Model of the Stock Market

Fys3150

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Abstract

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1 Introduction

The aim of this project is to simulate transactions of money between financial agents, people, using Monte Carlo methods. The final goal is to extract a distribution of income as function of the income m. From Pareto's work `(V. Pareto, 1897)`, and from empirical studies, it is known that the higher end of the distribution of money, rich end, follows a power distribution.

$$\omega_m = m^{-1-\alpha}$$

with $\alpha \in [1, 2]$. We will follow the analysis made by Patriarca and collaborators.

We assume we have N agents that exchange money in pairs (i, j). We assume also that all agents start with the same amount of money $m_0 > 0$. At a given 'time step', we choose a pair of agents (i, j) and let a transaction take place. This means that agent i's money m_i changes to m'_i and similarly we have $m_j \to m'_j$. Money is conserved during a transaction:

$$m_i + m_j = m'_i + m'_j. \tag{1}$$

In order to decide which agent gets what, we draw a random number ϵ. The change is done via a random reassignement (a random number) ϵ, meaning that

$$m'_i = \epsilon(m_i + m_j), \tag{2}$$

leading to

$$m'_j = (1 - \epsilon)(m_i + m_j). \tag{3}$$

The number ϵ is extracted from a uniform distribution. In this simple model, no agents are left with a debt, that is $m \geq 0$. Due to the conservation law above, one can show that the system relaxes toward an equilibrium state given by a Gibbs distribution

$$w_m = \beta \exp(-\beta m),$$

with

$$\beta = \frac{1}{\langle m \rangle},$$

and $\langle m \rangle = \sum_i m_i/N = m_0$, the average money. It means that after equilibrium has been reached that the majority of agents is left with a small number of money, while the number of richest agents, those with m larger than a specific value m', exponentially decreases with m'.

In each simulation, we need a sufficiently large number of transactions, say 10^7. Our aim is find the final equilibrium distribution w_m. In order to do that we would need several runs of the above simulations, at least $10^3 - 10^4$ runs (experiments).

2 Theory

Now there are multiple models to explore. Our first model picks two entirely random agents, i and j, and makes them trade a random amount decided my the factor ϵ (see equation 2 and 3). The next three models add one more realistic aspect each.

2.1 Saving factor λ

As our model is now, nothing is stopping our agents from throwing away all their money. In order to make our agents a bit more rational, we will limit the amount of money they trade away. The conservation law of Eq. (1) holds, but the money to be shared in a transaction between agent i and agent j is now $(1-\lambda)(m_i+m_j)$. This means that we have

$$m_i' = \lambda m_i + \epsilon(1-\lambda)(m_i+m_j), \tag{4}$$

and

$$m_j' = \lambda m_j + (1-\epsilon)(1-\lambda)(m_i+m_j), \tag{5}$$

which can be written as

$$m_i' = m_i + \delta m \tag{6}$$

and

$$m_j' = m_j - \delta m, \tag{7}$$

with

$$\delta m = (1-\lambda)(\epsilon m_j - (1-\epsilon)m_i), \tag{8}$$

showing how money is conserved during a transaction. If $\lambda = 0.8$, our agents will never trade more than 20%.

3 The Factors

4 Structure of Code

5 Method

References

[1] navn, *Hvor det er fra*, University of Oslo, 2016.

[2] Navn, subtitle, *title*, Grøndahl & Søn, pp. 28-33, 1881.

A stuff

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