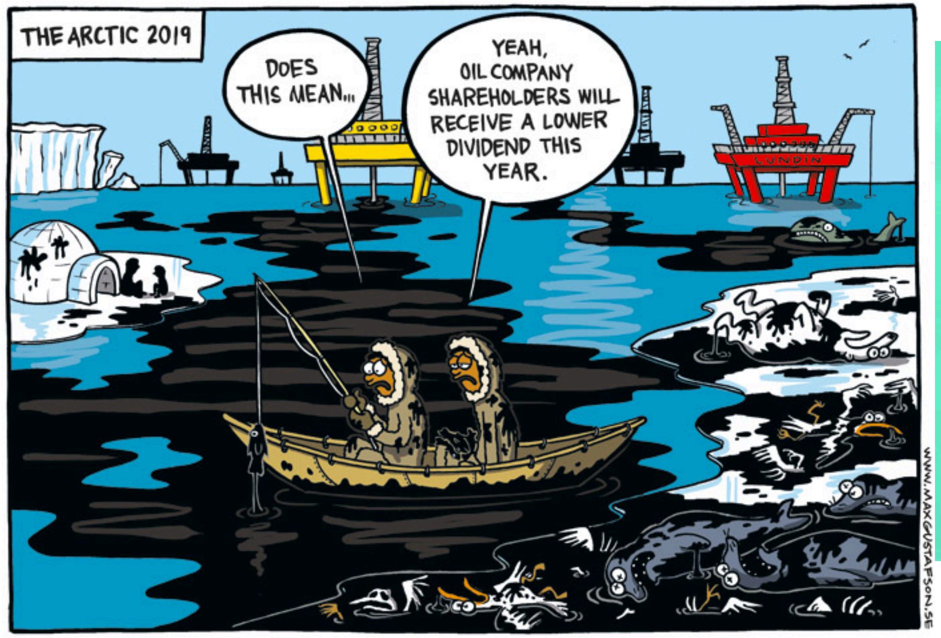
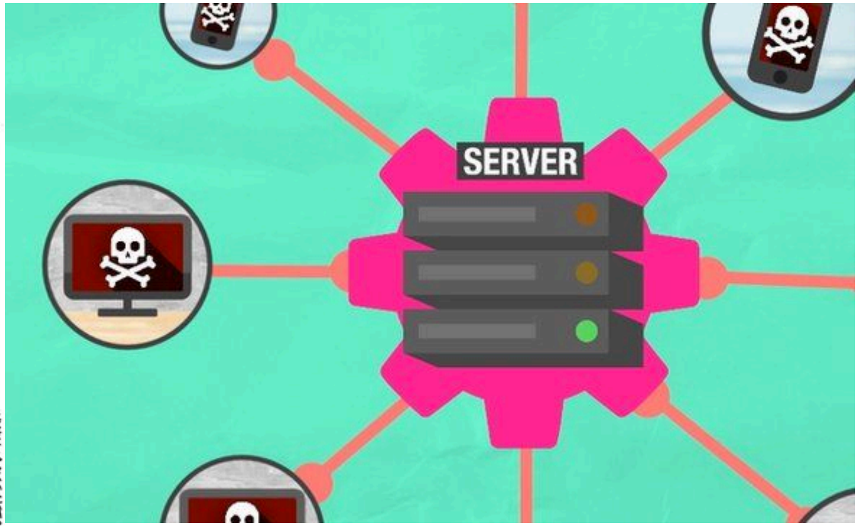
Selling a Single Item with Negative Externalities

To Regulate Production or Payments?

Tithi Chattopadhyay, Nick Feamster, **Matheus V. X. Ferreira**, Danny Yuxing Huang, S. Matthew Weinberg

Markets and their negative externalities



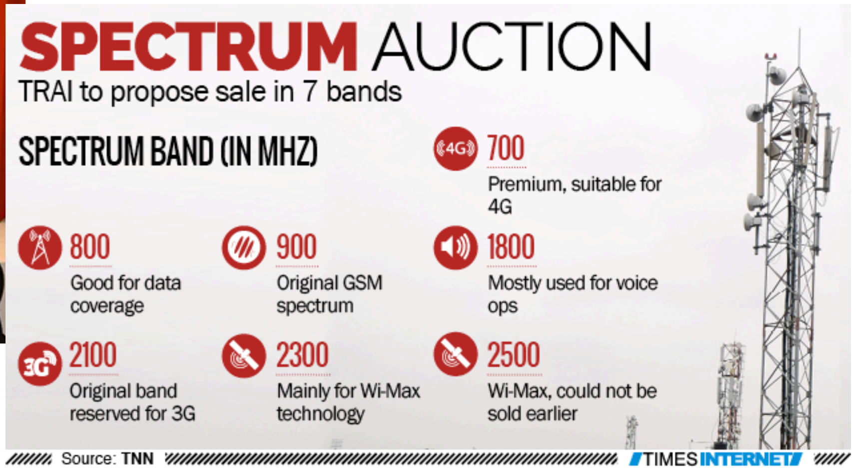


- Certain markets cause externalities to a third-party.
- There is an asymmetry on how effective the seller or the buyer is in mitigating externalities:
  - Car manufacturers should increase efficiency in reducing greenhouse gas emission.
  - Buyers of computer devices must use secure passwords to avoid devices being compromised and attacking other services in the internet.
- Motivating Questions: Over all possible regulations, which one minimizes negative externalities subject to minimum impact on seller’s profit?

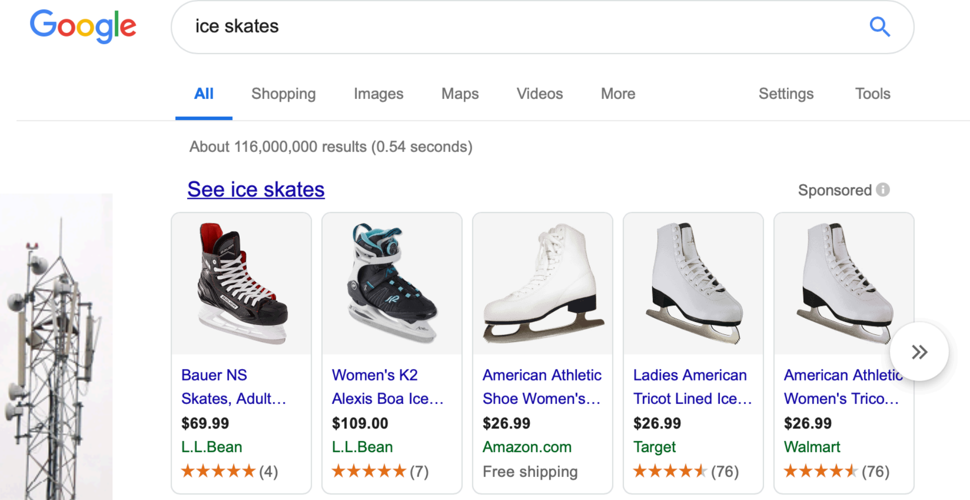
Auctions

Selling Items

Spectrum Auctions



Sponsored Search



Designing an Optimal Auction?

[Myerson’81]: Characterize the optimal price and allocation to maximize revenue in a single-item auction.

[Vickrey ’61, Clarke’ 71, Groves’73]: VCG Mechanism - Truthful and welfare maximizing mechanisms.

Externalities inside the Market

Positive Externalities

- Social Networks: the value for a phone increase if friends purchase the same phone Haghpanah, et al. (2013); Hartline, et al. (2008); Mirrokni, et al. (2012); Candogan, et al. (2012).

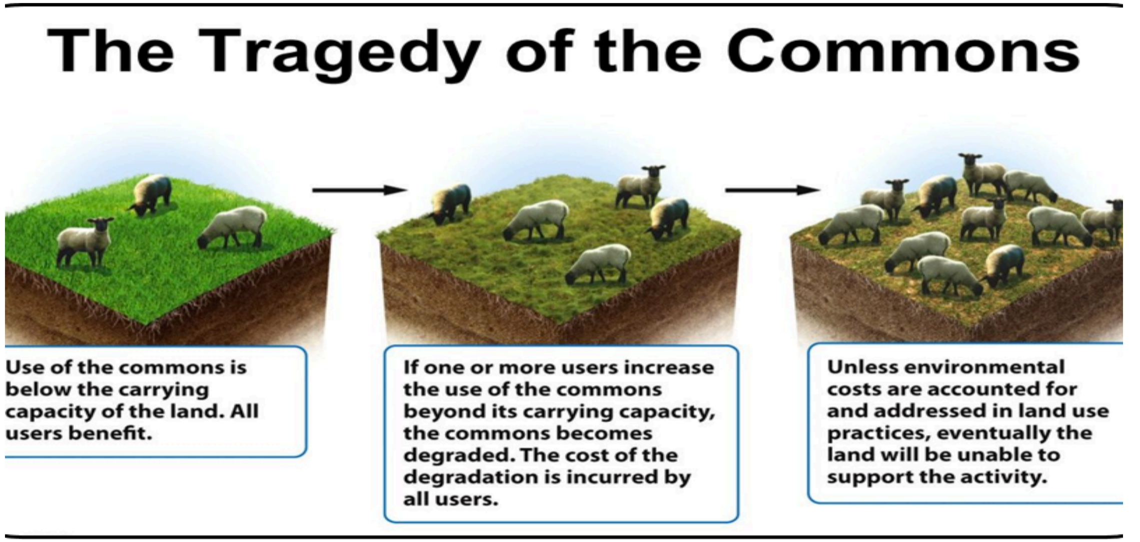
Negative Externalities

- Advertising Auctions: Bhattacharya, et al. (2011).
- Selling Innovations: Nuclear Weapons Jehiel, et al. (1996).

Tragedy of the Commons

- The externalities of a market affect common goods (e.g. clean air, security, wireless spectrum access)
- Common approach is to regulate the market that is causing externalities
- Large body of work: [Seabright'93], [Lehr and Crowcroft'05]; [Montero'08]; [Feldman et al.'13]; [Martimort and Sand-Zantman'16]

[Weitzman'74]: Regulate Prices (Taxes, Fines) or Quantities (Minimum Standard) and maximize BENEFITS – COST

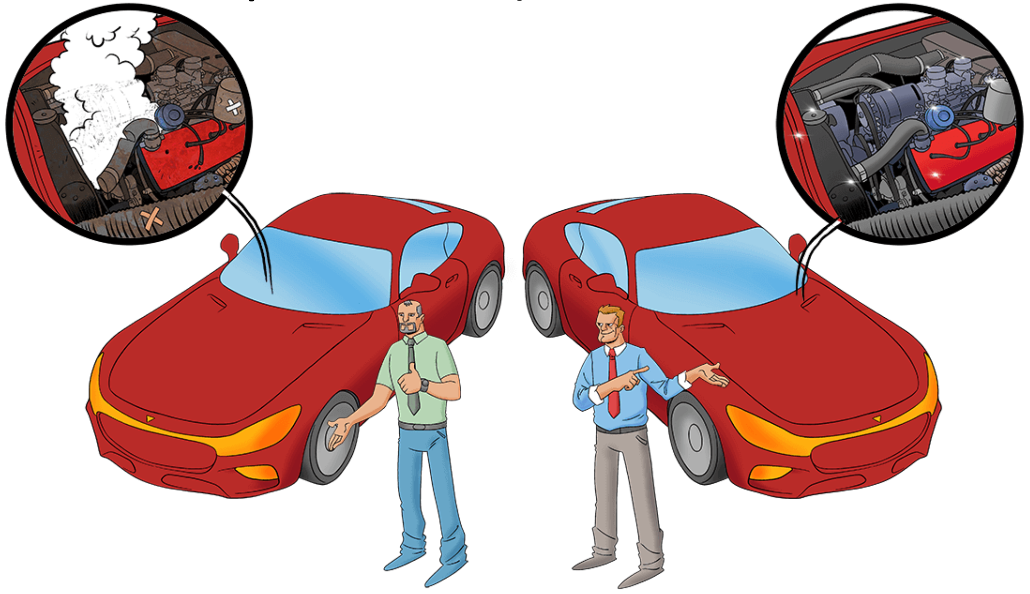


The Tragedy of the Commons in Information Security

- Software or Hardware provider sell insecure products.
- Consumers do not consider or take actions towards reducing security risks due to asymmetric information
  - Users cannot distinguish secure from insecure devices

[Akerlof'70]: Asymmetric information results in a bad equilibrium ("**The market for lemons**")

- Vulnerabilities result in harm to third-party services, loss of reputation to sellers and compromise security and privacy of users.



Regulating Prices vs Production

Production: The regulator requires minimum security from seller.

Examples: Healthcare organizations, financial institutions and federal agencies are required to protect their systems and information.

" WHEN IT COMES DOWN TO IT, JIM, SECURITY IS A PERSONAL RESPONSIBILITY."

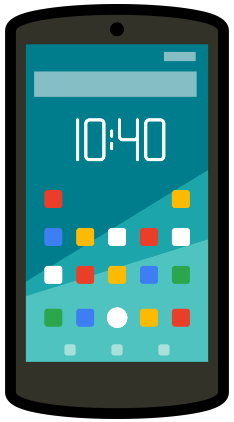
Prices: The regulator penalize users when items cause externalities.

Examples: Taxes or fines where users are made liable for software or devices that are compromised [Kunreuther and Heal`03]. Vendors offer discounts to users who adopt good security practices [August et al.'16].

Distribution of Buyers

- We consider regulating externalities in a market for single item and single buyer.
- Buyer has two parameters (value and efficiency) drawn independently:

$$(v, k) \leftarrow D_v \times D_k$$



Prices and Production regulation are no interchangeable and how effective one is in comparison with the other depends on how efficient the buyer is in decreasing externalities.

Efficiency (k): How efficient a buyer is in securing the device where **k >> 1** is very efficient and **k < 1** is inefficient

- Some buyers are more efficient than others.
- Depends on the application domain (IoT devices, technology, Oil licensing, etc…).

Value (v): How much the buyer values the item when there is no externalities and no regulation.

Externality

- Seller has an investment **c** towards security.
- Buyer spends effort **h** securing the item.
- Externalities are modeled as the probability of a device being compromised.
- Externalities are model as a exponentially decreasing quantity because reducing risk becomes harder as the investments increase.

Buyer's Efficiency

Seller's Effort

Buyer's Effort

$$Effort(k, c, h) := c + kh$$

Probability of an Externality

$$Risk(k, c, h) := e^{-Effort(k,c,h)}$$

Regulation

- A regulation is a combination of minimum security investments for the seller and fines to the buyer.
- Regulating Production: mandatory security investment **c** (equivalent to seller's cost).
- Regulating Prices: buyer receives a fine **y** if device is compromised.
    - Can be indirect economic incentives: insurance, discounts.
- Seller's price **p.**
- A regulation is simple if either **c = 0** (fine policy) or **y = 0** (cost policy).
- [Weitzman'74]: Study which simple policy is optimal for welfare maximization.

Regulation: $s := (y, c, p)$

How regulation affects buyer's utility

- The fine **y** induces the buyer to adopt security practices.

Buyer's Penalty

Buyer's Effort

Price

Buyer's Utility $$u(t,s) := v - yRisk(k,c,h) - h - p$$

Optimal Buyer's Effort $$h^*(t,s) := \max\left(0, \frac{\ln(yk) - c}{k}\right)$$

Buyer's Externality under Optimal Effort $$risk(t,s) := \min\left(e^{-c}, \frac{1}{yk}\right)$$

- We model externalities as the expected risk conditioned on purchases.

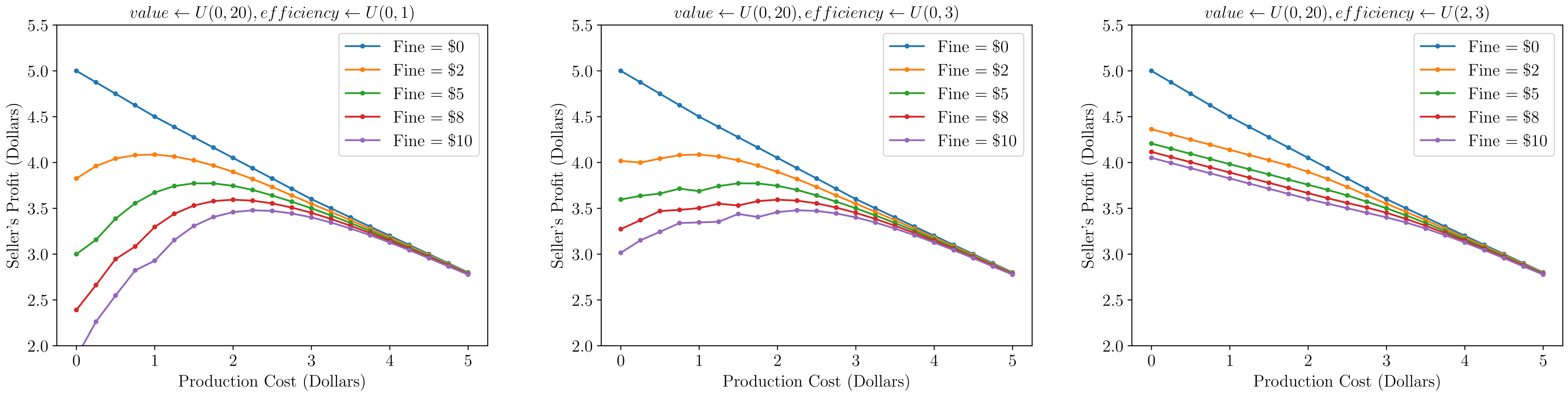
$$Ext_D(s) := \frac{\mathbb{E}_{t\leftarrow D}[risk(t,s)\cdot \mathbb{I}(u(t,s)\geq 0)]}{Pr_{t\leftarrow D}[u(t,s)\geq 0]}$$

How regulation affects seller's profit

- Regulating prices **y**, decreases buyer's utility which decrease their probability of purchasing the item.
- Regulating production **c**, decreases seller's net profit per sale

$$Profit_D(s) := (p - c)Pr_{t \leftarrow D}[u(t, s) \geq 0]$$

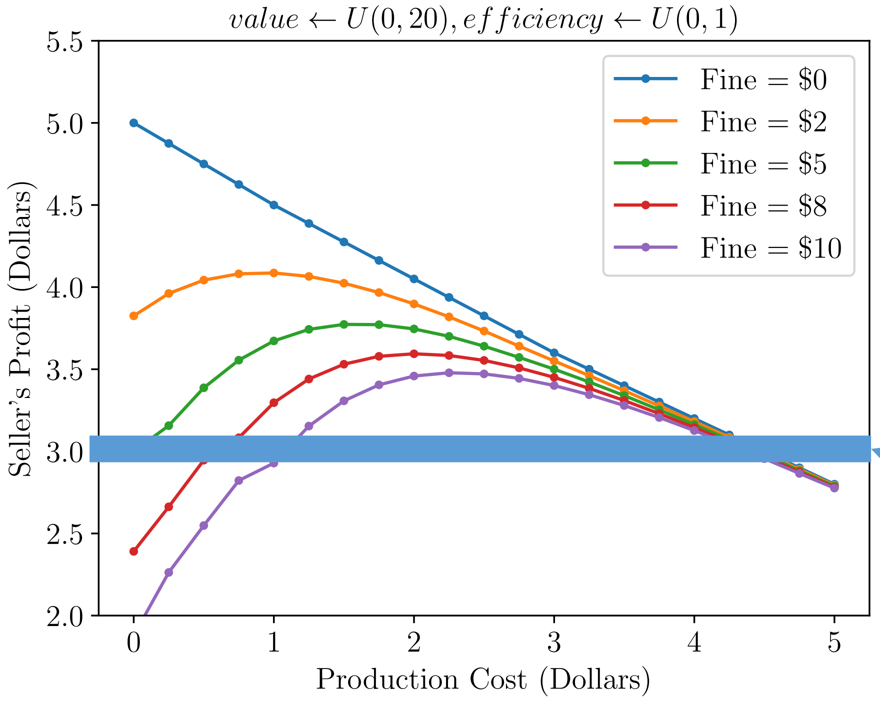
- Optimal profit for different combinations of regulation:



Defining the Optimal Regulation

- Any regulation directly or indirectly affects buyer's profit. To minimize the impact on seller, we constrain on regulations that provide minimum profit **R**.

$$Profit_D(s) := (p-c)Pr_{t\leftarrow D}[u(t,s)\geq 0]$$



$$Ext_D(s) := \frac{\mathbb{E}_{t\leftarrow D}[risk(t,s)\cdot \mathbb{I}(u(t,s)\geq 0)]}{Pr_{t\leftarrow D}[u(t,s)\geq 0]}$$

Def (Optimal Regulation):

$$OPT(D,R) := \arg\min_{s|Profit_D(s)\geq R} Ext_D(s)$$

Minimum Profit of \$3

Example (Regulating Production is Optimal)

- Assume any value distribution **D**, efficiency distribution with all density in 0 and some regulation **s = (y, c, p)**

Buyer is Inefficient in Reducing Externalities

$$D_k = \{0\}$$

$$h^*(t,s) = \max\left(0, \frac{\ln(yk) - c}{k}\right)$$

$$risk(t,s) = \max\left(e^{-c}, \frac{1}{yk}\right)$$

$$u(t,s) = v - ye^{-c} - p$$

$$Ext_D(s) = e^{-c}$$

It is in buyer's best interest to never put in effort to reduce externalities since any effort **h > 0** has no impact on externalities

$$\Longrightarrow$$

Externality only depends on the seller's effort

$$\Longrightarrow$$

Any price regulation **y > 0** can only hurt buyer's utility and has no impact on externality

$$\Longrightarrow$$

The externality of the population only depends on seller's investment

- **Conclusion:** Since any fine **y > 0** can only hurt revenue and has no impact on externalities. The optimal regulation only regulates production.

When Regulating Only Production is Optimal?

- ***Theorem**: For all value distributions **V**, and profit constraint **R**, there is a cutoff **T = T(V, R)** such that for all efficiency distribution **K** supported on **[0, T(V, R)]**, regulating quantities is optimal.*

Let **s** be the optimal regulation on a point mass on **T** where **s** regulates only quantities.

$$u(t, s) = v - ye^{-c} - p$$

Suppose for contradiction **s'** is a better regulation on **K**, then **s'** is a better regulation on **{T}**

$$\begin{aligned} Ext_{V\times\{T\}}(s') &\leq Ext_{V\times K}(s') \\ &< Ext_{V\times K}(s) \\ &= Ext_{V\times\{T\}}(s) \end{aligned}$$

Increasing efficiency can only decrease externality

Because **s** regulates only quantities, externalities do not depend on the distribution

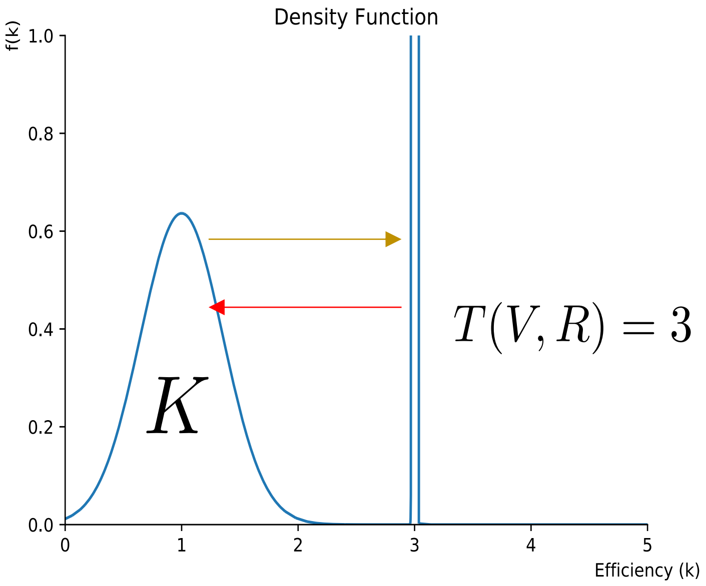
Since **y = 0**, the buyer's utility does not depen on the efficiency distribution.

$$\Longrightarrow$$

Since **s** is feasible (and optimal) on a point mass on **T**, then **s** is feasible on **K**.

$$\Longrightarrow$$

s must be optimal on **K**



What about distributions supported above the cutoff?

***Def (Profit-Maximizing)**: A regulation **s = (y, c, p)** is profit-maximizing if the price is a best-response to the seller.*

- There is an instance **(V x K, R)** where the optimal policy is not simple even though **K** is supported on **[T(V, R), ∞)**. Such policies are not profit maximizing.

$$V = \begin{cases} 1 & \text{w.p. } 1/2 \\ 16/15 & \text{w.p. } 1/2 \end{cases} \qquad \begin{matrix} \mathcal{K} = \{3\} \\ R = 1/2 \end{matrix}$$

- If **K** is a point mass and **K = 3** the optimal regulation regulates only production and for **K > 3** the optimal regulation regulates only prices.

$$\mathcal{K} = \begin{cases} 3 & \text{w.p. } 1/2 \\ x \to \infty & \text{w.p. } 1/2 \end{cases}$$

What about distributions supported above the cutoff?

Consider the following uniform distribution over buyers:

(v, k): (1, 3) (16/15, 3) (1, x) (16/15, x)

Efficient Buyers

Profit constraint R = 1/2

Externalities are affected by the population that purchase. The optimal regulation can reduce externalities by forcing inefficient buyers out of the market

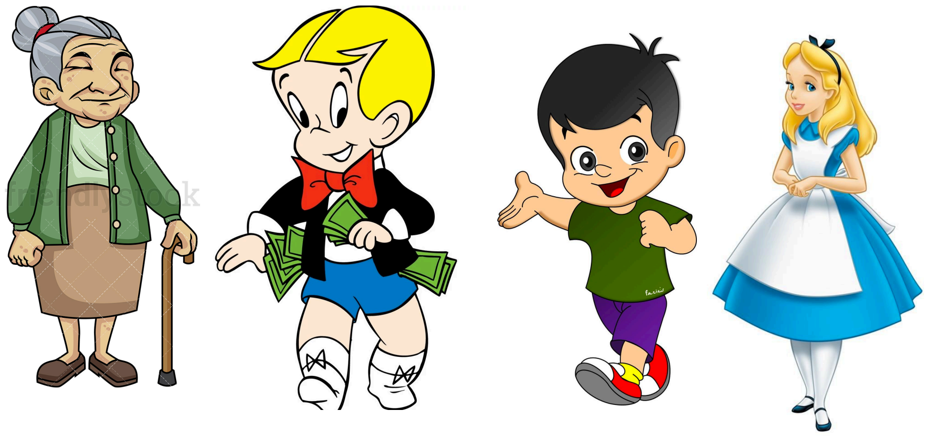
Selling only (1, x) and (16/15, x) is not feasible (Price < 1 and sell with probability ½)

Selling to everyone implies the optimal regulation is the same regulation as if everyone had efficiency 3

Selling to everyone but (1, 3), price must be at least 2/3 (Prob. of sale is ¾). This limits how big fines can be; which implies the optimal regulation is mixed between fines and quantities.

When Regulating Only Prices is Optimal?

- The optimal regulation charges price **1 - ε** to not sell to (1, 3) and is a mix between fines and cost.
- **Remark**: such regulations is not profit-maximizing. Even though the regulation achieves the minimum profit of ½, a seller willing to maximize profit will sell at a lower price 14/15 so that (1, 3) purchase giving higher profit.



(v, k): (1, 3) (16/15, 3) (1, x) (16/15, x)

***Theorem**: For all value distributions **V** and profit constraint **R**, there is a cutoff **T = T(V, R)** such that for all efficiency distributions **K** supported on **[T, ∞]**, the optimal regulation regulates only prices or it is not profit-maximizing.*

Can simple regulations approximate mixed regulations?

- Recap: a simple regulation regulates **only prices** or **only production**.
- Hope for an approximation: externalities increase by at most a constant factor.
- Answer:
  - **No**, if we are not willing to relax the profit constraint **R**.
  - **Yes**, if we are willing to relax the profit constraint **Ω(R)**.

$$Ext_D(s') \leq 40/3 Ext_D(s)$$

Simple regulation

Some arbitrary regulation

$$Prof_D(s') \geq \frac{1}{8} Prof_D(s)$$

Constructing a bad distribution

- For sufficiently large **x > 0**, we construct a distribution where any simple policy will increase externalities by at least **x** times the externalities of the optimal mixed regulation.
- We construct an instance where all feasible regulation must sell to everyone.

$$V = 2e^{x/2}(x + e^{-x}) \quad \mathcal{K} = \begin{cases} 0 & \text{w.p. } e^{-x/2} \\ e^{xe^{x/2}} & \text{w.p. } 1 - e^{-x/2} \end{cases}$$

$$R = V - e^{-x} - x$$

$$v(1 - e^{-x/2}) = 2e^{x/2}(x + e^{-x})(1 - e^{-x/2})$$

Maximum revenue by not selling to everyone

$$= (x + e^{-x})(2e^{x/2} - 2)$$
$$< (x + e^{-x})(2e^{x/2} - 1)$$
$$= R$$

- Consider policy $(\boldsymbol{y} = \mathbf{1}, \boldsymbol{c} = \boldsymbol{x})$ and observe the seller maximize profit by selling at price **p = R - c**

$$u(v, 0, s) = (v - e^{x}) - (R - c) = 0$$

Everyone purchase and the profit is **R**

Externality is at most $\boldsymbol{e^{-\frac{x}{2}} e^{-x} + e^{-xe^{x/2}}}$

Constructing a bad distribution

- The highest cost for a cost policy is $\boldsymbol{x} + \boldsymbol{e^{-x}}$ (externality at least $\boldsymbol{e^{-x-1}}$)

Optimal price is equals to the only possible value since buyer's utility does not depend on their efficiency when **y = 0**

$$p = v$$

$$Rev_D(s) = v - c = v - (x - e^{-x}) = R$$

$$Ext_D(s) = e^{-c} = e^{-x+e^{-x}}$$

- The highest fine for a fine policy is $\boldsymbol{x} + \boldsymbol{e^{-x}}$ (externality is at least $\boldsymbol{e^{-x/2}}$)

Since we must sell to everyone, we must sell to the buyer with efficiency **0**

$$u(v, 0, s) = v - y - p$$

$$p = v - y$$

$$Rev_D(s) = v - y = v - (x - e^{-x}) = R$$

$$Ext_D(s) = e^0 e^{-x/2} + \frac{1}{ye^{xe^{x/2}}}(1 - e^{-x/2})$$

$$> e^{-x/2}$$

- However, the mixed regulation with **y = 1**, **c = x** has externalities at least **x** times smaller

$$Ext_D(y = 1, c = x) < e^{-x/2}e^{-x} + e^{-xe^{x/2}}$$

Relaxing the profit constraints towards an approximation

- The previous example shows that if the profit constraint cannot be violated we cannot hope for any approximation with simple regulations.

$$R = (x + e^{-x})(2e^{x/2} - 1)$$

Profit when only Alice purchase:

$$\begin{aligned} 2e^{x/2}(x+e^{-x})(1-e^{-x/2}) &= (x+e^{-x})(2e^{x/2}-2) \\ &< (x+e^{-x})(2e^{x/2}-1) \\ &= R \end{aligned}$$

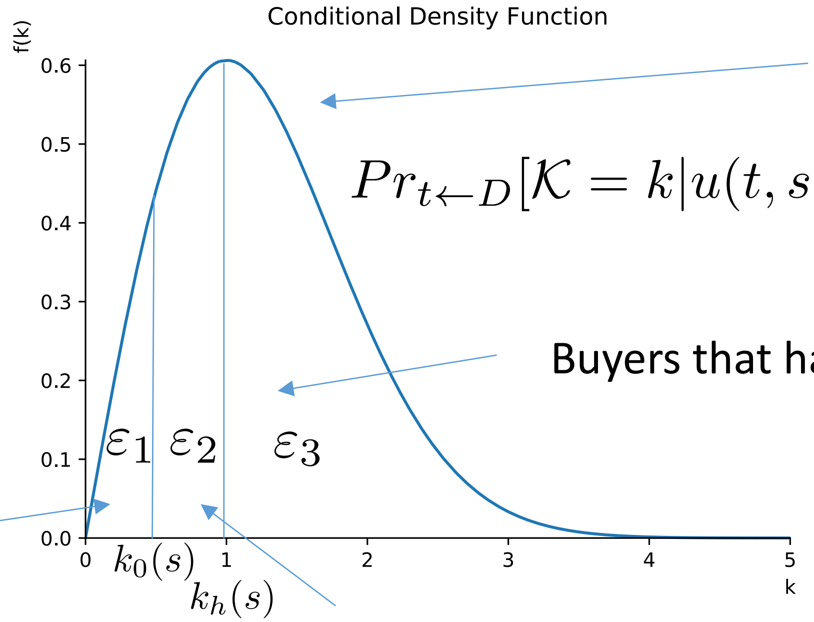
$$(2e^{x/2}(x+e^{-x}), 0) \qquad (2e^{x/2}(x+e^{-x}), e^{xe^{x/2}})$$

$$p_1 = e^{-x/2} \qquad p_2 = 1 - e^{-x/2}$$

The inefficient buyer has a negligible contribution to profit. Losing a small fraction of the profit by not selling to the inefficient buyer greatly improve externalities of regulating only prices

Bicriteria Approximation: Regulating the population that purchase

- Given some arbitrary regulation **s**, we will construct a simple regulation **s'** targeting the population that already purchase the item under regulation **s**. This new regulation can potentially decrease profit; however, we can guarantee that we still get a constant fraction of the old profit.



Efficiency distribution of buyers that purchase

$$Pr_{t \leftarrow D}[\mathcal{K} = k | u(t,s) \geq 0]$$

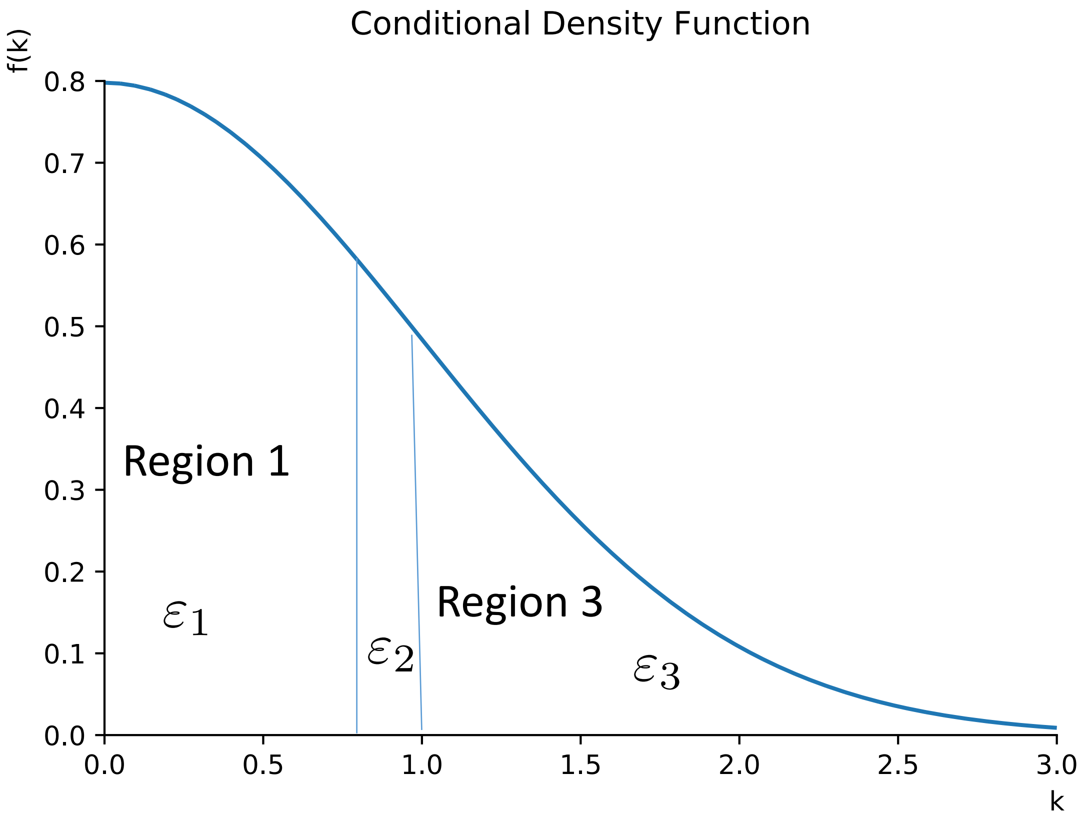
Buyers that have efficiency **k > 1** and put in effort **h > 0**

Buyers that put in no effort **h = 0**

Buyers that have efficiency **k ≦ 1** but put in effort **h > 0**

Simple Regulations in Region 1

- If a large fraction of the population does not put in effort then removing fines will have small effect on externalities.



$$Cost^1(s) := (0, c, p)$$

$$Ext_D(Cost^1(s)) = e^{-c}$$

$$\begin{aligned} Ext_D(s) &= E_{t \leftarrow D}[risk(t,s) \mid u(t,s) \geq 0] \\ &\geq Pr[k \leq k_0(s) \mid u(t,s) \geq 0] E_{t \leftarrow D}[risk(t,s) \mid u(t,s) \geq 0, k \leq k_0(s)] \\ &= \varepsilon_1 e^{-c} \end{aligned}$$

Simple Regulations in Region 2

- The externalities on region 2 are at least $risk(t,s) = \frac{1}{yk} \geq \frac{1}{y}$
- However, if y >> c, the same strategy of region 1 would fail.

$$Cost^2(s) := (0, c + \ell(\sigma, s), p + \ell(\sigma, s))$$

$$Pr_{t \leftarrow D}[u(t, Cost^2(s)) \geq 0] = (\varepsilon_1 + \varepsilon_2) Pr_{t \leftarrow D}[u(t,s) \geq 0]$$

- We claim $c + \ell(\sigma, s) \leq c + \ell(1, s) = c + 1 + \ln y - c = 1 + \ln y$ then:

$$Ext_D(Cost^2(s)) = e^{-c-\ell(\sigma,s)} \leq e^{-1-\ln y} \leq \frac{1}{y}$$

$$Ext_D(s) \geq Pr_{t \leftarrow D}[k \in \text{Region } 2] \cdot E_{t \leftarrow D}[r(t,s) | k \in \text{Region } 2]$$

$$\geq \frac{\varepsilon_2}{y}$$

Simple Regulations in Region 2

- If $\sigma > 1$, we reach a contradiction because the probability of sale would be too big

$$
\begin{aligned}
(\varepsilon_1 + \varepsilon_2) Pr_{t \leftarrow D}[v \geq p + \ell(k,s)] &= Pr_{t \leftarrow D}[v \geq p + \ell(\sigma,s)] \\
&> Pr_{t \leftarrow D}[v \geq p + \ell(1,s)] \\
&\geq Pr_{t \leftarrow D}[v \geq p + \ell(k,s) | k \leq 1] \\
&= \frac{Pr_{t \leftarrow D}[k \leq 1 | u(t,s) \geq 0] Pr_{t \leftarrow D}[u(t,s) \geq 0]}{Pr_{t \leftarrow D}[k \leq 1]} \\
&\geq Pr_{t \leftarrow D}[k \leq 1 | u(t,s) \geq 0] Pr_{t \leftarrow D}[u(t,s) \geq 0] \\
&= (\varepsilon_1 + \varepsilon_2) Pr_{t \leftarrow D}[v \geq p + \ell(k,s)] \Rightarrow\!\Leftarrow
\end{aligned}
$$

Simple Regulations in Region 3

- In this case, simple regulations can either regulate only production or only prices.
- In the first step, we blowup fines preserving constant fraction of the sales:

$$Blowup(s) := (ye^{\sigma}, 0, p - c)$$

- The goal is to decrease utility of inefficient buyers and reduce externalities
- Bad cases:
  - If **K** has small expected value, then we are unable to induce a large blowup.
  - If **V** is heavy tail we cannot drive inefficient buyers out of the market.
- In the bad cases, we can derive cost policies that have good externality guarantees.

Example (The Optimal regulation depends on the value distribution)

- Assume the buyer has value drawn from the equal revenue distribution which is heavy-tail and fix some regulation **s = (y = 0, c, p)**

Equal Revenue

$$Pr[v \geq x] = \begin{cases} 1, \ x \in [0,1) \\ 1 - \frac{1}{x}, \ x \geq 1 \end{cases}$$

$$\begin{aligned} Profit_D(s) &= (p-c)Pr[v \geq p] \\ &= (p-c)(1-(1-1/p)) \\ &= 1 - c/p \end{aligned}$$

$$Ext_D(s) = e^{-c}$$

For every profit constraint **R < 1**, for every externality target **E** there is a regulation (**y = 0**, **c > 0**) that gets revenue **> R** and externality **< E**.

- In this example, the optimal regulation does not converge to a finite value because the buyer's value has infinite expected which can be converted to reduce externality.

Open Questions

- Can the approximation rations be improved?
  - It is not clear how to get better than ½ profit approximation even when we are ok in having higher externalities.
- Considering competing sellers and multiple items.

Questions?