A First Amplitude

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Abstract

We work through an elementary S-matrix calculation in scalar Yukawa theory following the popular exposition by David Tong. For pedagogical purposes, we proceed slowly and begin with a self-contained review of the necessary background in QFT.

0 Review of prequisite facts

0.1 The Interaction Picture

Frequently we are interested in systems with Hamiltonians of the form

$$H = H_0 + H_{\text{int}}$$

where H_0 corresponds to a free theory with known spectrum. In that case it is helpful to transition into the *interaction picture*, defined in terms of the Schrödinger picture as follows. First, H_0 is defined to agree with the Schrödinger picture:

$$(H_0)_I = (H_0)_S.$$

Operators in general time evolve as

$$\mathcal{O}_I(t) = e^{iH_0 t} \mathcal{O}_s e^{-iH_0 t}.$$

States evolve as

$$\left|\psi(t)\right\rangle_I = e^{iH_0 t} \left|\psi(t)\right\rangle_S.$$

The dynamics of the states are then governed by the interaction Hamiltonian:

$$i\frac{d\,|\psi(t)\rangle_I}{dt} = H_I(t)\,|\psi(t)\rangle_I$$

where we use the shorthand

$$H_I(t) \equiv (H_{\text{int}})_I(t).$$

0.2 Dyson's formula

It's useful to write the dynamics of a theory in terms of a unitary time-evolution operator U which acts as

$$|\psi(t)\rangle_I = U(t,t_0)\,|\psi(t_0)\rangle_I\,.$$

This is soluble in terms of the familiar time-ordering operator as

$$U(t,t_o) = T\left(-i\int_{t_0}^{f} H_I(t')dt'\right).$$