CS / ISyE 730 - Spring 2016 - Homework 2

(assigned 1/30/16, due in class 2/8/16)

1. Defining the tangent cone as the set of limiting feasible directions, prove that for the polyhedral set $C := \{x \,|\, Dx \le d,\ Gx = g\}$ and a point $x_0 \in C$ with active set $\mathcal{A} := \{i \,|\, D_i x_0 = d_i\}$, we have that

$$T_C(x_0) = \{z \,|\, D_{\mathcal{A}} z \le 0,\ Gz = 0\}.$$

2. Prove the following inhonogeneous version of Farkas's Lemma: Given matrix A, vectors b and c, and scalar β, exactly one of these two statements is true:

   I. $b^T x > \beta$, $Ax \le c$ for some x;

   II. For some y, EITHER $A^T y = b$, $c^T y \le \beta$, and $y \ge 0$ OR $A^T y = 0$, $c^T y < 0$, and $y \ge 0$.

3. Consider the following constraint set $\Omega \subset \mathbf{R}$:

$$\Omega := \{0\} \cup \{x \,|\, \sin(1/x) = 0\},$$

   which has elements 0 and $1/(j\pi)$ for any nonzero integer j.

   (a) What is the tangent cone to Ω at 0? What is the normal cone?

   (b) What is the tangent cone to Ω at any feasible point other than 0? What is the normal cone?

   (c) If f is a smooth function, what condition is required to make 0 a local solution of $\min_{x\in\Omega} f(x)$?

   (d) What condition is required to make x a local minimum of this problem at any feasible point x other that 0?