Consider the goodness of fit statistic:

$$\mathrm{KS}_n := \sup_x |Y_n(x;\hat{\theta})|, \quad Y_n(x;\hat{\theta}) = \sqrt{n}(F_n(x) - F(x;\hat{\theta}_n)),$$

where F_n denotes the empirical distribution function based on $X_1, \ldots, X_n$. We note that

$$Y_n(x;\hat{\theta}) = \sqrt{n}(F_n(x) - F(x)) - \sqrt{n}(F(x;\hat{\theta}_n) - F(x)),$$

where $F(x)$ is the true cdf (under the null $F(x) = F(x;\theta_0)$ for some true parameter θ_0).

Let us first consider the case where X_i's are i.i.d. Denote by F_n^* the empirical distribution of the bootstrap samples $X_1^*, \ldots, X_n^*$ and let $\hat{\theta}_n^*$ be the parameter estimate based on the bootstrap samples. Using the bootstrap (asymptotic) theory, we can approximate the distribution of

$$\sqrt{n}(F_n(x) - F(x))$$

by that of

$$\sqrt{n}(F_n^*(x) - F_n(x)),$$

and the distribution of

$$\sqrt{n}(F(x;\hat{\theta}_n) - F(x))$$

by that of

$$\sqrt{n}(F(x;\hat{\theta}_n^*) - F(x;\hat{\theta}_n)).$$

Therefore, if we define

$$\begin{aligned} Y_n^*(x) =& \sqrt{n}(F_n^*(x) - F_n(x)) - \sqrt{n}(F(x;\hat{\theta}_n^*) - F(x;\hat{\theta}_n)) \\ =& \sqrt{n}(F_n^*(x) - F(x;\hat{\theta}_n^*)) - \sqrt{n}(F_n(x) - F(x;\hat{\theta}_n)), \end{aligned}$$

then $\mathrm{KS}_n^* := \sup_x |Y_n^*(x)|$ is the bootstrap statistic that is expected to approximate the distribution of KS_n. We note that the term $\sqrt{n}(F_n(x) - F(x;\hat{\theta}_n))$ is exactly the bias term considered in Babu and Rao (2004).

We now consider the case where X_i's are realizations from a time series and $X_1^*, \ldots, X_n^*$ are generated by block bootstrap. In this case, we can approximate the distribution of

$$\sqrt{n}(F_n(x) - F(x))$$

by that of

$$\sqrt{n}(F_n^*(x) - E^*[F_n^*(x)]),$$

and the distribution of

$$\sqrt{n}(F(x;\hat{\theta}_n) - F(x))$$

by that of

$$\sqrt{n}(F(x;\hat{\theta}_n^*) - F(x;E^*[\hat{\theta}_n^*])).$$

Here E^* denotes the expectation with respect to the bootstrap distribution (i.e., the randomness due to the resampling using block bootstrap) conditional on the observations $X_1, \ldots, X_n$. We can compute $E^*[F_n^*(x)]$ and $E^*[\hat{\theta}_n^*]$ numerically (they can also be computed analytically, depending on the types of block bootstrap we use). For example, let $X_{1,b}^*, \ldots, X_{n,b}^*$ be the bth block bootstrap sample for $1 \leq b \leq B$. One can compute $F_{n,b}^*$ and $\hat{\theta}_{n,b}^*$ based on $X_{1,b}^*, \ldots, X_{n,b}^*$. Then

$$E^*[F_n^*(x)] \approx \frac{1}{B}\sum_{b=1}^{B} F_{n,b}^*(x),$$

$$E^*[\hat{\theta}_n^*] \approx \frac{1}{B}\sum_{b=1}^{B} \hat{\theta}_{n,b}^*.$$

In this case, we can define

$$\begin{aligned} Y_n^*(x) =& \sqrt{n}(F_n^*(x) - E^*[F_n^*(x)]) - \sqrt{n}(F(x;\hat{\theta}_n^*) - F(x;E^*[\hat{\theta}_n^*]) \\ =& \sqrt{n}(F_n^*(x) - F(x;\hat{\theta}_n^*)) - \sqrt{n}(E^*[F_n^*(x)] - F(x;E^*[\hat{\theta}_n^*])), \end{aligned}$$

and $\mathrm{KS}_n^* = \sup_x |Y_n^*(x)|$.