^n \cdot 2^{2n} \, x^{2n}}{(2n)!} = \frac{2^4 x^4}{4!} - \frac{2^6 x^6}{6!} + \ldots$$

$$\frac{\cos 2x - 1 + 2x^2}{x^4} = \sum_{n=2}^{\infty} \frac{(-1)^n \cdot 2^{2n} \, x^{2n-4}}{(2n)!} = \frac{2^4}{4!} - \frac{2^6 x^2}{6!} + \ldots$$

$$\lim_{x \to 0} \frac{2^4}{4!} - \frac{2^6 x^2}{6!} + \frac{2^8 x^4}{8!} + \ldots = \frac{2^4}{4!} = \frac{\cancel{2}.\cancel{2}.2.\cancel{2}}{\cancel{4}.3.\cancel{2}} = \frac{2}{3}$$

b) $\displaystyle\lim_{x \to 0} \frac{\operatorname{sen}(x^2) + \cos(x^3) - x^2 - 1}{x^6}$

Data

$$\cos(x^3) + \operatorname{sen}(x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{6n}}{(2n)!} + \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{4n+2}}{(2n+1)!}$$

$$\cos(x^3) + \operatorname{sen}(x^2) - 1 - x^2 = \sum_{n=1}^{\infty} \frac{(-1)^n x^{6n}}{(2n)!} + \sum_{n=1} \frac{(-1)^n x^{4n+2}}{(2n+1)!}$$

$$\frac{\cos(x^3) + \operatorname{sen}(x^2) - 1 - x^2}{x^6} = \sum_{n=1}^{\infty} \frac{(-1)^n x^{6n-6}}{(2n)!} + \sum_{n=1}^{\infty} \frac{(-1)^n x^{4n-4}}{(2n+1)!}$$

$$\lim_{x\to 0} -\frac{1}{2!} + \frac{1}{3!} + \frac{x^6}{4!} + \frac{x^4}{5!} + \ldots = -\frac{1}{2} - \frac{1}{6} = -\frac{2}{3}$$

c) $\lim_{x\to 0} \frac{\ln(1+x^2)}{1-\cos x}$

$$\ln(1+x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{n+1} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n}}{n}$$

$$1 - \cos x = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n}}{(2n)!}$$

$$\lim_{x\to 0} \frac{\ln(1+x^2)}{1-\cos x} = \frac{x^2 - \frac{x^4}{2} + \frac{x^6}{3} + \ldots}{\frac{x^2}{2} - \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots} = 2$$

d) $\lim_{x\to 0} \frac{\ln(1+x^2) - 3\operatorname{sen}(2x^2)}{x^2}$

$$\ln(1+x^2) - 3\operatorname{sen}(2x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{n+1} - \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 3 \cdot 2^{2n+1} x^{4n+2}}{(2n+1)!}$$

$$\frac{\ln(1+x^2) - 3\operatorname{sen}(2x^2)}{x^2} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n+1} - \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 3 \cdot 2^{n+1} x^{4n}}{(2n+1)!}$$

Data

$$\lim_{x\to 0} \frac{1}{1} - \frac{6}{2} - x^2 + \frac{12x^2}{3!} + \ldots = -5$$

e) $$\lim_{x\to 0} \frac{\ln(1+x^3) - e^{x^3} + 1}{x^6}$$

$$\ln(1+x^3) - e^{x^3} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+3}}{n+1} - \sum_{n=0}^{\infty} \frac{x^{3n}}{n!}$$

$$\ln(1+x^3) - e^{x^3} + 1 = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{3n}}{n} - \sum_{n=1}^{\infty} \frac{x^{3n}}{n!}$$

$$\frac{\ln(1+x^3) - e^{x^3} + 1}{x^6} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{3n-6}}{n} - \sum_{n=1}^{\infty} \frac{x^{3n-6}}{n!}$$

Para $n \geq 2$

$$\lim_{x\to 0} -\frac{1}{2} - \frac{1}{2} + \frac{x^3}{3} - \frac{x^3}{3!} + \ldots = -1$$

f) $$\lim_{x\to 0} \frac{x^2 \operatorname{sen}(x^2) + e^{x^4} - 1}{\ln(1+x^4)}$$

$n = m-1$

$2n = 2m-2$

$$x^2 \operatorname{sen}(x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+4}}{(2n+1)!}$$

$$x^2 \operatorname{sen}(x^2) + e^{x^4} - 1 = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+4}}{(2n+1)!} + \sum_{n=1}^{\infty} \frac{x^{4n}}{n!}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n+2}}{(2n-1)!} + \sum_{n=1}^{\infty} \frac{x^{4n}}{n!}$$

$$\ln(1+x^4) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+4}}{n+1} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{4n}}{n} \quad |x|$$

Data

$$\lim_{x\to 0} \frac{x^2 \operatorname{sen}(x^2) + e^{x^4} - 1}{\ln(1+x^2)} = \frac{(x^4 + x^4) + \left(\frac{x^8}{2!} + \frac{x^6}{3!}\right) + \dots}{x^4 - \frac{x^8}{2} + \dots} = 2$$

g) $\lim_{x\to 0} \frac{\cos(2x^2) - e^{x^4}}{x \operatorname{sen}(x^3)}$

$$\cos(2x^2) - e^{x^4} = \sum_{n=0} (-1)^n \frac{4^n x^{4n}}{(2n)!} - \sum_{n=0} \frac{x^{4n}}{n!}$$

$$x \operatorname{sen}(x^3) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+4}}{(2n+1)!}$$

$$\lim_{x\to 0} \frac{\cos(2x^2) - e^{x^4}}{x \operatorname{sen}(x^3)} = \frac{(1-1) - \left(\frac{4x^4}{2} + x^4\right) + \dots}{x^4 - \frac{x^{10}}{3!} + \dots} = -3$$

h) $\lim_{x\to 0} \frac{\operatorname{sen}(x^8) + \cos(3x^4) - 1}{e^{x^8} - 1}$

$$\operatorname{sen} x^8 + \cos(3x^4) - 1 = \sum_{n=0}^{\infty} \frac{(-1)^n x^{16n+8}}{(2n+1)!} + \sum_{n=1}^{\infty} \frac{(-1)^n 9^n x^{8n}}{(2n)!}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{16n-8}}{(2n-1)!} + \frac{(-1)^n 9^n x^{8n}}{(2n)!}$$

$$e^{x^8} - 1 = \sum_{n=1} \frac{x^{8n}}{n!}$$

$$\lim_{x\to 0} \frac{\operatorname{sen}(x^8) + \cos(3x^4) - 1}{e^{x^8} - 1} = \frac{\left(x^8 - \frac{9x^8}{2}\right) + \left(\frac{81x^{16}}{4!} - \frac{x^{24}}{3!}\right) + \dots}{x^8 + \frac{x^{16}}{2!} + \dots} = -\frac{7}{2}$$

Data

(38) a) $\sum_{n=0}^{\infty} e^{nk} = 9$

$1 + e^{K} + e^{2K} + e^{3K} + \dots = 9$

$q = e^{K}$

$S = \frac{1}{1-e^{k}} = 9 \Rightarrow 9 - 9e^{K} = 1$

$9e^{K} = 8$

$e^{K} = \frac{8}{9} \Rightarrow K = \ln\left(\frac{8}{9}\right)$

b) $\lim_{x \to 0} \frac{e^{-x^4} - \cos(x^2)}{x^4} = K$

$e^{-x^4} - \cos(x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{n!} - \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{(2n)!}$

$\frac{e^{-x^4} - \cos(x^2)}{x^4} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n-4}}{n!} - \frac{(-1)^n x^{4n-4}}{(2n)!}$

Para $n \geq 1$

$\lim_{x \to 0} -1 + \frac{1}{2} + \frac{x^4}{2!} - \frac{x^4}{4!} + \dots = -\frac{1}{2} = K$

(39) a) $f(x) = \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$

b) $f(x) = \frac{1}{\sqrt{1+x}}$

$(1+x)^n = 1 + \sum_{K=1}^{\infty} \frac{n(n-1)(n-2)\cdots(n-K+1)}{K!} x^K$

Data

Para $n = -\frac{1}{2}$

$$\frac{1}{\sqrt{1+x}} = 1 + \sum_{k=1}^{\infty} \frac{(-1/2 - k + 1)!}{k!} x^k = 1 + \sum_{k=1}^{\infty} \left(\frac{-2k+1}{2}\right)! \frac{1}{k!} x^k$$

$$= 1 + \sum_{k=1}^{\infty} \frac{(-1)^k (2k-1)!\, x^k}{2^k k!}$$

c) $f(x) = \frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n \cdot x^{2n}$

d) $f(x) = \frac{1}{\sqrt{1-x^2}} = 1 + \sum_{k=1}^{\infty} \frac{(2k-1)!\, x^{2k}}{2^k k!}$

e) $f(x) = \int \frac{\operatorname{sen} x}{x} dx$

$$\frac{\operatorname{sen}(x)}{x} = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n}}{(2n+1)!}$$

$$\int \frac{\operatorname{sen}(x)}{x} dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n}}{(2n+1)!} dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n+1}}{(2n+1)(2n+1)!} + K$$

f) $f(x) = \int e^{-x^2} dx$

$$e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n}}{n!}$$

$$\int e^{-x^2} dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n}}{n!} dx = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n+1}}{(2n+1)\, n!} + K$$

Data

g) $f(x)=\int \frac{\ln(1+x)}{x}\,dx$

$\frac{\ln(1+x)}{x}=\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{n+1}$

$\int \frac{\ln(1+x)}{x}\,dx=\int \sum_{n=0}^{\infty}\frac{(-1)^n x^n}{n+1}\,dx=\sum_{n=0}^{\infty}\frac{(-1)^n x^{n+1}}{(n+1)^2}+K$

h) $f(x)=\ln\left(\frac{1+x}{1-x}\right)=\ln(1+x)-\ln(1-x)$

$\ln(1+x)-\ln(1-x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{n+1}}{n+1}-\sum_{n=0}^{\infty}\frac{(-1)^{2n+1}x^{n+1}}{n+1}$

$=\sum_{n=0}^{\infty}\frac{(-1)^n x^{n+1}+x^{n+1}}{n+1}$

$=\sum_{n=0}^{\infty}\frac{x^{n+1}((-1)^n+1)}{n+1}$

$=2x+0+\frac{2x^3}{3}+\ldots$

$=\sum_{n=0}^{\infty}\frac{2x^{2n+1}}{2n+1}$

i) $f(x)=\arcsen x$

$f'(x)=\frac{1}{\sqrt{1-x^2}}$

$\begin{cases}\arcsen(0)=0+K\\ K=0\end{cases}$

$\frac{1}{\sqrt{1-x^2}}=1+\sum_{n=1}^{\infty}\frac{(2n-1)!!\,x^{2n}}{2^n\,n!}$

$\int\frac{1}{\sqrt{1-x^2}}\,dx=\int 1+\sum_{n=1}^{\infty}\frac{(2n-1)!!\,x^{2n}}{2^n\,n!}\,dx$

$\arcsen x=x+\sum_{n=1}^{\infty}\frac{(2n-1)!!\,x^{2n+1}}{2^n(2n+1)\,n!}$

Data

j) $f(x) = \arccos x$

$$f'(x) = \frac{-1}{\sqrt{1-x^2}} = -(\operatorname{arcsen} x)'$$

$$\arccos x = -\left(x + \sum_{n=1}^{\infty} \frac{(2n-1)!\, x^{2n+1}}{2^n (2n+1) n!}\right)$$

$$= -x - \sum_{n=1}^{\infty} \frac{(2n-1)!\, x^{2n+1}}{2^n (2n+1) . n!}$$

K) $f(x) = \operatorname{arctg} x$

$$f'(x) = \frac{1}{1+x^2}$$

$$\left\} \begin{array}{l} \operatorname{arctg}(0) = 0 + K \\ K = 0 \end{array}\right.$$

$$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n}$$

$$\int \frac{1}{1+x^2} dx = \int \sum_{n=0}^{\infty} (-1)^n x^{2n} dx$$

$$\operatorname{arctg} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$$

l) $f(x) = \sqrt[3]{1+x}$

$$(x+1)^n = 1 + \sum_{k=1}^{\infty} \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!} x^k$$

$$(x+1)^{1/3} = 1 + \sum_{k=1}^{\infty} \left(\frac{1}{3} - k + 1\right)! \cdot \frac{x^k}{k!}$$

$$= 1 + \sum_{k=1}^{\infty} \left(\frac{4-3k}{3}\right)! \frac{x^k}{k!}$$

$$= 1 + \sum_{k=1}^{\infty} \frac{(-1)^k (3k-4)!\, x^k}{3^k\, k!}$$

Data

(10) $f(x) = \int_0^t \frac{1}{\sqrt[3]{1+x^4}}\,dx$

$(1+x^4)^n = 1 + \sum_{K=1}^{\infty} \frac{(n-K+1)!}{K!} x^{4K}$

$\frac{1}{(1+x^4)^{1/3}} = 1 + \sum_{K=1}^{\infty} \left(\frac{-1}{3} - K + 1\right)! \frac{x^{4K}}{K!}$

$= 1 + \sum_{K=1}^{\infty} \left(\frac{-3K+2}{3}\right)! \frac{x^{4K}}{K!}$

$= 1 + \sum_{K=1}^{\infty} (-1)^K \frac{(3K-2)!\, x^{4K}}{3^K K!}$

$\int_0^t \frac{1}{\sqrt[3]{1+x^4}}\,dx = \int_0^t 1 + \sum_{K=1}^{\infty} (-1)^K \frac{(3K-2)!\, x^{4K}}{3^K K!}\,dx$

$= \left(x + \sum_{K=1}^{\infty} \frac{(-1)^K (3K-2)!\, x^{4K+1}}{(4K+1) . 3^K . K!}\right)\Big|_0^t$

$= t + \sum_{K=1}^{\infty} \frac{(-1)^K (3K-2)!\, t^{4K+1}}{3^K (4K+1) K!}$

$\int_0^t \frac{1}{\sqrt[3]{1+x^4}}\,dx = t - \frac{t^5}{35} + \frac{4!\, t^9}{3^2 . 9 . 2!} - \frac{8!\, t^{13}}{3^3 . 13 . 3!} + \frac{10!\, t^{15}}{3^4 . 17 . 4!}$

(9) Continuação:

K) $g(x) = \int_0^x t^2 e^{-t^2}\,dt$

$t^2 e^{-t^2} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{n!}$

$\int_0^x t^2 e^{-t^2}\,dt = \int_0^x \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{n!}\,dt = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{(2n+3)n!}$

∴ Verdadeira

Data

l) $\sum_{n=1}^{\infty} (-1)\, 3^n x^n$

$L = \lim_{n \to +\infty} \left| \frac{(-1)\, 3^{n+1}\, x^{n+1}}{(-1) \cdot 3^n \cdot x^n} \right| = 3|x|$

$\therefore$ a série converge se $L < 1 \Rightarrow 3|x| < 1 \Rightarrow -\frac{1}{3} < x < \frac{1}{3}$

Para $x = -1/3$

$\sum_{n=1}^{\infty} (-1)\, 3^n \cdot \left(\frac{-1}{3}\right)^n = \sum_{n=1}^{\infty} (-1)^{n+1}$

Para n ímpar

$L = \lim_{n \to +\infty} 1 = 1 \neq 0$

$\therefore$ a série diverge, pelo critério do termo geral

Para $x = 1/3$

$\sum_{n=1}^{\infty} (-1) \cdot 3^n \cdot \frac{1}{3^n} = \sum_{n=1}^{\infty} (-1)$

$L = \lim_{n \to +\infty} -1 = -1 \neq 0$

$\therefore$ a série diverge, pelo critério do termo geral

Assim, $R = \frac{1}{3}$, $I = \left(-\frac{1}{3}, \frac{1}{3}\right)$

Data

$\sum_{n=1}^{\infty} (-1) . 3^n x^n = -3x - 3^2x^2 - 3^3x^3 - \dots$

$q = \frac{-3^{\cancel{2}} x^{\cancel{2}}}{\cancel{-3x}} = 3x$

$S = \frac{-3x}{1+3x}$ $\therefore$ Verdadeira

m) $\sum_{n=1}^{\infty} (u_{n+1} - u_n)$

u_n (converge) $\Rightarrow u_{n+1} \leq u_n$
$u_{n+1} - u_n \leq 0$

Como $(u_{n+1} - u_n)$ é menor que uma constante, a série converge $\Rightarrow$ Verdadeira

n) $\sum_{n=0}^{\infty} \frac{(-1)^n . (3x-5)^{2n}}{2^{2n} (n!)^2}$

$L = \lim_{n \to +\infty} \left| \frac{(-1)^{n+1} . \cancel{(3x-5)}^{2n+2}}{\cancel{2}^{2n+2} . (n+1)!^2} . \frac{\cancel{2^{2n}} . (n!)^2}{(-1)^n . \cancel{(3x-5)}^{2n}} \right|$

$L = \lim_{n \to +\infty} \frac{(3x-5)^2}{4 . (n+1)^2} = 0 < 1$

$\therefore$ a série converge $\forall x \in \mathbb{R} \Rightarrow R = +\infty, I = (-\infty, +\infty)$

Verdadeira