Math251
Practice Exam #05

1. Re-write each radical expression using only positive exponents.

   a) $\sqrt[5]{x^2}$

   $= x^{\frac{2}{5}}$

   b) $\sqrt[3]{27x^{12}y^9}$

   $= 3x^4 y^3$

   c) $\sqrt[3]{\sqrt[5]{x^2}}$

   $= \sqrt[3]{x^{2/5}}$

   $= x^{\frac{2/5}{3}}$

   $= x^{\frac{2}{5} \cdot \frac{1}{3}}$

   $= x^{\frac{2}{15}}$

2. Simplify the following.

   a) $6\sqrt{18} - 5\sqrt{32}$

   $= 6\sqrt{9 \cdot 2} - 5\sqrt{16 \cdot 2}$

   $= 6 \cdot 3\sqrt{2} - 5 \cdot 4\sqrt{2}$

   $= 18\sqrt{2} - 20\sqrt{2}$

   $= \boxed{-2\sqrt{2}}$

   b) $5\sqrt{6} - 3\sqrt{24} + 6\sqrt{54}$

   $= 5\sqrt{6} - 3\sqrt{4 \cdot 6} + 6\sqrt{9 \cdot 6}$

   $= 5\sqrt{6} - 3 \cdot 2\sqrt{6} + 6 \cdot 3\sqrt{6}$

   $= 5\sqrt{6} - 6\sqrt{6} + 18\sqrt{6}$

   $= -\sqrt{6} + 18\sqrt{6}$

   $= \boxed{17\sqrt{6}}$

   c) $3\sqrt{3} \cdot 6\sqrt{3}$

   $= 3 \cdot 6 \cdot \sqrt{3} \cdot \sqrt{3}$

   $= 18 \cdot 3$

   $= \boxed{54}$

3. Simplify the radical expressions.

   a) $\sqrt[3]{32x^5y^3z^9}$

   $= \sqrt[3]{2^5 x^5 y^3 z^9}$

   $= \sqrt[3]{2^3 2^2 x^3 x^2 y^3 z^9}$

   $= 2xyz^3 \sqrt[3]{2^2 x^2}$

   $= \boxed{2xyz^3 \sqrt[3]{4x^2}}$

   b) $\sqrt[4]{32x^5y^3z^9}$

   $= \sqrt[4]{2^5 x^5 y^3 z^9}$

   $= \sqrt[4]{2^4 \cdot 2 x^4 x y^3 z^8 z}$

   $= \boxed{2xz^2 \sqrt[4]{2xy^3z}}$

   c) $\sqrt[5]{32x^5y^3z^9}$

   $= \sqrt[5]{2^5 \cdot x^5 \cdot y^3 \cdot z^9}$

   $= \sqrt[5]{2^5 x^5 \cdot y^3 z^5 \cdot z^4}$

   $= \boxed{2xz \sqrt[5]{y^3z^4}}$

4. Solve for x.

a) $x^2 = 25$

$$\sqrt{x^2} = \pm\sqrt{25}$$

$$\boxed{x = \pm 5}$$

b) $x^2 - 3x = 0$

$$x(x-3) = 0$$

$$x = 0 \qquad x - 3 = 0$$
$$ \qquad +3 \quad +3$$

$$\boxed{x = 0} \qquad \boxed{x = 3}$$

c) $2x^2 - 5x - 3 = 0$

$a \cdot c = -6$

$-6 \quad 1$ sum -5

$$2x^2 - 6x + x - 3 = 0$$
$$2x(x-3) + 1(x-3) = 0$$
$$(x-3)(2x+1) = 0$$
$$x - 3 = 0 \qquad 2x + 1 = 0$$
$$\boxed{x = 3} \qquad 2x = -1$$
$$\boxed{x = -\frac{1}{2}}$$

5. Solve for x.

a) $\sqrt{4x+1} = 3$

$$(\sqrt{4x+1})^2 = 3^2$$
$$4x + 1 = 9$$
$$\quad -1 \quad -1$$
$$\frac{4x}{4} = \frac{8}{4}$$

$$\boxed{x = 2}$$

b) $\sqrt{7-3x} - 4 = 0$
$$\qquad +4 \ +4$$
$$\sqrt{7-3x} = 4$$
$$(\sqrt{7-3x})^2 = 4^2$$
$$7 - 3x = 16$$
$$-7 \qquad -7$$
$$\frac{-3x}{-3} = \frac{9}{-3}$$

$$\boxed{x = -3}$$

c) $\sqrt{x} = 4$

$$(\sqrt{x})^2 = 4^2$$

$$\boxed{x = 16}$$

6. Solve by completing the square.

a) $x^2 - 4x + 2 = 0$

$$x^2 - 4x + 2 = 0$$
$$\quad -2 \ -2$$
$$x^2 - 4x = -2$$
$$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$$
$$x^2 - 4x + 4 = -2 + 4$$
$$(x-2)^2 = 2$$
$$\sqrt{(x-2)^2} = \pm\sqrt{2}$$
$$x - 2 = \pm\sqrt{2}$$
$$+2 \qquad +2$$

$$\boxed{x = 2 \pm \sqrt{2}}$$

b) $18x^2 - 6x - 1 = 0$

$$18x^2 - 6x - 1 = 0$$
$$\qquad +1 \ +1$$
$$\frac{18x^2}{18} - \frac{6x}{18} = \frac{1}{18}$$
$$x^2 - \frac{1}{3}x = \frac{1}{18}$$
$$\left(\frac{b}{2}\right)^2 = \left[\frac{-\frac{1}{3}}{2}\right]^2 = \left(-\frac{1}{6}\right)^2 = \frac{1}{36}$$
$$x^2 - \frac{1}{3}x + \frac{1}{36} = \frac{1}{18} + \frac{1}{36}$$
$$\left(x - \frac{1}{6}\right)^2 = \frac{2}{36} + \frac{1}{36}$$
$$\left(x - \frac{1}{6}\right)^2 = \frac{3}{36}$$
$$\left(x - \frac{1}{6}\right)^2 = \frac{1}{12}$$
$$\sqrt{\left(x - \frac{1}{6}\right)^2} = \pm\sqrt{\frac{1}{12}}$$
$$x - \frac{1}{6} = \pm\frac{1}{\sqrt{12}}$$
$$x - \frac{1}{6} = \pm\frac{1}{\sqrt{4 \cdot 3}}$$
$$x - \frac{1}{6} = \pm\frac{1}{2\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$$
$$x - \frac{1}{6} = \pm\frac{\sqrt{3}}{6}$$
$$+\frac{1}{6} \qquad +\frac{1}{6}$$
$$x = \frac{1}{6} \pm \frac{\sqrt{3}}{6} = \boxed{\frac{1 \pm \sqrt{3}}{6}}$$

c) $2x^2 - 6x + 2 = 0$

$$2x^2 - 6x + 2 = 0$$
$$\qquad -2 \ -2$$
$$\frac{2x^2}{2} - \frac{6x}{2} = \frac{-2}{2}$$
$$x^2 - 3x = -1$$
$$\left(\frac{b}{2}\right)^2 = \left(\frac{-3}{2}\right)^2 = \frac{9}{4}$$
$$x^2 - 3x + \frac{9}{4} = -1 + \frac{9}{4}$$
$$\left(x - \frac{3}{2}\right)^2 = -\frac{4}{4} + \frac{9}{4}$$
$$\left(x - \frac{3}{2}\right)^2 = \frac{5}{4}$$
$$\sqrt{\left(x - \frac{3}{2}\right)^2} = \pm\sqrt{\frac{5}{4}}$$
$$x - \frac{3}{2} = \pm\frac{\sqrt{5}}{\sqrt{4}}$$
$$x - \frac{3}{2} = \pm\frac{\sqrt{5}}{2}$$
$$+\frac{3}{2} \qquad +\frac{3}{2}$$
$$x = \frac{3}{2} \pm \frac{\sqrt{5}}{2} = \boxed{\frac{3 \pm \sqrt{5}}{2}}$$

7. Rationalize the denominators.

a) $\dfrac{3}{\sqrt{2}}$

$= \dfrac{3}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{3\sqrt{2}}{\sqrt{4}}$

$= \boxed{\dfrac{3\sqrt{2}}{2}}$

b) $\dfrac{3}{2-\sqrt{2}}$

$= \dfrac{3}{(2-\sqrt{2})} \cdot \dfrac{(2+\sqrt{2})}{(2+\sqrt{2})}$

$= \dfrac{3(2+\sqrt{2})}{2^2-(\sqrt{2})^2}$

$= \dfrac{3(2+\sqrt{2})}{4-2}$

$= \boxed{\dfrac{3(2+\sqrt{2})}{2}}$

c) $\dfrac{3}{\sqrt[3]{4}}$

$= \dfrac{3}{\sqrt[3]{2^2}}$

$= \dfrac{3}{\sqrt[3]{2^2}} \cdot \dfrac{\sqrt[3]{2}}{\sqrt[3]{2}}$

$= \dfrac{3\sqrt[3]{2}}{\sqrt[3]{2^3}}$

$= \boxed{\dfrac{3\sqrt[3]{2}}{2}}$

8. Simplify the radical expressions.

a) $\sqrt{24}$

$\sqrt{24} = \sqrt{4 \cdot 6}$

$= \boxed{2\sqrt{6}}$

b) $\sqrt[3]{54}$

$\sqrt[3]{54}$

$= \sqrt[3]{27 \cdot 2}$

$= \boxed{3\sqrt[3]{2}}$

c) $\left(\sqrt{3}-\sqrt{4}\right)\left(\sqrt{3}+\sqrt{5}\right)$

$= \left(\sqrt{3}-2\right)\left(\sqrt{3}+\sqrt{5}\right)$

$= \sqrt{3}\left(\sqrt{3}-2\right)+\sqrt{5}\left(\sqrt{3}-2\right)$

$= \boxed{3-2\sqrt{3}+\sqrt{15}-2\sqrt{5}}$

9. Solve by using the quadratic equation. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

a) $x^2 - 4x + 2 = 0$ 　　　　b) $2x^2 - 5x - 3 = 0$ 　　　　c) $2x^2 - 6x + 2 = 0$

a) $x^2 - 4x + 2 = 0$

$a = 1$
$b = -4$
$c = 2$

$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$

$= \dfrac{4 \pm \sqrt{16 - 8}}{2} = \dfrac{4 \pm \sqrt{8}}{2}$

$= \dfrac{4 \pm \sqrt{4 \cdot 2}}{2} = \dfrac{4 \pm 2\sqrt{2}}{2} = \dfrac{2(2 \pm \sqrt{2})}{2}$

$$\boxed{x = 2 \pm \sqrt{2}}$$

b) $2x^2 - 5x - 3 = 0$

$a = 2$
$b = -5$
$c = -3$

$x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$

$= \dfrac{5 \pm \sqrt{25 + 24}}{4} = \dfrac{5 \pm \sqrt{49}}{4} = \dfrac{5 \pm 7}{4}$

$\Rightarrow \dfrac{5 + 7}{4}$ & $\dfrac{5 - 7}{4} \Rightarrow \dfrac{12}{4}$ & $\dfrac{-2}{4} \Rightarrow 3$ & $-\dfrac{1}{2}$

$$\boxed{\begin{array}{l} x = 3 \\ x = -\dfrac{1}{2} \end{array}}$$

c) $2x^2 - 6x + 2 = 0$

$a = 2$
$b = -6$
$c = 2$

$x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(2)}}{2(2)}$

$= \dfrac{6 \pm \sqrt{36 - 16}}{4} = \dfrac{6 \pm \sqrt{20}}{4}$

$= \dfrac{6 \pm \sqrt{4 \cdot 5}}{4} = \dfrac{6 \pm 2\sqrt{5}}{4} = \dfrac{2(3 \pm \sqrt{5})}{4}$

$$\boxed{x = \dfrac{3 \pm \sqrt{5}}{2}}$$