# Math251

## Practice Exam #03

1. Find each square.

a) $(a-b)^2$

$=(a-b)(a-b)$

$=a(a-b)-b(a-b)$

$=a^2-ab-ab+b^2$

$=\boxed{a^2-2ab+b^2}$

b) $\left(4a-\frac{5}{4}b\right)^2$

$=\left(4a-\frac{5}{4}b\right)\left(4a-\frac{5}{4}b\right)$

$=4a\left(4a-\frac{5}{4}b\right)-\frac{5}{4}b\left(4a-\frac{5}{4}b\right)$

$=16a^2-5ab-5ab+\frac{25}{16}b^2$

$=\boxed{16a^2-10ab+\frac{25}{16}b^2}$

c) $-a(a-b)^2$

$=-a(a-b)(a-b)$

$=-a[a(a-b)-b(a-b)]$

$=-a[a^2-ab-ab+b^2]$

$=-a[a^2-2ab+b^2]$

$=\boxed{-a^3+2a^2b-ab^2}$

2. Find each product.

a) $(9x-2y)(9x+2y)$

$=9x(9x-2y)+2y(9x-2y)$

$=81x^2-18xy+18xy-4y^2$

$=\boxed{81x^2-4y^2}$

b) $(x-3)(x+3)$

$=x(x-3)+3(x-3)$

$=x^2-3x+3x-9$

$=\boxed{x^2-9}$

c) $a(ab-c)(ab+c)$

$=a[ab(ab-c)+c(ab-c)]$

$=a[a^2b^2-abc+abc-c^2]$

$=a[a^2b^2-c^2]$

$=\boxed{a^3b^2-ac^2}$

3. Simplify each expression. Write answers with only positive exponents.

a) $\dfrac{\left(x^{-5}\right)^{-4}}{x^{-2}}$

$= \dfrac{x^{20}}{x^{-2}} = x^{20} \cdot x^{2}$

$= \boxed{x^{22}}$

b) $\dfrac{\left(b^2+a^2\right)^{-4}}{\left(b^2+a^2\right)^{-7}}$

$= \dfrac{\left(b^2+a^2\right)^{7}}{\left(b^2+a^2\right)^{4}}$

$= \boxed{\left(b^2+a^2\right)^{3}}$

c) $\dfrac{\left(x^3y^{-4}z^2\right)^{-1}}{\left(x^{-1}y^{-2}z^{-1}\right)^{-2}}$

$= \dfrac{x^{-3}y^{4}z^{-2}}{x^{2}y^{4}z^{2}}$

$= \dfrac{y^4}{x^2 x^3 \cdot y^4 \cdot z^2 z^2} = \dfrac{y^4}{x^5 y^4 \cdot z^4}$

$= \boxed{\dfrac{1}{x^5 z^4}}$

4. Simplify.

a) $\left(4x^2-3x+2\right)-\left(-9x^2-5x+1\right)$

$= 4x^2-3x+2+9x^2+5x-1$

$= \boxed{13x^2+2x+1}$

b) $a^2b-4a-ab^4+4b^3$

$= a(ab-4)-b^3(ab-4)$

$= \boxed{(ab-4)(a-b^3)}$

5. Factor completely.

a) $2x^2+15x+7$

$a=2$, $b=15$, $c=7$

$a \cdot c = 14$

| | sum |
|---|---|
| 14, 1 | 15 |

$= 2x^2+14x+x+7$

$= 2x(x+7)+1(x+7)$

$= \boxed{(x+7)(2x+1)}$

b) $6x^2-13x-8$

$a=6$, $b=-13$, $c=-8$

$a \cdot c = -48$

| | sum |
|---|---|
| $-48 \cdot 1$ | $-47$ |
| $-24 \cdot 2$ | $-22$ |
| $-12 \cdot 4$ | $-8$ |
| $-6 \cdot 8$ | $2$ |
| $-3 \cdot 16$ | $13$ |
| $3 \cdot -16$ | $-13$ |

$= 6x^2+3x-16x-8$

$= 3x(2x+1)-8(2x+1)$

$= \boxed{(2x+1)(3x-8)}$

c) $16x^2+65x+4$

$a=16$, $b=65$, $c=4$

$a \cdot c = 64$

| | sum |
|---|---|
| 64, 1 | 65 |

$= 16x^2+64x+x+4$

$= 16x(x+4)+1(x+4)$

$= \boxed{(x+4)(16x+1)}$

6. Factor Completely.

a) $14x^3y^4 + 3x^2y^5 - 2xy^6$

$= xy^4(14x^2 + 3xy - 2y^2)$

| $a = 14$ | $a \cdot c = -28$ | sum |
|---|---|---|
| $b = 3$ | $-28 \cdot 1$ | $-27$ |
| $c = -2$ | $-14 \cdot 2$ | $-12$ |
| | $-7 \cdot 4$ | $-3$ |
| | $7 \cdot -4$ | $3$ |

$= xy^4[14x^2 + 7xy - 4xy - 2y^2]$

$= xy^4[7x(2x+y) - 2y(2x+y)]$

$= \boxed{xy^4(2x+y)(7x-2y)}$

b) $2y^3z - 3y^2z^2 - 27yz^3$

$= yz[2y^2 - 3yz - 27z^2]$

| $a = 2$ | $a \cdot c = -54$ | sum |
|---|---|---|
| $b = -3$ | $-54 \cdot 1$ | $-53$ |
| $c = -27$ | $-27 \cdot 2$ | $-25$ |
| | $-9 \cdot 6$ | $-3$ |

$= yz[2y^2 - 9yz + 6yz - 27z^2]$

$= yz[y(2y-9z) + 3z(2y-9z)]$

$= \boxed{yz(2y-9z)(y+3z)}$

7. Solve for x.

a) $x^2 - 3ax - 10a^2 = 0$

| $a = 1$ | $a \cdot c = -10$ | sum |
|---|---|---|
| $b = -3$ | $-10 \cdot 1$ | $-9$ |
| $c = -10$ | $-5 \cdot 2$ | $-3$ |

$x^2 - 5ax + 2ax - 10a^2 = 0$

$x(x-5a) + 2a(x-5a) = 0$

$(x-5a)(x+2a) = 0$

$x - 5a = 0$ (+5a +5a) $\quad x + 2a = 0$ (−2a −2a)

$\boxed{x = 5a} \qquad \boxed{x = -2a}$

b) $3x(x-5) = -12$

$3x^2 - 15x = -12$

$\frac{1}{3}[3x^2 - 15x] = \frac{1}{3}[-12]$

$x^2 - 5x = -4$

$+4 \quad +4$

$x^2 - 5x + 4 = 0$

| $a = 1$ | $a \cdot c = 4$ | sum |
|---|---|---|
| $b = -5$ | $-4 \cdot -1$ | $-5$ |
| $c = 4$ | | |

$a = 1$: shortcut! $(x-4)(x-1) = 0$

$x - 4 = 0$ $\quad x - 1 = 0$

$\boxed{x = 4} \qquad \boxed{x = 1}$

8. Factor Completely. Use the note below for parts b) and c).

**Note:** $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ and $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

a) $x^4 - 81$

$= (x^2)^2 - 9^2$

$= (x^2+9)(x^2-9)$

$= \boxed{(x^2+9)(x+3)(x-3)}$

b) $y^3 + 8$

$= y^3 + 2^3$

$= (y+2)(y^2 - 2 \cdot y + 2^2)$

$= \boxed{(y+2)(y^2 - 2y + 4)}$

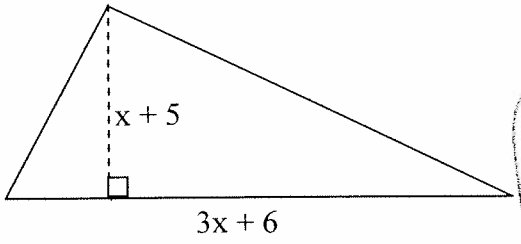
c) $27y^3 - 8$

$= (3y)^3 - 2^3$

$= (3y-2)[(3y)^2 + (3y)(2) + 2^2]$

$= \boxed{(3y-2)(9y^2 + 6y + 4)}$

9. The area of this triangle is 60 square inches. Find the height and the base of this triangle. Set up an equation an solve. $A = \frac{1}{2}bh$




$60 = \frac{1}{2}(3x+6)(x+5)$

$60 = \frac{1}{2}[x(3x+6)+5(3x+6)]$

$60 = \frac{1}{2}[3x^2+6x+15x+30]$

$60 = \frac{1}{2}[3x^2+21x+30]$

★ mult both sides by 2: $120 = 3x^2+21x+30$

★ divide both sides by 3: $40 = x^2+7x+10$

$-40 \quad -40$

$0 = x^2+7x-30$

$x^2+7x-30=0$

$a=1$, $b=7$, $c=-30$

$a \cdot c = -30$

| | sum |
|---|---|
| $-30 \cdot 1$ | $-29$ |
| $-15 \cdot 2$ | $-13$ |
| $-5 \cdot 6$ | $1$ |
| $-10 \cdot 3$ | $-7$ |
| $10 \cdot -3$ | $7$ |

$a=1$: short cut!

$(x+10)(x-3)=0$

$x+10=0 \quad x-3=0$

$-10 \quad -10 \quad +3 \quad +3$

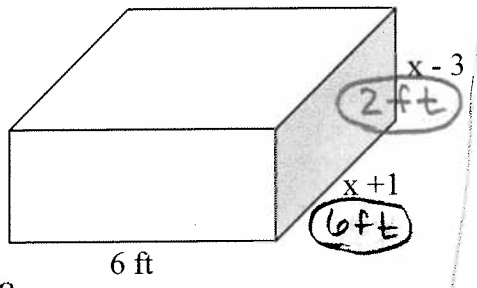
$x=-10 \quad x=3$

$x=3$: $b = 3(3)+6 = 9+6 = 15$

$h = (3)+5 = 8$

$b=15,\ h=8$

10. The box below has a volume of 72 $ft^3$. Set up an equation and solve for x. Then label the width and the height of the box.




$V = lwh$

$72 = 6(x+1)(x-3)$

★ divide both sides by 6: $12 = (x+1)(x-3)$

$12 = x(x+1)-3(x+1)$

$12 = x^2+x-3x-3$

$12 = x^2-2x-3$

$-12 \quad -12$

$0 = x^2-2x-15$

$x^2-2x-15=0$

$a=1$, $b=-2$, $c=-15$

$a \cdot c = -15$

| | sum |
|---|---|
| $-15 \cdot 1$ | $-14$ |
| $-5 \cdot 3$ | $-2$ |

$a=1$: Short cut!

$(x-5)(x+3)=0$

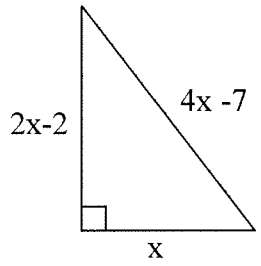
$x-5=0 \quad x+3=0$

$+5 \quad +5 \quad -3 \quad -3$

$x=5 \quad x=-3$

11. Write out an equation that relates the three sides of the right triangle below. Then solve this equation for x. Then label the length of the sides of the triangle.

$c^2 = a^2 + b^2$




$(4x-7)^2 = (2x-2)^2 + x^2$

$(4x-7)(4x-7) = (2x-2)(2x-2) + x^2$

$4x(4x-7)-7(4x-7) = 2x(2x-2)-2(2x-2) + x^2$

$16x^2-28x-28x+49 = 4x^2-4x-4x+4+x^2$

$16x^2-56x+49 = 5x^2-8x+4$

$-5x^2+8x-4 \quad -5x^2+8x-4$

$11x^2-48x+45=0$

$a=11$
$b=-48$
$c=45$

$a \cdot c = 495$

| | SUM |
|---|---|
| 495 1 | 496 |
| 165 3 | 168 |
| 55 9 | 64 |
| 11 45 | 56 |
| 33 15 | 48 |
| −33 −15 | −48 |

$11x^2-33x-15x+45=0$

$11x(x-3)-15(x-3)=0$

$(x-3)(11x-15)=0$

$x-3=0$ $\quad 11x-15=0$

$+3 \quad +3 \quad +15 \quad +15$

$x=3 \quad \frac{11x}{11}=\frac{15}{11}$

$x=\frac{15}{11}$

If $x=\frac{15}{11}$, then

$4x-7 = 4\left(\frac{15}{11}\right)-7$

$= \frac{60}{11}-7$

$= \frac{60}{11}-\frac{77}{11}$

$= -\frac{17}{11}$

Not Possible!

SOLUTION

So, $x=3$.