Revisit: KKT optimality conditions

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1 To read

1.1 Read these sections in B & V's Convex Optimization.

- 3.4: Quasiconvex functions. Include optimality conditions for quasiconvex problems. The objective function is quasiconvex and the feasible set formed by constraints can be replaced with convex functions.

- 4.2.2: Local and global optima. I may need to introduce these concepts in the introduction part of my article. Also, Amir Beck's book also presents these concepts well.

- 4.2.3: An optimality criterion for differentiable f_0.
  This is an optimality criterion. What I want to do is to compare this with the optimality conditions within open and closed domains for unconstrained problems and constrained problems. Note that there is no convexity assumption on the objective function. This is another core work in this article.

  `https://inst.eecs.berkeley.edu/~ee127/sp21/livebook/l_sdual_main.html` "For convex problems where strong duality holds, and both primal and dual optimal values are attained, we can proceed to derive necessary and sufficient conditions for optimality, **which are more amenable to algorithms than the supporting hyperplane conditions given here.**" This is a good comment on the optimality criterion. Some examples are available at `https://inst.eecs.berkeley.edu/~ee127/sp21/livebook/l_cp_pbs.html`

- 5.2.4: A nonconvex quadratic problem with strong duality. Figure out this example. Also, read B.1 for "Single constraint quadratic optimization".

  Normally, strong duality requires convexity of the original problem. This section suggests that convexity of a problem itself is not necessary for strong duality, but convexity together with Slater's condition are sufficient. A counterexample is given in this part and the proof for sufficiency is available in 5.3.2. These will be an important part of my article.

- 5.3.2: Proof of strong duality under constraint qualification

- 5.5.1: Certificate of suboptimality and stopping criteria

- 5.5.2: Complementary slackness

- 5.5.3: KKT optimality conditions (for nonconvex problems and convex problems)

1.2 Read Ryan Tibshirani's lecture slides on KKT

I may make comments on his slides in my article.

- When subdifferentials are employed in the stationarity, KKT conditions are always sufficient. See slides 5 and 9.
- On Slide 9, when f is differentiable, $\partial f = \{\nabla\}$ does not necessarily hold. It only holds for convex f. I will illustrate this point in my article. This is critical for the sufficiency of KKT conditions.

1.3 Read Amir Beck's Introduction to Nonlinear Optimization Theory, Algorithms, and Applications with MATLAB by Amir Beck.

- Refer to the definition for stationary points. I need to introduce this concepts in my article.
- Read its Chapter 2 for optimality
- Read Chapter 10 and 11 for KKT condtions

1.4 Read Vandenburghe's lecture slides on subgradients.

- `http://www.seas.ucla.edu/~vandenbe/236C/lectures/subgradients.pdf` Optimality conditions with subdifferentials and a concise for unconstrained problems. Also, I give more details on this in my notes on optimization theory.

1.5 VECTOR CALCULUS, LINEAR ALGEBRA, AND DIFFERENTIAL FORMSA-UNIFIED APPROACH

- Chapter 1: Five big theorems (The existence of a convergent subsequence in a compact set; Continuous functions on compact sets)
- Chapter 3.6-3.7: Critical points; Behavior of a function near a saddle

1.6 Read Matrix differential calculus

Chapter 7 Static optimization

1.7 Peruse web resources

- `https://inst.eecs.berkeley.edu/~ee227a/fa10/login/l_dual_strong.html` It talks about strong duality with convex quadratic problems. I'll put that nonconvex quadratic problem with strong duality and convex quadratic problems together. At last, this link gives some examples on strong duality. The first one is a counterexample which may be used in my article, i.e., convexity does not necessarily imply strong duality. The example of e^{-x} in this link does not satisfy Slater's condition.
- `https://math.stackexchange.com/questions/3783596/sufficiency-of-kkt-in-nonconvex-case` It talks about the nonconvex case with subdifferentials, which will be a core of the article I am going to work on.
- `https://math.stackexchange.com/questions/3616646/question-about-kkt-conditions-and-strong-duali` It presents some important questions on KKT condtitions and contains some good examples in its answers.

- https://math.stackexchange.com/questions/3680509/how-to-use-the-kkt-conditions-for-a-not-diffe
  To read the theorems on stationarity for some special cases for complementing this article. Specifically, I need to refer to Ruszczyński, Andrzej's Nonlinear Optimization book for nondifferentiable functions.

- https://math.stackexchange.com/questions/218337/sufficient-condition-for-kkt-problems
  It contains an answer to the proof of sufficiency.

- https://math.stackexchange.com/questions/2513300/is-kkt-conditions-necessary-and-sufficient-fo
  It contains a useful example for understanding KKT points.

- http://mdav.ece.gatech.edu/ece-6270-spring2021/notes/14-kkt-conditions.pdf It provides notes on KKT.

- https://www.stat.cmu.edu/~ryantibs/convexopt/lectures/dual-gen.pdf See Page 13 for a nonconvex quadratic minimization problem. How can we get the dual?

1.8 Writing

In terms of writing, I would like to follow Boyd's Proximal Algorithms.

2 Introduction

Bibliography