CS240: Homework 1

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Problem 1

Note that the original statement has the form $A \implies (B \vee C)$

a)

- i) $B \vee C \implies A$
- ii) Converse
- iii) Not equivalent
- vi) Notice that when $B = T, C = T, A = F$, then $(A \implies (B \vee C)) \equiv (F \implies (T \vee T)) \equiv F \implies T \equiv T$ but$((B \vee C) \implies A) \equiv ((T \vee T) \implies F) \equiv F$. Hence, $((B \vee C) \implies A) \not\equiv A \implies B \vee C$

b)

- i) $(\neg B \vee \neg C) \implies \neg A$
- ii) Neither
- iii) Not equivalent.
- vi) Suppose that $A = T, B = T, C = F$. Then $(\neg B \vee \neg C) \implies \neg A \equiv (\neg T \vee \neg F) \implies \neg T \equiv T \implies F \equiv F$. But $A \implies (B \vee C) \equiv T \implies T \vee F \equiv T \implies T \equiv T$. Because the 2 expressions do not agree on the same propsitional values, $A \implies (B \vee C) \not\equiv (\neg B \vee \neg C) \implies \neg A$.

c)

- i) $A \wedge \neg B \wedge \neg C$
- ii) Neither
- iii) Not equivalent
- vi) Suppose $A = F, B = T, C = T$. Then $A \wedge \neg B \wedge \neg C \equiv F \wedge \neg T \wedge \neg T \equiv F$ But$A \implies (B \vee C) \equiv F \implies (T \vee T) \equiv F \implies T \equiv T$. Because the 2 expressions do not agree on the same propsitional values, $A \implies (B \vee C) \not\equiv A \wedge \neg B \wedge \neg C$

d)

- i) $(\neg B \wedge \neg C) \implies \neg A$
- ii) Contrapositive, noting that by deMorgan's law, $(\neg B \wedge \neg C) \equiv \neg(B \vee C)$
- iii) Equivalent

vi) Here, we construct the truth table and show the equivalence of the 2 expression across all propositional values.

A	B	C	$A \implies (B \vee C)$	$(\neg B \wedge \neg C) \implies \neg A$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

e)

i) $(A \wedge \neg C) \implies B$

ii) Neither

iii) Equivalent

vi) Recall that $A \implies B \equiv \neg A \vee B$. Then using associativity and commutivity on $\vee$, $A \implies (B \vee C) \equiv \neg A \vee (B \vee C) \equiv (\neg A \vee C) \vee B$. Using deMorgan's law, $(\neg A \vee C) \vee B \equiv \neg(A \wedge \neg C) \vee B \equiv (A \wedge \neg C) \implies B$. Hence, we have shown that $(A \wedge \neg C) \implies B \equiv A \implies (B \vee C)$.

Problem 2

a)

Here, we construct the truth table:

P	Q	$P \wedge (Q \vee P)$
T	T	T
T	F	T
F	T	F
F	F	F

Notice that P and $P \wedge (Q \vee P)$ are equivalent on every permutation of propositional values, despite the value of Q. Hence, $P \wedge (Q \vee P) \equiv P$

b)

I claim that $(P \implies Q) \wedge (P \implies \neg Q) \equiv \neg P$. To prove this, we shall use a truth table:

P	Q	$(P \implies Q) \wedge (P \implies \neg Q)$	$\neg P$
T	T	$T \wedge F \equiv F$	F
T	F	$F \wedge T \equiv F$	F
F	T	$T \wedge T \equiv T$	T
F	F	$T \wedge T \equiv T$	T

As we can see, $(P \implies Q) \wedge (P \implies \neg Q)$ and $\neg P$ are equal across all propostional values, so $(P \implies Q) \wedge (P \implies \neg Q) \equiv \neg P$.

c)

Recall that $A \implies B \equiv \neg A \vee B$. So $(P \implies Q) \implies (P \implies R) \equiv (\neg P \vee Q) \implies (\neg P \vee R) \equiv \neg(\neg P \vee Q) \vee (\neg P \vee R)$. By deMorgan's law, $\neg(\neg P \vee Q) \vee (\neg P \vee R) \equiv (P \wedge \neg Q) \vee (\neg P \vee R) \equiv ((P \wedge \neg Q) \vee \neg P) \vee R$. Notice that $(P \wedge \neg Q) \vee \neg P \equiv (P \vee \neg P) \wedge (\neg P \vee \neg Q) \equiv T \wedge (\neg P \vee \neg Q) \equiv (\neg P \vee \neg Q) \equiv \neg(P \wedge Q)$. So $((P \wedge \neg Q) \vee \neg P) \vee R \equiv \neg(P \wedge Q) \vee R \equiv (P \wedge Q) \implies R$. So $(P \implies Q) \implies (P \implies R) \equiv (P \wedge Q) \implies R$

Problem 3

a)

$\exists x \in D\, [P(x) \wedge \neg H(x)]$

b)

$\forall x \in D\, [(P(x) \wedge H(x)) \implies \neg I(x)]$

c)

$\forall x \in D\, [I(x) \implies (P(x) \vee R(x))]$

d)

$\forall x \in D\, [\neg K(Darby, x) \implies \neg T(Darby, x)]$

e)

$\exists x \in D\, (R(x) \wedge (\forall y \in D\, [I(y) \implies K(x, y)])$

f)

Some politicians are honest, but not all people are politicians and are honest

g)

Everyone trusts every honest politician.

h)

If Jessie is honest, then there are some influence-peddlers that Jessie trusts

i)

There is not someone who trusts everyone.

j)

Everyone who is a registered lobbyist has someone other than themselves whom they trust

Problem 4

$\neg(\forall x \in Z)(\exists y \in Z)((x + y = 0) \wedge (y < x)) \equiv$
$(\exists x \in Z)\neg(\exists y \in Z)((x + y = 0) \wedge (y < x)) \equiv$
$(\exists x \in Z)(\forall y \in Z)\neg((x + y = 0) \wedge (y < x)) \equiv$
$(\exists x \in Z)(\forall y \in Z)(\neg(x + y = 0) \vee \neg(y < x)) \equiv$
$(\exists x \in Z)(\forall y \in Z)((x + y \neq 0) \vee (y \geq x))$

Problem 5

Set A as Alex's decision, B as Blair's, C as Casey's decision. Then we assume the following are all true:

If Alex orders dessert, so does Casey: $A \implies C$

Either Casey or Blair always orders dessert but never both at the same meal: $B \oplus C \equiv (B \vee C) \wedge \neg(B \wedge C)$ (that is, XOR)

Either Alex or Blair or both order dessert: $A \vee B$

If Blair orders dessert, so does Alex: $B \implies A$

Constructing the truth table:

A	B	C	$A \implies C$	$B \oplus C$	$A \vee B$	$B \implies A$
T	T	T	T	F	T	T
T	T	F	F	T	T	T
T	F	T	T	T	T	T
T	F	F	F	F	T	T
F	T	T	T	F	T	F
F	T	F	T	T	T	F
F	F	T	T	T	F	T
F	F	F	T	F	F	T

The only set of values on the variables which satisfies our assumption that the given propositions are all true are $A = T, B = F, C = T$.

That is, Alex and Casey get dessert while Blair does not.