Statement

The following seems to be the equivalent rewording using the exclusive OR on EITHER ... OR ..."

For given A, b, c, β, exactly one of the following is true:

(I) $\exists x^*$ s.t. $b^T x^* > \beta, Ax^* \leq c$

(II) For some y, exactly one of the following holds:

(IIa) $A^T y = b, y \geq 0$ and $c^T y \leq \beta$

(IIb) $A^T y = 0, y \geq 0$, and $c^T y < 0$

Counterexample

Consider $A = 0$,$b = 0$,$\beta = 0$,$c = -1$ (a trivial case with 1 by 1 matrices and vectors). Suppose the above "theorem" were true; then if (I) was false, then (II) must be true. Then (I) is false since $b^T x = 0x = 0 > \beta = 0$ is false for any x, making (II) true. So then exactly one of (IIa) or (IIb) holds, but not both. But notice that for $y = 1$:

(IIa) $A^T y = 0 * 1 = b = 0, y = 1 \geq 0$ and $c^T y = -1 * 1 = -1 \leq \beta = 0$

(IIb) $A^T y = 0 * 1 = 0, y = 1 \geq 0$, and $c^T y = -1 * 1 = -1 < 0$

Hence, both IIa and IIb are true, contradicting (II) statement that "For some y, exactly one of the following holds".