Proof of $\delta(g(t)) = \frac{\delta(t)}{|g'(0)|}$

Proof. By definition, we have that

$$\int_{\mathbb{R}} \delta(t) f(t) dt = f(0). \tag{1}$$

Without loss of generality, assume that $g(0) = 0$ and that g is locally smooth. Next, let $t = g(v)$, then $\frac{\partial t}{\partial v} = g'(v)$, and $dt = |g'(v)|dv$, where we assumed without loss of generality monotonicity around 0. Changing t for v in eq. (1), we have

$$\begin{aligned}
\int_{\mathbb{R}} \delta(g(v)) f(v) dv &= \int_{g(\mathbb{R})} \delta(t) f(g^{-1}(t)) dg^{-1}(t) && (2) \\
&= \int_{g(\mathbb{R})} \delta(t) \frac{f(g^{-1}(t))}{|g'(g^{-1}(t))|} dt && (3) \\
&= \frac{f(0)}{|g'(0)|} && (4) \\
&= \int_{\mathbb{R}} \frac{\delta(t)}{|g'(0)|} f(v) dv && (5)
\end{aligned}$$

Thus $\delta(g(v)) = \frac{\delta(t)}{|g'(0)|}$. □