Using Scilab to Create an Efficient Distribution Network

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Executive summary

Distribution costs often represent a significant percentage of a product or service-driven organization's overall operating expenses. Reducing costs in this area can substantially increase profit margins. While this problem is relatively simple to solve for small-scale transportation networks where there are few supply and demand components, the difficulty increases substantially with increased network complexity. In this paper, a medium scale transportation problem with 5 supply components and 8 demand components will be considered. The Scilab programming language is used to determine the optimal transportation schedule that has the lowest-cost while fulfilling all demand requirements. Although Scilab is a sophisticated language used for complex numerical analysis, setting-up this problem is simple and the solution produced is easy to interpret.

Introduction

Linear programming (LP) is a mathematical technique for maximizing or minimizing a linear relation subject to a set of linear constraints (Anton, 2010). Soviet economist and mathematician Leonid Kantorovich is acknowledged as being the first to publish a linear programming model. Kantorovich's 1939 work, *The Mathematical Method of Production Planning*, demonstrated applications of LP to various economic planning problems (Kantorovich, 1975). LP was popularized by George Dantzig in 1947 with the publication of his "simplex method" which is used in diverse applications such as resource allocation, production scheduling, and investment planning (Levy, 2005). LP and the simplex method are used extensively to determine optimal product distribution schedules for simple and complex transportation networks.

The Transportation Problem

Transportation models, commonly referred to as *transportation problems* (TP), assist in determining the minimum-cost plan for transporting products from a number of sources to a number of destinations within the constraints of source supply and destination demand. If a company manufactures and distributes a product at *m* different supply centers (SC), denoted mathematically by $i = 1,...,m$, at the *i*th SC, the supply produced is S_i. The product is distributed to *n* geographically disparate demand centers (DC) denoted by $j = 1,...,n$. At the *j*th DC, demand is D_j. The problem is determining a balance between S_i and D_j at a minimum cost. Assuming that a linear relationship exists between the cost and number of products shipped where the cost of shipping one unit from SC *i* to DC *j* is c_{ij} and the number of units shipped from SC *i* to DC *j* is x_{ij}, the minimum-cost plan can be represented by the following equation:

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} c_{ij} \tag{1}$$

Since the quantity shipped from the SC cannot exceed the available supply, and the demand must be met at each DC, the following constraints must be imposed:

$$\sum_{j=1}^{n} x_{ij} \leq S_i \text{ for all } i = 1, \ldots, m \tag{2}$$

and

$$\sum_{i=1}^{m} x_{ij} \geq D_j \text{ for all } j = 1, \ldots, n \tag{3}$$

When the conventional assumption of a balanced system is used, where total supply equals total demand, the following *balanced transportation equation* is applicable:

$$\sum_{i=1}^{m} S_i = \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n} D_j \tag{4}$$

In the event where an unbalanced system is encountered where total demand is not equal to total supply, dummy sources or dummy destinations can be added to the system. The balanced transportation model can be expressed by the following linear programming problem:

$$\begin{aligned} &\text{minimize} \qquad x_0 = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \\ &\text{with constraints} \quad \begin{cases} \sum_{j=1}^{n} x_{ij} = S_i, \quad 1 \leq i \leq m \\ \sum_{i=1}^{m} x_{ij} = D_j, \quad 1 \leq j \leq n \\ x_{ij} \geq 0 \end{cases} \end{aligned} \tag{5}$$

Using matrices, the same problem can be expressed as

$$\text{minimize} \quad f\left(\vec{x}\right)=\vec{c}^{\,T}\vec{x}$$
$$\text{with constraints} \quad \begin{cases} A\vec{x}=\vec{b} \\ G\vec{x}\le\vec{h} \\ \vec{x}_L\le\vec{x}\le\vec{x}_U \end{cases} . \tag{6}$$

Here, $\vec{x}$ contains the *n* design variables to be minimized ($x_1, x_2, ..., x_n$), $\vec{c}$ contains the *n* coefficients of the objective function, *A* is an $n\times m$ matrix, $\vec{b}$ contains *m* components, *G* is a $p\times q$ matrix, $\vec{h}$ contains *p* components, $\vec{x}_L$ and $\vec{x}_U$ are the lower and upper bounds of the constraints, respectively.

Example Problem

Presented here, is a small-scale transportation problem involving the MicroWidget Company. The goal is to minimize transportation costs while meeting daily warehouse demands. The factory production capabilities, delivery distances, and demands are shown in Table 1.

Table 1
Supply and demand of widgets for the MicroWidget Company

Factory	Distance to warehouse (miles)		Maximum deliveries per day per factory
	Warehouse A	Warehouse B	
1	20	25	10
2	32	23	8
Warehouse demand (deliveries/day)	10	9	

Roundtrip transportation costs are shown in Table 2.

Table 2
Round-trip transportation costs at $3.00 per mile

Factory	Warehouse A	Warehouse B
1	$ 120	$ 150
2	$ 192	$ 138

Using this data from Tables 1 and 2, the problem can be setup for cost-minimization:

$$\text{Let } x_{ij} = \text{Round-trip costs from Factory } i \text{ to Warehouse } j$$

The objective function is

$$120x_{11} + 150x_{12} + 192x_{21} + 138x_{22} \tag{7}$$

with the following constraints:

$$x_{11} + x_{21} \geq 10 \qquad \text{Deliveries to Warehouse A} \tag{8}$$

$$x_{12} + x_{22} \geq 9 \qquad \text{Deliveries to Warehouse B} \tag{9}$$

$$x_{11} + x_{12} \leq 10 \qquad \text{Deliveries from Factory 1} \tag{10}$$

$$x_{21} + x_{22} \leq 8 \qquad \text{Deliveries from Factory 2} \tag{11}$$

$$x_{ij} \geq 0 \tag{12}$$

Scilab and the linpro function

Scilab is a high-level programming language with interfaces for hundreds of mathematical functions (INRIA, 2011 & ENPC, 2007). The `linpro` (linear programming) function used to solve the TPs found in this paper is found in Quapro, a Scilab complimentary module for linear and linear quadratic programming (Steer, Mendez, Renteria, & Delebecque, 2010).

The following variable assignments are required by Scilab's `linpro` function to solve the MicroWidget Company's TP:

$$c = \begin{bmatrix} 120 \\ 150 \\ 192 \\ 138 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 10 \\ 9 \\ 10 \\ 8 \end{bmatrix}, \quad cL = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad cU = \begin{bmatrix} Inf \\ Inf \\ Inf \\ Inf \end{bmatrix} \tag{13}$$

The Scliab script used to solve this TP (Figure 1) was adopted from examples provided in Winston's text, *Operational Research*: *Applications and Algorithms* (as cited in Baudin and Paul, 2011). The results of running this script are shown in Figure 2.

```
c=[120 150 192 138]';

A=[1 0 1 0;0 1 0 1;1 1 0 0;0 0 1 1];

A=-A;

b=[10 9 10 8]';

b=-b;

[n,p]=size(A);

cL=zeros(p,1);

cU=%inf*ones(p,1);

[xopt,lagr,fopt]=linpro(c,A,b,cL,cU);
```

Figure 1. Scilab script for the MicroWidget Company cost-minimization problem.

```
fopt  =

   2442.
 lagr  =

   0.
 - 12.
 - 72.
   0.
   120.
   138.
   0.
   0.
 xopt  =

   10.
   0.
   0.
   9.
```

Legend
fopt = The optimal value of the function.
lagr = A vector of the Larange multipliers.
xopt = The value of the variables that minimize the objective function.

Figure 2. Output of the Scilab "linpro" function.

The output `fopt` gives a daily total optimized transportation cost of $2,442. The vector of Larange multipliers, `lagr`, provides information about the effect of lower and upper constraints (x_L and x_U) on the given TP. Zeros in the `lagr` vector indicate that the solution was found without using that particular constraint (Urroz, 2001). The first four `lagr` values, `0`, `-12`, `-72`, and `0`, indicate that none of the lower-bound constraints were used in determining the solution. The next four values, `120`, `138`, `0`, `0`, indicate that only the first and second (equations 8 and 9 respectively) upper-bound constraints were used. The `xopt` indicates that 10 deliveries should be sent from Factory 1 to Warehouse A and 9 deliveries should be sent from Factory 2 to Warehouse B. The results are summarized in Table 3.

Table 3

MicroWidget Company transportation schedule

Factory	Warehouse	Deliveries per day	Cost per delivery	Total Cost
1	A	10	$ 120	$ 1200
1	B	0	150	0
2	A	0	192	0
2	B	9	138	1242
				$ 2442

Medium-Scale Transportation Problem Solved with Scilab

The MacroWidget Company has 5 factories that distribute products to 8 regional distribution centers, as illustrated in Figure 3.

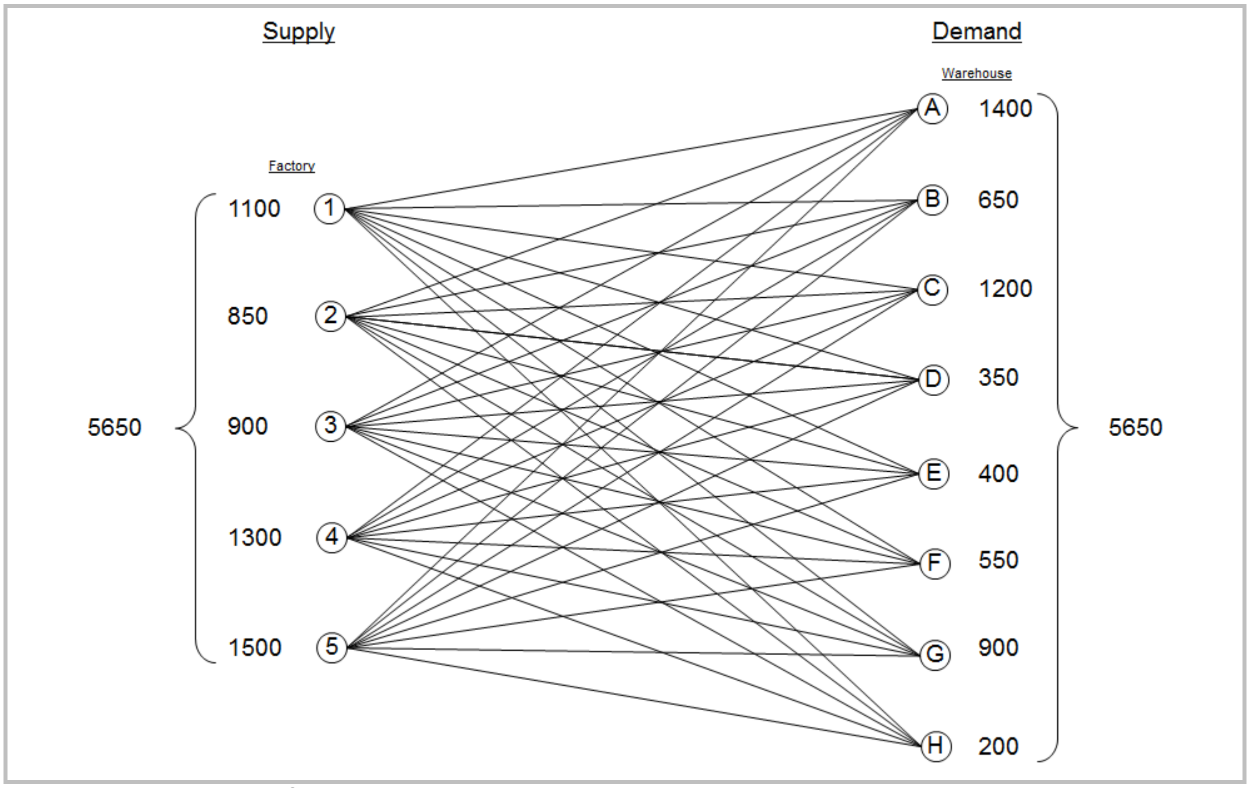

Figure 3. The MacroWidget Company's distribution network. Costs have been excluded for clarity.

Round-trip costs between each factory and warehouse (x_{ij}) are given in the following objective function:

$$\begin{aligned}&300x_{11}+250x_{12}+175x_{13}+125x_{14}+370x_{15}+290x_{16}+310x_{17}+425x_{18}+\\&464x_{21}+264x_{22}+492x_{23}+345x_{24}+394x_{25}+232x_{26}+246x_{27}+286x_{28}+\\&325x_{31}+394x_{32}+343x_{33}+193x_{34}+408x_{35}+216x_{36}+221x_{37}+287x_{38}+\\&297x_{41}+372x_{42}+130x_{43}+171x_{44}+410x_{45}+264x_{46}+196x_{47}+379x_{48}+\\&452x_{51}+417x_{52}+270x_{53}+461x_{54}+291x_{55}+335x_{56}+261x_{57}+366x_{58},\end{aligned} \tag{14}$$

with constraints

$$x_{11}+x_{12}+x_{13}+x_{14}+x_{15}+x_{16}+x_{17}+x_{18}\leq 1100 \tag{15}$$

$$x_{21}+x_{22}+x_{23}+x_{24}+x_{25}+x_{26}+x_{27}+x_{28}\leq 850 \tag{16}$$

$$x_{31}+x_{32}+x_{33}+x_{34}+x_{35}+x_{36}+x_{37}+x_{38}\leq 900 \tag{17}$$

$$x_{41}+x_{42}+x_{43}+x_{44}+x_{45}+x_{46}+x_{47}+x_{48}\leq 1300 \tag{18}$$

$$x_{51}+x_{52}+x_{53}+x_{54}+x_{55}+x_{56}+x_{57}+x_{58}\leq 1500 \tag{19}$$

$$x_{11}+x_{21}+x_{31}+x_{41}+x_{51}+x_{61}+x_{71}+x_{81}\geq 1400 \tag{20}$$

$$x_{11}+x_{12}+x_{13}+x_{14}+x_{15}+x_{16}+x_{17}+x_{18}\leq 1100 \tag{21}$$

$$x_{21}+x_{22}+x_{23}+x_{24}+x_{25}+x_{26}+x_{27}+x_{28}\leq 650 \tag{22}$$

$$x_{31}+x_{32}+x_{33}+x_{34}+x_{35}+x_{36}+x_{37}+x_{38}\leq 1200 \tag{23}$$

$$x_{41}+x_{42}+x_{43}+x_{44}+x_{45}+x_{46}+x_{47}+x_{48}\leq 350 \tag{24}$$

$$x_{51}+x_{52}+x_{53}+x_{54}+x_{55}+x_{56}+x_{57}+x_{58}\leq 400 \tag{25}$$

$$x_{61}+x_{62}+x_{63}+x_{64}+x_{65}+x_{66}+x_{67}+x_{68}\leq 550 \tag{26}$$

$$x_{71}+x_{72}+x_{73}+x_{74}+x_{75}+x_{76}+x_{77}+x_{78}\leq 900 \tag{27}$$

$$x_{81}+x_{82}+x_{83}+x_{84}+x_{85}+x_{86}+x_{87}+x_{88}\leq 200 \tag{28}$$

For brevity, only the `fopt` result of the `linpro` function is shown (Figure 4). The optimized distribution schedule shown in Table 4 was produced using the `xopt` vector.

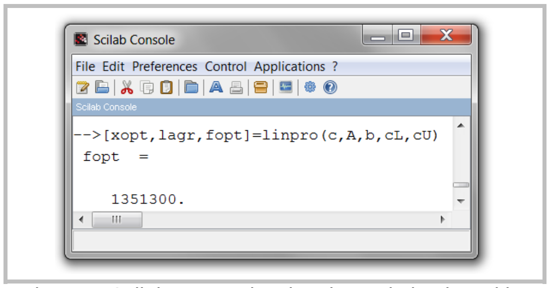

Figure 4. Scilab output showing the optimized weekly shipping costs for the MacroWidget Company.

Table 4

MacroWidget Company transportation schedule

Factory	Warehouse	Deliveries per week	Cost per delivery	Total Cost
1	A	750	$ 300	$ 225,000
1	B	0	250	0
1	C	0	175	0
1	D	350	125	43,750
1	E	0	370	0
1	F	0	290	0
1	G	0	310	0
1	H	0	425	0
2	A	0	464	0
2	B	650	264	171,600
2	C	0	492	0
2	D	0	345	0
2	E	0	394	0
2	F	200	232	46,400
2	G	0	246	0
2	H	0	286	0
3	A	550	325	178,750
3	B	0	394	0
3	C	0	343	0
3	D	0	193	0
3	E	0	408	0
3	F	350	216	75,600
3	G	0	221	0
3	H	0	287	0
4	A	100	297	29,700
4	B	0	372	0
4	C	1200	130	156,000
4	D	0	171	0
4	E	0	410	0
4	F	0	264	0
4	G	0	196	0
4	H	0	379	0
5	A	0	452	0
5	B	0	417	0
5	C	0	270	0
5	D	0	461	0
5	E	400	291	116,400
5	F	0	335	0
5	G	900	261	234,900
5	H	200	366	73,200
				$ 1,351,300

Conclusion

Linear programming is an invaluable tool with widespread economic and scientific applications. It has been shown how technology can be used to optimize linear programming models such as the TP considered in this paper. Although Scilab simplifies this process considerably, careful attention must be given when programming the required matrices as these can become quite large.

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