

Math251

Practice Exam #04

SOLUTIONS

1. Multiply or divide as indicated.

$$\begin{aligned} \text{a) } & \frac{8x-24}{3x+12} \cdot \frac{7x+28}{5x-15} \\ & = \frac{8(x-3)}{3(x+4)} \cdot \frac{7(x+4)}{5(x-3)} \\ & = \frac{8(x-3) \cdot 7(x+4)}{3(x+4) \cdot 5(x-3)} \\ & = \boxed{\frac{56}{15}} \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{x^2-3x-10}{x^2-4x-5} \div \frac{4x+8}{8x+8} \\ & = \frac{x^2-3x-10}{x^2-4x-5} \cdot \frac{8x+8}{4x+8} \\ & = \frac{(x-5)(x+2)}{(x-5)(x+1)} \cdot \frac{8(x+1)}{4(x+2)} \\ & = \frac{(x-5)(x+2) \cdot 8(x+1)}{(x-5)(x+1) \cdot 4(x+2)} = \frac{2}{1} = \boxed{2} \end{aligned}$$

Note: For both trinomials, $a=1$, so we can use the shortcut!

2. Add or Subtract as indicated and simplify.

$$\begin{aligned} \text{a) } & \frac{-2p}{p-4} - \frac{8}{4-p} \\ & = \frac{-2p}{p-4} - \frac{8}{-(p-4)} \\ & = \frac{-2p}{p-4} + \frac{8}{p-4} \\ & = \frac{-2p+8}{p-4} = \frac{-2(p-4)}{p-4} \\ & = \boxed{-2} \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{1}{2x^2+13x+6} - \frac{1}{2x^2-x-1} \\ & \quad \swarrow \quad \searrow \\ & \quad \begin{array}{l} a=2 \\ b=13 \\ c=6 \\ a \cdot c = 12 \\ \begin{array}{c} \wedge \\ 12 \quad 1 \end{array} \quad \text{sum} \end{array} \quad \begin{array}{l} a=2 \\ b=-1 \\ c=-1 \\ a \cdot c = -2 \\ \begin{array}{c} \wedge \\ -2 \quad 1 \end{array} \quad \text{sum} \end{array} \\ & \quad \begin{array}{l} 2x^2+12x+x+6 \\ = 2x(x+6)+1(x+6) \\ = (2x+1)(x+6) \end{array} \quad \begin{array}{l} 2x^2-2x+x-1 \\ = 2x(x-1)+1(x-1) \\ = (2x+1)(x-1) \end{array} \\ & = \frac{1}{(2x+1)(x+6)} - \frac{1}{(2x+1)(x-1)} \\ & \quad \text{LCD} = (2x+1)(x+6)(x-1) \\ & = \frac{(x-1) - (x+6)}{(2x+1)(x+6)(x-1)} \\ & = \frac{x-1-x-6}{(2x+1)(x+6)(x-1)} = \boxed{\frac{-7}{(2x+1)(x+6)(x-1)}} \end{aligned}$$

3. Simplify each complex fraction.

a) $\frac{\frac{2}{x} - 2}{5 - \frac{5}{x}}$ LCD = x

$$\frac{x \left(\frac{2}{x} - 2 \right)}{x \left(5 - \frac{5}{x} \right)} = \frac{2 - 2x}{5x - 5}$$

$$= \frac{2(1-x)}{5(x-1)} = \frac{-2(x-1)}{5(x-1)}$$

$$= \boxed{-\frac{2}{5}}$$

b) $\frac{\frac{1}{x^3y} + \frac{2}{xy^2}}{\frac{4}{xy} + \frac{1}{x^2y}}$ LCD = x^3y^2

$$= \frac{x^3y^2 \left[\frac{1}{x^3y} + \frac{2}{xy^2} \right]}{x^3y^2 \left[\frac{4}{xy} + \frac{1}{x^2y} \right]}$$

$$= \frac{y + 2x^2}{4x^2y + xy} = \boxed{\frac{y + 2x^2}{xy(4x+1)}}$$

4. Solve each equation.

a) $\frac{3x}{4} - \frac{x-5}{3} = x$ LCD = 12

$$12 \left[\frac{3x}{4} - \frac{(x-5)}{3} \right] = 12[x]$$

$$3(3x) - 4(x-5) = 12x$$

$$9x - 4(x-5) = 12x$$

$$9x - 4x + 20 = 12x$$

$$5x + 20 = 12x$$

$$\begin{array}{r} 5x + 20 = 12x \\ -5x \quad \quad -5x \\ \hline \end{array}$$

$$\frac{20}{7} = \frac{7x}{7}$$

$$\boxed{x = \frac{20}{7}}$$

b) $\frac{2x}{x-3} + \frac{1}{x+3} = \frac{2x}{x^2-9}$

$$\frac{2x}{x-3} + \frac{1}{x+3} = \frac{2x}{(x+3)(x-3)}$$

$$\text{LCD} = (x+3)(x-3)$$

$$(x+3)(x-3) \left[\frac{2x}{x-3} + \frac{1}{x+3} \right] = (x+3)(x-3) \left[\frac{2x}{(x+3)(x-3)} \right]$$

$$2x(x+3) + (x-3) = 2x$$

$$2x^2 + 6x + x - 3 = 2x$$

$$2x^2 + 7x - 3 = 2x$$

$$\begin{array}{r} 2x^2 + 7x - 3 = 2x \\ -2x \quad \quad -2x \\ \hline \end{array}$$

$$2x^2 + 5x - 3 = 0$$

$a=2$	$a.c = -6$	Sum
$b=5$	$\begin{array}{c} \wedge \\ -6 \quad 1 \end{array}$	-5
$c=-3$	$\begin{array}{c} 6 \quad -1 \end{array}$	5

$$2x^2 + 6x - x - 3 = 0$$

$$2x(x+3) - 1(x+3) = 0$$

$$(x+3)(2x-1) = 0$$

$$x+3=0 \quad 2x-1=0$$

$$x = -3 \quad 2x = 1 \Rightarrow x = \frac{1}{2}$$

Note: This is an EXTRANEous solution \rightarrow

5. A large pump can empty a pool in 6 hours, and a smaller pump can do it in 24 hours. How long will it take the two pumps working together to empty half of the pool?

Complete the table and set up an equation to solve this problem

	Rate	Time	Task
large	$\frac{1}{6}$	t	$\frac{t}{6}$
small	$\frac{1}{24}$	t	$\frac{t}{24}$
			$\frac{1}{2}$

$$\frac{t}{6} + \frac{t}{24} = \frac{1}{2}$$

$$24 \left[\frac{t}{6} + \frac{t}{24} \right] = 24 \left[\frac{1}{2} \right]$$

$$4t + t = 12$$

$$5t = 12$$

$$t = \frac{12}{5} \text{ hrs.}$$

6. If x varies inversely as y , and $x = \frac{1}{10}$ when $y = 5$, find y when $x = 6$.

$$x = \frac{k}{y}$$

$$\frac{1}{10} = \frac{k}{5}$$

$$5 = 10k$$

$$k = \frac{5}{10}$$

$$k = \frac{1}{2}$$

$$x = \frac{\frac{1}{2}}{y}$$

$$6 = \frac{\frac{1}{2}}{y}$$

$$6y = \frac{1}{2}$$

$$y = \frac{1}{12}$$

7. Sally can row 5 mph in still water. It takes her the same amount of time to row 30 miles upstream as it does to row 90 miles downstream. How fast is the current?

Complete the table and set up an equation to solve this problem.

	Rate \times Time = Distance		
upstream	$5 - x$	$\frac{30}{5 - x}$	30
downstream	$5 + x$	$\frac{90}{5 + x}$	90

$$R_{\text{Boat}} = 5 \text{ mph}$$

$$R_{\text{Current}} = x$$

$$\frac{30}{5 - x} = \frac{90}{5 + x}$$

$$\text{LCD} = (5 - x)(5 + x)$$

$$(5 - x)(5 + x) \left[\frac{30}{5 - x} \right] = (5 - x)(5 + x) \left[\frac{90}{5 + x} \right]$$

$$30[5 + x] = 90[5 - x]$$

$$150 + 30x = 450 - 90x$$

$$+ 90x \quad \quad + 90x$$

$$150 + 120x = 450$$

$$- 150 \quad \quad - 150$$

$$120x = 300$$

$$\frac{120}{120} \quad \frac{300}{120}$$

$$x = \frac{5}{2} \text{ mph}$$