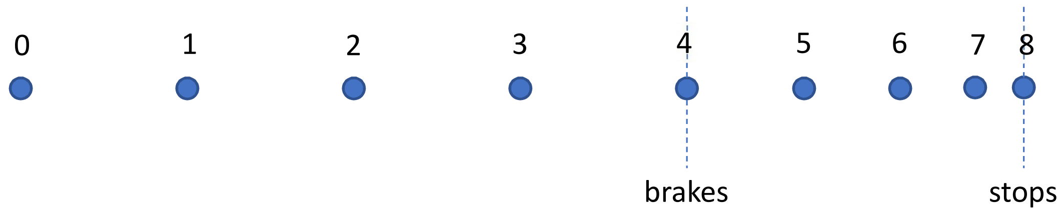
SLE123 Physics for the Life Sciences

1.2 Velocity: Rate of Change of Position – ANSWERS

1. The dots are equally spaced until the brakes are applied to the car. Equidistant dots indicate constant average speed. On braking, the dots get closer as the average speed decreases.



2. Both speed and velocity are ratios with a time interval in the denominator, but speed is a scalar because it is the ratio of the scalar distance over the time interval while velocity is a vector because it is the ratio of the vector displacement over the time interval. Speed and velocity have the same SI units, but one must specify the direction when giving a velocity. An example of speed would be that your hair grows (the end of a strand of hair moves relative to your scalp) at a speed of about 20 mm/month. An example of velocity (where direction matters) would be when you spring off a diving board. Your velocity could initially be 2.0 m/s up, while later it could be 2.0 m/s down.

3. Because the dots are getting farther apart to the right (and the numbers are increasing to the right) we know that the object is speeding up. The choice that best fits that is a car pulling away (to the right) from a stop sign. So the correct choice is C. An ice skater gliding (choice A) would likely have nearly constant velocity (constant spacing between dots). The motion diagram for a plane braking (choice B) might look like the given diagram with the dots numbered in reverse order. The pool ball reversing direction (choice D) would have dot numbers increasing in one direction at first but then going the other way.

4. Because of the numbering of the dots, we see the object is moving to the left. It is slowing because the dots are getting closer together. The choice that fits this scenario is a cyclist moving to the left and braking to a stop. So choice B is correct. If the dots were numbered in reverse order then choice C would be correct.

5. The speed is the distance divided by the time.

$$speed = \frac{distance}{time} = \frac{0.3\ km}{5.0\ min} = \frac{300m}{300s} = 1.0\, m/s$$

6. The speed is the distance divided by the time.

$$speed = \frac{distance}{time} = \frac{110m}{240s} = 0.46\, m/s$$

7. We are asked to rank in order three different speeds, so we simply compute each one.

$$speed = \frac{distance}{time}$$

Toy car $$\frac{0.15m}{2.5s} = 0.060\, m/s$$

Ball $$\frac{2.3m}{0.55s} = 4.2\, m/s$$

Bicycle $$\frac{0.60m}{0.075s} = 8.0\, m/s$$

Cat $$\frac{8.0m}{2.0s} = 4.0\, m/s$$

So the order from fastest to slowest is bicycle, ball, cat, and toy car.

8. Since the dots are spaced at equal intervals of time, and there is one dot between the time 10 s and 30 s, the spacing between the dots indicate a 10 s time interval. The dot between 10 s and 30 s will mark a time of 20 s. The horse is moving to the left, as time increases to the left, so the rightmost dot must be at 0 s. We will use the definition of velocity:

$$v = \frac{\Delta x}{\Delta t}$$

a. $x_f = 500$ m, $x_i = 600$ m, $t_f = 10$ s, $t_i = 0$ s

$$v = \frac{\Delta x}{\Delta t} = \frac{500m - 600m}{10s - 0s} = \frac{-100m}{10s} = -10\, m/s$$

b. $x_f = 300$ m, $x_i = 350$ m, $t_f = 40$ s, $t_i = 30$ s

$$v = \frac{\Delta x}{\Delta t} = \frac{300m - 350m}{40s - 30s} = \frac{-50m}{10s} = -5\, m/s$$

c. $x_f = 50$ m, $x_i = 250$ m, $t_f = 70$ s, $t_i = 50$ s

$$v = \frac{\Delta x}{\Delta t} = \frac{50m - 250m}{70s - 50s} = \frac{-200m}{20s} = -10\, m/s$$

9. Average velocity is defined as the displacement x Δ divided by the time interval t Δ. We are given Δt = 35 s, but we will do a preliminary calculation to find the displacement.

$$\Delta x = x_f - x_i = -47m - (-12m) = -35m$$

$$v = \frac{\Delta x}{\Delta t} = \frac{-35m}{35s} = -1.0\, m/s$$

The negative sign tells us that Harry is walking to the left.

10. x_f = 3 m, x_i = -12 m, Δt = 10 s

$$v = \frac{\Delta x}{\Delta t} = \frac{3m - (-12)m}{10s} = \frac{15m}{10s} = +1.5\, m/s$$

It's important to keep track of signs on positions and displacements in equations. The positive sign tells us that the dog is trotting to the right.

11. In this problem we are given x_i = 2.1 m and x_f = 7.3 m as well as v = 0.35 m/s and asked to solve for Δt.

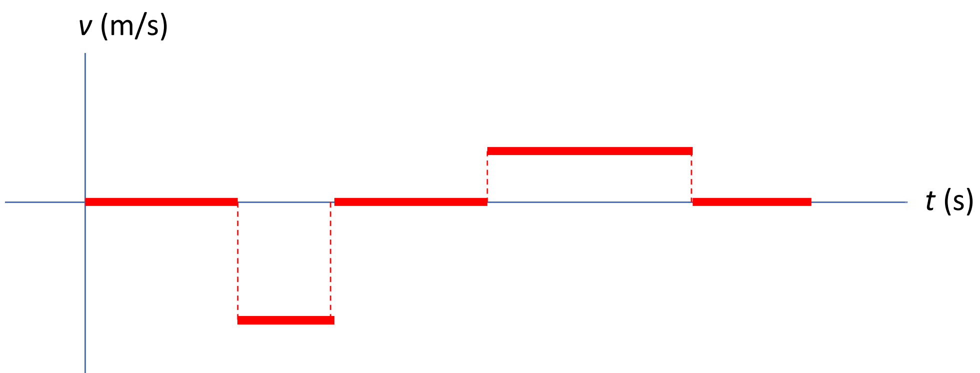
$$\Delta t = \frac{\Delta x}{v} = \frac{x_f - x_i}{v} = \frac{7.3m - 2.1m}{0.35\, m/s} = \frac{5.2m}{0.35\, m/s} = 15s$$

12. a. The object is at rest during segments B and D. Notice that the graph is a horizontal line while object is at rest.

b. The object is moving to the right whenever x is increasing. That is only during segment A. Don't confuse something going right on the graph (such as segments C and E) with the object physically moving to the right (as in segment A). Just because t is increasing doesn't mean x is.

c. The speed is the magnitude of the slope of the graph. Both segments C and E have negative slope, but C's slope is steeper, so the object has a greater speed during segment C than during segment E.

13. There are five different segments of the motion, since the lines on the position-versus-time graph have different slopes between five different time periods. The velocity-versus-time graph would look like the following diagram:



14. a. For the velocity to be constant, the velocity-versus-time graph must have zero slope. Looking at the graph, there are three time-intervals where the graph has zero slope: segment A, segment D and segment F.

    b. For an object to be speeding up, the magnitude of the velocity of the object must be increasing. When the slope of the lines on the graph is nonzero, the object is changing speed. Consider segment B. The velocity is positive while the slope of the line is negative - the object is slowing down. At the start of segment B, we can see the velocity is +2 m/s, while at the end of segment B the velocity is 0 m/s. During segment E the slope of the line is positive but the velocity is negative - the object is slowing here also. Looking at the graph at the beginning of segment E the velocity is –2 m/s (which has a magnitude of 2 m/s, the negative just gives direction). At the end of segment E the velocity is 0 m/s, so the object has slowed down. Consider segment C. Here the slope of the line is negative and the velocity is negative - the object is speeding up. The object is gaining velocity in the negative direction. At the beginning of that segment the velocity is 0 m/s, and at the end the velocity is –2 m/s, (which has a magnitude of 2 m/s, the negative just gives direction).

    c. In the analysis for part b, we found that the object is slowing down during segments B and E.

    d. An object standing still has zero velocity. The only time this is true on the graph is during segment F, where the line has zero slope, and is along $v = 0$ m/s. The velocity is also zero for an instant at time $t = 5$ s between segments B and C.

    e. For an object to be moving to the right, the convention is that the velocity is positive. In terms of the graph, positive values of velocity are above the time axis. The velocity is positive for segments A and B. The velocity must also be greater than zero. Segment F represents a velocity of 0 m/s.

15. The velocity of an object is given by the physical slope of the line on the position-versus-time graph. Since the graph has constant slope, the velocity is constant. We can calculate the slope, choosing any two points on the line since the velocity is constant. In particular, at $t_1 = 0$ s the position is $x_1 = 5$ m. At time $t_2 = 3$ s the position is $x_2 = 15$ m.

$$v = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{15m - 5m}{3s - 0s} = \frac{10m}{3s} = +3.3\, m/s$$

    The correct choice is C.

16. Note that the slope of the position-versus-time graph at every point gives the velocity at that point. The graph has a distinct slope and hence distinct velocity in the time intervals: from t = 0 to t = 20 s; from 20 s to 30 s; and from 30 s to 40 s.

    The slope at t = 10 s is

$$v = \frac{\Delta x}{\Delta t} = \frac{100m - 50m}{20s} = +2.5\, m/s$$

The slope at t = 25 s is

$$v = \frac{\Delta x}{\Delta t} = \frac{100m - 100m}{10s} = 0\,m/s$$

The slope at t = 35 s is

$$v = \frac{\Delta x}{\Delta t} = \frac{0m - 100m}{10s} = -10\,m/s$$

17. Firstly, compute the time the faster runner takes to finish the race.

$$t = \frac{8.00km}{14.0\,km/h} = 0.571h$$

Then see how far the slower runner has gone in that amount of time.

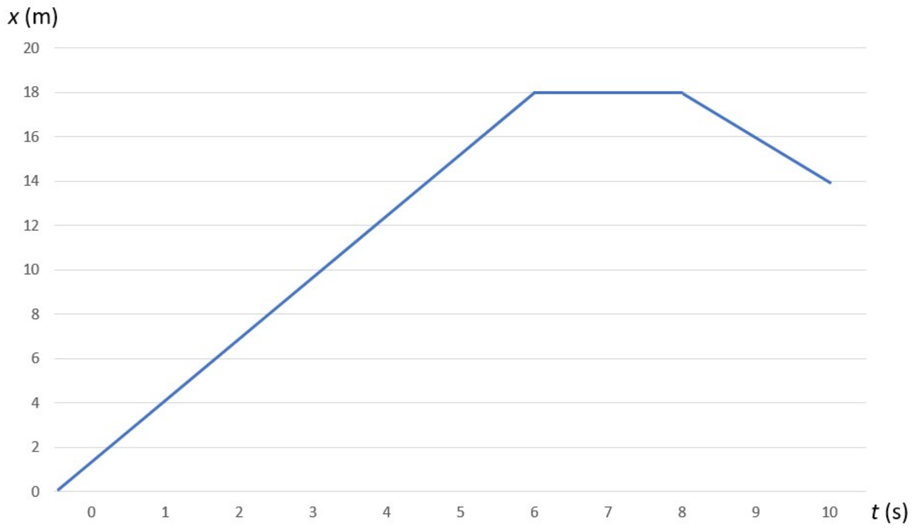
$$d = (11.0\,km/h) \times (0.571h) = 6.29km$$

This leaves the slower runner

$$8.00km - 6.29km = 1.71km$$

from the finish line as the faster runner crosses the line.

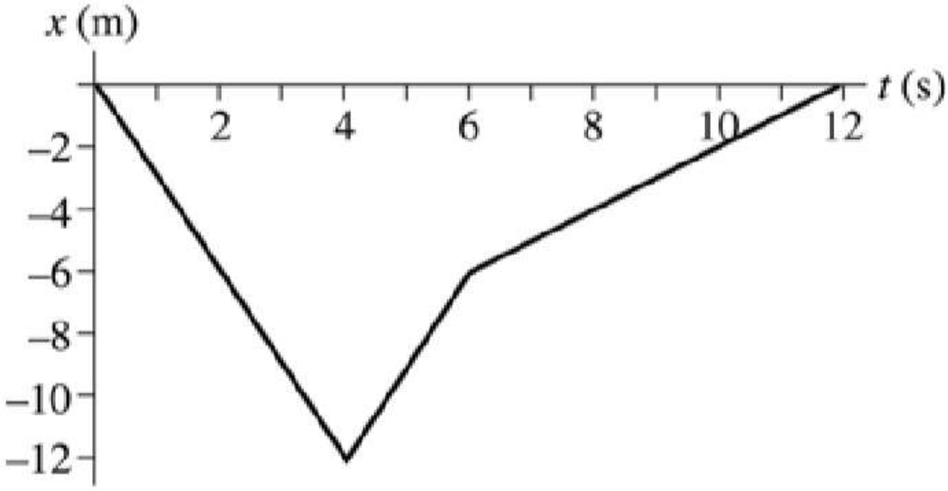
18. To get a position from a velocity graph we count the area under the curve.
    a.



b. We need to count the area under the velocity graph (area below the *x*-axis is subtracted). There are 18 m of area above the axis and 4 m of area below. 18 m – 4 m = 14m.

19. To get a position from a velocity graph we count the area under the curve.

a.



b. We need to count the area under the velocity graph (area below the *x*-axis is subtracted). There are 12 m of area below the axis and 12 m of area above. 12 m – 12 m = 0m.

20. Assume that the ball travels in a horizontal line at a constant v_x. It doesn't really, but if it is a straight drive then it is a fair approximation.

$$\Delta t = \frac{\Delta x}{v} = \frac{20.12m}{241.4\, km/h} = \frac{20.12m}{67\, m/s} = 0.3s$$