Exercícios Cap 1

① $f(x) = x+2$; $g(x) = x^2+x$; $x \in [1,3]$

$\Delta x = \dfrac{2}{n}$

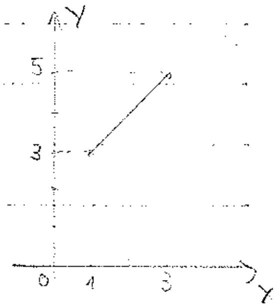
$x_0 = 1$

$x_1 = 1+\Delta x$

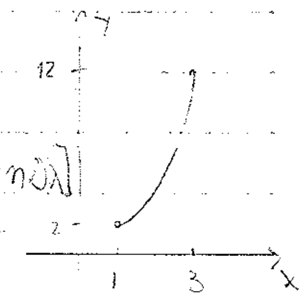
$x_2 = 1+2\Delta x$

$x_n = 1+n\Delta x$

$$\begin{aligned}
\bar{S}(f) &= f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x \\
&= [f(1+\Delta x) + f(1+2\Delta x) + \dots + f(1+n\Delta x)]\Delta x \\
&= [1+\Delta x+2 + 1+2\Delta x+2 + \dots + 1+n\Delta x+2]\Delta x \\
&= [3n + \Delta x(1+2+\dots+n)]\Delta x \\
&= \left[3n + \frac{\cancel{2}}{\cancel{n}} \cdot \frac{\cancel{n}(n+1)}{\cancel{2}}\right]\frac{2}{n} \\
&= 6 + 2 + \frac{2}{n} \\
&= 8 + \frac{2}{n}
\end{aligned}$$



$$\begin{aligned}
\bar{S}(g) &= g(x_1)\Delta x + g(x_2)\Delta x + \dots + g(x_n)\Delta x \\
&= [g(1+\Delta x) + g(1+2\Delta x) + \dots + g(1+n\Delta x)]\Delta x \\
&= \Delta x[(1+\Delta x)^2 + 1+\Delta x + (1+2\Delta x)^2 + 1+2\Delta x + \dots + (1+n\Delta x)^2 + 1+n\Delta x] \\
&= [n + \Delta x(1+2+\dots+n) + 1+2\Delta x+\Delta x^2 + 1+4\Delta x+2^2\Delta x^2 + \dots \\
&\quad 1+2n\Delta x+n^2\Delta x^2]\Delta x \\
&= \left[2n + \Delta x \cdot \frac{n(n+1)}{2} + 2\Delta x(1+2+\dots+n) + \Delta x^2(1^2+2^2+\dots+n^2)\right]\Delta x \\
&= \left[2n + 3 \cdot \frac{\cancel{2}}{\cancel{n}} \cdot \frac{\cancel{n}(n+1)}{\cancel{2}} + \frac{\cancel{4}^{2}}{n^{\cancel{2}}} \cdot \frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}_{3}}\right]\frac{2}{n} \\
&= \left[2n + 3n + 3 + \frac{2(2n^2+3n+1)}{3n}\right]\frac{2}{n} \\
&= 10 + \frac{6}{n} + \frac{8}{3} + \frac{4}{n} + \frac{4}{3n^2} \\
&= \frac{38}{3} + \frac{10}{n} + \frac{4}{3n^2}
\end{aligned}$$



Exercícios Cap 1

① $f(x) = x+2$; $g(x) = x^2+x$; $x \in [1,3]$ $\qquad \Delta x = \frac{2}{n}$

$\bar{S}(f) = f(x_1)\Delta x + f(x_2)\Delta x + \ldots + f(x_n)\Delta x$ $\qquad x_0 = 1$

$= [f(1+\Delta x) + f(1+2\Delta x) + \ldots + f(1+n\Delta x)]\Delta x$ $\qquad x_1 = 1+\Delta x$

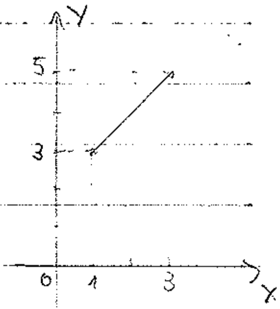
$= [1+\Delta x+2 + 1+2\Delta x+2 + \ldots + 1+n\Delta x+2]\Delta x$ $\qquad x_2 = 1+2\Delta x$

$= [3n + \Delta x(1+2+\ldots+n)]\Delta x$ $\qquad x_n = 1+n\Delta x$

$$= \left[3n + \frac{\cancel{2}}{\cancel{n}} \cdot \frac{n(n+1)}{\cancel{2}}\right]\frac{2}{n}$$

$$= 6 + 2 + \frac{2}{n}$$

$$= 8 + \frac{2}{n}$$



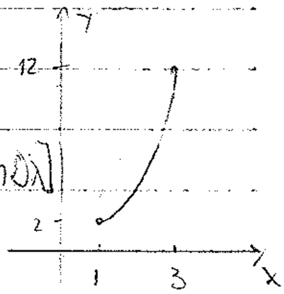
$\bar{S}(g) = g(x_1)\Delta x + g(x_2)\Delta x + \ldots + g(x_n)\Delta x$

$= [g(1+\Delta x) + g(1+2\Delta x) + \ldots + g(1+n\Delta x)]\Delta x$

$= \Delta x[(1+\Delta x)^2 + 1+\Delta x + (1+2\Delta x)^2 + 1+2\Delta x + \ldots + (1+n\Delta x)^2 + 1+n\Delta x]$

$= [n + \Delta x(1+2+\ldots+n) + 1+2\Delta x+\Delta x^2 + 1+4\Delta x+2^2\Delta x^2 + \ldots +$

$1+2n\Delta x + n^2\Delta x^2]\Delta x$



$$= \left[2n + \Delta x \cdot \frac{n(n+1)}{2} + 2\Delta x(1+2+\ldots+n) + \Delta x^2(1^2+2^2+\ldots+n^2)\right]\Delta x$$

$$= \left[2n + 3 \cdot \frac{\cancel{2}}{\cancel{n}} \cdot \frac{\cancel{n}(n+1)}{\cancel{2}} + \frac{4}{n^{\cancel{2}}} \cdot \frac{\cancel{n}(n+1)\cdot(2n+1)}{6\,3}\right]\frac{2}{n}$$

$$= \left[2n + 3n + 3 + \frac{2(2n^2+3n+1)}{3n}\right]\frac{2}{n}$$

$$= 10 + \frac{6}{n} + \frac{8}{3} + \frac{4}{n} + \frac{4}{3n^2}$$

$$= \frac{38}{3} + \frac{10}{n} + \frac{4}{3n^2}$$

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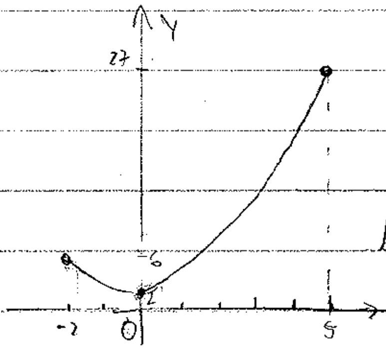
② $f:[-2,5]\to\mathbb{R}$, $f(x)=x^2+2$



a) $[-2,0]$

$x_0=-2$

$x_1=-2+\Delta x$

$\Delta x=\frac{2}{n}$ $\quad x_2=-2+2\Delta x$

$x_n=-2+n\Delta x$

b) $[0,5]$

$x_0=0$

$x_1=\Delta x$

$x_2=2\Delta x$ $\quad \Delta x=\frac{5}{n}$

$x_n=n\Delta x$

a) $\underline{S_1(f)} = f(x_1)\Delta x+f(x_2)\Delta x+\ldots+f(x_n)\Delta x$

$= [f(-2+\Delta x)+f(-2+2\Delta x)+\ldots+f(-2+n\Delta x)]\Delta x$

$= [(-2+\Delta x)^2+2+(-2+2\Delta x)^2+2+\ldots+(-2+n\Delta x)^2+2]\Delta x$

$= [2n+4-4\Delta x+\Delta x^2+4-8\Delta x+2^2\Delta x^2+\ldots+4-4n\Delta x+n^2\Delta x^2]\Delta x$

$= [2n+4n-4\Delta x(1+2+\ldots+n)+\Delta x^2(1^2+2^2+\ldots+n^2)]\Delta x$

$= \left[6n-4\cdot\frac{\cancel{2}}{n}\cdot\frac{\cancel{n}(n+1)}{\cancel{2}}+\frac{\cancel{4}2}{n^2}\cdot\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}3}\right]\frac{2}{n}$

$= \left[6n-4n-4+\frac{2}{3n}(2n^2+3n+1)\right]\frac{2}{n}$

$= 4-\frac{8}{n}+\frac{8}{3}+\frac{4}{n}+\frac{4}{3n^2}$

$= \frac{20}{3}-\frac{4}{n}+\frac{4}{3n^2}$

b) $\underline{S_2(f)} = f(x_0)\Delta x+f(x_1)\Delta x+\ldots+f(x_{n-1})\Delta x$

$= [f(0)+f(\Delta x)+\ldots+f((n-1)\Delta x)]\Delta x$

$= [0^2+2+\Delta x^2+2+\ldots+(n-1)^2\Delta x^2+2]\Delta x$

$= [2n+\Delta x^2(1^2+2^2+\ldots+(n-1)^2)]\Delta x$

$= \left[2n+\frac{25}{n^{\cancel{2}}}\cdot\frac{\cancel{n}(n-1)(2n-1)}{6}\right]\frac{5}{n}$

$= 10+\frac{125}{6n^2}(2n^2-3n+1)$

$$S_2(f) = 10 + \frac{125}{3} - \frac{125}{2n} + \frac{125}{6n^2}$$
$$= \frac{155}{3} - \frac{125}{2n} + \frac{125}{6n^2}$$

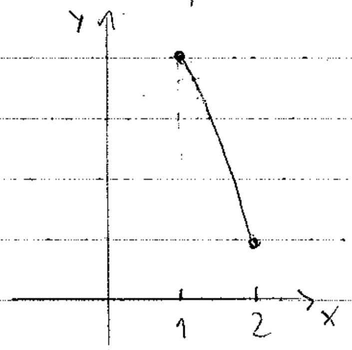
$[-2,5]$

$S_1(f) + S_2(f)$

$$\frac{20}{3} - \frac{4}{n} + \frac{4}{3n^2} + \frac{155}{3} - \frac{125}{2n} + \frac{125}{6n^2}$$

$$\frac{175}{3} - \frac{133}{2n} + \frac{133}{6n^2}$$

③ $f(x) = 5 - x^2, \ x \in [1,2]$



$x_0 = 1 \qquad \Delta x = \frac{1}{n}$

$x_1 = 1 + \Delta x$

$x_2 = 1 + 2\Delta x$

$x_n = 1 + n\Delta x$

$$\overline{S}(f) = f(x_0)\Delta x + f(x_1)\Delta x + \dots + f(x_{n-1})\Delta x$$
$$= [f(1) + f(1+\Delta x) + \dots + f(1+(n-1)\Delta x)]\Delta x$$
$$= [5 - 1^2 + 5 - (1+\Delta x)^2 + \dots + 5 - (1+(n-1)\Delta x)^2]\Delta x$$
$$= [5n - 1^2 - 1^2 - 2\Delta x - \Delta x^2 + \dots + (-1) - 2(n-1)\Delta x - (n-1)^2\Delta x^2]\Delta x$$
$$= [5n - n - 2\Delta x(1 + 2 + \dots + (n-1)) - \Delta x^2(1^2 + 2^2 + \dots + (n-1)^2)]\Delta x$$
$$= \left[4n - \frac{\cancel{2}}{\cancel{n}} \cdot \frac{\cancel{n}(n-1)}{\cancel{2}} - \frac{1}{n^2} \cdot \frac{\cancel{n} \cdot (n-1) \cdot (2n-1)}{6}\right] \cdot \frac{1}{n}$$
$$= \left[4n - n + 1 - \frac{1}{6n}(2n^2 - 3n + 1)\right] \frac{1}{n}$$
$$= 3 + \frac{1}{n} - \frac{1}{3} + \frac{1}{2n} - \frac{1}{6n^2} = \frac{8}{3} + \frac{3}{2n} - \frac{1}{6n^2}$$

$$\underline{S}(f) = f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x$$
$$= [f(1+\Delta x) + f(1+2\Delta x) + \dots + f(1+n\Delta x)]\Delta x$$
$$= [5-(1+\Delta x)^2 + 5-(1+2\Delta x)^2 + \dots + 5-(1+n\Delta x)^2]\Delta x$$
$$= [5n - 1^2 - 2\Delta x - \Delta x^2 - 1 - 4\Delta x - 2^2\Delta x^2 + \dots + (-1) - 2n\Delta x - n^2\Delta x^2]\Delta$$
$$= [5n - n - 2\Delta x(1+2+\dots+n) - \Delta x^2(1^2+2^2+\dots+n^2)]\Delta x$$
$$= \left[4n - \frac{2}{\cancel{n}} \cdot \frac{\cancel{n}(n+1)}{2} - \frac{1}{n^2} \cdot \frac{\cancel{n}(n+1)(2n+1)}{6}\right] \cdot \frac{1}{n}$$
$$= \left[4n - n - 1 - \frac{1}{6n}(2n^2+3n+1)\right] \cdot \frac{1}{n}$$
$$= 3 - \frac{1}{n} - \frac{1}{6n^2}(2n^2+3n+1)$$
$$= 3 - \frac{1}{n} - \frac{1}{3} - \frac{1}{2n} - \frac{1}{6n^2}$$
$$= \frac{8}{3} - \frac{3}{2n} - \frac{1}{6n^2}$$

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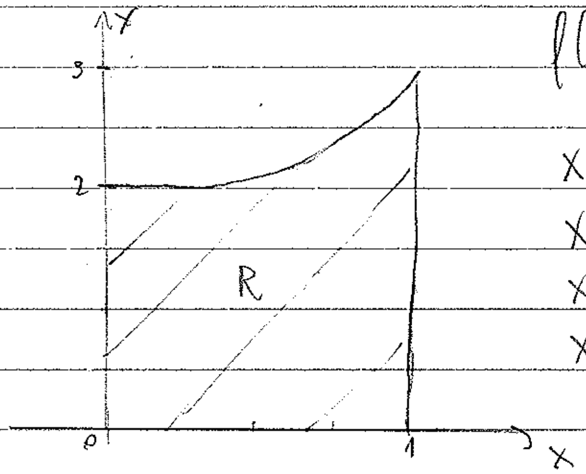


$f(x) = x^4 + 2$

$x_0 = 0$ $\qquad \Delta x = \frac{1}{n}$

$x_1 = \Delta x$

$x_2 = 2\Delta x$

$x_n = n\Delta x$

$$\overline{S}(f) = f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x$$
$$= [f(\Delta x) + f(2\Delta x) + \dots + f(n\Delta x)]\Delta x$$
$$= [\Delta x^4 + 2 + 2^4\Delta x^4 + 2 + \dots + n^4\Delta x^4 + 2]\Delta x$$
$$= [2n + \Delta x^4(1^4 + 2^4 + \dots + n^4)]\Delta x$$
$$= \left[2n + \frac{1}{n^4} \cdot \frac{n(n+1)(6n^3+9n^2+n-1)}{30}\right] \cdot \frac{1}{n}$$

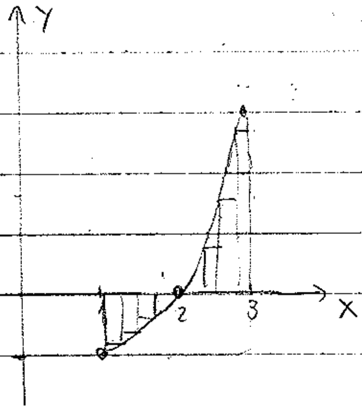
$$S(f) = 2 + \frac{1}{30n^4}\left[6n^4 + 9n^3 + n^2 - \cancel{n} + 6n^3 + 9n^2 + \cancel{n} - 1\right]$$

$$= 2 + \frac{1}{30n^4}\left(6n^4 + 15n^3 + 11n^2 - 1\right)$$

$$= 2 + \frac{1}{5} + \frac{1}{2n} + \frac{10}{30n^2} - \frac{1}{30n^4}$$

$$= \frac{11}{5} + \frac{1}{2n} + \frac{1}{3n^2} - \frac{1}{30n^4}$$

⑤ $\int_1^3 (x^2 - 2x)\,dx$



$\Delta x = \frac{1}{n}$

$x_0 = 1$

$x_1 = 1 + \Delta x$

$x_2 = 1 + 2\Delta x$

$x_n = 1 + n\Delta x$

$$S_1(f) = f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x$$

$$= \left[f(1+\Delta x) + f(1+2\Delta x) + \dots + f(1+n\Delta x)\right]\Delta x$$

$$= \Delta x\left[(1+\Delta x)^2 - 2(1+\Delta x) + (1+2\Delta x)^2 - 2(1+2\Delta x) + \dots + (1+n\Delta x)^2 - 2(1+n\Delta x)\right]$$

$$= \Delta x\left[\cancel{1} + \cancel{2\Delta x} + \Delta x^2 - \cancel{2} - \cancel{2\Delta x} + \cancel{1} + \cancel{4\Delta x} + 2^2\Delta x^2 - 2 - \cancel{4\Delta x} + \dots + \cancel{1} + 2n\cancel{\Delta x} + n^2\Delta x^2 - \cancel{2} - 2n\Delta x\right]$$

$$= \Delta x\left[n - 2n + \Delta x^2\left(1^2 + 2^2 + \dots + n^2\right)\right]$$

$$= \left[-n + \frac{1}{n^2} \cdot \frac{n\cdot(n+1)(2n+1)}{6}\right] \cdot \frac{1}{n}$$

$$= -1 + \frac{1}{6n^2}\left(2n^2 + 3n + 1\right)$$

$$= -1 + \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2} = -\frac{2}{3} + \frac{1}{2n} + \frac{1}{6n^2}$$

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$\Delta x = \frac{1}{n}$ $\quad x_0 = 2$

$x_1 = 2 + \Delta x$

$x_2 = 2 + 2\Delta x$

$x_n = 2 + n\Delta x$

$$\underline{S}_2(f) = f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_{n-1})\Delta x$$

$$= [f(2) + f(2+\Delta x) + f(2+2\Delta x) + \dots + f(2+(n-1)\Delta x)]\Delta x$$

$$= [2^2 - 2.2 + (2+\Delta x)^2 - 2(2+\Delta x) + (2+2\Delta x)^2 - 2(2+2\Delta x) + \dots + (2+(n-1)\Delta x)^2 - 2(2+(n-1)\Delta x)]\Delta x$$

$$= [\cancel{4} + 4\Delta x + \Delta x^2 - \cancel{4} - 2\Delta x + \cancel{4} + 8\Delta x + 2^2\Delta x^2 - \cancel{4} - 4\Delta x + \dots + \cancel{4} + 4(n-1)\Delta x + (n-1)^2\Delta x^2 - \cancel{4} - 2(n-1)\Delta x]\Delta x$$

$$= [2\Delta x(1+2+\dots+(n-1)) + \Delta x^2(1^2+2^2+\dots+(n-1)^2)]\Delta x$$

$$= \left[\frac{\cancel{2}}{\cancel{n}} \cdot \frac{(n-1).\cancel{n}}{\cancel{2}} + \frac{1}{n^2} \cdot \frac{(n-1).n(2n-1)}{6}\right]\frac{1}{n}$$

$$= 1 - \frac{1}{n} + \frac{1}{6n^2}(2n^2 - 3n + 1)$$

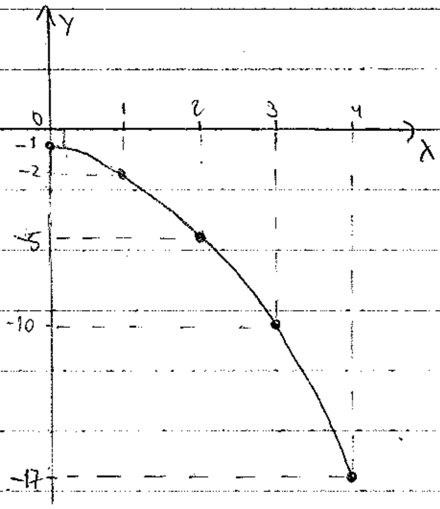
$$= 1 - \frac{1}{n} + \frac{1}{3} - \frac{1}{2n} + \frac{1}{6n^2}$$

$$= \frac{4}{3} - \frac{3}{2n} + \frac{1}{6n^2}$$

$$\underline{S}_T = \underline{S}_1 + \underline{S}_2 = -\frac{2}{3} + \frac{1}{2n} + \frac{1}{6n^2} + \frac{4}{3} - \frac{3}{2n} + \frac{1}{6n^2} = \frac{2}{3} - \frac{1}{n} + \frac{1}{3n^2}$$

$$\int_1^3 (x^2 + 2x)\,dx = \lim_{n \to +\infty}\left(\frac{2}{3} - \cancel{\frac{1}{n}}_{0} + \cancel{\frac{1}{3n^2}}_{0}\right) = \frac{2}{3}$$

⑥ $\int_0^4 (-x^2-1)\,dx$



$\Delta x = \dfrac{4}{n}$

$x_0 = 0$

$x_1 = \Delta x$

$x_2 = 2\Delta x$

$x_n = n\Delta x$

$$\overline{S}(f) = f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x$$
$$= \left[f(\Delta x) + f(2\Delta x) + \dots + f(n\Delta x)\right]\Delta x$$
$$= \left[-\Delta x^2 - 1 - 2^2\Delta x^2 - 1 + \dots - n^2\Delta x^2 - 1\right]\Delta x$$
$$= \left[-n - \Delta x^2\left(1^2 + 2^2 + \dots + n^2\right)\right]\Delta x$$
$$= \left[-n - \frac{\cancel{16}\,8}{n^2}\,\frac{n(n+1)\cdot(2n+1)}{\cancel{6}\,3}\right]\cdot\frac{4}{n}$$
$$= -4 - \frac{32}{3n^2}\left(2n^2 + 3n + 1\right)$$
$$= -4 - \frac{64}{3} - \frac{32}{n} - \frac{32}{3n^2}$$
$$= -\frac{76}{3} - \frac{32}{n} - \frac{32}{3n^2}$$

$$\int_0^4 (-x^2-1)\,dx = \lim_{n\to+\infty}\left(-\frac{76}{3} - \underbrace{\frac{32}{n}}_{\to 0} - \underbrace{\frac{32}{3n^2}}_{\to 0}\right) = -\frac{76}{3}$$

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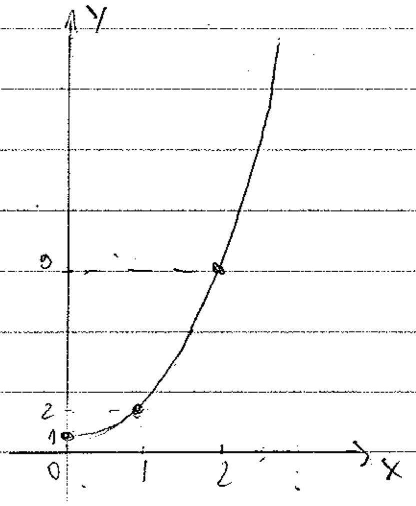
⑦ $f(x) = x^3 + 1,\ x \in [0, b]$



$x_0 = 0$ $\qquad \Delta x = \dfrac{b}{n}$

$x_1 = \Delta x$

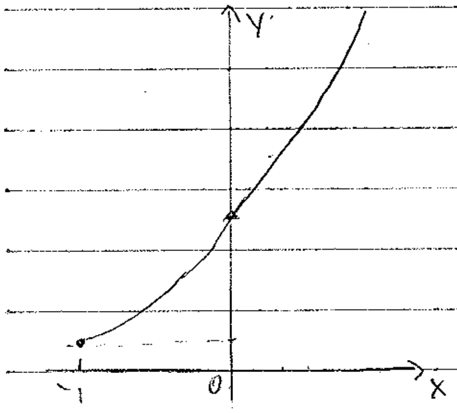
$x_2 = 2\Delta x$

$x_n = n\Delta x$

$$\bar{S}(f) = f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x$$
$$= [f(\Delta x) + f(2\Delta x) + \dots + f(n\Delta x)]\,\Delta x$$
$$= [\Delta x^3 + 1 + 2^3\Delta x^3 + 1 + \dots + n^3\Delta x^3 + 1]\,\Delta x$$
$$= [n + \Delta x^3\,[1^3 + 2^3 + \dots + n^3]]\,\Delta x$$
$$= \left[n + \frac{b^3}{n^3} \cdot \frac{n^2 \cdot (n+1)^2}{4}\right] \cdot \frac{b}{n}$$
$$= b + \frac{b^4}{4n^2} \cdot (n^2 + 2n + 1)$$
$$= b + \frac{b^4}{4} + \frac{b^4}{2n} + \frac{b^4}{4n^2}$$
$$= \frac{4b + b^4}{4} + \frac{b^4}{2n} + \frac{b^4}{4n^2}$$

$$\int_0^b (x^3+1)\,dx = \lim_{n \to +\infty} \left(\frac{4b + b^4}{4} + \frac{\cancel{b^4}}{2n_{\to 0}} + \frac{\cancel{b^4}}{4n^2_{\to 0}} \right) = \frac{4b + b^4}{4}$$

⑧ $f(x)=e^{x}\ ,\ x\in[-1,2]$



$\Delta x=\frac{3}{n}$

$S_n=\frac{a_1(1-q^n)}{1-q}$

$x_0=-1$

$x_1=-1+\Delta x$

$x_2=-1+2\Delta x$

$x_n=-1+n\Delta x$

$$\bar{S}(f)=f(x_1)\Delta x+f(x_2)\Delta x+\dots+f(x_n)\Delta x$$
$$=[f(-1+\Delta x)+f(-1+2\Delta x)+\dots+f(-1+n\Delta x)]\Delta x$$
$$=[e^{-1+\Delta x}+e^{-1+2\Delta x}+\dots+e^{-1+n\Delta x}]\Delta x$$
$$=[e^{-1}.e^{\Delta x}+e^{-1}.e^{2\Delta x}+\dots+e^{-1}e^{n\Delta x}]\Delta x$$
$$=[e^{-1}e^{\Delta x}(1+e^{\Delta x}+e^{2\Delta x}+\dots+e^{(n-1)\Delta x}]\Delta x$$
$$=\left[e^{-1}.e^{\Delta x}.\frac{e^{n\Delta x}-1}{e^{\Delta x}-1}\right].\Delta x$$
$$=\frac{e^{\frac{3}{n}}}{e}.\frac{e^{3}-1}{e^{\frac{3}{n}}-1}.\frac{3}{n}$$
$$=\frac{e^{3}-1}{e}.\frac{3.e^{\frac{3}{n}}}{n(e^{\frac{3}{n}}-1)}$$

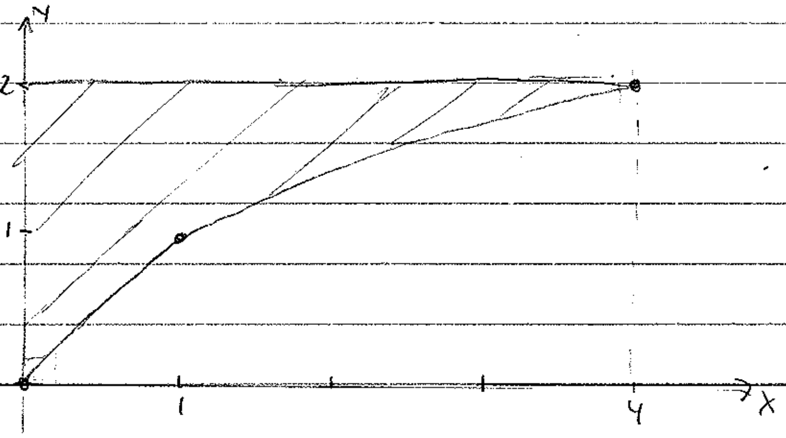
$$\int_{-1}^{2}e^{x}dx=\lim_{n\to+\infty}\frac{e^{3}-1}{e}.\frac{3e^{\frac{3}{n}}}{n(e^{\frac{3}{n}}-1)}$$
$$=\frac{e^{3}-1}{e}\lim_{n\to+\infty}\frac{3}{n}.\frac{e^{\frac{3}{n}}}{e^{\frac{3}{n}}-1}$$
$$=\frac{e^{3}-1}{e}\lim_{t\to0^{+}}t.\frac{e^{t}}{e^{t}-1}$$
$$=\frac{e^{3}-1}{e}\lim_{t\to0^{+}}\frac{\cancel{e^{t}}+t.\cancel{e^{t}}}{\cancel{e^{t}}}$$

$$\int_{-1}^{2}e^{x}dx=\frac{e^{3}-1}{e}\lim_{t\to0^{+}}1+t$$

$$\int_{-1}^{2}e^{x}dx=\frac{e^{3}-1}{e}$$

$$\left.\frac{3}{n}=t\ \right\}\ \begin{matrix}n\to+\infty\\ t\to0^{+}\end{matrix}$$

(9) $x = y^2$, $y \in [0,2]$



$$\Delta y = \frac{2}{n}$$

$y_0 = 0$

$y_1 = \Delta y$

$y_2 = 2\Delta y$

$y_n = n\Delta y$

$$\underline{S} = f(y_0)\Delta y + f(y_1)\Delta y + f(y_2)\Delta y + \dots + f(y_{n-1})\Delta y$$

$$= [f(0) + f(\Delta y) + f(2\Delta y) + \dots + f((n-1)\Delta y)]\Delta y$$

$$= [0 + \Delta y^2 + 2^2\Delta y^2 + \dots + (n-1)^2\Delta y^2]\Delta y$$

$$= [\Delta y^2(1^2 + 2^2 + \dots + (n-1)^2)]\Delta y$$

$$= \Delta y^3 \cdot \frac{(n-1)\cdot n \cdot (2n-1)}{6}$$

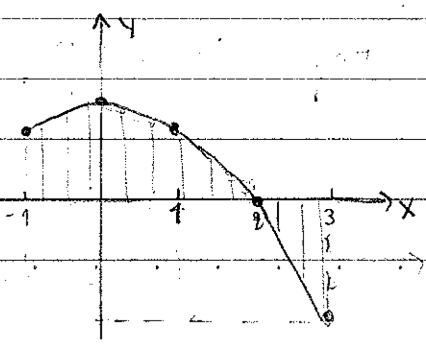
$$= \frac{\cancel{8}^{4}}{n^{\cancel{3}}} \cdot \frac{\cancel{n}\,(2n^2 - 3n + 1)}{\cancel{6}\,3}$$

$$= \frac{8}{3} - \frac{4}{n} + \frac{4}{3n^2}$$

$$\int_0^2 y^2 = \lim_{n \to +\infty}\left(\frac{8}{3} - \underbrace{\frac{4}{n}}_{0} + \underbrace{\frac{4}{3n^2}}_{0}\right) = \frac{8}{3}$$

(10) $I = \int_{-1}^{3} (4 - x^2)\,dx$

a)



Deve ser dividida em três parcelas ($[-1,0]$, $[0,2]$, $[2,3]$) devido a mudança do comportamento da questão (crescente, decrescente, negativa).

b) [0,2]

$\Delta x = \dfrac{2}{n}$ $\quad x_0 = 0$

$x_1 = \Delta x$

$x_2 = 2\Delta x$

$x_n = n\Delta x$

$$S(f) = f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x$$
$$= [f(\Delta x) + f(2\Delta x) + \dots + f(n\Delta x)]\Delta x$$
$$= [4 - \Delta x^2 + 4 - 2^2\Delta x^2 + \dots + 4 - n^2\Delta x^2]\Delta x$$
$$= [4n - \Delta x^2(1^2 + 2^2 + \dots + n^2)]\Delta x$$
$$= \left[4n - \frac{\cancel{4}2}{n^{\cancel{2}}} \cdot \frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}3}\right] \cdot \frac{2}{n}$$
$$= 8 - \frac{4}{3n^2}(2n^2 + 3n + 1)$$
$$= 8 - \frac{8}{3} - \frac{4}{n} - \frac{4}{3n^2}$$
$$= \frac{16}{3} - \frac{4}{n} - \frac{4}{3n^2}$$

$$\int_0^2 (4 - x^2)\,dx = \lim_{n \to +\infty} \left(\frac{16}{3} - \cancel{\frac{4}{n}}_{0} - \cancel{\frac{4}{3n^2}}_{0}\right) = \frac{16}{3}$$

c) Não, pois o intervalo [2,3] é negativo, deve-se, portanto, inverter o sinal assim,

$$A = \int_{-1}^{2} (4 - x^2)\,dx - \int_{2}^{3} (4 - x^2)\,dx$$

/ /

(11) a) $\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_{0}^{a} f(x)dx$

f é par, então $f(x) = f(-x)$. Substituindo:

$$\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(-x)dx + \int_{0}^{a} f(x)dx$$

$u = -x$
$du = -dx$
$x=0 \,\}\, x=-a$
$u=0 \,\}\, u=a$

$$= -\int_{a}^{0} f(u)du + \int_{0}^{a} f(x)dx \qquad u$$

$$= \int_{0}^{a} f(u)du + \int_{0}^{a} f(u)du$$

$$= 2\int_{0}^{a} f(u)du$$

b) $\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_{0}^{a} f(x)dx$

f é ímpar, então $f(x) = -f(-x)$. Substituindo

$$\int_{-a}^{a} f(x)dx = \int_{-a}^{0} -f(-x)dx + \int_{0}^{a} f(x)dx$$

$$= -\int_{-a}^{0} f(-x)dx + \int_{0}^{a} f(x)dx$$

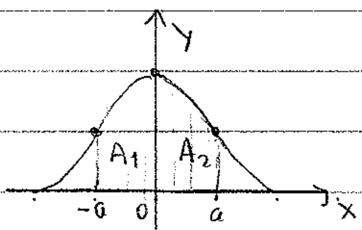
$u = -x$
$du = -dx$
$x=0 \,\}\, x=-a$
$u=0 \,\}\, u=a$

$$= \int_{a}^{0} f(u)du + \int_{0}^{a} f(x)dx$$

$$= -\int_{0}^{a} f(u)du + \int_{0}^{a} f(u)du$$

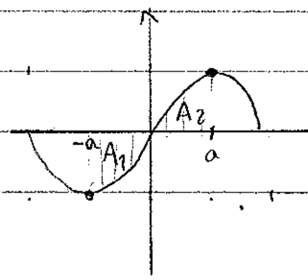
$$= 0$$

c)



$A_1 = A_2 = A$

$\therefore A_1 + A_2 = 2A$



$A_1 = -A_2$

$|A_1| = |A_2| = A$

$A_1 + A_2 = -A + A = 0$

⑫ TVM: $\exists c \in [6,12] \,/\, \int_6^{12} \frac{t}{20}(t-12)(t-24)\,dt = (12-6)\,f(c)$

$$\frac{1}{20}\int_6^{12} t(t^2-36t+288)\,dt = 6\,f(c)$$

$$\frac{1}{20}\int_6^{12} (t^3-36t^2+288t)\,dt = 6\,f(c)$$

$$\frac{1}{20}\left(\int_6^{12} t^3\,dt - 36\int_6^{12} t^2\,dt + 288\int_6^{12} t\,dt\right) = 6\,f(c)$$

$$\frac{1}{20}\left(\frac{t^4}{4}\Big|_6^{12} - 36\,\frac{t^3}{3}\Big|_6^{12} + 288\,\frac{t^2}{2}\Big|_6^{12}\right) = 6\,f(c)$$

$$\left(\frac{t^4}{80} - \frac{6}{10}t^3 + \frac{72}{10}t^2\right)\Big|_6^{12} = 6\,f(c)$$

$$\left(\frac{t^4}{80} - \frac{6t^3}{10} + \frac{6^2.2t^2}{10}\right)\Big|_6^{12} = 6\,f(c)$$

$$\left(\frac{t^4 - 6.2^3t^3 + 6^2.2^4t^2}{80}\right)\Big|_6^{12} = 6\,f(c)$$

$$\frac{t^2}{2^4.5}\left(t^2 - 6.2^3t + 6^2 2^4\right)\Big|_6^{12} = 6\,f(c)$$

$$\frac{2^2.6^2}{2^4.5}\left(2^2 6^2 - \cancel{6^2.2^4} + \cancel{6^2.2^4}\right) - \frac{6^2}{80}\left(6^2 - 6^2.2^3 + 6^2.2^4\right) = 6\,f(c)$$

$$\frac{6^4}{5} - \frac{6^4}{80}\left(1-2^3+2^4\right) = 6\,f(c)$$

$$\frac{6^4}{5} - \frac{6^4.9}{80} = 6\,f(c)$$

$$\frac{6^4}{5}\left(1 - \frac{9}{16}\right) = 6\,f(c)$$

$$\frac{6^3}{5}\cdot\frac{7}{16} = f(c)$$

$$\frac{\cancel{2^3}.3^3.7}{5.2^4} = f(c) \Rightarrow f(c) = 18{,}9\,°F$$

/ /

(13) $\int_0^t f(x)\,dx = t^3$

$F(x)\Big|_0^t = t^3$

$F(t) - F(0) = t^3$

$f(t) = 3t^2$

(14) $\int_0^x f(t)\,dt = [f(x)]^2$

$F(t)\Big|_0^x = [f(x)]^2$

$F(x) - F(0) = [f(x)]^2$

$\cancel{f(x)} = 2\,\cancel{f(x)}\cdot f'(x)$

$\int f'(x) = \int \frac{1}{2}$

$f(x) = \frac{x}{2}$

(15) $g(x) = \int_{x^3}^{x^5} f(t)\,dt$

$g(x) = F(t)\Big|_{x^3}^{x^5}$

$g(x) = F(x^5) - F(x^3)$

$g'(x) = f(x^5) - f(x^3)$

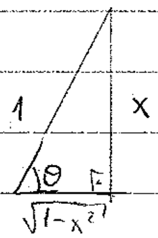
$g'(1) = f(1) - f(1)$

$g'(1) = 0$

(16) $f(x) = \int_0^x \frac{dg}{dt}(t)\,dt$

$f(x) = g(x) - g(0)$ R: c)

(17) $\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \lim_{b \to 1^-} \int_0^b \frac{dx}{\sqrt{1-x^2}}$



$\operatorname{sen}\theta = x$

$dx = \cos\theta\,d\theta$

$\sqrt{1-x^2} = \cos\theta$

$\theta = \operatorname{arcsen} x$

$= \lim_{b \to 1^-} \int_0^b \frac{\cancel{\cos\theta}\,d\theta}{\cancel{\cos\theta}}$

$= \lim_{b \to 1^-} \operatorname{arcsen} x\Big|_0^b$

$= \lim_{b \to 1^-} \operatorname{arcsen}(b) - \operatorname{arcsen}(0)$

$= \frac{\pi}{2}$

18 a) $\frac{1}{2}\int_{1}^{\sqrt{2}} \frac{2x}{e^{x^2}}\,dx$ $\quad u=x^2$, $du=2x\,dx$

$\frac{1}{2}\int \frac{du}{e^{u}}$

$-\frac{1}{2e^{u}}$

$-\frac{1}{2e^{x^2}}\Big|_{1}^{\sqrt{2}}$

$-\frac{1}{2e^{2}}+\frac{1}{2e}$

b) $\frac{1}{3}\int_{-1}^{1} \frac{3x^2}{\sqrt{x^3+9}}\,dx$ $\quad u=x^3+9$, $du=3x^2\,dx$

$\frac{1}{3}\int \frac{du}{\sqrt{u}}$

$\frac{2}{3}\sqrt{u}$

$\frac{2\sqrt{x^3+9}}{3}\Big|_{-1}^{1}$

$\frac{2\sqrt{10}}{3}-\frac{2\sqrt{8}}{3}$

$\frac{2\sqrt{10}}{3}-\frac{4\sqrt{2}}{3}$

c) $\int_{0}^{\frac{\pi}{4}} tg^2x\, sec^2x\,dx$ $\quad u=tg\,x$, $du=sec^2x\,dx$

$\int u^2\,du$

$\frac{u^3}{3}$

$\frac{tg^3x}{3}\Big|_{0}^{\frac{\pi}{4}}$

$\frac{1}{3}$

d) $\int_{0}^{1} x\,sen\,x\,dx$

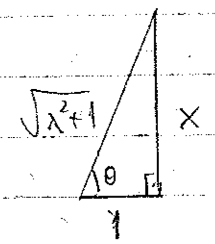
u (derivo)	dv (integro)
+ x	sen x
− 1	− cos x
	− sen x

$-x\cos x+sen\,x\Big|_{0}^{1}$

$-\cos(1)+sen(1)+0\cos(0)-sen(0)$

$sen(1)-\cos(1)$

e) $\int_{\frac{3}{4}}^{\frac{4}{3}} \frac{dx}{x\sqrt{1+x^2}}$



$tg\,\theta=x$

$sec^2\theta\,d\theta=dx$

$\sqrt{x^2+1}=sec\,\theta$

$\int \frac{sec^{\cancel{2}}\theta\,d\theta}{tg\,\theta\cdot\cancel{sec\,\theta}}$

$\displaystyle\int \frac{1}{\cancel{\cos\theta}\cdot\frac{\operatorname{sen}\theta}{\cancel{\cos\theta}}}\,d\theta$

$\int \operatorname{cossec}\theta\,d\theta$

$\ln|\operatorname{cossec}\theta - \operatorname{cotg}\theta|$

$\ln\left|\frac{\sqrt{x^2+1}}{x} - \frac{1}{x}\right| \Big|_{3/4}^{4/3}$

$\ln\left|\sqrt{\frac{16}{9}+1}\cdot\frac{3}{4} - \frac{3}{4}\right| - \ln\left|\sqrt{\frac{9}{16}+1}\cdot\frac{4}{3} - \frac{4}{3}\right|$

$\ln\left|\frac{5}{\cancel{3}}\cdot\frac{\cancel{3}}{4} - \frac{3}{4}\right| - \ln\left|\frac{5}{\cancel{4}}\cdot\frac{\cancel{4}}{3} - \frac{4}{3}\right|$

$\ln\left|\frac{1}{2}\right| - \ln\left|\frac{1}{3}\right|$

$\ln 3 - \ln 2$

$0{,}405$

f) $\displaystyle\int_0^3 \frac{x\,dx}{\sqrt{x+1}}$ $\qquad u = x+1$, $du = dx$

$\displaystyle\int \frac{(u-1)}{\sqrt{u}}\,du$

$\displaystyle\int \frac{u}{\sqrt{u}}\,du - \int \frac{du}{\sqrt{u}}$

$\int u^{1/2}\,du - \int u^{-1/2}\,du$

$\frac{2}{3}u^{3/2} - 2u^{1/2}$

$\frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2}\Big|_0^3$

$\frac{2}{3}\cdot 4^{3/2} - 2\cdot 4^{1/2} - \frac{2}{3}1^{3/2} + 2\cdot 1^{1/2}$

$\frac{2}{3}\cdot 2^3 - 2\cdot 2 - \frac{2}{3} + 2$

$\frac{2}{3}(8-1) - 2$

$\frac{14-6}{3}$

$\frac{8}{3}$

g) $\int_1^2 \left(\sqrt{x} + \frac{1}{\sqrt[3]{x}} + \sqrt[4]{x}\right) dx$

$\int_1^2 x^{1/2}\, dx + \int_1^3 x^{-1/3} dx + \int_1^3 x^{1/4}\, dx$

$\left(\frac{2}{3}x^{3/2} + \frac{3}{2} \cdot x^{2/3} + \frac{4}{5} x^{5/4}\right)\Big|_1^2$

$\frac{2}{3} \cdot 2^{3/2} + \frac{3}{2} \cdot 2^{2/3} + \frac{4}{5} \cdot 2^{5/4} - \frac{2}{3} - \frac{3}{2} - \frac{4}{5}$

$\frac{2}{3}(2\sqrt{2}-1) + \frac{3}{2}(\sqrt[3]{4}-1) + \frac{4}{5}(2\sqrt[4]{2}-1)$

$3,203$

h) $\int_0^{\frac{\pi}{3}} tg\, x\, dx$ $\quad u = \cos x$

$du = -\operatorname{sen} x\, dx$

$\int_0^{\frac{\pi}{3}} \frac{\operatorname{sen} x\, dx}{\cos x}$

$-\int \frac{du}{u}$

$-\ln|u|$

$-\ln|\cos x|\Big|_0^{\frac{\pi}{3}}$

$-\ln|\cos(\frac{\pi}{3})| + \ln|\cos(0)|$

$-\ln\left(\frac{1}{2}\right)$

$\ln 2$

i) $\int_1^4 \frac{x}{\sqrt{2+4x}} dx$ $\quad u = 1+2x$

$du = 2dx$

$\frac{1}{4\sqrt{2}}\int_1^4 \frac{2x \cdot 2dx}{\sqrt{1+2x}}$

$\frac{1}{4\sqrt{2}}\int \frac{(u-1)\,du}{\sqrt{u}}$

$\frac{1}{4\sqrt{2}}\int u^{1/2} du - \frac{1}{4\sqrt{2}}\int u^{-1/2} du$

$\frac{1}{6\sqrt{2}} \cdot u^{3/2} - \frac{1}{2\sqrt{2}} u^{1/2}$

$\frac{1}{6\sqrt{2}}(1+2x)^{3/2} - \frac{1}{2\sqrt{2}}(1+2x)^{1/2}\Big|_1^4$

$\frac{1}{6\sqrt{2}} \cdot 9^{3/2} - \frac{1}{2\sqrt{2}} \cdot 9^{1/2} - \frac{1}{6\sqrt{2}} \cdot 3^{3/2} + \frac{1}{2\sqrt{2}} \cdot 3^{1/2}$

$\frac{3^2}{2\sqrt{2}} - \frac{3}{2\sqrt{2}} \cancel{-\frac{\sqrt{3}}{2\sqrt{2}}} \cancel{+\frac{\sqrt{3}}{2\sqrt{2}}}$

$\frac{3}{\sqrt{2}}$

19) a) $\int_0^1 \left(x+\sqrt{x}-\frac{1}{\sqrt[3]{x}}\right)dx$

$\lim_{a\to 0^-} \int_a^1 \left(x+\sqrt{x}-\frac{1}{\sqrt[3]{x}}\right)dx$

$\lim_{a\to 0^-} \left(\int_a^1 x\,dx + \int_a^1 \sqrt{x}\,dx - \int \frac{dx}{\sqrt[3]{x}}\right)$

$\lim_{a\to 0^-} \left(\frac{x^2}{2}+\frac{2x^{3/2}}{3}-\frac{3x^{2/3}}{2}\right)\Big|_a^1$

$\lim_{a\to 0^-} \frac{1}{2}+\frac{2}{3}-\frac{3}{2}-\frac{a^2}{2}-\frac{2a^{3/2}}{3}+\frac{3a^{2/3}}{2}$

$-1+\frac{2}{3}$

$-\frac{1}{3}$

b) $\int_0^2 x^2 \ln x\,dx$

$\lim_{a\to 0^-} \int_a^2 x^2 \ln x\,dx$

$u=\ln x \qquad \int dv=\int x^2\,dx$

$du=\frac{dx}{x} \qquad v=\frac{x^3}{3}$

$\lim_{a\to 0^-} \int_a^2 x^2\ln x\,dx = \lim_{a\to 0^-} \frac{x^3\ln x}{3}\Big|_a^2 - \int_a^2 \frac{x^3}{3}\frac{dx}{x}$

$= \lim_{a\to 0^-} \frac{x^3\ln x}{3}\Big|_a^2 - \frac{1}{3}\int_a^2 x^2\,dx$

$= \lim_{a\to 0^-} \frac{x^3\ln x}{3}\Big|_a^2 - \frac{x^3}{9}\Big|_a^2$

$$\lim_{a\to 0^-} \frac{2^3 \ln 2}{3} - \frac{a^3 \ln a}{3} - \frac{2^3}{9} + \frac{\cancel{a^3}}{9} \to 0$$

$$\lim_{a\to 0^-} \frac{2^3 \ln 2}{3} - \frac{2^3}{9} - \frac{1}{3} \lim_{a\to 0^-} a^3 \ln a$$

$$\frac{2^3 \ln 2}{3} - \frac{2^3}{9} - \frac{1}{3} \lim_{a\to 0^-} \frac{\ln a}{\frac{1}{a^3}}$$

$$\frac{2^3 \ln 2}{3} - \frac{2^3}{9} - \frac{1}{3} \lim_{a\to 0^-} \frac{\frac{1}{a}}{-\frac{3}{a^4}}$$

$$\frac{2^3 \ln 2}{3} - \frac{2^3}{9} + \cancel{\frac{1}{9} \lim_{a\to 0^-} a^3} \to 0$$

$$\frac{2^3 \ln 2}{3} - \frac{2^3}{9}$$

c) $\int_1^{+\infty} \frac{1}{x^2} \cos\left(\frac{1}{x}\right) dx$

$$\lim_{b\to +\infty} \int_1^b \frac{1}{x^2} \cos\left(\frac{1}{x}\right) dx$$

$$w = \frac{1}{x}$$

$$dw = -\frac{1}{x^2} dx$$

$$\lim_{b\to +\infty} -\int \cos w \, dw$$

$$\lim_{b\to +\infty} - \operatorname{sen} w$$

$$\lim_{b\to +\infty} - \operatorname{sen}\left(\frac{1}{x}\right)\Big|_1^b$$

$$\lim_{b\to +\infty} - \cancel{\operatorname{sen}\left(\frac{1}{b}\right)} \to 0 + \operatorname{sen}(1)$$

$\operatorname{sen}(1)$

d) $\int_0^{\frac{\sqrt{2}}{2}} \frac{dx}{\sqrt{1-x^2}}$

$$\operatorname{arcsen} x \Big|_0^{\frac{\sqrt{2}}{2}}$$

$$\operatorname{arcsen}\left(\frac{\sqrt{2}}{2}\right) - \operatorname{arcsen} 0$$

$$\frac{\pi}{4}$$

e) $\int_{-\infty}^0 x.e^x dx$

$u = x \qquad \int dv = \int e^x dx$

$du = dx \qquad v = e^x$

$$\lim_{a\to -\infty} \int_a^0 x.e^x dx = \lim_{a\to -\infty} x.e^x \Big|_a^0 - \int_a^0 e^x dx$$

/ /

$$\lim_{a\to-\infty} x.e^x\Big|_a^0 - e^x\Big|_a^0$$

$$\lim_{a\to-\infty} 0.e^0 - a.e^a - e^0 + e^a$$

$$-1+\lim_{a\to-\infty} e^a - a.e^a$$

$$-1+\lim_{a\to-\infty} e^a - a\lim_{a\to-\infty} e^a$$

$$-1+0-a0$$

$$-1$$

f) $\int_{-\infty}^{+\infty} x\, e^{-|x-4|}\,dx$ $|x-4| = \begin{cases} x-4, & \text{se } x\geq 4 \\ 4-x, & \text{se } x<4 \end{cases}$

$$\lim_{a\to-\infty}\int_a^4 x.e^{(x-4)}\,dx + \lim_{b\to+\infty}\int_4^b x.e^{4-x}\,dx$$

$u = x-4$ $v = 4-x$

$du = dx$ $dv = -dx$

$$\lim_{a\to-\infty}\int (u+4).e^u\,du + \lim_{b\to+\infty}\int (4-v)\,e^v.-dv$$

$$\lim_{a\to-\infty}\left(\int u.e^u\,du + 4\int e^u\,du\right) + \lim_{b\to+\infty}\left(\int v e^v\,dv - 4\int e^v\,dv\right)$$

$$\lim_{a\to-\infty}(u.e^u - e^u + 4e^u) + \lim_{b\to+\infty}(v.e^v - e^v - 4e^v)$$

$$\lim_{a\to-\infty}(u.e^u + 3e^u) + \lim_{b\to+\infty}(v.e^v - 5e^v)$$

$$\lim_{a\to-\infty} e^u(3+u) + \lim_{b\to+\infty} e^v(v-5)$$

$$\lim_{a\to-\infty} e^{x-4}(x-1)\Big|_a^4 + \lim_{b\to+\infty} e^{4-x}(-x-1)\Big|_4^b$$

$\lim_{a\to-\infty} e^{0}.3 - e^{a-4}(a-1) - \lim_{b\to+\infty} e^{4-b}(b+1) - e^{0}.5$

$3 - \lim_{a\to-\infty} e^{a-4}(a-1) - \lim_{b\to+\infty} e^{4-b}(b+1) + 5$

$8 - (a-1)\lim_{a\to-\infty} e^{a-4} - (b+1)\lim_{b\to+\infty} e^{4-b}$

$8 - (a-1).0 - (b+1).0$

8

g) $\int_1^5 \frac{1}{\sqrt{5-x}}\,dx$ $\quad u=5-x$, $du=-dx$	h) $\int_0^{+\infty} \frac{dx}{e^x}$ $\quad u=-x$, $du=-dx$
$\lim_{b\to5^-} \int_1^b \frac{1}{\sqrt{5-x}}\,dx$	$\lim_{b\to+\infty} -\int e^u\,du$
$\lim_{b\to5^-} -\int \frac{du}{\sqrt{u}}$	$\lim_{b\to+\infty} -e^u$
$\lim_{b\to5^-} -2u^{1/2}$	$\lim_{b\to+\infty} -e^{-x}\Big\|_0^b$
$\lim_{b\to5^-} -2\sqrt{5-x}\Big\|_1^b$	$\lim_{b\to+\infty} -e^{-b} + e^{-0}$
$\lim_{b\to5^-} -2\sqrt{5-b} + 2\sqrt{4}$	$\underline{1}$
$4 - 2\lim_{b\to5^-}\sqrt{5-b}$	
4	

i) $-\frac{1}{2}\int_0^4 \frac{-2x}{\sqrt{16-x^2}}\,dx$ $\qquad u = 16-x^2$, $du = -2x\,dx$

$\lim_{b\to 4^-} -\frac{1}{2}\int \frac{du}{\sqrt{u}}$

$\lim_{b\to 4^-} -\frac{1}{\cancel{2}}\cdot\cancel{2}\cdot u^{1/2}$

$-\lim_{b\to 4^-} \sqrt{16-x^2}\,\Big|_0^b$

$-\lim_{b\to 4^-} \sqrt{16-b^2} - \sqrt{16}$

4

j) $\int_0^{+\infty} \frac{x}{e^x}\,dx$ $\qquad u = -x$, $du = -dx$

$\lim_{b\to+\infty} \int -u\cdot e^u \cdot -du$

$\lim_{b\to+\infty} u e^u - e^u$

$\lim_{b\to+\infty} \frac{-x}{e^x} - \frac{1}{e^x}\Big|_0^b$

$\lim_{b\to+\infty} \frac{-b}{e^b} - \cancel{\frac{1}{e^b}}_{\to 0} + \cancel{\frac{0}{e^0}}_{\to 0} + \frac{1}{e^0}$

$1 - \lim_{b\to+\infty} \frac{b}{e^b}$

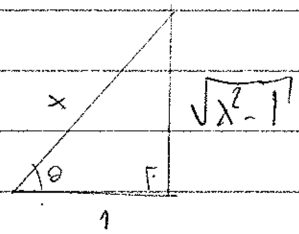
$1 - \lim_{b\to+\infty} \frac{1}{e^b}$

1

k) $\int_1^{+\infty} \frac{dx}{x\sqrt{x^2-1}}$ $\qquad \sec\theta = x$, $dx = tg\theta\sec\theta\,d\theta$ $\qquad tg\theta = \sqrt{x^2-1}$

$\lim_{a\to 1^-} \int_a^c \frac{dx}{x\sqrt{x^2-1}} + \lim_{b\to+\infty} \int_c^b \frac{dx}{x\sqrt{x^2-1}}$

$\lim_{a\to 1^-} \int \frac{\cancel{tg\theta\sec\theta}\,d\theta}{\cancel{\sec\theta\, tg\theta}} + \lim_{b\to+\infty} \int \frac{\cancel{tg\theta\sec\theta}\,d\theta}{\cancel{\sec\theta}\, tg\theta}$



$\lim_{a\to 1^-} \theta + \lim_{b\to+\infty} \theta$

$\lim_{a\to 1^-} arctg(\sqrt{x^2-1})\,\Big|_a^c + \lim_{b\to+\infty} arctg(\sqrt{x^2-1})\,\Big|_c^b$

$\lim_{a\to 1^-} \cancel{arctg(\sqrt{c^2-1})} - \cancel{arctg(\sqrt{a^2-1})}_{\to 0} + \lim_{b\to+\infty} arctg(\sqrt{b^2-1}) - \cancel{arctg(\sqrt{c^2-1})}$

$\frac{\pi}{2}$

l) $\int_0^1 \frac{dx}{\sqrt{1-x}}$ $\quad u = 1-x$, $du = -dx$

$\lim_{b\to 1^-} -\int \frac{du}{\sqrt{u}}$

$\lim_{b\to 1^-} -2\sqrt{u}$

$\lim_{b\to 1^-} -2\sqrt{1-x}\Big|_0^b$

$\lim_{b\to 1^-} -2\sqrt{1-b} + 2\sqrt{1-0}$

2

m) $\int_{-\infty}^{1} e^x\,dx$

$\lim_{a\to -\infty} e^x\Big|_a^1$

$\lim_{a\to -\infty} e^1 - e^a$

e

o) $\int_0^1 \frac{1}{x^3}\,dx$

$\lim_{a\to 0^-} \int_a^1 \frac{1}{x^3}\,dx$

$\lim_{a\to 0^-} -\frac{1}{4x^4}\Big|_a^1$

$\lim_{a\to 0^-} -\frac{1}{4} + \frac{1}{4a^4}$

$+\infty$

A integral diverge

n) $\int_{-1}^{1} \frac{dx}{x^4}$

$\int_{-1}^{0} \frac{dx}{x^4} + \int_0^1 \frac{dx}{x^4}$

$\lim_{b\to 0^-} \int_{-1}^{b} \frac{dx}{x^4} + \lim_{a\to 0^+} \int_a^1 \frac{dx}{x^4}$

$\lim_{b\to 0^-} -\frac{1}{5x^5}\Big|_{-1}^{b} + \lim_{a\to 0^+} -\frac{1}{5x^5}\Big|_a^1$

$\lim_{b\to 0^-} -\frac{1}{5b^5} + \cancel{\frac{1}{5}} + \lim_{a\to 0^+} \cancel{-\frac{1}{5}} + \frac{1}{5a^5}$

$-\infty + \infty$

Indeterminação

p) $\int_{-2}^{+\infty} \frac{dx}{(x+1)^2}$

$\int_{-2}^{-1} \frac{dx}{(x+1)^2} + \int_{-1}^{+\infty} \frac{dx}{(x+1)^2}$

$\lim_{b\to -1^-} \int_{-2}^{b} \frac{dx}{(x+1)^2} + \lim_{a\to -1^+} \int_a^c \frac{dx}{(x+1)^2} + \lim_{d\to +\infty} \int_c^d \frac{dx}{(x+1)^2}$

$\lim_{b\to -1^-} \frac{-1}{3(x+1)^3}\Big|_{-2}^{b} + \lim_{a\to -1^+} \frac{-1}{3(x+1)^3}\Big|_a^c + \lim_{d\to +\infty} \frac{-1}{3(x+1)^3}\Big|_c^d$

$\lim_{b\to -1^-} \frac{-1}{3(b+1)^3} - \frac{1}{3} + \lim_{a\to -1^+} \cancel{\frac{-1}{3(c+1)^3}} + \frac{1}{3(a+1)^3} +$

$+ \lim_{d\to +\infty} \cancel{\frac{-1}{3(d+1)^3}}_{\to 0} + \cancel{\frac{1}{3(c+1)^3}}$

$-\infty + \infty$

Indeterminação

/ /

(20) $\int_0^{+\infty} \frac{700}{e^{t/5}}\,dt \qquad u = -\frac{t}{5}$

$700 \int_0^{+\infty} e^{-t/5}\,dt \qquad dt = -5\,du$

$700 \lim_{b \to +\infty} \int_0^b e^{-t/5}\,dt$

$-3500 \lim_{b \to +\infty} \int e^u\,du$

$-3500 \lim_{b \to +\infty} e^u$

$-3500 \lim_{b \to +\infty} e^{-\frac{t}{5}} \Big|_0^b$

$-3500 \left(\lim_{b \to +\infty} e^{\cancel{-\frac{b}{5}}} - e^0 \right)$

3500 milhares de m^3

(22) $\int_e^{+\infty} \frac{1}{x(\ln x)^p}\,dx$

$\lim_{b \to +\infty} \int_e^b \frac{1}{x(\ln x)^p}\,dx \qquad u = \ln x \qquad du = \frac{dx}{x}$

$\lim_{b \to +\infty} \int \frac{du}{u^p}$

$\lim_{b \to +\infty} -\frac{1}{p-1} \cdot \frac{1}{u^{p-1}}$

$-\frac{1}{p-1} \lim_{b \to +\infty} \frac{1}{(\ln x)^{p-1}} \Big|_e^b$

$-\frac{1}{p-1} \lim_{b \to +\infty} \frac{1}{(\ln b)^{p-1}} - \frac{1}{1^{p-1}}$

$p > 1$, conforme justificativa da questão "21"

(21) $\int_1^{+\infty} \frac{1}{x^p}\,dx$

$\lim_{b \to +\infty} \int_1^b \frac{1}{x^p}\,dx$

$\lim_{b \to +\infty} \frac{-1}{p-1} \cdot \frac{1}{x^{p-1}} \Big|_1^b$

$-\frac{1}{p-1} \cdot \lim_{b \to +\infty} \frac{1}{b^{p-1}} - \frac{1}{1^{p-1}}$

Para I convergir é necessário que $p-1>0$, pois isso mantém b no denominador, fazendo com que $\frac{1}{b^{p-1}} \to 0$. Assim I converge $\forall p > 1$.

/ /

(23) a) $\int_{1}^{+\infty} \frac{x}{e^{x^2}}\,dx$ $\quad u=x^2$, $du=2x\,dx$

$\lim\limits_{b\to+\infty} \frac{1}{2}\int_{1}^{b} \frac{2x}{e^{x^2}}\,dx$

$\frac{1}{2}\lim\limits_{b\to+\infty} \int \frac{du}{e^u}$

$\frac{1}{2}\lim\limits_{b\to+\infty} -\frac{1}{e^u}$

$-\frac{1}{2}\lim\limits_{b\to+\infty} \frac{1}{e^{x^2}}\Big|_{1}^{b}$

$-\frac{1}{2}\lim\limits_{b\to+\infty} \cancel{\frac{1}{e^{b^2}}}_{\to 0} - \frac{1}{e}$

$\frac{1}{2e}$

b) $\int_{-\infty}^{+\infty} \frac{\operatorname{arctg} x}{x^2+1}\,dx$

$\lim\limits_{a\to-\infty}\int_{a}^{c} \frac{\operatorname{arctg} x}{x^2+1}\,dx + \lim\limits_{b\to+\infty}\int_{c}^{b} \frac{\operatorname{arctg} x}{x^2+1}\,dx$

$u=\operatorname{arctg} x \Rightarrow du=\frac{dx}{x^2+1}$

$\lim\limits_{a\to-\infty}\int u\,du + \lim\limits_{b\to+\infty}\int u\,du$

$\lim\limits_{a\to-\infty}\frac{u^2}{2} + \lim\limits_{b\to+\infty}\frac{u^2}{2}$

$\frac{1}{2}\lim\limits_{a\to-\infty}\operatorname{arctg}^2 x\,\Big|_{a}^{c} + \frac{1}{2}\lim\limits_{b\to+\infty}\operatorname{arctg}^2 x\,\Big|_{c}^{b}$

$\frac{1}{2}\lim\limits_{a\to-\infty}\cancel{\operatorname{arctg}^2 c}-\operatorname{arctg}^2 a + \frac{1}{2}\lim\limits_{b\to+\infty}\operatorname{arctg}^2 b - \cancel{\operatorname{arctg}^2 c}$

$-\frac{1}{2}\cdot\frac{\pi^2}{4} + \frac{1}{2}\cdot\frac{\pi^2}{4} = 0$

c) $\int_{-\infty}^{\frac{\pi}{2}} \operatorname{sen}(2x)\,dx$

$\lim\limits_{a\to-\infty}\int_{a}^{\frac{\pi}{2}} \operatorname{sen}(2x)\,dx$

$\lim\limits_{a\to-\infty} -\frac{1}{2}\cos(2x)\Big|_{a}^{\frac{\pi}{2}}$

$\lim\limits_{a\to-\infty} -\frac{1}{2}\cos(\pi) + \frac{1}{2}\cos(2a)$

a integral diverge

d) $\int_{0}^{1} x\ln x\,dx$

$\lim\limits_{a\to 0^-}\int_{a}^{1} x\ln x\,dx$

$u=\ln x$ $\quad \int dv=\int x\,dx$

$du=\frac{dx}{x}$ $\quad v=\frac{x^2}{2}$

$\lim\limits_{a\to 0^-}\frac{x^2\ln x}{2}\Big|_{a}^{1} - \int_{a}^{1}\frac{x^2}{2}\frac{dx}{x}$

$\lim\limits_{a\to 0^-}\left(\frac{x^2\ln x}{2} - \frac{x^2}{4}\right)\Big|_{a}^{1}$

$\lim_{a\to 0^-} \frac{\cancel{1^2}\ln 1}{2} - \frac{1^2}{4} - \frac{a^2\ln a}{2} + \frac{\cancel{a^2}}{4}$

$-\frac{1}{4} - \frac{1}{2}\lim_{a\to 0^-} a^2 . \ln(a)$

$-\frac{1}{4} - \frac{1}{2}\lim_{a\to 0^-} \frac{\ln(a)}{\frac{1}{a^2}}$

$-\frac{1}{4} - \frac{1}{2}\lim_{a\to 0^-} \frac{\frac{1}{a}}{\frac{-2}{a^3}}$

$-\frac{1}{4} + \frac{1}{4}\cancel{\lim_{a\to 0^-} a^2}$

$-\frac{1}{4}$

e) $2\int_0^9 \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx$ $\quad u = \sqrt{x}$ $\quad du = \frac{dx}{2\sqrt{x}}$

$\lim_{a\to 0^-} 2\int e^u du$

$\lim_{a\to 0^-} 2e^u$

$\lim_{a\to 0^-} 2e^{\sqrt{x}}\Big|_a^9$

$\lim_{a\to 0^-} 2.e^3 - 2e^{\sqrt{a}}$

$2e^3 - 2$

f) $-\int_0^{\pi} \frac{-\cos x}{\sqrt{1-\operatorname{sen} x}}\,dx$ $\quad u = 1-\operatorname{sen} x$ $\quad du = -\cos x\,dx$

$\lim_{b\to\frac{\pi}{2}^-} -\int_0^b \frac{-\cos x\,dx}{\sqrt{1-\operatorname{sen} x}} - \lim_{a\to\frac{\pi}{2}^+}\int_a^{\pi} \frac{-\cos x\,dx}{\sqrt{1-\operatorname{sen} x}}$

$-\lim_{b\to\frac{\pi}{2}^-}\int \frac{du}{\sqrt{u}} - \lim_{a\to\frac{\pi}{2}^+}\int \frac{du}{\sqrt{u}}$

$-\lim_{b\to\frac{\pi}{2}^-} 2\sqrt{u} - \lim_{a\to\frac{\pi}{2}^+} 2\sqrt{u}$

$-2\lim_{b\to\frac{\pi}{2}^-} \sqrt{1-\operatorname{sen} x}\Big|_0^b - 2\lim_{a\to\frac{\pi}{2}^+} \sqrt{1-\operatorname{sen} x}\Big|_a^{\pi}$

$-2\lim_{b\to\frac{\pi}{2}^-} \sqrt{1-\operatorname{sen} b} - \sqrt{1-\operatorname{sen} 0} - 2\lim_{a\to\frac{\pi}{2}^+}$

$\sqrt{1-\operatorname{sen}\pi} - \cancel{\sqrt{1-\operatorname{sen} a}}$

$-2-2=0$

g) $\int_1^{+\infty} \frac{\ln\left(\frac{1}{x}\right)}{x^2}\,dx$ $\quad v = \frac{1}{x}$

$\lim_{b\to+\infty} -\int_1^b -\frac{\ln\left(\frac{1}{x}\right)}{x^2}\,dx$ $\quad dv = -\frac{dx}{x^2}$

$-\lim_{b\to+\infty} \int \ln(v)\,dv$

$u = \ln(v)$ $\quad \int dv = \int dv$

$du = \frac{dv}{v}$ $\quad v = v$

$-\lim_{b\to+\infty} v\ln v - v$

$\lim_{b\to+\infty} \left(\frac{1}{x} - \frac{\ln\left(\frac{1}{x}\right)}{x}\right)\Big|_1^b$

$$\lim_{b\to+\infty} \frac{\cancel{1}}{b}\searrow 0 - \frac{\ln\left(\frac{1}{b}\right)}{b} - \frac{1}{1} + \frac{\cancel{\ln 1}}{1}\searrow 0$$

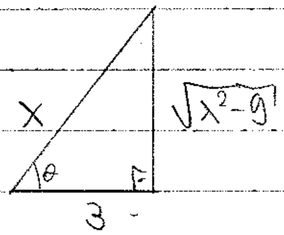
$$-1 - \lim_{b\to+\infty} \frac{\ln\left(\frac{1}{b}\right)}{b}$$

$$-1 - \lim_{b\to+\infty} \frac{-\frac{1}{b^2}}{\frac{1}{b}}$$

$$-1 + \lim_{b\to+\infty} \frac{1}{b}\searrow 0$$

$$-1$$

h) $\displaystyle\int_3^6 \frac{dx}{x^3\sqrt{x^2-9}}$



$\sec\theta = \frac{x}{3}$

$$\lim_{a\to 3^-} \int_a^6 \frac{dx}{x^3\sqrt{x^2-9}}$$

$dx = 3\sec\theta\, tg\,\theta\, d\theta$

$$\lim_{a\to 3^-} \int_a^6 \frac{\cancel{3\sec\theta\, tg\,\theta}\, d\theta}{27\sec^{\cancel{3}\,2}\theta\ \cancel{3\,tg\,\theta}}$$

$tg\,\theta = \frac{\sqrt{x^2-9}}{3}$

$$\frac{1}{27}\lim_{a\to 3^-}\int_a^6 \cos^2\theta\, d\theta$$

$sen\,\theta = \frac{\sqrt{x^2-9}}{x}$

$$\frac{1}{27}\lim_{a\to 3^-}\int_a^6 \left(\frac{1}{2} + \frac{1}{2}\cos(2\theta)\right)d\theta$$

$\cos\theta = \frac{3}{x}$

$$\frac{1}{54}\lim_{a\to 3^-}\int_a^6 d\theta + \int_a^6 \cos(2\theta)\,d\theta$$

$$\frac{1}{54}\lim_{a\to 3^-} \theta + \frac{1}{2}\, sen(2\theta)$$

$$\frac{1}{54}\lim_{a\to 3^-} arctg\left(\frac{\sqrt{x^2-9}}{3}\right)\Bigg|_a^6 + \frac{1}{\cancel{2}}\,\cancel{2}\, sen\,\theta\cos\theta$$

$$\frac{1}{54}\lim_{a\to 3^-} arctg\left(\frac{\sqrt{x^2-9}}{3}\right)\Bigg|_a^6 + \frac{3\sqrt{x^2-9}}{x^2}\Bigg|_a^6$$

/ /

$\frac{1}{54}\lim_{a\to 3^-} \operatorname{arctg}\sqrt{3} - \operatorname{arctg}\left(\cancel{\frac{\sqrt{a^2-9}}{3}}\right)_{\to 0} + \frac{\sqrt{3}}{4} - \cancel{\frac{3\sqrt{a^2-9}}{a^2}}_{\to 0}$

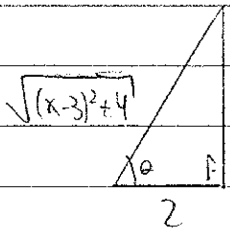
$\frac{1}{54}\cdot\frac{\Pi}{3} + \frac{1}{54}\cdot\frac{\sqrt{3}}{4}$

$\frac{\pi}{162} + \frac{\sqrt{3}}{216}$

0,027

i) $\int_1^3 \sqrt{x^2-6x+13}\,dx$

$\int_1^3 \sqrt{(x-3)^2+4}\,dx$



$tg\theta = \frac{x-3}{2}$

$2\sec^2\theta\, d\theta = dx$

$\int 2\sec\theta \cdot 2\sec^2\theta\, d\theta$

$4\int \sec^3\theta\, d\theta$

$\sec\theta = \frac{\sqrt{(x-3)^2+4}}{2}$

$u = \sec\theta \qquad \int dv = \int \sec^2\theta\, d\theta$

$du = \sec\theta\, tg\theta\, d\theta \qquad v = tg\theta$

$\int \sec^3\theta\, d\theta = tg\theta \sec\theta - \int tg^2\theta \sec\theta\, d\theta$

$\int \sec^3\theta\, d\theta = tg\theta \sec\theta - \int (\sec^2\theta - 1)\sec\theta\, d\theta$

$\int \sec^3\theta\, d\theta = tg\theta \sec\theta - \int \sec^3\theta\, d\theta + \int \sec\theta\, d\theta$

$2\int \sec^3\theta\, d\theta = tg\theta \sec\theta + \ln|\sec\theta + tg\theta|$

$2\, tg\theta \sec\theta + 2\ln|\sec\theta + tg\theta|$

$\left(\frac{1}{2}(x-3)\sqrt{(x-3)^2+4} + 2\ln\left|\frac{(x-3)+\sqrt{(x-3)^2+4}}{2}\right|\right)\Bigg|_1^3$

$0 + \cancel{2\ln 1}_{0} - \frac{(-2)\sqrt{(-2)^2+4}}{2} - 2\ln\left|\frac{-2+\sqrt{(-2)^2+4}}{2}\right|$

$2\sqrt{2} - 2\ln|\sqrt{2}-1| = 4,59$

(24) $L(f(x)) = \int_0^{+\infty} e^{-sx} f(x)\,dx$

a) $f(x) = e^{ax}$

$$L(e^{ax}) = \int_0^{+\infty} e^{-sx} \cdot e^{ax}\,dx$$

$$L(e^{ax}) = \lim_{b\to+\infty} \int_0^b e^{ax-sx}\,dx$$

$$L(e^{ax}) = \lim_{b\to+\infty} \int_0^b e^{(a-s)x}\,dx$$

$$L(e^{ax}) = \lim_{b\to+\infty} \frac{1}{(a-s)} e^{(a-s)x} \Big|_0^b$$

$$L(e^{ax}) = \lim_{b\to+\infty} \frac{1}{(a-s)} e^{(a-s)b} - \frac{1}{(a-s)}$$

p/ $a-s<0$

$s>a$

$$L(e^{ax}) = \frac{1}{s-a}$$

b) $f(x) = \cos x$

$$L(\cos x) = \int_0^{+\infty} e^{-sx} \cos x\,dx$$

$$L(\cos x) = \lim_{b\to+\infty} \int_0^b e^{-sx} \cos x\,dx$$

$u = \cos x$ $\qquad \int dv = \int e^{-sx}\,dx$

$du = -\operatorname{sen} x\,dx$ $\qquad v = -\frac{1}{s} e^{-sx}$

$$\int e^{-sx}\cos x\,dx = -\frac{\cos x}{s\,e^{sx}} - \frac{1}{s}\int e^{-sx}\operatorname{sen} x\,dx$$

$w = \operatorname{sen} x$

$dw = \cos x\,dx$

$$\int e^{-sx}\operatorname{sen} x\,dx = -\frac{\operatorname{sen} x}{s\,e^{sx}} + \frac{1}{s}\int e^{-sx}\cos x\,dx$$

$$\int e^{-sx}\cos x\,dx = -\frac{\cos x}{s\,e^{sx}} + \frac{\operatorname{sen} x}{s^2 e^{sx}} - \frac{1}{s^2}\int e^{-sx}\cos x\,dx$$

$$\frac{s^2+1}{\cancel{s^2}}\int e^{-sx}\cos x\,dx = \frac{-s\cos x + \operatorname{sen} x}{\cancel{s^2}\, e^{sx}}$$

$$\int e^{-sx}\cos x\,dx = \frac{\operatorname{sen} x - s\cos x}{(s^2+1)e^{sx}}$$

$$L(\cos x) = \lim_{b\to+\infty} \frac{\operatorname{sen} x - s\cos x}{(s^2+1)\cdot e^{sx}}\Big|_0^b$$

$$L(\cos x) = \lim_{b\to+\infty} \frac{\operatorname{sen} b - s\cos b}{(s^2+1)e^{sx}} - \frac{(\operatorname{sen} 0 - s\cos 0)}{(s^2+1)e^{s\cdot 0}}$$

$$= \frac{s}{s^2+1} + \lim_{b\to+\infty} \frac{\operatorname{sen} b - s\cos b}{(s^2+1)e^{sx}}$$

p/ $s>0$

$$L(\cos x) = \frac{s}{s^2+1}$$

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c) $f(x) = \operatorname{sen} x$

$$L(\operatorname{sen} x) = \int_0^{+\infty} e^{-sx} \operatorname{sen} x \, dx$$

$$L(\operatorname{sen} x) = \lim_{b \to +\infty} \int_0^b e^{-sx} \operatorname{sen} x \, dx$$

$u = \operatorname{sen} x \qquad \int dv = \int e^{-sx}$

$du = \cos x \, dx \qquad v = -\frac{1}{s} e^{-sx}$

$$\int e^{-sx} \operatorname{sen} x \, dx = -\frac{\operatorname{sen} x}{s e^{sx}} + \frac{1}{s} \int \cos x \, e^{-sx} \, dx$$

$$\int e^{-sx} \operatorname{sen} x \, dx = -\frac{\operatorname{sen} x}{s e^{sx}} + \frac{\operatorname{sen} x}{s(s^2+1) e^{sx}} - \frac{\cos x}{(s^2+1) e^{sx}}$$

$$L(\operatorname{sen} x) = \lim_{b \to +\infty} \frac{\operatorname{sen} x}{(s^3+s) e^{sx}} - \frac{\operatorname{sen} x}{s e^{sx}} - \frac{\cos x}{(s^2+1) e^{sx}} \Bigg|_0^b$$

$$L(\operatorname{sen} x) = \lim_{b \to +\infty} \frac{\operatorname{sen} b}{(s^3+s) e^{sb}} - \frac{\operatorname{sen} b}{s \cdot e^{sb}} - \frac{\cos x}{(s^2+1) e^{sb}} - \cancel{\frac{\operatorname{sen} 0}{(s^3+s) e^{s.0}}}_{0} + \frac{\operatorname{sen} 0}{s . e^{s.0}}_{0} + \frac{\cos 0}{(s^2+1) e^{s.0}}$$

p/ $s > 0$

$$L(\operatorname{sen} x) = \frac{1}{s^2+1}$$

$$\textcircled{25}\ \Gamma(x) = \int_0^{+\infty} t^{x-1} e^{-t}\, dt \quad , \forall\, x > 0$$

a) $\Gamma(1) = \int_0^{+\infty} t^0 e^{-t}\, dt$

$$\Gamma(1) = \lim_{b \to +\infty} \int_0^b e^{-t}\, dt$$

$$\Gamma(1) = \lim_{b \to +\infty} -\frac{1}{e^t} \Big|_0^b$$

$$\Gamma(1) = \lim_{b \to +\infty} -\frac{1}{e^b} + \frac{1}{e^0}$$

$$\Gamma(1) = \underline{1}$$

$\Gamma(2) = \int_0^{+\infty} t^1 e^{-t}\, dt$

u (derivo)	dv (integro)
	e^{-t}
$\oplus$ t	$-e^{-t}$
$\ominus$ 1	e^{-t}

$$\Gamma(2) = \lim_{b \to +\infty} \int_0^b t e^{-t}\, dt$$

$$\Gamma(2) = \lim_{b \to +\infty} -\frac{t}{e^t} - \frac{1}{e^t} \Big|_0^b$$

$$\Gamma(2) = \lim_{b \to +\infty} -\frac{b}{e^b} - \cancel{\frac{1}{e^b}}_{\to 0} + \cancel{\frac{0}{e^0}} + \frac{1}{e^0}$$

$$\Gamma(2) = 1 - \lim_{b \to +\infty} \frac{b}{e^b}$$

$$\Gamma(2) = 1 - \lim_{b \to +\infty} \frac{1}{e^b}$$

$$\Gamma(2) = \underline{1}$$

b) $\Gamma(n+1) = n\,\Gamma(n)$

$$\lim_{b \to +\infty} \int_0^b t^n e^{-t}\, dt = n \lim_{b \to +\infty} \int_0^b t^{n-1} e^{-t}\, dt$$

$u = t^n \qquad \int dv = \int e^{-t}\, dt \qquad w = t^{n-1}$

$du = n\, t^{n-1} \qquad v = -e^{-t} \qquad dw = (n-1)\, t^{n-2}$

$$\int t^n e^{-t} dt = -t^n e^{-t} + n \int t^{n-1} e^{-t}\, dt$$

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$$\int t^{n-1} e^{-t}\,dt = -t^{n-1} e^{-t} + (n-1)\int t^{n-2} e^{-t}\,dt$$

$$\int t^{n} e^{-t}\,dt = -t^{n} e^{-t} - n t^{n-1} e^{-t} + n(n-1)\int t^{n-2} e^{-t}\,dt$$

$z = t^{n-2}$

$dz = (n-2)t^{n-3}$

$$\int t^{n-2} e^{-t} = -t^{n-2} e^{-t} + (n-2)\int t^{n-3} e^{-t}\,dt$$

$$\int t^{n} e^{-t}\,dt = -t^{n} e^{-t} - n t^{n-1} e^{-t} - n(n-1) t^{n-2} e^{-t} + n(n-1)(n-2)\int t^{n-3} e^{-t}\,dt$$

$$\int t^{n} e^{-t}\,dt = -e^{-t}\left(t^{n} + n t^{n-1} + n(n-1)t^{n-2} + \dots + 0\right)$$

$$\int t^{n} e^{-t}\,dt = -t^{n} e^{-t}\left(1 + \frac{n}{t} + \frac{n(n-1)}{t^2} + \dots + \frac{0}{t^n}\right)$$

$$\int t^{n-1} e^{-t}\,dt = -t^{n-1} e^{-t} - (n-1)t^{n-2} e^{-t} + (n-1)(n-2)\int t^{n-3} e^{-t}\,dt$$

$$= -t^{n} e^{-t}\left(\frac{1}{t} + \frac{(n-1)}{t^2} + \dots + \_\right)$$

$$n\int t^{n-1} e^{-t}\,dt = -t^{n} e^{-t}\left(\frac{n}{t} + \frac{n(n-1)}{t^2} + \dots + 1\right)$$

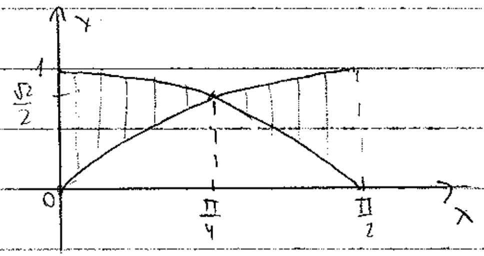
$$\Gamma(n+1) = n\Gamma(n)$$

(26) $\int_0^1 (x^4 - 5x^2 + 4)\,dx - \int_1^2 (x^2 - 5x^2 + 4)\,dx$

$$\frac{x^5}{5} - \frac{5}{3}x^3 + 4x \Big|_0^1 - \left(\frac{x^5}{5} - \frac{5}{3}x^3 + 4x\right)\Big|_1^2$$

$$\frac{1}{5} - \frac{5}{3} + \cancel{4} - \frac{32}{5} + \frac{40}{3} - 8 + \frac{1}{5} - \frac{5}{3} + \cancel{4} = -\frac{30}{5} + \frac{30}{3} = -6 + 10 = 4 = \frac{60}{15}$$

(17) a) $y = \operatorname{sen} x,\ y = \cos x,\ x = 0$ e $x = \frac{\pi}{2}$



$\operatorname{sen} x = \cos x$

$\operatorname{tg} x = 1$

$x = \frac{\pi}{4}$

$$A = \int_0^{\frac{\pi}{4}} (\cos x - \operatorname{sen} x)\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\operatorname{sen} x - \cos x)\,dx$$

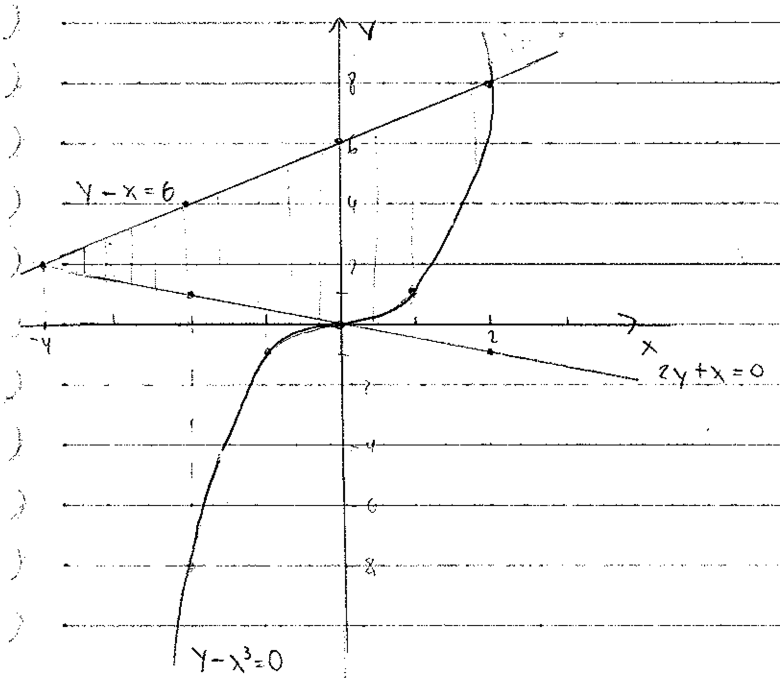
$$A = \operatorname{sen} x\Big|_0^{\frac{\pi}{4}} + \cos x\Big|_0^{\frac{\pi}{4}} - \cos x\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} - \operatorname{sen} x\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$A = \operatorname{sen}\left(\frac{\pi}{4}\right) - \operatorname{sen}(0) + \cos\left(\frac{\pi}{4}\right) - \cos(0) - \cos\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{4}\right) - \operatorname{sen}\left(\frac{\pi}{2}\right) + \operatorname{sen}\left(\frac{\pi}{4}\right)$$

$$A = \frac{\sqrt{2}}{2} - 0 + \frac{\sqrt{2}}{2} - 1 - 0 + \frac{\sqrt{2}}{2} - 1 + \frac{\sqrt{2}}{2}$$

$A = 2\sqrt{2} - 2$ u.a

b) $y - x = 6,\ y - x^3 = 0$ e $2y + x = 0$



$y = 6 + x,\ y = x^3,\ y = -\frac{x}{2}$

$6 + x = -\frac{x}{2}$

$12 + 2x = -x$

$12 = -3x$

$x = -4$

$x^3 = -\frac{x}{2}$

$x\left(x^2 + \frac{1}{2}\right) = 0$

$x = 0$

$x \in \mathbb{C}$

$6 + x = x^3$

$x = 2$

$x \in \mathbb{C}$

$A = \int_{-4}^{0} \left(6+x-\left(-\frac{x}{2}\right)\right) dx + \int_{0}^{2} (6+x-x^3)\, dx$

$A = 6x + \frac{x^2}{2} + \frac{x^2}{4} \Big|_{-4}^{0} + 6x + \frac{x^2}{2} - \frac{x^4}{4} \Big|_{0}^{2}$

$A = -6(-4) - \frac{(-4)^2}{2} - \frac{(-4)^2}{4} + 6 \cdot 2 + \frac{2^2}{2} - \frac{2^4}{4}$

$A = 24 - \cancel{8} - \cancel{4} + \cancel{12} + 2 - 4$

$A = 22$ u.a

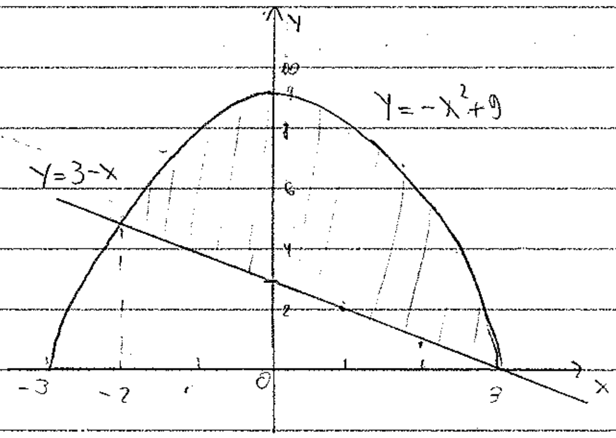
c) $y = -x^2 + 9$ e $y = 3 - x$

$-x^2 + 9 = 3 - x \qquad x_1 = -2$

$x^2 - x - 6 = 0 \qquad x_2 = 3$

$3 + (-2) = 1$

$3 \cdot (-2) = -6$



$A = \int_{-2}^{3} (-x^2 + 9 - 3 + x)\, dx$

$A = -\frac{x^3}{3} + 6x + \frac{x^2}{2} \Big|_{-2}^{3}$

$A = -\frac{3^3}{3} + 6.3 + \frac{3^2}{2} + \frac{(-2)^3}{3} - 6(-2) - \frac{(-2)^2}{2}$

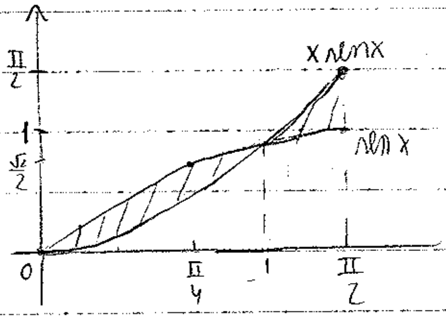
$A = -9 + 18 + \frac{9}{2} - \frac{8}{3} + 12 - 2$

$A = 19 + \frac{9}{2} - \frac{8}{3} = \frac{114 + 27 - 16}{6} = \frac{125}{6}$ u.a

d) $y = \operatorname{sen} x$, $y = x \operatorname{sen} x$, $x = 0$ e $x = \frac{\pi}{2}$

$\operatorname{sen} x = x \operatorname{sen} x$

$x = 1$



$\int x \operatorname{sen} x \, dx$

u	dv
⊕ x	sen x
⊖ 1	−cos x
	−sen x

$A = \int_0^1 (\operatorname{sen} x - x \operatorname{sen} x)dx + \int_1^{\frac{\pi}{2}} (x \operatorname{sen} x - \operatorname{sen} x)dx$

$A = -\cos x + x\cos x - \operatorname{sen} x \Big|_0^1 - x\cos x + \operatorname{sen} x + \cos x \Big|_1^{\frac{\pi}{2}}$

$A = -\cancel{\cos 1} + \cancel{1 \cdot \cos 1} - \operatorname{sen} 1 + \cos 0 - \cancel{0 \cdot \cos 0} + \cancel{\operatorname{sen} 0} - \cancel{\frac{\pi}{2} \cos\left(\frac{\pi}{2}\right)} +$
$\operatorname{sen}\left(\frac{\pi}{2}\right) + \cancel{\cos\left(\frac{\pi}{2}\right)} + \cancel{1 \cos 1} - \operatorname{sen} 1 - \cancel{\cos 1}$

$A = -\operatorname{sen} 1 + 1 + 1 - \operatorname{sen} 1$

$A = 2 - 2\operatorname{sen} 1$ u.a

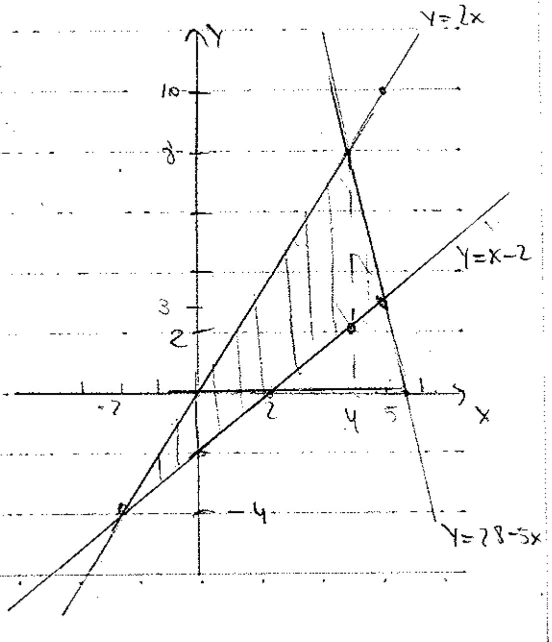
$A = 0{,}32$ u.a

e) $28 - y - 5x = 0$, $x - y - 2 = 0$, $y = 2x$ e $y = 0$

$28 - 5x = 2x$
$7x = 28$
$x = 4$
$y = 8$

$28 - 5x = x - 2$
$30 = 6x$
$x = 5$
$y = 3$

$x - 2 = 2x$
$x = -2$
$y = -4$



$A = \int_{-2}^{4} (x+2)\,dx + \int_{4}^{5} (30-6x)\,dx$

$A = \frac{x^2}{2} + 2x \Big|_{-2}^{4} + 30x - 3x^2 \Big|_{4}^{5}$

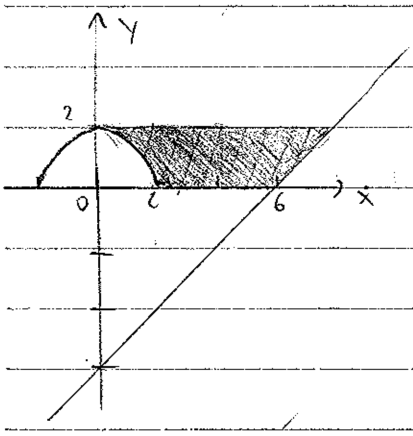
$A = \frac{4^2}{2} + 2.2 - \frac{(-2)^2}{2} - 2.(-2) + 30.5 - 3.5^2 - 30.4 + 3.4^2$

$A = -8 + 4 - 2 + 4 + 150 - 75 - 120 + 48$

$A = 17$ u.a

(28) $A = \int_{0}^{2} ((y+6) - (\sqrt{4-y^2}))\,dy$

$x = y+6 \qquad x^2+y^2=4$



(29) a) $y = x+3$ e $x = -y^2+3$

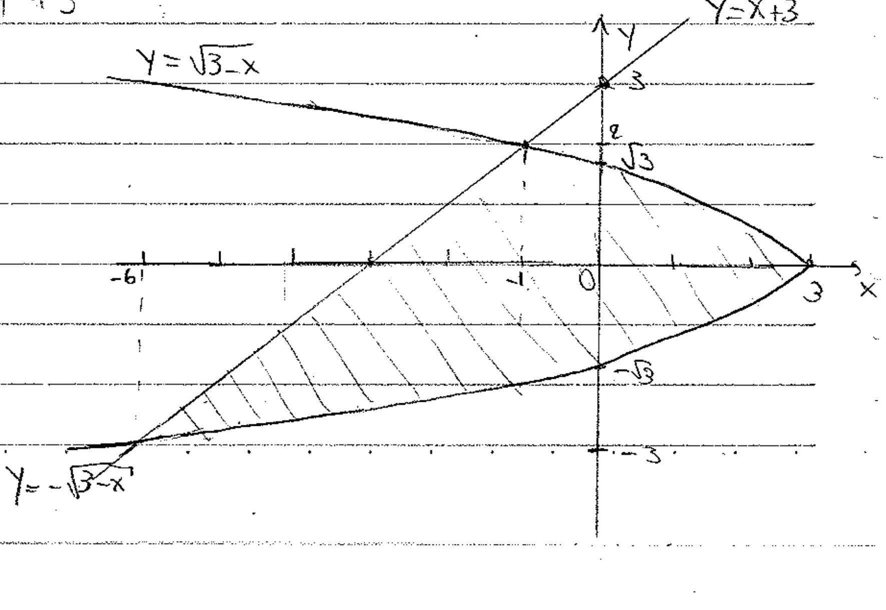
$(x+3)^2 = 3-x \qquad x_1 = -1$

$x^2+6x+9 = 3-x \qquad y_1 = 2$

$x^2+7x+6=0$

$-1 + -6 = -7 \qquad x_2 = -6$

$-1 \cdot -6 = 6 \qquad y_2 = -3$



i) $A = \int_{-6}^{-1} (x+3) + (\sqrt{3-x})\, dx + \int_{-1}^{3} 2\sqrt{3-x}\, dx$

$\int \sqrt{3-x}\, dx$

$= -\int \sqrt{u}\, du$

$-\frac{2}{3} u^{3/2}$

$-\frac{2}{3} \sqrt{(3-x)^3}$

$A = \frac{x^2}{2} + 3x - \frac{2}{3}\sqrt{(3-x)^3}\Big|_{-6}^{-1} - \frac{4}{3}\sqrt{(3-x)^3}\Big|_{-1}^{3}$

$A = \frac{1}{2} - 3 - \frac{16}{3} - \cancel{18} + \cancel{18} + 18 + \frac{32}{3}$

$A = 15 + \frac{1}{2} + \frac{16}{3}$

$A = \frac{90+3+32}{6}$

$A = \frac{125}{6}$ u.a

ii) $A = \int_{-3}^{2} ((-y^2+3) - (y-3))\, dy$

$A = \int_{-3}^{2} (-y^2 - y + 6)\, dy$

$A = -\frac{y^3}{3} - \frac{y^2}{2} + 6y \Big|_{-3}^{2}$

$A = -\frac{8}{3} - 2 + 12 - 9 + \frac{9}{2} + 18$

$A = \frac{9}{2} - \frac{8}{3} + 19$

$A = \frac{27-16+114}{6}$

$A = \frac{125}{6}$ u.a

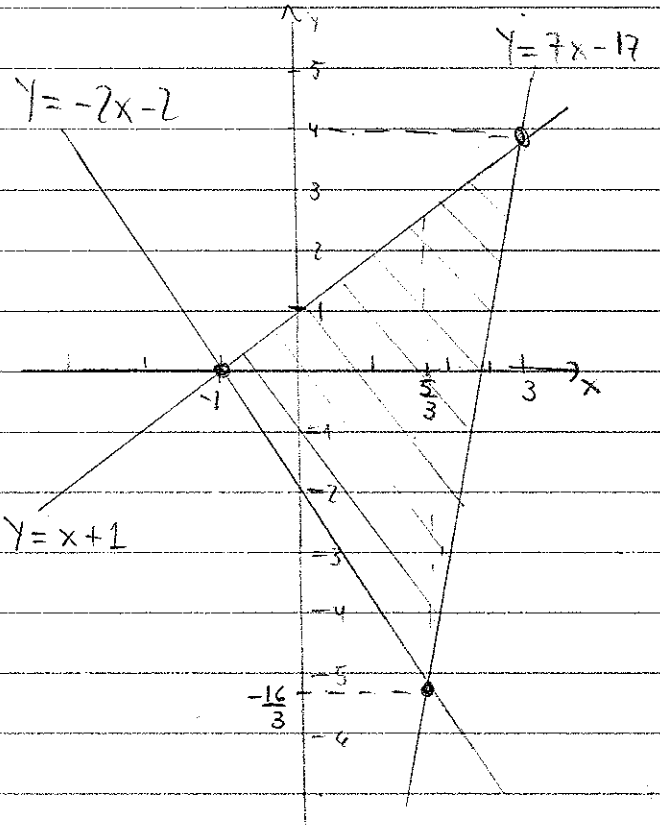
b) $2x+y=-2$, $x-y=-1$ e $7x-y=17$

$-2-2x = x+1$ $\qquad -2-2x = 7x-17$ $\qquad x+1 = 7x-17$

$3x=-3$ $\qquad 9x = 15$ $\qquad 18 = 6x$

$x=-1$ $\qquad x = \frac{5}{3}$ $\qquad x = 3$

$y=0$ $\qquad y = -\frac{16}{3}$ $\qquad y = 4$



i) $A = \int_{-1}^{\frac{5}{3}} (x+1 + 2x+2)\,dx + \int_{\frac{5}{3}}^{3} (x+1-7x+17)\,dx$

$A = \int_{-1}^{\frac{5}{3}} (3x+3)\,dx + \int_{\frac{5}{3}}^{3} (18-6x)\,dx$

$= \frac{3x^2}{2} + 3x \Big|_{-1}^{\frac{5}{3}} + 18x - 3x^2 \Big|_{\frac{5}{3}}^{3}$

$= \frac{\cancel{3}}{2} \cdot \frac{25}{\cancel{9}\,3} + 5 - \frac{3}{2} + \cancel{3} + \cancel{54} - \cancel{27} - \cancel{30} + \frac{25}{3}$

$= \frac{25}{6} + \frac{25}{3} - \frac{3}{2} + 5$

$= \frac{25+50-9+30}{6} = \frac{96}{6} = 16\,u.a$

ii) $A = \int_{-\frac{16}{3}}^{0} \left(\frac{y}{7} + \frac{17}{7}\right) + \left(\frac{y}{2} + 1\right) dy + \int_{0}^{4} \left(\left(\frac{y}{7} + \frac{17}{7}\right) - y + 1\right) dy$

$A = \frac{y^2}{14} + \frac{17}{7}y + \frac{y^2}{4} + y \Big|_{-\frac{16}{3}}^{0} + \frac{y^2}{14} + \frac{17}{7}y - \frac{y^2}{2} + y \Big|_{0}^{4}$

$$A = -\frac{1}{14}\cdot\frac{16^2}{9} + \frac{17}{7}\cdot\frac{16}{3} - \frac{1}{4}\cdot\frac{16^2}{9} + \frac{16}{3} + \frac{16}{14} + \frac{17.4}{7} - \frac{16}{2} + 4$$

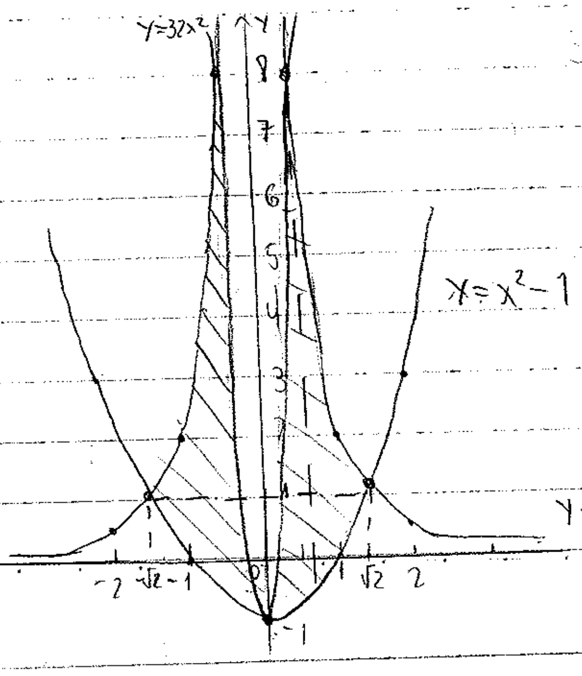
$$A = -\frac{128}{63} + \frac{272}{21} - \frac{64}{9} + \frac{16}{3} + \frac{8}{7} + \frac{68}{7} - 8 + 4$$

$$A = \frac{688}{63} - \frac{16}{9} + \frac{76}{7} - 4$$

$$A = \frac{688 - 112 + 684 - 252}{63} = \frac{1008}{63} = 16\,u.a$$

c) $Y = x^2 - 1$, $Y = \frac{2}{x^2}$ e $Y = 32x^2$

$x^2 - 1 = \frac{2}{x^2}$	$x^2 = 2$	$x^2 - 1 = 32x^2$	$\frac{2}{x^2} = \overset{16}{\cancel{32}}x^2$
$x^4 - x^2 = 2$	$x = \pm\sqrt{2}$	$31x^2 = -1$	$x^4 = \frac{1}{16}$
$x^4 - x^2 - 2 = 0$	$Y = 1$	$x^2 = -\frac{1}{32}$	$x = \pm\frac{1}{2}$
$x^2 = t$		$x \notin \mathbb{R}$	$Y = 8$
$t^2 - t - 2 = 0$	$x^2 = -1$		
$2 + (-1) = 1$	$x \notin \mathbb{R}$		
$2 \cdot (-1) = -2$			



i) $A = 2\int_0^{1/2}(32x^2 - x^2 + 1)\,dx +$

$2\int_{1/2}^{\sqrt{2}}\left(\frac{2}{x^2} - x^2 + 1\right)dx$

$A = 2\int_0^{1/2}(31x^2 + 1)\,dx + 2\int_{1/2}^{\sqrt{2}}\left(\frac{2}{x^2} - x^2 + 1\right)dx$

$A = 2\left(\frac{31}{3}x^3 + x\right)\Big|_0^{1/2} + 2\left(-\frac{2}{x} - \frac{x^3}{3} + x\right)\Big|_{1/2}^{\sqrt{2}}$

$A = \left(\frac{62}{3}x^3 + 2x\right)\Big|_0^{1/2} + \left(2x - \frac{4}{x} - \frac{2}{3}x^3\right)\Big|_{1/2}^{\sqrt{2}}$

$$A = \frac{\cancel{64}^{31}}{3} \cdot \frac{1}{\cancel{8}4} + \cancel{2} \cdot \frac{1}{\cancel{2}} + \cancel{2\sqrt{2} \cdot \frac{4}{\sqrt{2}}} - \frac{2}{3}\sqrt{2^3} - \cancel{8} \cdot \frac{1}{\cancel{8}} + 8 + \frac{\cancel{2}^{1}}{3} \cdot \frac{1}{\cancel{8}4}$$

$$A = \frac{31}{12} + \frac{1}{12} + 8 - \frac{4}{3}\sqrt{2}$$

$$A = \frac{8}{3} + 8 - \frac{4\sqrt{2}}{3}$$

$$A = \frac{32 - 4\sqrt{2}}{3} \text{ u.a}$$

$$ii)\ A = 2\int_{-1}^{1}\left(\sqrt{y+1} - \frac{\sqrt{y}}{4\sqrt{2}}\right)dy + 2\int_{1}^{8}\left(\frac{\sqrt{2}}{\sqrt{y}} - \frac{\sqrt{y}}{4\sqrt{2}}\right)dy$$

$$A = 2\left(\frac{2}{3}\sqrt{(y+1)^3} - \frac{2}{3} \cdot \frac{\sqrt{y^3}}{4\sqrt{2}}\right)\Bigg|_{-1}^{1} + 2\left(2\sqrt{2}\cdot\sqrt{y} - \frac{2}{3}\frac{\sqrt{y^3}}{4\sqrt{2}}\right)\Bigg|_{1}^{8}$$

$$A = \left(\frac{4}{3}\sqrt{(y+1)^3} - \frac{1}{3\sqrt{2}}\sqrt{y^3}\right)\Bigg|_{-1}^{1} + \left(4\sqrt{2}\sqrt{y} - \frac{1}{3\sqrt{2}}\sqrt{y^3}\right)\Bigg|_{1}^{8}$$

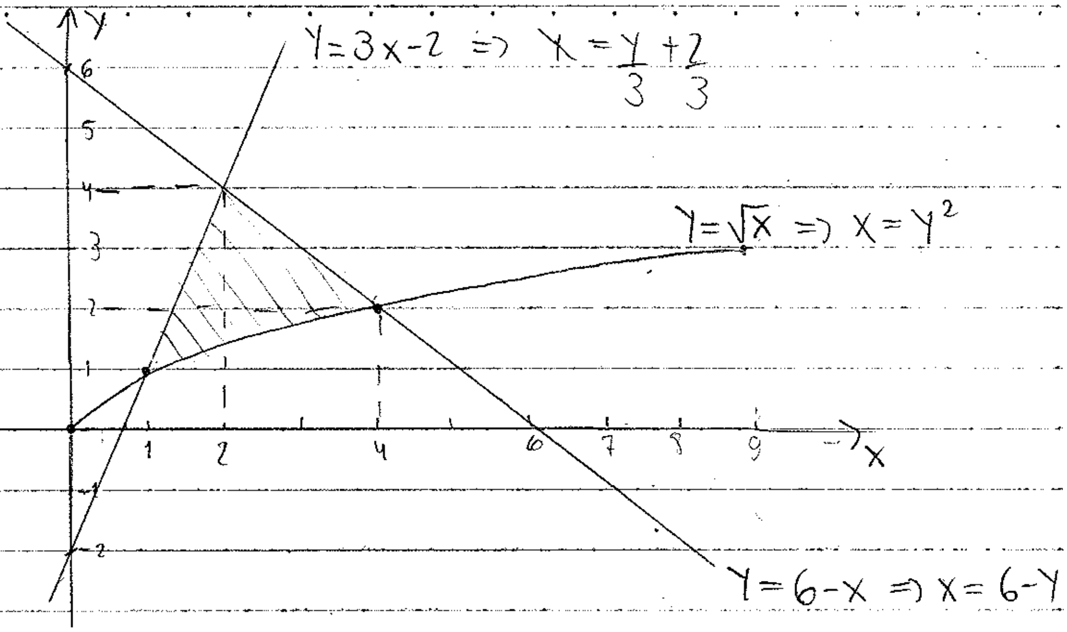
$$A = \frac{8\sqrt{2}}{3} - \cancel{\frac{1}{3\sqrt{2}}} + 16 - \frac{16}{3} - 4\sqrt{2} + \cancel{\frac{1}{3\sqrt{2}}}$$

$$A = \frac{8\sqrt{2} + 48 - 16 - 12\sqrt{2}}{3}$$

$$A = \frac{32 - 4\sqrt{2}}{3} \text{ u.a}$$

d) $y + x = 6$, $y = \sqrt{x}$ e $y + 2 = 3x$

$6 - x = 3x - 2$	$6 - x = \sqrt{x}$	$x_1 = 9$	$\sqrt{x} = 3x - 2$
$4x = 8$	$36 - 12x + x^2 = x$	$y_1 = 3$	$x = 9x^2 - 12x + 4$
$x = 2$	$x^2 - 13x + 36 = 0$		$9x^2 - 13x + 4 = 0$
$y = 4$	$x = \frac{13 \pm \sqrt{169 - 4.1.36}}{2}$	$x_2 = 4$	$x = \frac{13 \pm \sqrt{169 - 4.4.9}}{2}$
	$x = \frac{13 \pm 5}{2}$	$y_2 = 2$	$x = \frac{13 \pm 5}{2}$



i) $A = \int_1^2 (3x-2-\sqrt{x})\,dx + \int_2^4 (6-x-\sqrt{x})\,dx$

$A = \left(\frac{3}{2}x^2 - 2x - \frac{2}{3}\sqrt{x^3}\right)\Big|_1^2 + \left(6x - \frac{x^2}{2} - \frac{2}{3}\sqrt{x^3}\right)\Big|_2^4$

$A = \cancel{6} - 4 - \cancel{\frac{4}{3}\sqrt{2}} - \frac{3}{2} + \cancel{2} + \frac{2}{3} + 24 - \cancel{8} - \frac{16}{3} - 12 + 2 + \cancel{\frac{4\sqrt{2}}{3}}$

$A = 10 - \frac{3}{2} - \frac{14}{3}$

$A = \frac{60-9-28}{6} = \frac{23}{6}$ u.a

ii) $A = \int_1^2 \left(y^2 - \frac{y}{3} - \frac{2}{3}\right) dy + \int_2^4 \left(6 - y - \frac{y}{3} - \frac{2}{3}\right) dy$

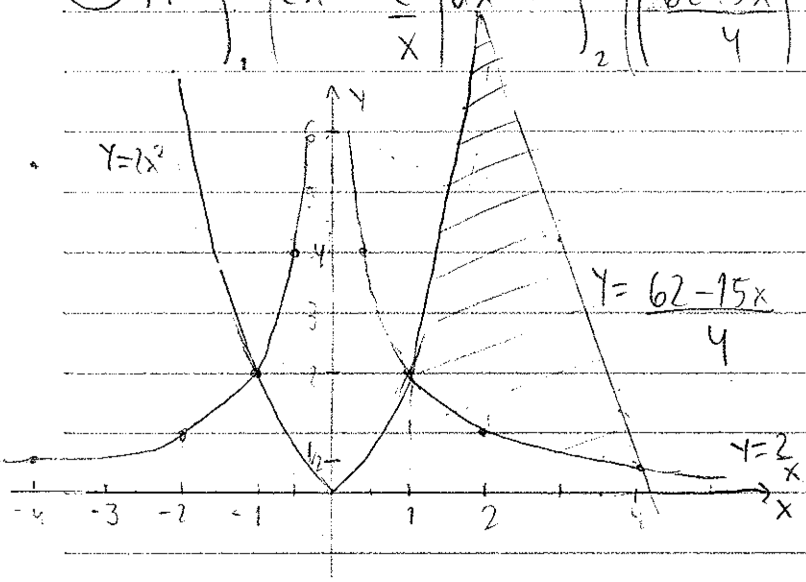
$A = \left(\frac{y^3}{3} - \frac{y^2}{6} - \frac{2}{3}x\right)\Big|_1^2 + \left(\frac{16}{3}y - \frac{2}{3}y^2\right)\Big|_2^4$

$A = \frac{8}{3} - \cancel{\frac{2}{3}} - \frac{4}{3} - \frac{1}{3} + \frac{1}{6} + \cancel{\frac{2}{3}} + \cancel{\frac{64}{3}} - \cancel{\frac{32}{3}} - \cancel{\frac{32}{3}} + \frac{8}{3}$

$A = \frac{11}{3} + \frac{1}{6} = \frac{23}{6}$ u.a

/ /

(30) $$A=\int_1^2\left(2x^2-\frac{2}{x}\right)dx+\int_2^4\left(\left(\frac{62-15x}{4}\right)-\frac{2}{x}\right)dx$$



$$A=\int_{1/2}^{2}\left(\frac{62-4y}{15}-\frac{2}{y}\right)dy+\int_2^8\left(\frac{62-4y}{15}-\frac{\sqrt{y}}{\sqrt{2}}\right)dy$$

(31) $x=\dfrac{4}{x-1}$

$x^2-x-4=0$

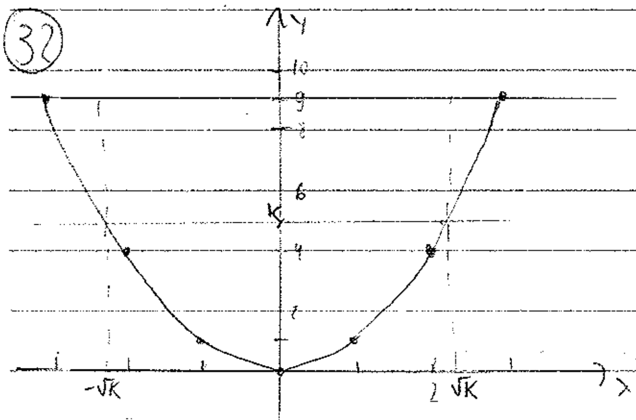
$x=\dfrac{1\pm\sqrt{1-4\cdot1\cdot-4}}{2}$

$x=\dfrac{1\pm\sqrt{17}}{2}$

a) $$A=\int_1^2\left(x^2-x\right)dx+\int_2^{\frac{1+\sqrt{17}}{2}}\left(\frac{4}{x-1}-x\right)dx$$

b) $$A=\int_1^{\frac{1+\sqrt{17}}{2}}\left(y-\sqrt{y}\right)dy+\int_{\frac{1\pm\sqrt{17}}{2}}^{4}\left(\frac{4}{y}+1-\sqrt{y}\right)dy$$

(32)



$$A=\int_{-3}^{3}\left(9-x^2\right)dx$$

$$A=9x-\frac{x^3}{3}\Big|_{-3}^{3}$$

$A=27-9+27-9$

$A=36u.a$

$$\frac{A}{2} = \int_{-\sqrt{K}}^{\sqrt{K}} (K - x^2)\,dx$$

$$18 = Kx - \frac{x^3}{3}\Big|_{-\sqrt{K}}^{\sqrt{K}}$$

$$18 = K\sqrt{K} - \frac{\sqrt{K^3}}{3} + K\sqrt{K} - \frac{\sqrt{K^3}}{3}$$

$$18 = 2K\sqrt{K} - \frac{2K\sqrt{K}}{3}$$

$$18 = \frac{6K\sqrt{K} - 2K\sqrt{K}}{3}$$

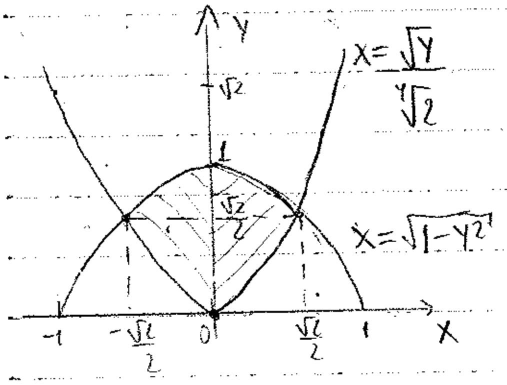
$$54 = 4K\sqrt{K}$$

$$\frac{27}{2} = \sqrt{K^3}$$

$$\frac{3^6}{2^2} = K^3$$

$$K = \frac{3^2}{\sqrt[3]{4}} = \frac{9}{\sqrt[3]{4}}$$

33) $A = \int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} \left(\sqrt{1-x^2} - \sqrt{2}\,x^2\right) dx$



a) $A = 2\int_{0}^{\frac{\sqrt{2}}{2}} \frac{\sqrt{y}}{\sqrt[4]{2}}\,dy + 2\int_{\frac{\sqrt{2}}{2}}^{1} \sqrt{1-y^2}\,dy$

b) $\begin{cases} x = \cos t \\ y = \operatorname{sen} t \end{cases}$ $\qquad$ $\begin{cases} x = t \\ y = \sqrt{2}\,t^2 \end{cases}$

$dx = -\operatorname{sen} t\,dt \qquad dx = dt$

$\phi(\alpha) = -\frac{\sqrt{2}}{2} \Rightarrow \cos\alpha = -\frac{\sqrt{2}}{2} \Rightarrow \alpha = 135° = \frac{3\pi}{4}$

$\phi(\beta) = \frac{\sqrt{2}}{2} \Rightarrow \cos\beta = \frac{\sqrt{2}}{2} \Rightarrow \beta = 45° = \frac{\pi}{4}$

$\begin{cases} y = \psi(t) \\ x = \phi(t) \\ dx = \phi'(t) \end{cases}$

$$A = \int_{\frac{3\pi}{4}}^{\frac{\pi}{4}} -\operatorname{sen}^2 t\,dt - \int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} \sqrt{2}\,t^2\,dt$$

/ /

(34) $Y(t) = 3\,\text{sen}\,t \qquad x^2 + y^2 = 9$

$\int dx = \int -3\,\text{sen}\,t\,dt$

$x(t) = 3\cos t$

$a = x(\alpha) = x(\pi) = -3$

$b = x(\beta) = x(0) = 3$

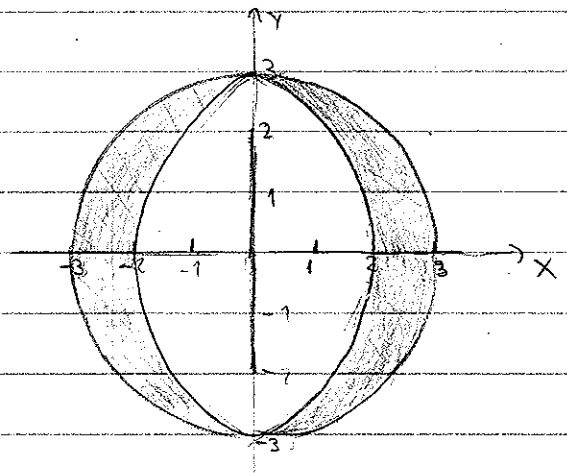
$Y(t) = 3\,\text{sen}\,t \qquad \dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$

$\int dx = \int -2\,\text{sen}\,t\,dt$

$x(t) = 2\cos t$

$a = x(\alpha) = x(\pi) = -2$

$b = x(\beta) = x(0) = 2$



(35) $\begin{cases} x = a(t - \text{sen}\,t) \Rightarrow dx = a(1-\cos t)\,dt \\ y = a(1-\cos t) \end{cases}$

$A = \int_0^{2\pi} a^2 (1-\cos t)^2\,dt$

$A = a^2 \int_0^{2\pi} (1 - 2\cos t + \cos^2 t)\,dt$

$A = a^2 (t - 2\,\text{sen}\,t)) \Big|_0^{2\pi} + \dfrac{a^2}{2} \int_0^{2\pi} (1 + \cos(2t))\,dt$

$A = a^2 . 2\pi + \dfrac{a^2 t}{2} \Big|_0^{2\pi} + \dfrac{a^2}{4}\,\text{sen}(2t) \Big|_0^{2\pi}$

$A = a^2 2\pi + a^2 \pi$

$A = 3\pi a^2$ u.a

36) $x^{2/3} + y^{2/3} = a^{2/3}$

$(a\cos^3 t)^{2/3} + (a\,\text{sen}^3 t)^{2/3} = a^{2/3}$

$a^{2/3}\cos^2 t + a^{2/3}\,\text{sen}^2 t = a^{2/3}$

$\begin{cases} x = a\cos^3 t \Rightarrow dx = -3a\,\text{sen}\,t\cos^2 t\,dt \\ y = -a\,\text{sen}^3 t \end{cases}$

$A = \int_0^{2\pi} -a\,\text{sen}^3 t\,(-3a\,\text{sen}\,t\cos^2 t)\,dt$

$A = -3a^2 \int_0^{2\pi} \text{sen}^4 t \cos^2 t\,dt$

$A = \frac{a^2}{64}\left(12t - 3\,\text{sen}(2t) - 3\,\text{sen}(4t) + \text{sen}(6t)\right)\Big|_0^{2\pi}$

$A = \frac{a^2}{64} \cdot 24\pi$

$A = \frac{3\pi a^2}{8}$ u.a

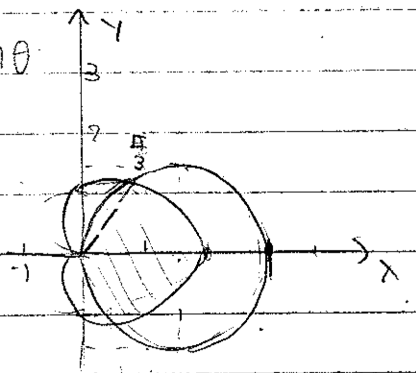
37) a) $r = 3\cos\theta$ e $r = 1 + \cos\theta$

$3\cos\theta = 1 + \cos\theta$

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$



$r = 3\cos\theta$

$r = 3\frac{x}{r}$

$r^2 = 3x$

$x^2 + y^2 = 3x$

$x^2 - 3x + y^2 = 0$

$(x - \frac{3}{2})^2 + y^2 = \frac{9}{4}$

$A = 2 \cdot \frac{1}{2}\int_0^{\frac{\pi}{3}} (1 + \cos\theta)^2\,d\theta + 2 \cdot \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (3\cos\theta)^2\,d\theta$

$A = \int_0^{\frac{\pi}{3}} (1 + 2\cos\theta + \cos^2\theta)\,d\theta + 9\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^2\theta\,d\theta$

$A = (\theta + 2\,\text{sen}\,\theta)\Big|_0^{\frac{\pi}{3}} + \frac{1}{2}\int_0^{\frac{\pi}{3}} (1 + \cos(2\theta))\,d\theta + \frac{9}{2}\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} (1 + \cos(2\theta))\,d\theta$

/ /

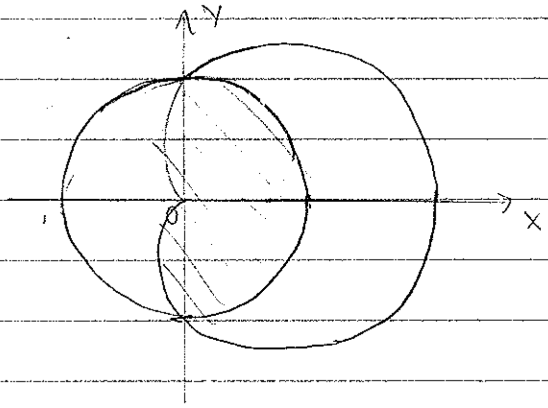
$A = \frac{\pi}{3} + 2 \cdot \frac{\sqrt{3}}{2} + \left(\frac{\theta}{2} + \frac{1}{4} \operatorname{sen}(2\theta)\right)\Big|_0^{\frac{\pi}{3}} + \left(\frac{9\theta}{2} + \frac{9}{4} \operatorname{sen}(2\theta)\right)\Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}}$

$A = \frac{\pi}{3} + \cancel{\sqrt{3}} + \frac{\pi}{6} + \cancel{\frac{\sqrt{3}}{8}} + \frac{9\pi}{4} - \frac{3\pi}{2} - \cancel{\frac{9\sqrt{3}}{8}}$

$A = \frac{3\pi}{6} + \frac{3\pi}{4}$

$A = \frac{6\pi + 9\pi}{12} = \frac{15\pi}{12} = \frac{5\pi}{4}$ u.a

b) $r = 1 + \cos\theta$ e $r = 1$



$1 + \cos\theta = 1$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$

$A = 2 \cdot \frac{1}{2} \int_0^{\frac{\pi}{2}} 1^2 \, d\theta + 2 \cdot \frac{1}{2} \int_{\frac{\pi}{2}}^{\pi} (1 + \cos\theta)^2 \, d\theta$

$A = \theta \Big|_0^{\frac{\pi}{2}} + \left(\frac{3\theta}{2} + 2 \operatorname{sen}\theta + \frac{1}{4} \operatorname{sen}(2\theta)\right)\Big|_{\frac{\pi}{2}}^{\pi}$

$A = \frac{\pi}{2} + \frac{3\pi}{2} - \frac{3\pi}{4} - 2$

$A = \frac{5\pi}{4} - 2$ u.a

c) $r = \operatorname{sen}\theta$ e $r = 1 - \cos\theta$

$r = \operatorname{sen}\theta$

$r = \frac{y}{r}$

$r^2 = y$

$x^2 + y^2 - y = 0$

$x^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}$

$\operatorname{sen}\theta = 1 - \cos\theta$

$(\cos\theta + \operatorname{sen}\theta)^2 = 1^2$

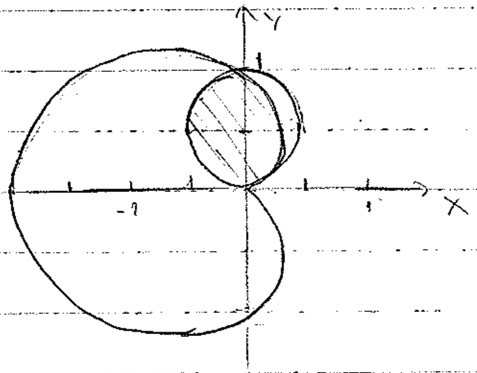
$\cos^2\theta + 2\operatorname{sen}\theta\cos\theta + \operatorname{sen}^2\theta = 1$

$\cancel{1} + 2\operatorname{sen}\theta\cos\theta = \cancel{1}$

$\operatorname{sen}\theta = 0$ ou $\cos\theta = 0$

$\theta = 0$ ou $\theta = \frac{\pi}{2}$

$\therefore \theta = K\frac{\pi}{2}$



$$A = \frac{1}{2}\int_0^{\frac{\pi}{2}} (1 - \cos\theta)^2\, d\theta + \frac{1}{2}\int_{\frac{\pi}{2}}^{\pi} \operatorname{sen}^2\theta\, d\theta$$

$$A = \frac{1}{2}\int_0^{\frac{\pi}{2}} (1 - 2\cos\theta + \cos^2\theta)\, d\theta + \frac{1}{4}\int_{\frac{\pi}{2}}^{\pi} (1 - \cos(2\theta))\, d\theta$$

$$A = \left(\frac{\theta}{2} - \operatorname{sen}\theta + \frac{\theta}{4} + \frac{1}{8}\operatorname{sen}(2\theta)\right)\Big|_0^{\frac{\pi}{2}} + \left(\frac{\theta}{4} - \frac{1}{8}\operatorname{sen}(2\theta)\right)\Big|_{\frac{\pi}{2}}^{\pi}$$

$$A = \frac{\pi}{4} - 1 + \cancel{\frac{\pi}{8}} + \frac{\pi}{4} \cancel{- \frac{\pi}{8}}$$

$$A = \frac{\pi - 1}{2} \text{ u.a}$$

/ /

d) $r^2 = \cos(2\theta)$ e $r^2 = \operatorname{sen}(2\theta)$

P/ $\cos(2\theta) \neq 0$

$\cos(2\theta) = \operatorname{sen}(2\theta)$

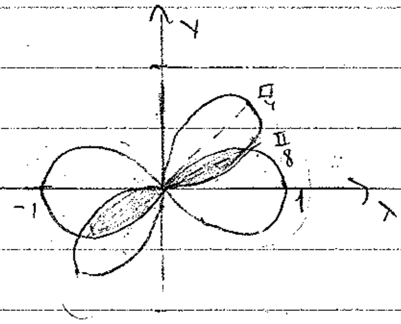
$\operatorname{tg}(2\theta) = 1$

$2\theta = \frac{\pi}{4} \therefore \theta = \frac{\pi}{8}$

P/ $\cos(2\theta) = 0$

$2\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$



$$A = 2 \cdot \frac{1}{2} \int_0^{\frac{\pi}{8}} \operatorname{sen}(2\theta)\, d\theta + 2 \cdot \frac{1}{2} \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \cos(2\theta)\, d\theta$$

$$A = -\frac{1}{2} \cos(2\theta) \Big|_0^{\frac{\pi}{8}} + \frac{1}{2} \operatorname{sen}(2\theta) \Big|_{\frac{\pi}{8}}^{\frac{\pi}{4}}$$

$$A = -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} + \frac{1}{2} - \frac{1}{2} \frac{\sqrt{2}}{2}$$

$$A = 1 - \frac{\sqrt{2}}{2} \text{ u.a}$$

e) $r = 2(1 + \operatorname{sen}\theta)$ e $r = 2(1 + \cos\theta)$

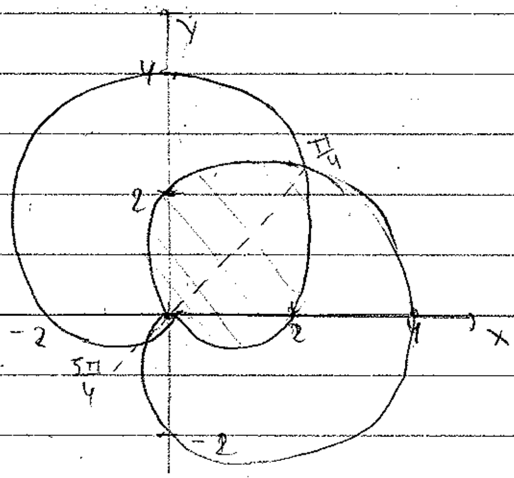
$2(1 + \operatorname{sen}\theta) = 2(1 + \cos\theta)$

$\cancel{1} + \operatorname{sen}\theta = \cancel{1} + \cos\theta$

$\operatorname{sen}\theta = \cos\theta$

$\operatorname{tg}\theta = 1$

$\theta = \frac{\pi}{4}$ ou $\frac{5\pi}{4}$



$A = 2 \cdot \frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (2+2\cos\theta)^2 \, d\theta$

$A = 4 \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (1+2\cos\theta+\cos^2\theta)\, d\theta$

$A = 4\theta + 8\,\text{sen}\,\theta + 2\theta + \text{sen}(2\theta) \Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$

$A = 6 \cdot \frac{5\pi}{4} - 8 \cdot \frac{\sqrt{2}}{2} + \cancel{1} - 6\frac{\pi}{4} - \frac{8\sqrt{2}}{2} - \cancel{1}$

$A = \frac{24\pi}{4} - 8\sqrt{2}$

$A = 6\pi - 8\sqrt{2}$ u.a

(38) $r = 6\cos\theta$ (interior) e $r = 2(1+\cos\theta)$ (exterior)

$6\cos\theta = 2(1+\cos\theta)$

$3\cos\theta = 1+\cos\theta$

$2\cos\theta = 1$

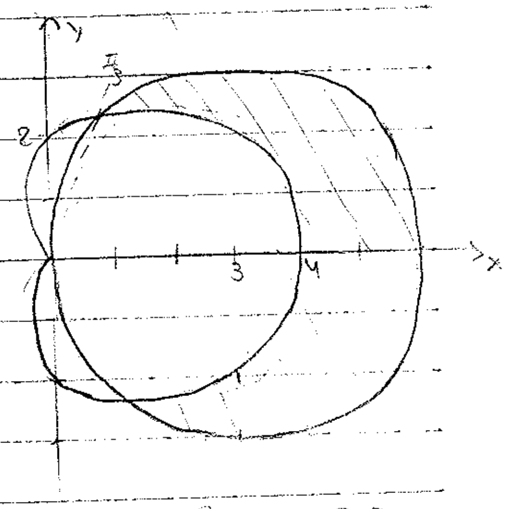
$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

$r^2 = 6x$

$x^2+y^2 = 6x$

$(x-3)^2+y^2 = 9$



$A = 2 \cdot \frac{1}{2} \int_0^{\frac{\pi}{3}} (6\cos\theta)^2 - 2^2(1+\cos\theta)^2 \, d\theta$

$A = \int_0^{\frac{\pi}{3}} 36\cos^2\theta - 4(1+2\cos\theta+\cos^2\theta)\, d\theta$

$A = \int_0^{\frac{\pi}{3}} 32\cos^2\theta - 8\cos\theta - 4 \, d\theta$

$16\theta + 8\,\text{sen}(2\theta) - 8\,\text{sen}\,\theta - 4\theta \Big|_0^{\frac{\pi}{3}}$

$A = 12\frac{\pi}{3} + \cancel{8\frac{\sqrt{3}}{2}} - \cancel{8\frac{\sqrt{3}}{2}}$

$A = 4\pi$ u.a

/ /

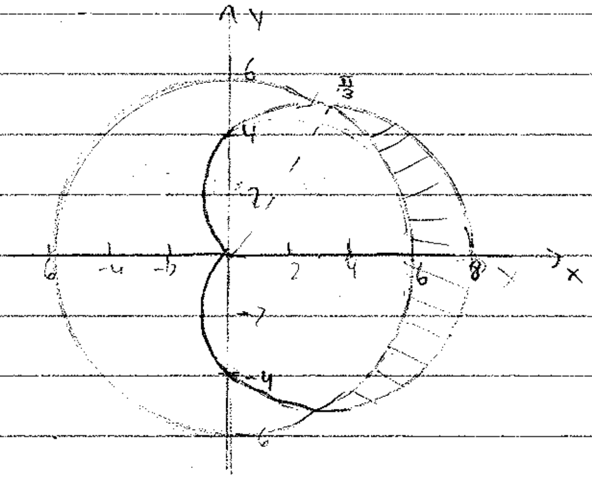
(39) $r = 4(1+\cos\theta)$ (interior) e $r = 6$ (exterior)

$4(1+\cos\theta) = 6$

$1+\cos\theta = \frac{3}{2}$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$



$A = 2 \cdot \frac{1}{2} \int_0^{\frac{\pi}{3}} 4^2(1+\cos\theta)^2 - 6^2 \, d\theta$

$A = \int_0^{\frac{\pi}{3}} 16(1+2\cos\theta+\cos^2\theta) - 36 \; d\theta$

$A = \int_0^{\frac{\pi}{3}} 16\cos^2\theta + 32\cos\theta - 20 \; d\theta$

$A = 8\theta + 4\,\text{sen}(2\theta) + 32\,\text{sen}\,\theta - 20\theta \Big|_0^{\frac{\pi}{3}}$

$A = 4\frac{\sqrt{3}}{2} + 32\frac{\sqrt{3}}{2} - \cancel{12}^{4}\frac{\pi}{3}$

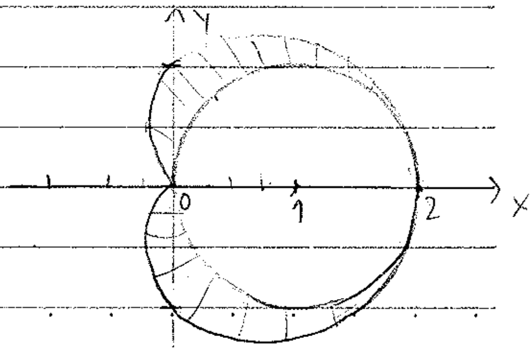
$A = 18\sqrt{3} - 4\pi$ u.a

(40) $r = 1+\cos\theta$ (interior) $r = 2\cos\theta$ (exterior)

$1+\cos\theta = 2\cos\theta$ $\qquad$ $r^2 = 2r\cos\theta$

$\cos\theta = 1$ $\qquad$ $x^2+y^2 = 2x$

$\theta = 0$ ou $\theta = 2\pi$ $\qquad$ $(x-1)^2 + y^2 = 1$



$$A = 2 \cdot \frac{1}{2} \int_0^{\pi} (1+\cos\theta)^2 - \frac{1}{2} \int_0^{\pi} 4\cos^2\theta \, d\theta$$

$$A = \int_0^{\pi} (1 + 2\cos\theta + \cos^2\theta)\, d\theta - 2\int_0^{\pi} \cos^2 \, d\theta$$

$$A = \cancel{\theta} + 2\,\text{sen}\,\theta + \frac{\theta}{2} + \frac{1}{4}\text{sen}(2\theta) - \cancel{\theta} - \frac{1}{2}\text{sen}(2\theta) \Big|_0^{\pi}$$

$$A = \frac{\pi}{2} \text{ u.a}$$

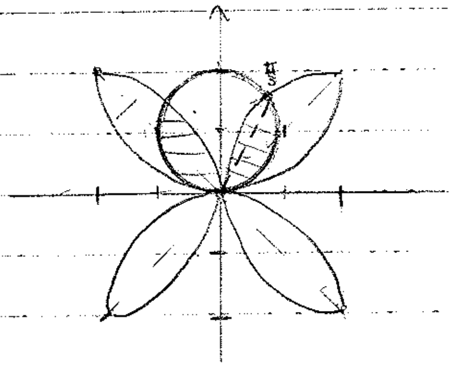
(41) $r = \text{sen}(2\theta)$ e $r = \text{sen}\,\theta \Rightarrow$ interior

$\text{sen}(2\theta) = \text{sen}(\theta)$

$2\,\cancel{\text{sen}\,\theta}\cos\theta = \cancel{\text{sen}\,\theta}$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$



$r = \text{sen}\,\theta$

$r^2 = r\,\text{sen}\,\theta$

$x^2 + y^2 = y$

$x^2 + (y - 1/2)^2 = \frac{1}{4}$

$\frac{1}{2} - \frac{\cos(4\theta)}{2}$

$$A = 2 \cdot \frac{1}{2} \int_0^{\frac{\pi}{3}} \text{sen}^2\theta \, d\theta + 2 \cdot \frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \text{sen}^2(2\theta)\, d\theta$$

$$A = \frac{\theta}{2} - \frac{1}{4}\text{sen}(2\theta)\Big|_0^{\frac{\pi}{3}} + \frac{\theta}{2} - \frac{1}{8}\text{sen}(4\theta)\Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$A = \cancel{\frac{\pi}{6}} - \frac{\sqrt{3}}{8} + \frac{\pi}{4} - \cancel{\frac{\pi}{6}} - \frac{\sqrt{3}}{16}$$

$$A = \frac{\pi}{4} - \frac{3\sqrt{3}}{16} \text{ u.a}$$

/ /

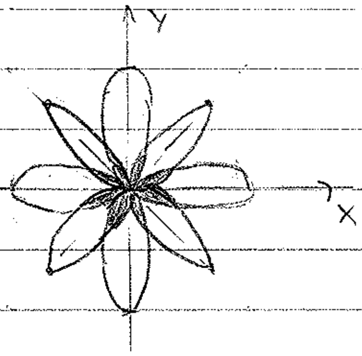
(42) $r = \operatorname{sen}(2\theta)$ e $r = \cos(2\theta)$

$\operatorname{sen}(2\theta) = \cos(2\theta)$

$tg(2\theta) = 1$

$2\theta = \frac{\pi}{4}$

$\theta = \frac{\pi}{8}$



$$A = 8 \cdot \frac{1}{2} \int_0^{\frac{\pi}{8}} \operatorname{sen}^2(2\theta)\, d\theta + 8 \cdot \frac{1}{2} \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \cos^2(2\theta)\, d\theta$$

$$A = 2\theta - \frac{1}{2}\operatorname{sen}(4\theta) \Big|_0^{\frac{\pi}{8}} + 2\theta + \frac{1}{2}\operatorname{sen}(4\theta) \Big|_{\frac{\pi}{8}}^{\frac{\pi}{4}}$$

$$A = \cancel{\frac{\pi}{4}} - \frac{1}{2} + \frac{\pi}{2} - \cancel{\frac{\pi}{4}} - \frac{1}{2}$$

$$A = \frac{\pi}{2} - 1 \text{ u.a}$$

(43) $r = \operatorname{sen}(2\theta)$ e $r = \sqrt{3}\cos(2\theta)$

$\operatorname{sen}(2\theta) = \sqrt{3}\cos(2\theta)$

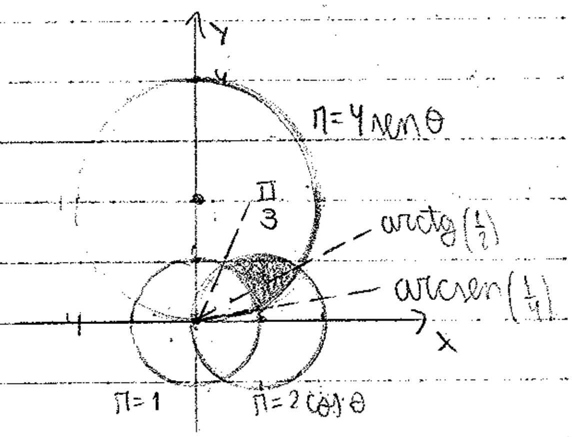
$tg(2\theta) = \sqrt{3}$

$2\theta = \frac{\pi}{3}$

$\theta = \frac{\pi}{6}$

$$A = \frac{3}{2} \int_0^{\frac{\pi}{4}} \cos^2(2\theta)\, d\theta + \frac{1}{2} \int_0^{\frac{\pi}{6}} \operatorname{sen}^2(2\theta)\, d\theta + \frac{3}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos^2(2\theta)\, d\theta$$

44) $r = 2\cos\theta$ e $r = 4\,\text{sen}\,\theta$ (interior), $r = 1$ (exterior)



$r^2 = 2r\cos\theta$ $\qquad$ $r^2 = 4r\,\text{sen}\,\theta$

$x^2+y^2 = 2x$ $\qquad$ $x^2+y^2 = 4y$

$(x-1)^2 + y^2 = 1$ $\qquad$ $x^2 + (y-2) = 4$

$2\cos\theta = 4\,\text{sen}\,\theta$ $\qquad$ $2\cos\theta = 1$

$\text{tg}\,\theta = \frac{1}{2}$ $\qquad$ $\cos\theta = \frac{1}{2}$

$\theta = \text{arctg}\left(\frac{1}{2}\right)$ $\qquad$ $\theta = \frac{\pi}{3}$ $\qquad$ $4\,\text{sen}\,\theta = 1$

$\text{sen}\,\theta = \frac{1}{4} \Rightarrow \theta = \text{arcsen}\left(\frac{1}{4}\right)$

a) $\frac{1}{2}\int_{\text{arcsen}(\frac{1}{4})}^{\text{arctg}(\frac{1}{2})} (16\,\text{sen}^2\theta - 1)\,d\theta + \frac{1}{2}\int_{\text{arctg}(\frac{1}{2})}^{\frac{\pi}{3}} (4\cos^2\theta - 1)\,d\theta$

45) a) $r = 1$ e $r = 2\cos(2\theta)$

$1 = 2\cos(2\theta)$

$\cos(2\theta) = \frac{1}{2}$

$2\theta = \frac{\pi}{3}$

$\theta = \frac{\pi}{6}$

$A = \frac{1}{2}\int_0^{\frac{\pi}{6}} (4\cos^2(2\theta) - 1)\,d\theta + 8.\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (1 - 4\cos^2(2\theta))\,d\theta$

$A = 2\int_0^{\frac{\pi}{6}} \cos^2(2\theta) - \frac{\theta}{2}\Big|_0^{\frac{\pi}{6}} + 4\theta\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} - 16\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos^2(2\theta)\,d\theta$

$A = \frac{\theta}{2} + \frac{1}{4}\,\text{sen}(4\theta)\Big|_0^{\frac{\pi}{6}} - \left(-4\theta + 2\,\text{sen}(4\theta)\right)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}}$

$A = \frac{\pi}{12} + \frac{\sqrt{3}}{8} - \pi + \frac{2\pi}{3} + \sqrt{3}$

$A = \frac{\pi - 12\pi + 8\pi}{12} + \frac{9\sqrt{3}}{8}$

$A = \frac{9\sqrt{3}}{8} - \frac{\pi}{4}$ u.a

b) $r = 2e^{\frac{\theta}{4}}$

$A = \frac{1}{2}\int_{\frac{5\pi}{2}}^{\frac{9\pi}{2}} 4\sqrt{e^{\theta}}\,d\theta - \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{5\pi}{2}} 4\sqrt{e^{\theta}}\,d\theta$

$A = 4\sqrt{e^{\theta}}\Big|_{\frac{5\pi}{2}}^{\frac{9\pi}{2}} - 4\sqrt{e^{\theta}}\Big|_{\frac{\pi}{2}}^{\frac{5\pi}{2}}$

$A = 4e^{\frac{9\pi}{4}} - 4e^{\frac{5\pi}{4}} - 4e^{\frac{5\pi}{4}} + 4e^{\frac{\pi}{4}}$

$A = 4e^{\frac{9\pi}{4}} - 8e^{\frac{5\pi}{4}} + 4e^{\frac{\pi}{4}}$ u.a

c) $r = \operatorname{sen}(3\theta)$ e $r = \cos(3\theta)$

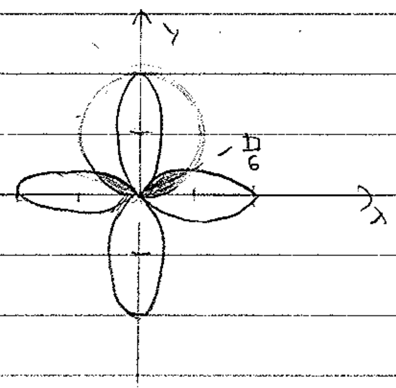
$\operatorname{sen}(3\theta) = \cos(3\theta)$	$A = \frac{3}{2}\int_0^{\frac{\pi}{12}} \operatorname{sen}^2(3\theta)\,d\theta + \frac{3}{2}\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \cos^2(3\theta)\,d\theta$
$tg(3\theta) = 1$	
$3\theta = \frac{\pi}{4}$	$A = \frac{3}{4}\int_0^{\frac{\pi}{12}} (1-\cos(6\theta))\,d\theta + \frac{3}{4}\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} (1+\cos(6\theta))\,d\theta$
$\theta = \frac{\pi}{12}$	$A = \frac{3\theta}{4} - \frac{1}{8}\operatorname{sen}(6\theta)\Big\|_0^{\frac{\pi}{12}} + \frac{3\theta}{4} + \frac{1}{8}\operatorname{sen}(6\theta)\Big\|_{\frac{\pi}{12}}^{\frac{\pi}{6}}$
	$A = \cancel{\frac{\pi}{16}} - \frac{1}{8} + \frac{\pi}{8} - \cancel{\frac{\pi}{16}} - \frac{1}{8} = \frac{\pi}{8} - \frac{1}{4}$ u.a

46) $r = \operatorname{sen}\theta$ e $r = \cos(2\theta)$

$r^2 = r\operatorname{sen}\theta$

$x^2 + y^2 = y$

$x^2 + (y - 1/2)^2 = \frac{1}{4}$



$\operatorname{sen}\theta = \cos(2\theta)$

$\operatorname{sen}\theta = \cos^2\theta - \operatorname{sen}^2\theta$

$\operatorname{sen}\theta = 1 - \operatorname{sen}^2\theta - \operatorname{sen}^2\theta$

$2\operatorname{sen}^2\theta + \operatorname{sen}\theta - 1 = 0$

$\operatorname{sen}\theta = \frac{-1 \pm \sqrt{1-4.2.-1}}{4} = \frac{-1\pm 3}{4}$ ⟨ -1, $\frac{1}{2}$

$\operatorname{sen}\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

(47) $r = 2(1+\cos\theta)$, $r = 4\cos(3\theta)$ e $r = 2$

$2(1+\cos\theta) = 2$

$\cos\theta = 0$

$\theta = \dfrac{\pi}{2}$

$2(1+\cos\theta) = 4\cos(3\theta)$

$1+\cos\theta = 2\cos(3\theta)$

$1+\cos\theta = 8\cos^3\theta - 6\cos\theta$

$8\cos^3\theta - 7\cos\theta - 1 = 0$

$\cos\theta = 1$

$\theta = 0$

$4\cos(3\theta) = 2$

$\cos(3\theta) = \dfrac{1}{2}$

$3\theta = \dfrac{\pi}{3}$

$\theta = \dfrac{\pi}{9}$

$4\cos(3\theta) = 0$

$3\theta = \dfrac{\pi}{2} \Rightarrow \theta = \dfrac{\pi}{6}$

$$A = \frac{1}{2}\int_0^{\frac{\pi}{9}} \left(4(1+\cos\theta)^2 - 16\cos^2(3\theta)\right)d\theta + \frac{1}{2}\int_{\frac{\pi}{9}}^{\frac{\pi}{2}} \left(4(1+\cos\theta)^2 - 4\right)d\theta +$$

$$+\frac{1}{2}\int_0^{\frac{\pi}{9}} 4\,d\theta + \frac{1}{2}\int_{\frac{\pi}{9}}^{\frac{\pi}{6}} 16\cos^2(3\theta)\,d\theta$$

$$A = 2\int_0^{\frac{\pi}{9}} (1-\cos\theta)^2 - 4\cos^2(3\theta)\,d\theta + 2\int_{\frac{\pi}{9}}^{\frac{\pi}{2}} (1+\cos\theta)^2 - 1\,d\theta + 2\int_0^{\frac{\pi}{9}} d\theta + 8\int_{\frac{\pi}{9}}^{\frac{\pi}{6}} \cos^2(3\theta)\,d\theta$$

(44) b) $$\int_{\arcsen(\frac{1}{4})}^{\arctg(\frac{1}{2})} \sqrt{16\cos^2\theta + 16\sen^2\theta}\,d\theta + \int_{\arctg(\frac{1}{2})}^{\frac{\pi}{3}} \sqrt{4\cos^2\theta + 4\sen^2\theta}\,d\theta +$$

$$+\int_{\arcsen(\frac{1}{4})}^{\frac{\pi}{3}} \sqrt{1}\,d\theta =$$

$$\int_{\arcsen(\frac{1}{4})}^{\arctg(\frac{1}{2})} 4\,d\theta + \int_{\arctg(\frac{1}{2})}^{\frac{\pi}{3}} 2\,d\theta + \int_{\arcsen(\frac{1}{4})}^{\frac{\pi}{3}} d\theta$$

(48) $x = \dfrac{1}{3}y^3 + \dfrac{1}{4y}$

$x' = y^2 - \dfrac{1}{4y^2}$

$$l = \int_2^5 \sqrt{1+\left(y^2 - \frac{1}{4y^2}\right)^2}\, dy$$

$$l = \int_2^5 \sqrt{1 + y^4 - \frac{1}{2} + \frac{1}{16y^4}}\, dy$$

$$l = \int_2^5 \sqrt{y^4 + \frac{1}{2} + \frac{1}{16y^4}}\, dy$$

$$l = \int_2^5 \sqrt{\left(y^2 + \frac{1}{4y^2}\right)^2}\, dy$$

$$l = \int_2^5 \left| y^2 + \frac{1}{4y^2} \right| dy$$

$$l = \int_2^5 y^2 + \frac{1}{4y^2}\, dy$$

$$l = \frac{y^3}{3} - \frac{1}{4y}\Big|_2^5$$

$$l = \frac{125}{3} - \frac{1}{20} - \frac{8}{3} + \frac{1}{8}$$

$$l = \frac{117}{3} + \frac{5-2}{40}$$

$$l = 39 + \frac{3}{40} \text{ u.c}$$

$$l = \frac{1563}{40} \text{ u.c}$$

$$t = \frac{10}{6} tg\theta \Rightarrow tg^2\theta = \frac{36t^2}{100}$$

$$dt = \frac{10}{6} sec^2\theta$$

$$u = sec\theta$$

$$du = tg\theta\, sec\theta$$

b) $\begin{cases} x = 3 + t^2 \\ y = 6 + 2t^2 \end{cases}, t \in [1,5]$

$$l = \int_1^5 \sqrt{(2t)^2 + (4t)^2}\, dt$$

$$l = \int_1^5 \sqrt{4t^2 + 16t^2}\, dt$$

$$l = \int_1^5 \sqrt{20}\, t\, dt$$

$$l = \frac{\sqrt{20}\, t^2}{2}\Big|_1^5$$

$$l = \frac{25\sqrt{20} - \sqrt{20}}{2}$$

$$l = \frac{24\sqrt{20}}{2}$$

$$l = 24\sqrt{5} \text{ u.c}$$

c) $\begin{cases} x = 5t^2 \\ y = 2t^3 \end{cases}, t \in [0,1]$

$$l = \int_0^1 \sqrt{(10t)^2 + (6t^2)^2}\, dt$$

$$l = \int_0^1 \sqrt{100t^2 + 36t^4}\, dt$$

$$l = \int_0^1 t\sqrt{100 + 36t^2}\, dt$$

$$l = \int \frac{10}{6} tg\theta \sqrt{100 + \cancel{36} \cdot \frac{100\, tg^2\theta}{\cancel{36}}} \cdot \frac{10}{6} sec^2\theta\, d\theta$$

$$l = \int \frac{1000}{36} tg\theta\, sec\theta\, sec^2\theta\, d\theta$$

$$l = \int \frac{1000}{36} u^2\, du$$

$$l = \frac{1000}{108} u^3$$

$$l = \frac{1000}{108}\left(\sqrt{1 + \frac{36t^2}{100}}\right)^3\Big|_0^1$$

$$l = \frac{1}{108}\left(\sqrt{100+36t^2}\right)^3 \Big|_0^1$$

$$l = \frac{1}{108}\left(\sqrt{136}\right)^3 - \frac{1}{108}\left(\sqrt{100}\right)^3$$

$$l = \frac{8}{108}\left(\sqrt{34}\right)^3 - \frac{1000}{108}$$

$$l = \frac{68\sqrt{34}}{27} - \frac{250}{27} \text{ u.c}$$

d) $\begin{cases} x = e^t \cos t \\ y = e^t \operatorname{sen} t \end{cases}, t \in [0, \frac{\pi}{2}]$

$x' = e^t \cos t - e^t \operatorname{sen} t$

$y' = e^t \cos t - e^t \operatorname{sen} t$

$$l = \int_0^{\frac{\pi}{2}} \sqrt{(e^t \cos t - e^t \operatorname{sen} t)^2 + (e^t \cos t + e^t \operatorname{sen} t)^2}\, dt$$

$$l = \int_0^{\frac{\pi}{2}} \sqrt{e^{2t}\cos^2 t - \cancel{2e^{2t}\cos t \operatorname{sen} t} + e^{2t}\operatorname{sen}^2 t + e^{2t}\cos^2 t + \cancel{2e^{2t}\cos t \operatorname{sen} t} + e^{2t}\operatorname{sen}^2 t}\, dt$$

$$l = \int_0^{\frac{\pi}{2}} \sqrt{2e^{2t}\cos^2 t + 2e^{2t}\operatorname{sen}^2 t}\, dt$$

$$l = \int_0^{\frac{\pi}{2}} \sqrt{2}\, e^t\, dt$$

$$l = \sqrt{2}\, e^t \Big|_0^{\frac{\pi}{2}}$$

$$l = \sqrt{2}\, e^{\frac{\pi}{2}} - \sqrt{2}$$

$$l = \sqrt{2}\left(e^{\frac{\pi}{2}} - 1\right) \text{ u.c}$$

e) $r = e^{-\theta}$, $\theta \in [0, 2\pi]$

$r' = -e^{-\theta}$

$$l = \int_0^{2\pi} \sqrt{(e^{-\theta})^2 + (-e^{-\theta})^2}\, d\theta$$

$$l = \int_0^{2\pi} \sqrt{e^{-2\theta} + e^{-2\theta}}\, d\theta$$

$$l = \int_0^{2\pi} \sqrt{2}\, e^{-\theta}\, d\theta$$

$$l = -\sqrt{2}\, e^{-\theta} \Big|_0^{2\pi}$$

$$l = -\sqrt{2}\, e^{-2\pi} + \sqrt{2}$$

$$l = \sqrt{2}\left(1 - e^{-2\pi}\right) \text{ u.c}$$

1) $r = \cos^2\left(\frac{\theta}{2}\right), \quad \theta \in [0,\pi] \qquad \cos^2\left(\frac{\theta}{2}\right) = \frac{1}{2}\cos\theta + \frac{1}{2}$

$$r' = 2\cos\left(\frac{\theta}{2}\right) \cdot \frac{1}{2} \cdot -\operatorname{sen}\left(\frac{\theta}{2}\right)$$

$$r' = -\frac{1}{2}\operatorname{sen}(\theta)$$

$$l = \int_0^{\pi} \sqrt{\left(-\frac{1}{2}\operatorname{sen}\theta\right)^2 + \left(\frac{1}{2}\cos\theta + \frac{1}{2}\right)^2}\, d\theta$$

$$l = \int_0^{\pi} \sqrt{\frac{1}{4}\operatorname{sen}^2\theta + \frac{1}{4}\cos^2\theta + \frac{1}{2}\cos\theta + \frac{1}{4}}\, d\theta$$

$$l = \int_0^{\pi} \sqrt{\frac{1}{4}(\operatorname{sen}^2\theta + \cos^2\theta) + \frac{1}{2}\cos\theta + \frac{1}{4}}\, d\theta$$

$$l = \int_0^{\pi} \sqrt{\frac{1}{2}\cos\theta + \frac{1}{2}}\, d\theta$$

$$l = \int_0^{\pi} \sqrt{\cos^2\left(\frac{\theta}{2}\right)}\, d\theta$$

$$l = \int_0^{\pi} \cos\left(\frac{\theta}{2}\right) d\theta$$

$$l = 2\operatorname{sen}\left(\frac{\theta}{2}\right)\Big|_0^{\pi}$$

$l = 2$ u.c

49) $\begin{cases} x(t) = 1 + \cos(3\sqrt{t}) \\ y(t) = 3 - \operatorname{sen}(3\sqrt{t}) \end{cases}$ $\quad A(2,3)$ e $B(0,3)$

$2 = 1 + \cos(3\sqrt{t})$	$0 = 1 + \cos(3\sqrt{t})$	$x' = \frac{-3}{2\sqrt{t}}\operatorname{sen}(3\sqrt{t})$	$y' = -\frac{3}{2\sqrt{t}}\cos(3\sqrt{t})$
$\cos(3\sqrt{t}) = 1$	$\cos(3\sqrt{t}) = -1$		
$t = 0$	$t = \frac{\pi^2}{9}$		

$$d = \int_0^{\frac{\pi^2}{9}} \sqrt{\frac{9}{4t} \operatorname{sen}^2(3\sqrt{t}) + \frac{9}{4t} \cos^2(3\sqrt{t})}\, dt$$

$$d = \int_0^{\frac{\pi^2}{9}} \frac{3}{2\sqrt{t}}\, dt$$

$$d = 3\sqrt{t}\,\Big|_0^{\frac{\pi^2}{9}}$$

$$d = \frac{\cancel{3}\pi}{\cancel{3}}$$

$d = \pi$ u.c

(50) $\begin{cases} x(t) = 2\cos t + 2t \operatorname{sen} t \\ y(t) = 2\operatorname{sen} t - 2t\cos t \end{cases}$ $t \in [0, \frac{\pi}{2}]$

$x' = -2\cancel{\operatorname{sen} t} + 2\cancel{\operatorname{sen} t} + 2t\cos t = 2t\cos t$

$y' = 2\cancel{\cos} t - 2\cancel{\cos} t + 2t\operatorname{sen} t = 2t\operatorname{sen} t$

$d = \int_0^{\frac{\pi}{2}} \sqrt{4t^2\cos^2 t + 4t^2 \operatorname{sen}^2 t}\, dt$

$d = \int_0^{\frac{\pi}{2}} 2t\, dt$

$d = t^2 \Big|_0^{\frac{\pi}{2}}$

$d = \frac{\pi^2}{4}$ u.c

(51) $\begin{cases} x(t) = 4t^3 + 1 \\ y(t) = 2t^{\frac{9}{2}} \end{cases}$ $A(5,2)$ e $B(33, 33\sqrt{2})$

$x' = 12t^2$	$x(\alpha) = 5$	$x(\beta) = 33$
$y' = 9t^{\frac{7}{2}}$	$4\alpha^3 + 1 = 5$	$4\beta^3 + 1 = 33$
	$\alpha^3 = 1$	$\beta^3 = 8$
	$\alpha = 1$	$\beta = 2$

$$d = \int_1^2 \sqrt{144t^4 + 81t^7}\, dt \qquad t = tg^{\frac{2}{3}}\theta \sqrt[3]{\frac{144}{81}} \Rightarrow$$

$$d = \int_1^2 t^2 \sqrt{144 + 81t^3}\, dt$$

$$t = \left(\frac{4}{3} tg\theta\right)^{2/3}$$

$$u = sec\theta$$

$$du = tg\theta\, sec\theta\, d\theta \qquad dt = \frac{2}{3}\left(\frac{4}{3} tg\theta\right)^{-1/3} \cdot \frac{4}{3} sec^2\theta\, d\theta$$

$$d = \int \left(\frac{4}{3}\right)^{4/3} tg^{4/3}\theta \cdot 12\, sec\theta \cdot \frac{8}{9} \cdot \left(\frac{4}{3}\right)^{-1/3} \cdot tg^{-1/3}\theta\, sec^2\theta\, d\theta$$

$$d = \int \frac{8}{9} \cdot \frac{4}{\cancel{3}} \cdot \overset{4}{\cancel{12}}\, tg\theta\, sec\theta\, sec^2\theta\, d\theta$$

$$d = \frac{128}{9}\int u^2\, du \qquad t^3 = \left(\frac{4}{3} tg\theta\right)^2$$

$$d = \frac{128}{27} u^3 \qquad \frac{9}{16} t^3 = tg^2\theta$$

$$d = \frac{128}{27}\left(\frac{9}{16} t^3 + 1\right)^{3/2}\Big|_1^2 \qquad sec^2\theta = \frac{9}{16} t^3 + 1$$

$$d = \frac{128}{27} \cdot \frac{(9t^3+16)^{3/2}}{64}\Big|_1^2 \qquad sec^3\theta = \left(\frac{9}{16} t^3 + 1\right)^{3/2}$$

$$d = \frac{2}{27}(9t^3+16)^{3/2}\Big|_1^2$$

$$d = \frac{2}{27} \cdot 88\sqrt{88} - \frac{2}{27} \cdot 125$$

$$d = \frac{352\sqrt{22}}{27} - \frac{250}{27}\ u.c$$

52) $\begin{cases} x(t) = 1 + 2\cos(3t^{5/2}) \\ y(t) = 5 - 2\operatorname{sen}(3t^{5/2}) \end{cases}$, $t \in [0,4]$

$$x' = -\cancel{2}\operatorname{sen}(3t^{5/2}) \cdot 3 \cdot \frac{5}{\cancel{2}} t^{3/2} = -15t^{3/2}\operatorname{sen}(3t^{5/2})$$

$$y' = -\cancel{2}\cos(3t^{5/2}) \cdot 3 \cdot \frac{5}{\cancel{2}} t^{3/2} = -15t^{3/2}\cos(3t^{5/2})$$

$$d = \int_0^4 \sqrt{225t^3\operatorname{sen}^2(3t^{5/2}) + 222t^3\cos^2(3t^{5/2})}\, dt$$

$$d = \int_0^4 15t^{3/2}\, dt$$

$$d = \cancel{15}^{3} \cdot \frac{2}{\cancel{5}} t^{5/2} \Big|_0^4$$

$$d = 6t^{5/2}\Big|_0^4$$

$$d = 192 \text{ u.c}$$

53) $\begin{cases} x(t) = 3e^{-t}\cos 6t \\ y(t) = 3e^{-t}\operatorname{sen} 6t \end{cases}$, $t \in [0, +\infty)$

$$x' = -3e^{-t}\cos 6t - 18e^{-t}\operatorname{sen} 6t$$

$$y' = -3e^{-t}\operatorname{sen} 6t + 18e^{-t}\cos 6t$$

$$l = \int_0^\infty \sqrt{(-3e^{-t}\cos 6t - 18e^{-t}\operatorname{sen} 6t)^2 + (-3e^{-t}\operatorname{sen} 6t + 18e^{-t}\cos 6t)^2}\, dt$$

$$l = \int_0^\infty \sqrt{9e^{-2t}\cos^2 6t + \cancel{108e^{-2t}\cos 6t\operatorname{sen} 6t} + 324e^{-2t}\operatorname{sen}^2 6t + 9e^{-2t}\operatorname{sen}^2 6t + 324e^{-2t}\cos^2 6t - \cancel{108e^{-2t}\cos 6t\operatorname{sen} 6t}}\, dt$$

$$l = \int_0^\infty \sqrt{9e^{-2t} + 324e^{-2t}}\, dt$$

$$l = \int_0^\infty e^{-t}\sqrt{333}\, dt$$

$$l = -\frac{\sqrt{333}}{e^t}\Big|_0^\infty$$

$$l = \sqrt{333} \text{ u.c}$$

/ /

(54) $r_1 = \sqrt{3}\,\text{sen}\,\theta$ $\qquad r_2 = 3\cos\theta$

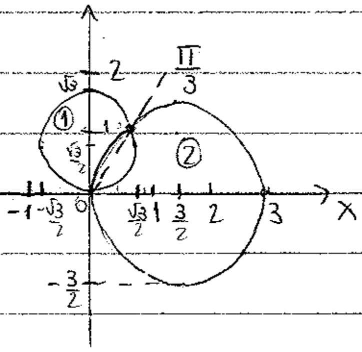
$r^2 = \sqrt{3}\, r\, \text{sen}\,\theta$ $\qquad r^2 = 3\, r \cos\theta$

$x^2 + y^2 - \sqrt{3}y = 0$ $\qquad x^2 + y^2 - 3x = 0$

$x^2 + \left(y - \frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$ $\qquad \left(x - \frac{3}{2}\right)^2 + y^2 = \frac{9}{4}$

$r_1' = \sqrt{3}\cos\theta$

$r_2' = -3\,\text{sen}\,\theta$



$\sqrt{3}\,\text{sen}\,\theta = 3\cos\theta$

$\text{tg}\,\theta = \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\cancel{\sqrt{3}}}$

$\theta = \text{arctg}(\sqrt{3})$

$\theta = \frac{\pi}{3}$

$l = \int_0^{\frac{\pi}{3}} \sqrt{3\,\text{sen}^2\theta + 3\cos^2\theta}\, d\theta + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sqrt{9\cos^2\theta + 9\,\text{sen}^2\theta}\, d\theta$

$l = \int_0^{\frac{\pi}{3}} \sqrt{3}\, d\theta + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 3\, d\theta$

$l = \sqrt{3}\,\theta \Big|_0^{\frac{\pi}{3}} + 3\,\theta \Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}}$

$l = \frac{\sqrt{3}\pi}{3} + \frac{3\pi}{2} - \frac{\cancel{3}\pi}{\cancel{3}}$

$l = \frac{\sqrt{3}\pi}{3} + \frac{\pi}{2}$ u.c

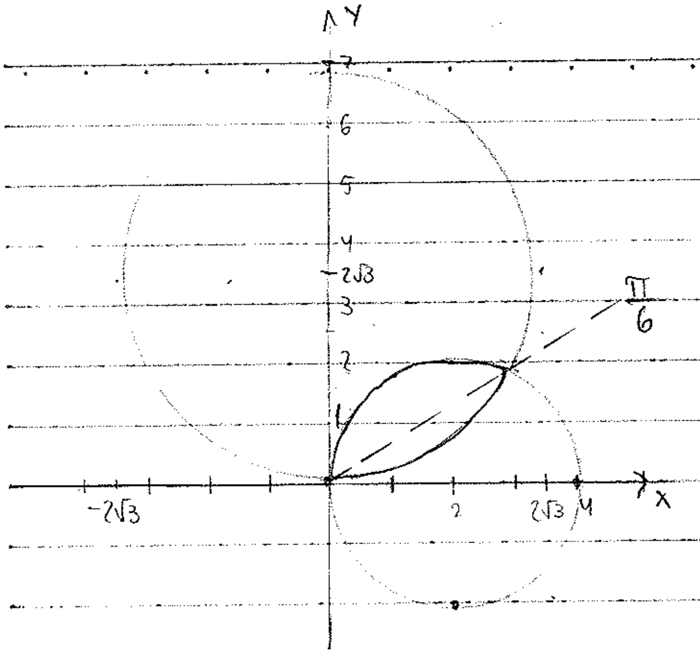
(55) $r_1 = 4\sqrt{3}\,\text{sen}\,\theta$ $\qquad r_2 = 4\cos\theta$

$r' = 4\sqrt{3}\cos\theta$ $\qquad r' = -4\,\text{sen}\,\theta$

$r^2 = 4\sqrt{3}\, r\, \text{sen}\,\theta$ $\qquad r^2 = 4 r\cos\theta$

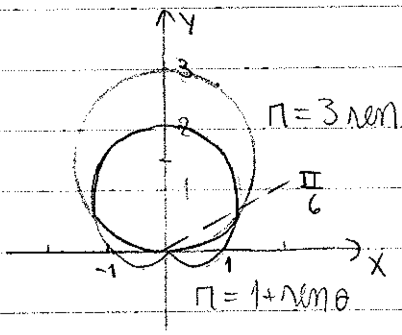
$x^2 + y^2 = 4\sqrt{3}y$ $\qquad x^2 + y^2 = 4x$

$x^2 + (y - 2\sqrt{3})^2 = 12$ $\qquad (x-2)^2 + y^2 = 4$



56) $r = 1 + \operatorname{sen}\theta$ $\qquad r = 3\operatorname{sen}\theta$

$r' = \cos\theta$ $\qquad r' = 3\cos\theta$



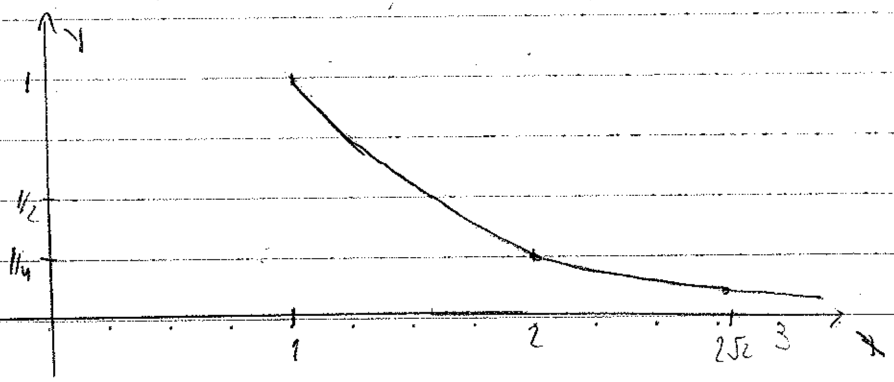
$1 + \operatorname{sen}\theta = 3\operatorname{sen}\theta$

$\operatorname{sen}\theta = \dfrac{1}{2}$

$\theta = \dfrac{\pi}{6}$

$$l = 2\int_0^{\frac{\pi}{6}} \sqrt{9\cos^2\theta + 9\operatorname{sen}^2\theta}\, d\theta + 2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{\cos^2\theta + (1+\operatorname{sen}\theta)^2}\, d\theta$$

57) $y = \dfrac{1}{x^2}$, $x \geq 1$



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$$V = \pi \int_1^{+\infty} \left(\frac{1}{x^2}\right)^2 dx$$

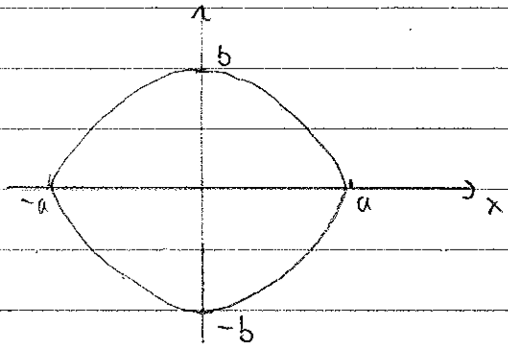
$$V = \pi \int_1^{+\infty} \frac{1}{x^4} dx$$

$$V = -\frac{\pi}{3x^3} \Big|_1^{+\infty}$$

$$V = -\cancel{\frac{\pi}{3\infty}}_{0} + \frac{\pi}{3}$$

$$V = \frac{\pi}{3} \text{ u.v}$$

(58) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



$$b^2x^2 + a^2y^2 = a^2b^2$$

$$y^2 = \frac{a^2b^2 - b^2x^2}{a^2}$$

$$V = \pi \int_{-a}^{a} \left(b^2 - \frac{b^2}{a^2}x^2\right) dx$$

$$V = \pi \left(b^2x - \frac{b^2}{3a^2}x^3\right)\Big|_{-a}^{a}$$

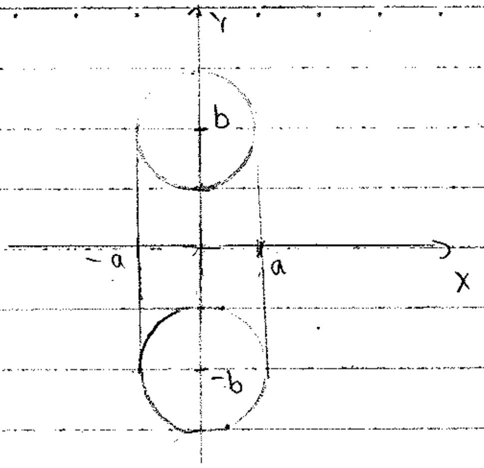
$$V = \pi\left(ab^2 - \frac{ab^2}{3}\right) - \pi\left(-ab^2 + \frac{ab^2}{3}\right)$$

$$V = 2\pi ab^2 - \frac{2}{3}\pi ab^2$$

$$V = \frac{4}{3}\pi ab^2 \text{ u.v}$$

(59) $x^2+(y-b)^2=a^2$

$y = b \pm \sqrt{a^2-x^2}$



$x = a\,\mathrm{sen}\,\theta$

$dx = a\cos\theta\, d\theta$

$\dfrac{x^2}{a^2} = \mathrm{sen}^2\theta$

$\cos^2\theta = 1 - \dfrac{x^2}{a^2}$

$$V = 2\pi \int_0^a \left(b+\sqrt{a^2-x^2}\right)^2 - \left(b-\sqrt{a^2-x^2}\right)^2 dx$$

$$V = 2\pi \int_0^a \left(\cancel{b^2} + 2b\sqrt{a^2-x^2} + \cancel{a^2} - \cancel{x^2} - \cancel{b^2} + 2b\sqrt{a^2-x^2} - \cancel{a^2} + \cancel{x^2}\right) dx$$

$$V = 8b\pi \int_0^a \sqrt{a^2-x^2}\, dx$$

$$V = 8b\pi \int \sqrt{a^2-a^2\,\mathrm{sen}^2\theta}\; a\cos\theta\, d\theta$$

$$V = 8a^2b\pi \int \cos^2\theta\, d\theta$$

$$V = 4a^2b\pi \int (\cos(2\theta)+1)\, d\theta$$

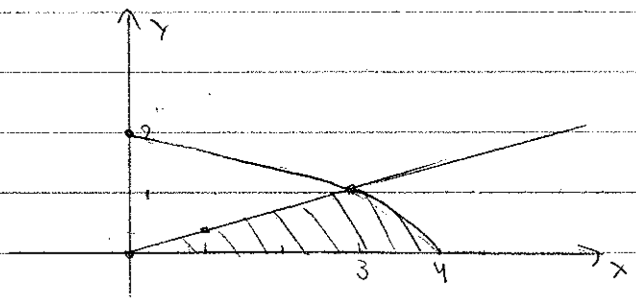
$$V = 4a^2b\pi \left(\frac{1}{2}\mathrm{sen}(2\theta) + \theta\right)$$

$$V = 4a^2b\pi \left(\frac{x\sqrt{a^2-x^2}}{a^2} + \mathrm{arcsen}\left(\frac{x}{a}\right)\right)\Bigg|_0^a$$

$$V = 4a^2b\pi \cdot \frac{\pi}{2}$$

$$V = 2\pi^2 a^2 b \;\; u.v$$

60 a) $y=\sqrt{4-x}$, $3y=x$, $y=0$, rot eixo x



$\frac{x}{3}=\sqrt{4-x}$

$x^2=9(4-x)$

$x^2=36-9x$

$x^2+9x-36=0$

$3+(-12)=-9$

$3\cdot(-12)=-36$

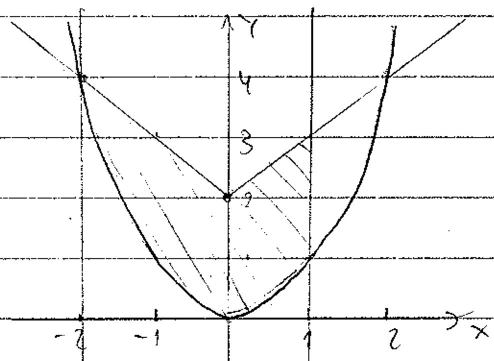
$V=\pi\int_0^3 \frac{x^2}{9}\,dx+\pi\int_3^4 4-x\,dx$

$V=\frac{\pi x^3}{27}\Big|_0^3+4x\pi-\frac{\pi x^2}{2}\Big|_3^4$

$V=\pi+16\pi-8\pi-12\pi+\frac{9}{2}\pi$

$V=\frac{3}{2}\pi$ u.v

b) $y=|x|+2$, $y=x^2$, $x=-2$, $x=1$, rot eixo x



$V=\pi\int_{-2}^0 (|x|+2)^2-x^4\,dx+\pi\int_0^1 (|x|+2)^2-x^4\,dx$

$V=\pi\int_{-2}^0 x^2+4|x|+4-x^4\,dx+\pi\int_0^1 x^2+4|x|+4-x^4\,dx$

$V=\pi\int_{-2}^0 -x^4+x^2-4x+4\,dx+\pi\int_0^1 -x^4+x^2+4x+4\,dx$

$$V = \pi\left(-\frac{x^5}{5}+\frac{x^3}{3}-2x^2+4x\right)\Big|_{-2}^{0}+\pi\left(-\frac{x^5}{5}+\frac{x^3}{3}+2x^2+4x\right)\Big|_{0}^{1}$$

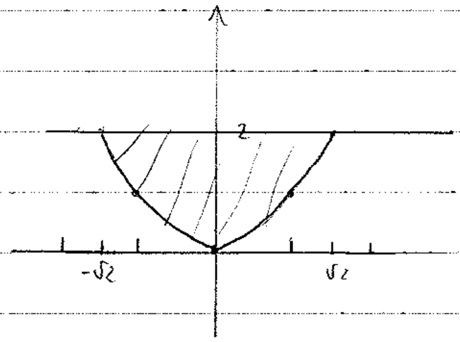
$$V = -\pi\left(\frac{32}{5}-\frac{8}{3}-8-8\right)+\pi\left(-\frac{1}{5}+\frac{1}{3}+2+4\right)$$

$$V = -\frac{32}{5}\pi+\frac{8}{3}\pi+16\pi-\frac{\pi}{5}+\frac{\pi}{3}+6\pi$$

$$V = -\frac{33}{5}\pi+25\pi$$

$$V = \frac{92}{5}\pi \text{ u.v}$$

c) $y = x^2$ e $y = 2$, rot $y = 2$



$$V = 2\pi\int_{-\sqrt{2}}^{0}(2-x^2)^2\,dx$$

$$V = 2\pi\int_{-\sqrt{2}}^{0} x^4-4x^2+4\,dx$$

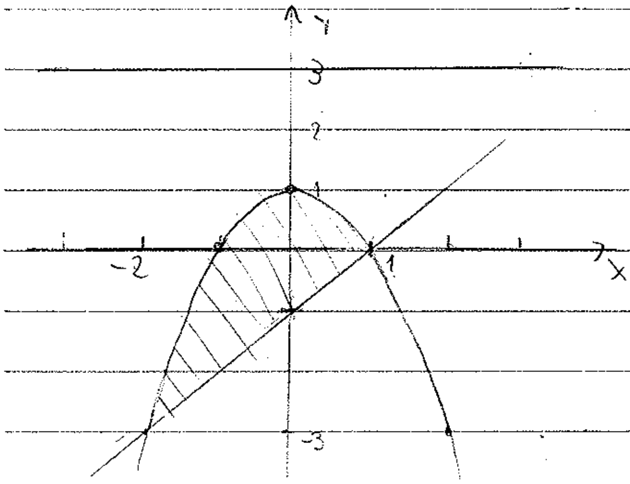
$$V = 2\pi\left(\frac{x^5}{5}-\frac{4}{3}x^3+4x\right)\Big|_{-\sqrt{2}}^{0}$$

$$V = -2\pi\left(-\frac{4\sqrt{2}}{5}+\frac{8\sqrt{2}}{3}-4\sqrt{2}\right)$$

$$V = -\frac{2\pi}{15}\left(-12\sqrt{2}+40\sqrt{2}-60\sqrt{2}\right)$$

$$V = \frac{64}{15}\sqrt{2}\pi \text{ u.v}$$

d) $y = 1 - x^2$ e $x - y = 1$, rot $y = 3$



$1 - x^2 = x - 1$

$x^2 + x - 2 = 0$

$-2 + 1 = -1$

$-2 \cdot 1 = -2$

$$V = \pi \int_{-2}^{1} (3 - (x-1))^2 - (3 - (1 - x^2))^2 \, dx$$

$$V = \pi \int_{-2}^{1} (4 - x)^2 - (x^2 + 2)^2 \, dx$$

$$V = \pi \int_{-2}^{1} 16 - 8x + x^2 - (x^4 + 4x^2 + 4) \, dx$$

$$V = \pi \int_{-2}^{1} -x^4 - 3x^2 - 8x + 12 \, dx$$

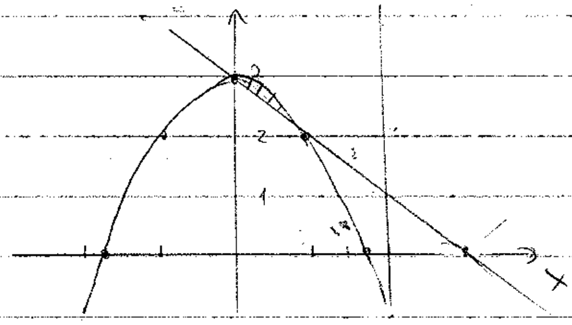
$$V = \pi \left(\frac{-x^5}{5} - x^3 - 4x^2 + 12x \right) \Bigg|_{-2}^{1}$$

$$V = \pi \left(\frac{-1}{5} - 1 - 4 + 12 \right) - \pi \left(\frac{32}{5} + 8 - 16 - 24 \right)$$

$$V = \frac{-\pi}{5} + 7\pi - \frac{32\pi}{5} + 32\pi$$

$$V = \frac{162}{5} \pi \text{ u.v}$$

e) $x+y=3$ e $y+x^2=3$, rot $x=2$



$\cancel{3}-x=\cancel{3}-x^2$

$x=x^2$

$x^2-x=0$

$x(x-1)=0$

$x=0$; $x=1$

$$V=\pi\int_2^3 (2-(3-y))^2-(2-(\sqrt{3-y}))^2\,dy$$

$$V=\pi\int_2^3 y^2-2y+1-(4-4\sqrt{3-y}+3-y)\,dy$$

$$V=\pi\int_2^3 y^2-y-6+4\sqrt{3-y}\,dy$$

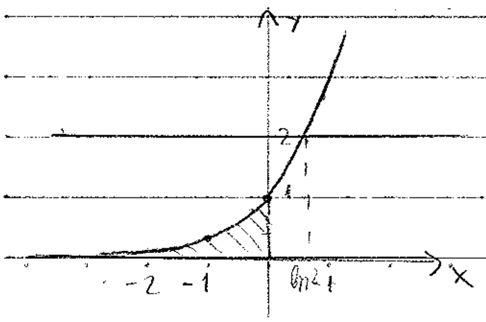
$$V=\pi\left(\frac{y^3}{3}-\frac{y^2}{2}-6y-\frac{8}{3}(3-y)^{3/2}\right)\Big|_2^3$$

$$V=\pi\left(9-\frac{9}{2}-18\right)-\pi\left(\cancel{\frac{8}{3}}-2-12-\cancel{\frac{8}{3}}\right)$$

$$V=-9\pi-\frac{9}{2}\pi+14\pi$$

$$V=\frac{\pi}{2}\ uv$$

(61) $y=e^x$, $y\geq 0$ e $x\leq 0$, rot $y=2$



$$V=\pi\int_{-\infty}^{0}(0-2)^2-(2-e^x)^2\,dx$$

$$V=\pi\int_{-\infty}^{0}4-(4-4e^x+e^{2x})\,dx$$

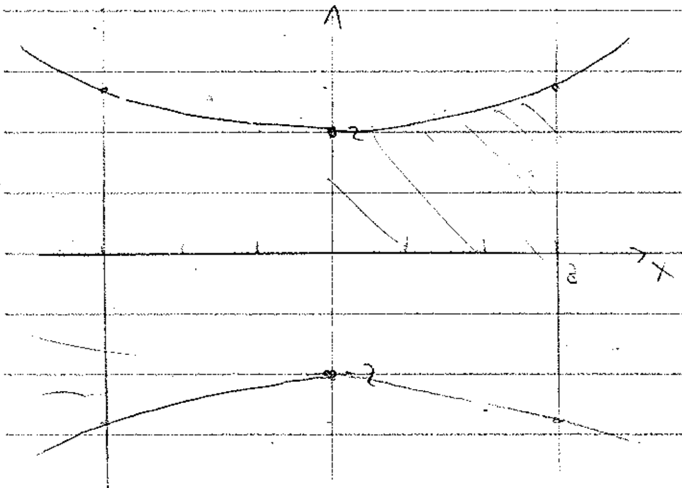
$$V = \pi \int_{-\infty}^{0} 4e^{x} - e^{2x} \, dx$$

$$V = \pi \left(4e^{x} - \frac{1}{2} e^{2x} \right) \Big|_{-\infty}^{0}$$

$$V = \pi \left(4 - \frac{1}{2} \right) = \frac{7}{2} \pi \text{ u.v}$$

(62) $x = -3, x = 3, \; 9y^2 - 4x^2 = 36$, rot eixo x

$$\frac{y^2}{4} - \frac{x^2}{9} = 1$$



$$V = 2\pi \int_{0}^{3} 4 + \frac{4x^2}{9} \, dx$$

$$V = 8\pi \left(x + \frac{x^3}{27} \right) \Big|_{0}^{3}$$

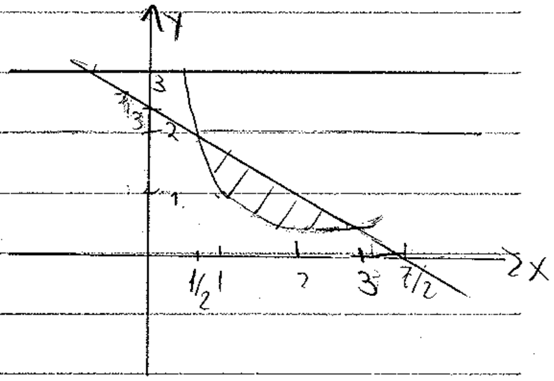
$$V = 8\pi (3 + 1)$$

$$V = 32\pi \text{ u.v}$$

(63) $x = \frac{7 - 3y}{2}$ $\qquad x = \frac{1}{y}$

$2x - 7 = -3y$ $\qquad y = \frac{1}{x}$

$y = \frac{7}{3} - \frac{2x}{3}$



$$V = \pi \int_{1/2}^{3} \left(3 - \frac{1}{x} \right)^2 - \left(3 - \frac{7}{3} + \frac{2x}{3} \right)^2 dx$$

$$V = \pi \int_{1/2}^{3} 9 - \frac{6}{x} + \frac{1}{x^2} - \left(\frac{2}{3}(1-x)\right)^2 \, dx$$

$$V = \pi \int_{1/2}^{3} 9 - \frac{6}{x} + \frac{1}{x^2} - \frac{4}{9}(1-2x+x^2) \, dx$$

$$V = \pi \int_{1/2}^{3} 9 - \frac{6}{x} + \frac{1}{x^2} - \frac{4}{9} + \frac{8x}{9} - \frac{4}{9}x^2 \, dx$$

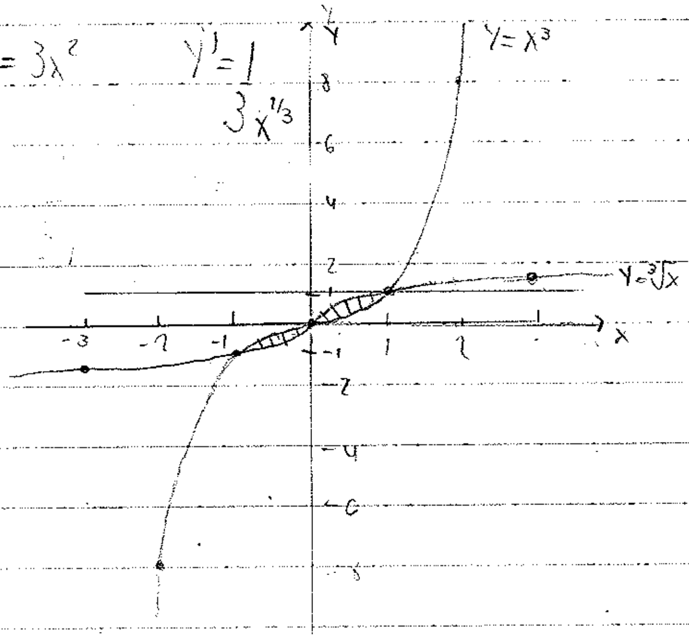
$$V = \pi \int_{1/2}^{3} \frac{77}{9} - \frac{6}{x} + \frac{1}{x^2} + \frac{8x}{9} - \frac{4}{9}x^2 \, dx$$

$$V = \pi \left(\frac{77}{9}x - \frac{1}{x} + \frac{4}{9}x^2 - \frac{4}{27}x^3 - 6\ln x \right) \Bigg|_{1/2}^{3}$$

$V =$

6) $y = x^3$ e $x = y^3$

$y' = 3x^2$ $\quad$ $y' = \dfrac{1}{3x^{1/3}}$



$x^3 = \sqrt[3]{x}$

$x^9 = x$

$x^9 - x = 0$

$x(x^8 - 1) = 0$

$x = 0$

$x = \pm 1$

a) $L = 2\int_0^1 \sqrt{1 + \left(\dfrac{1}{3x^{2/3}}\right)^2} + \sqrt{1 + (3x^2)^2} \, dx$

$L = 2\int_0^1 \sqrt{1 + \dfrac{1}{9\sqrt[3]{x}}} + \sqrt{1 + 9x^4} \, dx$

/ /

b) $V = 2\pi \int_0^1 (\sqrt[3]{y})^2 - (y^3)^2 \, dy$

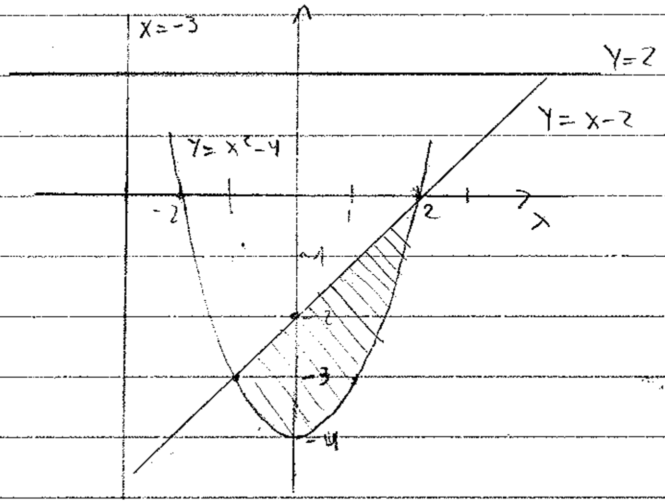
$V = 2\pi \int_0^1 y^{2/3} - y^6 \, dy$

$V = 2\pi \left(\frac{3}{5} y^{5/3} - \frac{y^7}{7} \right) \Big|_0^1$

$V = 2\pi \left(\frac{3}{5} - \frac{1}{7} \right) = 2\pi \left(\frac{21-5}{35} \right) = \frac{32}{35} \pi$ u.v

c) $V = \pi \int_{-1}^{0} (1-\sqrt[3]{x})^2 - (1-x^3)^2 dx + \pi \int_0^1 (1-x^3)^2 - (1-\sqrt[3]{x})^2 \, dx$

(65) $y = x^2 - 4$ e $y = x - 2$



$x^2 - 4 = x - 2$

$x^2 - x - 2 = 0$

$-1 + 2 = 1$

$-1 \cdot 2 = -2$

a) rot eixo x

$V = \pi \int_{-1}^{2} (x^2-4)^2 - (x-2)^2 \, dx$

$V = \pi \int_{-1}^{2} x^4 - 8x^2 + 16 - x^2 + 4x - 4 \, dx$

$$V=\pi\int_{-1}^{2} x^4-9x^2+4x+12\,dx$$

b) rot $y=2$

$$V=\pi\int_{-1}^{2} (2-(x^2-4))^2-(2-(x-2))^2\,dx$$

$$V=\pi\int_{-1}^{2} (2-x^2+4)^2-(2-x+2)^2\,dx$$

$$V=\pi\int_{-1}^{2} (6-x^2)^2-(4-x)^2\,dx$$

$$V=\pi\int_{-1}^{2} 36-12x^2+x^4-16+8x-x^2\,dx$$

$$V=\pi\int_{-1}^{2} x^4-13x^2+8x+20\,dx$$

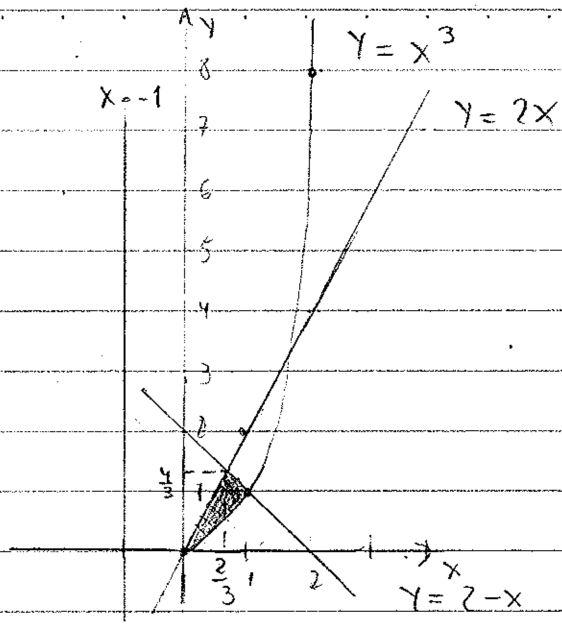
c) rot $x=-3$

$$V=\pi\int_{-4}^{-3} (\sqrt{y+4}+3)^2-(3-\sqrt{y+4})^2\,dy+\pi\int_{-3}^{0}(\sqrt{y+4}+3)^2-(y+2+3)^2\,dy$$

$$V=\pi\int_{-4}^{-3} \cancel{y}+\cancel{4}+6\sqrt{y+4}+\cancel{9}-(\cancel{9}-6\sqrt{y+4}+\cancel{y}+\cancel{4})\,dy+\pi\int_{-3}^{0} y+4+6\sqrt{y+4}+9-(y^2+10y+25)\,dy$$

$$V=\pi\int_{-4}^{-3} 12\sqrt{y+4}\,dy+\pi\int_{-3}^{0} -y^2-9y-12+6\sqrt{y+4}\,dy$$

(66)



$2x = 2 - x$

$3x = 2$

$x = \frac{2}{3}$

a) rot eixo x

$$V = \pi\int_0^{2/3} (2x)^2 - (x^3)^2 \, dx + \pi\int_{2/3}^{1} (2-x)^2 - (x^3)^2 \, dx$$

$$V = \pi\int_0^{2/3} 4x^2 - x^6 \, dx + \pi\int_{2/3}^{1} 4 - 4x + x^2 - x^6 \, dx$$

$$V = \pi\left(\frac{4x^3}{3} - \frac{x^7}{7}\right)\Big|_0^{2/3} + \pi\left(4x - 2x^2 + \frac{x^3}{3} - \frac{x^7}{7}\right)\Big|_{2/3}^{1}$$

$$V = \frac{32\pi}{81} - \cancel{\frac{2^7}{7 \cdot 3^7}\pi} + 4\pi - 2\pi + \frac{\pi}{3} - \frac{\pi}{7} - \frac{8}{3}\pi + \frac{8}{9}\pi - \frac{8}{81}\pi + \cancel{\frac{2^7}{7 \cdot 3^7}\pi}$$

$$V = \frac{24\pi}{81} - \frac{7}{3}\pi + \frac{8}{9}\pi - \frac{\pi}{7} + 2\pi$$

$$V = \frac{8}{27}\pi - \frac{13\pi}{9} + \frac{13}{7}\pi$$

$$V = \frac{56\pi - 273\pi + 351\pi}{189}$$

$$V = \frac{134}{189}\pi \text{ u.v}$$

$$V = \pi \int_{-1}^{2} x^4 - 9x^2 + 4x + 12 \, dx$$

b) rot $y = 2$

$$V = \pi \int_{-1}^{2} (2 - (x^2 - 4))^2 - (2 - (x-2))^2 \, dx$$

$$V = \pi \int_{-1}^{2} (2 - x^2 + 4)^2 - (2 - x + 2)^2 \, dx$$

$$V = \pi \int_{-1}^{2} (6 - x^2)^2 - (4 - x)^2 \, dx$$

$$V = \pi \int_{-1}^{2} 36 - 12x^2 + x^4 - 16 + 8x - x^2 \, dx$$

$$V = \pi \int_{-1}^{2} x^4 - 13x^2 + 8x + 20 \, dx$$

c) rot $x = -3$

$$V = \pi \int_{-4}^{-3} (\sqrt{y+4} + 3)^2 - (3 - \sqrt{y+4})^2 \, dy + \pi \int_{-3}^{0} (\sqrt{y+4} + 3)^2 - (y+2+3)^2 \, dy$$

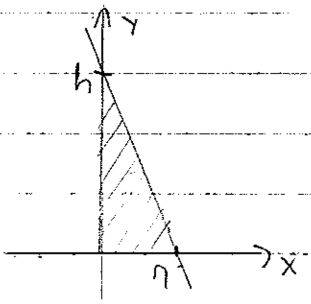
$$V = \pi \int_{-4}^{-3} \cancel{y} + \cancel{4} + 6\sqrt{y+4} + \cancel{9} - (\cancel{9} - 6\sqrt{y+4} + \cancel{y} + \cancel{4}) \, dy + \pi \int_{-3}^{0} y + 4 + 6\sqrt{y+4} + 9 - (y^2 + 10y + \text{[illegible]}$$

$$V = \pi \int_{-4}^{-3} 12\sqrt{y+4} \, dy + \pi \int_{-3}^{0} -y^2 - 9y - 12 + 6\sqrt{y+4} \, dy$$

b) rot $x = -1$

$$V = \pi \int_0^1 (\sqrt[3]{y} + 1)^2 - \left(\frac{y}{2} + 1\right)^2 dy + \pi \int_1^{4/3} (2 - y + 1)^2 - \left(\frac{y}{2} + 1\right)^2 dy$$

(67) $V = \dfrac{\pi r^2 h}{2}$



$$y = h - \frac{h}{r}x$$

$$ry = rh - hx$$

$$x = r - \frac{r}{h}y$$

$$V = \pi \int_0^h \left(r - \frac{r}{h}y\right)^2 dy$$

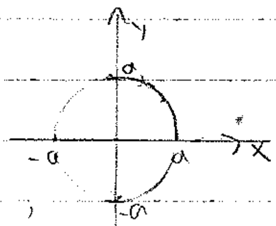
$$V = \pi \int_0^h r^2 - 2\frac{r^2}{h}y + \frac{r^2}{h^2}y^2 \, dy$$

$$V = \pi \left(r^2 y - \frac{r^2}{h}y^2 + \frac{r^2}{3h^2}y^3\right)\Bigg|_0^h$$

$$V = \pi \left(\cancel{r^2 h} - \cancel{\frac{r^2}{h}h^2} + \frac{r^2}{3\cancel{h^2}}h^3\right)$$

$$V = \frac{\pi r^2 h}{3} \text{ u.v}$$

(68) $V = \dfrac{4}{3}\pi a^3$



$$x^2 + y^2 = a$$

$$y^2 = a^2 - x^2$$

$$V = \pi \int_{-a}^{a} a^2 - x^2 \, dx$$

$$V = \pi \left(a^2 x - \frac{x^3}{3}\right)\Bigg|_{-a}^{a}$$

$$V = \pi\left(a^3 - \frac{a^3}{3}\right) - \pi\left(-a^3 + \frac{a^3}{3}\right)$$

$$V = a^3\pi - \frac{a^3}{3}\pi + a^3\pi - \frac{a^3}{3}\pi$$

$$V = \frac{4}{3}\pi a^3 \text{ u.v}$$