Cancelling gives $a^2 = \dfrac{rp}{(x+y)}$

Taking the square root of both sides gives

$$\boldsymbol{a = \sqrt{\left(\dfrac{rp}{x+y}\right)}}$$

Whenever the letter required as the new subject occurs more than once in the original formula, after rearranging, **factorizing** will always be needed.

Problem 20. Make b the subject of the formula $a = \dfrac{x-y}{\sqrt{bd+be}}$

Rearranging gives $\dfrac{x-y}{\sqrt{bd+be}} = a$

Multiplying both sides by $\sqrt{bd+be}$ gives

$$x - y = a\sqrt{bd+be}$$

or $a\sqrt{bd+be} = x - y$

Dividing both sides by a gives $\sqrt{bd+be} = \dfrac{x-y}{a}$

Squaring both sides gives $bd + be = \left(\dfrac{x-y}{a}\right)^2$

Factorizing the LHS gives $b(d+e) = \left(\dfrac{x-y}{a}\right)^2$

Dividing both sides by $(d+e)$ gives

$$\boldsymbol{b = \dfrac{\left(\dfrac{x-y}{a}\right)^2}{(d+e)}} \quad \text{or} \quad \boldsymbol{b = \dfrac{(x-y)^2}{a^2(d+e)}}$$

Problem 21. If $a = \dfrac{b}{1+b}$, make b the subject of the formula

Rearranging gives $\dfrac{b}{1+b} = a$

Multiplying both sides by $(1+b)$ gives

$$b = a(1+b)$$

Removing the bracket gives $b = a + ab$

Rearranging to obtain terms in b on the LHS gives

$$b - ab = a$$

Factorizing the LHS gives $b(1-a) = a$

Dividing both sides by $(1-a)$ gives $\boldsymbol{b = \dfrac{a}{1-a}}$

Problem 22. Transpose the formula $V = \dfrac{Er}{R+r}$ to make r the subject

Rearranging gives $\dfrac{Er}{R+r} = V$

Multiplying both sides by $(R+r)$ gives

$$Er = V(R+r)$$

Removing the bracket gives $Er = VR + Vr$

Rearranging to obtain terms in r on the LHS gives

$$Er - Vr = VR$$

Factorizing gives $r(E-V) = VR$

Dividing both sides by $(E-V)$ gives $\boldsymbol{r = \dfrac{VR}{E-V}}$

Problem 23. Transpose the formula $y = \dfrac{pq^2}{r+q^2} - t$ to make q the subject

Rearranging gives $\dfrac{pq^2}{r+q^2} - t = y$

and $\dfrac{pq^2}{r+q^2} = y + t$

Multiplying both sides by $(r+q^2)$ gives

$$pq^2 = (r+q^2)(y+t)$$

Removing brackets gives $pq^2 = ry + rt + q^2y + q^2t$

Rearranging to obtain terms in q on the LHS gives

$$pq^2 - q^2y - q^2t = ry + rt$$

Factorizing gives $q^2(p - y - t) = r(y+t)$

Dividing both sides by $(p-y-t)$ gives

$$q^2 = \dfrac{r(y+t)}{(p-y-t)}$$

Taking the square root of both sides gives

$$\boldsymbol{q = \sqrt{\left(\dfrac{r(y+t)}{p-y-t}\right)}}$$

Problem 24. Given that $\dfrac{D}{d} = \sqrt{\left(\dfrac{f+p}{f-p}\right)}$ express p in terms of D, d and f

Rearranging gives $\sqrt{\left(\dfrac{f+p}{f-p}\right)} = \dfrac{D}{d}$

Squaring both sides gives $\left(\dfrac{f+p}{f-p}\right) = \dfrac{D^2}{d^2}$

Cross-multiplying, i.e. multiplying each term by $d^2(f-p)$, gives

$$d^2(f+p) = D^2(f-p)$$

Removing brackets gives $d^2 f + d^2 p = D^2 f - D^2 p$

Rearranging, to obtain terms in p on the LHS gives

$$d^2 p + D^2 p = D^2 f - d^2 f$$

Factorizing gives $p(d^2 + D^2) = f(D^2 - d^2)$

Dividing both sides by $(d^2 + D^2)$ gives

$$\boldsymbol{p = \frac{f(D^2 - d^2)}{(d^2 + D^2)}}$$

Now try the following Practice Exercise

Practice Exercise 48 Further transposing formulae (answers on page 345)

Make the symbol indicated the subject of each of the formulae shown in problems 1 to 7 and express each in its simplest form.

1. $y = \dfrac{a^2 m - a^2 n}{x}$ (a)
2. $M = \pi(R^4 - r^4)$ (R)
3. $x + y = \dfrac{r}{3+r}$ (r)
4. $m = \dfrac{\mu L}{L + rCR}$ (L)
5. $a^2 = \dfrac{b^2 - c^2}{b^2}$ (b)
6. $\dfrac{x}{y} = \dfrac{1+r^2}{1-r^2}$ (r)
7. $\dfrac{p}{q} = \sqrt{\left(\dfrac{a+2b}{a-2b}\right)}$ (b)
8. A formula for the focal length, f, of a convex lens is $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$. Transpose the formula to make v the subject and evaluate v when $f = 5$ and $u = 6$.
9. The quantity of heat, Q, is given by the formula $Q = mc(t_2 - t_1)$. Make t_2 the subject of the formula and evaluate t_2 when $m = 10$, $t_1 = 15$, $c = 4$ and $Q = 1600$.
10. The velocity, v, of water in a pipe appears in the formula $h = \dfrac{0.03Lv^2}{2dg}$. Express v as the subject of the formula and evaluate v when $h = 0.712$, $L = 150$, $d = 0.30$ and $g = 9.81$
11. The sag, S, at the centre of a wire is given by the formula $S = \sqrt{\left(\dfrac{3d(l-d)}{8}\right)}$. Make l the subject of the formula and evaluate l when $d = 1.75$ and $S = 0.80$.
12. In an electrical alternating current circuit the impedance Z is given by $Z = \sqrt{\left\{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2\right\}}$. Transpose the formula to make C the subject and hence evaluate C when $Z = 130$, $R = 120$, $\omega = 314$ and $L = 0.32$
13. An approximate relationship between the number of teeth, T, on a milling cutter, the diameter of cutter, D, and the depth of cut, d, is given by $T = \dfrac{12.5\,D}{D + 4d}$. Determine the value of D when $T = 10$ and $d = 4$ mm.
14. Make λ, the wavelength of X-rays, the subject of the following formula: $\dfrac{\mu}{\rho} = \dfrac{CZ^4\sqrt{\lambda^5}\,n}{a}$

Chapter 13

Solving simultaneous equations

13.1 Introduction

Only one equation is necessary when finding the value of a **single unknown quantity** (as with simple equations in Chapter 11). However, when an equation contains **two unknown quantities** it has an infinite number of solutions. When two equations are available connecting the same two unknown values then a unique solution is possible. Similarly, for three unknown quantities it is necessary to have three equations in order to solve for a particular value of each of the unknown quantities, and so on.

Equations which have to be solved together to find the unique values of the unknown quantities, which are true for each of the equations, are called **simultaneous equations**.

Two methods of solving simultaneous equations analytically are:

(a) by **substitution**, and

(b) by **elimination**.

(A graphical solution of simultaneous equations is shown in Chapter 19.)

13.2 Solving simultaneous equations in two unknowns

The method of solving simultaneous equations is demonstrated in the following worked problems.

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Problem 1. Solve the following equations for x and y, (a) by substitution and (b) by elimination

$$x + 2y = -1 \qquad (1)$$

$$4x - 3y = 18 \qquad (2)$$

(a) By substitution

From equation (1): $x = -1 - 2y$

Substituting this expression for x into equation (2) gives

$$4(-1 - 2y) - 3y = 18$$

This is now a simple equation in y.

Removing the bracket gives

$$-4 - 8y - 3y = 18$$

$$-11y = 18 + 4 = 22$$

$$y = \frac{22}{-11} = -2$$

Substituting $y = -2$ into equation (1) gives

$$x + 2(-2) = -1$$

$$x - 4 = -1$$

$$x = -1 + 4 = 3$$

Thus, $\boldsymbol{x = 3}$ and $\boldsymbol{y = -2}$ is the solution to the simultaneous equations.

Check: in equation (2), since $x = 3$ and $y = -2$,

$$\text{LHS} = 4(3) - 3(-2) = 12 + 6 = 18 = \text{RHS}$$

(b) By elimination

$$x + 2y = -1 \qquad (1)$$

$$4x - 3y = 18 \qquad (2)$$

If equation (1) is multiplied throughout by 4, the coefficient of x will be the same as in equation (2), giving

$$4x + 8y = -4 \qquad (3)$$

Subtracting equation (3) from equation (2) gives

$$\begin{aligned} 4x - 3y &= 18 \qquad (2)\\ 4x + 8y &= -4 \qquad (3)\\ \hline 0 - 11y &= 22 \end{aligned}$$

Hence, $\boldsymbol{y} = \dfrac{22}{-11} = \mathbf{-2}$

(Note: in the above subtraction,

$$18 - -4 = 18 + 4 = 22.)$$

Substituting $y = -2$ into either equation (1) or equation (2) will give $\boldsymbol{x = 3}$ as in method (a). The solution $\boldsymbol{x = 3, y = -2}$ is the only pair of values that satisfies both of the original equations.

Problem 2. Solve, by a substitution method, the simultaneous equations

$$3x - 2y = 12 \qquad (1)$$

$$x + 3y = -7 \qquad (2)$$

From equation (2), $x = -7 - 3y$

Substituting for x in equation (1) gives

$$3(-7 - 3y) - 2y = 12$$

i.e. $$-21 - 9y - 2y = 12$$

$$-11y = 12 + 21 = 33$$

Hence, $y = \dfrac{33}{-11} = -3$

Substituting $y = -3$ in equation (2) gives

$$x + 3(-3) = -7$$

i.e. $$x - 9 = -7$$

Hence $$x = -7 + 9 = 2$$

Thus, $\boldsymbol{x = 2, y = -3}$ is the solution of the simultaneous equations. (Such solutions should always be checked by substituting values into each of the original two equations.)

Problem 3. Use an elimination method to solve the following simultaneous equations

$$3x + 4y = 5 \qquad (1)$$

$$2x - 5y = -12 \qquad (2)$$

If equation (1) is multiplied throughout by 2 and equation (2) by 3, the coefficient of x will be the same in the newly formed equations. Thus,

2 × equation (1) gives $\quad 6x + 8y = 10 \qquad (3)$

3 × equation (2) gives $\quad 6x - 15y = -36 \qquad (4)$

Equation (3) – equation (4) gives

$$0 + 23y = 46$$

i.e. $$\boldsymbol{y} = \frac{46}{23} = \mathbf{2}$$

(Note $+8y - -15y = 8y + 15y = 23y$ and $10 - -36 = 10 + 36 = 46$.)

Substituting $y = 2$ in equation (1) gives

$$3x + 4(2) = 5$$

from which $$3x = 5 - 8 = -3$$

and $$\boldsymbol{x = -1}$$

Checking, by substituting $x = -1$ and $y = 2$ in equation (2), gives

$$\text{LHS} = 2(-1) - 5(2) = -2 - 10 = -12 = \text{RHS}$$

Hence, $\boldsymbol{x = -1}$ and $\boldsymbol{y = 2}$ is the solution of the simultaneous equations.

The elimination method is the most common method of solving simultaneous equations.

Problem 4. Solve

$$7x - 2y = 26 \qquad (1)$$

$$6x + 5y = 29 \qquad (2)$$

When equation (1) is multiplied by 5 and equation (2) by 2, the coefficients of y in each equation are numerically the same, i.e. 10, but are of opposite sign.

$5 \times$ equation (1) gives	$35x - 10y = 130$	(3)
$2 \times$ equation (2) gives	$12x + 10y = 58$	(4)

Adding equations (3) and (4) gives $\quad 47x + 0 = 188$

Hence, $\quad \boldsymbol{x} = \dfrac{188}{47} = \mathbf{4}$

Note that when the signs of common coefficients are **different** the two equations are **added** and when the signs of common coefficients are the **same** the two equations are **subtracted** (as in Problems 1 and 3).

Substituting $x = 4$ in equation (1) gives

$$\begin{aligned} 7(4) - 2y &= 26 \\ 28 - 2y &= 26 \\ 28 - 26 &= 2y \\ 2 &= 2y \end{aligned}$$

Hence, $\quad \boldsymbol{y = 1}$

Checking, by substituting $x = 4$ and $y = 1$ in equation (2), gives

$$\text{LHS} = 6(4) + 5(1) = 24 + 5 = 29 = \text{RHS}$$

Thus, the solution is $\boldsymbol{x = 4}$, $\boldsymbol{y = 1}$.

Now try the following Practice Exercise

Practice Exercise 49 Solving simultaneous equations (answers on page 345)

Solve the following simultaneous equations and verify the results.

1. $2x - y = 6$, $x + y = 6$
2. $2x - y = 2$, $x - 3y = -9$
3. $x - 4y = -4$, $5x - 2y = 7$
4. $3x - 2y = 10$, $5x + y = 21$
5. $5p + 4q = 6$, $2p - 3q = 7$
6. $7x + 2y = 11$, $3x - 5y = -7$
7. $2x - 7y = -8$, $3x + 4y = 17$
8. $a + 2b = 8$, $b - 3a = -3$
9. $a + b = 7$, $a - b = 3$
10. $2x + 5y = 7$, $x + 3y = 4$
11. $3s + 2t = 12$, $4s - t = 5$
12. $3x - 2y = 13$, $2x + 5y = -4$
13. $5m - 3n = 11$, $3m + n = 8$
14. $8a - 3b = 51$, $3a + 4b = 14$
15. $5x = 2y$, $3x + 7y = 41$
16. $5c = 1 - 3d$, $2d + c + 4 = 0$

13.3 Further solving of simultaneous equations

Here are some further worked problems on solving simultaneous equations.

Problem 5. Solve

$$3p = 2q \qquad (1)$$
$$4p + q + 11 = 0 \qquad (2)$$

Rearranging gives

$$3p - 2q = 0 \qquad (3)$$
$$4p + q = -11 \qquad (4)$$

Multiplying equation (4) by 2 gives

$$8p + 2q = -22 \qquad (5)$$

Adding equations (3) and (5) gives

$$\begin{aligned} 11p + 0 &= -22 \\ p &= \frac{-22}{11} = -2 \end{aligned}$$

Substituting $p = -2$ into equation (1) gives

$$\begin{aligned} 3(-2) &= 2q \\ -6 &= 2q \\ q &= \frac{-6}{2} = -3 \end{aligned}$$

Checking, by substituting $p = -2$ and $q = -3$ into equation (2), gives

$$\text{LHS} = 4(-2) + (-3) + 11 = -8 - 3 + 11 = 0 = \text{RHS}$$

Hence, the solution is $\boldsymbol{p = -2, q = -3}$.

Problem 6. Solve

$$\frac{x}{8}+\frac{5}{2}=y \qquad (1)$$

$$13-\frac{y}{3}=3x \qquad (2)$$

Whenever fractions are involved in simultaneous equations it is often easier to firstly remove them. Thus, multiplying equation (1) by 8 gives

$$8\left(\frac{x}{8}\right)+8\left(\frac{5}{2}\right)=8y$$

i.e. $$x+20=8y \qquad (3)$$

Multiplying equation (2) by 3 gives

$$39-y=9x \qquad (4)$$

Rearranging equations (3) and (4) gives

$$x-8y=-20 \qquad (5)$$

$$9x+y=39 \qquad (6)$$

Multiplying equation (6) by 8 gives

$$72x+8y=312 \qquad (7)$$

Adding equations (5) and (7) gives

$$73x+0=292$$

$$\boldsymbol{x}=\frac{292}{73}=\mathbf{4}$$

Substituting $x=4$ into equation (5) gives

$$4-8y=-20$$

$$4+20=8y$$

$$24=8y$$

$$\boldsymbol{y}=\frac{24}{8}=\mathbf{3}$$

Checking, substituting $x=4$ and $y=3$ in the original equations, gives

(1): $\text{LHS}=\frac{4}{8}+\frac{5}{2}=\frac{1}{2}+2\frac{1}{2}=3=y=\text{RHS}$

(2): $\text{LHS}=13-\frac{3}{3}=13-1=12$

$\text{RHS}=3x=3(4)=12$

Hence, the solution is $\boldsymbol{x=4,\ y=3}$.

Problem 7. Solve

$$2.5x+0.75-3y=0$$

$$1.6x=1.08-1.2y$$

It is often easier to remove decimal fractions. Thus, multiplying equations (1) and (2) by 100 gives

$$250x+75-300y=0 \qquad (1)$$

$$160x=108-120y \qquad (2)$$

Rearranging gives

$$250x-300y=-75 \qquad (3)$$

$$160x+120y=108 \qquad (4)$$

Multiplying equation (3) by 2 gives

$$500x-600y=-150 \qquad (5)$$

Multiplying equation (4) by 5 gives

$$800x+600y=540 \qquad (6)$$

Adding equations (5) and (6) gives

$$1300x+0=390$$

$$\boldsymbol{x}=\frac{390}{1300}=\frac{39}{130}=\frac{3}{10}=\mathbf{0.3}$$

Substituting $x=0.3$ into equation (1) gives

$$250(0.3)+75-300y=0$$

$$75+75=300y$$

$$150=300y$$

$$\boldsymbol{y}=\frac{150}{300}=\mathbf{0.5}$$

Checking, by substituting $x=0.3$ and $y=0.5$ in equation (2), gives

$\text{LHS}=160(0.3)=48$

$\text{RHS}=108-120(0.5)=108-60=48$

Hence, the solution is $\boldsymbol{x=0.3,\ y=0.5}$

Now try the following Practice Exercise

Practice Exercise 50 Solving simultaneous equations (answers on page 345)

Solve the following simultaneous equations and verify the results.

1. $7p + 11 + 2q = 0$
   $-1 = 3q - 5p$
2. $\frac{x}{2} + \frac{y}{3} = 4$
   $\frac{x}{6} - \frac{y}{9} = 0$
3. $\frac{a}{2} - 7 = -2b$
   $12 = 5a + \frac{2}{3}b$
4. $\frac{3}{2}s - 2t = 8$
   $\frac{s}{4} + 3y = -2$
5. $\frac{x}{5} + \frac{2y}{3} = \frac{49}{15}$
   $\frac{3x}{7} - \frac{y}{2} + \frac{5}{7} = 0$
6. $v - 1 = \frac{u}{12}$
   $u + \frac{v}{4} - \frac{25}{2} = 0$
7. $1.5x - 2.2y = -18$
   $2.4x + 0.6y = 33$
8. $3b - 2.5a = 0.45$
   $1.6a + 0.8b = 0.8$

13.4 Solving more difficult simultaneous equations

Here are some further worked problems on solving more difficult simultaneous equations.

Problem 8. Solve

$$\frac{2}{x} + \frac{3}{y} = 7 \qquad (1)$$

$$\frac{1}{x} - \frac{4}{y} = -2 \qquad (2)$$

In this type of equation the solution is easier if a substitution is initially made. Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$

Thus equation (1) becomes $\quad 2a + 3b = 7 \qquad (3)$

and equation (2) becomes $\quad a - 4b = -2 \qquad (4)$

Multiplying equation (4) by 2 gives

$$2a - 8b = -4 \qquad (5)$$

Subtracting equation (5) from equation (3) gives

$$0 + 11b = 11$$

i.e. $\quad \boldsymbol{b = 1}$

Substituting $b = 1$ in equation (3) gives

$$2a + 3 = 7$$

$$2a = 7 - 3 = 4$$

i.e. $\quad \boldsymbol{a = 2}$

Checking, substituting $a = 2$ and $b = 1$ in equation (4), gives

$$\text{LHS} = 2 - 4(1) = 2 - 4 = -2 = \text{RHS}$$

Hence, $\boldsymbol{a = 2}$ and $\boldsymbol{b = 1}$.

However, since $\quad \frac{1}{x} = a, \quad x = \frac{1}{a} = \frac{1}{2}$ or 0.5

and since $\quad \frac{1}{y} = b, \quad y = \frac{1}{b} = \frac{1}{1} = 1$

Hence, the solution is $\boldsymbol{x = 0.5}$, $\boldsymbol{y = 1}$.

Problem 9. Solve

$$\frac{1}{2a} + \frac{3}{5b} = 4 \qquad (1)$$

$$\frac{4}{a} + \frac{1}{2b} = 10.5 \qquad (2)$$

Let $\frac{1}{a} = x \quad$ and $\quad \frac{1}{b} = y$

then $\quad \frac{x}{2} + \frac{3}{5}y = 4 \qquad (3)$

$$4x + \frac{1}{2}y = 10.5 \qquad (4)$$

To remove fractions, equation (3) is multiplied by 10, giving

$$10\left(\frac{x}{2}\right) + 10\left(\frac{3}{5}y\right) = 10(4)$$

i.e. $\quad 5x + 6y = 40 \qquad (5)$

Multiplying equation (4) by 2 gives

$$8x + y = 21 \qquad (6)$$

Multiplying equation (6) by 6 gives

$$48x + 6y = 126 \qquad (7)$$

Subtracting equation (5) from equation (7) gives

$$43x + 0 = 86$$
$$\boldsymbol{x} = \frac{86}{43} = \mathbf{2}$$

Substituting $x = 2$ into equation (3) gives

$$\frac{2}{2} + \frac{3}{5}y = 4$$
$$\frac{3}{5}y = 4 - 1 = 3$$
$$\boldsymbol{y} = \frac{5}{3}(3) = \mathbf{5}$$

Since $\quad \frac{1}{a} = x, \quad a = \frac{1}{x} = \frac{1}{2}$ or 0.5

and since $\quad \frac{1}{b} = y, \quad b = \frac{1}{y} = \frac{1}{5}$ or 0.2

Hence, the solution is $\boldsymbol{a = 0.5}$, $\boldsymbol{b = 0.2}$, which may be checked in the original equations.

Problem 10. Solve

$$\frac{1}{x+y} = \frac{4}{27} \qquad (1)$$
$$\frac{1}{2x-y} = \frac{4}{33} \qquad (2)$$

To eliminate fractions, both sides of equation (1) are multiplied by $27(x+y)$, giving

$$27(x+y)\left(\frac{1}{x+y}\right) = 27(x+y)\left(\frac{4}{27}\right)$$

i.e. $\quad 27(1) = 4(x+y)$

$$27 = 4x + 4y \qquad (3)$$

Similarly, in equation (2) $\quad 33 = 4(2x - y)$

i.e. $\quad 33 = 8x - 4y \qquad (4)$

Equation (3) + equation (4) gives

$$60 = 12x \text{ and } \boldsymbol{x} = \frac{60}{12} = \mathbf{5}$$

Substituting $x = 5$ in equation (3) gives

$$27 = 4(5) + 4y$$

from which $\quad 4y = 27 - 20 = 7$

and $\quad \boldsymbol{y} = \frac{7}{4} = \mathbf{1\frac{3}{4}}$ or $\mathbf{1.75}$

Hence, $\boldsymbol{x = 5}$, $\boldsymbol{y = 1.75}$ is the required solution, which may be checked in the original equations.

Problem 11. Solve

$$\frac{x-1}{3} + \frac{y+2}{5} = \frac{2}{15} \qquad (1)$$
$$\frac{1-x}{6} + \frac{5+y}{2} = \frac{5}{6} \qquad (2)$$

Before equations (1) and (2) can be simultaneously solved, the fractions need to be removed and the equations rearranged.

Multiplying equation (1) by 15 gives

$$15\left(\frac{x-1}{3}\right) + 15\left(\frac{y+2}{5}\right) = 15\left(\frac{2}{15}\right)$$

i.e. $\quad 5(x-1) + 3(y+2) = 2$

$$5x - 5 + 3y + 6 = 2$$
$$5x + 3y = 2 + 5 - 6$$

Hence, $\quad 5x + 3y = 1 \qquad (3)$

Multiplying equation (2) by 6 gives

$$6\left(\frac{1-x}{6}\right) + 6\left(\frac{5+y}{2}\right) = 6\left(\frac{5}{6}\right)$$

i.e. $\quad (1-x) + 3(5+y) = 5$

$$1 - x + 15 + 3y = 5$$
$$-x + 3y = 5 - 1 - 15$$

Hence, $\quad -x + 3y = -11 \qquad (4)$

Thus the initial problem containing fractions can be expressed as

$$5x + 3y = 1 \qquad (3)$$
$$-x + 3y = -11 \qquad (4)$$

Subtracting equation (4) from equation (3) gives

$$6x + 0 = 12$$
$$\boldsymbol{x} = \frac{12}{6} = \mathbf{2}$$

Substituting $x = 2$ into equation (3) gives

$$5(2) + 3y = 1$$
$$10 + 3y = 1$$
$$3y = 1 - 10 = -9$$
$$\boldsymbol{y} = \frac{-9}{3} = \mathbf{-3}$$

Checking, substituting $x = 2$, $y = -3$ in equation (4) gives

$$\text{LHS} = -2 + 3(-3) = -2 - 9 = -11 = \text{RHS}$$

Hence, the solution is $\boldsymbol{x = 2}$, $\boldsymbol{y = -3}$.

Now try the following Practice Exercise

Practice Exercise 51 Solving more difficult simultaneous equations (answers on page 345)

In problems 1 to 7, solve the simultaneous equations and verify the results

1. $\frac{3}{x} + \frac{2}{y} = 14$
   $\frac{5}{x} - \frac{3}{y} = -2$
2. $\frac{4}{a} - \frac{3}{b} = 18$
   $\frac{2}{a} + \frac{5}{b} = -4$
3. $\frac{1}{2p} + \frac{3}{5q} = 5$
   $\frac{5}{p} - \frac{1}{2q} = \frac{35}{2}$
4. $\frac{5}{x} + \frac{3}{y} = 1.1$
   $\frac{3}{x} - \frac{7}{y} = -1.1$
5. $\frac{c+1}{4} - \frac{d+2}{3} + 1 = 0$
   $\frac{1-c}{5} + \frac{3-d}{4} + \frac{13}{20} = 0$
6. $\frac{3r+2}{5} - \frac{2s-1}{4} = \frac{11}{5}$
   $\frac{3+2r}{4} + \frac{5-s}{3} = \frac{15}{4}$
7. $\frac{5}{x+y} = \frac{20}{27}$
   $\frac{4}{2x-y} = \frac{16}{33}$
8. If $5x - \frac{3}{y} = 1$ and $x + \frac{4}{y} = \frac{5}{2}$, find the value of $\frac{xy+1}{y}$

13.5 Practical problems involving simultaneous equations

There are a number of situations in engineering and science in which the solution of simultaneous equations is required. Some are demonstrated in the following worked problems.

Problem 12. The law connecting friction F and load L for an experiment is of the form $F = aL + b$ where a and b are constants. When $F = 5.6\,\text{N}$, $L = 8.0\,\text{N}$ and when $F = 4.4N$, $L = 2.0\,\text{N}$. Find the values of a and b and the value of F when $L = 6.5\,\text{N}$

Substituting $F = 5.6$ and $L = 8.0$ into $F = aL + b$ gives

$$5.6 = 8.0a + b \qquad (1)$$

Substituting $F = 4.4$ and $L = 2.0$ into $F = aL + b$ gives

$$4.4 = 2.0a + b \qquad (2)$$

Subtracting equation (2) from equation (1) gives

$$1.2 = 6.0a$$
$$\boldsymbol{a} = \frac{1.2}{6.0} = \boldsymbol{\frac{1}{5}} \text{ or } \mathbf{0.2}$$

Substituting $a = \frac{1}{5}$ into equation (1) gives

$$5.6 = 8.0\left(\frac{1}{5}\right) + b$$
$$5.6 = 1.6 + b$$
$$5.6 - 1.6 = b$$

i.e. $\boldsymbol{b = 4}$

Checking, substituting $a = \frac{1}{5}$ and $b = 4$ in equation (2), gives

$$\text{RHS} = 2.0\left(\frac{1}{5}\right) + 4 = 0.4 + 4 = 4.4 = \text{LHS}$$

Hence, $\boldsymbol{a = \frac{1}{5}}$ and $\boldsymbol{b = 4}$

When $\boldsymbol{L = 6.5}$, $F = aL + b = \frac{1}{5}(6.5) + 4 = 1.3 + 4$, i.e. $\boldsymbol{F = 5.30}\,\textbf{N}$.

Problem 13. The equation of a straight line, of gradient m and intercept on the y-axis c, is $y = mx + c$. If a straight line passes through the point where $x = 1$ and $y = -2$, and also through the point where $x = 3.5$ and $y = 10.5$, find the values of the gradient and the y-axis intercept

Substituting $x = 1$ and $y = -2$ into $y = mx + c$ gives

$$-2 = m + c \qquad (1)$$

Substituting $x = 3.5$ and $y = 10.5$ into $y = mx + c$ gives

$$10.5 = 3.5m + c \qquad (2)$$

Subtracting equation (1) from equation (2) gives

$$12.5 = 2.5m, \quad \text{from which,} \quad \boldsymbol{m} = \frac{12.5}{2.5} = \mathbf{5}$$

Substituting $m = 5$ into equation (1) gives

$$-2 = 5 + c$$

$$\boldsymbol{c} = -2 - 5 = \mathbf{-7}$$

Checking, substituting $m = 5$ and $c = -7$ in equation (2), gives

$$\text{RHS} = (3.5)(5) + (-7) = 17.5 - 7 = 10.5 = \text{LHS}$$

Hence, the **gradient $m = 5$** and the **y-axis intercept $c = -7$**.

Problem 14. When Kirchhoff's laws are applied to the electrical circuit shown in Figure 13.1, the currents I_1 and I_2 are connected by the equations

$$27 = 1.5I_1 + 8(I_1 - I_2) \qquad (1)$$

$$-26 = 2I_2 - 8(I_1 - I_2) \qquad (2)$$

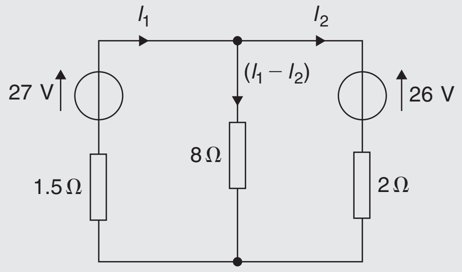

Figure 13.1

Solve the equations to find the values of currents I_1 and I_2

Removing the brackets from equation (1) gives

$$27 = 1.5I_1 + 8I_1 - 8I_2$$

Rearranging gives

$$9.5I_1 - 8I_2 = 27 \qquad (3)$$

Removing the brackets from equation (2) gives

$$-26 = 2I_2 - 8I_1 + 8I_2$$

Rearranging gives

$$-8I_1 + 10I_2 = -26 \qquad (4)$$

Multiplying equation (3) by 5 gives

$$47.5I_1 - 40I_2 = 135 \qquad (5)$$

Multiplying equation (4) by 4 gives

$$-32I_1 + 40I_2 = -104 \qquad (6)$$

Adding equations (5) and (6) gives

$$15.5I_1 + 0 = 31$$

$$\boldsymbol{I_1} = \frac{31}{15.5} = \mathbf{2}$$

Substituting $I_1 = 2$ into equation (3) gives

$$9.5(2) - 8I_1 = 27$$

$$19 - 8I_2 = 27$$

$$19 - 27 = 8I_2$$

$$-8 = 8I_2$$

and $\qquad \boldsymbol{I_2 = -1}$

Hence, the solution is $\boldsymbol{I_1 = 2}$ and $\boldsymbol{I_2 = -1}$ (which may be checked in the original equations).

Problem 15. The distance s metres from a fixed point of a vehicle travelling in a straight line with constant acceleration, a m/s^2, is given by $s = ut + \frac{1}{2}at^2$, where u is the initial velocity in m/s and t the time in seconds. Determine the initial velocity and the acceleration given that $s = 42$ m when $t = 2$ s, and $s = 144$ m when $t = 4$ s. Also find the distance travelled after 3 s