

# Math251

## Practice Exam #03

1. Find each square.

a)  $(a-b)^2$

$$\begin{aligned} &= (a-b)(a-b) \\ &= a(a-b) - b(a-b) \\ &= a^2 - ab - ab + b^2 \\ &= \boxed{a^2 - 2ab + b^2} \end{aligned}$$

b)  $\left(4a - \frac{5}{4}b\right)^2$

$$\begin{aligned} &= \left(4a - \frac{5}{4}b\right)\left(4a - \frac{5}{4}b\right) \\ &= 4a\left(4a - \frac{5}{4}b\right) - \frac{5}{4}b\left(4a - \frac{5}{4}b\right) \\ &= 16a^2 - 5ab - 5ab + \frac{25}{16}b^2 \\ &= \boxed{16a^2 - 10ab + \frac{25}{16}b^2} \end{aligned}$$

c)  $-a(a-b)^2$

$$\begin{aligned} &= -a(a-b)(a-b) \\ &= -a[a(a-b) - b(a-b)] \\ &= -a[a^2 - ab - ab + b^2] \\ &= -a[a^2 - 2ab + b^2] \\ &= \boxed{-a^3 + 2a^2b - ab^2} \end{aligned}$$

2. Find each product.

a)  $(9x-2y)(9x+2y)$

$$\begin{aligned} &= 9x(9x-2y) + 2y(9x-2y) \\ &= 81x^2 - 18xy + 18xy - 4y^2 \\ &= \boxed{81x^2 - 4y^2} \end{aligned}$$

b)  $(x-3)(x+3)$

$$\begin{aligned} &= x(x-3) + 3(x-3) \\ &= x^2 - 3x + 3x - 9 \\ &= \boxed{x^2 - 9} \end{aligned}$$

c)  $a(ab-c)(ab+c)$

$$\begin{aligned} &= a[ab(ab-c) + c(ab-c)] \\ &= a[a^2b^2 - abc + abc - c^2] \\ &= a[a^2b^2 - c^2] \\ &= \boxed{a^3b^2 - ac^2} \end{aligned}$$

3. Simplify each expression. Write answers with only positive exponents.

$$\begin{aligned} \text{a) } \frac{(x^{-5})^{-4}}{x^{-2}} \\ = \frac{x^{20}}{x^{-2}} = x^{20} \cdot x^2 \\ = \boxed{x^{22}} \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{(b^2 + a^2)^{-4}}{(b^2 + a^2)^{-7}} \\ = \frac{(b^2 + a^2)^7}{(b^2 + a^2)^4} \\ = \boxed{(b^2 + a^2)^3} \end{aligned}$$

$$\begin{aligned} \text{c) } \frac{(x^3 y^{-4} z^2)^{-1}}{(x^{-1} y^{-2} z^{-1})^{-2}} \\ = \frac{x^{-3} y^4 z^{-2}}{x^2 y^4 z^2} \\ = \frac{y^4}{x^2 x^3 y^4 z^2 z^2} = \frac{y^4}{x^5 y^4 z^4} \\ = \boxed{\frac{1}{x^5 z^4}} \end{aligned}$$

4. Simplify.

$$\begin{aligned} \text{a) } (4x^2 - 3x + 2) - (-9x^2 - 5x + 1) \\ = 4x^2 - 3x + 2 + 9x^2 + 5x - 1 \\ = \boxed{13x^2 + 2x + 1} \end{aligned}$$

$$\begin{aligned} \text{b) } a^2 b - 4a - ab^4 + 4b^3 \\ = a(ab - 4) - b^3(ab - 4) \\ = \boxed{(ab - 4)(a - b^3)} \end{aligned}$$

5. Factor completely.

$$\begin{aligned} \text{a) } 2x^2 + 15x + 7 \\ \begin{array}{l|l} a=2 & a \cdot c = 14 \\ b=15 & \\ c=7 & \end{array} \quad \begin{array}{c} \triangle \\ 14 \quad 1 \end{array} \quad \begin{array}{c} \text{sum} \\ 15 \end{array} \\ = 2x^2 + 14x + x + 7 \\ = 2x(x+7) + 1(x+7) \\ = \boxed{(x+7)(2x+1)} \end{aligned}$$

$$\begin{aligned} \text{b) } 6x^2 - 13x - 8 \\ \begin{array}{l|l} a=6 & a \cdot c = -48 \\ b=-13 & \\ c=-8 & \end{array} \quad \begin{array}{c} \wedge \\ -48 \cdot 1 \\ -24 \cdot 2 \\ -12 \cdot 4 \\ -6 \cdot 8 \\ -3 \cdot 16 \\ 3 \cdot -16 \end{array} \quad \begin{array}{c} \text{sum} \\ -47 \\ -22 \\ -8 \\ 2 \\ 13 \\ -13 \end{array} \\ = 6x^2 + 3x - 16x - 8 \\ = 3x(2x+1) - 8(2x+1) \\ = \boxed{(2x+1)(3x-8)} \end{aligned}$$

$$\begin{aligned} \text{c) } 16x^2 + 65x + 4 \\ \begin{array}{l|l} a=16 & a \cdot c = 64 \\ b=65 & \\ c=4 & \end{array} \quad \begin{array}{c} \wedge \\ 64 \quad 1 \end{array} \quad \begin{array}{c} \text{sum} \\ 65 \end{array} \\ = 16x^2 + 64x + x + 4 \\ = 16x(x+4) + 1(x+4) \\ = \boxed{(x+4)(16x+1)} \end{aligned}$$

6. Factor Completely.

$$\begin{aligned} \text{a) } & 14x^3y^4 + 3x^2y^5 - 2xy^6 \\ & = xy^4(14x^2 + 3xy - 2y^2) \end{aligned}$$

$$\begin{array}{c|c|c} a=14 & a \cdot c = -28 & \text{sum} \\ b=3 & \begin{array}{c} \wedge \\ -28 \end{array} & -27 \\ c=-2 & \begin{array}{c} -14 \quad 2 \\ -7 \quad 4 \\ 7 \quad -4 \end{array} & -12 \end{array}$$

$$\begin{aligned} & = xy^4[14x^2 + 7xy - 4xy - 2y^2] \\ & = xy^4[7x(2x+y) - 2y(2x+y)] \\ & = \boxed{xy^4(2x+y)(7x-2y)} \end{aligned}$$

$$\begin{aligned} \text{b) } & 2y^3z - 3y^2z^2 - 27yz^3 \\ & = yz[2y^2 - 3yz - 27z^2] \end{aligned}$$

$$\begin{array}{c|c|c} a=2 & a \cdot c = -54 & \text{sum} \\ b=-3 & \begin{array}{c} \wedge \\ -54 \end{array} & -53 \\ c=-27 & \begin{array}{c} -54 \quad 1 \\ -27 \quad 2 \\ -9 \quad 6 \end{array} & -25 \end{array}$$

$$\begin{aligned} & = yz[2y^2 - 9yz + 6yz - 27z^2] \\ & = yz[y(2y-9z) + 3z(2y-9z)] \\ & = \boxed{yz(2y-9z)(y+3z)} \end{aligned}$$

7. Solve for x.

$$\text{a) } x^2 - 3ax - 10a^2 = 0$$

$$\begin{array}{c|c|c} a=1 & a \cdot c = -10 & \text{sum} \\ b=-3 & \begin{array}{c} \wedge \\ -10 \end{array} & -9 \\ c=-10 & \begin{array}{c} -5 \quad 2 \end{array} & -3 \end{array}$$

$$\begin{aligned} & x^2 - 5ax + 2ax - 10a^2 = 0 \\ & x(x-5a) + 2a(x-5a) = 0 \\ & (x-5a)(x+2a) = 0 \end{aligned}$$

$$\begin{array}{c} x-5a=0 \\ +5a \quad +5a \\ \hline \boxed{x=5a} \end{array} \quad \begin{array}{c} x+2a=0 \\ -2a \quad -2a \\ \hline \boxed{x=-2a} \end{array}$$

$$\text{b) } 3x(x-5) = -12$$

$$3x^2 - 15x = -12$$

$$\frac{1}{3}[3x^2 - 15x] = \frac{1}{3}[-12]$$

$$\begin{array}{c} x^2 - 5x = -4 \\ +4 \quad +4 \end{array}$$

$$x^2 - 5x + 4 = 0$$

$$\begin{array}{c|c|c} a=1 & a \cdot c = 4 & \text{sum} \\ b=-5 & \begin{array}{c} \wedge \\ -4 \end{array} & -5 \\ c=4 & \begin{array}{c} -4 \quad -1 \end{array} & -5 \end{array}$$

$$a=1: \text{Shortcut! } (x-4)(x-1) = 0$$

$$\begin{array}{c} x-4=0 \\ \boxed{x=4} \end{array} \quad \begin{array}{c} x-1=0 \\ \boxed{x=1} \end{array}$$

8. Factor Completely. Use the note below for parts b) and c).

**Note:**  $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$  and  $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$$\begin{aligned} \text{a) } & x^4 - 81 \\ & = (x^2)^2 - 9^2 \\ & = (x^2 + 9)(x^2 - 9) \\ & = \boxed{(x^2 + 9)(x+3)(x-3)} \end{aligned}$$

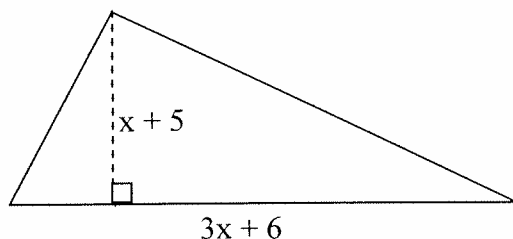
$$\begin{aligned} \text{b) } & y^3 + 8 \\ & = y^3 + 2^3 \\ & = (y+2)(y^2 - 2y + 2^2) \\ & = \boxed{(y+2)(y^2 - 2y + 4)} \end{aligned}$$

$$\begin{aligned} \text{c) } & 27y^3 - 8 \\ & = (3y)^3 - 2^3 \\ & = (3y-2)[(3y)^2 + (3y)(2) + 2^2] \\ & = \boxed{(3y-2)(9y^2 + 6y + 4)} \end{aligned}$$

9. The area of this triangle is 60 square inches. Find the height and the base of this

triangle. Set up an equation and solve.

$$A = \frac{1}{2}bh$$



$$60 = \frac{1}{2}(3x+6)(x+5)$$

$$60 = \frac{1}{2}[x(3x+6) + 5(3x+6)]$$

$$60 = \frac{1}{2}[3x^2 + 6x + 15x + 30]$$

$$60 = \frac{1}{2}[3x^2 + 21x + 30]$$

\* mult both sides by 2:  $120 = 3x^2 + 21x + 30$

\* divide both sides by 3:  $40 = x^2 + 7x + 10$

$$\begin{array}{r} 40 = x^2 + 7x + 10 \\ -40 \quad \quad -40 \\ \hline \end{array}$$

$$0 = x^2 + 7x - 30$$

$$x^2 + 7x - 30 = 0$$

a=1	a·c = -30	sum
b=7	-30 1	-29
c=-30	-15 2	-13
	-5 6	1
	-10 3	-7
	10 -3	7

a=1: short cut!

$$(x+10)(x-3) = 0$$

$$\begin{array}{r} x+10=0 \\ -10 \quad -10 \\ \hline \end{array}$$

$$x = -10$$

$$\begin{array}{r} x-3=0 \\ +3 \quad +3 \\ \hline \end{array}$$

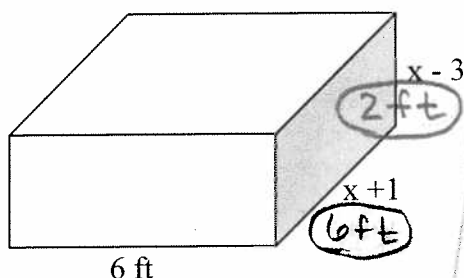
$$x = 3$$

x=3: b = 3(3)+6 = 9+6 = 15  
h = (3)+5 = 8

$$\boxed{b=15, h=8}$$

10. The box below has a volume of 72 ft<sup>3</sup>. Set up an equation and solve for x. Then label

the width and the height of the box.



$$V = l \cdot w \cdot h$$

$$72 = 6(x+1)(x-3)$$

\* divide both sides by 6:  $12 = (x+1)(x-3)$

$$12 = x(x+1) - 3(x+1)$$

$$12 = x^2 + x - 3x - 3$$

$$12 = x^2 - 2x - 3$$

$$\begin{array}{r} 12 = x^2 - 2x - 3 \\ -12 \quad \quad -12 \\ \hline \end{array}$$

$$0 = x^2 - 2x - 15$$

$$x^2 - 2x - 15 = 0$$

a=1	a·c = -15	sum
b=-2	-15 1	-14
c=-15	-5 3	-2

a=1: Short cut!

$$(x-5)(x+3) = 0$$

$$\begin{array}{r} x-5=0 \\ +5 \quad +5 \\ \hline \end{array}$$

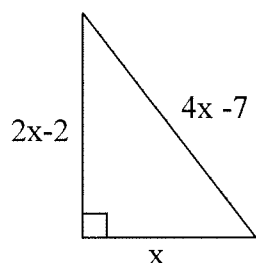
$$\boxed{x=5}$$

$$\begin{array}{r} x+3=0 \\ -3 \quad -3 \\ \hline \end{array}$$

$$x = -3$$

11. Write out an equation that relates the three sides of the right triangle below. Then solve this equation for  $x$ . Then label the length of the sides of the triangle.

$$c^2 = a^2 + b^2$$



$$(4x-7)^2 = (2x-2)^2 + x^2$$

$$(4x-7)(4x-7) = (2x-2)(2x-2) + x^2$$

$$4x(4x-7) - 7(4x-7) = 2x(2x-2) - 2(2x-2) + x^2$$

$$16x^2 - 28x - 28x + 49 = 4x^2 - 4x - 4x + 4 + x^2$$

$$16x^2 - 56x + 49 = 5x^2 - 8x + 4$$

$$11x^2 - 48x + 45 = 0$$

$$\begin{aligned} a &= 11 \\ b &= -48 \\ c &= 45 \end{aligned}$$

$$a \cdot c = 495$$

		SUM
495	1	496
165	3	168
55	9	64
11	45	56
33	15	48
-33	-15	-48

If  $x = \frac{15}{11}$ , then

$$4x - 7 = 4\left(\frac{15}{11}\right) - 7$$

$$= \frac{60}{11} - 7$$

$$= \frac{60}{11} - \frac{77}{11}$$

$$= -\frac{17}{11}$$

Not Possible!

$$11x^2 - 33x - 15x + 45 = 0$$

$$11x(x-3) - 15(x-3) = 0$$

$$(x-3)(11x-15) = 0$$

$$x-3=0$$

$$+3 \quad +3$$

$$x = 3$$

$$11x-15=0$$

$$+15 \quad +15$$

$$\frac{11x}{11} = \frac{15}{11}$$

$$x = \frac{15}{11}$$

**SOLUTION**

So,  $x = 3$ .

