CS240: Homework 11

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Problem 1

A)

$\epsilon|1|0|(01)|(10)|(11)|(00)$

B)

$00(0|1)^*0$

C)

Assuming that there does not have to be atleast a single "1" (e.g. the empty string is valid):
$(0|100)^*$

D)

$(0^*10^*10^*)^*$
Note by order of operations that this is equivalent to: $((0)^*1(0)^*1(0)^*)^*$

Problem 2

A)

There 26 total capital letters, so we have 26 choices for each initial in a 3-letter initial; hence, we have $26 * 26 * 26 = 17576$ possibilities.

B)

We have 26 choices for the first, 25 for the second (since we can't choose whichever the first was), and 24 for the last one (again, since we can't choose the 2 letters from earlier), so $26 * 25 * 24 = 15600$ possibilities.

C)

If A is the first letter, than we only have freedom in the last 2 letters; each has 26 possible letters, so we have $26 * 26 = 676$ possibilities.

D)

The intials which have atleast 1 “A” can be broken down into several catagories: Initials with exactly 1 “A”, Initials with exactly 2 “A”s, Initials with all 3 “A”s.

Clearly, there is only 1 string with all As (namely AAA), so we have 1 for that catagory.

Next, for exactly 2 As, notice that the string can take one of the following forms: XAA, AXA, AAX. Here, X is any non-A letter. In each case, we have 25 choices for X, so we have a total of $25 + 25 + 25 = 75$ choices for strings with exactly 2 As.

Next, for exactly 1 “A”, notice that the string can take one of the following forms: XXA, AXX, XAX. Here, X is any non-A letter. In each case, we have 25 choices for X and thus each form has $25 * 25$ total possibilities; so we have a total of $25 * 25 + 25 * 25 + 25 * 25 = 1875$ choices for strings with exactly 1 “A”.

In total, we have $1 + 75 + 75 + 1875$.

Problem 3

A)

We assume ordering does not apply to poker hands.

If the hand has exactly 2 queens, we have to choose 2 cards from the 4 total queens, so the number of ways to pick 2 queens from 4 is $\left(\begin{array}{c} 4 \\ 2 \end{array} \right) = 6$. The rest of 3 cards which can be any among the rest of the $52 - 4 = 48$ non-queen cards. So we have $\left(\begin{array}{c} 48 \\ 3 \end{array} \right) = 17296$ choices here. Total we have $6 * 17296$ choices.

If the hand has exactly 3 queens, we have to choose 3 cards from the 4 total queens, so the number of ways to pick 3 queens from 4 is $\left(\begin{array}{c} 4 \\ 3 \end{array} \right) = 4$. The rest of 2 cards which can be any among the rest of the $52 - 4 = 48$ non-queen cards. So we have $\left(\begin{array}{c} 48 \\ 2 \end{array} \right) = 1128$. choices here. Total we have $6 * 1128$ choices.

If the hand has 4 queens, then we have 48 possible choices for the last card, so we have 48 choices here.

Total, we have $6 * 17296 + 6 * 1128 + 48 = 110592$ choices for poker hands with atleast 2 queens.

B)

The hands with the two of clubs without the 2 of spades has 50 total cards (excluding the afformentioned 2) to choose the other 4 cards from, so there are , $\left(\begin{array}{c} 50 \\ 4 \end{array} \right) = 230300$ such hands.

The hands with the two of spades without the 2 of clubs has 50 total cards (excluding the afformentioned 2) to choose the other 4 cards from, so there are , $\left(\begin{array}{c} 50 \\ 4 \end{array} \right) = 230300$ such hands.

The hands with the two of spades and the 2 of clubs has 50 total cards (excluding the afformentioned 2) to choose the other 3 cards from, so there are , $\left(\begin{array}{c} 50 \\ 3 \end{array} \right) = 19600$ such hands.

Note these hands are exclusive from the others. In total, we have $230300 + 230300 + 19600 = 480200$

Problem 4

A)

Assuming anyone can be goalkeeper, we can choose any 11 of the 15 players where ordering on the choice does not matter; hence, we have $\begin{pmatrix} 15 \\ 11 \end{pmatrix} = 1365$ possible configurations.

B)

If 11 players are on the field, any choice of 5 from the 11 can be defenders, any choice of 3 from the remaining 6 can be midfielders and any choice of 2 of the remaining 3 can be forwards (last one becomes goalie). So we have $\begin{pmatrix} 11 \\ 5 \end{pmatrix}\begin{pmatrix} 6 \\ 3 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix} = (462)(20)(3) = 27720$

C)

If 11 players are on the field, any choice of 4 from the 11 can be defenders, any choice of 4 from the remaining 7 can be midfielders and any choice of 2 of the remaining 3 can be forwards (last one becomes goalie). So we have $\begin{pmatrix} 11 \\ 4 \end{pmatrix}\begin{pmatrix} 7 \\ 4 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix} = (330)(35)(3) = 34650$.