Week 5 Homework – CMSC405

1. Show the results and intermediate steps for a translation of (20,40,-10), a rotation of 135 degrees about the z-axis applied to a starting point of (45,-95,20). Perform a separate operation applying scale with scale factors of sx=2.0, sy=1.6 and sz=1.0. You should use 4x4 matrix math for your calculations. Note: Use the $P_2 = T^{-1}$ R(z) T P1 approach for a general 3D rotation about the z-axis and $P_2 = T^{-1}$ S T P1 approach for a general scaling.

Setup starting point, rotation matrix, and translation matrix:

$$P_1 = \begin{bmatrix} 45 \\ -95 \\ 20 \\ 1 \end{bmatrix} \tag{1}$$

$$R(z) = \begin{bmatrix} \cos\left(\frac{3\pi}{4}\right) & -\sin\left(\frac{3\pi}{4}\right) & 0 & 0 \\ \sin\left(\frac{3\pi}{4}\right) & \cos\left(\frac{3\pi}{4}\right) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$T = \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & -10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 20 & 40 & -10 & 1 \end{bmatrix} \tag{4}$$

Using $P_2 = T^{-1}R(z)TP_1$:

$$P_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 45 & -95 & 20 & 1 \end{bmatrix} \begin{bmatrix} \cos\left(\frac{3\pi}{4}\right) & -\sin\left(\frac{3\pi}{4}\right) & 0 & 0 \\ \sin\left(\frac{3\pi}{4}\right) & \cos\left(\frac{3\pi}{4}\right) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 45 \\ 0 & 1 & 0 & -95 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 45 \\ -95 \\ 20 \\ 1 \end{bmatrix} = \begin{bmatrix} -7.0710678 \\ 84.852814 \\ 10 \\ 3153.6912 \end{bmatrix} \tag{5}$$

$$P_2 = (-7.0710678, 84.852814, 10)$$

Setup the scale transformation matrix:

$$S = \begin{bmatrix} 2.0 & 0 & 0 & 0 \\ 0 & 1.6 & 0 & 0 \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

Using $T^{-1}STP_1$:

$$P_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 20 & 40 & -10 & 1 \end{bmatrix} \begin{bmatrix} 2.0 & 0 & 0 & 0 \\ 0 & 1.6 & 0 & 0 \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & -10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 45 \\ -95 \\ 20 \\ 1 \end{bmatrix} = \begin{bmatrix} 130 \\ -88 \\ 10 \\ -1019 \end{bmatrix} \tag{7}$$

$P_2 = (130, -88, 10)$

2. Using quaternions, determine the final transformed location of point P1= (5, 9, -10), after a 45-degree rotation about the z-axis, 90-degree rotation about the x-axis and 75-degree rotation about the y-axis. Be sure to show your work including the quaternion values for all steps.

Using the general quaternion form:

$$q = \cos\left(\frac{\theta}{2}\right) + i\left(x\sin\left(\frac{\theta}{2}\right)\right) + j\left(y\sin\left(\frac{\theta}{2}\right)\right) + k\left(z\sin\left(\frac{\theta}{2}\right)\right) \tag{1}$$

Substituting z, y, and x with $(0,0,1)$ for a z-axis rotation and θ with $\frac{\pi}{4}$:

$$\begin{aligned} q &= \cos\left(\frac{\pi}{8}\right) + i\left(0\sin\left(\frac{\pi}{8}\right)\right) + j\left(0\sin\left(\frac{\pi}{8}\right)\right) + k\left(1\sin\left(\frac{\pi}{8}\right)\right) \\ &= 0.92388 + i0 + j0 + k0.382683 \\ &= 0.92388 + k0.382683 \end{aligned} \tag{2}$$

Determining *P'* using qPq':

$$q^{-1} = 0.92388 - k0.382683 \tag{3}$$

$$P = 0 + i5 + j9 - k10 \tag{4}$$

$$\begin{aligned} P' &= qPq' \\ &= (0.92388 + k0.382683)(i5 + j9 - k10)(0.92388 - k0.382683) \\ &= (i4.6194 + j8.31492 - k9.2388 + ki1.91342 + kj3.44415 - kk3.82683)(0.92388 - k0.382683) \\ &= (i4.6194 + j8.31492 - k9.2388 + j1.91342 - i3.44415 + 3.82683)(0.92388 - k0.382683) \\ &= (i1.17525 + j10.2283 - k9.2388 + 3.82683)(0.92388 - k0.382683) \\ &= i1.08579 + j9.44972 - k8.53554 + 3.53553 - ik0.44974 - jk3.91419 + kk3.53553 - k1.46446 \\ &= i1.08579 + j9.44972 - k8.53554 + 3.53553 + j0.44974 - i3.91419 - 3.53553 - k1.46446 \\ &= -i2.8284 + j9.89946 - 10 \end{aligned} \tag{5}$$

$$P' = \begin{bmatrix} -2.8284 \\ 9.8995 \\ -10.0000 \\ 1.0000 \end{bmatrix} \tag{6}$$

Substituting z, y, and x with $(1,0,0)$ for a x-axis rotation and θ with $\frac{\pi}{2}$:

$$
\begin{aligned}
q &= \cos\left(\frac{\pi}{4}\right) + i\left(1\sin\left(\frac{\pi}{4}\right)\right) + j\left(0\sin\left(\frac{\pi}{4}\right)\right) + k\left(0\sin\left(\frac{\pi}{4}\right)\right) \\
&= 0.9999 + i0.0137 + j0 + k0 \\
&= 0.9999 + i0.0137
\end{aligned}
\tag{7}
$$

Determining *P'* using qPq':

$$q^{-1} = 0.9999 - i0.0137 \tag{8}$$

$$P = 0 - i2.8284 + j9.8995 - k10 \tag{9}$$

$$
\begin{aligned}
P' &= qPq' \\
&= (0.9999 + i0.0137)(-i2.8284 + j9.8995 - k10)(0.9999 - i0.0137) \\
&= (-i2.8281 + j9.8985 - k9.999 - ii0.0387 + ij0.1356 - ik0.137)(0.9999 - i0.0137) \\
&= (-i2.8281 + j9.8985 - k9.999 + 0.0387 + k0.1356 + j0.137)(0.9999 - i0.0137) \\
&= (-i2.8281 + j10.0355 - k9.8634 + 0.0387)(0.9999 - i0.0137) \\
&= -i2.8278 + j10.0344 - k9.8624 + 0.0386 + ii0.0387 - ji0.1374 + ki0.1351 - i0.0005 \\
&= -i2.8278 + j10.0344 - k9.8624 + 0.0386 - 0.0387 + k0.1374 + j0.1351 - i0.0005 \\
&= -i2.8283 + j10.1695 - k9.725 - 0.0001
\end{aligned}
\tag{10}
$$

$$
P' = \begin{bmatrix} -2.8283 \\ 10.1695 \\ -9.7250 \\ 1.0000 \end{bmatrix}
\tag{11}
$$

Substituting z, y, and x with $(0,1,0)$ for a y-axis rotation and θ with $\frac{5\pi}{12}$:

$$
\begin{aligned}
q &= \cos\left(\frac{5\pi}{24}\right) + i\left(0\sin\left(\frac{5\pi}{24}\right)\right) + j\left(1\sin\left(\frac{5\pi}{24}\right)\right) + k\left(0\sin\left(\frac{5\pi}{24}\right)\right) \\
&= 0.793353 + i0 + j0.6087 + k0 \\
&= 0.793353 + j0.6087
\end{aligned}
\tag{12}
$$

Determining *P'* using qPq':

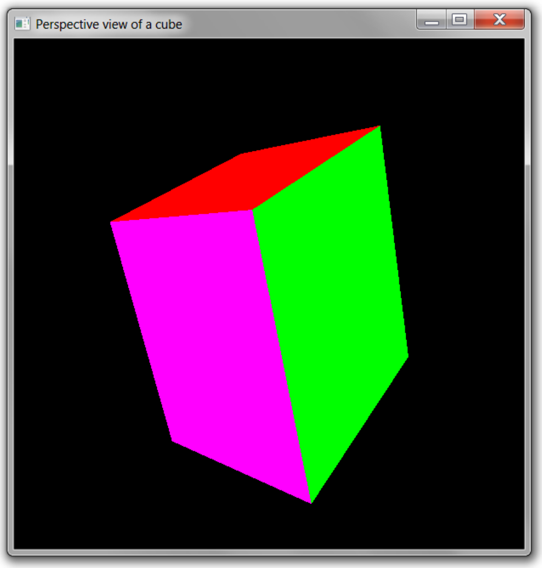
$$q^{-1} = 0.793353 - j0.6087 \tag{13}$$

$$P = 0 - i2.8283 + j10.1695 - k9.725 \tag{14}$$

$$
\begin{aligned}
P' &= qPq' \\
&= (0.7934 + j0.6087)(-i2.8283 + j10.1695 - k9.725)(0.7934 - j0.6087) \\
&= (-i2.2439 + j8.0685 - k7.7158 - ji1.7246 + jj6.1902 - jk5.9196)(0.7934 - j0.6087) \\
&= (-i2.2439 + j8.0685 - k7.7158 + k1.7246 - 6.1902 - i5.9196)(0.7934 - j0.6087) \\
&= (-i8.1635 + j8.0685 - k5.9912 - 6.1902)(0.7934 - j0.6087) \\
&= -i6.4769 + j6.4015 - k4.7534 - 4.9113 + ij4.9691 - jj4.9113 + kj3.6468 + j3.7680 \\
&= -i6.4769 + j6.4015 - k4.7534 - 4.9113 + k4.9691 + 4.9113 - i3.6468 + j3.7680 \\
&= -i10.1237 + j10.1695 + k0.2157
\end{aligned}
\tag{15}
$$

$$P' = \begin{bmatrix} -10.1237 \\ 10.1695 \\ 0.2157 \\ 1.0000 \end{bmatrix} \quad (16)$$

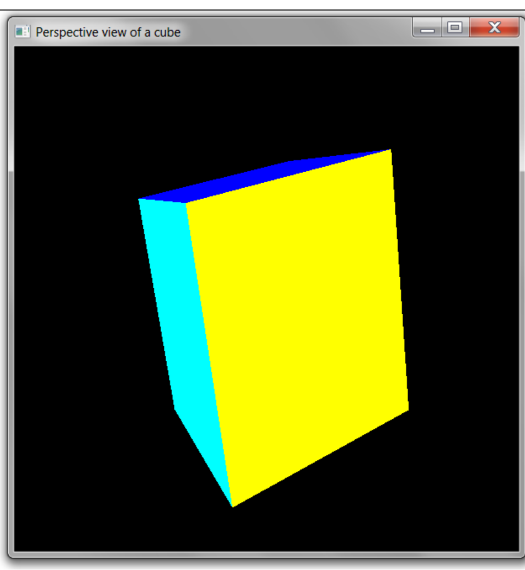
3. Using OpenGL and your programming environment, create and provide 3D views for a 100 by 100 by 100 cube. You should use QL_QUADS to create each of the cube sides. Each side should be a different color (of your choice) and have text or a bitmap pattern of your choice. Your code should display each of the 6-sides using perspectives of your choice. However; each perspective should retain the 3D perspective. Hint: You can expand from the existing code example on pages 346-347 and build your cube one side at a time. No animation is required. You can provide the 6 perspectives by manually changing the parameters and submitting the snapshots in your document along with the parameter changes; or you can use C++ code to display each perspective in a loop. Either approach is acceptable.



<u>Rotation Parameters</u>

```
glTranslatef(0.0, 0.0, -250.0);
glRotatef(30.0, 1.0, 1.0, 1.0);
glRotatef(30.0, 0.0, 1.0, 0.0);
```

Figure 1. Screen capture and rotation parameters for mathewyamasaki5.cpp



<u>Rotation Parameters</u>

```
glTranslatef(0.0, 0.0, -250.0);
glRotatef(30.0, 1.0, 2.0, 1.0);
glRotatef(180.0, 3.0, 3.0, 0.0);
```

Figure 2. Screen capture and rotation parameters for mathewyamasaki5.cpp