

$$\begin{aligned}
Y(N, C_2, H_2, W_2) &:= Conv2d \left[ X(N, C_1, H_1, W_1) \right] \\
&= \bigwedge_{c_2 \in C_2} \left[ \sum_{c_1 \in C_1} \overbrace{\omega(c_2, c_1, K, K)}^{\text{weights}} * \overbrace{X(N, c_1, H_1, W_1)}^{\text{data/features}} + \overbrace{b(N, c_2, H_2, W_2)}^{\text{bias}} \right] \\
&= \bigwedge_{c_2 \in C_2} \left[ \sum_{c_1 \in C_1} \omega_{c_2 c_1} * X_{c_1} + b_{c_2} \right] \\
H_2 &= \left\lfloor \frac{H_1 + 2p - d(K - 1) - 1}{s} + 1 \right\rfloor \\
W_2 &= \left\lfloor \frac{W_1 + 2p - d(K - 1) - 1}{s} + 1 \right\rfloor
\end{aligned}$$

- N = batch size (GPU parallel computation)
- C = channels / num of filters / RGB(3)
- H = height (row indexes)
- W = width (column indexes)
- p = padding = 0
- d = dilation = 1
- s = stride = 1

$$\omega(c_2, c_1, K, K) * X(N, c_1, H_1, W_1) = \bigwedge_{i_1 \in \hat{H}_1} \bigwedge_{j_1 \in \hat{W}_1} \left[ \sum_{i' \in [i_1 \pm \frac{K-1}{2}]} \sum_{j' \in [j_1 \pm \frac{K-1}{2}]} \omega(c_2, c_1, i', j') \cdot X(N, c_1, i', j') \right]$$

so we can have

$$\begin{aligned}
Y(N, c_2, i_2, j_2) &= \bigwedge_{c_2 \in C_2} \bigwedge_{i_2 \in H_2} \bigwedge_{j_2 \in W_2} \left[ \sum_{c_1 \in C_1} \sum_{i' \in [i_1(i_2) \pm \frac{K-1}{2}]} \sum_{j' \in [j_1(j_2) \pm \frac{K-1}{2}]} \omega(c_2, c_1, i', j') \cdot X(N, c_1, i', j') \right] \\
\text{其中 : } i_1(i_2) &= i_2 + \frac{K-1}{2}, j_1(j_2) = j_2 + \frac{K-1}{2}
\end{aligned}$$

finally :

$$\begin{aligned}
&Y(N, C_2, H_2, W_2) \\
&= \bigwedge_{c_2 \in C_2} \bigwedge_{i_2 \in H_2} \bigwedge_{j_2 \in W_2} \left[ \sum_{c_1 \in C_1} \sum_{k \in [0, K-1]} \sum_{k \in [0, K-1]} \omega(c_2, c_1, i_2 + k, j_2 + k) \cdot X(N, c_1, i_2 + k, j_2 + k) \right]
\end{aligned}$$