66) $f(x,y) = d^2(P,A) + d^2(P,B) + d^2(P,C)$

$$= x^2+y^2 + (x-4)^2 + y^2 + (x-3)^2 + (y-3)^2$$

$$= x^2 + 2y^2 + x^2 - 8x + 1 + x^2 - 6x + 9 + y^2 - 6y + 9$$

$$= 3x^2 - 14x + 3y^2 - 6y + 22$$

$$\frac{\partial f}{\partial x} = 6x - 14 \qquad \frac{\partial f}{\partial y} = 6y - 6 \qquad \frac{\partial^2 f}{\partial x \partial y} = 0$$

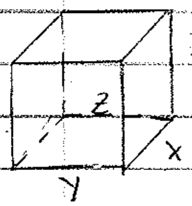
$$\frac{\partial^2 f}{\partial x^2} = 6 \qquad \frac{\partial^2 f}{\partial y^2} = 6$$

$$\begin{cases} 6x - 14 = 0 \\ 6y - 6 = 0 \end{cases} \Rightarrow x = \frac{7}{3}, \; y = 1$$

$$\Delta(x,y) = \begin{vmatrix} 6 & 0 \\ 0 & 6 \end{vmatrix} = 36$$

$\Delta(P) > 0$, $f_{xx}(P) > 0$, $P\left(\frac{7}{3}, 1\right)$ é um ponto de mínimo

67)



$$A = xy + 2yz + 2xz = 300$$

$$2z(x+y) = 300 - xy$$

$$z = \frac{300 - xy}{2(x+y)}$$

$$V = xyz = xy.\frac{(300 - xy)}{2(x+y)} = \frac{300xy - x^2y^2}{2(x+y)}$$

$$\frac{\partial V}{\partial x} = \frac{(300y - 2xy^2)\,\cancel{2}(x+y) - (300xy - x^2y^2)\cancel{2}}{2^2(x+y)^2}$$

$$\frac{\partial V}{\partial x} = \frac{\cancel{300xy} + 300y^2 - 2x^2y^2 - 2xy^3 - \cancel{300xy} + \cancel{x^2y^2}}{2(x+y)^2}$$

$$\frac{\partial V}{\partial x} = \frac{300y^2 - x^2y^2 - 2xy^3}{2(x+y)^2}$$

$$\frac{\partial V}{\partial x} = \frac{y^2(300 - x^2 - 2xy)}{2(x+y)^2}$$

$$\frac{\partial V}{\partial y} = \frac{(300x - 2x^2y)\cancel{2}(x+y) - (300xy - x^2y^2).\cancel{2}}{2^{\cancel{2}}(x+y)^2}$$

$$\frac{\partial V}{\partial y} = \frac{300x^2 + \cancel{300xy} - 2x^3y - 2x^2y^2 - \cancel{300xy} + \cancel{x^2y^2}}{2(x+y)^2}$$

$$\frac{\partial V}{\partial y} = \frac{300x^2 - x^2y^2 - 2x^3y}{2(x+y)^2}$$

$$\frac{\partial V}{\partial y} = \frac{x^2(300 - y^2 - 2xy)}{2(x+y)^2}$$

$$\begin{cases} \frac{\partial V}{\partial x} = 0 \Rightarrow 300 - x^2 - 2xy = 0 \\ \frac{\partial V}{\partial y} = 0 \Rightarrow 300 - y^2 - 2xy = 0 \end{cases}$$

$-x^2 + y^2 = 0$

$x^2 = y^2$

$x = \pm y$

$x = y = 10$

$300 - y^2 - 2y^2 = 0$

$3y^2 = 300$

$y = \pm 10$

$y = 10$

$$z = \frac{300 - 10.10}{2.(10+10)}$$

$z = 5$

$$\frac{\partial^2 V}{\partial x^2} = \frac{(-\cancel{2}xy^2 - \cancel{2}y^3)\cancel{2}(x+y)^2 - \cancel{2}^2 y\cancel{(x+y)}(300y^2 - x^2y^2 - 2xy^3)}{2^{\cancel{2}}(x+y)^{3\cancel{4}}}$$

$$\frac{\partial^2 V}{\partial x^2} = \frac{-\cancel{x^2y^2} - \cancel{xy^3} - xy^3 - y^4 - 300y^2 + \cancel{x^2y^2} + 2\cancel{xy^3}}{(x+y)^3}$$

$$\frac{\partial^2 V}{\partial x^2} = \frac{-y^2(y^2 + 300)}{(x+y)^3}$$

$$\frac{\partial^2 V}{\partial y^2} = \frac{-x^2(x^2+300)}{(x+y)^3}$$

$$\frac{\partial^2 V}{\partial y \partial x} = \frac{(-3y^3-600y)(x+y)^{\cancel{3}} - (-y^2(y^2+300))\,3\cancel{(x+y)^2}}{(x+y)^{\cancel{6}\,4}}$$

$$\frac{\partial^2 V}{\partial y \partial x} = \frac{-3xy^3 - \cancel{3y^4} - 600xy - \cancel{600y^2} + \cancel{3y^4} + \overset{300}{\cancel{900}}y^2}{(x+y)^4}$$

$$\frac{\partial^2 V}{\partial y \partial x} = \frac{3y(100y - 200x - xy^2)}{(x+y)^4}$$

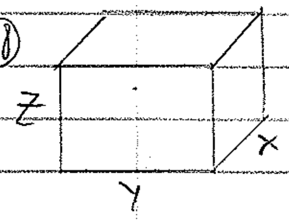
$$\frac{\partial^2 V}{\partial x \partial y} = \frac{3x(100x - 200y - x^2y)}{(x+y)^4}$$

$$\Delta(10,10) = \begin{vmatrix} -2{,}5 & -0{,}375 \\ -0{,}375 & -2{,}5 \end{vmatrix} = 6{,}109 > 0$$

$V_{xx}(10,10) = -2{,}5 < 0$

$\therefore$ P(10,10) é um ponto de máximo

(68)



$V = xyz = 128$

$z = \dfrac{128}{xy}$

$$C = \frac{1}{2}\,2xz + \frac{1}{2}\,2yz + 1.2xy$$

$$C = z(x+y) + 2xy$$

$$C = \frac{128\,(x+y)}{xy} + 2xy$$

$$\frac{\partial C}{\partial x} = 2y + 128\,\frac{(1.xy - (x+y).y)}{x^2y^2} = 2y - \frac{128}{x^2}$$

$$\frac{\partial C}{\partial y} = 2x - \frac{128}{y^2} \qquad \frac{\partial^2 C}{\partial y^2} = \frac{256}{y^3}$$

$$\frac{\partial^2 C}{\partial x^2} = \frac{256}{x^3} \qquad \frac{\partial^2 C}{\partial x \partial y} = 2$$

$$\frac{\partial C}{\partial x} = 0 \Rightarrow 2y - \frac{128}{x^2} = 0 \Rightarrow 2y = \frac{128}{x^2} \Rightarrow y = \frac{64}{x^2}$$

$$\frac{\partial C}{\partial y} = 0 \Rightarrow 2x - \frac{128}{y^2} = 0 \Rightarrow 2x = \frac{128}{y^2} \Rightarrow x = \frac{64}{y^2}$$

$$x = \frac{64}{\left(\frac{64}{x^2}\right)^2} \Rightarrow \cancel{x} = \cancel{64} \cdot \frac{x^{\cancel{4}\,3}}{64^{\cancel{2}}} \Rightarrow x^3 = 2^6 \qquad y = \frac{64}{4^2} = 4$$

$$x = 4$$

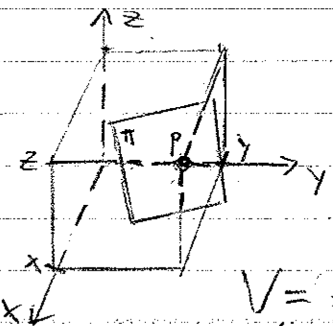
$$z = \frac{128}{xy} = \frac{128}{4 \cdot 4} = 8$$

$$\Delta(4,4) = \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} = 12 > 0$$

$$C_{xx}(4,4) = 4 > 0$$

$\therefore$ P(4,4) é um ponto de mínimo

(69)



$$\pi: 3x + 2y + z - 6 = 0$$

$$P = (x, y, z)$$

$$P = (x, y, 6 - 3x - 2y)$$

$$V = xyz = xy(6 - 3x - 2y)$$

$$V = 6xy - 3x^2y - 2xy^2$$

$\frac{\partial V}{\partial x} = 6y - 6xy - 2y^2$ $\frac{\partial^2 V}{\partial x^2} = -6y$ $\frac{\partial^2 V}{\partial x \partial y} = 6 - 6x - 4y$

$\frac{\partial V}{\partial y} = 6x - 3x^2 - 4xy$ $\frac{\partial^2 V}{\partial y^2} = -4x$

$\frac{\partial V}{\partial x} = 0 \Rightarrow 6y - 6xy - 2y^2 = 0 \Rightarrow 6 - 6x - 2y = 0$

$\frac{\partial V}{\partial y} = 0 \Rightarrow 6x - 3x^2 - 4xy = 0 \Rightarrow 6 - 3x - 4y = 0$

$6x + 2y = 3x + 4y$ $6 - \cancel{3} \cdot \frac{2y}{\cancel{3}} - 4y = 0$

$3x = 2y$

$x = \frac{2}{3}y = \frac{2}{3}$ $6 - 6y = 0$

$y = 1$

$z = 6 - 3x - 2y = 6 - \cancel{3} \cdot \frac{2}{\cancel{3}} - 2 \cdot 1 = 2$

$\Delta(2/3, 1) = \begin{vmatrix} -6 & -2 \\ -2 & -8/3 \end{vmatrix} = {}^2\cancel{6} \cdot \frac{8}{\cancel{3}} + 4 = 20 > 0$

$V_{xx}(2/3, 1) = -6 < 0$

$\therefore P(2/3, 1)$ é um ponto de máximo

$V = xyz = \frac{2}{3} \cdot 1 \cdot 2 = \frac{4}{3}$ u.v

70) $L(x,y) = R(x,y) - C(x,y)$

$L(x,y) = 10\cancel{0}x - x^2 + 2000y + xy - x^2 - 10\cancel{0}x - y^2 + xy$

$L(x,y) = 2000y + 2xy - 2x^2 - y^2$

$\frac{\partial L}{\partial x} = 2y - 4x$ $\frac{\partial L}{\partial y} = 2000 + 2x - 2y$ $\frac{\partial^2 L}{\partial x \partial y} = 2$

$\frac{\partial^2 L}{\partial x^2} = -4$ $\frac{\partial^2 L}{\partial y^2} = -2$

$\frac{\partial L}{\partial x} = 0 \Rightarrow 2y - 4x = 0 \Rightarrow y = 2x = 2000$

$\frac{\partial L}{\partial y} = 0 \Rightarrow 2000 + 2x - 2y = 0$

$y = x + 1000$

$2x = x + 1000$

$x = 1000$

$P(1000, 2000)$

$\Delta(x,y) = \begin{vmatrix} -4 & 2 \\ 2 & -2 \end{vmatrix} = 8 - 4 = 4 > 0$

$L_{xx} = -4 < 0$

$\therefore P(1000,2000)$ é um ponto de máximo

(71) $L(x,y) = xp_1 + yp_2 - C(x,y)$

$L(x,y) = x(120-2x) + y(200-y) - (x^2+2y^2+2xy)$

$L(x,y) = 120x - 2x^2 + 200y - y^2 - x^2 - 2y^2 - 2xy$

$L(x,y) = 120x + 200y - 3x^2 - 3y^2 - 2xy$

$\frac{\partial L}{\partial x} = 120 - 6x - 2y = 0 \Rightarrow y = 60 - 3x = 30$

$\frac{\partial L}{\partial y} = 200 - 6y - 2x = 0 \Rightarrow x = 100 - 3y$

$x = 100 - 3(60 - 3x)$

$x = 100 - 180 + 9x$

$8x = 80$

$x = 10$ $\quad P(10;30)$

$\frac{\partial^2 L}{\partial x^2} = -6 \quad \frac{\partial^2 L}{\partial x \partial y} = -2$

$\frac{\partial^2 L}{\partial y^2} = -6$

$$\Delta(x,y) = \begin{vmatrix} -6 & 2 \\ 2 & -6 \end{vmatrix} = 36 - 4 = 32 > 0$$

$$\frac{\partial^2 L}{\partial x^2} = -6 < 0$$

$\therefore$ P(10,30) é um ponto de máximo

(72) $L(x,y) = (3200 - 50x + 25y)(x-40) + (25x - 25y)(y-50)$

$L(x,y) = 25(128 - 2x + y)(x-40) + 25(x-y)(y-50)$

$L(x,y) = 25(128x - 2x^2 + xy - 5120 + 80x - 40y) + 25(xy - 50x - y^2 + 50y)$

$L(x,y) = 25(208x - 2x^2 + xy - 40y - 5120 + xy - 50x - y^2 + 50y)$

$L(x,y) = 25(158x + 10y + 2xy - 2x^2 - y^2 - 5120)$

$$\frac{\partial L}{\partial x} = 3950 + 50y - 100x = 0 \Rightarrow 79 + y - 2x = 0 \Rightarrow y = 2x - 79$$

$$\frac{\partial L}{\partial y} = 250 + 50x - 50y = 0 \Rightarrow 5 + x - y = 0 \Rightarrow y = x + 5$$

$y = x + 5$ $\qquad$ $2x - 79 = x + 5$

$y = 89$ $\qquad$ $x = 84$

P(84,89)

$$\frac{\partial^2 L}{\partial x^2} = -100 \qquad \frac{\partial^2 L}{\partial x \partial y} = 50$$

$$\frac{\partial^2 L}{\partial y^2} = -50$$

$$\Delta(x,y) = \begin{vmatrix} -100 & 50 \\ 50 & -50 \end{vmatrix} = 5000 - 2500 = 2500 > 0$$

$L_{xx} = -100 < 0$

$\therefore$ P(84,89) é um ponto de máximo

73) $Q = \{(x,y,z) \in \mathbb{R}^3 / z = x+y+5\}$

$Q = \{(x,y,x+y+5) \in \mathbb{R}^3\}$

$$f = [d(P,Q)]^2 = (x-3)^2 + (y+2)^2 + (x+y+5-1)^2$$

$$= x^2 - 6x + 9 + y^2 + 4y + 4 + x^2 + 2x(y+4) + (y+4)^2$$

$$= 2x^2 + y^2 - 6x + 4y + 13 + 2xy + 8x + y^2 + 8y + 16$$

$$= 2x^2 + 2y^2 + 2x + 12y + 2xy + 29$$

$$\frac{\partial f}{\partial x} = 4x + 2 + 2y = 0 \Rightarrow y = -1 - 2x$$

$$\frac{\partial f}{\partial y} = 4y + 12 + 2x = 0 \Rightarrow x = -6 - 2y$$

$x = -6 - 2(-1-2x)$ $\quad$ $y = -1 - 2x$ $\quad$ $z = x+y+5$

$x = -6 + 2 + 4x$ $\quad$ $y = -1 - 2.\frac{4}{3}$ $\quad$ $z = \frac{4}{3} - \frac{11}{3} + 5$

$3x = 4$

$x = \frac{4}{3}$ $\quad$ $y = -\frac{11}{3}$ $\quad$ $z = \frac{8}{3}$

$$\frac{\partial^2 f}{\partial x^2} = 4 \qquad \frac{\partial^2 f}{\partial y^2} = 4 \qquad \frac{\partial^2 f}{\partial x \partial y} = 2$$

$$\Delta(x,y) = \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} = 16 - 4 = 12 > 0$$

$$\frac{\partial^2 f}{\partial x^2} = 4 > 0$$

$\therefore Q\left(\frac{4}{3}, -\frac{11}{3}, \frac{8}{3}\right)$ é um ponto de mínimo

(74) $z^2 = 4 - x^2 - y^2$

$T(x,y,z) = xyz^2 = xy(4 - x^2 - y^2) = 4xy - x^3y - xy^3$

$\dfrac{\partial T}{\partial x} = 4y - 3x^2y - y^3 = 0 \Rightarrow y(4 - 3x^2 - y^2) = 0$

$\dfrac{\partial T}{\partial y} = 4x - x^3 - 3xy^2 = 0 \Rightarrow x(4 - x^2 - 3y^2) = 0$

$\cancel{4} - 3x^2 - y^2 = \cancel{4} - x^2 - 3y^2$ $\qquad$ $4y - 3x^2y - y^3 = 0$

$\cancel{2}x^2 = \cancel{2}y^2$ $\qquad$ $4y - 3y^3 - y^3 = 0$

$x = \pm y$ $\qquad$ $4y - 4y^3 = 0$

$x = 0$ ou $x = \pm 1$ $\qquad$ $y(4 - 4y^2) = 0$

$y = 0$ ou $y = \pm 1$

$P_1(0,0)$, $P_2(-1,-1)$, $P_3(-1,1)$, $P_4(1,-1)$, $P_5(1,1)$

$\dfrac{\partial^2 T}{\partial x^2} = -6xy$ $\qquad$ $\dfrac{\partial^2 T}{\partial y^2} = -6xy$ $\qquad$ $\dfrac{\partial^2 T}{\partial x \partial y} = 4 - 3x^2 - 3y^2$

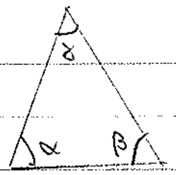
$\Delta(x,y) = \begin{vmatrix} -6xy & 4-3x^2-3y^2 \\ 4-3x^2-3y^2 & -6xy \end{vmatrix} = 36x^2y^2 - (4 - 3x^2 - 3y^2)^2$

$\Delta_1 = -16 < 0 \therefore P_1$ é um ponto de sela

$\Delta_2 = \Delta_3 = \Delta_4 = \Delta_5 = 32 > 0$

$T_{xx2} = T_{xx5} = -6 < 0 \therefore P_2$ e P_5 são pontos de máximo

$T_{xx3} = T_{xx4} = 6 > 0 \therefore P_3$ e P_4 são pontos de mínimo

(75) 

$$\alpha+\beta+\gamma=\pi$$

$$f=\cos\alpha+\cos\beta+\cos\gamma$$
$$f=\cos\alpha+\cos\beta+\cos(\pi-(\alpha+\beta))$$
$$f=\cos\alpha+\cos\beta-\cos(\alpha+\beta)$$
$$f=\cos\alpha+\cos\beta-(\cos\alpha\cos\beta-\operatorname{sen}\alpha\operatorname{sen}\beta)$$
$$f=\cos\alpha+\cos\beta-\cos\alpha\cos\beta+\operatorname{sen}\alpha\operatorname{sen}\beta$$

$$\frac{\partial f}{\partial\alpha}=-\operatorname{sen}\alpha+\operatorname{sen}\alpha\cos\beta+\cos\alpha\operatorname{sen}\beta=-\operatorname{sen}\alpha+\operatorname{sen}(\alpha+\beta)$$
$$\frac{\partial f}{\partial\beta}=-\operatorname{sen}\beta+\operatorname{sen}\beta\cos\alpha+\cos\beta\operatorname{sen}\alpha=-\operatorname{sen}\beta+\operatorname{sen}(\alpha+\beta)$$

$$-\operatorname{sen}\alpha+\operatorname{sen}(\alpha+\beta)=0 \qquad -\operatorname{sen}\beta+\operatorname{sen}(\alpha+\beta)=0$$
$$\operatorname{sen}\alpha=\operatorname{sen}(\alpha+\beta) \qquad \operatorname{sen}\beta=\operatorname{sen}(\alpha+\beta)$$

$$\operatorname{sen}\alpha=\operatorname{sen}\beta$$

$\alpha=\beta$ ou $\alpha=\beta+\pi$ (não convém)

$$\operatorname{sen}\alpha=\operatorname{sen}(2\alpha) \qquad \operatorname{sen}\alpha\neq0$$
$$\cancel{\operatorname{sen}\alpha}=2\cancel{\operatorname{sen}\alpha}\cos\alpha \qquad \alpha\neq0,\pi$$
$$\cos\alpha=\frac{1}{2}$$
$$\alpha=\frac{\pi}{3}=\beta=\gamma$$

$$\frac{\partial^2 f}{\partial\alpha^2}=-\cos\alpha+\cos(\alpha+\beta) \qquad \frac{\partial^2 f}{\partial\beta^2}=-\cos\beta+\cos(\alpha+\beta)$$
$$\frac{\partial^2 f}{\partial\alpha\partial\beta}=\cos(\alpha+\beta)$$

$$\Delta(\alpha,\beta)=\begin{vmatrix}-\cos\alpha+\cos(\alpha+\beta) & \cos(\alpha+\beta)\\ \cos(\alpha+\beta) & -\cos\beta+\cos(\alpha+\beta)\end{vmatrix}$$
$$=\cos\alpha\cos\beta-\cos\alpha\cdot\cos(\alpha+\beta)-\cos\beta\cos(\alpha+\beta)$$
$$=\cos\alpha\cos\beta-\cos(\alpha+\beta)(\cos\alpha+\cos\beta)$$

$$\Delta(\alpha,\alpha) = \cos^2\alpha - \cos(2\alpha) . 2\cos\alpha$$
$$= \cos^2\alpha - (\cos^2\alpha - \operatorname{sen}^2\alpha) . 2\cos\alpha$$
$$= \cos^2\alpha - 2\cos^3\alpha + 2\operatorname{sen}^2\alpha \cos\alpha$$
$$= \left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right)^3 + 2 . \left(\frac{\sqrt{3}}{2}\right)^2 . \frac{1}{2}$$
$$= \cancel{\frac{1}{4}} - \cancel{\frac{1}{4}} + \frac{3}{4} > 0$$

$$f_{\alpha\alpha} = -\frac{1}{2} + \left(-\frac{1}{2}\right) = -1 < 0$$

$\therefore P\left(\frac{\pi}{3}, \frac{\pi}{3}\right)$ é um ponto de máximo

$$f_{MÁX} = 3\cos\left(\frac{\pi}{3}\right) = \frac{3}{2}$$

76 $S: z^3 - (x^2 + y^2)z + 2 = 0$, $P(1,2,2)$

$$\frac{\partial z}{\partial x} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial z}} = \frac{-(-2xz)}{3z^2 - x^2 - y^2} = \frac{-(-2.1.2)}{3.4 - 1 - 4} = \frac{4}{7}$$

$$\frac{\partial z}{\partial y} = -\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial z}} = \frac{-(-2yz)}{3z^2 - x^2 - y^2} = \frac{-(-2.2.2)}{3.4 - 1 - 4} = \frac{8}{7}$$

$$\vec{n} = \left(\frac{-4}{7}, \frac{-8}{7}, 1\right) \qquad -\frac{4}{7} . 1 - \frac{8}{7} . 2 + 2 + d = 0 \Rightarrow d = \frac{6}{7}$$

$$\pi: -\frac{4}{7}x - \frac{8}{7}y + z + d = 0 \Rightarrow \pi: -4x - 8y + 7z + 6 = 0$$

(77) $x^2+z^3-z-xy\,\text{sen}\,z=1$, $P(1,1,0)$

$$\frac{\partial z}{\partial x}=-\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial z}}=-\frac{(2x-y\,\text{sen}\,z)}{3z^2-1-xy\cos z}=-\frac{(2.1-\cancel{1.\text{sen}0}_0)}{3.0-1-1.1\cos 0}=\frac{-2}{-2}=1$$

$$\frac{\partial z}{\partial y}=-\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial z}}=-\frac{(-x\,\text{sen}\,z)}{3z^2-1-xy\cos z}=\frac{1.\text{sen}0}{3.0-1-1.1.\cos 0}=0$$

$\vec{n}=(-1,0,1)$ $\qquad -1+0+d=0 \Rightarrow d=1$

$\pi: -x+z+d=0 \Rightarrow \pi: -x+z+1=0$

(78) $x=F(u,v) \Rightarrow F(u,v)-x=0$

$u=x^2+y,\ v=y^2$

$$\frac{\partial y}{\partial x}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}=-\frac{\left(\frac{\partial F}{\partial u}\cdot\frac{\partial u}{\partial v}-1\right)}{\frac{\partial F}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial F}{\partial v}\cdot\frac{\partial v}{\partial y}}=\frac{1-2x\frac{\partial F}{\partial u}}{\frac{\partial F}{\partial u}+2y\frac{\partial F}{\partial v}}$$

(79) $F(xy,z)=0 \Rightarrow F(u,v)=0 \begin{cases} u=xy \\ v=z \end{cases}$

$$\frac{\partial z}{\partial x}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}=-\frac{\frac{\partial F}{\partial u}\cdot\frac{\partial u}{\partial x}}{\frac{\partial F}{\partial v}\cdot\frac{\partial v}{\partial z}}=\frac{-y.\frac{\partial F}{\partial u}}{\frac{\partial F}{\partial v}}$$

$$\frac{\partial z}{\partial y} = -\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}} = -\frac{\frac{\partial F}{\partial u}\cdot\frac{\partial u}{\partial y}}{\frac{\partial F}{\partial v}\cdot\frac{\partial v}{\partial z}} = \frac{-x\frac{\partial F}{\partial u}}{\frac{\partial F}{\partial v}}$$

$$x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = -xy\frac{\frac{\partial F}{\partial u}}{\frac{\partial F}{\partial v}} + xy\frac{\frac{\partial F}{\partial u}}{\frac{\partial F}{\partial v}} = 0$$

Lista 3

1 a) $$\int_0^1\int_x^{3x+1} xy\,dy\,dx = \int_0^1 \frac{xy^2}{2}\Big|_x^{3x+1} dx = \int_0^1 \frac{x(3x+1)^2}{2} - \frac{x}{2}\cdot x^2\,dx$$

$$\int_0^1 \frac{x(9x^2+6x+1)}{2} - \frac{x^3}{2}\,dx = \int_0^1 4x^3+3x^2+\frac{x}{2}\,dx$$

$$\frac{4}{4}x^4 + \frac{3}{3}x^3 + \frac{1}{2}\cdot\frac{x^2}{2}\Big|_0^1 = 1+1+\frac{1}{4} = \frac{9}{4}$$

b) $$\int_0^1\int_y^{3y+1} xy^2\,dx\,dy = \int_0^1 \frac{x^2y^2}{2}\Big|_y^{3y+1} dy = \int_0^1 \frac{(3y+1)^2}{2}y^2 - \frac{y^2y^2}{2}\,dy$$

$$\int_0^1 \frac{(9y^2+6y+1).y^2}{2} - \frac{y^4}{2}\,dy = \int_0^1 4y^4+3y^3+\frac{y^2}{2}\,dy$$

$$\frac{4}{5}y^5 + \frac{3}{4}y^4 + \frac{1}{6}y^3\Big|_0^1 = \frac{4}{5}+\frac{3}{4}+\frac{1}{6} = \frac{48+45+10}{60} = \frac{103}{60}$$

c) $$\int_0^4\int_0^1 x e^{xy}\,dy\,dx = \int_0^4 e^{xy}\Big|_0^1 dx = \int_0^4 e^x - 1\,dx = e^x - x\Big|_0^4$$

$$e^4 - 4 - 1 = e^4 - 5$$

d) $$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\int_0^{4\cos\theta} \cos\theta\,\operatorname{sen}\theta\, r e^{r^2}\,dr\,d\theta\cdot\frac{2}{2} = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{2}\cos\theta\,\operatorname{sen}\theta\cdot e^{r^2}\Big|_0^{4\cos\theta} d\theta$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{2}\cos\theta \operatorname{sen}\theta \cdot e^{16\cos^2\theta} - \frac{1}{2}\cos\theta\operatorname{sen}\theta \, d\theta \cdot \frac{64}{64}$$

$u = 16\cos^2\theta$
$du = -32\cos\theta\operatorname{sen}\theta \, d\theta$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{32\cos\theta\operatorname{sen}\theta \, e^{16\cos^2\theta}}{64} - \frac{32}{64}\cos\theta\operatorname{sen}\theta \, d\theta$$

$$-\frac{1}{64} \cdot e^{16\cos^2\theta} + \frac{1}{64} \cdot 16\cos^2\theta \Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$-\frac{1}{64} e^{16\cos^2(\frac{\pi}{2})}{}_{0} + \cancel{\frac{1}{4}\cos^2\left(\frac{\pi}{2}\right)}_{0} + \frac{1}{64} e^{16\cos^2(\frac{\pi}{6})} - \frac{1}{4}\cos^2\left(\frac{\pi}{6}\right)$$

$$-\frac{1}{64} + \frac{1}{64} \cdot e^{12} - \frac{3}{16} \cdot \frac{4}{4} = \frac{e^{12} - 13}{64}$$

e) $$\int_0^{\pi}\int_0^{y^2} \cos\frac{x}{y} \, dx\,dy = \int_0^{\pi} y\operatorname{sen}\left(\frac{x}{y}\right)\Big|_0^{y^2} dy = \int_0^{\pi} y\operatorname{sen} y \, dy$$

u	dv
$\oplus$ y	$\operatorname{sen} y$
$\ominus$ 1	$-\cos y$
	$-\operatorname{sen} y$

$$y\cos y - \operatorname{sen} y \Big|_0^{\pi} = \pi\cos\pi - \operatorname{sen}\pi = \pi$$

f) $$\int_0^{\ln 2}\int_0^{y} x y^5 e^{x^2y^2} \, dx\,dy \cdot \frac{2}{2} = \int_0^{\ln 2} \frac{y^3}{2} e^{x^2y^2}\Big|_0^{y} dy$$

$u = e^{x^2y^2}$
$du = 2xy^2 e^{x^2y^2} dx$

$$\int_0^{\ln 2} \frac{y^3}{2} e^{y^2y^2} - \frac{y^3}{2} \, dy = \int_0^{\ln 2} \frac{y^3}{2} e^{y^4} - \frac{y^3}{2} \, dy \cdot \frac{4}{4}$$

$$\frac{e^{y^4}}{8} - \frac{y^4}{8}\Big|_0^{\ln 2} = \frac{e^{\ln^4 2}}{8} - \frac{\ln^4 2}{8} - \frac{1}{8} = \frac{1}{8}\left(e^{\ln^4 2} - \ln^4 2 - 1\right)$$

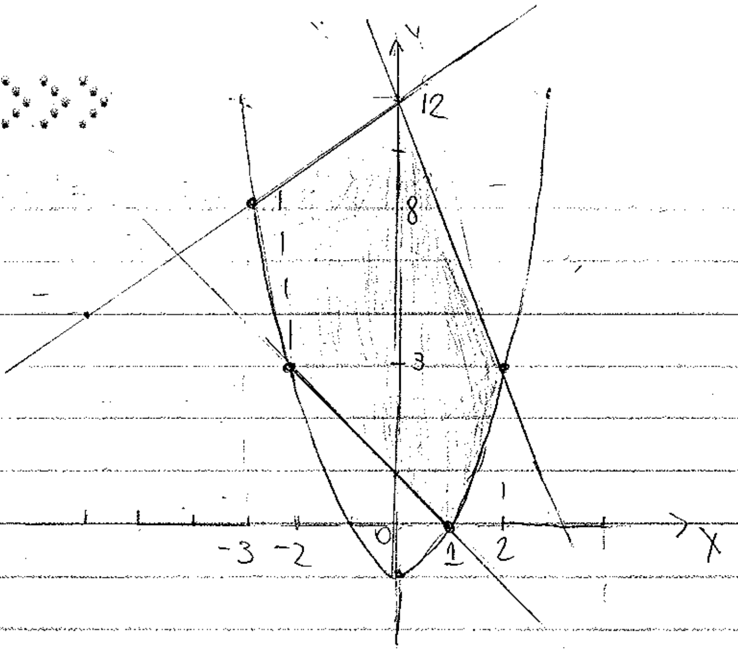
(2) a) $y = x^2 - 1$; $y = 1 - x$, $y = \frac{4x}{3} + 12$, $y = 12 - \frac{9}{2}x$

$x^2 - 1 = 1 - x$
$x^2 + x - 2 = 0$
$1 + -2 = -1$
$1 \cdot -2 = -2$

$x^2 - 1 = \frac{4x}{3} + 12$
$3x^2 - 4x - 39 = 0$

$x = \frac{4 \pm \sqrt{16 - 4 \cdot 3 \cdot (-39)}}{2 \cdot 3}$
$x = \frac{4 \pm 22}{6}$, $x_1 = \frac{13}{3}$, $x_2 = -3$

$\frac{4x}{3} + 12 = 12 - \frac{9}{2}x$
$\frac{4x}{3} = -\frac{9}{2}x$
$x = 0$



$$A=\int_{-3}^{-2}\int_{x^2-1}^{\frac{4}{3}x+12} dy\,dx+\int_{-2}^{0}\int_{1-x}^{\frac{4}{3}x+12} dy\,dx+\int_{0}^{1}\int_{1-x}^{12-\frac{9}{2}x} dy\,dx+\int_{1}^{2}\int_{x^2-1}^{12-\frac{9}{2}x} dy\,dx$$

$$A=\int_{0}^{3}\int_{1-y}^{\sqrt{y+1}} dx\,dy+\int_{3}^{8}\int_{-\sqrt{y+1}}^{\frac{2}{9}(12-y)} dx\,dy+\int_{8}^{12}\int_{\frac{3}{4}(y-12)}^{\frac{2}{9}(12-y)} dx\,dy$$

b) $y=\frac{4}{3}x+\frac{8}{3}$, $y=\frac{x}{2}-2$, $y=\frac{16}{3}-\frac{4}{3}x$, $y=-2-x$

$\frac{x}{2}-2=\frac{16}{3}-\frac{4}{3}x$

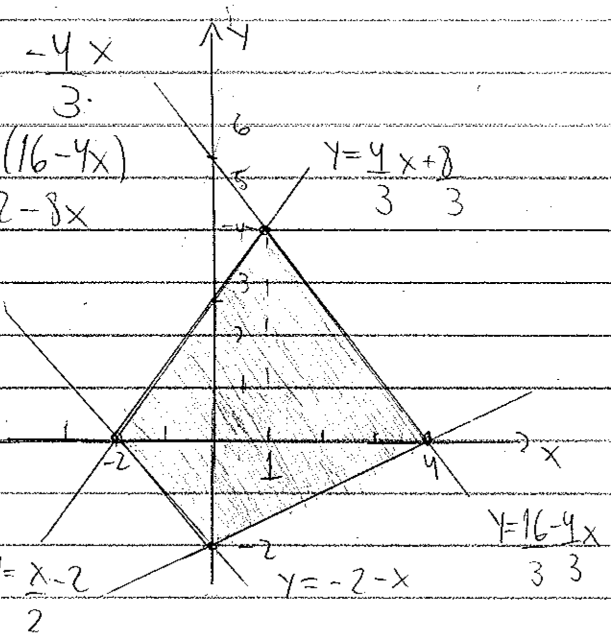
$3(x-4)=2(16-4x)$

$3x-12=32-8x$

$11x=44$

$x=4$

$y=0$



$\frac{4}{3}x+\frac{8}{3}=\frac{16}{3}-\frac{4}{3}x$

$\frac{8}{3}x=\frac{8}{3}$

$x=1 \Rightarrow y=4$

$\frac{x}{2}-2=-2-x$

$x=0 \Rightarrow y=-2$

$-2-x=\frac{4}{3}x+\frac{8}{3}$

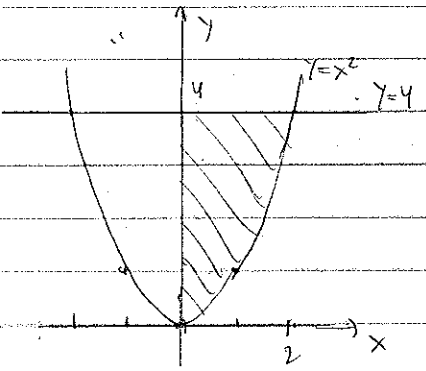
$-6-3x=4x+8$

$x=-2 \Rightarrow y=0$

$$A=\int_{-2}^{0}\int_{-2-x}^{\frac{4}{3}x+\frac{8}{3}} dy\,dx+\int_{0}^{1}\int_{\frac{x}{2}-2}^{\frac{4}{3}x+\frac{8}{3}} dy\,dx+\int_{1}^{4}\int_{\frac{x}{2}-2}^{\frac{16}{3}-\frac{4}{3}x} dy\,dx$$

$$A=\int_{-2}^{0}\int_{-2-y}^{2y+4} dx\,dy+\int_{0}^{4}\int_{\frac{3}{4}y+2}^{4-\frac{3}{4}y} dx\,dy$$

3a) $\int_{0}^{2}\int_{x^2}^{4} x\,\text{sen}(y^2)\,dy\,dx$



$y \in [0,4]$

$x \in [0,\sqrt{y}]$

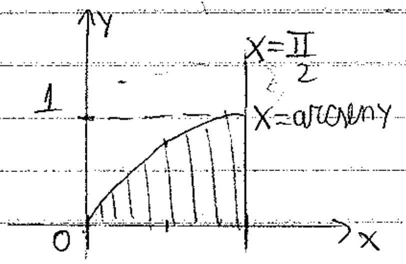
$$\int_{0}^{4}\int_{0}^{\sqrt{y}} x\,\text{sen}(y^2)\,dx\,dy=\int_{0}^{4}\frac{x^2}{2}\,\text{sen}(y^2)\Big|_{0}^{\sqrt{y}}dy$$

$$\int_{0}^{4}\frac{y}{2}\,\text{sen}(y^2)\,dy\cdot\frac{2}{2}=\frac{-\cos(y^2)}{4}\Big|_{0}^{4}=\frac{-\cos 16+1}{4}$$

b) $\int_{0}^{1}\int_{\text{arcsen}\,y}^{\frac{\pi}{2}} \cos x\sqrt{1+\cos^2 x}\,dx\,dy$

$u=1+\cos^2 x$

$du=-2\cos x\,\text{sen}\,x$



$$\int_{0}^{\frac{\pi}{2}}\int_{0}^{\text{sen}\,x}\cos x\sqrt{1+\cos^2 x}\,dy\,dx$$

$$\int_{0}^{\frac{\pi}{2}} y\cos x\sqrt{1+\cos^2 x}\Big|_{0}^{\text{sen}\,x}dx$$

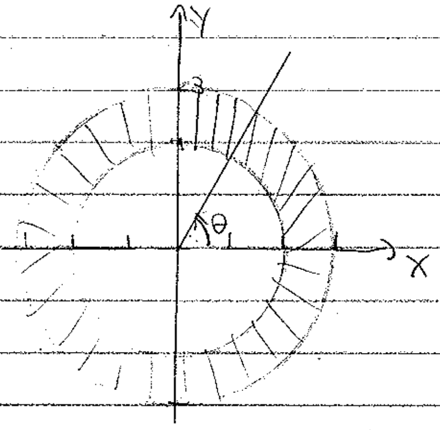
$$\int_{0}^{\frac{\pi}{2}}\text{sen}\,x\cos x\sqrt{1+\cos^2 x}\,dx\cdot\frac{-2}{-2}$$

$x \in [0,\pi/2]$

$y \in [0,\text{sen}\,x]$

$$-\frac{1}{3}(1+\cos^2 x)^{3/2}\Big|_{0}^{\frac{\pi}{2}}=\frac{2\sqrt{2}-1}{3}$$

4) a) $\iint_R \sqrt{14-x^2-y^2}\,dx\,dy \quad , R: 4 \leq x^2+y^2 \leq 9$



$r \in [2,3]$
$\theta \in [0,2\pi]$

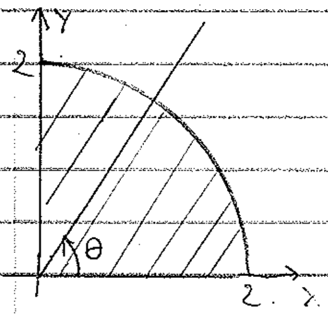
$\sqrt{14-x^2-y^2} = \sqrt{14-r^2}$

$\int_0^{2\pi}\int_2^3 \sqrt{14-r^2}\, r\,dr\,d\theta \quad \frac{-2}{-2}$

$\int_0^{2\pi} -\frac{1}{3}(14-r^2)^{3/2}\Big|_2^3 \,d\theta$

$\int_0^{2\pi}\left(-\frac{1}{3}\cdot 5^{3/2} + \frac{1}{3}10^{3/2}\right)d\theta = \frac{5^{3/2}}{3}\left(2^{3/2}-1\right)\cdot 2\pi$

b) $\iint_R \sqrt{14-x^2-y^2}\,dx\,dy \quad , R: x^2+y^2 \leq 4 \; , x \geq 0 \; , y \geq 0$



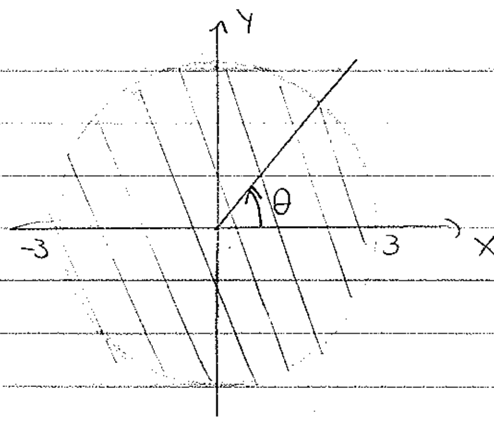
$r \in [0,2]$
$\theta \in [0,\pi/2]$

$\sqrt{14-x^2-y^2} = \sqrt{14-r^2}$

$\int_0^{\frac{\pi}{2}}\int_0^2 \sqrt{14-r^2}\, r\,dr\,d\theta \cdot \frac{-2}{-2} = \int_0^{\frac{\pi}{2}} -\frac{1}{3}(14-r^2)^{3/2}\Big|_0^2 \,d\theta$

$\int_0^{\frac{\pi}{2}}\left(-\frac{10^{3/2}}{3} + \frac{14^{3/2}}{3}\right)d\theta = \frac{14^{3/2}-10^{3/2}}{3}\theta\Big|_0^{\frac{\pi}{2}} = \frac{(14^{3/2}-10^{3/2})}{6}\pi$

c) $\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} e^{-x^2-y^2}\,dy\,dx$



$r \in [0,3]$

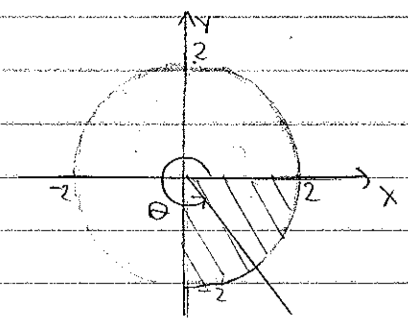
$\theta \in [0,2\pi]$

$e^{-x^2-y^2} = e^{-r^2}$

$$\int_0^{2\pi}\int_0^3 r \cdot e^{-r^2}\, dr\, d\theta \quad \frac{-2}{-2}$$

$$\int_0^{2\pi} \frac{e^{-r^2}}{-2}\Big|_0^3 d\theta = \int_0^2 \left(-\frac{e^{-9}}{2}+\frac{1}{2}\right) d\theta = \frac{(1-e^{-9})\theta}{2}\Big|_0^{2\pi} = (1-e^{-9})\pi$$

d) $$\int_0^2\int_{-\sqrt{4-x^2}}^0 \frac{1}{4+\sqrt{x^2+y^2}}\, dy\, dx$$



$r \in [0,2]$

$\theta \in [3\pi/2, 2\pi]$

$$\frac{1}{4+\sqrt{x^2+y^2}} = \frac{1}{4+r}$$

$$\int_{\frac{3\pi}{2}}^{2\pi}\int_0^2 \frac{1}{4+r} \cdot r\, dr\, d\theta$$

$4+r=u$ $\quad r=2,\ u=6$

$dr=du$ $\quad r=0,\ u=4$

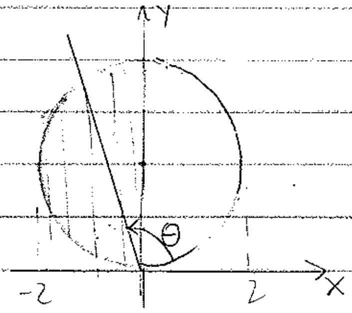
$$\int_{\frac{3}{2}\pi}^{2\pi}\int_4^6 \frac{u-4}{u}\, du\, d\theta$$

$$\int_{\frac{3}{2}\pi}^{2\pi} u - 4\ln u\Big|_4^6 d\theta = \int_{\frac{3}{2}\pi}^{2\pi} 6-4\ln 6-4+4\ln 4\, d\theta$$

$$2-4(\ln 6-\ln 4)\,\theta\Big|_{\frac{3}{2}\pi}^{2\pi} = \left[2-4\ln\left(\frac{3}{2}\right)\right]\left(2\pi-\frac{3\pi}{2}\right)$$

$$\pi - 2\pi\ln\left(\frac{3}{2}\right)$$

e) $\displaystyle\int_{-2}^{0}\int_{2-\sqrt{4-x^2}}^{2+\sqrt{4-x^2}} \frac{xy}{\sqrt{x^2+y^2}}\,dy\,dx$



$y = 2+\sqrt{4-x^2}$ $\qquad r \in [0, 4\,\text{sen}\,\theta]$

$y-2 = \sqrt{4-x^2}$

$(y-2)^2 = 4-x^2$ $\qquad \theta \in [\pi/2, \pi]$

$x^2+(y-2)^2 = 4$

$x^2+y^2-4y+\cancel{4} = \cancel{4}$ $\qquad \dfrac{xy}{\sqrt{x^2+y^2}} = \dfrac{r\cos\theta\,\cancel{r}\,\text{sen}\,\theta}{\cancel{r}}$

$x^2+y^2 = 4y$ $\qquad = r\cos\theta\,\text{sen}\,\theta$

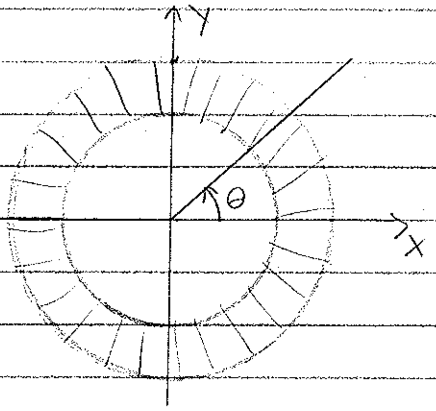
$r^{\cancel{2}} = 4\cancel{r}\,\text{sen}\,\theta$

$r = 4\,\text{sen}\,\theta$

$$\int_{\frac{\pi}{2}}^{\pi}\int_{0}^{4\,\text{sen}\,\theta} r^2\cos\theta\,\text{sen}\,\theta\,dr\,d\theta = \int_{\frac{\pi}{2}}^{\pi} \frac{r^3}{3}\cos\theta\,\text{sen}\,\theta\,\Big|_0^{4\,\text{sen}\,\theta} d\theta$$

$$\int_{\frac{\pi}{2}}^{\pi} \frac{64}{3}\cos\theta\,\text{sen}^4\theta\,d\theta = \frac{64}{15}\,\text{sen}^5\theta\,\Big|_{\frac{\pi}{2}}^{\pi} = -\frac{64}{15}$$

f) $\displaystyle\iint_R \frac{1}{(x^2+y^2)^3}\,dx\,dy$, $R: 4 \le x^2+y^2 \le 9$



$r \in [2,3]$

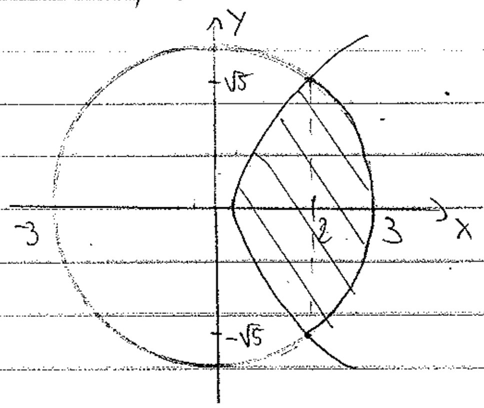
$\theta \in [0, 2\pi]$

$\dfrac{1}{(x^2+y^2)^3} = \dfrac{1}{r^6}$

$$\int_0^{2\pi}\int_2^3 \frac{1}{r^6}\cdot r\,dr\,d\theta = \int_0^{2\pi}\int_2^3 r^{-5}\,dr\,d\theta = \int_0^{2\pi} -\frac{1}{4}r^{-4}\Big|_2^3 d\theta$$

$$\int_0^{2\pi}\left(-\frac{1}{4}\cdot 3^{-4} + \frac{1}{4}\,2^{-4}\right)d\theta = \left(\frac{1}{64} - \frac{1}{324}\right)2\pi = \frac{\pi}{32} - \frac{\pi}{162}$$

⑤ $x^2+y^2=9,\ y^2+1=3x$



$9-x^2=3x-1$

$x^2+3x-10=0$

$-5+2=-3$

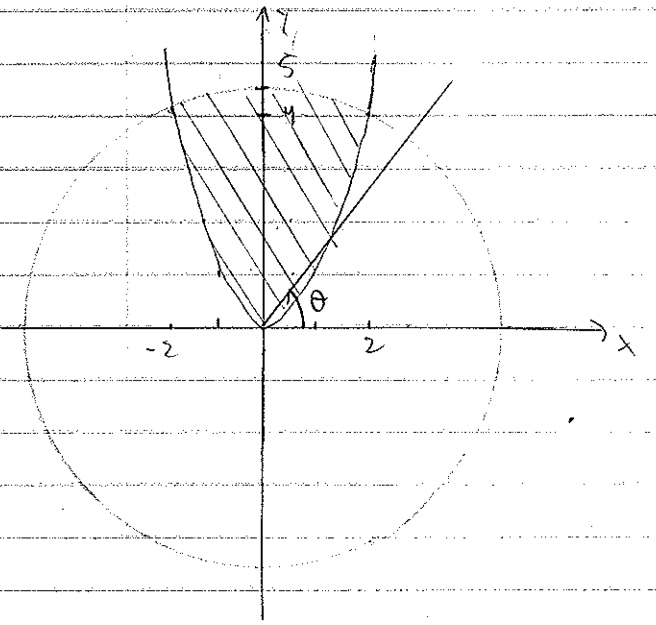
$-5\cdot2=-10$

$y=\pm\sqrt5$

a) $A=2\int_{1/3}^{2}\int_{0}^{\sqrt{3x-1}} dy\,dx + 2\int_{2}^{3}\int_{0}^{\sqrt{9-x^2}} dy\,dx$

b) $A=2\int_{0}^{\sqrt5}\int_{\frac{y^2+1}{3}}^{\sqrt{9-y^2}} dx\,dy$

⑥ $x^2+y^2=20,\ y=x^2$



$y^2+y=20$

$y^2+y-20=0$

$-5+4=-1$

$-5\cdot4=-20$

$y=4$

$x=\pm2$

a) $A=2\int_{0}^{2}\int_{x^2}^{\sqrt{20-x^2}} dy\,dx$ ‖ b) $A=2\int_{0}^{4}\int_{0}^{\sqrt{y}} dx\,dy + 2\int_{4}^{\sqrt{20}}\int_{0}^{\sqrt{20-y^2}} dx\,dy$

P3 - Bena - 2,3,6

c) $x^2+y^2=20 \qquad y=x^2$

$r^2=20$

$r=\sqrt{20}$

$r\,\text{sen}\,\theta = r^2\cos^2\theta$

$r=\dfrac{\text{sen}\,\theta}{\cos^2\theta}$

$\dfrac{\text{sen}\,\theta}{\cos^2\theta}=\sqrt{20}$

$\dfrac{\text{sen}\,\theta}{1-\text{sen}^2\theta}=\sqrt{20}$

$\text{sen}\,\theta=\sqrt{20}-\sqrt{20}\,\text{sen}^2\theta$

$\sqrt{20}\,\text{sen}^2\theta+\text{sen}\,\theta-\sqrt{20}=0$

$\Delta=1^2+4\cdot\sqrt{20}\cdot\sqrt{20}$

$\Delta=81$

$\text{sen}\,\theta=\dfrac{-1\pm 9}{2\sqrt{20}}$

$\text{sen}\,\theta=\dfrac{4}{\sqrt{20}}=\dfrac{2}{\sqrt{5}}$

$\theta=\text{arcsen}\left(\dfrac{2}{\sqrt{5}}\right)$

$$A=2\int_0^{\text{arcsen}\left(\frac{2}{\sqrt5}\right)}\int_0^{\frac{\text{sen}\,\theta}{\cos^2\theta}} r\,dr\,d\theta+2\int_{\text{arcsen}\left(\frac{2}{\sqrt5}\right)}^{\frac{\pi}{2}}\int_0^{\sqrt{20}} r\,dr\,d\theta$$

7) $$I=\int_1^2\int_0^{\sqrt{2x-x^2}}\frac{\sqrt{x^2+y^2}}{x+y}\,dy\,dx$$

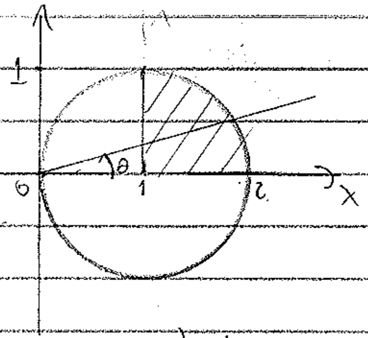
$y=\sqrt{2x-x^2}$

$y^2=2x-x^2$

$x^2-2x+y^2=0$

$(x-1)^2+y^2=1$

$x=1\pm\sqrt{1-y^2}$



$\dfrac{\sqrt{x^2+y^2}}{x+y}=\dfrac{r}{r\cos\theta+r\,\text{sen}\,\theta}=\dfrac{1}{\cos\theta+\text{sen}\,\theta}$

a) $$I=\int_0^1\int_1^{1+\sqrt{1-y^2}}\frac{\sqrt{x^2+y^2}}{x+y}\,dx\,dy$$

$\dfrac{1}{\cos\theta}=2\cos\theta$

$\cos^2\theta=\dfrac{1}{2}$

$\cos\theta=\dfrac{\sqrt2}{2}$

$\theta=\dfrac{\pi}{4}$

$x=1$

$r\cos\theta=1$

$r=\dfrac{1}{\cos\theta}$

$r=\sec\theta$

$x^2+y^2=2x$

$r^2=2r\cos\theta$

$r=2\cos\theta$

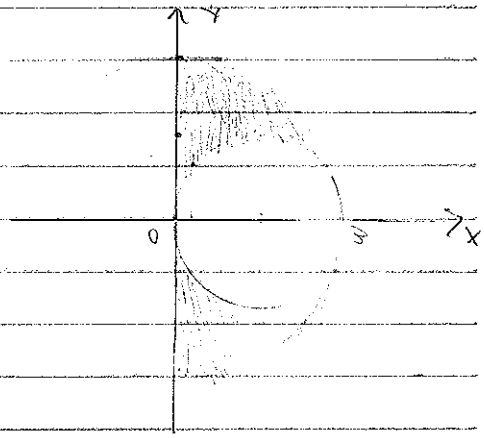
b) $$I=\int_0^{\frac{\pi}{4}}\int_{\sec\theta}^{2\cos\theta}\frac{r}{\cos\theta+\text{sen}\,\theta}\,dr\,d\theta$$

(8) $I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{3\cos\theta}^{3} \operatorname{sen}\theta \, dr\, d\theta$

$r=3$ $\qquad$ $r=3\cos\theta$

$x^2+y^2=9$ $\qquad$ $x^2+y^2=3x$

$\left(x-\frac{3}{2}\right)^2+y^2=\frac{9}{4}$



$\frac{\operatorname{sen}\theta}{r}\cdot\frac{r}{r}=\frac{y}{x^2+y^2}$

$I=\int_0^3\int_{\sqrt{3x-x^2}}^{\sqrt{9-x^2}}\frac{y}{x^2+y^2}\,dy\,dx+\int_0^3\int_{-\sqrt{9-x^2}}^{-\sqrt{3x-x^2}}\frac{y}{x^2+y^2}\,dy\,dx$

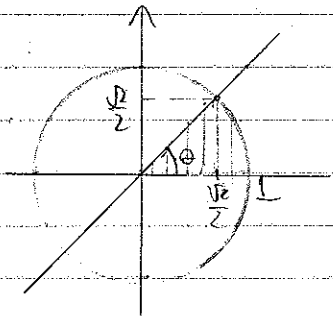
(9) $I=\int_0^{\frac{\sqrt{2}}{2}}\int_y^{\sqrt{1-y^2}}\frac{2x+4y}{\sqrt{x^2+y^2}}\,dx\,dy$

$x=\sqrt{1-y^2}$ $\qquad$ $x=y$

$x^2+y^2=1$ $\qquad$ $r\cos\theta=r\operatorname{sen}\theta$

$r^2=1$ $\qquad$ $tg\,\theta=1$

$r=1$ $\qquad$ $\theta=\frac{\pi}{4}$



$\frac{2r\cos\theta+4r\operatorname{sen}\theta}{r}$

$2\cos\theta+4\operatorname{sen}\theta$

a) $I=\int_0^{\frac{\sqrt{2}}{2}}\int_0^{x}\frac{2x+4y}{\sqrt{x^2+y^2}}\,dy\,dx+\int_{\frac{\sqrt{2}}{2}}^{1}\int_0^{\sqrt{1-x^2}}\frac{2x+4y}{\sqrt{x^2+y^2}}\,dy\,dx$

b) $I=\int_0^{\frac{\pi}{4}}\int_0^1(2\cos\theta+4\operatorname{sen}\theta)\,r\,dr\,d\theta$

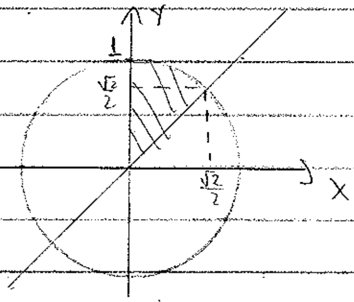
c) $I=\int_0^{\frac{\pi}{4}}2(\cos\theta+2\operatorname{sen}\theta)\frac{r^2}{2}\Big|_0^1\,d\theta=\int_0^{\frac{\pi}{4}}\cos\theta+2\operatorname{sen}\theta\,d\theta$

$$I = \text{sen}\,\theta - 2\cos\theta \Big|_0^{\frac{\pi}{4}} = \frac{\sqrt{2}}{2} - 2\frac{\sqrt{2}}{2} - 0 + 2.1 = \frac{4-\sqrt{2}}{2}$$

(10) $I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^1 r^3 \, dr \, d\theta$

$r = 1$: $\quad r^2 = x^2+y^2$

$x^2+y^2 = 1$

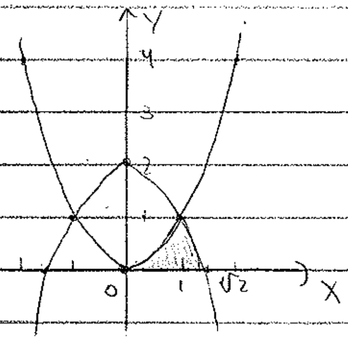


a) $I = \int_0^{\frac{\sqrt{2}}{2}} \int_x^{\sqrt{1-x^2}} x^2+y^2 \, dy\, dx$

b) $I = \int_0^{\frac{\sqrt{2}}{2}} \int_0^{y} x^2+y^2 \, dx\, dy + \int_{\frac{\sqrt{2}}{2}}^{1} \int_0^{\sqrt{1-y^2}} x^2+y^2 \, dx\, dy$

(11) $I = \int_0^1 \int_0^{x^2} x\cos(1-y)^2 \, dy\, dx + \int_1^{\sqrt{2}} \int_0^{2-x^2} x\cos(1-y)^2 \, dy\, dx$

a)



b) $I = \int_0^1 \int_{\sqrt{y}}^{\sqrt{2-y}} x\cos(1-y)^2 \, dx\, dy$

$= \int_0^1 \frac{x^2\cos(1-y)^2}{2} \Big|_{\sqrt{y}}^{\sqrt{2-y}} dy$

$= \int_0^1 \frac{(2-y)\cos(1-y)^2}{2} - \frac{y\cos(1-y)^2}{2} \, dy$

$= \int_0^1 \cos(1-y)^2 - y\cos(1-y)^2 \, dy$

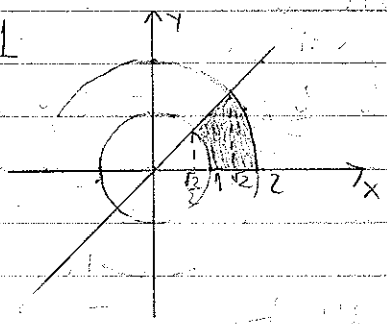
$= \int_0^1 (1-y)\cos(1-y)^2 \, dy \cdot \frac{-2}{-2}$

$= -\frac{\text{sen}(1-y)^2}{2} \Big|_0^1 = \frac{\text{sen}\,1}{2}$

(12) $I = \int_{\frac{\sqrt{2}}{2}}^{1} \int_{\sqrt{1-x^2}}^{x} xy\,dy\,dx + \int_{1}^{\sqrt{2}} \int_{0}^{x} xy\,dy\,dx + \int_{\sqrt{2}}^{2} \int_{0}^{\sqrt{4-x^2}} xy\,dy\,dx$

$$\begin{cases} y = x \Rightarrow tg\,\theta = 1 \\ x^2+y^2=1 \Rightarrow r=1 \\ x^2+y^2=4 \Rightarrow r=2 \end{cases}$$

$xy = r^2 \operatorname{sen}\theta \cos\theta$



$I = \int_{0}^{\frac{\pi}{4}} \int_{1}^{2} r^3 \operatorname{sen}\theta \cos\theta \,dr\,d\theta$

(13) $I = \int_{0}^{1} \int_{0}^{1-\sqrt{1-y^2}} \frac{\sqrt{x^2+y^2}}{x^2+y^2}\,dx\,dy + \int_{1}^{2} \int_{0}^{\sqrt{2y-y^2}} \frac{\sqrt{x^2+y^2}}{x^2+y^2}\,dx\,dy$

$x = 1-\sqrt{1-y^2}$

$-(x-1) = \sqrt{1-y^2}$

$(x-1)^2 = 1-y^2$

$(x-1)^2+y^2=1$

$x^2-2x+\cancel{1}+y^2=\cancel{1}$

$x^2+y^2=2x$

$r^2 = 2r\cos\theta$

$r = 2\cos\theta$

$x = \sqrt{2y-y^2}$

$x^2 = 2y-y^2$

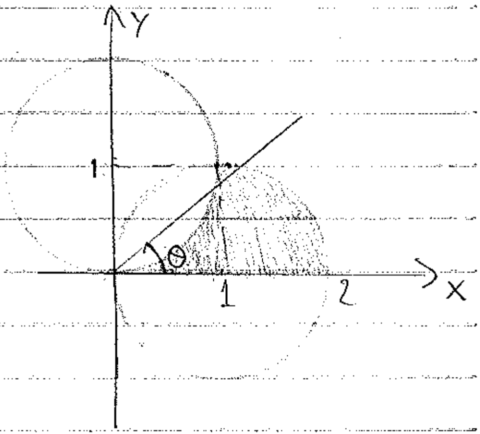
$x^2+(y-1)^2=1$

$x^2+y^2-2y+\cancel{1}=\cancel{1}$

$x^2+y^2=2y$

$r^2 = 2r\operatorname{sen}\theta$

$r = 2\operatorname{sen}\theta$



$\frac{\sqrt{x^2+y^2}}{x^2+y^2} = \frac{\cancel{r}}{r^{\cancel{2}}} = \frac{1}{r}$

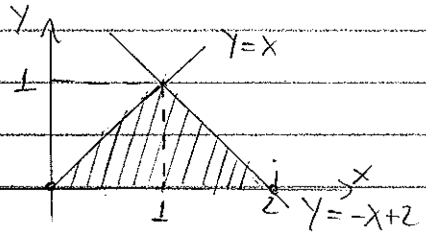
a) $I = \int_{0}^{1} \int_{-\sqrt{2x-x^2}}^{1+\sqrt{1-x^2}} \frac{\sqrt{x^2+y^2}}{x^2+y^2}\,dy\,dx$

b) $I = \int_{0}^{\frac{\pi}{4}} \int_{2\operatorname{sen}\theta}^{2\cos\theta} dr\,d\theta$

c) $I = \int_0^{\frac{\pi}{4}} r \Big|_{2\,sen\theta}^{2\cos\theta} d\theta = \int_0^{\frac{\pi}{4}} (2\cos\theta - 2\,sen\theta)\, d\theta$

$= 2\,sen\theta + 2\cos\theta \Big|_0^{\frac{\pi}{4}} = \cancel{2}\cdot\frac{\sqrt{2}}{\cancel{2}} + \cancel{2}\frac{\sqrt{2}}{\cancel{2}} - 2 = 2\sqrt{2} - 2$

(14) $\iint_D (x+3y)\,dA$, D: (0,0), (1,1), (2,0)



$I = \int_0^1 \int_y^{2-y} (x+3y)\,dx\,dy = \int_0^1 \frac{x^2}{2} + 3xy \Big|_y^{2-y} dy$

$= \int_0^1 \frac{(2-y)^2}{2} + 3(2-y)y - \frac{y^2}{2} - 3yy \; dy$

$= \int_0^1 2 - 2y + \frac{\cancel{y^2}}{\cancel{2}} + 6y - 3y^2 - \frac{\cancel{y^2}}{\cancel{2}} - 3y^2 \; dy$

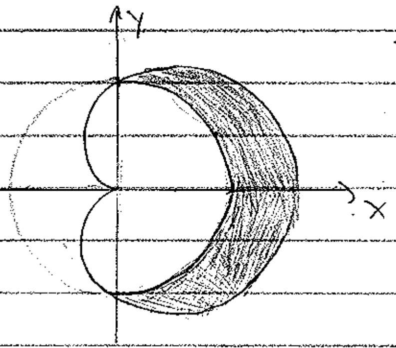
$= \int_0^1 2 + 4y - 6y^2 \; dy = 2y + 2y^2 - 2y^3 \Big|_0^1 = 2$

(15) $\iint_D \frac{1}{\sqrt{x^2+y^2}}\, dA$, D: $x \geq 0$, $r = 1+\cos\theta$, $r = 1$

$1+\cos\theta = 1$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$



$I = 2\int_0^{\frac{\pi}{2}} \int_1^{1+\cos\theta} dr\,d\theta$

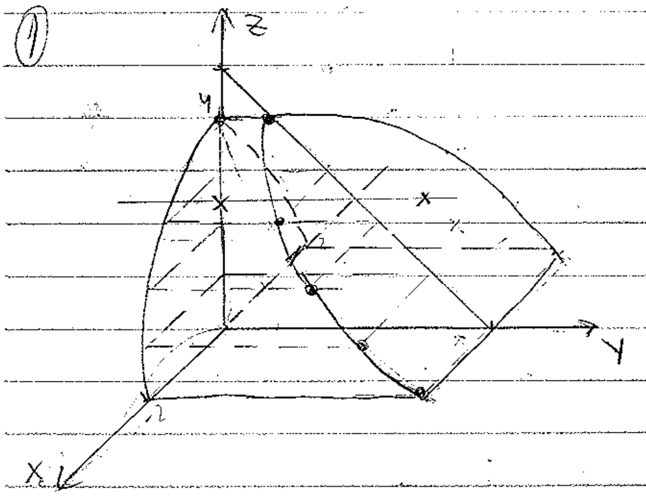
$I = 2\int_0^{\frac{\pi}{2}} r \Big|_1^{1+\cos\theta} d\theta$

$I = 2\int_0^{\frac{\pi}{2}} \cancel{1} + \cos\theta - \cancel{1}\; d\theta$

$I = 2\,sen\theta \Big|_0^{\frac{\pi}{2}} = 2$

Lista 4

1



$y=0,\ z=0,\ y+z=5,\ z=4-x^2$

$$I=\int_{-2}^{2}\int_{0}^{4-x^2}\int_{0}^{5-z}(x-1)\,dy\,dz\,dx$$

$$=\int_{-2}^{2}\int_{0}^{4-x^2}(x-1)y\Big|_{0}^{5-z}dz\,dx=\int_{-2}^{2}\int_{0}^{4-x^2}(x-1)(5-z)\,dz\,dx$$

$$=\int_{-2}^{2}(x-1)5z-(x-1)\cdot\frac{z^2}{2}\Big|_{0}^{4-x^2}dx$$

$$=\int_{-2}^{2}(x-1)5(4-x^2)-(x-1)\frac{(4-x^2)^2}{2}\,dx$$

$$=\int_{-2}^{2}20x-20-5x^3+5x^2-\frac{(x-1)}{2}(16-8x^2+x^4)\,dx$$

$$=\int_{-2}^{2}20x-20-5x^3+5x^2-8x+8+4x^3-4x^2-\frac{x^5}{2}+\frac{x^4}{2}\,dx$$

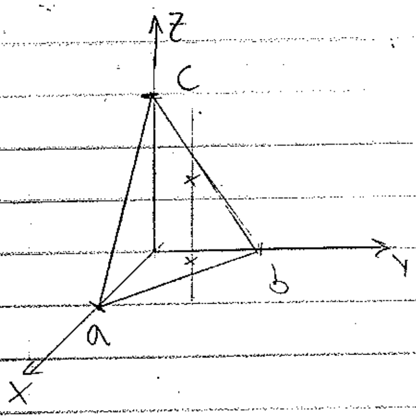
$$=\int_{-2}^{2}-\frac{x^5}{2}+\frac{x^4}{2}-x^3+x^2+12x-12\,dx$$

$$=\frac{x^6}{12}+\frac{x^5}{10}-\frac{x^4}{4}+\frac{x^3}{3}+6x^2-12x\Big|_{-2}^{2}$$

$$=\cancel{\frac{64}{12}}+\frac{32}{10}-\cancel{\frac{16}{4}}+\frac{8}{3}+\cancel{24}-24-\cancel{\frac{64}{12}}+\frac{32}{10}+\cancel{\frac{16}{4}}+\frac{8}{3}-\cancel{24}-24$$

$$=\frac{\overset{32}{64}}{\underset{5}{10}}+\frac{16}{3}-48=\frac{96+80-720}{15}=-\frac{544}{15}$$

② $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1,\ x=0,\ y=0,\ z=0$



$$\frac{y}{b}=1-\frac{x}{a}$$

$$V=\int_0^a\int_0^{-\frac{b}{a}x+b}\int_0^{c-\frac{c}{a}x-\frac{c}{b}y} dz\,dy\,dx=\int_0^a\int_0^{-\frac{b}{a}x+b} z\Big|_0^{c-\frac{c}{a}x-\frac{c}{b}y} dy\,dx$$

$$=\int_0^a\int_0^{-\frac{b}{a}x+b} c-\frac{c}{a}x-\frac{c}{b}y\,dy\,dx=\int_0^a cy-\frac{cxy}{a}-\frac{cy^2}{2b}\Big|_0^{-\frac{b}{a}x+b}dx$$

$$=\int_0^a c\left(\frac{-b}{a}x+b\right)-\frac{cx}{a}\left(\frac{-b}{a}x+b\right)-\frac{c}{2b}\left(\frac{-b}{a}x+b\right)^2dx$$

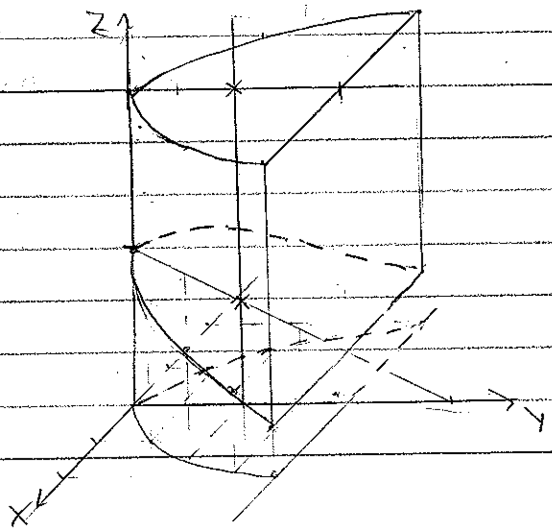
$$=\int_0^a -\frac{bc}{a}x+bc+\frac{bc}{a^2}x^2-\frac{bc}{a}x-\frac{c}{2b}\left(\frac{b^2x^2}{a^2}-\frac{2b^2}{a}x+b^2\right)dx$$

$$=\int_0^a \frac{2bc}{2a^2}x^2-\frac{2bc}{a}x+bc-\frac{bc}{2a^2}x^2+\frac{bc}{a}x-\frac{bc}{2}\,dx$$

$$=\int_0^a \frac{bc}{2a^2}x^2-\frac{bc}{a}x+\frac{bc}{2}\,dx$$

$$=\frac{bc}{6a^2}x^3-\frac{bc}{2a}x^2+\frac{bc}{2}x\Big|_0^a=\frac{abc}{6}-\cancel{\frac{abc}{2}}+\cancel{\frac{abc}{2}}=\frac{abc}{6}$$

③ $y=4,\ y+2z=6,\ z=6,\ y=x^2$



$$M=\int_{-2}^{2}\int_{x^2}^{4}\int_{\frac{6-y}{2}}^{6} 2y+z \, dz\,dy\,dx =$$

$$=\int_{-2}^{2}\int_{x^2}^{4} 2yz+\frac{z^2}{2}\Big|_{\frac{6-y}{2}}^{6} dy\,dx$$

$$=\int_{-2}^{2}\int_{x^2}^{4} 12y+18-y(6-y)-\frac{(6-y)^2}{8}\,dy\,dx$$

$$=\int_{-2}^{2}\int_{x^2}^{4} 12y+18-6y+y^2-\frac{9}{2}+\frac{3}{2}y-\frac{y^2}{8}\,dy\,dx$$

$$=\int_{-2}^{2}\int_{x^2}^{4} \frac{7}{8}y^2+\frac{15}{2}y+\frac{27}{2}\,dy\,dx$$

$$=\int_{-2}^{2} \frac{7}{24}y^3+\frac{15}{4}y^2+\frac{27}{2}y\Big|_{x^2}^{4} dx$$

$$=\int_{-2}^{2} \frac{7}{24}.64+\frac{15}{4}.16+\frac{27}{2}.4-\frac{7}{24}x^6-\frac{15}{4}x^4-\frac{27}{2}x^2\,dx$$

$$=\int_{-2}^{2} \frac{56}{3}+60+54-\frac{7}{24}x^6-\frac{15}{4}x^4-\frac{27}{2}x^2\,dx$$

$$=-\frac{\cancel{7}}{24}.\frac{x^7}{\cancel{7}}-\frac{\cancel{15}^{3}}{4}.\frac{x^5}{\cancel{5}}-\frac{\cancel{27}^{9}}{2}.\frac{x^3}{\cancel{3}}+\frac{398}{3}.x\Big|_{-2}^{2}$$

$$=-\frac{128}{24}-\frac{96}{4}-\frac{72}{2}+\frac{796}{3}-\frac{128}{24}-\frac{96}{4}-\frac{72}{2}+\frac{796}{3}$$

$$=-\frac{128}{12}-48-72+\frac{1592}{3}=\frac{-128-576-864+6368}{12}=400\ u.m.$$

④ $V=\int_0^2\int_0^{\frac{2-z}{2}}\int_0^{4-x^2} dy\,dx\,dz$

$$V=\int_0^1\int_0^{8x-4x^2}\int_0^{2-2x} dz\,dy\,dx+\int_0^1\int_{8x-4x^2}^{4}\int_0^{\sqrt{4-y}} dz\,dy\,dx$$

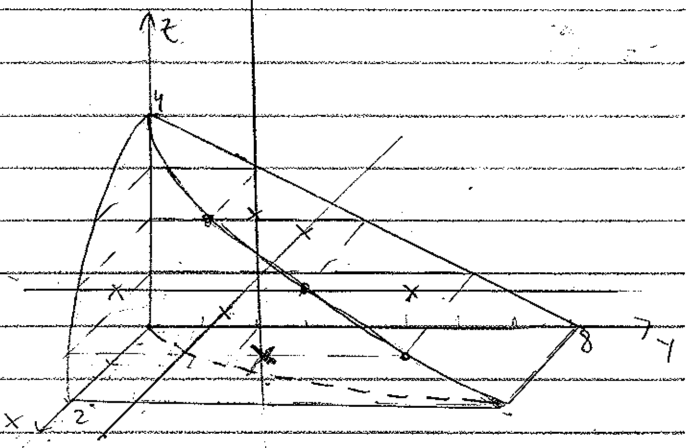
$$V=\int_0^4\int_0^{\frac{2-\sqrt{4-y}}{2}}\int_0^{2-2x} dz\,dx\,dy+\int_0^4\int_{\frac{2-\sqrt{4-y}}{2}}^{1}\int_0^{2-2x} dz\,dx\,dy$$

$$V=\int_0^2\int_0^{4-z^2}\int_0^{\frac{2-z}{2}} dx\,dy\,dz$$

$$\begin{cases} z=2-2x \\ y=4-z^2 \\ x=0 \\ y=0 \\ z=0 \end{cases}$$

$$V=\int_0^4\int_0^{\sqrt{4-y}}\int_0^{\frac{2-z}{2}} dx\,dz\,dy$$

(5) $\begin{cases} y=8-2z \\ x=\sqrt{4-z} \\ z=4 \\ x=0 \\ y=0 \\ z=0 \end{cases}$ $\Rightarrow x=\sqrt{\frac{y}{2}}$



$$V=\int_0^2\int_0^{4-x^2}\int_0^{8-2z} dy\,dz\,dx$$

$z=4-x^2$

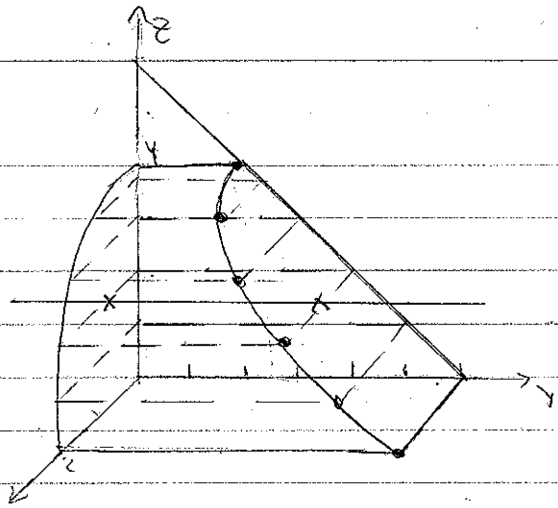
$$V=\int_0^8\int_0^{4-\frac{y}{2}}\int_0^{\sqrt{4-z}} dx\,dz\,dy$$

$$V=\int_0^4\int_0^{8-2z}\int_0^{\sqrt{4-z}} dx\,dy\,dz$$

$$V=\int_0^2\int_0^{2x^2}\int_0^{4-x^2} dz\,dy\,dx+\int_0^2\int_{2x^2}^{8}\int_0^{4-\frac{y}{2}} dz\,dy\,dx$$

$$V=\int_0^8\int_0^{\sqrt{\frac{y}{2}}}\int_0^{4-\frac{y}{2}} dz\,dx\,dy+\int_0^8\int_{\sqrt{\frac{y}{2}}}^{2}\int_0^{4-x^2} dz\,dx\,dy$$

⑥ $\begin{cases} z = 4 - x^2 \\ z = 6 - y \\ y = 2 + x^2 \\ y = 6 \\ y = 0 \\ x = 0 \\ z = 0 \end{cases}$

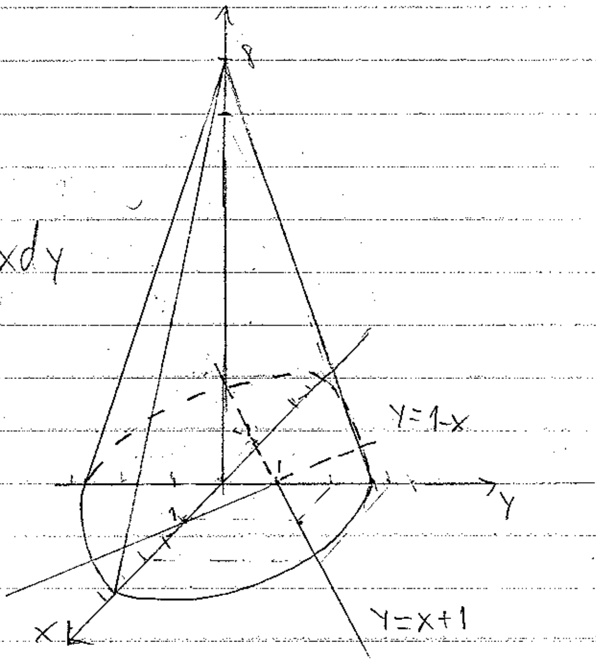


$$V = \int_0^2 \int_0^{4-x^2} \int_0^{6-z} dy\, dz\, dx$$

⑦ $z = 8 - x^2 - y^2$

$y = x + 1$

$y = 1 - x$



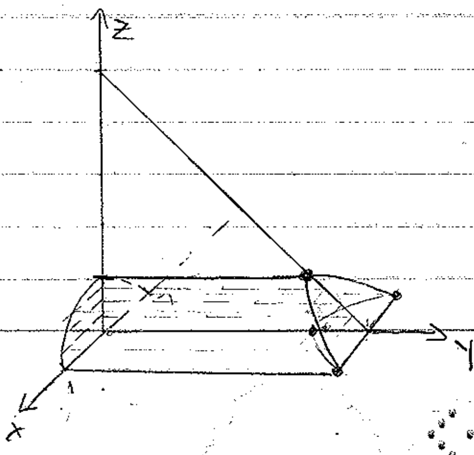
$$I = \int_0^1 \int_{y-1}^{1-y} \int_0^{8-x^2-y^2} y\, dz\, dx\, dy$$

⑧ $z = 1 - x^2$

$y = x^2 + 4$

$y = 5$

$z = 5 - y$



$$I=\int_{-1}^{1}\int_{0}^{1-x^2}\int_{0}^{3-z} dy\,dz\,dx$$

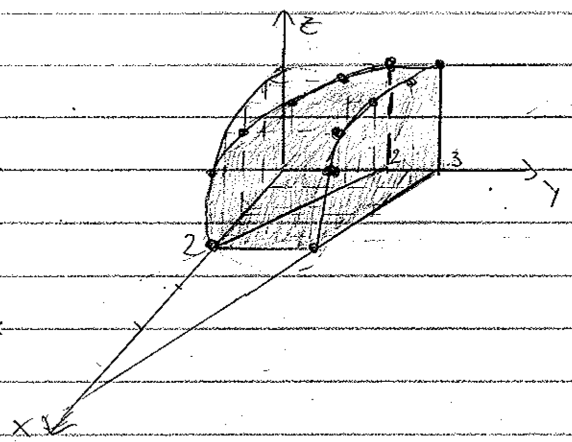
$$I=\int_{0}^{1}\int_{0}^{3-z}\int_{-\sqrt{1-z}}^{\sqrt{1-z}} dx\,dy\,dz$$

$$I=\int_{0}^{1}\int_{-\sqrt{1-z}}^{\sqrt{1-z}}\int_{0}^{5-z} dy\,dx\,dz$$

(9) $x^2+z^2=4$

$x+y=2$

$x+2y=6$



$$M=\int_{0}^{2}\int_{2-x}^{3-\frac{x}{2}}\int_{0}^{\sqrt{4-x^2}} 12z\,dz\,dy\,dx$$

$$=\int_{0}^{2}\int_{2-x}^{3-\frac{x}{2}} 12\frac{z^2}{2}\Big|_{0}^{\sqrt{4-x^2}} dy\,dx$$

$$=\int_{0}^{2}\int_{2-x}^{3-\frac{x}{2}} 6(4-x^2)\,dy\,dx$$

$$=\int_{0}^{2} 24y-6x^2y\Big|_{2-x}^{3-\frac{x}{2}} dx$$

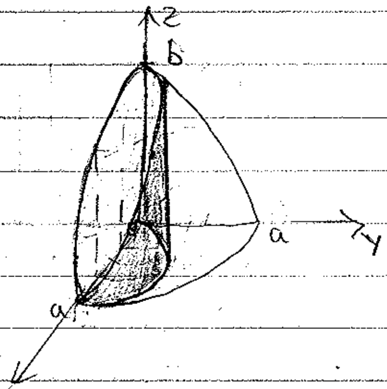
$$=\int_{0}^{2} 72-12x-18x^2+3x^3-48+24x+12x^2-6x^3\,dx$$

$$=\int_{0}^{2} -3x^3-6x^2+12x+24\,dx$$

$$=-\frac{3}{4}x^4-2x^3+6x^2+24x\Big|_{0}^{2}=44\text{ u.m}$$

⑩ $b^2(x^2+y^2)+a^2z^2=a^2b^2$ $\qquad x^2+y^2=ax$

$$\frac{x^2}{a^2}+\frac{y^2}{a^2}+\frac{z^2}{b^2}=1 \qquad \left(x-\frac{a}{2}\right)^2+y^2=\frac{a^2}{4}$$



$$b^2r^2+a^2z^2=a^2b^2$$

$$r^2=ar\cos\theta \Rightarrow r=a\cos\theta$$

$$V=4\int_0^{\frac{\pi}{2}}\int_0^{a\cos\theta}\int_0^{\sqrt{b^2-\frac{b^2}{a^2}r^2}} r\,dz\,dr\,d\theta \qquad u=b^2-\frac{b^2}{a^2}r^2$$

$$V=4\int_0^{\frac{\pi}{2}}\int_0^{a\cos\theta} rz\Big|_0^{\sqrt{b^2-\frac{b^2}{a^2}r^2}} dr\,d\theta \qquad du=-\frac{2b^2}{a^2}r\,dr$$

$$V=4\int_0^{\frac{\pi}{2}}\int_0^{a\cos\theta} r\sqrt{b^2-\frac{b^2}{a^2}r^2}\,dr\,d\theta \qquad r\,dr=\frac{a^2\,du}{-2b^2}$$

$$V=4\int_0^{\frac{\pi}{2}} -\frac{a^2}{3b^2}\left(b^2-\frac{b^2}{a^2}r^2\right)^{3/2}\Big|_0^{a\cos\theta} d\theta \qquad v=\cos\theta$$

$$V=4\int_0^{\frac{\pi}{2}} -\frac{a^2}{3b^2}\left(b^2-b^2\cos^2\theta\right)^{3/2}+\frac{a^2b}{3}\,d\theta \qquad dv=-\operatorname{sen}\theta\,d\theta$$

$$V=4\int_0^{\frac{\pi}{2}} -\frac{a^2}{3\cancel{b^2}}b^3\operatorname{sen}^3\theta+\frac{a^2}{3}\,d\theta \qquad \int\operatorname{sen}^3\theta\,d\theta$$

$$\int\operatorname{sen}\theta(1-\cos^2\theta)\,d\theta$$

$$V=4\left[-\frac{a^2b}{3}\left(-\cos\theta+\frac{\cos^3\theta}{3}\right)+\frac{a^2b\theta}{3}\right]\Big|_0^{\frac{\pi}{2}} d\theta \qquad \int-(1-v^2)\,dv$$

$$-v+\frac{v^3}{3}$$

$$V=\frac{4}{3}a^2b\left(\cos\theta-\frac{\cos^3\theta}{3}\right)\Big|_0^{\frac{\pi}{2}}+\frac{4a^2b\theta}{3}\Big|_0^{\frac{\pi}{2}} \qquad -\cos\theta+\frac{\cos^3\theta}{3}$$

$$V=-\frac{4}{3}a^2b\left(1-\frac{1}{3}\right)+\frac{2a^2b\pi}{3}$$

$$V=\frac{2a^2b(3\pi-4)}{9}\ \text{u.v}$$