Exercícios de CDI-2

Cap2

(24) $z = 4x^2 - 2y^2$, $P(-1,1,2)$

$\frac{\partial z}{\partial x} = 8x \qquad \frac{\partial z}{\partial y} = -4y$

$\vec{n} = (-8x_0, 4y_0, 1) = (8,4,1)$

$\pi: 8x + 4y + z + d = 0$

$-8 + 4 + 2 + d = 0$

$d = 2$

$\therefore \pi: 8x + 4y + z + 2 = 0$

(25) $z = -12x^2 + 3y^2$, $P(1,4,36)$

$\frac{\partial z}{\partial x} = -24x \qquad \frac{\partial z}{\partial x} = 6y$

$\vec{n} = (24, -24, 1)$

$\pi: 24x - 24y + z + d = 0$

$24 - 96 + 36 + d = 0$

$d = 36$

$\pi: 24x - 24y + z + 36 = 0$

(26) $z = 3x^2 - y^2$ e $\pi // 6x + 4y - z - 5 = 0$

$\vec{n} = K(6,4,-1)$

$\frac{\partial z}{\partial x} = 6x \qquad \frac{\partial z}{\partial y} = -2y$

$\vec{n} = (-6x_0, 2y_0, 1)$

$\begin{cases} -6x_0 = 6K \\ 2y_0 = 4K \\ 1 = -K \end{cases} \quad \therefore K = -1;\ x_0 = 1,\ y_0 = -2$

$z = 3x_0^2 - y_0^2 = 3 - 4 = -1$

$\therefore P(1,-2,-1)$

(27) $P(1,1,1)$ e $\pi // 2x + y + 3z - 6 = 0$

$\vec{n} = K(2,1,3) = \left(-\frac{\partial f}{\partial x}(1,1), -\frac{\partial f}{\partial y}(1,1), 1\right) \Rightarrow 3K = 1$

$K = \frac{1}{3}$

$\frac{\partial f}{\partial x}(1,1) = -\frac{2}{3} \qquad \frac{\partial f}{\partial y} = -\frac{1}{3}$

(28) $f(x,y) = \dfrac{x^3}{x^2+y^2}$, $P(a, b, f(a,b))$, $O(0,0,0)$

$$\frac{\partial f}{\partial x} = \frac{3x^2(x^2+y^2) - x^3(2x)}{(x^2+y^2)^2} = \frac{3x^4+3x^2y^2-2x^4}{(x^2+y^2)^2} = \frac{x^4+3x^2y^2}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y} = \frac{0\cdot(x^2+y^2) - 2y \cdot x^3}{(x^2+y^2)^2} = \frac{-2x^3y}{(x^2+y^2)^2}$$

$$\vec{n} = \left(-\frac{a^4-3a^2b^2}{(a^2+b^2)^2}, \frac{-2a^3b}{(a^2+b^2)^2}, 1\right)$$

$$\pi: -\left(\frac{a^4+3a^2b^2}{(a^2+b^2)^2}\right)x + \left(\frac{2a^3b}{(a^2+b^2)^2}\right)y + z + d = 0 \Rightarrow O \in \pi \therefore d = 0$$

$$\pi: (a^4+3a^2b^2)x - 2a^3by - (a^2+b^2)^2 z = 0$$

(29) $P_1(1,1,2)$ e $P_2(-1,1,1) \in \pi$, $f(x,y) = xy$, $P(x_0, y_0, z_0)$

$f_x = y$ e $f_y = x$

$\vec{n} = (-y_0, -x_0, 1)$

$\pi: -y_0x - x_0y + z + d = 0$

$$\begin{cases} -y_0 \cdot 1 - x_0 \cdot 1 + 2 + d = 0 \\ -y_0 \cdot (-1) - x_0 \cdot 1 + 1 + d = 0 \\ -y_0 \cdot x_0 - \cancel{x_0 \cdot y_0} + \cancel{x_0 y_0} + d = 0 \end{cases} \Rightarrow -y_0 - \cancel{x_0} + 2 = y_0 - \cancel{x_0} + 1$$

$2y_0 = 1$

$y_0 = \dfrac{1}{2}$

$\therefore d = x_0 y_0 = 3 \cdot \dfrac{1}{2} = \dfrac{3}{2}$

$-y_0 - x_0 + 2 + x_0 y_0 = 0$

$-\dfrac{1}{2} - x_0 + 2 + x_0 \cdot \dfrac{1}{2} = 0$

$-\dfrac{x_0}{2} + \dfrac{3}{2} = 0$

$\therefore x_0 = 3$

$\pi: -\dfrac{1}{2}x - 3y + z + \dfrac{3}{2} = 0$

$\pi: x + 6y - 2z - 3 = 0$

30) $f(x,y) = 2 + x^2 + y^2$ e $g(x,y) = -x^2 - y^2$

a) $P(1,2,7)$ | $\pi_1 // \pi_2$

$\frac{\partial f}{\partial x} = 2x$ $\frac{\partial f}{\partial y} = 2y$ | $\frac{\partial g}{\partial x} = -2x$ $\frac{\partial g}{\partial y} = -2y$

$\vec{n}_1 = (-2,-4,1)$ | $\vec{n}_2 = K(-2,-4,1) = (2x_0, 2y_0, 1)$

$\pi_1: -2x - 4y + z + d = 0$
$-2 - 8 + 7 + d = 0$
$d = 3$
$\pi_1: 2x + 4y - z - 3 = 0$

$\begin{cases} 2x_0 = -2K \Rightarrow x_0 = -1 \\ 2y_0 = -4K \Rightarrow y_0 = -2 \\ 1 = K \quad z_0 = g(x_0,y_0) = -5 \end{cases}$

$\therefore P(-1,-2,-5)$

31) $f(x,y) = \sqrt{100 + 4y^2 - 25x^2}$

a) $100 + 4y^2 - 25x^2 \geq 0$
$25x^2 - 4y^2 \leq 100$
$\frac{x^2}{4} - \frac{y^2}{25} \leq 1$

$D = \left\{ (x,y) \in \mathbb{R}^2 \,/\, \frac{x^2}{4} - \frac{y^2}{25} \leq 1 \right\}$

a região sobre ou no interior da hipérbole

b) $\vec{n} // \vec{v}$, $\vec{v} = (0,1,2)$

$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{100+4y^2-25x^2}} \cdot -50x = \frac{-25x}{\sqrt{100+4y^2-25x^2}}$

$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{100+4y^2-25x^2}} \cdot 8y = \frac{4y}{\sqrt{100+4y^2-25x^2}}$

$\vec{n} = K(0,1,2) = \left(\frac{25x_0}{\sqrt{100+4y_0^2-25x_0^2}}, \frac{-4y_0}{\sqrt{100+4y_0^2-25x_0^2}}, \frac{1}{0} \right)$

$\therefore x_0 = 0 \quad K = \frac{1}{2}$

$$\frac{-4y_0}{\sqrt{100+4y_0^2-25x_0^2}} = \frac{1}{2} \qquad z_0 = \sqrt{100+4.\frac{5}{3}}$$

$$-8y_0 = \sqrt{100+4y_0^2} \qquad z_0 = \sqrt{\frac{320}{3}} = 8\sqrt{\frac{5}{3}}$$

$$64y_0^2 = 100+4y_0^2$$

$$60y_0^2 = 100$$

$$y_0 = \pm\sqrt{\frac{5}{3}} \qquad P_1 = \left(0, \sqrt{\frac{5}{3}}, 8\sqrt{\frac{5}{3}}\right) \text{ e } P_2\left(0, -\sqrt{\frac{5}{3}}, 8\sqrt{\frac{5}{3}}\right)$$

(32) $f(x,y) = \sqrt{36-9x^2-4y^2}$, $C = S \cap y=2$

a) $36-9x^2-4y^2 \geq 0$ $\quad D = \left\{(x,y)\in\mathbb{R}^2 \,/\, \frac{x^2}{4}+\frac{y^2}{9}\leq 1\right\}$

$9x^2+4y^2 \leq 36$

$\frac{x^2}{4}+\frac{y^2}{9} \leq 1$

↳ região sobre ou no interior da elipse

b) $P(1,2,\sqrt{11})$

$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{36-9x^2-4y^2}} \cdot -2.9x^2 \qquad \frac{\partial f}{\partial x}(1,2) = \frac{-9}{\sqrt{36-9-16}} = \frac{-9}{\sqrt{11}}$$

$$t: \begin{cases} y = 2 \\ z - \sqrt{11} = \frac{-9}{\sqrt{11}}(x-1) \end{cases}$$

(33) $w = (x^2+y^2+z^2)^K$

$$\frac{\partial w}{\partial x} = K(x^2+y^2+z^2)^{K-1}. 2x$$

$$\frac{\partial^2 w}{\partial x^2} = 2K(x^2+y^2+z^2)^{K-1} + 2Kx(K-1).(x^2+y^2+z^2)^{K-2}. 2x$$

$$\frac{\partial^2 w}{\partial x^2} = 2K(x^2+y^2+z^2)^{K-1} + 4x^2K(K-1)(x^2+y^2+z^2)^{K-2}$$

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial y^2} = 0$$

$$6K(x^2+y^2+z^2)^{K-1} + 4K(K-1)(x^2+y^2+z^2)^{K-2}(x^2+y^2+z^2) = 0$$

$$6K(x^2+y^2+z^2)^{K-1} + 4K(K-1)(x^2+y^2+z^2)^{K-1} = 0$$

$$6K + 4K^2 - 4K = 0$$

$$2K(2K+1) = 0$$

$$K = 0 \text{ e } K = -\frac{1}{2}$$

(34) $z = f(u)$, $u = x + ay^2$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y} = \frac{\partial z}{\partial u} \cdot 2ay$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} = \frac{\partial z}{\partial u} \cdot 1$$

$$\frac{\partial z}{\partial y} - 2ay\frac{\partial z}{\partial x} = 2ay\frac{\partial z}{\partial u} - 2ay\frac{\partial z}{\partial u} = 0$$

(35) $f(x-y, y-z, z-x) = f(u,v,t)$

$$\begin{cases} u = x - y \\ v = y - z \\ t = z - x \end{cases}$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial x} + \frac{\partial f}{\partial t} \cdot \frac{\partial t}{\partial x}$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u} \cdot 1 + \frac{\partial f}{\partial v} \cdot 0 + \frac{\partial f}{\partial t} \cdot -1$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u} - \frac{\partial f}{\partial t}$$

$$\frac{\partial f}{\partial y}=\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial y}+\frac{\partial f}{\partial t}\cdot\frac{\partial t}{\partial y}$$

$$\frac{\partial f}{\partial y}=\frac{\partial f}{\partial u}\cdot(-1)+\frac{\partial f}{\partial v}\cdot 1+\frac{\partial f}{\partial t}\cdot 0$$

$$\frac{\partial f}{\partial y}=-\frac{\partial f}{\partial u}+\frac{\partial f}{\partial v}$$

$$\frac{\partial f}{\partial z}=\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial z}+\frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial z}+\frac{\partial f}{\partial t}\cdot\frac{\partial t}{\partial z}$$

$$\frac{\partial f}{\partial z}=\frac{\partial f}{\partial u}\cdot 0+\frac{\partial f}{\partial v}\cdot -1+\frac{\partial f}{\partial t}\cdot 1$$

$$\frac{\partial f}{\partial z}=-\frac{\partial f}{\partial v}+\frac{\partial f}{\partial t}$$

$$\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}+\frac{\partial f}{\partial z}=\frac{\partial f}{\partial u}-\frac{\partial f}{\partial t}-\frac{\partial f}{\partial u}+\frac{\partial f}{\partial v}-\frac{\partial f}{\partial v}+\frac{\partial f}{\partial t}=0$$

36) $f\left(\frac{y-x}{xy},\frac{z-y}{yz}\right)=f(u,v)$

$$\begin{cases} u=\frac{y-x}{xy}=\frac{1}{x}-\frac{1}{y} \\ v=\frac{z-y}{yz}=\frac{1}{y}-\frac{1}{z} \end{cases}$$

$$\frac{\partial f}{\partial x}=\frac{\partial f}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial f}{\partial v}\frac{\partial v}{\partial x}=\frac{\partial f}{\partial u}\cdot\frac{-1}{x^2}+\frac{\partial f}{\partial v}\cdot 0=\frac{-1}{x^2}\frac{\partial f}{\partial u}$$

$$\frac{\partial f}{\partial y}=\frac{\partial f}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial y}=\frac{\partial f}{\partial u}\cdot\frac{1}{y^2}+\frac{\partial f}{\partial v}\cdot\frac{-1}{y^2}=\frac{1}{y^2}\left(\frac{\partial f}{\partial u}-\frac{\partial f}{\partial v}\right)$$

$$\frac{\partial f}{\partial z}=\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial z}+\frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial z}=\frac{\partial f}{\partial u}\cdot 0+\frac{\partial f}{\partial v}\cdot\frac{1}{z^2}=\frac{1}{z^2}\frac{\partial f}{\partial v}$$

$$x^2\frac{\partial f}{\partial x}+y^2\frac{\partial f}{\partial y}+z^2\frac{\partial f}{\partial z}=-\frac{\partial f}{\partial u}+\frac{\partial f}{\partial u}-\frac{\partial f}{\partial v}+\frac{\partial f}{\partial v}=0$$

37 $w=x^3 f\left(\frac{y}{x},\frac{x}{z},\frac{z}{x}\right)=x^3 f(u,v,t)$

$\left\{u=\frac{y}{x};\ v=\frac{x}{z};\ t=\frac{z}{x}\right.$

$$\frac{\partial w}{\partial x}=3x^2 f(u,v,t)+x^3\frac{\partial f}{\partial x}$$

$$\frac{\partial w}{\partial x}=3x^2 f(u,v,t)+x^3\left(\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial x}+\frac{\partial f}{\partial t}\cdot\frac{\partial t}{\partial x}\right)$$

$$\frac{\partial w}{\partial x}=3x^2 f(u,v,t)+x^3\left(\frac{\partial f}{\partial u}\cdot\frac{-y}{x^2}+\frac{\partial f}{\partial v}\cdot\frac{1}{z}+\frac{\partial f}{\partial t}\cdot\frac{-z}{x^2}\right)$$

$$\frac{\partial w}{\partial x}=3x^2 f(u,v,t)-xy\frac{\partial f}{\partial u}+\frac{x^3}{z}\frac{\partial f}{\partial v}-xz\frac{\partial f}{\partial t}$$

$$\frac{\partial w}{\partial y}=x^3\left(\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial y}+\frac{\partial f}{\partial t}\cdot\frac{\partial t}{\partial y}\right)$$

$$\frac{\partial w}{\partial y}=x^3\left(\frac{\partial f}{\partial u}\cdot\frac{1}{x}+\frac{\partial f}{\partial v}\cdot 0+\frac{\partial f}{\partial t}\cdot 0\right)$$

$$\frac{\partial w}{\partial y}=x^2\frac{\partial f}{\partial u}$$

$$\frac{\partial w}{\partial z}=x^3\left(\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial z}+\frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial z}+\frac{\partial f}{\partial t}\cdot\frac{\partial t}{\partial z}\right)$$

$$\frac{\partial w}{\partial z}=x^3\left(\frac{\partial f}{\partial u}\cdot 0+\frac{\partial f}{\partial v}\cdot\frac{-x}{z^2}+\frac{\partial f}{\partial t}\cdot\frac{1}{x}\right)$$

$$\frac{\partial w}{\partial z}=-\frac{x^4}{z^2}\frac{\partial f}{\partial v}+x^2\frac{\partial f}{\partial t}$$

norma

$$x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z}$$

$$3x^3 f(u,v,t) - \cancel{x^2 y\frac{\partial f}{\partial u}} + \cancel{\frac{x^4}{z}\frac{\partial f}{\partial v}} - \cancel{x^2 z\frac{\partial f}{\partial t}} + \cancel{x^2 y\frac{\partial f}{\partial u}} - \cancel{\frac{x^4}{z}\frac{\partial f}{\partial v}} + \cancel{x^2 z\frac{\partial f}{\partial t}}$$

$$3x^3 f(u,v,t)$$

$$3w$$

(38) $w = f(x^2 - at) + g(x + at^2)$

$$\begin{cases} u = x^2 - at \\ v = x + at^2 \end{cases}$$

$$\frac{\partial w}{\partial t} = \frac{\partial f}{\partial t} + \frac{\partial g}{\partial t} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial t} + \frac{\partial g}{\partial v}\cdot\frac{\partial v}{\partial t} = \frac{\partial f}{\partial u}\cdot -a + \frac{\partial g}{\partial v}\cdot 2at$$

$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial w}{\partial t}\right) = \frac{\partial}{\partial t}\left(-a\frac{\partial f}{\partial u} + 2at\frac{\partial g}{\partial v}\right)$$

$$= -a\frac{\partial}{\partial t}\left(\frac{\partial f}{\partial u}\right) + 2a\left(\frac{\partial}{\partial t}\left(t\frac{\partial g}{\partial v}\right)\right)$$

$$= -a\frac{\partial}{\partial u}\left(\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial t}\right) + 2a\left(\frac{\partial g}{\partial v} + t\frac{\partial}{\partial t}\left(\frac{\partial g}{\partial v}\right)\right)$$

$$= -a\frac{\partial}{\partial u}\left(\frac{\partial f}{\partial u}\cdot -a\right) + 2a\frac{\partial g}{\partial v} + 2at\frac{\partial}{\partial v}\left(\frac{\partial g}{\partial v}\cdot\frac{\partial v}{\partial t}\right)$$

$$= a^2\frac{\partial^2 f}{\partial u^2} + 2a\frac{\partial g}{\partial v} + 2at\cdot\frac{\partial}{\partial v}\cdot\left(\frac{\partial g}{\partial v}\cdot 2at\right)$$

$$= a^2\frac{\partial^2 f}{\partial u^2} + 2a\frac{\partial g}{\partial v} + 4a^2t^2\frac{\partial^2 g}{\partial v^2}$$

$$\frac{\partial w}{\partial x} = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial x} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial g}{\partial v}\cdot\frac{\partial v}{\partial x} = \frac{\partial f}{\partial u}\cdot 2x + \frac{\partial g}{\partial v}$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}\right) = \frac{\partial}{\partial x}\left(2x\,\frac{\partial f}{\partial u} + \frac{\partial g}{\partial v}\right) = 2\frac{\partial}{\partial x}\left(x\frac{\partial f}{\partial u}\right) + \frac{\partial}{\partial x}\left(\frac{\partial g}{\partial v}\right)$$

$$= 2\left(\frac{\partial f}{\partial u} + x\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial u}\right)\right) + \frac{\partial}{\partial v}\left(\frac{\partial g}{\partial v}\cdot\frac{\partial v}{\partial x}\right)$$

$$= 2\left(\frac{\partial f}{\partial u} + x\frac{\partial}{\partial u}\left(\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x}\right)\right) + \frac{\partial}{\partial v}\left(\frac{\partial g}{\partial v}\cdot 1\right)$$

$$= 2\left(\frac{\partial f}{\partial u} + x\frac{\partial}{\partial u}\left(\frac{\partial f}{\partial u}\cdot 2x\right)\right) + \frac{\partial^2 g}{\partial v^2}$$

$$= 4x^2\frac{\partial^2 f}{\partial u^2} + 2\frac{\partial f}{\partial u} + \frac{\partial^2 g}{\partial v^2}$$

(39) $w = f(u) + g(v)$

$$\begin{cases} u(x,t) = x^2 + t^2 \\ v(x,t) = x^2 - t^2 \end{cases}$$

$$\frac{\partial w}{\partial x} = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial x} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial g}{\partial v}\cdot\frac{\partial v}{\partial x} = \frac{\partial f}{\partial u}\cdot 2x + \frac{\partial g}{\partial v}\cdot 2x = 2x\left(\frac{\partial f}{\partial u} + \frac{\partial g}{\partial v}\right)$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial}{\partial x}\left(2x\left(\frac{\partial f}{\partial u} + \frac{\partial g}{\partial v}\right)\right) = 2\left(\frac{\partial f}{\partial u} + \frac{\partial g}{\partial v}\right) + 2x\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial u} + \frac{\partial g}{\partial v}\right)$$

$$= 2\frac{\partial f}{\partial u} + 2\frac{\partial g}{\partial v} + 2x\frac{\partial}{\partial u}\left(\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x}\right) + 2x\frac{\partial}{\partial v}\left(\frac{\partial g}{\partial v}\cdot\frac{\partial v}{\partial x}\right)$$

$$= 2\frac{\partial f}{\partial u} + 2\frac{\partial g}{\partial v} + 4x^2\frac{\partial^2 f}{\partial u^2} + 4x^2\frac{\partial^2 g}{\partial v^2}$$

$$\frac{\partial w}{\partial t} = \frac{\partial f}{\partial t} + \frac{\partial g}{\partial t} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial t} + \frac{\partial g}{\partial v}\cdot\frac{\partial v}{\partial t} = \frac{\partial f}{\partial u}\cdot 2t + \frac{\partial g}{\partial v}\cdot -2t = 2t\left(\frac{\partial f}{\partial u} - \frac{\partial g}{\partial v}\right)$$

$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial t}\left(2t\left(\frac{\partial f}{\partial u} - \frac{\partial g}{\partial v}\right)\right) = 2\left(\frac{\partial f}{\partial u} - \frac{\partial g}{\partial v}\right) + 2t\left(\frac{\partial}{\partial t}\left(\frac{\partial f}{\partial u} - \frac{\partial g}{\partial v}\right)\right)$$

norma

$$\frac{\partial^2 w}{\partial t^2} = 2\frac{\partial f}{\partial u} - 2\frac{\partial g}{\partial v} + 2t\frac{\partial}{\partial u}\left(\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial t}\right) - 2t\frac{\partial}{\partial v}\left(\frac{\partial g}{\partial v}\cdot\frac{\partial v}{\partial t}\right)$$

$$= 2\frac{\partial f}{\partial u} - 2\frac{\partial g}{\partial v} + 4t^2\frac{\partial^2 f}{\partial u^2} + 4t^2\frac{\partial^2 g}{\partial v^2}.$$

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial t^2} = 2\frac{\partial f}{\partial u} + \cancel{2\frac{\partial f}{\partial g}} + 4x^2\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 g}{\partial v^2}\right) + 2\frac{\partial f}{\partial u} - \cancel{2\frac{\partial g}{\partial \theta}} + 4t^2\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 g}{\partial v^2}\right)$$

$$= 4\frac{\partial f}{\partial u} + 4(x^2+t^2)\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 g}{\partial v^2}\right)$$

40 $w = f(x,y)$

$$\begin{cases} x(r,\theta) = r\cos\theta \\ y(r,\theta) = r\,\text{sen}\,\theta \end{cases}$$

$$\frac{\partial w}{\partial r} = \frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial r} = \frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\cdot\text{sen}\,\theta$$

$$\frac{\partial w}{\partial \theta} = \frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial \theta} = \frac{\partial f}{\partial x}\, r(-\text{sen}\,\theta) + \frac{\partial f}{\partial y}\cdot r\cos\theta$$

$$\frac{\partial w}{\partial x} = \frac{\partial f}{\partial x}$$

$$\frac{\partial w}{\partial y} = \frac{\partial f}{\partial y}$$

$$\left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2 =$$

$$\cos^2\theta\left(\frac{\partial f}{\partial x}\right)^2 + 2\cos\theta\,\text{sen}\,\theta\,\frac{\partial f}{\partial x}\cdot\frac{\partial f}{\partial y} + \text{sen}^2\theta\left(\frac{\partial f}{\partial y}\right)^2 + \frac{1}{r^2}\left(r^2\text{sen}^2\theta\left(\frac{\partial f}{\partial x}\right)^2 -\right.$$

$$\left. -2r^2\cos\theta\,\text{sen}\,\theta\,\frac{\partial f}{\partial x}\frac{\partial f}{\partial y} + r^2\cos^2\theta\left(\frac{\partial f}{\partial y}\right)^2\right)$$

$$\cos^2\theta\left(\frac{\partial f}{\partial x}\right)^2 + \text{sen}(2\theta)\cancel{\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}} + \text{sen}^2\theta\left(\frac{\partial f}{\partial y}\right)^2 + \text{sen}^2\theta\left(\frac{\partial f}{\partial x}\right)^2 - \text{sen}(2\theta)\cancel{\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}} +$$

$$+\cos^2\theta\left(\frac{\partial f}{\partial y}\right)^2$$

$$\left(\frac{\partial f}{\partial x}\right)^2(\cos^2\theta + \text{sen}^2\theta) + \left(\frac{\partial f}{\partial y}\right)^2(\cos^2\theta + \text{sen}^2\theta)$$

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2$$

$$\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2$$

(41) $g(t) = t \cdot \frac{\partial f}{\partial y}(2t, t^3) = t \cdot \frac{\partial f}{\partial y}(x,y)$ $\quad \begin{cases} x = 2t \\ y = t^3 \end{cases}$

$$\frac{dg}{dt} = \frac{\partial f}{\partial y} + t \cdot \frac{d}{dt}\left(\frac{\partial f}{\partial y}\right)$$

$$\frac{dg}{dt} = \frac{\partial f}{\partial y} + t \cdot \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial f}{\partial y}\cdot\frac{dy}{dt}\right)$$

$$\frac{dg}{dt} = \frac{\partial f}{\partial y} + t \cdot \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\cdot 2 + \frac{\partial f}{\partial y}\cdot 3t^2\right)$$

$$\frac{dg}{dt} = \frac{\partial f}{\partial y} + 2t\frac{\partial^2 f}{\partial x \partial y} + 3t^3\frac{\partial^2 f}{\partial y^2}$$

(42) $F(x,y) = f(\text{sen}\,x, \cos y)$

$\begin{cases} u = \text{sen}\,x \\ v = \cos y \end{cases}$ $\quad \frac{\partial F}{\partial x} = \frac{\partial f}{\partial x} \quad \frac{\partial F}{\partial y} = \frac{\partial f}{\partial y} \quad \frac{\partial f}{\partial u}(0,1) = \frac{\partial f}{\partial v}(0,1) = 2$

$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} = 2\cos x = 2$ $\quad\quad \text{sen}\,x = 0$, $x = 0$

$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial y} = 2 \cdot -\text{sen}\,y = 0$ $\quad\quad \cos y = 1$, $y = 0$

(43) $g(x,y) = f(u,v)$, $P(0,1,10)$

$u = \sqrt{x^3} + \ln y + 1$ $(x_0,y_0) = (0,1)$
$v = \cos x + \sqrt{y^2+3}$ $(u_0,v_0) = (1,3)$

$$\frac{\partial g}{\partial x} = \frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial x} = \frac{\partial f}{\partial u}\cdot\frac{3\sqrt{x}}{2} + \frac{\partial f}{\partial v}\cdot -\text{sen}\, x$$

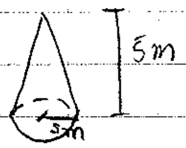
$$\frac{\partial g}{\partial y} = \frac{\partial f}{\partial y} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial y} = \frac{\partial f}{\partial u}\cdot\frac{1}{y} + \frac{\partial f}{\partial v}\cdot\frac{y}{\sqrt{y^2+3}}$$

$$\vec{n} = \left(\text{sen}\, x_0 \cdot \frac{\partial f}{\partial v}(x_0,y_0) - \frac{3}{2}\sqrt{x_0}\,\frac{\partial f}{\partial u}(u_0,v_0),\ -\frac{1}{y_0}\frac{\partial f}{\partial u}(u_0,v_0) - \frac{y_0}{\sqrt{y_0^2+3}}\frac{\partial f}{\partial v}(u_0,v_0), 1\right)$$

$$\vec{n} = \left(0, -6 - \frac{1}{\cancel{2}}\cdot\cancel{2}, 1\right) = (0,-7,1)$$

$\pi: -7y + z + d = 0$
$-7 + 10 + d = 0$
$d = -3$
$\pi: 7y - z + 3 = 0$

(44)



$\frac{dV}{dt} = 4\ m^3/min$

$\frac{d2R}{dt} = 2\ cm/min \therefore \frac{dR}{dt} = 1cm/min = 0{,}01\ m/min$

$V = \frac{1}{3}\pi R^2 h$

$$\frac{\partial V}{\partial t} = \frac{\partial V}{\partial R}\cdot\frac{\partial R}{\partial t} + \frac{\partial V}{\partial h}\cdot\frac{\partial h}{\partial t}$$

$$4 = \frac{2}{3}\pi R h \cdot 0{,}01 + \frac{1}{3}\pi R^2 \cdot \frac{\partial h}{\partial t}$$

$$4 = \frac{2}{\cancel{3}}\pi\cdot\cancel{3}\cdot 5\cdot 0{,}01 + \frac{1}{\cancel{3}}\pi\cdot\cancel{3}\cdot 3\cdot\frac{\partial h}{\partial t}$$

$$4 = 0{,}1\pi + 3\pi\,\frac{\partial h}{\partial t}$$

$$\frac{\partial h}{\partial t} = \frac{4 - 0{,}1\pi}{3\pi} = 0{,}39\ m/min$$

Exercícios (folha)

① $\frac{dV}{dt} = 5$ V/min , $\frac{dR}{dt} = -3\ \Omega$/min , $R = 60$ e $V = 120$

$I = \frac{V}{R}$

$\frac{dI}{dt} = \frac{\partial I}{\partial V} \cdot \frac{dV}{dt} + \frac{\partial I}{\partial R} \cdot \frac{dR}{dt}$

$\frac{dI}{dt} = \frac{1}{R} \cdot 5 + \frac{V}{-R^2} \cdot -3$

$\frac{dI}{dt} = \frac{5}{60} + \frac{360}{60 \cdot 60}$

$\frac{dI}{dt} = \frac{1}{12} + \frac{1}{10} = \frac{11}{60}$ A/min

② $h = 20$ cm, $R = 8$ cm

a) $\frac{dR}{dt} = -3$ cm/1 $\frac{dh}{dt} = 5$ cm/1

$V = \frac{1}{3} \pi R^2 h$

$\frac{dV}{dt} = \frac{\partial V}{\partial R} \cdot \frac{dR}{dt} + \frac{\partial V}{\partial h} \cdot \frac{dh}{dt} = \frac{2\pi R h \cdot -\cancel{3}}{\cancel{3}} + \frac{1}{3} \pi R^2 \cdot 5 = -2\pi 8 \cdot 20 + \frac{5\pi 64}{3}$

$\frac{dV}{dt} = \frac{-960\pi + 320\pi}{3} = \frac{-640\pi}{3} \cong -670$ cm³/1

b) $A_L = -\frac{\pi}{2}$ cm²/1 $R = -0{,}25$ cm/1

$A_L = \pi R g = \pi R \sqrt{h^2 + R^2}$

$$\frac{dA}{dt} = \frac{\partial A}{\partial R} \cdot \frac{dR}{dt} + \frac{\partial A}{\partial h} \cdot \frac{dh}{dt}$$

$$-\frac{\pi}{2} = \left(\pi\sqrt{R^2+h^2} + \pi R \cdot \frac{1}{\cancel{2}\sqrt{h^2+R^2}} \cdot \cancel{2}R\right) \cdot -\frac{1}{4} + \left(\pi R + \frac{\cancel{2}h}{\cancel{2}\sqrt{R^2+h^2}}\right)\frac{dh}{dt}$$

$$-\frac{\pi}{2} = \frac{-\pi(R^2+h^2) - \pi R^2}{4\sqrt{R^2+h^2}} + \left(\pi R + \frac{h}{\sqrt{R^2+h^2}}\right)\frac{dh}{dt}$$

$$-\frac{\pi}{2} = \frac{-\pi(64+400) - \pi 64}{4\sqrt{64+400}} + \left(\pi 8 + \frac{20}{\sqrt{64+400}}\right)\frac{dh}{dt}$$

$$\frac{dh}{dt} = -\frac{\pi}{2} + 19{,}25\left(\frac{1}{26{,}06}\right) = 0{,}69 \text{ cm/h}$$

④ $dr < \frac{3}{100} r \quad dh < \frac{7}{1000} h$

$$dV = \frac{\partial V}{\partial r} dr + \frac{\partial V}{\partial h} dh = 2\pi r h \, dr + \pi r^2 dh$$

$$dV < 2\pi r h \frac{3r}{100} + \pi r^2 \frac{0{,}7h}{100} = \frac{6{,}7}{100} \pi r^2 h = \frac{6{,}7}{100} V$$

Erro máximo é 6,7%

③ $I = \sqrt[3]{(5{,}02)^2 + (3{,}97)^2}$

$f(x,y) = \sqrt[3]{x^2+y^2}$ $\quad \begin{cases} x = 5 & y = 4 \\ dx = 0{,}02 & dy = -0{,}03 \end{cases}$

$$f(5+0{,}02;\ 4-0{,}03) - f(5,4) = df$$

$$f(5{,}02;\ 3{,}97) = f(5,4) + df = f(5,4) + \frac{\partial f}{\partial x} \cdot dx + \frac{\partial f}{\partial y} \cdot dy$$

$$df = \frac{2x}{3(x^2+y^2)^{2/3}} \cdot 0{,}02 + \frac{2y}{3(x^2+y^2)^{2/3}}(-0{,}03)$$

$$df = \frac{0,04x - 0,06y}{3(x^2+y^2)^{2/3}} = \frac{0,04.5 - 0,06.4}{3(5^2+4^2)^{2/3}} = \frac{-0,04}{3\ 11,89} = -0,001121$$

$$I = (5^2+4^2)^{1/3} - \frac{0,04}{3(5^2+4^2)^{2/3}} = 3,447$$

⑤ $h = 20\,cm$ e $r = 4\,cm$ $\quad V = \pi R^2 h$

a) $dV = 2\pi Rh\, dR + \pi R^2\, dh$ | $dV = 2\pi Rh\, dR + \pi R^2\, dh$

$dV = 160\pi\, dR + 16\pi\, dh$ | $dV = 250\pi\, dR + 625\pi\, dh$

∴ Será mais sensível a R | ∴ Será mais sensível a h

b) $dV = \frac{\partial V}{\partial R} dR + \frac{\partial V}{\partial h} dh$

$$dV = 2\pi Rh \frac{2}{100} R + \pi R^2 \left(\frac{-3}{100} h\right)$$

$$dV = \frac{4}{100} \pi R^2 h - \frac{3}{100} \pi R^2 h$$

$$dV = \frac{1}{100} \pi R^2 h = \frac{1}{100} V$$

$dV = 10,05\ cm^3$

$dV = 1\%\, V$

c) $dV = 2\pi Rh\, dR + \pi R^2\, dh$

$$0 = 2\cancel{\pi}\cancel{R}h . 0,3 + \cancel{\pi} R^{\cancel{2}}\, dh$$

$$dh = \frac{-0,6h}{R} = \frac{-0,6.20}{4} = -3\,cm$$

d) $C = 0,8 A \qquad A = 2\pi R^2 + 2\pi Rh$

$$dC = 0,8\, dA = 0,8\left(\frac{\partial A}{\partial R} dR + \frac{\partial A}{\partial h} dh\right)$$

$dC = 0{,}8(4\pi R + 2\pi h)dR + 0{,}8 \cdot 2\pi R \cdot dh$

$dC = 0{,}8 \cdot (4\pi \cdot 4 + 2\pi \cdot 20) \cdot \frac{5}{100} \cdot 4 + 0{,}8 \cdot 2\pi 4 \cdot \frac{3}{100} \cdot 20$

$dC = 0{,}8 \cdot \frac{1}{5} \cdot 56\pi + 0{,}8 \cdot \frac{24}{5}\pi$

$dC = 8{,}96\pi + 3{,}84\pi$

$dC = 7{,}8\pi$

$dC = 24{,}5$

R\$ 24,50

482,55 — 100%

24,50 — x %

$x = 5{,}1\%$

⑥ $d(P,Q) = \sqrt{(1-x)^2 + (7-y)^2 + (-3-z)^2}$

$d(P,Q) = \sqrt{(1-x)^2 + (7-y)^2 + (-3+3x-5y-8)^2}$

$d(P,Q) = \sqrt{(1-x)^2 + (7-y)^2 + (3x-5y-11)^2}$

$$\frac{\partial d}{\partial x} = \frac{\cancel{2}(1-x)\cdot(-1) + \cancel{2}(3x-5y-11)\cdot 3}{\cancel{2}\sqrt{(1-x)^2 + (7-y)^2 + (3x-5y-11)^2}} = \frac{(x-1) + 3(3x-5y-11)}{\sqrt{(1-x)^2 + (7-y)^2 + (3x-5y-11)^2}}$$

$$\frac{\partial d}{\partial y} = \frac{\cancel{2}(7-y)\cdot(-1) + \cancel{2}(3x-5y-11)\cdot -5}{\cancel{2}\sqrt{(1-x)^2 + (7-y)^2 + (3x-5y-11)^2}} = \frac{(y-7) - 5(3x-5y-11)}{\sqrt{(1-x)^2 + (7-y)^2 + (3x-5y-11)^2}}$$

$\frac{\partial d}{\partial x} = 0$

$x - 1 + 9x - 15y - 33 = 0$

$10x - 15y - 34 = 0$

$5x = \frac{15}{2}y + 17$

$\frac{\partial d}{\partial y} = 0$

$y - 7 - 15x + 25y + 55 = 0$

$26y - 15x + 48 = 0$

$5x = \frac{26}{3}y + 16$

$$\frac{15}{2}y + 17 = \frac{26}{3} + 16$$

$$\frac{15}{2}y - \frac{26}{3}y + 1 = 0$$

$$\frac{45y - 52y + 6}{6} = 0$$

$$-7y + 6 = 0$$

$$y = \frac{6}{7}$$

$$5x = \frac{15}{\cancel{2}} \cdot \frac{\cancel{6}^{3}}{7} + 17$$

$$x = \frac{9}{7} + \frac{17}{5} = \frac{45 + 119}{35} = \frac{164}{35}$$

$$z = 8 - 3x + 5y = 8 - 3 \cdot \frac{164}{35} + 5 \cdot \frac{6}{7} = \frac{280 - 492 + 150}{35} = \frac{-62}{35}$$

$$Q = \left(\frac{164}{35}, \frac{6}{7}, \frac{-62}{35} \right)$$

(7) $T(x,y,z) = xy^2z$

$T(x,z) = -xz(4x^2 + 5z^2 - 20)$

$T(x,z) = -(4x^3z + 5xz^3 - 20xz)$

$$\frac{\partial T}{\partial x} = -(12x^2z + 5z^3 - 20z) \qquad \frac{\partial T}{\partial z} = -(4x^3 + 15xz^2 - 20x)$$

$$\frac{\partial^2 T}{\partial x^2} = -24xz \qquad \frac{\partial T}{\partial z^2} = -30xz$$

$$\frac{\partial T}{\partial z \partial x} = -(12x^2 + 15z^2 - 20)$$

$\frac{\partial T}{\partial x} = 0$ $\quad -(12x^2z + 5z^3 - 20z) = 0$

$-z(12x^2 + 5z^2 - 20) = 0$

$z = 0$ ou $12x^2 + 5z^2 - 20 = 0$

$\frac{\partial T}{\partial z} = 0$ $\quad -(4x^3 + 15xz^2 - 20x) = 0$

$-x(4x^2 + 15z^2 - 20) = 0$

$x = 0$ ou $4x^2 + 15z^2 - 20 = 0$

$$\begin{cases} 12x^2+5z^2-20=0 \\ 4x^2+15z^2-20=0 \end{cases}$$

$$\begin{cases} -36x^2-15z^2+60=0 \\ 4x^2+15z^2-20=0 \end{cases}$$

$$-32x^2+40=0$$

$$x^2=\frac{5}{4}$$

$$x=\pm\frac{\sqrt{5}}{2}$$

$$\cancel{4}\cdot\frac{5}{\cancel{4}}+15z^2-20=0$$

$$15z^2=15$$

$$z^2=1$$

$$z=\pm 1$$

De $x=0, z=0 \Rightarrow y=\pm\sqrt{20}$

De $z=\pm 1, x=\pm\frac{\sqrt{5}}{2} \Rightarrow y=\pm\sqrt{10}$

$P_1(0,\sqrt{20},0)$, $P_2(0,-\sqrt{20},0)$, $P_3\left(-\frac{\sqrt{5}}{2},-\sqrt{10},-1\right)$, $P_4\left(-\frac{\sqrt{5}}{2},-\sqrt{10},1\right)$, $P_5\left(-\frac{\sqrt{5}}{2},\sqrt{10},-1\right)$, $P_6\left(-\frac{\sqrt{5}}{2},\sqrt{10},1\right)$, $P_7\left(\frac{\sqrt{5}}{2},-\sqrt{10},-1\right)$, $P_8\left(\frac{\sqrt{5}}{2},-\sqrt{10},1\right)$, $P_9\left(\frac{\sqrt{5}}{2},\sqrt{10},-1\right)$, $P_{10}\left(\frac{\sqrt{5}}{2},\sqrt{10},1\right)$

$$\Delta(x,z)=\begin{vmatrix} -24xz & -12x^2-15z^2+20 \\ -12x^2-15z^2+20 & -30xz \end{vmatrix}=720x^2z^2-(-12x^2-15z^2+20)^2$$

$$\Delta_1=\Delta_2=-400<0$$

$$\Delta_3=\Delta_4=\Delta_5=\Delta_6=\Delta_7=\Delta_8=\Delta_9=\Delta_{10}=800>0$$

$T_{xx1}=T_{xx2}=0$ — P_1 e P_2 são pontos de sela

$T_{xx3}=T_{xx5}=T_{xx8}=T_{xx10}<0$ — P_3, P_5, P_8, P_{10} são pontos de máximo

$T_{xx4}=T_{xx6}=T_{xx7}=T_{xx9}>0$ — P_4, P_6, P_7, P_9 são pontos de mínimo

$T_{mín}=-5\sqrt{5}$

$T_{máx}=5\sqrt{5}$

Cap 2

(45) $R = \frac{E}{I}$, $E = 120V$ e $I = 15A$, $\frac{\partial E}{\partial t} = 0{,}1 V/s$ e $\frac{\partial I}{\partial t} = -0{,}05 A/s$

$$\frac{\partial R}{\partial t} = \frac{\partial R}{\partial E} \cdot \frac{\partial E}{\partial t} + \frac{\partial R}{\partial I} \frac{\partial I}{\partial t} = \frac{1}{I} \cdot 0{,}1 + \left(\frac{-E}{I^2}\right) \cdot (-0{,}05)$$

$$\frac{\partial R}{\partial t} = \frac{0{,}1}{15} + \frac{0{,}05 \cdot 120}{15^2} = \frac{1{,}5+6}{15^2} = \frac{\cancel{7{,}5} \cdot 2}{\cancel{15} \cdot 15 \cdot 2} = \frac{1}{30} = 0{,}033\ \Omega/s$$

(46) $I = \frac{V}{\sqrt{R^2+10L^2}}$, $V = 210V$, $R = 3\Omega$, $L = 2H$, $\frac{\partial R}{\partial t} = -0{,}1 \Omega/s$, $\frac{\partial L}{\partial t} = 0{,}05 H/s$

$V = I\sqrt{R^2+10L^2}$

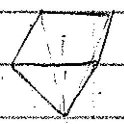
$$\frac{\partial V}{\partial t} = \frac{\partial V}{\partial I} \cancel{\frac{\partial I}{\partial t}}_{=0} + \frac{\partial V}{\partial R} \cdot \frac{\partial R}{\partial t} + \frac{\partial V}{\partial L} \cdot \frac{\partial L}{\partial t}$$

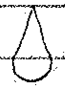
$$I = \frac{210}{\sqrt{9+10 \cdot 4}} = \frac{210}{7} = 30A$$

$$\frac{\partial V}{\partial t} = \frac{IR}{\sqrt{R^2+10L^2}} (-0{,}1) + \frac{10IL}{\sqrt{R^2+10L^2}} \cdot 0{,}05$$

$$\frac{\partial V}{\partial t} = \frac{30 \cdot 3(-0{,}1) + 10 \cdot 30 \cdot 2 \cdot 0{,}05}{\sqrt{9+10 \cdot 4}} = \frac{30-9}{\sqrt{49}} = \frac{21}{7} = 3\ V/s$$

(47) $\frac{\partial V}{\partial t} = -40\pi\ cm^3/min$, $V = 243\pi\ cm^3$





$h = 3cm$, $\frac{\partial R}{\partial t} = 0{,}3\ cm/min$, $\frac{\partial h}{\partial t} = ?$ $\Rightarrow V = 81\pi\ cm^3$

$V = \frac{\pi}{3} R^2 h \Rightarrow h = \frac{3V}{\pi R^2}$

$81\cancel{\pi} = \frac{\cancel{\pi}}{\cancel{3}} R^2 \cdot \cancel{3}$

$R = 9cm$

$$\frac{\partial h}{\partial t} = \frac{\partial h}{\partial V}\cdot\frac{\partial V}{\partial t} + \frac{\partial h}{\partial R}\cdot\frac{\partial R}{\partial t}$$

$$\frac{\partial h}{\partial t} = \frac{3}{\cancel{\pi}R^2}\cdot 40\cancel{\pi} + \frac{(-6V)}{\pi R^3}\cdot 0{,}3$$

$$\frac{\partial h}{\partial t} = \frac{3\cdot 40}{9^2} - \frac{6\cdot\cancel{81}\cancel{\pi}}{\cancel{\pi}\,9^{\cancel{3}}}\cdot\frac{3}{10}$$

$$\frac{\partial h}{\partial t} = \frac{120}{81} - \frac{18}{90} = \frac{40}{27} - \frac{2}{10} \cong 1{,}28\ cm/min$$

(48) $PV = KT$ $\quad V = 120\ cm^3$ $\quad \frac{dV}{\partial t} = 2\ cm^3/s$ $\quad \frac{\partial P}{\partial T} = -0{,}1\ din/cm^2 s$

$PV = 10T$ $\quad P = 8\ din/cm^2$

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial V}\cdot\frac{\partial V}{\partial t} + \frac{\partial T}{\partial P}\cdot\frac{\partial P}{\partial t} = \frac{P}{10}\cdot 2 + \frac{V}{10}\cdot(-0{,}1) = \frac{8\cdot 2}{10} - \frac{12\cancel{0}}{10\cancel{0}} = 0{,}4\ K/s$$

(49) $P = \frac{V^2}{R}$, $V = 200V$ e $R = 20\Omega$

a) $dP = 0$ e $dV = -0{,}2V$

$$dP = \frac{\partial P}{\partial V}\cdot dV + \frac{\partial P}{\partial R}\cdot dR$$

$$0 = \frac{2V}{R}\cdot(-0{,}2) + \frac{(-V^2)}{R^2}\ dR$$

$$\frac{V^{\cancel{2}}}{R^{\cancel{2}}}\ dR = -\frac{0{,}4\cancel{V}}{\cancel{R}}$$

$$dR = -\frac{0{,}4R}{V} = \frac{-0{,}4\cdot 2\cancel{0}}{20\cancel{0}} = -0{,}04\ \Omega$$

b) $dP = \frac{2V}{R}dV - \frac{V^2}{R^2}dR$

$$\frac{3}{100}P = \frac{2V}{R}dV - \frac{V^2}{R^{\cancel{2}}}\cdot\frac{-1}{100}\cdot\cancel{R}$$

$\frac{\partial h}{\partial t} = \frac{\partial h}{\partial V} \cdot \frac{\partial V}{\partial t} + \frac{\partial h}{\partial R} \cdot \frac{\partial R}{\partial t}$

$\frac{\partial h}{\partial t} = \frac{3}{\cancel{\pi} R^2} \cdot 40\cancel{\pi} + \frac{(-6V)}{\pi R^3} \cdot 0{,}3$

$\frac{\partial h}{\partial t} = \frac{3 \cdot 40}{9^2} - \frac{6 \cdot \cancel{81}\cancel{\pi}}{\cancel{\pi}\, 9^{\cancel{3}}} \cdot \frac{3}{10}$

$\frac{\partial h}{\partial t} = \frac{120}{81} - \frac{18}{90} = \frac{40}{27} - \frac{2}{10} \approx 1{,}28 \text{ cm/min}$

(48) $PV = KT \quad V = 120 \, m^3 \quad \frac{\partial V}{\partial t} = 2 \, cm^3/h \quad \frac{\partial P}{\partial t} = -0{,}1 \text{ din/cm}^2 h$

$PV = 10T \quad P = 8 \text{ din/cm}^2$

$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial V} \cdot \frac{\partial V}{\partial t} + \frac{\partial T}{\partial P} \cdot \frac{\partial P}{\partial t} = \frac{P}{10} \cdot 2 + \frac{V}{10} \cdot (-0{,}1) = \frac{8 \cdot 2}{10} - \frac{12\cancel{0}}{10\cancel{0}} = 0{,}4 \text{ K/h}$

(49) $P = \frac{V^2}{R}$, $V = 200V$ e $R = 20\Omega$

a) $dP = 0$ e $dV = -0{,}2V$

$dP = \frac{\partial P}{\partial V} \cdot dV + \frac{\partial P}{\partial R} \cdot dR$

$0 = \frac{2V}{R} \cdot (-0{,}2) + \frac{(-V^2)}{R^2} \, dR$

$\frac{V^{\cancel{2}}}{R^{\cancel{2}}} \, dR = -\frac{0{,}4\cancel{V}}{\cancel{R}}$

$dR = -\frac{0{,}4R}{V} = -\frac{0{,}4 \cdot 2\cancel{0}}{20\cancel{0}} = -0{,}04 \, \Omega$

b) $dP = \frac{2V}{R} \, dV - \frac{V^2}{R^2} \, dR$

$\frac{3}{100} P = \frac{2V}{R} \, dV - \frac{V^2}{R^{\cancel{2}}} \cdot \frac{-1}{100} \cdot \cancel{R}$

$$\frac{2V}{R}dV = \frac{3}{100} \cdot \frac{V^2}{R} - \frac{1}{100}\frac{V^2}{R}$$

$$\frac{2\cancel{V}}{\cancel{R}}dV = \frac{2}{100}\frac{V^{\cancel{2}}}{\cancel{R}}$$

$$dV = \frac{1}{100}V$$

$$dV = 1\%\, V$$

(50) $A = \frac{xy \operatorname{sen}\theta}{2}$, $x = 40\,cm$, $y = 50\,cm$, $\theta = \frac{\pi}{6}\,rad$

a) $\frac{\partial x}{\partial t} = 3\,cm/s$, $\frac{\partial \theta}{\partial t} = 0{,}05\,rad/s$, $\frac{\partial y}{\partial t} = -2\,cm/s$

$$\frac{\partial A}{\partial t} = \frac{\partial A}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial A}{\partial y}\cdot\frac{\partial y}{\partial t} + \frac{\partial A}{\partial \theta}\cdot\frac{\partial \theta}{\partial t}$$

$$\frac{\partial A}{\partial t} = \frac{y \operatorname{sen}\theta}{2}\cdot 3 + \frac{x \operatorname{sen}\theta}{2}\cdot(-2) + \frac{xy\cos\theta}{2}\,0{,}05$$

$$\frac{\partial A}{\partial t} = 50\cdot\frac{1}{2}\cdot\frac{3}{2} + 40\cdot\frac{1}{2}\cdot(-1) + 40\cdot 50\cdot\frac{\sqrt{3}}{2}\cdot\frac{0{,}05}{2}$$

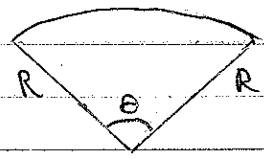
$$\frac{\partial A}{\partial t} = \frac{75}{2} - 20 - 25\sqrt{3} = \frac{35}{2} + 25\sqrt{3} = 60{,}80\,cm^2/s$$

b) $dA = \frac{\partial A}{\partial x}dx + \frac{\partial A}{\partial y}dy + \frac{\partial A}{\partial \theta}d\theta$

$$dA = \frac{y \operatorname{sen}\theta}{2}dx + \frac{x \operatorname{sen}\theta}{2}dy + \frac{xy\cos\theta}{2}d\theta$$

$$dA = 25\,dx + 20\,dy + 100\sqrt{3}\,d\theta$$

∴ O valor da área é mais sensível à θ, pois o coeficiente de $d\theta$ é o maior.

(51) 

$\pi R^2 — 360 \qquad \theta = 80°$

$A — \theta \qquad R = 20\,m$

$A = \dfrac{\theta \pi R^2}{360}$

$dA = \dfrac{\partial A}{\partial R}\, dR + \dfrac{\partial A}{\partial \theta} . d\theta$

$0 = \dfrac{\theta \pi R}{180} . dR + \dfrac{\pi R^2}{360} . (-1)$

$\dfrac{\cancel{\pi} R^{\cancel{2}}}{2\cancel{360}} = \dfrac{\theta \cancel{\pi} \cancel{R}}{\cancel{180}}\, dR$

$dR = \dfrac{R}{2\theta} = \dfrac{2\cancel{0}}{2.8\cancel{0}} = \dfrac{1}{8} = 0{,}125\, m$

(52) $PV = 8{,}31T$, $T = 300K$, $V = 100L$, $\dfrac{\partial T}{\partial t} = 0{,}1 K/s$, $\dfrac{\partial V}{\partial t} = 0{,}2 L/s$

$\dfrac{\partial P}{\partial t} = \dfrac{\partial P}{\partial T} . \dfrac{\partial T}{\partial t} + \dfrac{\partial P}{\partial V} \dfrac{\partial V}{\partial t} = \dfrac{8{,}31}{V} . 0{,}1 + \left(\dfrac{-8{,}31T}{V^2}\right) . 0{,}2$

$\dfrac{\partial P}{\partial t} = \dfrac{0{,}831}{100} - \dfrac{498{,}6}{10000} = 0{,}00831 - 0{,}04986 = 0{,}04155\, KPa/s$

(53) $Q = \sqrt{\dfrac{2KM}{h}}$, $K = 2$, $M = 20$, $h = 0{,}05 \Rightarrow Q = 40$

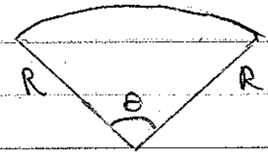
a) $dQ = \dfrac{\partial Q}{\partial K} dK + \dfrac{\partial Q}{\partial M} . dM + \dfrac{\partial Q}{\partial h} . dh$

$dQ = \sqrt{\dfrac{2M}{h}} . \dfrac{1}{2\sqrt{K}}\, dK + \sqrt{\dfrac{2K}{h}} . \dfrac{1}{2\sqrt{M}}\, dM + \sqrt{2KM} . \dfrac{-1}{2h^{3/2}}\, dh$

$dQ = \sqrt{\dfrac{M}{2Kh}} . dK + \sqrt{\dfrac{K}{2Mh}}\, dM - \sqrt{\dfrac{KM}{2h^3}}\, dh$

$dQ = 10dK + dM - 400dh$

∴ Q é mais sensível a h.

(51) 

$\pi R^2 — 360 \qquad \theta = 80^\circ$

$A — \theta \qquad R = 20\,m$

$A = \frac{\theta \pi R^2}{360}$

$dA = \frac{\partial A}{\partial R} dR + \frac{\partial A}{\partial \theta} . d\theta$

$0 = \frac{\theta \pi R}{180} . dR + \frac{\pi R^2}{360} . (-1)$

$\frac{\cancel{\pi} R^{\cancel{2}}}{2\cancel{360}} = \frac{\theta \cancel{\pi} \cancel{R}}{\cancel{180}} dR$

$dR = \frac{R}{2\theta} = \frac{2\cancel{0}}{2.8\cancel{0}} = \frac{1}{8} = 0{,}125\,cm$

(52) $PV = 8{,}31T$, $T = 300K$, $V = 100L$, $\frac{dT}{dt} = 0{,}1K/s$, $\frac{dV}{dt} = 0{,}2L/s$

$\frac{dP}{dt} = \frac{\partial P}{\partial T} . \frac{dT}{dt} + \frac{\partial P}{\partial V} \frac{dV}{dt} = \frac{8{,}31}{V} . 0{,}1 + \frac{(-8{,}31T)}{V^2} . 0{,}2$

$\frac{dP}{dt} = \frac{0{,}831}{100} - \frac{498{,}6}{10000} = 0{,}00831 - 0{,}04986 = 0{,}04155\,kPa/s$

(53) $Q = \sqrt{\frac{2KM}{h}}$, $K = 2$, $M = 20$, $h = 0{,}05 \Rightarrow Q = 40$

a) $dQ = \frac{\partial Q}{\partial K} dK + \frac{\partial Q}{\partial M} . dM + \frac{\partial Q}{\partial h} . dh$

$dQ = \sqrt{\frac{2M}{h}} . \frac{1}{2\sqrt{K}} dK + \sqrt{\frac{2K}{h}} . \frac{1}{2\sqrt{M}} dM + \sqrt{2KM} . \frac{-1}{2h^{3/2}} dh$

$dQ = \sqrt{\frac{M}{2Kh}} . dK + \sqrt{\frac{K}{2Mh}} dM - \sqrt{\frac{KM}{2h^3}} dh$

$dQ = 10dK + dM - 400dh$

∴ Q é mais sensível a h.

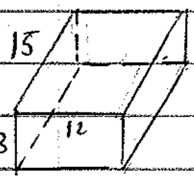
b) $dQ = \frac{1}{10}Q$, $dK = \frac{1}{10}K$, $dh = 0$

$\frac{1}{10}.Q = \cancel{10}\frac{1}{\cancel{10}}K + dM$

$dM = \frac{Q - 10K}{10} = \frac{40 - 20}{10} = 2$

$\therefore dM = 10\% M$

(54)



$A = xy + 2xz + 2yz$

$C = 12A$ $\quad dx = dy = dz = 0,05m$

$C = 12xy + 24xz + 24yz$

$dC = \frac{\partial C}{\partial x}.dx + \frac{\partial C}{\partial y}dy + \frac{\partial C}{\partial z}dz$

$dC = (12y + 24z + 12x + 24z + 24x + 24y)dx$

$dC = (36x + 36y + 48z)dx$

$dC = (3x + 3y + 4z).12.0,05$

$dC = (3.15 + 3.12 + 4.3).0,6$

$dC = 55,8$

$dC = R\$ 55,80$

(55) $P = \frac{V^2}{R}$, $V = 120V$, $R = 12\Omega$, $dV = -0,001V$, $dR = 0,02\,\Omega$

$dP = \frac{\partial P}{\partial V}.dV + \frac{\partial P}{\partial R}dR = \frac{2V}{R}(-0,001) + \frac{(-V^2)}{R^2}.0,02$

$dP = 2.\frac{120}{12}.(-0,001) - \frac{120^2}{12^2}.0,02 = -0,02 - 2 = -2,02\,W$

(56) $R = 12\,cm,\ h = 8\,cm,\ dR = 0{,}5\,cm,\ dh = -0{,}2\,cm,\ V = \frac{\pi R^2 h}{3}$

$$dV = \frac{\partial V}{\partial R} \cdot dR + \frac{\partial V}{\partial h} \cdot dh = \frac{2\pi R h}{3} dR + \frac{\pi R^2}{3} dh$$

$$dV = \frac{2\pi \cdot \cancel{12}^{4} \cdot 8 \cdot 1}{\cancel{3} \cdot \cancel{2}} + \pi \cdot \frac{12 \cdot \cancel{12}^{4}}{\cancel{3}}\left(\frac{-1}{5}\right) = 32\pi - \frac{48\pi}{5} = \frac{112\pi}{5} = 70{,}4\,cm^3$$

$$\Delta V = V - V_0 = \frac{\pi \cdot 12{,}5^2 \cdot 7{,}8}{3} - \frac{\pi \cdot 12^2 \cdot 8}{3} \cong 69{,}9\,cm^3$$

(57) $h = 4\,m,\ R = 3\,m,\ dh = 3\%\,h,\ dR = 5\%\,R,\ g = 5\,m$

$A = \pi R g$

$g = \sqrt{R^2 + h^2}$

$$dg = \frac{\partial g}{\partial R} \cdot dR + \frac{\partial g}{\partial h}\, dh$$

$C = 150\pi R g$

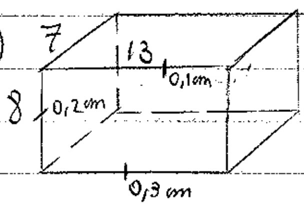
$$dg = \frac{R}{\sqrt{R^2+h^2}} \cdot \frac{5R}{100} + \frac{h}{\sqrt{R^2+h^2}} \cdot \frac{3h}{100}$$

$$dg = \frac{5R^2 + 3h^2}{100\sqrt{R^2+h^2}} = \frac{5 \cdot 9 + 3 \cdot 16}{100 \cdot 5} = 0{,}186$$

$$dC = \frac{\partial C}{\partial R} \cdot dR + \frac{\partial C}{\partial g} \cdot dg = 150\pi g\, dR + 150\pi R \cdot dg$$

$$dC = 150\pi\left(5 \cdot \frac{5}{100} \cdot 3 + 3 \cdot 0{,}186\right) = 150\pi \cdot 1{,}308 = 616{,}38$$

(58)



$V = xyz$

$$dV = \frac{\partial V}{\partial x}\, dx + \frac{\partial V}{\partial y} \cdot dy + \frac{\partial V}{\partial z}\, dz$$

$dx = dy = 2 \cdot 0{,}2 = 0{,}4\,cm$

$dz = 0{,}1 + 0{,}3 = 0{,}4\,cm$

$dV = yz\,dx + xz\,dy + xy\,dz$
$dV = (13.8 + 7.8 + 7.13)0,4$
$dV = 100,4 cm^3$

(59) $r = 4cm$, $h = 20cm$

$V = \pi R^2 h$

a) $dV = \frac{\partial V}{\partial R} dR + \frac{\partial V}{\partial h} \cdot dh = 2\pi R h\, dR + \pi R^2 dh = 160\pi dR + 16\pi dh$

∴ Deve se preocupar mais com variações no raio.

b) $C = 0,20 A$, $dR = \frac{-3}{100} R$, $dh = \frac{2}{100} h$

$A = 2\pi R^2 + 2\pi R h$

$dC = \frac{\partial C}{\partial R} \cdot dR + \frac{\partial C}{\partial h} \cdot dh$

$dC = 0,20(4\pi R + 2\pi h) \cdot \frac{-3}{100} R + 0,20 \cdot 2\pi R \cdot \frac{2}{100} h$

$dC = -0,20 \cdot (4\pi \cdot 4 + 2\pi 20) \cdot \frac{3}{100} \cdot 4 + 0,20 \cdot 2\pi \cdot 4 \cdot \frac{2}{100} \cdot 20$

$dC = -0,024(56\pi) + 0,64\pi = -0,704\pi = -2,2116$

$dC = -R\$2,21$

(60) a) $(1,1)^{3,02}$

$f(x,y) = x^y$
$x = 1 \quad dx = 0,1$
$y = 3 \quad dy = 0,02$

$$f(x+dx,\, y+dy) \approx f(x,y) + df = f(x,y) + \frac{\partial f}{\partial x}\, dx + \frac{\partial f}{\partial y}\, dy$$

$$(1{,}1)^{3{,}02} \approx 1^3 + y\, x^{y-1} \cdot 0{,}1 + y \cdot x^y \ln x \cdot 0{,}02$$

$$\approx 1 + 3 \cdot 1^{3-1} \cdot 0{,}1 + 3 \cdot 1^3 \cancel{\ln 1} \cdot 0{,}02$$

$$\approx 1{,}3$$

b) $\dfrac{\cos(e^{0,2}-1)}{\sqrt{9{,}4}}$

$$f(x,y) = \frac{\cos(e^x - 1)}{\sqrt{y}}$$

$x = 0 \quad dx = 0{,}2$

$y = 9 \quad dy = 0{,}4$

$$\frac{\cos(e^{0,2}-1)}{\sqrt{9{,}4}} \approx \frac{\cos(e^0 - 1)}{\sqrt{9}} + \frac{-\cancel{\operatorname{sen}(e^x-1)}\cdot e^x}{\sqrt{y}} \cdot 0{,}2 + \frac{-\cos(e^x-1)}{2\, y^{3/2}} \cdot 0{,}4$$

$$\approx \frac{1}{3} - \frac{1}{\cancel{2} \cdot 9^{3/2}} \cdot \frac{\cancel{4}\,2}{10} = \frac{90-2}{270} = \frac{88}{270} = \frac{44}{135}$$

c) $\sqrt{(3{,}02)^2 + (1{,}97)^2 + (5{,}99)^2}$

$$f(x,y,z) = \sqrt{x^2 + y^2 + z^2}$$

$x = 3 \quad dx = 0{,}02$

$y = 2 \quad dy = -0{,}03$

$z = 6 \quad dz = -0{,}01$

$$\sqrt{(3{,}02)^2 + (1{,}97)^2 + (5{,}99)^2} \approx \sqrt{3^2 + 2^2 + 6^2} + \frac{x\,dx + y\,dy + z\,dz}{\sqrt{x^2+y^2+z^2}}$$

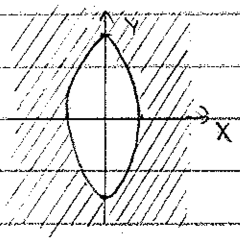
$$\approx 7 + \frac{3 \cdot 0{,}02 - 2 \cdot 0{,}03 - 6 \cdot 0{,}01}{7}$$

$$\approx 7 - \frac{0{,}06}{7} = 7 - \frac{6}{700} = \frac{4894}{700} = \frac{2447}{350}$$

61) $f(x,y) = 2\sqrt{x^2 + \frac{y^2}{9} - 1}$

a) $x^2 + \frac{y^2}{9} - 1 \geq 0$

$x^2 + \frac{y^2}{9} \geq 1$



$Df = \left\{(x,y) \in \mathbb{R}^2 \,/\, x^2 + \frac{y^2}{9} \geq 1\right\}$

Pontos sobre e exteriores a elipse $x^2 + \frac{y^2}{9} = 1$

b) $f(1{,}98;\, 3{,}3)$

$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} \cdot dy$

$df = \cancel{2} \cdot \frac{1}{\cancel{2}\sqrt{x^2 + \frac{y^2}{9} - 1}} \cdot 2x \cdot (-0{,}02) + \cancel{2} \cdot \frac{1}{\cancel{2}\sqrt{x^2 + \frac{y^2}{9} - 1}} \cdot \frac{2}{9} y \cdot 0{,}3$

$df = \frac{2 \cdot 2 \cdot (-0{,}02)}{\sqrt{4 + \cancel{\frac{9}{9}} - \cancel{1}}} + \frac{2 \cdot 3 \cdot 0{,}3}{9\sqrt{4 + \cancel{\frac{9}{9}} - \cancel{1}}}$

$df = -0{,}04 + 0{,}1 = 0{,}06$

$f(1{,}98;\, 3{,}3) = f(2,3) + df = 4 + 0{,}06 = 4{,}06$

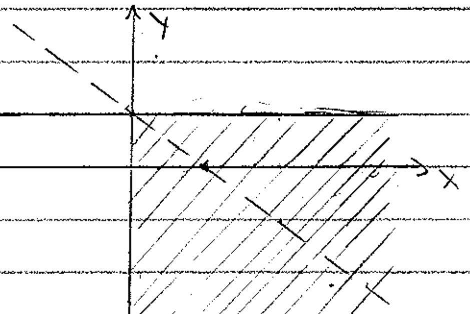
62) $f(x,y) = \frac{x+y-1}{\sqrt{x} - \sqrt{1-y}} = \frac{x - (1-y)}{\sqrt{x} - \sqrt{1-y}} = \frac{(\sqrt{x} + \sqrt{1-y})\cancel{(\sqrt{x} - \sqrt{1-y})}}{\cancel{(\sqrt{x} - \sqrt{1-y})}} = \sqrt{x} + \sqrt{1-y}$

a) $x \geq 0$ $\quad \sqrt{x} - \sqrt{1-y} \neq 0$

$1 - y \geq 0$ $\quad \sqrt{x} \neq \sqrt{1-y}$

$y \leq 1$ $\quad x \neq 1 - y$

$x + y \neq 1$



$Df = \{(x,y) \in \mathbb{R}^2 \,/\, x \geq 0,\ y \leq 1,\ x + y \neq 1\}$

b) $\lim_{(x,y)\to(4,-3)} \sqrt{x}+\sqrt{1-y} = \sqrt{4}+\sqrt{4} = 4$

c) $f(9{,}06,-3{,}04) \approx f(9,-3)+df$

$\approx \sqrt{9}+\sqrt{4}+\frac{1}{2\sqrt{x}}\cdot 0{,}06+\frac{1}{2\sqrt{1-y}}\cdot(-1)(-0{,}04)$

$\approx 3+2+0{,}01+0{,}01$

$\approx 5{,}02$

63 a) $z=-x^2-y^2+6x-4y-4 \qquad P(3,-2,9)$

$z=-(x^2-6x)-(y^2+4y)-4$

$z=-(x-3)^2+9-(y+2)^2+\cancel{4}-\cancel{4}$

$z=-(x-3)^2-(y+2)^2+9$

$\frac{\partial z}{\partial x}=-2(x-3)=-2x+6 \qquad \frac{\partial^2 z}{\partial x^2}=-2 \qquad \frac{\partial^2 z}{\partial x\partial y}=0$

$\frac{\partial z}{\partial y}=-2(y+2)=-2y-4 \qquad \frac{\partial^2 z}{\partial y^2}=-2$

$\vec{n}=(0,0,1)$

$\pi: z-9=0$

b) $\Delta(x,y)=\begin{vmatrix}-2 & 0\\ 0 & -2\end{vmatrix}=4$

$\Delta>0$, $z_{xx}=-2<0 \therefore P(3,-2,9)$ é um ponto de máximo

64 $f(x,y)=2\ln(x^2y)+\frac{1}{4}x^4-\frac{5}{2}x^2-y+5$

$\frac{\partial f}{\partial x}=2\cdot\frac{2x\cancel{y}}{x^2\cancel{y}}+\frac{\cancel{4}x^3}{\cancel{4}}-\frac{5}{\cancel{2}}\cdot\cancel{2}x=\frac{4}{x}+x^3-5x$

$$\frac{\partial f}{\partial y} = 2 \cdot \frac{x^2}{x^2 y} - 1 = \frac{2}{y} - 1 \qquad \frac{\partial^2 f}{\partial x \partial y} = 0$$

$$\frac{\partial^2 f}{\partial x^2} = -\frac{4}{x^2} + 3x^2 - 5 \qquad \frac{\partial^2 f}{\partial^2 y} = -\frac{2}{y^2}$$

$$\frac{\partial f}{\partial x} = 0 \Rightarrow \frac{4}{x} + x^3 - 5x = 0 \qquad x^2 = t$$

$$4 + x^4 - 5x^2 = 0$$

$$t^2 - 5t + 4 = 0$$

$$1 + 4 = 5$$

$$1 \cdot 4 = 4$$

$$x^2 = 1 \qquad x^2 = 4$$

$$x = \pm 1 \qquad x = \pm 2$$

$$\frac{\partial f}{\partial y} = 0 \Rightarrow \frac{2}{y} - 1 = 0$$

$$y = 2$$

$P_1(-2,2)$, $P_2(-1,2)$, $P_3(1,2)$, $P_4(2,2)$ são pontos críticos

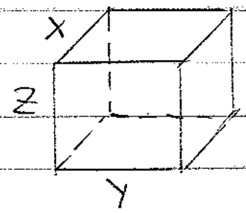
$$\Delta(x,y) = \begin{vmatrix} -\frac{4}{x^2} + 3x^2 - 5 & 0 \\ 0 & -\frac{2}{y^2} \end{vmatrix} = \left(-\frac{4}{x^2} + 3x^2 - 5\right) \cdot \left(\frac{-2}{y^2}\right)$$

$$\Delta_1 = \left(\frac{-4}{4} + 3 \cdot 4 - 5\right)\left(\frac{-2}{4}\right) = -3 < 0 \therefore \text{Ponto de sela}$$

$$\Delta_2 = \left(\frac{-4}{1} + 3 \cdot 1 - 5\right)\left(\frac{-2}{4}\right) = 3 > 0,\ f_{yy} < 0 \therefore \text{Ponto de máximo}$$

P_1 e P_4 são pontos de sela, enquanto P_2 e P_3 são pontos de máximo

65



$$V = xyz = 20$$
$$z = \frac{20}{xy}$$

$$C = 1.2yz + 1.2xz + 2xy + 3xy$$
$$C = 2yz + 2xz + 5xy$$
$$C = 2\cancel{y}.\frac{20}{x\cancel{y}} + 2x.\frac{20}{xy} + 5xy$$
$$C = \frac{40}{x} + \frac{40}{y} + 5xy.$$

$$\frac{\partial C}{\partial x} = -\frac{40}{x^2} + 5y \qquad \frac{\partial C}{\partial y} = -\frac{40}{y^2} + 5x \qquad \frac{\partial^2 C}{\partial x \partial y} = 5$$
$$\frac{\partial^2 C}{\partial x^2} = \frac{80}{x^3} \qquad \frac{\partial^2 C}{\partial^2 y} = \frac{80}{y^3}$$

$$\frac{\partial C}{\partial x} = 0 \Rightarrow -\frac{40}{x^2} + 5y = 0 \qquad \frac{\partial C}{\partial y} = 0 \Rightarrow -\frac{40}{y^2} + 5x = 0$$
$$5y = \frac{40}{x^2} \qquad 5x = \frac{40}{y^2}.$$
$$y = \frac{8}{x^2} \qquad x = \frac{8}{y^2}$$

$$x.\left(\frac{8}{x^2}\right)^2 = 8 \Rightarrow x\,\frac{8.8}{x^{4}} = 8 \Rightarrow x^3 = 8 \qquad y = 2$$
$$x = 2 \qquad z = 5$$

$P(2,2)$

$$\Delta(2,2) = \begin{vmatrix} 10 & 5 \\ 5 & 10 \end{vmatrix} = 75, \quad \frac{\partial^2 C}{\partial x^2} = 10 \quad \therefore$$ P(2,2) é um ponto de mínimo