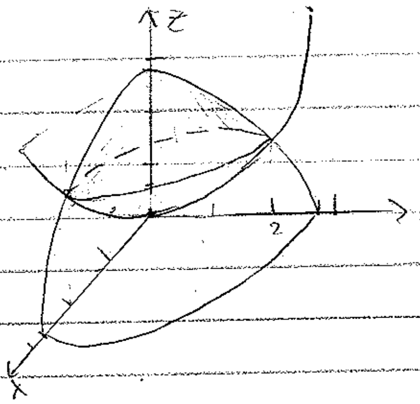
(11)



$$\begin{cases} x^2+y^2+z^2=8 \Rightarrow z=\sqrt{8-r^2} \\ x^2+y^2=2z \Rightarrow z=\dfrac{r^2}{2} \end{cases}$$

$z^2+2z-8=0$

$-4+2=-2$

$-4\cdot 2=-8$

$x^2+y^2=4$

$r^2=4$

$r=2$

$$V=\int_0^{2\pi}\int_0^2\int_{\frac{r^2}{2}}^{\sqrt{8-r^2}} r\,dz\,dr\,d\theta$$

$$V=\int_0^{2\pi}\int_0^2 rz\Big|_{\frac{r^2}{2}}^{\sqrt{8-r^2}} dr\,d\theta$$

$8-r^2=u$

$-2r\,dr=du$

$$V=\int_0^{2\pi}\int_0^2 \frac{-2r\sqrt{8-r^2}}{-2}-\frac{r^3}{2}\,dr\,d\theta$$

$\displaystyle\int\frac{\sqrt{u}\,du}{-2}$

$\displaystyle -\frac{1}{3}u^{3/2}$

$$V=\int_0^{2\pi} -\frac{1}{3}(8-r^2)^{3/2}-\frac{r^4}{8}\Big|_0^2\,d\theta$$

$$V=\int_0^{2\pi} -\frac{4^{3/2}}{3}-\frac{16}{8}+\frac{8^{3/2}}{3}\,d\theta$$

$$V=\int_0^{2\pi} -\frac{8}{3}-2+\frac{16\sqrt{2}}{3}\,d\theta$$

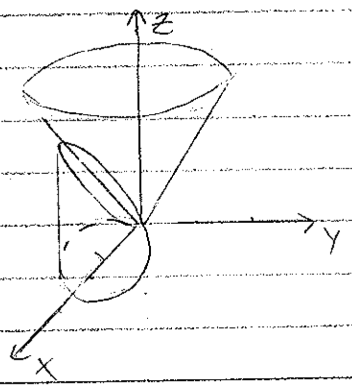
$$V=\frac{(-14+16\sqrt{2})\cdot 2\pi}{3}$$

$$V=\frac{4\pi(8\sqrt{2}-7)}{3}\ \text{u.v}$$

(12) $z=0$

$z^2=x^2+y^2 \Rightarrow z=r$

$x^2+y^2=2ax \Rightarrow r=2a\cos\theta$



$$V = 2\int_0^{\frac{\pi}{2}}\int_0^{2a\cos\theta}\int_0^{r} r\,dz\,dr\,d\theta$$

$$V = 2\int_0^{\frac{\pi}{2}}\int_0^{2a\cos\theta} rz\Big|_0^{r}\,dr\,d\theta$$

$$V = 2\int_0^{\frac{\pi}{2}}\int_0^{2a\cos\theta} r^2\,dr\,d\theta$$

$$V = 2\int_0^{\frac{\pi}{2}} \frac{r^3}{3}\Big|_0^{2a\cos\theta}\,d\theta$$

$$V = 2\int_0^{\frac{\pi}{2}} \frac{8a^3\cos^3\theta}{3}\,d\theta$$

$$V = 2\cdot\left[\frac{8a^3}{3}\left(\operatorname{sen}\theta - \frac{\operatorname{sen}^3\theta}{3}\right)\Big|_0^{\frac{\pi}{2}}\right]$$

$$V = 2\cdot\frac{8a^3}{3}\left(1 - \frac{1}{3}\right)$$

$$V = \frac{32a^3}{9}\ u.v$$

$\int \cos^3\theta\,d\theta$

$\int \cos\theta(1-\operatorname{sen}^2\theta)\,d\theta$

$u = \operatorname{sen}\theta$

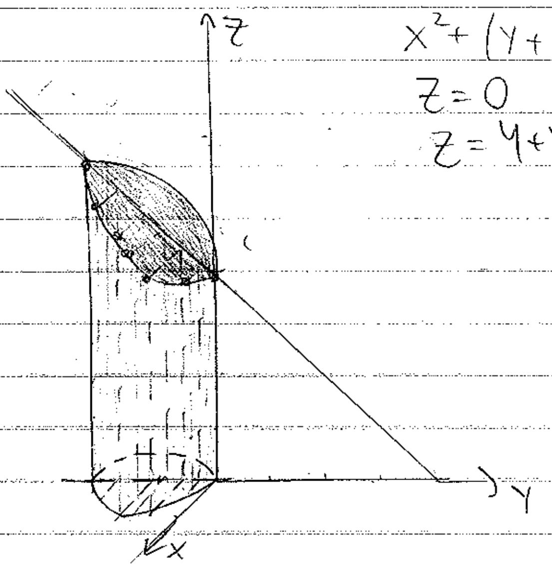
$du = \cos\theta\,d\theta$

$\int 1-u^2\,du$

$u - \frac{u^3}{3}$

$\operatorname{sen}\theta - \frac{\operatorname{sen}^3\theta}{3}$

(13)



$x^2+(y+1)^2 = 1 \Rightarrow x^2+y^2+2y = 0 \Rightarrow r = -2\operatorname{sen}\theta$

$z = 0$

$z = 4+y = 4+r\operatorname{sen}\theta$

$$V = \int_{\pi}^{2\pi}\int_0^{-2\operatorname{sen}\theta}\int_0^{4+r\operatorname{sen}\theta} r\,dz\,dr\,d\theta$$

$$V = \int_{\pi}^{2\pi}\int_0^{-2\operatorname{sen}\theta} rz\Big|_0^{4+r\operatorname{sen}\theta}\,dr\,d\theta$$

$$V = \int_{\pi}^{2\pi} \int_{0}^{-2\,\text{sen}\,\theta} 4r + r^2\,\text{sen}\,\theta \, dr\, d\theta$$

$$V = \int_{\pi}^{2\pi} 2r^2 + \frac{r^3}{3}\,\text{sen}\,\theta \Big|_{0}^{-2\,\text{sen}\,\theta} d\theta$$

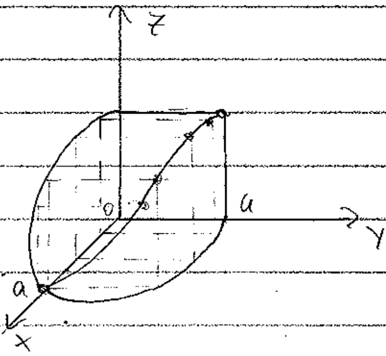
$$V = \int_{\pi}^{2\pi} 8\,\text{sen}^2\theta - \frac{8}{3}\,\text{sen}^4\theta \, d\theta$$

$$V = 4\theta - 4\,\text{sen}\,\theta\cos\theta - \theta + \frac{2}{3}\,\text{sen}(2\theta) - \frac{1}{12}\,\text{sen}(4\theta) \Big|_{\pi}^{2\pi}$$

$$V = 6\pi - 3\pi$$

$$V = 3\pi \ u.v$$

(14) $\begin{cases} x^2+y^2=a^2 \\ x^2+z^2=a^2 \end{cases}$



$$V = 8\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2}} dz\,dy\,dx$$

$$V = 8\int_0^a \int_0^{\sqrt{a^2-x^2}} z \Big|_0^{\sqrt{a^2-x^2}} dy\,dx$$

$$V = 8\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2}\, dy\,dx$$

$$V = 8\int_0^a y\sqrt{a^2-x^2} \Big|_0^{\sqrt{a^2-x^2}} dx$$

$$V = 8\int_0^a \sqrt{a^2-x^2}\cdot\sqrt{a^2-x^2}\, dx$$

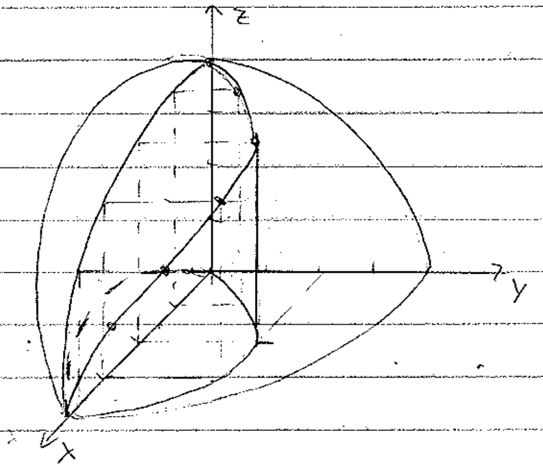
$$V = 8\int_0^a a^2 - x^2 \, dx$$

$$V = 8\left(a^2x - \frac{x^3}{3}\right)\Big|_0^a = 8\left(a^3 - \frac{a^3}{3}\right) = \frac{16}{3}a^3 \ u.v$$

(15) $r = 4\cos\theta \Rightarrow x^2+y^2 = 4x \Rightarrow (x-2)^2+y^2 = 4$

$z = 0$

$r^2 = 16 - z^2 \Rightarrow x^2+y^2+z^2 = 16.$



$$V = 2\int_0^{\frac{\pi}{2}}\int_0^{4\cos\theta}\int_0^{\sqrt{16-r^2}} r\,dz\,dr\,d\theta$$

$$V = 2\int_0^{\frac{\pi}{2}}\int_0^{4\cos\theta} rz\Big|_0^{\sqrt{16-r^2}} dr\,d\theta$$

$$V = 2\int_0^{\frac{\pi}{2}}\int_0^{4\cos\theta} \frac{-2r}{-2}\sqrt{16-r^2}\,dr\,d\theta$$

$$V = 2\int_0^{\frac{\pi}{2}} -\frac{1}{3}(16-r^2)^{3/2}\Big|_0^{4\cos\theta} d\theta$$

$$V = 2\int_0^{\frac{\pi}{2}} -\frac{4^3}{3}\operatorname{sen}^3\theta + \frac{4^3}{3}\,d\theta$$

$$V = \frac{128}{3}\int_0^{\frac{\pi}{2}} 1-\operatorname{sen}^3\theta\,d\theta$$

$$V = \frac{128}{3}\left(\theta + \cos\theta - \frac{\cos^3\theta}{3}\right)\Big|_0^{\frac{\pi}{2}}$$

$$V = \frac{64\pi}{3} - \frac{128}{3} + \frac{128}{9}$$

$$V = \frac{192\pi-256}{9}$$

$$V = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{4\cos\theta}\int_0^{\sqrt{16-r^2}} r\,dz\,dr\,d\theta$$

$\vdots$

$$V = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -\frac{4^3}{3}(\operatorname{sen}^2\theta)^{3/2} + \frac{4^3}{3}\,d\theta$$

$$V = \frac{4^3}{3}\theta\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{4^3}{3}\left[\int_{-\frac{\pi}{2}}^{0}\operatorname{sen}^3\theta\,d\theta - \int_0^{\frac{\pi}{2}}\operatorname{sen}^3\theta\,d\theta\right]$$

$$V = \frac{2^6}{3}\frac{\pi}{2} + \frac{2^6}{3}\frac{\pi}{2} + \frac{4^3}{3}\left[\left(\frac{\cos^3\theta-\cos\theta}{3}\right)\Big|_{-\frac{\pi}{2}}^{0} - \left(\frac{\cos^3\theta-\cos\theta}{3}\right)\Big|_0^{\frac{\pi}{2}}\right]$$

$$V = \frac{64\pi}{3} + \frac{64}{3}\left[\left(\frac{1}{3}-1\right) - \left(-\frac{1}{3}+1\right)\right]$$

$$V = \frac{64\pi}{3} + \frac{64}{3}\left(\frac{2}{3}-2\right)$$

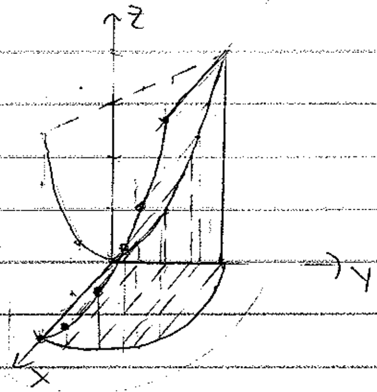
$$V = \frac{64\pi}{3} - \frac{256}{9}$$

$$V = \frac{192\pi-256}{9}$$

(16) $z=0$

$x^2+y^2=a^2 \Rightarrow r=a$

$z=x^2+y^2 \Rightarrow z=r^2$



$$M=\int_0^{2\pi}\int_0^a\int_0^{r^2}\frac{1}{\sqrt{1+r^4}}\, r\, dz\, dr\, d\theta$$

$u=1+r^4$

$du=4r^3 dr$

$$M=\int_0^{2\pi}\int_0^a \frac{r}{\sqrt{1+r^4}}\, z\Big|_0^{r^2} dr\, d\theta$$

$$\int \frac{u^{-1/2}}{4} du$$

$$M=\int_0^{2\pi}\int_0^a \frac{4r^3}{4\sqrt{1+r^4}}\, dr\, d\theta$$

$$\frac{2u^{1/2}}{4}$$

$$M=\int_0^{2\pi}\frac{\sqrt{1+r^4}}{2}\Big|_0^a d\theta$$

$$M=\int_0^{2\pi}\frac{\sqrt{1+a^4}}{2}-\frac{1}{2}\, d\theta$$

$$M=(\sqrt{1+a^4}-1)\pi$$

$$\cancel{\pi}(\sqrt{82}-1)=(\sqrt{1+a^4}-1)\cancel{\pi}$$

$$\sqrt{82}-\cancel{1}=\sqrt{1+a^4}-\cancel{1}$$

$$82=1+a^4$$

$$a^4=81$$

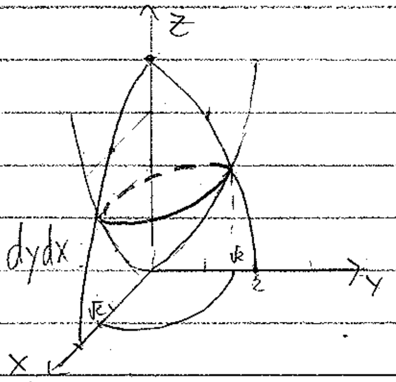
$$\therefore a=3$$

(17) $r=\sqrt{2} \Rightarrow x^2+y^2=2$

$z=r^2 \Rightarrow z=x^2+y^2$

$z=4-r^2 \Rightarrow x^2+y^2+z=4$

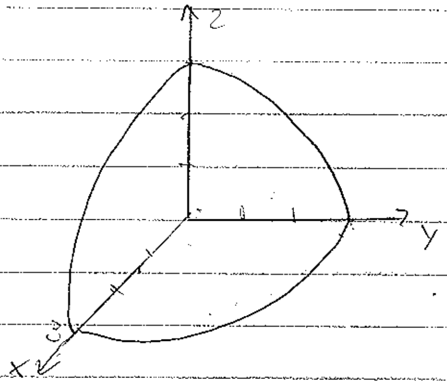
$$M=\int_{-\sqrt{2}}^{\sqrt{2}}\int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}}\int_{x^2+y^2}^{4-x^2-y^2}\frac{\sqrt{x^2+y^2-z+4}}{\sqrt{x^2+y^2}}\, dz\, dy\, dx$$



18 a) $I = 2\int_0^{\pi}\int_0^{\frac{\pi}{2}}\int_0^{3} \sqrt{9-\rho^2}\ \text{sen}\,\phi\ d\rho\, d\phi\, d\theta$

$\rho = 3 \Rightarrow x^2+y^2+z^2 = 9$

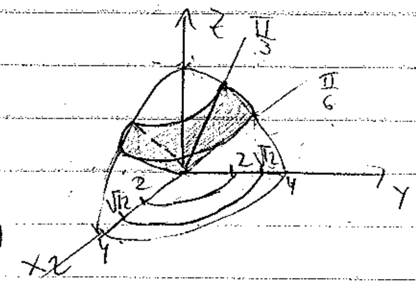
$\dfrac{\sqrt{9-x^2-y^2-z^2}}{x^2+y^2+z^2}\ \rho^2\ \text{sen}\,\phi$



$$I = \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{\sqrt{9-x^2-y^2}} \frac{\sqrt{9-x^2-y^2-z^2}}{x^2+y^2+z^2}\ dz\,dy\,dx$$

b) $I = \int_0^{\frac{\pi}{2}}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\int_0^{4} \sqrt{4-\rho^2}\ \rho\,\text{sen}\,\phi\ d\rho\, d\phi\, d\theta$

$\rho = 4 \Rightarrow x^2+y^2+z^2 = 16$



$\text{tg}\,\phi = \dfrac{\sqrt{x^2+y^2}}{z}$

$\text{tg}\,\dfrac{\pi}{3} = \dfrac{\sqrt{x^2+y^2}}{z}$ $\qquad$ $\text{tg}\,\dfrac{\pi}{6} = \dfrac{\sqrt{x^2+y^2}}{z}$

$x^2+y^2 = 3z^2$ $\qquad$ $x^2+y^2 = \dfrac{z^2}{3}$

$x^2+y^2 + \dfrac{x^2}{3} + \dfrac{y^2}{3} = 16$ $\qquad$ $x^2+y^2+3x^2+3y^2 = 16$

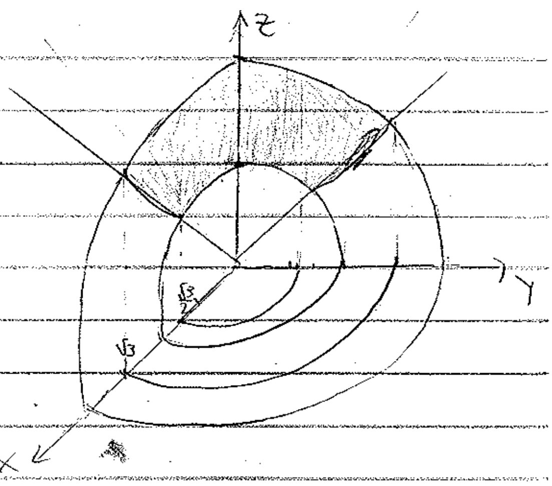
$\qquad\qquad\qquad\qquad\qquad\qquad x^2+y^2 = 4$

$x^2+y^2 = 12$

$$I = \int_0^{2}\int_0^{\sqrt{4-x^2}}\int_{\sqrt{3x^2+3y^2}}^{\sqrt{\frac{x^2+y^2}{3}}} \frac{\sqrt{4-x^2-y^2-z^2}}{\sqrt{x^2+y^2+z^2}}\ dz\,dy\,dx + \int_2^{\sqrt{12}}\int_{\sqrt{4-x^2}}^{\sqrt{12-x^2}}\int_{\sqrt{3x^2+y^2}}^{\sqrt{16-x^2-y^2}} \frac{\sqrt{4-x^2-y^2-z^2}}{\sqrt{x^2+y^2+z^2}}\ dz\,dy\,dx$$

(19) $\rho = 2 \Rightarrow x^2+y^2+z^2=4 \Rightarrow \boxed{z^2 = 4-r^2}$

$\rho = 1 \Rightarrow x^2+y^2+z^2=1 \Rightarrow \boxed{z^2 = 1-r^2}$



$tg\,\phi = \frac{r}{z} \Rightarrow \sqrt3 = \frac{r}{z} \Rightarrow z = \frac{r}{\sqrt3}$

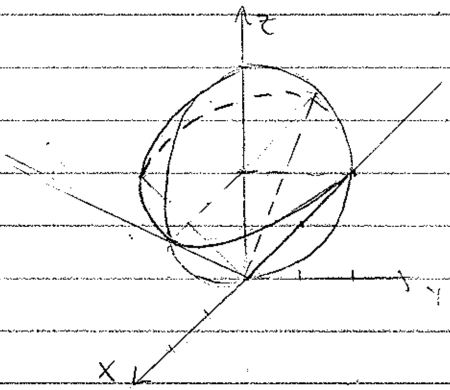
$3z^2 = x^2+y^2$

$x^2+y^2 = 3 \qquad x^2+y^2 = \frac{3}{4}$

$$V = \int_0^{2\pi}\int_0^{\frac{\sqrt3}{2}}\int_{\sqrt{1-r^2}}^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta + \int_0^{2\pi}\int_{\frac{\sqrt3}{2}}^{\sqrt3}\int_{\frac{r\sqrt3}{3}}^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$$

(20) $z^2 = x^2+y^2$ $\qquad d(x,y,z) = \sqrt{x^2+y^2+z^2}$

$x^2+y^2+z^2 = 4z \Rightarrow x^2+y^2+(z-2)^2 = 4$



$z^2 = x^2+y^2$ $\qquad x^2+y^2+z^2 = 4z$

$z = r$ $\qquad \rho^2 = 4\rho\cos\phi$

$\rho\cos\phi = \rho\,\text{sen}\,\phi$ $\qquad \rho = 4\cos\phi$

$tg\,\phi = 1$

$\phi = \frac{\pi}{4}$

$$M = \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{4\cos\phi} \rho^3\,\text{sen}\,\phi\,d\rho\,d\phi\,d\theta$$

$$M = \int_0^{2\pi}\int_0^{\frac{\pi}{4}} \frac{\rho^4}{4}\,\text{sen}\,\phi\,\Big|_0^{4\cos\phi} d\phi\,d\theta$$

$$M = \int_0^{2\pi}\int_0^{\frac{\pi}{4}} 64\cos^4\phi\,\text{sen}\,\phi\,.\,d\phi\,d\theta$$

$$M = \int_0^{2\pi} -\frac{64\cos^5\phi}{5}\Big|_0^{\frac{\pi}{4}} d\theta$$

$$M = \int_0^{2\pi} -\frac{64}{5}\cdot\frac{1}{2^{5/2}} + \frac{64}{5}\,d\theta$$

$$M = \frac{64}{5}\theta - \frac{64}{5\cdot 2^{5/2}}\theta\Big|_0^{2\pi}$$

$$M = \frac{128\pi}{5} - \frac{16\sqrt2\,\pi}{5}$$

$$M = \frac{16\pi(8-\sqrt2)}{5}\ \text{u.m}$$

(21) $I=\int_{-\sqrt{3}}^{\sqrt{3}}\int_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}}\int_{1}^{\sqrt{4-x^2-y^2}} \frac{z}{\sqrt{x^2+y^2}\,(x^2+y^2+z^2)^2}\, dz\, dy\, dx$

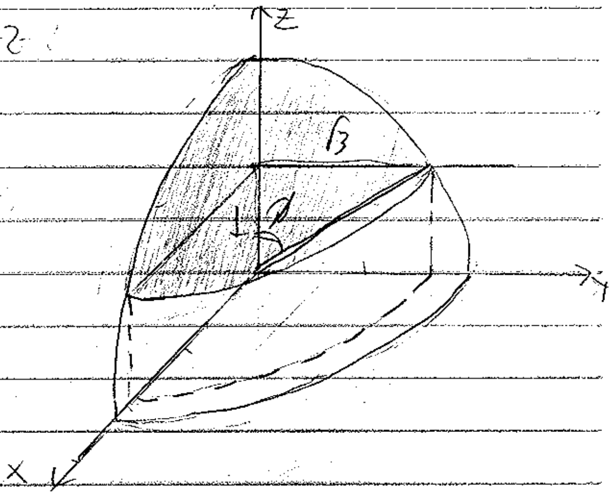
$z=\sqrt{4-x^2-y^2} \Rightarrow x^2+y^2+z^2=4 \Rightarrow \rho=2$

$z=1 \Rightarrow \rho=\sec\phi$

$y=\pm\sqrt{3-x^2} \Rightarrow x^2+y^2=3 \;;\; x=\pm\sqrt{3}$

$\frac{\rho\cos\phi}{\rho\,\text{sen}\,\phi\ \rho^4}$ $\qquad \phi=\text{arctg}\,\frac{\sqrt{3}}{1}=\frac{\pi}{3}$

$\text{tg}\,\phi=\sqrt{3}$



$I=\int_{0}^{2\pi}\int_{0}^{\frac{\pi}{3}}\int_{\sec\phi}^{2} \frac{\cos\phi}{\rho^2}\, d\rho\, d\phi\, d\theta$

$I=\int_{0}^{2\pi}\int_{0}^{\frac{\pi}{3}} -\frac{\cos\phi}{\rho}\Big|_{\sec\phi}^{2}\, d\phi\, d\theta$

$I=\int_{0}^{2\pi}\int_{0}^{\frac{\pi}{3}} -\frac{\cos\phi}{2}+\cos^2\phi\ \, d\phi\, d\theta$

$I=\int_{0}^{2\pi} -\frac{\text{sen}\,\phi}{2}+\frac{1\phi}{2}+\frac{\text{sen}\,2\phi}{4}\Big|_{0}^{\frac{\pi}{3}}\, d\theta$

$I=\int_{0}^{2\pi} -\frac{\sqrt{3}}{4}+\frac{\pi}{6}+\frac{\sqrt{3}}{8}\, d\theta$

$I=\frac{4\pi-3\sqrt{3}}{24}\,\theta\Big|_{0}^{2\pi}$

$I=\frac{4\pi^2-3\pi\sqrt{3}}{12}$

$I=\frac{\pi^2}{3}-\frac{\pi\sqrt{3}}{4}$

(22) $M = \int_0^{2\pi} \int_0^{\frac{\pi}{6}} \int_{\frac{\sqrt{3}}{\cos\phi}}^{\sqrt{\frac{5}{\cos^2\phi + 2\sin^2\phi}}} \rho \, d\rho \, d\phi \, d\theta$

$t = \frac{1}{\rho \sin\phi} = \frac{1}{r}$

$\rho^2 = \frac{5}{\cos^2\phi + 2\sin^2\phi}$ $\qquad \rho^2 = \frac{3}{\cos^2\phi}$ $\qquad \phi = \operatorname{arctg} \frac{r}{z}$

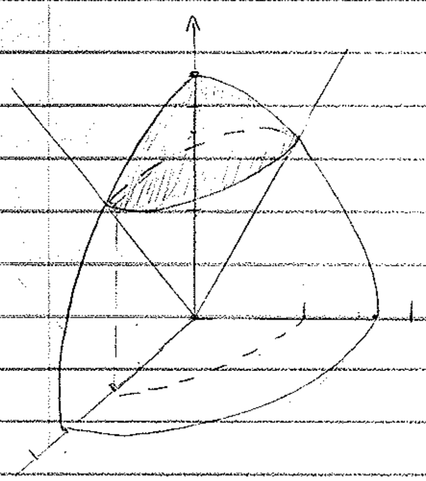
$\rho^2 = \frac{5}{1+\sin^2\phi}$ $\qquad \rho^2\cos^2\phi = 3$ $\qquad \frac{\pi}{6} = \operatorname{arctg}\frac{r}{z}$

$\rho^2 + \rho^2\sin^2\phi = 5$ $\qquad z^2 = 3$ $\qquad \frac{r}{z} = \frac{\sqrt{3}}{3}$

$x^2+y^2+z^2+x^2+y^2 = 5$ $\qquad z = \sqrt{3}$ $\qquad r = \frac{\sqrt{3}\,z}{3}$

$2x^2+2y^2+z^2 = 5$ $\qquad x^2+y^2 = \frac{z^2}{3}$

$\frac{x^2+y^2+z^2}{2} = \frac{5}{2} \Rightarrow z = \sqrt{5-2r^2}$



$2(x^2+y^2) = 5-z^2$

$2(x^2+y^2) = 5-3$

$x^2+y^2 = 1$

$r^2 = 1$

$r = 1$

$M = \int_0^{2\pi} \int_0^1 \int_{\sqrt{3}}^{\sqrt{5-2r^2}} dz \, dr \, d\theta$

(23) $M = \int_0^{2\pi} \int_0^{\sqrt{3}} \int_{\frac{r^2}{3}}^{\sqrt{10-3r^2}} (r+z)\, dz\, dr\, d\theta$

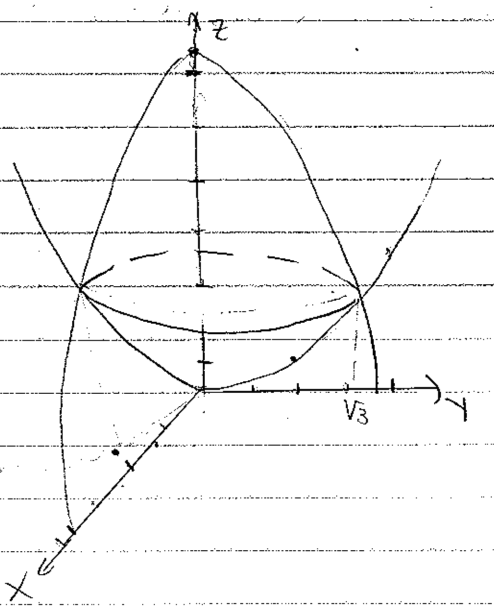
$\delta = \frac{r+z}{r} = 1 + \frac{z}{r} = 1 + \frac{\rho \cos\phi}{\rho \operatorname{sen}\phi} = 1 + \operatorname{cotg}\phi$

$z^2 = 10 - 3r^2$

$\rho^2 \cos^2\phi = 10 - 3\rho^2 \operatorname{sen}^2\phi$

$\rho = \frac{\sqrt{10}}{\sqrt{1+2\operatorname{sen}^2\phi}}$

$z = \frac{r^2}{3} \Rightarrow \rho \cos\phi = \frac{\rho^2 \operatorname{sen}^2\phi}{3} \Rightarrow \rho = \frac{3\cos\phi}{\operatorname{sen}^2\phi}$



$\frac{r^4}{9} = 10 - 3r^2 \qquad x = r^2$

$\frac{r^4}{9} + 3r^2 - 10 = 0$

$\frac{x^2}{9} + 3x - 10 = 0$

$x = \frac{-3 \pm \sqrt{9 - 4 \cdot \frac{1}{9} \cdot -10}}{2 \cdot \frac{1}{9}}$

$x = \frac{-3 \pm \frac{11}{3}}{\frac{2}{9}} \Rightarrow x = \frac{-27}{2} \pm \frac{33}{2}$

$\phi = \operatorname{arctg} \frac{r}{z} = \operatorname{arctg} \frac{\sqrt{3}}{1} = \frac{\pi}{3}$

$x' = -30, \; x'' = 3$
ñ convém

$\therefore r = \sqrt{3} \Rightarrow z = 1$

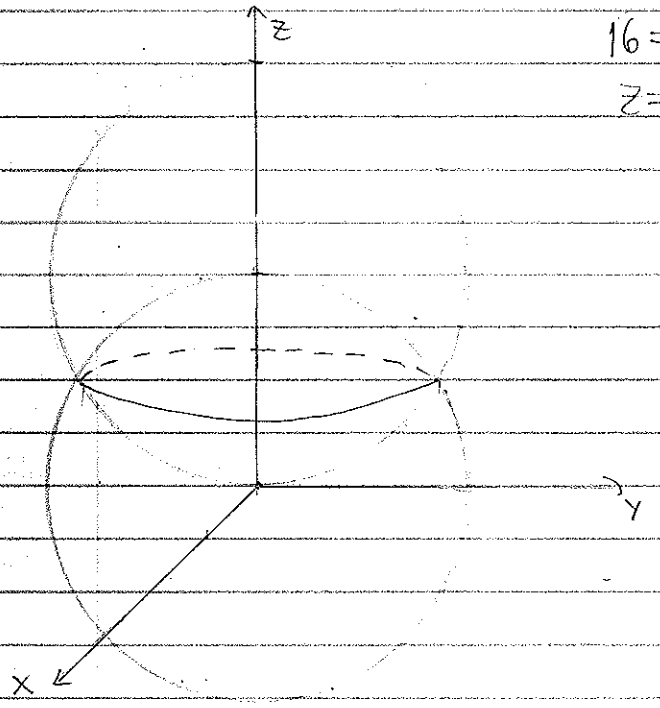
$M = \int_0^{2\pi} \int_0^{\frac{\pi}{3}} \int_0^{\frac{\sqrt{10}}{\sqrt{1+2\operatorname{sen}^2\phi}}} (1 + \operatorname{cotg}\phi)\, \rho^2 \operatorname{sen}\phi\, d\rho\, d\phi\, d\theta \; +$

$\int_0^{2\pi} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \int_0^{\frac{3\cos\phi}{\operatorname{sen}^2\phi}} (1 + \operatorname{cotg}\phi)\, \rho^2 \operatorname{sen}\phi\, d\rho\, d\phi\, d\theta$

(24) $x^2+y^2+z^2=16 \Rightarrow \rho=4$

$x^2+y^2+z^2=8z \Rightarrow \rho^2 = 8\rho\cos\phi \Rightarrow \rho = 8\cos\phi$

$x^2+y^2+(z-4)^2=16$



$16=8z$

$z=2$

$x^2+y^2=16-z^2$

$r^2=16-4$

$r=\sqrt{12}$

$r=2\sqrt{3}$

$\phi=\operatorname{arctg}\frac{2\sqrt{3}}{2}=\frac{\pi}{3}$

$$V=\int_0^{2\pi}\int_0^{\frac{\pi}{3}}\int_0^{4} \rho^2 \operatorname{sen}\phi \, d\rho \, d\phi \, d\theta + \int_0^{2\pi}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\int_0^{8\cos\phi} \rho^2 \operatorname{sen}\phi \, d\rho \, d\phi \, d\theta$$

$$V=\int_{-2\sqrt{3}}^{2\sqrt{3}}\int_{-\sqrt{12-x^2}}^{\sqrt{12-x^2}}\int_{4-\sqrt{16-x^2-y^2}}^{\sqrt{16-x^2-y^2}} dz\,dy\,dx$$

(25) $x^2+y^2+z^2=9 \Rightarrow z=\pm\sqrt{9-r^2}$

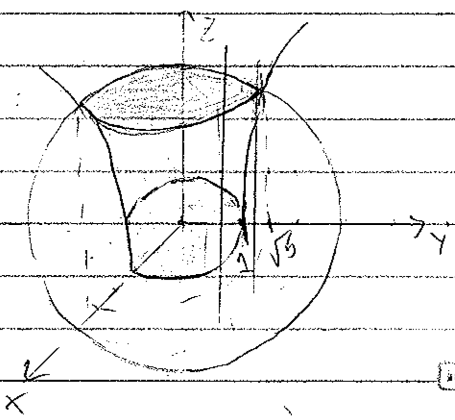
$x^2+y^2-z^2=1 \Rightarrow z=\pm\sqrt{r^2-1}$

$9-z^2=1+z^2$

$2z^2=8$

$z^2=4$

$x^2+y^2=5 \Rightarrow r=\sqrt{5}$



$$V = 4\int_0^{\frac{\pi}{2}}\int_0^1\int_0^{\sqrt{9-r^2}} r\,dz\,dr\,d\theta + 4\int_0^{\frac{\pi}{2}}\int_1^{\sqrt5}\int_{\sqrt{r^2-1}}^{\sqrt{9-r^2}} r\,dz\,dr\,d\theta$$

$$V = 4\int_0^{\frac{\pi}{2}}\int_0^1 rz\Big|_0^{\sqrt{9-r^2}} dr\,d\theta + 4\int_0^{\frac{\pi}{2}}\int_1^{\sqrt5} rz\Big|_{\sqrt{r^2-1}}^{\sqrt{9-r^2}} dr\,d\theta$$

$$V = 4\int_0^{\frac{\pi}{2}}\int_0^1 \frac{-2r}{-2}\sqrt{9-r^2}\,dr\,d\theta + 4\int_0^{\frac{\pi}{2}}\int_1^{\sqrt5} \frac{-2r}{-2}\sqrt{9-r^2} - \frac{2r}{2}\sqrt{r^2-1}\,dr\,d\theta$$

$$V = 4\int_0^{\frac{\pi}{2}} -\frac{1}{3}(9-r^2)^{3/2}\Big|_0^1 d\theta + 4\int_0^{\frac{\pi}{2}} -\frac{1}{3}(9-r^2)^{3/2} - \frac{1}{3}(r^2-1)^{3/2}\Big|_1^{\sqrt5} d\theta$$

$$V = 4\int_0^{\frac{\pi}{2}} -\frac{16\sqrt2}{3} + 9\,d\theta + 4\int_0^{\frac{\pi}{2}} -\frac{8}{3} - \frac{8}{3} + \frac{16\sqrt2}{3}\,d\theta$$

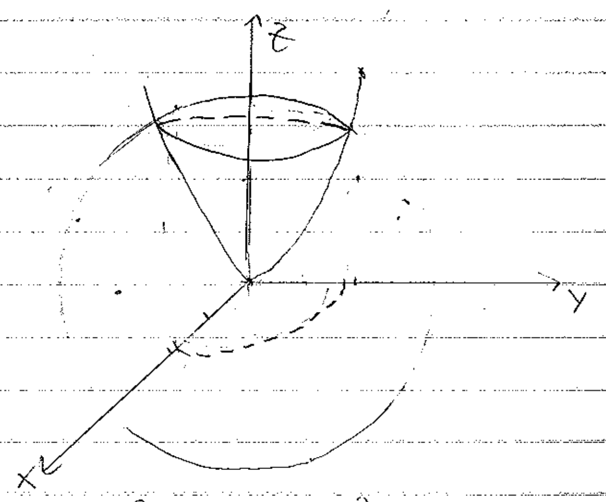
$$V = 4\left(-\frac{16\sqrt2}{3} + 9\right)\theta\Big|_0^{\frac{\pi}{2}} + 4\left(-\frac{16}{3} + \frac{16\sqrt2}{3}\right)\theta\Big|_0^{\frac{\pi}{2}}$$

$$V = \cancel{-\frac{32\sqrt2}{3}\pi} + 18\pi - \frac{32}{3}\pi + \cancel{\frac{32\sqrt2}{3}\pi}$$

$$V = 18\pi - \frac{32\pi}{3}\ \text{u.v}$$

26) $z = 2(x^2+y^2) \Rightarrow z = 2r^2$

$x^2+y^2+z^2 = 3 \Rightarrow z = \sqrt{3-r^2}$



$z = 2(3-z^2)$

$2z^2 + z - 6 = 0$

$z = \dfrac{-1 \pm \sqrt{1-4\cdot2\cdot-6}}{2\cdot2}$

$z = \dfrac{-1\pm7}{4}$

$z' = -2$, $z'' = \dfrac{3}{2} \Rightarrow r = \dfrac{\sqrt3}{2}$

ñ convém

$x^2+y^2 = 3 - z^2$

$x^2+y^2 = \dfrac{3}{4}$

$z = 2r^2$

$\rho\cos\phi = 2\rho^2 \operatorname{sen}^2\phi$

$\rho = \dfrac{\cos\phi}{2\operatorname{sen}^2\phi}$

$\phi = \operatorname{arctg}\dfrac{r}{z} = \operatorname{arctg}\dfrac{\sqrt3/2}{3/2} = \dfrac{\pi}{6}$

$$V = \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \int_{-\sqrt{\frac{3}{4}-x^2}}^{\sqrt{\frac{3}{4}-x^2}} \int_{2x^2+2y^2}^{\sqrt{3-x^2-y^2}} dz\,dy\,dx$$

$$V = \int_0^{2\pi} \int_0^{\frac{\sqrt{3}}{2}} \int_{2r^2}^{\sqrt{3-r^2}} r\,dz\,dr\,d\theta$$

$$V = \int_0^{2\pi} \int_0^{\frac{\pi}{6}} \int_0^{\sqrt{3}} \rho^2 \operatorname{sen}\phi\,d\rho\,d\phi\,d\theta + \int_0^{2\pi} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \int_0^{\frac{\cos\phi}{2\operatorname{sen}^2\phi}} \rho^2 \operatorname{sen}\phi\,d\rho\,d\phi\,d\theta$$

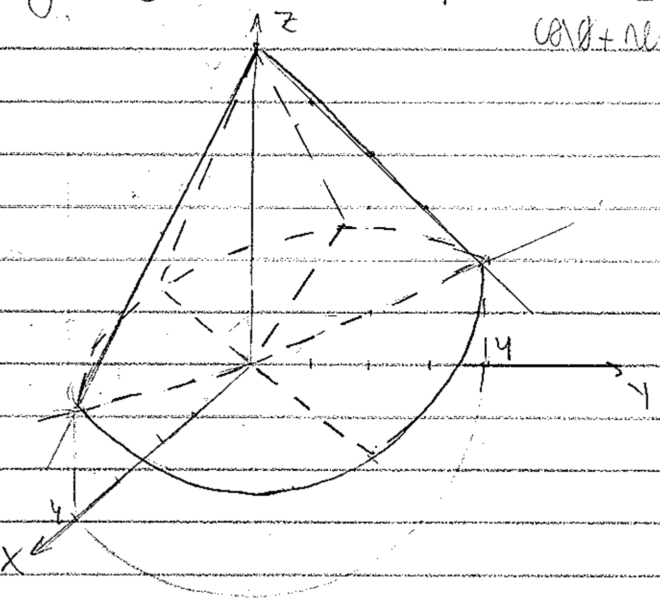
27) $2z = \sqrt{x^2+y^2} \Rightarrow z = \dfrac{r}{2}$

$z = 6 - \sqrt{x^2+y^2} \Rightarrow x^2 + y^2 = (z-6)^2 \Rightarrow z = 6 - r$

$2z = r$
$2\rho\cos\phi = \rho\operatorname{sen}\phi$
$\operatorname{tg}\phi = 2$

$z = 6 - r$
$\rho\cos\phi = 6 - \rho\operatorname{sen}\phi$
$\rho = \dfrac{6}{\cos\phi + \operatorname{sen}\phi}$

$z = 6 - 2z$
$3z = 6$
$z = 2$
$r = 4$
$x^2 + y^2 = 16$



$$V = \int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{\frac{\sqrt{x^2+y^2}}{2}}^{6-\sqrt{x^2+y^2}} dz\,dy\,dx$$

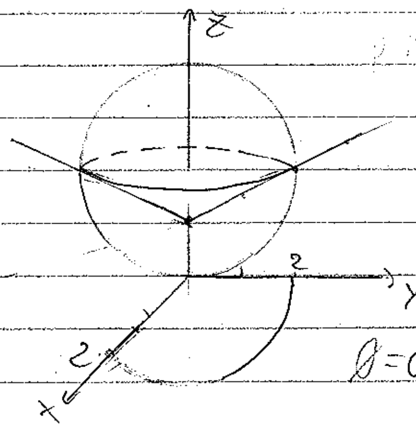
$$V = \int_0^{2\pi} \int_0^{4} \int_{\frac{r}{2}}^{6-r} r\,dz\,dr\,d\theta$$

$$V = \int_0^{2\pi} \int_0^{\operatorname{arctg} 2} \int_0^{\frac{6}{\cos\phi+\operatorname{sen}\phi}} \rho^2 \operatorname{sen}\phi\,d\rho\,d\phi\,d\theta$$

(28) $x^2+y^2+z^2=4z \Rightarrow x^2+y^2+(z-2)^2=4 \Rightarrow \rho=4\cos\phi \Rightarrow z=2\pm\sqrt{4-r^2}$

$z=1+\frac{1}{2}\sqrt{x^2+y^2} \Rightarrow (z-1)^2=\frac{x^2+y^2}{4} \Rightarrow z=1+\frac{r}{2} \Rightarrow \rho=\frac{2}{2\cos\phi-\operatorname{sen}\phi}$

$f(x,y,z)=\frac{(x^2+y^2)z^2}{\cos(x^2+y^2+z^2)}=\frac{r^2z^2}{\cos(r^2+z^2)}=\frac{\rho^4\operatorname{sen}^2\phi\cos^2\phi}{\cos\rho^2}$



$4(z-1)^2=4z-z^2$

$4z^2-8z+4=4z-z^2$

$5z^2-12z+4=0$

$z=\frac{12\pm\sqrt{144-4.5.4}}{2.5}$

$z=\frac{12\pm 8}{10}$

$z'=2,\ z''=\frac{2}{5}$ (ñ convém)

$x^2+y^2=4$

$\phi=\operatorname{arctg}\frac{r}{z}$

$\phi=\operatorname{arctg} 1$

$\phi=\pi/4$

$$V=\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{1+\frac{\sqrt{x^2+y^2}}{2}}^{2+\sqrt{4-x^2-y^2}}\frac{(x^2+y^2)z^2}{\cos(x^2+y^2+z^2)}\,dz\,dy\,dx$$

$$V=\int_{0}^{2\pi}\int_{0}^{2}\int_{1+\frac{r}{2}}^{2+\sqrt{4-r^2}}\frac{r^3\cdot z^2}{\cos(r^2+z^2)}\,dz\,dr\,d\theta$$

$$V=\int_{0}^{2\pi}\int_{0}^{\frac{\pi}{4}}\int_{\frac{2}{2\cos\phi-\operatorname{sen}\phi}}^{4\cos\phi}\frac{\rho^6\operatorname{sen}^4\phi\cos^2\phi}{\cos\rho^2}\,d\rho\,d\phi\,d\theta$$

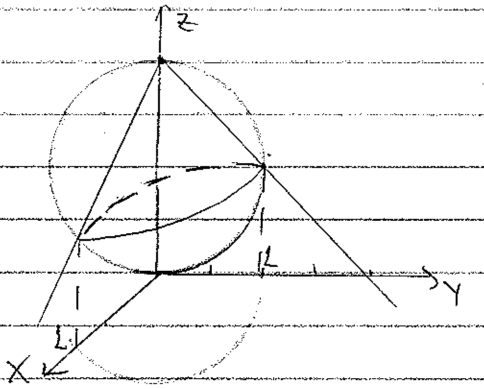
29) $x^2+y^2+z^2=2z \Rightarrow x^2+y^2+(z-1)^2=1 \Rightarrow z=1\pm\sqrt{1-x^2-y^2}$

$z=1\pm\sqrt{1-r^2}$

$\rho=2\cos\phi$

$z=2-\sqrt{x^2+y^2} \Rightarrow z=2-r \Rightarrow \rho=\dfrac{2}{\cos\phi+\operatorname{sen}\phi}$

$f(x,y,z)=\dfrac{e^{x^2+y^2+z^2}}{x+y+z}=\dfrac{e^{r^2+z^2}}{r(\cos\theta+\operatorname{sen}\theta)+z}=\dfrac{e^{\rho^2}}{\rho[\cos\phi+\operatorname{sen}\phi(\cos\theta+\operatorname{sen}\theta)]}$



$(z-2)^2=2z-z^2$

$z^2-4z+4=2z-z^2$

$2z^2-6z+4=0$

$z^2-3z+2=0$

$1+2=3$

$1\cdot2=2$

$z=1 \Rightarrow x^2+y^2=1 \Rightarrow r=1$

$$I=\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{1-\sqrt{1-x^2-y^2}}^{2-\sqrt{x^2+y^2}}\frac{e^{x^2+y^2+z^2}}{x+y+z}\,dz\,dy\,dx$$

$$I=\int_0^{2\pi}\int_0^{1}\int_{1-\sqrt{1-r^2}}^{2-r}\frac{e^{r^2+z^2}}{r(\cos\theta+\operatorname{sen}\theta)+z}\,r\,dz\,dr\,d\theta$$

$$I=\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\frac{2}{\cos\phi+\operatorname{sen}\phi}}\frac{e^{\rho^2}}{\rho[\cos\phi+\operatorname{sen}\phi(\cos\theta+\operatorname{sen}\theta)]}\,\rho^2\operatorname{sen}\phi\,d\rho\,d\phi\,d\theta+$$
$$\int_0^{2\pi}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^{2\cos\phi}\frac{e^{\rho^2}}{\rho[\cos\phi+\operatorname{sen}\phi(\cos\theta+\operatorname{sen}\theta)]}\,\rho^2\operatorname{sen}\phi\,d\rho\,d\phi\,d\theta$$

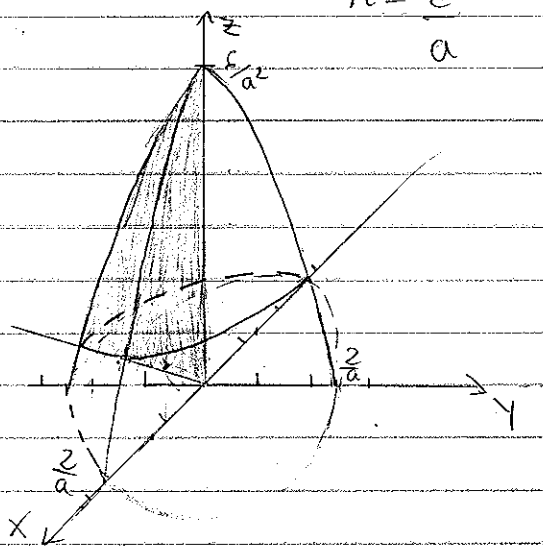
30) $$V=\int_0^{\frac{2}{a}}\int_{-\sqrt{\frac{4}{a^2}-x^2}}^{\sqrt{\frac{4}{a^2}-x^2}}\int_{a\sqrt{x^2+y^2}}^{6-a^2x^2-a^2y^2}dz\,dy\,dx$$

$z=a\sqrt{x^2+y^2} \Rightarrow \dfrac{z^2}{a^2}=x^2+y^2 \qquad z=6-a^2x^2-a^2y^2 \Rightarrow x^2+y^2=\dfrac{(z-6)}{-a^2}$

$$y = \pm\sqrt{\frac{4}{a^2} - x^2} \Rightarrow x^2 + y^2 = \frac{4}{a^2}$$

$$r = \frac{2}{a}$$

$$z^2 = 6 - z$$
$$z^2 + z - 6 = 0$$
$$-3 + 2 = -1$$
$$-3 \cdot 2 = -6$$
$$z = 2$$



a) $$V = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{\frac{2}{a}} \int_{ar}^{6-a^2r^2} r \, dz \, dr \, d\theta$$

b) $$V = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{\frac{2}{a}} r z \Big|_{ar}^{6-a^2r^2} dr \, d\theta$$

$$V = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{\frac{2}{a}} 6r - a^2r^3 - ar^2 \, dr \, d\theta$$

$$V = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 3r^2 - \frac{a^2r^4}{4} - \frac{ar^3}{3} \Big|_{0}^{\frac{2}{a}} d\theta$$

$$V = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{12}{a^2} - \frac{4}{a^2} - \frac{8}{3a^2} \, d\theta$$

$$V = \frac{(36a^2 - 12a^2 - 8)\pi}{3a^2} = \frac{16}{3}\pi$$

$$\begin{cases} 36a^2 - 12a^2 - 8 = 16 \\ 3a^2 = 3 \end{cases}$$
$$a = 1$$

Prova

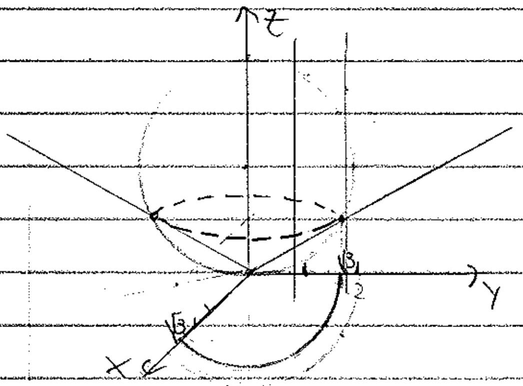
① $x^2+y^2+z^2=4z \Rightarrow x^2+y^2+(z-2)^2=4 \Rightarrow \begin{cases} \rho = 4\cos\phi \\ z = 2\pm\sqrt{4-x^2-y^2} \\ z = 2\pm\sqrt{4-r^2} \end{cases}$

$\sqrt{3}z=\sqrt{x^2+y^2} \Rightarrow \begin{cases} z=\sqrt{\dfrac{x^2+y^2}{3}} \\ z = \dfrac{r}{\sqrt{3}} \end{cases}$

$4z-z^2=3z^2$

$4z-4z^2=0$

$4z(1-z)=0 \quad \begin{cases} z=0 \\ z=1 \end{cases}$



$x^2+y^2=3$

$r=\sqrt{3}$

$\sqrt{3}\,\cancel{\rho}\cos\phi = \cancel{\rho}\,\text{sen}\,\phi$

$\text{tg}\,\phi=\sqrt{3}$

$\phi=\dfrac{\pi}{3}$

$$V=\int_0^{2\pi}\int_0^{\sqrt{3}}\int_{\frac{r}{\sqrt{3}}}^{2+\sqrt{4-r^2}} r\,dz\,dr\,d\theta+\int_0^{2\pi}\int_{\sqrt{3}}^{2}\int_{2-\sqrt{4-r^2}}^{2+\sqrt{4-r^2}} r\,dz\,dr\,d\theta$$

$$V=\int_0^{2\pi}\int_0^{\sqrt{3}} rz\Big|_{\frac{r}{\sqrt{3}}}^{2+\sqrt{4-r^2}}dr\,d\theta+\int_0^{2\pi}\int_{\sqrt{3}}^{2} rz\Big|_{2-\sqrt{4-r^2}}^{2+\sqrt{4-r^2}}dr\,d\theta$$

$$V=\int_0^{2\pi}\int_0^{\sqrt{3}} 2r+\frac{-2r\sqrt{4-r^2}}{-2}-\frac{r^2}{\sqrt{3}}\,dr\,d\theta+\int_0^{2\pi}\int_{\sqrt{3}}^{2}\cancel{2r}+\frac{-2r\sqrt{4-r^2}}{-2}-\cancel{2r}+\frac{-2r\sqrt{4-r^2}}{-2}\,dr\,d\theta$$

$$V=\int_0^{2\pi} r^2-\frac{1}{3}(4-r^2)^{3/2}-\frac{r^3}{3\sqrt{3}}\Big|_0^{\sqrt{3}}d\theta+\int_0^{2\pi}-\frac{1}{3}(4-r^2)^{3/2}-\frac{1}{3}(4-r^2)^{3/2}\Big|_{\sqrt{3}}^{2}d\theta$$

$$V=\int_0^{2\pi} 3-\frac{1}{3}-\frac{1}{3}+\frac{8}{3}\,d\theta+\int_0^{2\pi}\frac{2}{3}\,d\theta$$

$$V=\frac{13}{3}\theta\Big|_0^{2\pi}+\frac{2}{3}\theta\Big|_0^{2\pi}$$

$V=10\pi$ u.v

$$V=\int_0^{2\pi}\int_0^{\frac{\pi}{3}}\int_0^{4\cos\phi} \rho^2 \operatorname{sen}\phi \, d\rho \, d\phi \, d\theta$$

$$V=\int_0^{2\pi}\int_0^{\frac{\pi}{3}} \frac{\rho^3}{3}\operatorname{sen}\phi \Big|_0^{4\cos\phi} d\phi \, d\theta$$

$$V=\int_0^{2\pi}\int_0^{\frac{\pi}{3}} -\frac{64}{3}\cos^3\phi - \operatorname{sen}\phi \, d\phi \, d\theta$$

$$V=\int_0^{2\pi} -\frac{16}{3}\cos^4\phi \Big|_0^{\frac{\pi}{3}} d\theta$$

$$V=\int_0^{2\pi} -\frac{16}{3}\left(\frac{1}{2}\right)^4+\frac{16}{3}\, d\theta$$

$$V= 5\theta\Big|_0^{2\pi}$$

$V=10\pi$ u.v

② $$I=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_{2\cos\theta}^{\frac{2}{\cos\theta+\operatorname{sen}\theta}} r^3\cos\theta\operatorname{sen}\theta \, dr \, d\theta$$

a) $r=\dfrac{2}{\cos\theta+\operatorname{sen}\theta}$ $\qquad r=2\cos\theta$ $\qquad r^2\cos\theta\operatorname{sen}\theta$

$r\cos\theta+r\operatorname{sen}\theta=2$ $\qquad r^2=2r\cos\theta$ $\qquad XY$

$x+Y=2$ $\qquad x^2+Y^2=2x$

$(x-1)^2+Y^2=1$

$X^2+(2-x)^2=2x$

$x^2+4-4x+x^2-2x=0$

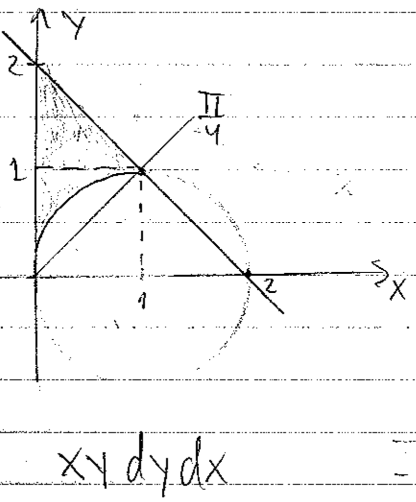
$x^2-3x+2=0$

$1+2=3$

$1\cdot 2=2$

$x=1,\ x=2$

$Y=1,\ Y=0$



$$I=\int_0^1\int_{\sqrt{2x-x^2}}^{2-x} xy \, dy \, dx$$

$$I = \int_0^1 \int_0^{1-\sqrt{1-y^2}} xy \, dx\,dy + \int_1^2 \int_0^{2-y} xy \, dx\,dy$$

b) $$I = \int_0^1 \int_{\sqrt{2x-x^2}}^{2-x} xy \, dy\,dx$$

$$I = \int_0^1 \frac{xy^2}{2} \Big|_{\sqrt{2x-x^2}}^{2-x} dx$$

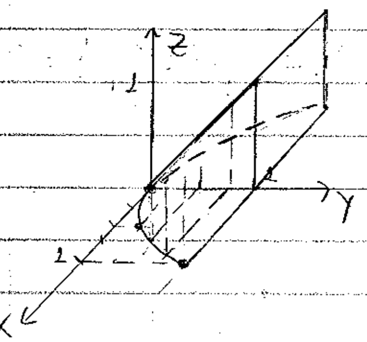
$$I = \int_0^1 \frac{x}{2}(2-x)^2 - \frac{x}{2}\left(\sqrt{2x-x^2}\right)^2 dx$$

$$I = \int_0^1 2x - 2x^2 + \frac{x^3}{2} - x^2 + \frac{x^3}{2} \, dx$$

$$I = x^2 - x^3 + \frac{x^4}{4} \Big|_0^1$$

$$I = \frac{1}{4}$$

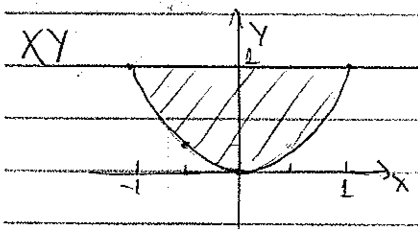
③ $z=0$, $y=1$, $y=z$, $y=x^2$

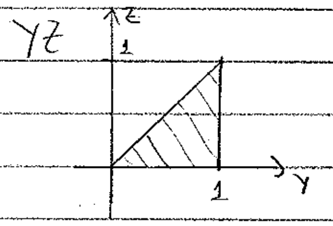


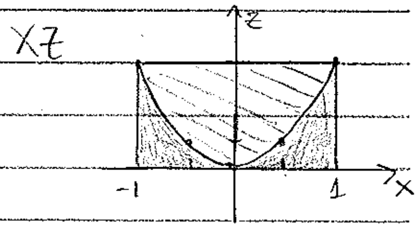
$$S = \int_{-1}^1 \int_{x^2}^1 \int_0^y dz\,dy\,dx$$

$$S = \int_0^1 \int_0^y \int_{-\sqrt{y}}^{\sqrt{y}} dx\,dz\,dy$$

$$S = 2\int_0^1 \int_0^{x^2} \int_{x^2}^1 dy\,dx\,dz + 2\int_0^1 \int_{x^2}^1 \int_z^1 dy\,dx\,dz$$

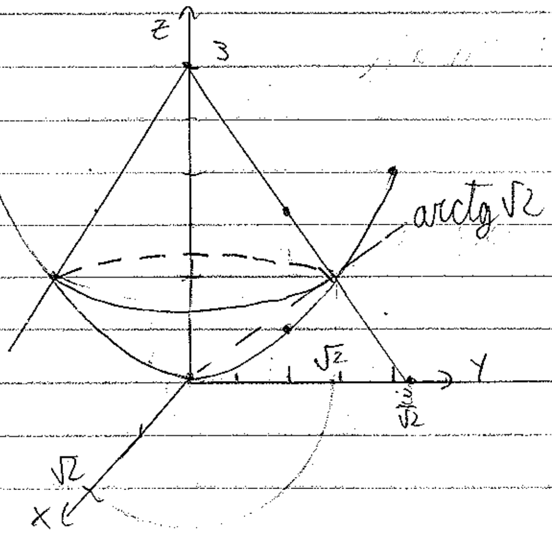






4) $z = 3 - \sqrt{2x^2+2y^2} \Rightarrow z = 3 - r\sqrt{2} \Rightarrow p = \dfrac{3}{\cos\phi + \sqrt{2}\,\text{sen}\,\phi}$

$z = \dfrac{x^2+y^2}{2} \Rightarrow z = \dfrac{r^2}{2} \Rightarrow p = \dfrac{2\cos\phi}{\text{sen}^2\phi}$



$\dfrac{r^2}{2} = 3 - r\sqrt{2}$

$r^2 + 2\sqrt{2}\,r - 6 = 0$

$r = \dfrac{-2\sqrt{2} \pm \sqrt{8 - 4 \cdot 1 \cdot -6}}{2}$

$r = \dfrac{-2\sqrt{2} \pm 4\sqrt{2}}{2}$

$r = -3\sqrt{2}$ ou $r = \sqrt{2}$

ñ convém $\quad z = 1$

$d(x,y,z) = (x+y)^2 \Rightarrow r^2(\cos\theta + \text{sen}\,\theta)^2 \Rightarrow p^2\,\text{sen}^2\phi(\cos\theta + \text{sen}\,\theta)^2$

$\phi = \text{arctg}\,\dfrac{r}{z} = \text{arctg}\,\sqrt{2}$

$$M = \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{\frac{x^2+y^2}{2}}^{3-\sqrt{2x^2+2y^2}} (x+y)^2 \, dz\,dy\,dx$$

$$M = \int_0^{2\pi} \int_0^{\sqrt{2}} \int_{\frac{r^2}{2}}^{3-r\sqrt{2}} r^3(\cos\theta + \text{sen}\,\theta)^2 \, dz\,dr\,d\theta$$

$$M = \int_0^{2\pi} \int_0^{\text{arctg}\sqrt{2}} \int_0^{\frac{3}{\cos\phi+\sqrt{2}\,\text{sen}\,\phi}} p^4\,\text{sen}^3\phi(\cos\theta+\text{sen}\,\theta)^2\,dp\,d\phi\,d\theta \; +$$

$$\int_0^{2\pi} \int_{\text{arctg}\sqrt{2}}^{\frac{\pi}{2}} \int_0^{\frac{2\cos\phi}{\text{sen}^2\phi}} p^4\,\text{sen}^3\phi(\cos\theta+\text{sen}\,\theta)^2\,dp\,d\phi\,d\theta$$

⑤ $V=\int_0^2\int_0^{\sqrt{4-x^2}} \frac{3\sqrt{4-x^2-y^2}}{2}\,dy\,dx$

$z=\frac{3}{2}\sqrt{4-(x^2+y^2)}$

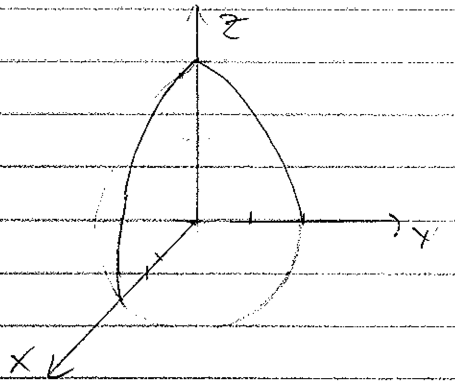
$\frac{2}{3}z=\sqrt{4-(x^2+y^2)}$

$\frac{4}{9}z^2=4-x^2-y^2$

$x^2+y^2+\frac{4}{9}z^2=4$

$\frac{x^2}{4}+\frac{y^2}{4}+\frac{z^2}{9}=1$

a)



b) $d(x,y,z)=\frac{1}{\sqrt{x^2+y^2+z^2}}=\frac{1}{\rho}$

$9x^2+9y^2+4z^2=36$

$9x^2+9y^2+9z^2=36+5z^2$

$9\rho^2=36+5\rho^2\cos^2\theta$

$M=\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^{\frac{6}{\sqrt{4+5\operatorname{sen}^2\theta}}} \rho\operatorname{sen}\phi\,d\rho\,d\phi\,d\theta$

$\rho=\frac{6}{\sqrt{9-5\cos^2\theta}}$

$\rho=\frac{6}{\sqrt{4+5\operatorname{sen}^2\theta}}$

Lista 5:

① a) $u_n=\frac{n}{4n+2}$ $\quad u=\left(\frac{1}{6},\frac{2}{10},\frac{3}{14},\frac{4}{18},\dots\right)$

$\lim_{n\to+\infty} u_n=\lim_{n\to+\infty}\frac{n}{4n+2}=\lim_{\substack{x\to+\infty\\x\in\mathbb{R}}}\frac{x}{4x+2}=\lim_{\substack{x\to+\infty\\x\in\mathbb{R}}}\frac{1}{4}=\frac{1}{4}$

b) $u_n=\frac{(-1)^n}{5-n}$ $\quad u=\left(\frac{-1}{4},\frac{1}{3},\frac{-1}{2},\frac{1}{1},\dots\right)$

$\lim_{n\to+\infty} u_n=\lim_{n\to+\infty}\left|\frac{(-1)^n}{5-n}\right|=\lim_{n\to+\infty}\frac{1}{|5-n|}=\lim_{\substack{x\to+\infty\\x\in\mathbb{R}}}\frac{1}{|5-x|}=0$

c) $u_n = \frac{(-1)^n \sqrt{n}}{n+1}$ $\quad u = \left(-\frac{1}{2}, \frac{\sqrt{2}}{3}, -\frac{\sqrt{3}}{4}, \frac{2}{5}, \dots\right)$

$\lim\limits_{n\to+\infty} u_n = \lim\limits_{n\to+\infty} \left|\frac{(-1)^n \sqrt{n}}{n+1}\right| = \lim\limits_{n\to+\infty} \frac{1.\sqrt{n}}{n+1} = \lim\limits_{\substack{x\to+\infty \\ x\in\mathbb{R}}} \frac{\sqrt{x}}{x+1} = \lim\limits_{\substack{x\to+\infty \\ x\in\mathbb{R}}} \frac{1}{2\sqrt{x}} = 0$

d) $u_n = \frac{100n}{n^{3/2}+4}$ $\quad u = \left(20, \frac{100}{\sqrt{2}+2}, \frac{300}{3\sqrt{3}+4}, \frac{100}{3}, \dots\right)$

$\lim\limits_{n\to+\infty} u_n = \lim\limits_{n\to+\infty} \frac{100n}{n^{3/2}+4} = \lim\limits_{\substack{x\to+\infty \\ x\in\mathbb{R}}} \frac{100x}{x^{3/2}+4} = \lim\limits_{\substack{x\to+\infty \\ x\in\mathbb{R}}} \frac{100}{\frac{3\sqrt{x}+4}{2}} = 0$

e) $u_n = \frac{n+1}{\sqrt{n}}$ $\quad u = \left(2, \frac{3}{\sqrt{2}}, \frac{4}{\sqrt{3}}, \frac{5}{2}, \dots\right)$

$\lim\limits_{n\to+\infty} u_n = \lim\limits_{n\to+\infty} \frac{n+1}{\sqrt{n}} = \lim\limits_{\substack{x\to+\infty \\ x\in\mathbb{R}}} \frac{x+1}{\sqrt{x}} = \lim\limits_{\substack{x\to+\infty \\ x\in\mathbb{R}}} \frac{1}{\frac{1}{2\sqrt{x}}} = \infty$

f) $u_n = \frac{\ln n}{n}$ $\quad u = \left(0, \frac{\ln 2}{2}, \frac{\ln 3}{3}, \frac{\ln 4}{4}, \dots\right)$

$\lim\limits_{n\to+\infty} u_n = \lim\limits_{n\to+\infty} \frac{\ln n}{n} = \lim\limits_{\substack{x\to+\infty \\ x\in\mathbb{R}}} \frac{\ln x}{x} = \lim\limits_{\substack{x\to+\infty \\ x\in\mathbb{R}}} \frac{\frac{1}{x}}{1} = 0$

g) $u_n = \ln\left(\frac{1}{n}\right)$ $\quad u = \left(0, \ln\left(\frac{1}{2}\right), \ln\left(\frac{1}{3}\right), \ln\left(\frac{1}{4}\right), \dots\right)$

$\lim\limits_{n\to+\infty} u_n = \lim\limits_{n\to+\infty} \ln\left(\frac{1}{n}\right) = -\infty$

h) $u_n = \frac{n^2}{5n+3}$ $\quad u = \left(\frac{1}{8}, \frac{4}{13}, \frac{9}{18}, \frac{16}{23}, \dots\right)$

$$\lim_{n\to+\infty} u_n = \lim_{n\to+\infty} \frac{n^2}{5n+3} = \lim_{\substack{x\to+\infty \\ x\in\mathbb{R}}} \frac{x^2}{5x+3} = \lim_{\substack{x\to+\infty \\ x\in\mathbb{R}}} \frac{2x}{5} = \infty$$

i) $u_n = \cos \frac{n\pi}{2}$ $\quad u = (0, -1, 0, 1, \dots)$

$$\lim_{n\to+\infty} u_n = \lim_{n\to+\infty} \cos \frac{n\pi}{2} = \nexists$$

j) $u_n = \operatorname{arctg} n$ $\quad u = (\operatorname{arctg} 1, \operatorname{arctg} 2, \operatorname{arctg} 3, \operatorname{arctg} 4, \dots)$

$$\lim_{n\to+\infty} u_n = \lim_{n\to+\infty} \operatorname{arctg} n = \frac{\pi}{2}$$

k) $u_n = \left(1 - \frac{2}{n}\right)^n$ $\quad u = \left(-1, 0, \frac{1}{27}, \frac{1}{16}, \dots\right)$

$$\lim_{n\to+\infty} u_n = \lim_{n\to+\infty} \left(1-\frac{2}{n}\right)^n = \lim_{\substack{x\to+\infty \\ x\in\mathbb{R}}} \left(1-\frac{2}{x}\right)^x = L$$

$$\ln L = \ln \lim_{x\to+\infty} \left(1-\frac{2}{x}\right)^x = \lim_{x\to+\infty} x \ln\left(1-\frac{2}{x}\right) = \lim_{x\to+\infty} \frac{\ln\left(1-\frac{2}{x}\right)}{\frac{1}{x}}$$

$$\ln L = \lim_{x\to+\infty} \frac{\frac{2}{x^2} \cdot \frac{x}{x-2}}{\frac{-1}{x^2}} = \lim_{x\to+\infty} \frac{2}{(x-2)x} \cdot -x^2 = \lim_{x\to+\infty} \frac{2x}{2-x}$$

$$\ln L = \lim_{x\to+\infty} \frac{2}{-1} = -2 \qquad \therefore L = e^{-2}$$

l) $u_n = \frac{n^2}{2^n}$ $\quad u = \left(\frac{1}{2}, 1, \frac{9}{8}, 1, \ldots\right)$

$\lim\limits_{n\to+\infty} u_n = \lim\limits_{n\to+\infty} \frac{n^2}{2^n} = \lim\limits_{\substack{x\to+\infty \\ x\in\mathbb{R}}} \frac{x^2}{2^x} = \lim\limits_{x\to+\infty} \frac{2x}{2^x \ln 2} = \lim\limits_{x\to+\infty} \frac{2}{2^x \ln^2 2} = 0$

m) $u_n = \frac{3n}{e^{2n}}$ $\quad u = \left(\frac{3}{e^2}, \frac{6}{e^4}, \frac{9}{e^6}, \frac{12}{e^8}, \ldots\right)$

$\lim\limits_{n\to+\infty} u_n = \lim\limits_{n\to+\infty} \frac{3n}{e^{2n}} = \lim\limits_{\substack{x\to+\infty \\ x\in\mathbb{R}}} \frac{3x}{e^{2x}} = \lim\limits_{x\to+\infty} \frac{3}{2.e^{2x}} = 0$

n) $u_n = 1+(-1)^n$ $\quad u = (0, 2, 0, 2, \ldots)$

$\lim\limits_{n\to+\infty} u_n = \lim\limits_{n\to+\infty} 1+(-1)^n = \nexists$

o) $u_n = \sqrt[n]{n}$ $\quad u = (1, \sqrt{2}, \sqrt[3]{3}, \sqrt[4]{4}, \ldots)$

$\lim\limits_{n\to+\infty} u_n = \lim\limits_{n\to+\infty} \sqrt[n]{n} = \lim\limits_{\substack{x\to+\infty \\ x\in\mathbb{R}}} x^{1/x} = L$

$\ln L = \ln \lim\limits_{x\to+\infty} x^{1/x} = \lim\limits_{x\to+\infty} \frac{1}{x} \ln x = \lim\limits_{x\to+\infty} \frac{\frac{1}{x}}{1} = 0$

$\ln L = 0 \quad \therefore \quad L = 1$

p) $u_n = 7^{-n} . 3^{n-1}$ $\quad u = \left(\frac{1}{7}, \frac{3}{49}, \frac{9}{343}, \frac{27}{2401}, \ldots\right)$

$\lim\limits_{n\to+\infty} u_n = \lim\limits_{n\to+\infty} \frac{3^{n-1}}{7^n} = \lim\limits_{\substack{x\to+\infty \\ x\in\mathbb{R}}} \frac{3^{x-1}}{7^x} = L$

$$\ln L = \ln \lim_{x \to +\infty} \frac{1}{7}\left(\frac{3}{7}\right)^{x-1} = \lim_{x \to +\infty} \frac{x-1}{7} \cdot \ln\left(\frac{3}{7}\right) = -\infty$$

$\ln L = -\infty \therefore L = 0$

② a) $\left\{\frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \frac{8}{81}, \ldots\right\} \Rightarrow U_n = \frac{2^{n-1}}{3^n}$

b) $\left\{\frac{1}{3}, -\frac{2}{9}, \frac{4}{27}, -\frac{8}{81}, \ldots\right\} \Rightarrow U_n = \frac{(-2)^{n-1}}{3^n}$

c) $\left\{\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \ldots\right\} \Rightarrow U_n = \frac{2n-1}{2n}$

d) $\left\{0, \frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \ldots\right\} \Rightarrow U_n = \frac{n-1}{n^2}$

③ a) $U_n = \frac{n}{2n-1}$ $\quad U = \left\{1, \frac{2}{3}, \frac{3}{5}, \ldots\right\}$ decrescente

b) $U_n = n - 2^n$ $\quad U = \{-1, -2, -5, \ldots\}$ decrescente

c) $U_n = \frac{n}{e^n}$ $\quad U = \left\{\frac{1}{e}, \frac{2}{e^2}, \frac{3}{e^3}, \ldots\right\}$ decrescente

d) $U_n = \frac{5^n}{2^{n^2}}$ $\quad U = \left\{\frac{5}{2}, \frac{25}{16}, \frac{125}{512}, \ldots\right\}$ decrescente

e) $U_n = \frac{10^n}{(2n)!}$ $\quad U = \left\{\frac{10}{2}, \frac{100}{24}, \frac{1000}{720}, \ldots\right\}$ decrescente

f) $u_n = \frac{n^n}{n!}$ $\quad u = \left\{ 1, \frac{4}{2}, \frac{27}{6}, \ldots \right\}$ crescente

g) $u_n = \frac{1}{n + \ln n}$ $\quad u = \left\{ \frac{1}{1}, \frac{1}{2+\ln 2}, \frac{1}{3+\ln 3}, \ldots \right\}$ decrescente

h) $u_n = \frac{n!}{3^n}$ $\quad u = \left\{ \frac{1}{3}, \frac{2}{9}, \frac{6}{27}, \frac{24}{81}, \ldots \right\}$ não-crescente

④ $1 \leq u_n \leq 5$ converge, pois é limitado e monótona. Além disso, $1 \leq \lim\limits_{n \to +\infty} u_n \leq 5$.

⑤ $u_n \leq 5$, deve convergir se for crescente, pois é limitada superiormente por 5, assim $\lim\limits_{n \to +\infty} u_n \leq 5$. Se u_n for decrescente nada se pode afirmar.

⑥ $u_{n+1} = \frac{1}{2}\left(u_n + \frac{K}{u_n}\right) = \frac{u_n^2 + K}{2u_n}$, $u_1 = \frac{1}{2}$

a) $u_2 = \frac{0{,}5^2 + 10}{2 \cdot 0{,}5} = 10{,}25$

$u_3 = \frac{10{,}25^2 + 10}{2 \cdot 10{,}25} = 5{,}613$

$u_4 = \frac{5{,}613^2 + 10}{2 \cdot 5{,}613} = 3{,}697$

$u_5 = \frac{3{,}697^2 + 10}{2 \cdot 3{,}697} = 3{,}201$

$u_6 = \frac{3{,}201^2 + 10}{2 \cdot 3{,}201} = 3{,}163$

b) $L = \lim_{n \to +\infty} u_n = \lim_{n \to +\infty} u_{n+1}$

$L = \frac{1}{2}\left(L + \frac{K}{L}\right)$ $\quad L^2 = K$

$L = \sqrt{K}$

$2L = L + \frac{K}{L}$

⑦ Fibonacci: $u_{n+1} = u_n + u_{n-1}$, $u_1 = u_2 = 1$

a) $F = \{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots\}$

b) $x_n = \frac{u_{n+1}}{u_n}$

$x_1 = 1$

$x_2 = 2$

$x_3 = \frac{3}{2} = 1{,}5$

$x_4 = \frac{5}{3} = 1{,}667$

$x_5 = \frac{8}{5} = 1{,}6$

c) $\varphi = \lim_{n \to +\infty} x_n = \lim_{n \to +\infty} \frac{u_{n+1}}{u_n}$

$\lim_{n \to +\infty} \frac{u_{n-1}}{u_n} = \frac{1}{\varphi}$

$\frac{u_{n+1}}{u_n} = 1 + \frac{u_{n-1}}{u_n}$

$\varphi = 1 + \frac{1}{\varphi}$

$\varphi^2 = \varphi + 1$

$\varphi^2 - \varphi + 1$

$\varphi = \frac{1 + \sqrt{5}}{2}$

⑧ a) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$

$$\frac{1}{(2n-1)(2n+1)} = \frac{A}{(2n-1)} + \frac{B}{(2n+1)} = \frac{A(2n+1) + B(2n-1)}{(2n-1)(2n+1)} = \frac{2n(A+B) + A - B}{(2n-1)(2n+1)}$$

$$\begin{cases} A + B = 0 \Rightarrow A = -B \Rightarrow B = -1/2 \\ A - B = 1 \Rightarrow A = 1/2 \end{cases}$$

$$\sum_{n=1}^{\infty} \frac{1}{2(2n-1)} - \frac{1}{2(2n+1)}$$

$$S_1 = \frac{1}{2} - \frac{1}{6}$$

$$S_2 = \left(\frac{1}{2} - \cancel{\frac{1}{6}}\right) + \cancel{\frac{1}{6}} - \frac{1}{10} = \frac{1}{2} - \frac{1}{10}$$

$$S_3 = \left(\frac{1}{2} - \cancel{\frac{1}{10}}\right) + \cancel{\frac{1}{10}} - \frac{1}{14} = \frac{1}{2} - \frac{1}{14}$$

$$\vdots$$

$$S_K = \frac{1}{2} - \frac{1}{4K+2} = \frac{1}{2}\left(1 - \frac{1}{2K+1}\right) = \frac{K}{2K+1} \quad \frac{1}{2}\left(\frac{2K+\cancel{1}-\cancel{1}}{2K+1}\right) \quad \frac{\cancel{2}K}{\cancel{2}(2K+1)}$$

$$\lim_{K\to+\infty} S_K = \lim_{K\to+\infty} \frac{1}{2} - \cancel{\frac{1}{4K+2}}_{\infty} = \frac{1}{2}$$

∴ a série converge para $1/2$

b) $$\sum_{n=1}^{\infty} \frac{8}{(4n-3)(4n+1)}$$

$$\frac{8}{(4n-3)(4n+1)} = \frac{A}{4n-3} + \frac{B}{4n+1} = \frac{(4n+1)A + (4n-3)B}{(4n-3)(4n+1)} = \frac{4n(A+B) + A - 3B}{(4n-3)(4n+1)}$$

$$\begin{cases} A+B=0 \Rightarrow A=-B \Rightarrow B=-2 \\ A-3B=8 \Rightarrow A+3A=8 \Rightarrow A=2 \end{cases}$$

$$\sum_{n=1}^{\infty} \frac{2}{4n-3} - \frac{2}{4n+1}$$

$$S_1 = 2 - \frac{2}{5}$$

$$S_2 = \left(2 - \cancel{\frac{2}{5}}\right) + \cancel{\frac{2}{5}} - \frac{2}{9} = 2 - \frac{2}{9}$$

$$S_3 = \left(2 - \cancel{\frac{2}{9}}\right) + \cancel{\frac{2}{9}} - \frac{2}{13} = 2 - \frac{2}{13}$$

$\vdots$

$$S_K = 2 - \frac{2}{4K+1} = \frac{8K}{4K+1}$$

$$\lim_{K\to+\infty} S_K = \lim_{K\to+\infty} 2 - \cancel{\frac{2}{4K+1}}_{0} = 2$$

$\therefore$ a série converge para 2

c) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$

$$\frac{2n+1}{n^2(n+1)^2} = \frac{An+B}{n^2} + \frac{Cn+D}{(n+1)^2} = \frac{(An+B)(n+1)^2 + (Cn+D)n^2}{n^2(n+1)^2}$$

$$= \frac{(An+B)(n^2+2n+1) + Cn^3 + Dn^2}{n^2(n+1)^2}$$

$$= \frac{An^3 + Bn^2 + 2An^2 + 2Bn + An + B + Cn^3 + Dn^2}{n^2(n+1)^2}$$

$$= \frac{(A+C)n^3 + (B+2A+D)n^2 + (2B+A)n + B}{n^2(n+1)^2}$$

$$\begin{cases} A+C=0 \Rightarrow C=0 \\ B+2A+D=0 \Rightarrow D=-1 \\ 2B+A=2 \Rightarrow A=0 \\ B=1 \end{cases}$$

$$\frac{2n+1}{n^2(n+1)^2} = \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

$$S_1 = 1 - \frac{1}{4}$$

$$S_2 = \left(1 - \cancel{\frac{1}{4}}\right) + \cancel{\frac{1}{4}} - \frac{1}{9} = 1 - \frac{1}{9}$$

$$S_3 = \left(1 - \cancel{\frac{1}{9}}\right) + \cancel{\frac{1}{9}} - \frac{1}{16} = 1 - \frac{1}{16}$$

$\vdots$

$$S_K = 1 - \frac{1}{(K+1)^2} = \frac{K(K+2)}{(K+1)^2}$$

$$\lim_{K \to +\infty} S_K = \lim_{K \to +\infty} 1 - \cancel{\frac{1}{(K+1)^2}}^{0} = 1$$ $\therefore$ a série converge para 1

d) $$\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right) = \sum_{n=1}^{\infty} \ln n - \ln(n+1)$$

$$S_1 = \cancel{\ln 1}^{0} - \ln 2 = -\ln 2 = \ln\left(\frac{1}{2}\right)$$

$$S_2 = -\cancel{\ln 2} + \cancel{\ln 2} - \ln 3 = -\ln 3 = \ln\left(\frac{1}{3}\right)$$

$\vdots$

$$S_K = \ln\left(\frac{1}{K+1}\right)$$

$$\lim_{K \to +\infty} S_K = \lim_{K \to +\infty} \ln\left(\frac{1}{K+1}\right) = -\infty$$

$\therefore$ a série diverge

e) $\sum_{n=1}^{\infty} \frac{2^{n-1}}{5^n} = \sum_{n=1}^{\infty} \frac{1}{5}\left(\frac{2}{5}\right)^{n-1}$

$$S_K = \frac{a\,(1-q^K)}{1-q} = \frac{\frac{1}{5}\left(1-\left(\frac{2}{5}\right)^K\right)}{1-\frac{2}{5}} = \frac{1-\frac{2^K}{5^K}}{3} = \frac{1}{3} - \frac{2^K}{3.5^K}$$

$$\lim_{K\to+\infty} S_K = \frac{a}{1-q} = \frac{\frac{1}{5}}{1-\frac{2}{5}} = \frac{1}{3}$$

∴ A série converge para 1/3

f) $$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}\,(\sqrt{n+1}+\sqrt{n})} \cdot \frac{(\sqrt{n+1}-\sqrt{n})}{(\sqrt{n+1}-\sqrt{n})} = \sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n(n+1)}\,(n+1-n)}$$

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}$$

$$S_1 = 1 - \frac{1}{\sqrt{2}}$$

$$S_2 = \left(1 - \cancel{\frac{1}{\sqrt{2}}}\right) + \cancel{\frac{1}{\sqrt{2}}} - \frac{1}{\sqrt{3}} = 1 - \frac{1}{\sqrt{3}}$$

⋮

$$S_K = 1 - \frac{1}{\sqrt{K+1}}$$

$$\lim_{K\to+\infty} S_K = \lim_{K\to+\infty} 1 - \cancel{\frac{1}{\sqrt{K+1}}}_{\infty} = 1$$

∴ A série converge para 1

$(n+2)(-7)$

g) $\sum_{n=1}^{\infty} \frac{1}{1.2.3.4.5\cdots n(n+2)} = \sum_{n=1}^{\infty} \frac{n+1^{+1-1}}{(n+2)!} = \sum_{n=1}^{\infty} \frac{1}{(n+1)!} - \frac{1}{(n+2)!}$

$S_1 = \frac{1}{2} - \frac{1}{6}$

$S_2 = \left(\frac{1}{2} - \cancel{\frac{1}{6}}\right) + \cancel{\frac{1}{6}} - \frac{1}{24} = \frac{1}{2} - \frac{1}{24}$

$S_3 = \left(\frac{1}{2} - \cancel{\frac{1}{24}}\right) + \cancel{\frac{1}{24}} - \frac{1}{120} = \frac{1}{2} - \frac{1}{120}$

$\vdots$

$S_k = \frac{1}{2} - \frac{1}{(k+2)!}$

$\lim_{k \to +\infty} S_k = \lim_{k \to +\infty} \frac{1}{2} - \cancel{\frac{1}{(k+2)!}}_{+\infty} = \frac{1}{2}$

$\therefore$ a série converge para $1/2$

h) $\sum_{n=1}^{\infty} \frac{3n+4}{n^3+3n^2+2n} = \sum_{n=1}^{\infty} \frac{3n+4}{n(n^2+3n+2)} = \sum_{n=1}^{\infty} \frac{3n+4}{n(n+1)(n+2)}$

$\frac{3n+4}{n(n+1)(n+2)} = \frac{A}{n} + \frac{B}{n+1} + \frac{C}{n+2} = \frac{A(n^2+3n+2) + B(n^2+2n) + C(n^2+n)}{n(n+1)(n+2)}$

$= \frac{(A+B+C)n^2 + (3A+2B+C)n + 2A}{n(n+1)(n+2)}$

$$\begin{cases} A+B+C=0 \Rightarrow C=-B-2 \Rightarrow C=-1 \\ 3A+2B+C=3 \Rightarrow 6+2B-B-2=3 \Rightarrow B=-1 \\ 2A=4 \Rightarrow A=2 \end{cases}$$

$\sum_{n=1}^{\infty} \frac{2}{n} - \frac{1}{n+1} - \frac{1}{n+2}$

$S_1 = 2 - \frac{1}{2} - \frac{1}{3}$

$S_2 = \left(2 - \frac{1}{2} - \frac{1}{3}\right) + 1 - \frac{1}{3} - \frac{1}{4} = \frac{5}{2} - \frac{2}{3} - \frac{1}{4}$

$S_3 = \left(\frac{5}{2} - \frac{1}{4} - \cancel{\frac{2}{3}}\right) + \cancel{\frac{2}{3}} - \frac{1}{4} - \frac{1}{5} = \frac{5}{2} - \frac{2}{4} - \frac{1}{5}$

$S_4 = \left(\frac{5}{2} - \cancel{\frac{2}{4}} - \frac{1}{5}\right) + \cancel{\frac{2}{4}} - \frac{1}{5} - \frac{1}{6} = \frac{5}{2} - \frac{2}{5} - \frac{1}{6}$

$\vdots$

$S_K = \frac{5}{2} - \frac{2}{K+1} - \frac{1}{K+2}$

$\lim_{K \to +\infty} S_K = \lim_{K \to +\infty} \frac{5}{2} - \cancel{\frac{2}{K+1}}_{0} - \cancel{\frac{1}{K+2}}_{0} = \frac{5}{2}$

$\therefore$ A série converge para $5/2$

⑨ a) Falsa, pois $u_n = (-1)^n$ é limitada $[-1,1]$, mas não é convergente

b) Falsa, novamente utilizando $u_n = (-1)^n$, que não é monótona

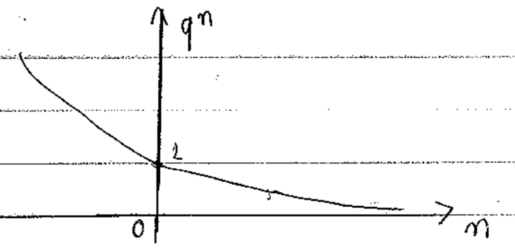
c) Falsa, por exemplo $u_n = \frac{1}{9-n^2}$ é convergente mas não é monótona

d) Falsa, pois $u_n = n - 2^n$ é decrescente e diverge

e) Verdadeiro, pois $u_n > 0$ é o limite inferior e como ela decresce, irá convergir para um valor maior que zero

f) Verdadeira, conforme o gráfico a seguir, no qual podemos perceber que se $n \to +\infty$, $q^n \to 0$

$|q|<1$



g) Falsa, $u_n = \frac{1}{n}$ converge para zero, no entanto, $\sum_{n=1}^{\infty} \frac{1}{n}$ diverge

h) Falsa, pois se $u_n = \frac{1}{n^2}$, $\sum_{n=1}^{\infty} \frac{1}{n^2}$ é convergente, mas $\sum_{n=1}^{\infty} \frac{1}{n}$ diverge.

i) Falsa, pois a série $\sum_{n=1}^{\infty} n(-1)^n$ é alternada e divergente

j) $\sum_{n=1}^{\infty} \frac{(n^3+1)^2}{(n^4+5)(n^2+1)}$

$$\lim_{n\to+\infty} \frac{n^6+2n^3+1}{n^6+n^4+5n^2+5} = \lim_{n\to+\infty} \frac{6n^{\cancel{5}\,4}+6\cancel{n^2}}{6n^{\cancel{5}\,4}+4n^{\cancel{3}\,2}+10\cancel{n}} = \lim_{n\to+\infty} \frac{24n^3+6}{24n^3+8n+10}$$

$$\lim_{n\to+\infty} \frac{72n^2}{72n^2+8} = \lim_{n\to+\infty} \frac{144\cancel{n}}{144\cancel{n}} = 1 \neq 0$$

Como $\lim_{n\to+\infty} u_n = 1$, a série diverge $\therefore$ Falsa

o) $\sum_{n=1}^{\infty} 2^{2n}\, 9^{1-n} = \sum_{n=1}^{\infty} 4\left(\frac{4}{9}\right)^{n-1}$

$$\lim_{k\to+\infty} S_k = \frac{a}{1-q} = \frac{4}{1-\frac{4}{9}} = \frac{36}{5}$$

Então a série converge para $36/5$ $\therefore$ Verdadeira

p) $\sum_{n=3}^{\infty} \frac{1}{n \ln n \ln(\ln n)}$

$\int_3^{\infty} \frac{dx}{x \ln x \ln(\ln x)}$ $\qquad \ln x = u \qquad \ln(u) = v$

$\qquad\qquad\qquad\qquad \frac{dx}{x} = du \qquad \frac{du}{u} = dv$

$\int \frac{du}{u \ln(u)}$

$\int \frac{dv}{v}$

$\ln v$

$\ln(\ln(\ln x)) \Big|_3^{\infty}$

$\ln \infty - \ln(\ln(\ln 3))$

∞

Então a série diverge $\therefore$ Falsa

10) $\sum_{n=1}^{\infty} \frac{4}{4n^2-1} = \sum_{n=1}^{\infty} \frac{1}{n^2 - 1/4} = \sum_{n=1}^{\infty} \frac{1}{(n+1/2)(n-1/2)}$

$\frac{1}{(n+1/2)(n-1/2)} = \frac{A}{(n+1/2)} + \frac{B}{(n-1/2)} = \frac{A(n-1/2)+B(n+1/2)}{(n+1/2)(n-1/2)}$

$= \frac{n(A+B) + 1/2(B-A)}{(n+1/2)(n-1/2)}$

$\begin{cases} A+B=0 \Rightarrow B=-A \Rightarrow A=-1 \\ 1/2(B-A)=1 \Rightarrow 1/2(B+B)=1 \Rightarrow B=1 \end{cases}$

$\sum_{n=1}^{\infty} \frac{2}{2n-1} - \frac{2}{2n+1}$

$S_1 = 2 - \frac{2}{3}$

$$S_2 = \left(2 - \cancel{\frac{2}{3}}\right) + \cancel{\frac{2}{3}} - \frac{2}{5} = 2 - \frac{2}{5}$$

$$S_3 = \left(2 - \cancel{\frac{2}{5}}\right) + \cancel{\frac{2}{5}} - \frac{2}{7} = 2 - \frac{2}{7}$$

$\vdots$

$$S_K = 2 - \frac{2}{2K+1} = \frac{4K}{2K+1}$$

$$\lim_{K \to +\infty} S_K = \lim_{K \to +\infty} 2 - \cancel{\frac{2}{2K+1}}_{\to 0} = 2$$

$\therefore$ a série converge para 2.

(11) a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n = \sum_{n=1}^{\infty} \frac{1}{5}\left(\frac{1}{5}\right)^{n-1}$

$$S_K = \frac{a(q^K - 1)}{q - 1}$$

$$\lim_{K \to +\infty} S_K = \frac{a}{1-q} = \frac{1/5}{1 - 1/5} = \frac{1}{4}$$

$\therefore$ a série converge para $1/4$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{5}{(5n+2)(5n+7)}$

$$\frac{5}{(5n+2)(5n+7)} = \frac{A}{(5n+2)} + \frac{B}{(5n+7)} = \frac{A(5n+7) + B(5n+2)}{(5n+2)(5n+7)}$$

$$= \frac{5(A+B)n + (7A+2B)}{(5n+2)(5n+7)}$$

$$\begin{cases} A+B=0 \Rightarrow B=-A=-1 \\ 7A+2B=5 \Rightarrow 5A=5 \Rightarrow A=1 \end{cases}$$

$$\sum_{n=1}^{\infty} \frac{1}{5n+2} - \frac{1}{5n+7}$$

$$S_1 = \frac{1}{7} - \frac{1}{13}$$

$$S_2 = \left(\frac{1}{7} - \cancel{\frac{1}{12}}\right) + \cancel{\frac{1}{12}} - \frac{1}{17} = \frac{1}{7} - \frac{1}{17}$$

$$S_3 = \left(\frac{1}{7} - \cancel{\frac{1}{17}}\right) + \cancel{\frac{1}{17}} - \frac{1}{22} = \frac{1}{7} - \frac{1}{22}$$

$\vdots$

$$S_K = \frac{1}{7} - \frac{1}{5K+7}$$

$$\lim_{K \to +\infty} S_K = \lim_{K \to +\infty} \frac{1}{7} - \cancel{\frac{1}{\underbrace{5K+7}_{\to \infty}}} = \frac{1}{7}$$

$\therefore$ a série converge para $1/7$

c) $$\sum_{n=1}^{\infty} \frac{1}{n^2+6n+8} = \sum_{n=1}^{\infty} \frac{1}{(n+2)(n+4)}$$

$$\frac{1}{(n+2)(n+4)} = \frac{A}{(n+2)} + \frac{B}{(n+4)} = \frac{A(n+4)+B(n+2)}{(n+2)(n+4)} = \frac{(A+B)n+(4A+2B)}{(n+2)(n+4)}$$

$$\begin{cases} A+B=0 \Rightarrow B=-A=-1/2 \\ 4A+2B=1 \Rightarrow 2A=1 \Rightarrow A=1/2 \end{cases}$$

$$\sum_{n=1}^{\infty} \frac{1}{2(n+2)} - \frac{1}{2(n+4)}$$

$S_1 = \frac{1}{6} - \frac{1}{10}$

$S_2 = \left(\frac{1}{6} - \frac{1}{10}\right) + \frac{1}{8} - \frac{1}{12}$

$S_3 = \left(\frac{1}{6} + \frac{1}{8} - \cancel{\frac{1}{10}} - \frac{1}{12}\right) + \cancel{\frac{1}{10}} - \frac{1}{14} = \frac{1}{6} + \frac{1}{8} - \frac{1}{12} - \frac{1}{14}$

$S_4 = \left(\frac{1}{6} + \frac{1}{8} - \cancel{\frac{1}{12}} - \frac{1}{14}\right) + \cancel{\frac{1}{12}} - \frac{1}{16} = \frac{1}{6} + \frac{1}{8} - \frac{1}{14} - \frac{1}{16}$

$\vdots$

$S_K = \frac{1}{6} + \frac{1}{8} - \frac{1}{2(n+3)} - \frac{1}{2(n+4)}$

$\lim_{K \to +\infty} S_K = \lim_{K \to +\infty} \frac{1}{6} + \frac{1}{8} - \cancel{\frac{1}{2(n+3)}} - \cancel{\frac{1}{2(n+4)}} = \frac{7}{24}$

d) $\sum_{n=1}^{\infty} \frac{-1}{\sqrt{n+1} + \sqrt{n}} \frac{(\sqrt{n+1} - \sqrt{n})}{(\sqrt{n+1} - \sqrt{n})} = \sum_{n=1}^{\infty} \frac{\sqrt{n} - \sqrt{n+1}}{\cancel{n}+1-\cancel{n}} = \sum_{n=1}^{\infty} \sqrt{n} - \sqrt{n+1}$

$S_1 = 1 - \sqrt{2}$

$S_2 = (1 - \cancel{\sqrt{2}}) + \cancel{\sqrt{2}} - \sqrt{3} = 1 - \sqrt{3}$

$S_3 = (1 - \cancel{\sqrt{3}}) + \cancel{\sqrt{3}} - \sqrt{4} = 1 - \sqrt{4}$

$\vdots$

$S_K = 1 - \sqrt{K+1}$

$\lim_{K \to +\infty} S_K = \lim_{K \to +\infty} 1 - \sqrt{K+1} = -\infty$

$\therefore$ a série diverge

(12) a) $\sum_{n=1}^{\infty} \frac{1}{n 3^n}$

$n 3^n \geq 3^n$

$\frac{1}{n3^n} \leq \frac{1}{3^n}$ $\quad \lim_{n\to+\infty} \sqrt[n]{u_n} = \lim_{n\to+\infty} \left(\frac{1}{3^n}\right)^{1/n} = \frac{1}{3} < 1$ (converge)

∴ a série dada converge, por comparação

b) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$

$n^2+1 \geq n^2$

$\frac{1}{n^2+1} \leq \frac{1}{n^2}$

$\frac{\sqrt{n}}{n^2+1} \leq \frac{\sqrt{n}}{n^2} = \frac{1}{n^{3/2}}$

Como $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ é convergente (série-p com $p = \frac{3}{2} > 1$) a série dada também converge, por comparação.

c) $\sum_{n=1}^{\infty} \frac{1}{n^n}$

$\frac{1}{n^n} \leq \frac{1}{n^2}$ (série-p com $p = 2 > 1 \Rightarrow$ convergente)

∴ a série dada converge, por comparação

d) $\sum_{n=1}^{\infty} \frac{n^2}{4n^3+1}$

$4n^3+1 \leq 4n^3 + n^3$

$\frac{n^2}{4n^3+1} \geq \frac{1}{5n}$

Como $\sum_{n=1}^{\infty} \frac{1}{5n}$ diverge (série-p com $p=1$), a série dado diverge por comparação

e) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+4n}}$

$\sqrt{n^2+4n} \leq \sqrt{n^2+4n^2}$

$\frac{1}{\sqrt{n^2+4n}} \geq \frac{1}{\sqrt{5}\,n}$ diverge (série harmônica)

$\therefore$ a série dada diverge, por comparação

f) $\sum_{n=1}^{\infty} \frac{|\operatorname{sen}(n)|}{2^n}$

$\frac{|\operatorname{sen}(2)|}{2^n} \leq \frac{1}{2^n} = \frac{1}{2}\left(\frac{1}{2}\right)^{n-1}$ convergente (série geom. com $|q| = \frac{1}{2} < 1$)

$\therefore$ a série da converge, por comparação

g) $\sum_{n=1}^{\infty} \frac{n!}{(n+2)!} = \sum_{n=1} \frac{\cancel{n!}}{(n+2)(n+1)\cancel{n!}} = \sum_{n=1} \frac{1}{n^2+3n+2}$

$n^2+3n+2 \geq n^2$

$\frac{1}{n^2+3n+2} \leq \frac{1}{n^2}$ convergente (série-p com $p = 2 > 1$)

$\therefore$ a série dada converge, por comparação.

h) $\sum_{n=1} \frac{1}{\sqrt{n^3+5}}$

$\sqrt{n^3+5} \geq \sqrt{n^3}$

$\dfrac{1}{\sqrt{n^3+5}} \leq \dfrac{1}{n^{3/2}}$ converge (série-p com $p = \frac{3}{2} > 1$)

$\therefore$ a série dada converge, por comparação.

i) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+5}}$

$n\sqrt{n^2+5} \geq n\sqrt{n^2}$

$\dfrac{1}{n\sqrt{n^2+5}} \leq \dfrac{1}{n^2}$ converge (série-p com $p=2>1$)

$\therefore$ a série dada converge, por comparação.

j) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n+5}}$

$n + \sqrt{n+5} \leq n+n+5$ $\qquad \cdot\, 2n+5 \leq 7n$

$\dfrac{1}{n+\sqrt{n+5}} \geq \dfrac{1}{2n+5} \geq \dfrac{1}{7n}$ diverge (série harmônica) $\qquad n \geq 1$

$\therefore$ a série dada diverge, por comparação.

k) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{4n^3+n+1}$

$4n^3+n+1 \geq 4n^3$

$\dfrac{n}{4n^3+n+1} \leq \dfrac{n}{4n^3} = \dfrac{1}{4n^2}$ converge (série-p com $p=2>1$)

$\therefore$ a série dada converge, por comparação.

norma

l) $\sum_{n=1}^{\infty} \frac{2^n}{(2n)!}$

$(2n)! \geq n!$

$\frac{2^n}{(2n)!} \leq \frac{2^n}{n!}$

$\lim_{n\to+\infty} \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} = \lim_{n\to+\infty} \frac{2 \cdot \cancel{2^n} \cdot \cancel{n!}}{(n+1) \cdot \cancel{n!} \cdot \cancel{2^n}} = 0 < 1$

Como $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ converge pelo critério da razão, a série dada converge, por comparação.

m) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}+\sqrt{n}}{\sqrt[3]{n}}$

$\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt[3]{n}} \geq \frac{\sqrt{n}+\sqrt{n}}{\sqrt[3]{n}} = \frac{2 \cdot n^{1/2}}{n^{1/3}} = 2n^{1/6}$

$\lim_{n\to+\infty} u_n = \lim_{n\to+\infty} 2n^{1/6} = +\infty \neq 0$

Como $\sum_{n=1}^{\infty} 2\sqrt[6]{n}$ diverge pelo critério do termo geral, a série dada diverge, por comparação.

n) $\sum_{n=1}^{\infty} \frac{1+n \cdot 4^{2n}}{n5^n}$

$\frac{1+n4^{2n}}{n5^n} \geq \frac{n4^{2n}}{n5^n} = \frac{16}{5}\left(\frac{16}{5}\right)^{n-1}$ diverge (série geom. com $|q| = \frac{16}{5} > 1$)

$\therefore$ a série dada diverge, por comparação.

o) $\sum_{n=1}^{\infty} \frac{2+\cos n}{n^2}$

$\frac{2+\cos n}{n^2} \leq \frac{1}{n^2}$ converge (série-p com $p = 2 > 1$)

$\therefore$ a série dada converge, por comparação

p) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+4}$

$n+4 \leq 5n$

$\frac{\sqrt{n}}{n+4} \geq \frac{\sqrt{n}}{5n} = \frac{1}{5\sqrt{n}}$ diverge $\left(\text{série-p com } p = \frac{1}{2} < 1\right)$

$\therefore$ a série dada diverge, por comparação.

q) $\sum_{n=1}^{\infty} \frac{1+2^n}{1+3^n}$

$1+3^n \geq 3^n$

$\frac{1+2^n}{1+3^n} \leq \frac{1+2^n}{3^n} = \frac{1}{3^n} + \left(\frac{2}{3}\right)^n = \frac{1}{3}\left(\frac{1}{3}\right)^{n-1} + \frac{2}{3}\left(\frac{2}{3}\right)^{n-1}$

Como $\sum_{n=1}^{\infty} \frac{1}{3}\left(\frac{1}{3}\right)^{n-1}$ e $\sum_{n=1}^{\infty} \frac{2}{3}\left(\frac{2}{3}\right)^{n-1}$ são série geom com $|q| < 1$

elas convergem, portanto a série dada converge, por comparação

r) $\sum_{n=1}^{\infty} \frac{n+\ln n}{n^3+1}$

$n^3+1 \geq n^3$ $\qquad$ $n+\ln n \leq n+n$

$\frac{n+\ln n}{n^3+1} \leq \frac{n+n}{n^3}$

$\frac{n+\ln n}{n^3+1} \leq \frac{2}{n^2}$ converge (série-p com $p=2>1$)

$\therefore$ a série dada converge, por comparação.

(13) a) $\sum_{n=1}^{\infty} \frac{n+1}{n^2 \cdot 2^n} =$

$L = \lim_{n \to +\infty} \frac{n+2}{(n+1)^2 \cdot 2^{n+1}} \cdot \frac{n^2 \cdot 2^n}{n+1} = \frac{n^3+2n}{2n^3+6n^2+6n+2} = \frac{1}{2}$

Como $L = 1/2 < 1$, a série converge, pelo critério da razão

b) $\sum_{n=1}^{\infty} \frac{n!}{e^n}$

$$L = \lim_{n\to+\infty} \frac{(n+1)!}{e^{n+1}} \cdot \frac{e^n}{n!} = \lim_{n\to+\infty} \frac{(n+1)\cancel{n!}}{e\cdot\cancel{e^n}} \frac{\cancel{e^n}}{\cancel{n!}} = +\infty$$

Como $L = +\infty > 1$ a série dada diverge, pelo critério da razão

c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+1).2^{n+1}}$

$$L = \lim_{n\to+\infty} \frac{(n+1).2^{n+1}}{(n+2).2^{n+2}} = \lim_{n\to+\infty} \frac{(n+1)}{2(n+1)} \frac{\cancel{2^{n+1}}}{\cancel{2^{n+1}}} = \frac{1}{2}$$

Como $L = 1/2 < 1$ a série converge, pelo critério da razão

d) $\displaystyle\sum_{n=1}^{\infty} \frac{3n}{\sqrt{n^3+1}}$

$$L = \lim_{n\to+\infty} \frac{3(n+1)}{\sqrt{(n+1)^3+1}} \cdot \frac{\sqrt{n^3+1}}{3n} = \lim_{n\to+\infty} \underbrace{\frac{3n+3}{3n}}_{1} \cdot \sqrt{\underbrace{\lim_{n\to+\infty} \frac{n^3+1}{n^3+3n^2+3n+2}}_{1}} = 1$$

Como $L = 1$, não é possível afirmar nada.

e) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{2^n(n^2+2)}$

$$L = \lim_{n\to+\infty} \frac{3^{n+1}}{2^{n+1}((n+1)^2+2)} \cdot \frac{2^n(n^2+2)}{3^n} = \lim_{n\to+\infty} \frac{3.\cancel{3^n}.\cancel{2^n}(n^2+2)}{2.\cancel{3^n}.\cancel{2^n}(n^2+2n+3)}$$

$$= \lim_{n\to+\infty} \frac{3n^2+6}{2n^2+4n+6} = \frac{3}{2}$$

Como $L = 3/2 > 1$ a série diverge, pelo critério da razão

f) $\sum_{n=1}^{\infty} \frac{n!}{2^n(2+n)!}$

$L = \lim_{n\to+\infty} \frac{(n+1)!}{2^{n+1}\cdot(n+3)!} \cdot \frac{2^n(n+2)!}{n!} = \lim_{n\to+\infty} \frac{(n+1)\cancel{n!}\,\cancel{(n+2)!}\,\cancel{2^n}}{\cancel{n!}\,(n+3)\cancel{(n+2)!}\,2.\cancel{2^n}} = \frac{1}{2}$

Como $L = 1/2 < 1$ a série converge, pelo critério da razão.

g) $\sum_{n=1}^{\infty} \frac{1}{n+5}$

$L = \lim_{n\to+\infty} \frac{n+5}{n+6} = 1$

Como $L = 1$ nada pode ser afirmado.

h) $\sum_{n=1}^{\infty} \frac{n+1}{n4^n}$

$L = \lim_{n\to+\infty} \frac{n+2}{(n+1).4^{n+1}} \cdot \frac{n.4^n}{n+1} = \lim_{n\to+\infty} \frac{(n^2+2n)\cancel{4^n}}{4(n^2+2n+1).\cancel{4^n}} = \frac{1}{4}$

Como $L = 1/4 < 1$ a série converge, pelo critério da razão

i) $\sum_{n=1}^{\infty} \frac{n}{4n^2+n+1}$

$L = \lim_{n\to+\infty} \frac{n+1}{4(n+1)^2+n+1+1} \cdot \frac{4n^2+n+1}{n} = \lim_{n\to+\infty} \frac{4n^3+n^2+n+4n^2+n+1}{(4(n^2+2n+1)+n+2)\,n}$

$= \lim_{n\to+\infty} \frac{4n^3+5n^2+2n+1}{4n^3+9n^2+6n} = 1$

Como $L = 1$, não é possível afirmar nada.

j) $\sum_{n=1}^{\infty} \frac{3n+1}{2^n}$

$L = \lim_{n\to+\infty} \frac{3(n+1)+1}{2^{n+1}} \cdot \frac{2^n}{3n+1} = \lim_{n\to+\infty} \frac{(3n+4)\cdot \cancel{2^n}}{(3n+1)\cdot 2\cdot \cancel{2^n}} = \frac{1}{2}$

Como $L = 1/2 < 1$ a série converge, pelo critério da razão

K) $\sum_{n=1}^{\infty} \frac{3^n}{n^2+2}$

$L = \lim_{n\to+\infty} \frac{3^{n+1}}{(n+1)^2+2} \cdot \frac{n^2+2}{3^n} = \lim_{n\to+\infty} \frac{3(n^2+2)\cdot \cancel{3^n}}{\cancel{3^n}(n^2+2n+3)} = 3$

Como $L = 3 > 1$ a série diverge, pelo critério da razão.

l) $\sum_{n=1}^{\infty} \frac{n!}{(n+2)^3}$

$L = \lim_{n\to+\infty} \frac{(n+1)!}{(n+3)^3} \cdot \frac{(n+2)^3}{n!} = \lim_{n\to+\infty} \frac{(n+1)\,\cancel{n!}\,(n^3+6n^2+12n+8)}{(n^3+9n^2+27n+27)\,\cancel{n!}}$

$= \lim_{n\to+\infty} \frac{n^4+6n^3+12n^2+8n+n^3+6n^2+12n+8}{n^3+9n^2+27n+27}$

$= \lim_{n\to+\infty} \frac{n^4+7n^3+18n^2+20n+8}{n^3+9n^2+27n+27}$

$= \lim_{n\to+\infty} \frac{4n^3+21n^2+36n+20}{3n^2+18n+27}$

$= \lim_{n\to+\infty} \frac{12n^2+42n+36}{6n+18}$

$= \lim_{n\to+\infty} \frac{24n+42}{6} = +\infty$

$\therefore$ Como $L = +\infty > 1$ a série diverge, pelo critério da razão

m) $\sum_{n=1}^{\infty} \frac{2^{n-1}}{5^n(n+1)}$

$L = \lim_{n\to+\infty} \frac{2^n}{5^{n+1}(n+2)} \cdot \frac{5^n(n+1)}{2^{n-1}} = \lim_{n\to+\infty} \frac{\cancel{2^n}\cdot\cancel{5^n}(n+1)}{5\cdot\cancel{5^n}\cdot\cancel{2^n}(n+2)} = \frac{2}{5}$

Como $L = 2/5 < 1$ a série converge, pelo critério da razão

(14) a) $\sum_{n=1}^{\infty} \frac{(\ln n)^n}{n^{n/2}}$

$L = \lim_{n\to+\infty} \left(\frac{\ln n^n}{n^{n/2}}\right)^{1/n} = \lim_{n\to+\infty} \frac{\ln n}{\sqrt{n}} = \lim_{n\to+\infty} \frac{1/n}{\frac{1}{2\sqrt{n}}} = \lim_{n\to+\infty} \frac{1}{2\sqrt{n}} = 0$

Como $L = 0 < 1$ a série converge, pelo critério da raiz

b) $\sum_{n=1}^{\infty} 2^n \left(\frac{n+1}{n^2}\right)^n$

$L = \lim_{n\to+\infty} \left(2^n\left(\frac{n+1}{n^2}\right)^n\right)^{1/n} = \lim_{n\to+\infty} 2\cdot\left(\frac{n+1}{n^2}\right) = \lim_{n\to+\infty} \frac{2}{2n} = 0$

Como $L = 0 < 1$ a série converge, pelo critério da raiz.

c) $\sum_{n=1}^{\infty} \left(\frac{n+1}{n^2 2^n}\right)^n$

$L = \lim_{n\to+\infty} \left(\left(\frac{n+1}{n^2 2^n}\right)^n\right)^{1/n} = \lim_{n\to+\infty} \frac{n+1}{n^2\cdot 2^n} = \lim_{n\to+\infty} \frac{1}{2n\cdot 2^n + n^2 2^n \ln 2} = 0$

Como $L = 0 < 1$ a série converge, pelo critério da raiz

d) $\sum_{n=1}^{\infty} \frac{n^{4n}-n}{(n^{10n}+1)^{1/2}}$

$(n^{10n}+1)^{1/2} \geq (n^{10n})^{1/2}$

$\frac{1}{(n^{10n}+1)^{1/2}} \leq \frac{1}{n^{5n}}$

$\frac{n^{4n}-n}{(n^{10n}-1)^{1/2}} \leq \frac{n^{4n}+n}{n^{5n}} \leq \frac{n^{4n}}{n^{5n}} = \frac{1}{n^n}$

$L = \lim_{n\to+\infty} \left(\frac{1}{n^n}\right)^{1/n} = \lim_{n\to+\infty} \frac{1}{n} = 0$

Como $L=0<1$, $\sum_{n=1}^{\infty} \frac{1}{n^n}$ é convergente pelo critério da raiz, portanto a série dada é convergente, por comparação.

(15) a) $\sum_{n=1}^{\infty} \frac{n}{e^n}$

$\lim_{b\to+\infty} \int_1^b x.e^{-x}\,dx = \lim_{b\to+\infty} -\frac{x}{e^x} - \frac{1}{e^x}\Big|_1^b = \lim_{b\to+\infty} -\frac{b}{e^b} \cancel{+\frac{1}{e^b}}_{\to 0} + \frac{1}{e} + \frac{1}{e}$

u	dv
$\oplus$ x	e^{-x}
$\ominus$ 1	$-e^{-x}$
	e^{-x}

$= \frac{2}{e} - \lim_{b\to+\infty} \frac{b}{e^b} = \frac{2}{e} - \lim_{b\to+\infty} \cancel{\frac{1}{e^b}}_{\to 0} = \frac{2}{e}$

∴ a série converge, pelo critério da integral.

b) $\sum_{n=1}^{\infty} \frac{\ln n}{n}$

$\begin{cases} u = \ln x \\ du = \frac{dx}{x} \end{cases}$

$\lim_{b\to+\infty} \int_1^b \frac{\ln x}{x}\,dx = \lim_{b\to+\infty} \frac{\ln^2 x}{2}\Big|_1^b = \lim_{b\to+\infty} \frac{\ln^2 b}{2} - \cancel{\frac{\ln^2 1}{2}}_{\to 0} = +\infty$

∴ a série diverge, pelo critério da integral

c) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

$$\begin{cases} u = \ln x \\ du = \frac{dx}{x} \end{cases}$$

$$\lim_{b \to +\infty} \int_2^b \frac{dx}{x \ln x} = \lim_{b \to +\infty} \ln(\ln x) \Big|_2^b = \lim_{b \to +\infty} \ln(\ln(b)) - \ln(\ln(2)) = +\infty$$

∴ A série diverge, pelo critério da integral

d) $\sum_{n=2}^{\infty} \frac{1}{(n+1)\sqrt{\ln(n+1)}}$

$$\begin{cases} u = x+1 & v = \ln u \\ du = dx & dv = \frac{du}{u} \end{cases}$$

$$\lim_{b \to +\infty} \int_1^b \frac{dx}{(x+1)\sqrt{\ln(x+1)}} = \lim_{b \to +\infty} \int \frac{du}{u\sqrt{\ln u}} = \lim_{b \to +\infty} \int \frac{dv}{\sqrt{v}} = \lim_{b \to +\infty} 2\sqrt{v}$$

$$= \lim_{b \to +\infty} 2\sqrt{\ln(x+1)} \Big|_1^b = \lim_{b \to +\infty} 2\sqrt{\ln(b+1)} - 2\sqrt{\ln 2}$$

$$= +\infty$$

∴ A série diverge, pelo critério do integral

e) $\sum_{n=1}^{\infty} \frac{\operatorname{arctg} n}{n^2+1}$

$$\begin{cases} u = \operatorname{arctg} x \\ du = \frac{dx}{x^2+1} \end{cases}$$

$$\lim_{b \to +\infty} \int_1^b \frac{\operatorname{arctg} x}{x^2+1} \, dx = \lim_{b \to +\infty} \frac{\operatorname{arctg}^2 x}{2} \Big|_1^b = \lim_{b \to +\infty} \frac{\operatorname{arctg}^2 b}{2} - \frac{\operatorname{arctg}^2 1}{2}$$

$$= \frac{1}{2}\frac{\pi^2}{4} - \frac{1}{2}\frac{\pi^2}{16} = \frac{4\pi^2 - \pi^2}{32} = \frac{3\pi^2}{32}$$

∴ A série converge, pelo critério da integral

f) $\sum_{n=1}^{\infty} \frac{n}{e^{n^2}}$

$$\begin{cases} u = e^{-x^2} \\ du = -2x e^{-x^2} dx \end{cases}$$

$$\lim_{b\to+\infty}\int_1^b x e^{-x^2}\,dx \cdot \frac{-2}{-2} = \lim_{b\to+\infty} -\frac{e^{-x^2}}{2}\Big|_1^b = \lim_{b\to+\infty} \cancel{-\frac{1}{2e^{b^2}}} + \frac{1}{2e} = \frac{1}{2e}$$

∴ a série converge, pelo critério do integral

g) $\sum_{n=1}^{\infty} \frac{n^2}{e^n}$

$$\lim_{b\to+\infty}\int_1^b x^2 \cdot e^{-x}\,dx = \lim_{b\to+\infty} -x^2e^{-x} - 2xe^{-x} - 2e^{-x}\Big|_1^b$$

$$= \lim_{b\to+\infty} -\frac{b^2}{e^b} - \frac{2b}{e^b} \cancel{-\frac{2}{e^b}} + \frac{1}{e} + \frac{2}{e} + \frac{2}{e}$$

u	dv
$\oplus\ x^2$	e^{-x}
$\ominus\ 2x$	$-e^{-x}$
$\oplus\ 2$	e^{-x}
	$-e^{-x}$

$$= \frac{5}{e} - \lim_{b\to+\infty} \frac{b^2+2b}{e^b} = \frac{5}{e} - \lim_{b\to+\infty} \frac{2b+2}{e^b}$$

$$= \frac{5}{e} - \lim_{b\to+\infty} \cancel{\frac{2}{e^b}} = \frac{5}{e}$$

∴ a série converge, pelo critério do integral

h) $\sum_{n=1}^{\infty} \frac{e^{\operatorname{arctg} n}}{n^2+1}$

$$\begin{cases} u = \operatorname{arctg} x \\ du = \frac{dx}{x^2+1} \end{cases}$$

$$\lim_{b\to+\infty}\int_1^b \frac{e^{\operatorname{arctg} x}}{x^2+1}\,dx = \lim_{b\to+\infty} e^{\operatorname{arctg} x}\Big|_1^b = \lim_{b\to+\infty} e^{\operatorname{arctg} b} - e^{\operatorname{arctg} 1} = e^{\frac{\pi}{2}} - e^{\frac{\pi}{4}}$$

∴ a série converge, pelo critério do integral

i) $\sum_{n=1}^{\infty} \frac{1}{4n+7}$

$$\begin{cases} u = 4x+7 \\ du = 4dx \end{cases}$$

$$\lim_{b\to+\infty}\int_1^b \frac{dx}{4x+7} \cdot \frac{4}{4} = \lim_{b\to+\infty} \frac{1}{4}\ln(4x+7)\Big|_1^b = \lim_{b\to+\infty} \frac{\ln(4b+7)}{4} - \frac{\ln 11}{4} = +\infty$$

∴ a série diverge, pelo critério do integral.

j) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$

$$\begin{cases} \theta = \operatorname{arctg} x & x = \operatorname{tg}\theta \\ & dx = \sec^2\theta\, d\theta \end{cases}$$

$$\lim_{b\to+\infty} \int_1^b \frac{dx}{x\sqrt{x^2+1}} = \lim_{b\to+\infty} \int_{\frac{\pi}{4}}^{\operatorname{arctg} b} \frac{\sec^2\theta\, d\theta}{\operatorname{tg}\theta\sqrt{\operatorname{tg}^2\theta+1}} = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sec^{\cancel{2}}\theta}{\operatorname{tg}\theta\, \cancel{\sec\theta}}\, d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{d\theta}{\operatorname{sen}\theta}$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \operatorname{cosec}\theta\, d\theta = \ln|\operatorname{cosec}\theta - \operatorname{cotg}\theta| \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \ln\left|\frac{1-\cos\theta}{\operatorname{sen}\theta}\right| \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \ln\left|\frac{1-0}{1}\right| - \ln\left|\frac{1-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}\right| = \underbrace{\cancel{\ln 1}}_{0} - \ln(\sqrt{2}-1) = -\ln(\sqrt{2}-1)$$

∴ a série converge, pelo critério do integral

k) $\sum_{n=1}^{\infty} \frac{1}{n(1+\ln^2 n)}$

$$\begin{cases} u = \ln x \\ du = \frac{dx}{x} \end{cases}$$

$$\lim_{b\to+\infty} \int_1^b \frac{dx}{x(1+\ln^2 x)} = \lim_{b\to+\infty} \int \frac{du}{1+u^2} = \lim_{b\to+\infty} \operatorname{arctg}(\ln^2 x) \Big|_1^b$$

$$= \lim_{b\to+\infty} \operatorname{arctg}(\ln^2 b) - \underbrace{\cancel{\operatorname{arctg}(\ln^2 1)}}_{0} = \frac{\pi}{2}$$

∴ a série converge, pelo critério do integral

16) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$

$$\begin{cases} u = \ln x \\ du = \frac{dx}{x} \end{cases}$$

$$\lim_{b\to+\infty} \int_2^b \frac{dx}{x(\ln x)^p} = \lim_{b\to+\infty} \int \frac{du}{u^p} = \begin{cases} \lim_{b\to+\infty} \frac{(\ln x)^{1-p}}{1-p} \Big|_2^b, \text{ se } p \neq 1 & (I) \\ \lim_{b\to+\infty} \ln(\ln x) \Big|_2^b, \text{ se } p = 1 & (II) \end{cases}$$

$$(I) \lim_{b\to+\infty} \frac{(\ln x)^{1-p}}{1-p} \Big|_2^b = \lim_{b\to+\infty} \frac{(\ln b)^{1-p}}{1-p} - \frac{(\ln 2)^{1-p}}{1-p} = \begin{cases} +\infty, \text{ se } p < 1 \\ \frac{(\ln 2)^{1-p}}{p-1}, \text{ se } p > 1 \end{cases}$$

(II) $\lim_{b\to+\infty} \ln(\ln x)\Big|_2^b = \lim_{b\to+\infty} \ln(\ln b) - \ln(\ln 2) = \infty$

$\therefore$ a série apenas convergirá para os valores de $p > 1$

17 a) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^n}{n!}$

$\left|\frac{(-1)^{n-1}\,2^n}{n!}\right| = \frac{2^n}{n!}$

$L = \lim_{n\to+\infty} \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} = \lim_{n\to+\infty} \frac{\cancel{2^n}\cdot 2\cdot \cancel{n!}}{(n+1)\cdot\cancel{n!}\cdot\cancel{2^n}} = 0$

Como $L = 0 < 1$, o módulo da série converge, pelo critério da razão, e, pelo teorema, a série dada converge.

$\therefore$ Absolutamente convergente

b) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{(2n-1)!}$

$\left|(-1)^{n-1} \cdot \frac{1}{(2n-1)!}\right| = \frac{1}{(2n-1)!}$

$L = \lim_{n\to+\infty} \frac{1}{(2n+1)!} \cdot (2n-1)! = \lim_{n\to+\infty} \frac{\cancel{(2n-1)!}}{(2n+1)\cdot 2n\,\cancel{(2n-1)!}} = 0$

Como $L = 0 < 1$, o módulo da série converge, pelo critério da razão, e, pelo teorema, a série dada converge.

$\therefore$ Absolutamente convergente

c) $\sum_{n=1}^{\infty} (-1)^{n-1} \dfrac{n^2}{n!}$

$$\left| (-1)^{n-1} \cdot \frac{n^2}{n!} \right| = \frac{n^2}{n!}$$

$$L = \lim_{n \to +\infty} \frac{(n+1)^2}{(n+1)!} \cdot \frac{n!}{n^2} = \lim_{n \to +\infty} \frac{(n^2+2n+2) \cdot \cancel{n!}}{n^2(n+1)\cancel{n!}} = \lim_{n \to +\infty} \frac{n^2+2n+2}{n^3+n^2}$$

$$= \lim_{n \to +\infty} \frac{2n+2}{3n^2+2n} = \lim_{n \to +\infty} \frac{2}{6n+2} = 0$$

Como $L = 0 < 1$, o módulo da série converge, pelo critério da razão, e, pelo teorema, a série dada converge.

$\therefore$ Absolutamente convergente.

d) $\sum_{n=1}^{\infty} (-1)^{n-1}\, n \left(\dfrac{2}{3}\right)^n$

$$\left| (-1)^{n-1} \cdot n \left(\frac{2}{3}\right)^n \right| = n \left(\frac{2}{3}\right)^n$$

$$L = \lim_{n \to +\infty} (n+1) \cdot \left(\frac{2}{3}\right)^{n+1} \cdot \frac{1}{n} \cdot \left(\frac{3}{2}\right)^n = \lim_{n \to +\infty} \frac{2(n+1) \cdot \cancel{2^n} \cdot \cancel{3^n}}{3 \cdot \cancel{3^n} \cdot n \cdot \cancel{2^n}} = \frac{2}{3}$$

Como $L = 2/3 < 1$, o módulo da série converge, pelo critério da razão, e, pelo teorema, a série dada converge.

$\therefore$ Absolutamente convergente.