Schmidt Decomposition of 4-Particle State

$$|W\rangle = \frac{1}{2\sqrt{2}} (|1100\rangle + \sqrt{2}|1010\rangle - |1001\rangle + |0011\rangle - \sqrt{2}|0101\rangle + |0110\rangle)$$

Density matrix $= |W\rangle\langle W|$

$$= \frac{1}{8} (|1100\rangle\langle 1100| + \sqrt{2}|1100\rangle\langle 1010| - |1100\rangle\langle 1001| + |1100\rangle\langle 0011| - \sqrt{2}|1100\rangle\langle 0101| + |1100\rangle\langle 0110|$$
$$+ \sqrt{2}|1010\rangle\langle 1100| + 2|1010\rangle\langle 1010| - \sqrt{2}|1010\rangle\langle 1001| + \sqrt{2}|1010\rangle\langle 0011| - 2|1010\rangle\langle 0101| + \sqrt{2}|1010\rangle\langle 0110|$$
$$- |1001\rangle\langle 1100| - \sqrt{2}|1001\rangle\langle 1010| + |1001\rangle\langle 1001| - |1001\rangle\langle 0011| + \sqrt{2}|1001\rangle\langle 0101| - |1001\rangle\langle 0110|$$
$$+ |0011\rangle\langle 1100| + \sqrt{2}|0011\rangle\langle 1010| - |0011\rangle\langle 1001| + \sqrt{2}|0011\rangle\langle 0011| - \sqrt{2}|0011\rangle\langle 0101| + |0011\rangle\langle 0110|$$
$$- \sqrt{2}|0101\rangle\langle 1100| - 2|0101\rangle\langle 1010| + \sqrt{2}|0101\rangle\langle 1001| - \sqrt{2}|0101\rangle\langle 0011| + 2|0101\rangle\langle 0101| - \sqrt{2}|0101\rangle\langle 0110|$$
$$+ |0110\rangle\langle 1100| + \sqrt{2}|0110\rangle\langle 1010| - |0110\rangle\langle 1001| + |0110\rangle\langle 0011| - \sqrt{2}|0110\rangle\langle 0101| + |0110\rangle\langle 0110|)$$

Partial Trace of subsystem 2 & 4 $\longrightarrow$

$=$

Take partial tra... s...

~~First~~ $\mathrm{Tr}_{24}\, \rho_{1234} = |1...$

$=$

Form matrix & find eig...

If square mat... is ...

If they are ...

If only 1 ...

$\Rightarrow$ Schmid...

Partial Trace of subsystem

2 & 4 $\longrightarrow$ $|abcd\rangle\langle efgh|$

$= |ac\rangle\langle ef| \underbrace{\langle bd|gh\rangle}$

$=$

$Tr(bd(gh)^T)$

to obtain reduced state of subsystem 1 & 3.

Take partial trace for both systems in this way

~~first~~ $Tr_{24}\, \rho_{1234} = |1100\rangle\langle 0101|$

$= |10\rangle\langle 01| \langle 10|01\rangle$

Form matrix of coefficients of partial trace & find eigenvalues

If square root of eigenvalues for both subsystem is non-0, then it's entangled

If they are non-0, & equal, then maximally entangled state

If only 1 eigenvalue is non-0, then it's separable

$\Rightarrow$ Schmidt Rank = Schmidt Num

$= 1$.