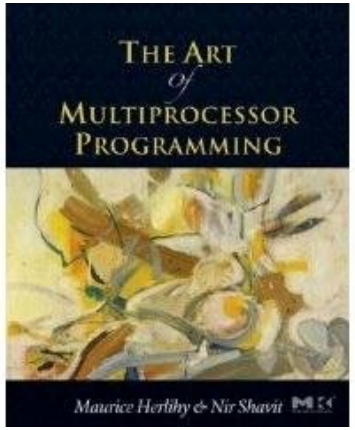
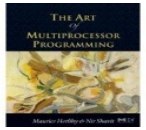
Foundations of Shared Memory



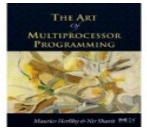
Hyungsoo Jung



Art of Multiprocessor Programming

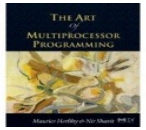
Last Lecture

- Defined concurrent objects using linearizability and sequential consistency
- Fact: implemented linearizable objects (Two thread FIFO Queue) in read-write memory without mutual exclusion
- Fact: hardware does not provide linearizable read-write memory



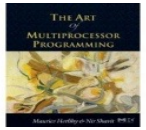
Fundamentals

- What is the weakest form of communication that supports mutual exclusion?
- What is the weakest shared object that allows shared-memory computation?

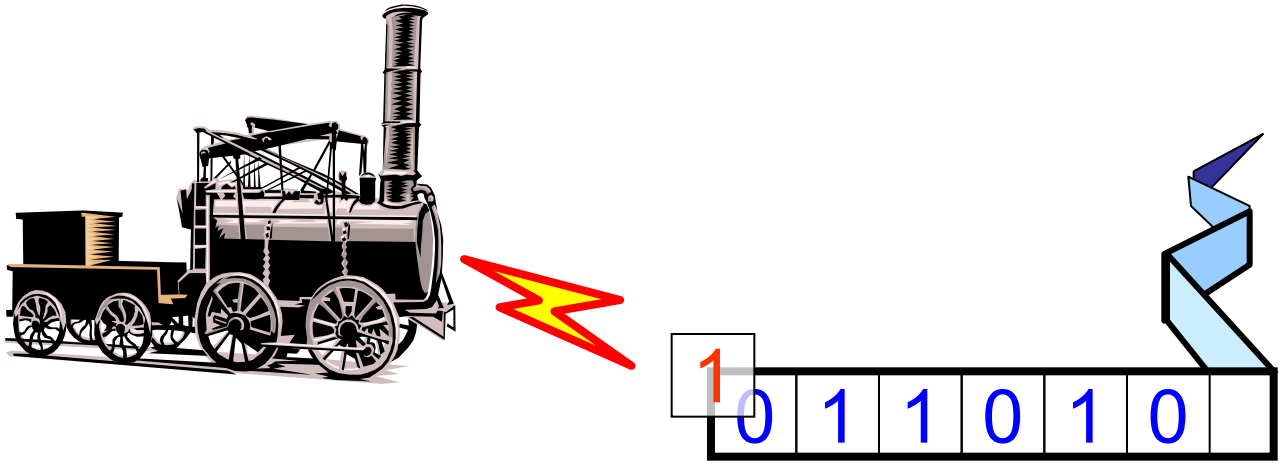


Alan Turing

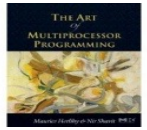
- Showed what is and is not computable on a sequential machine.
- Still best model there is.



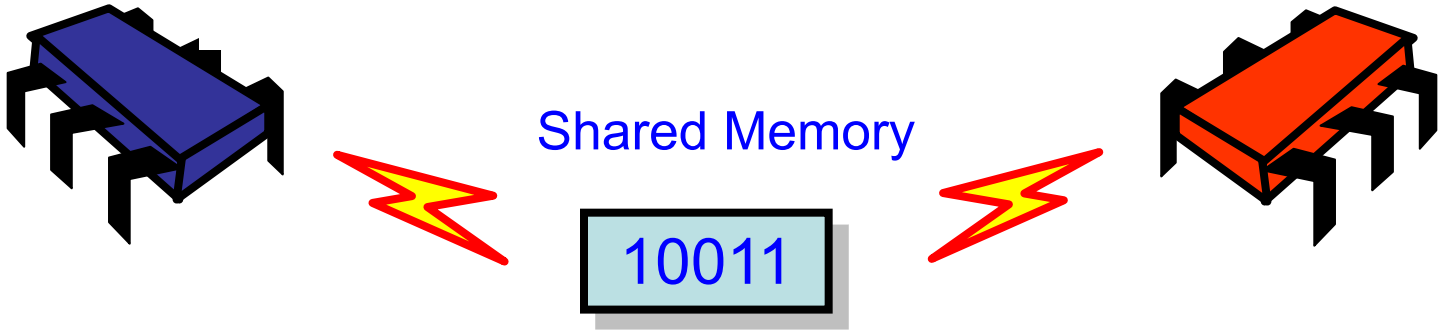
Turing Computability



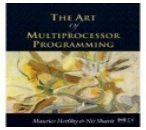
- Mathematical model of computation
- What is (and is not) computable
- Efficiency (mostly) irrelevant



Shared-Memory Computability?

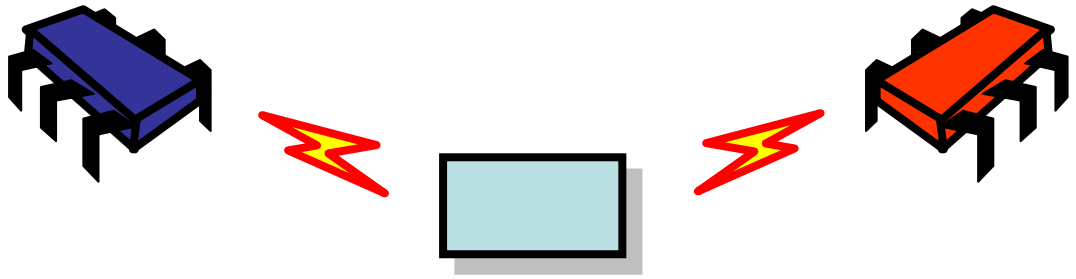


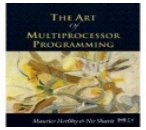
- Mathematical model of concurrent computation
- What is (and is not) concurrently computable
- Efficiency (mostly) irrelevant



Foundations of Shared Memory

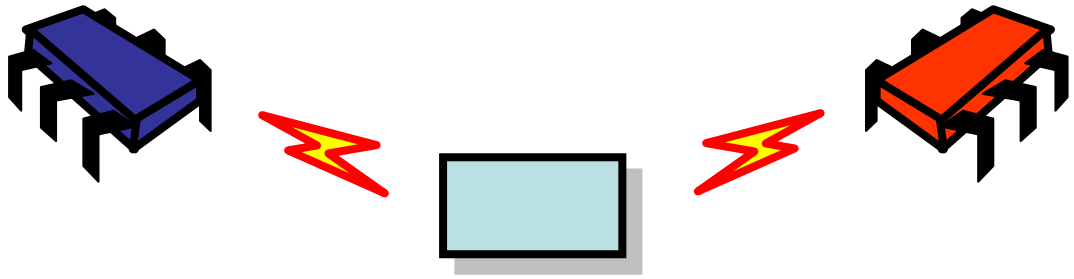
To understand modern multiprocessors we need to ask some basic questions …

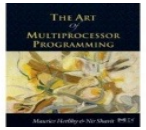




Foundations of Shared Memory

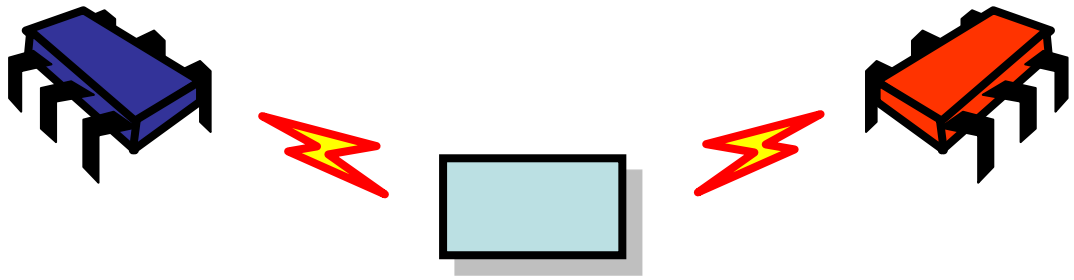
What is the weakest useful form of shared memory?

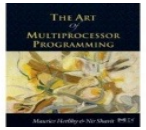




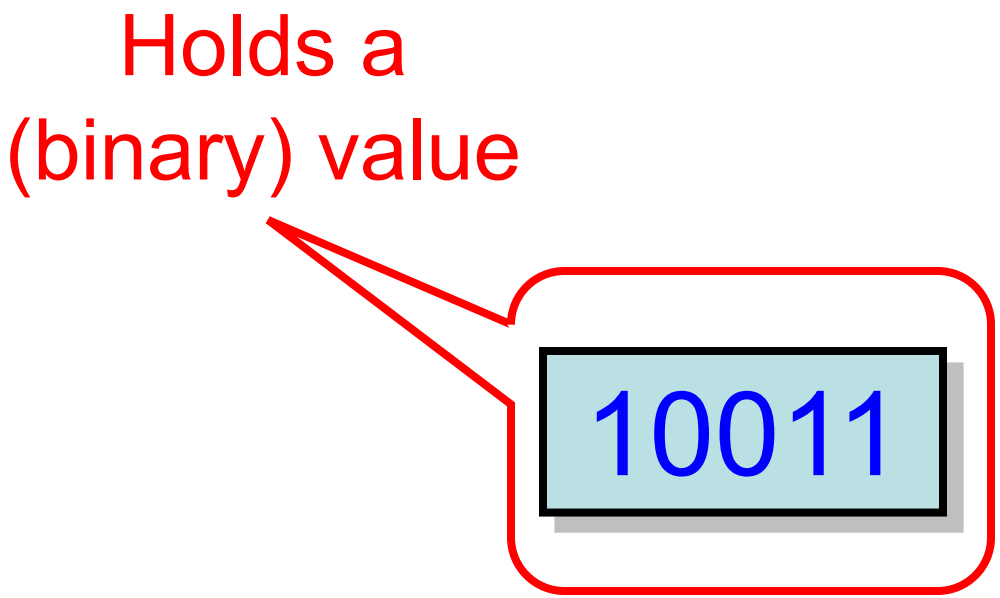
Foundations of Shared Memory

What can it do?

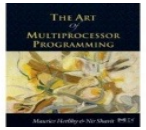




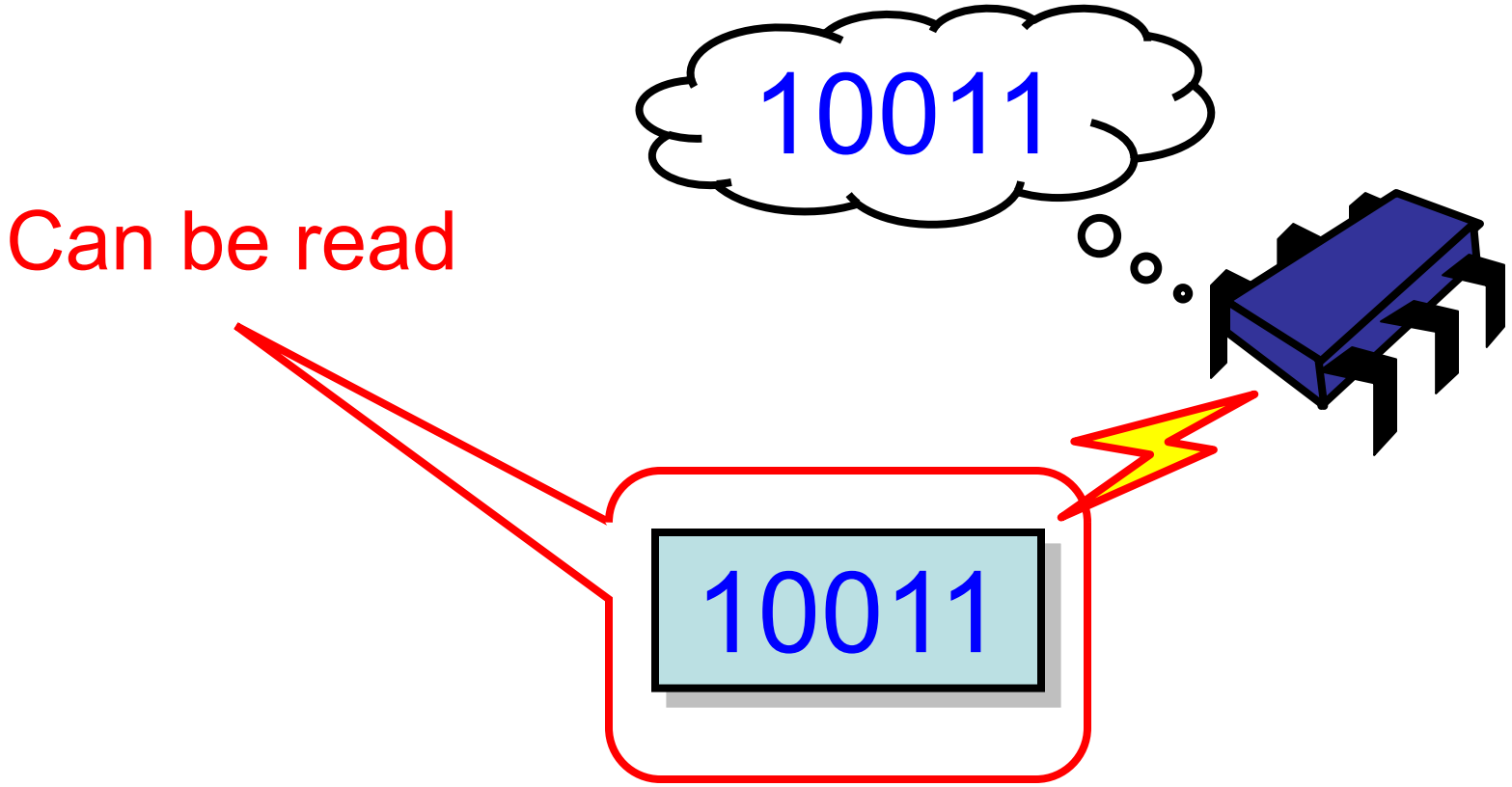
Register*

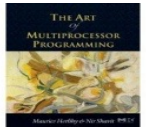


*** A memory location: name is historical**

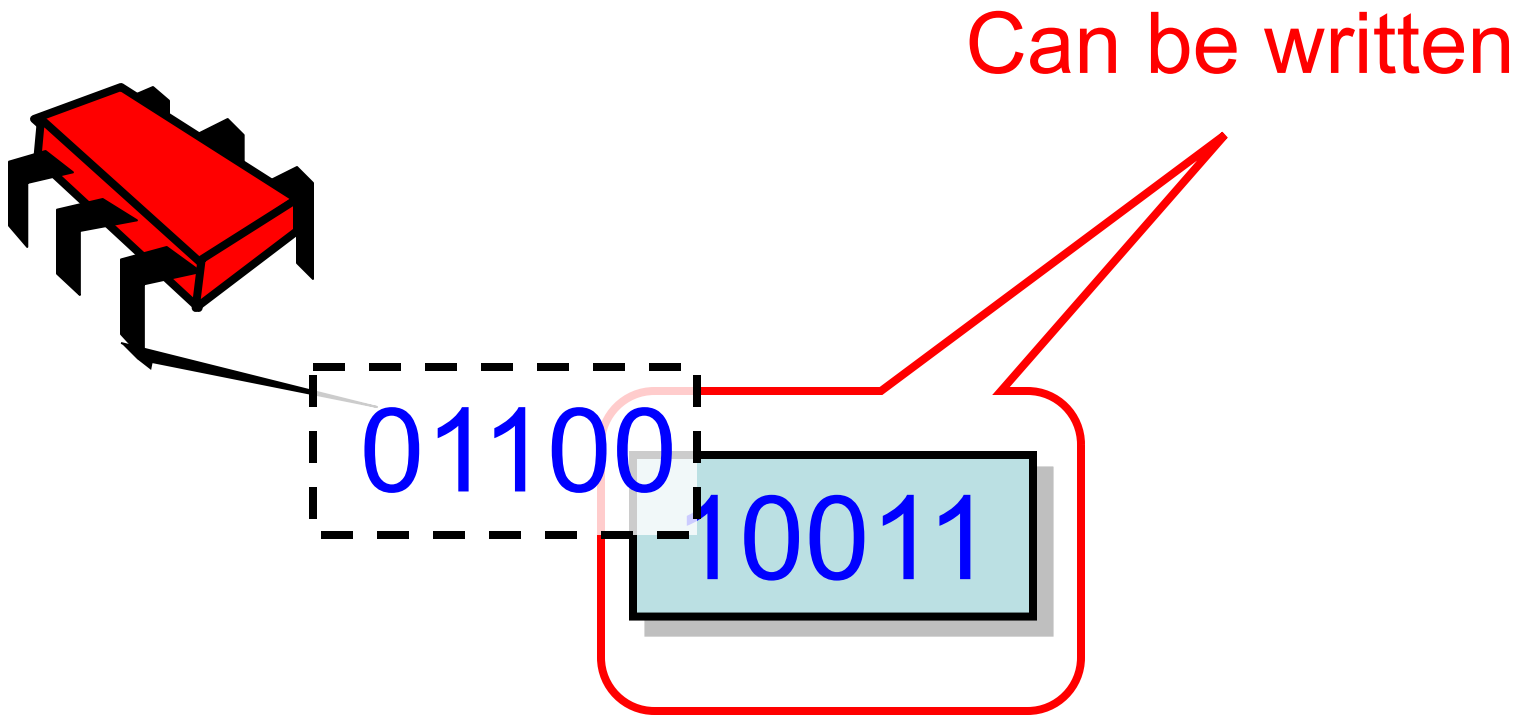


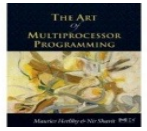
Register





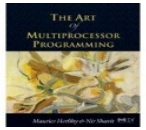
Register





Registers

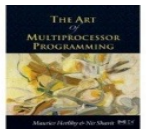
```
public interface Register<T> {
  public T read();
  public void write(T v);
}
```



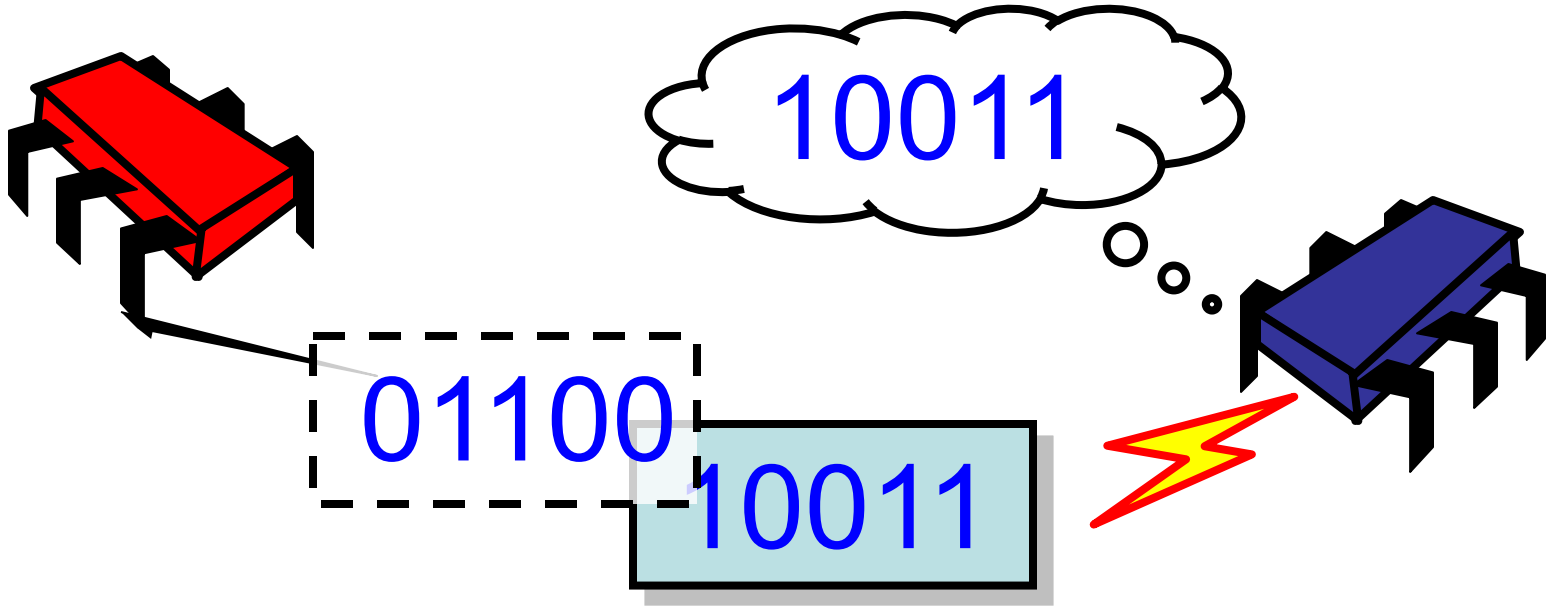
Registers

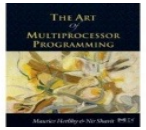
```
public interface Register<T> {
  public T read();
  public void write(T v);
}
```

Type of register
(usually Boolean or *m*-bit Integer)

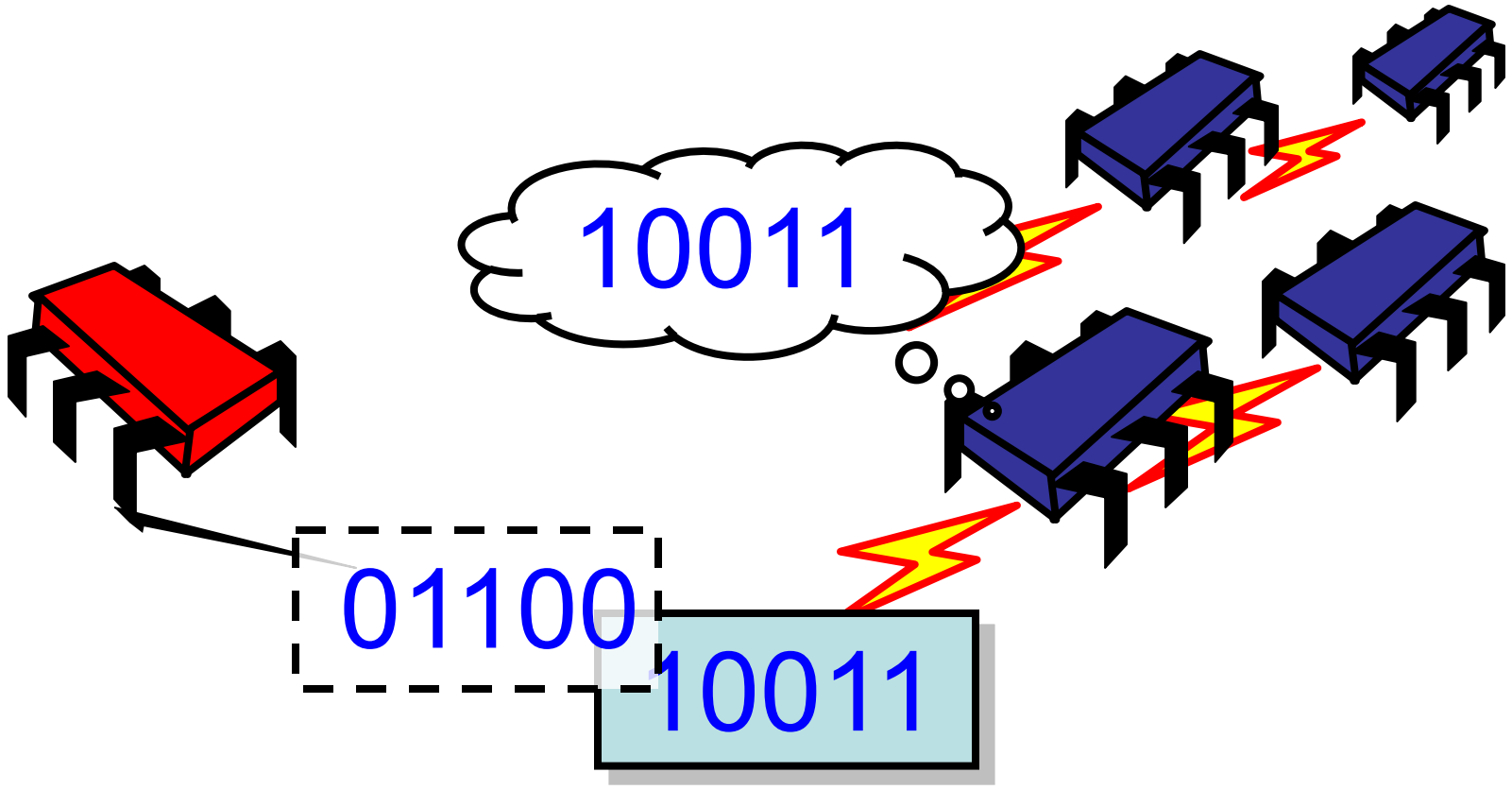


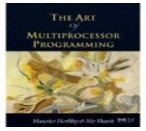
Single-Reader/Single-Writer Register



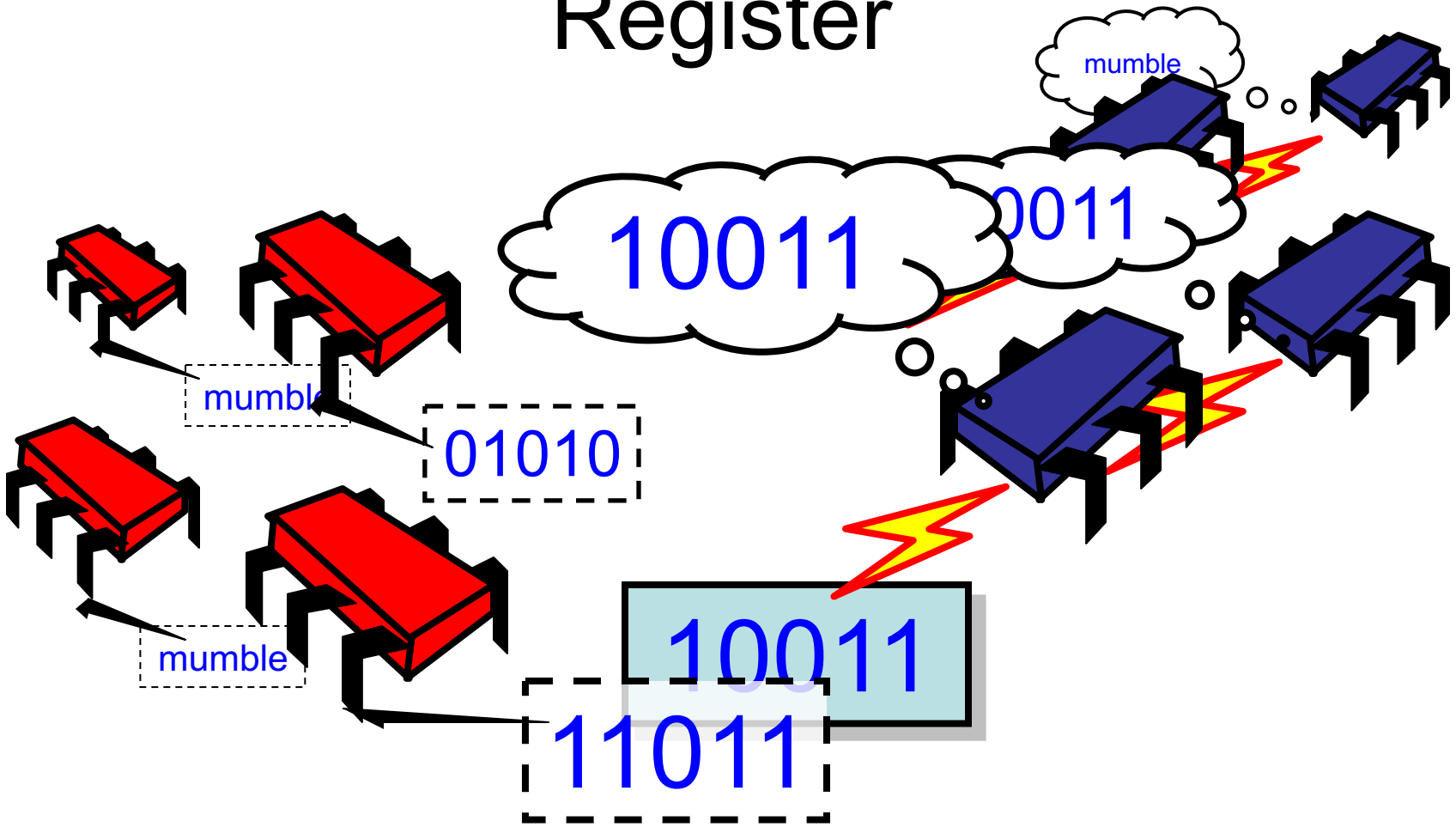


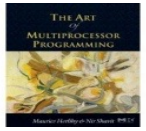
Multi-Reader/Single-Writer Register





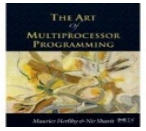
Multi-Reader/Multi-Writer Register





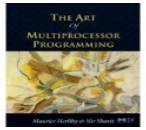
Jargon Watch

- SRSW
  - Single-reader single-writer
- MRSW
  - Multi-reader single-writer
- MRMW
  - Multi-reader multi-writer



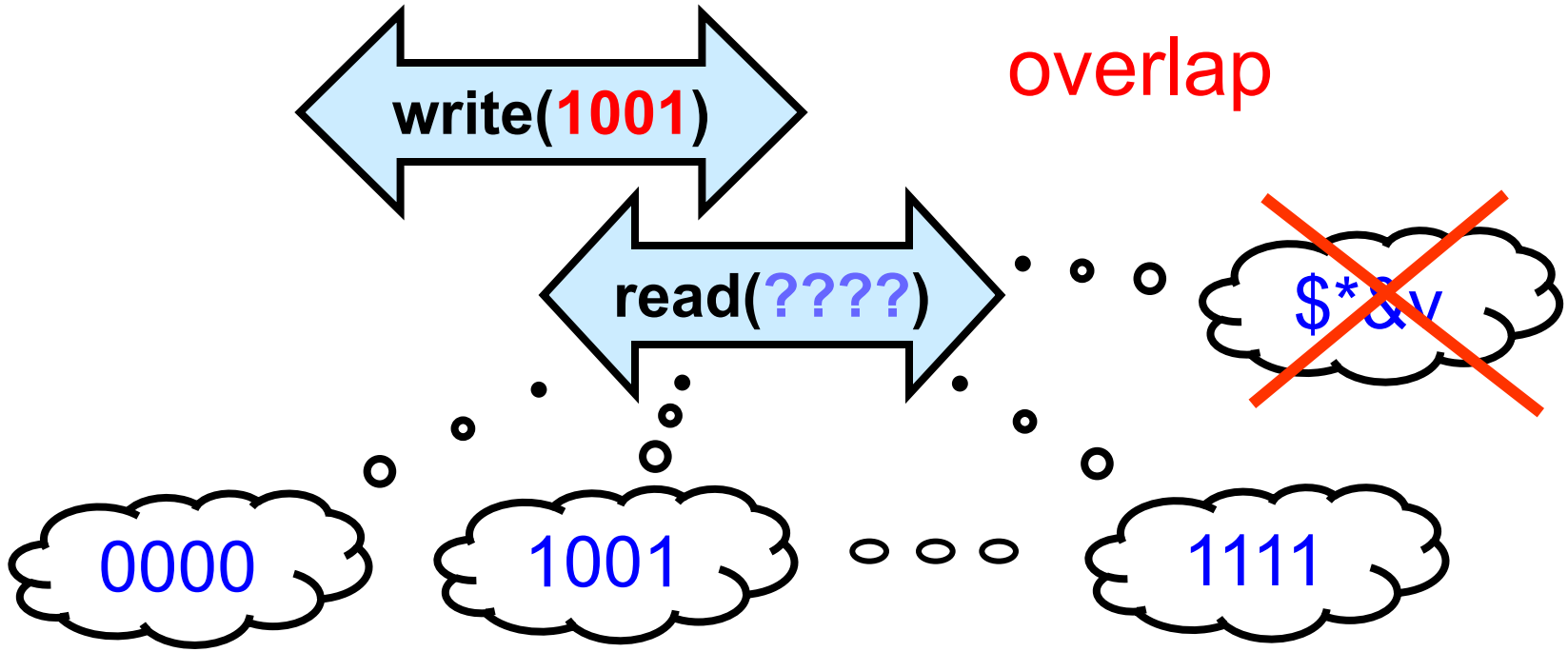
Safe Register

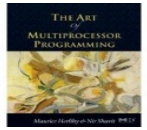
OK if reads
and writes
don't overlap



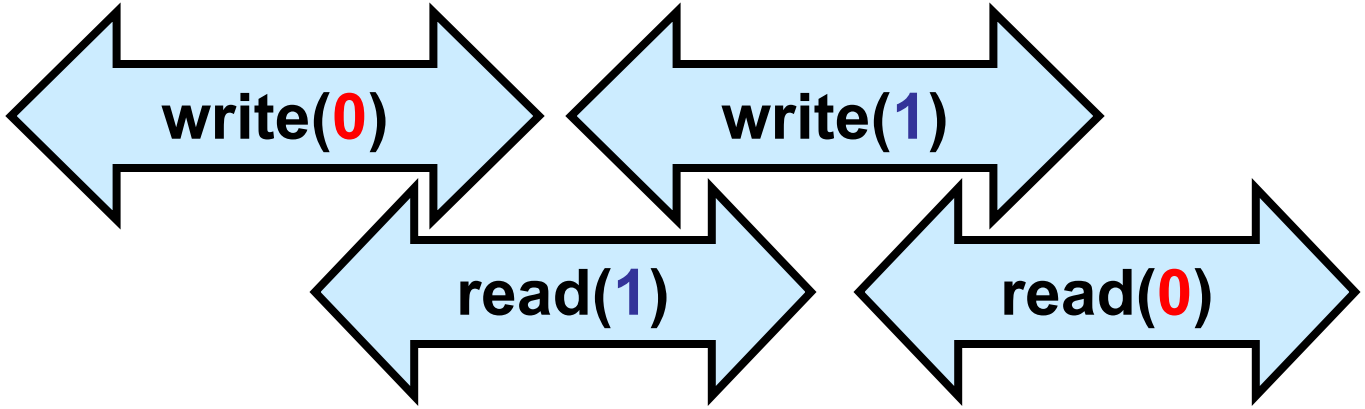
Safe Register

Some valid value if reads and writes do overlap

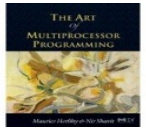




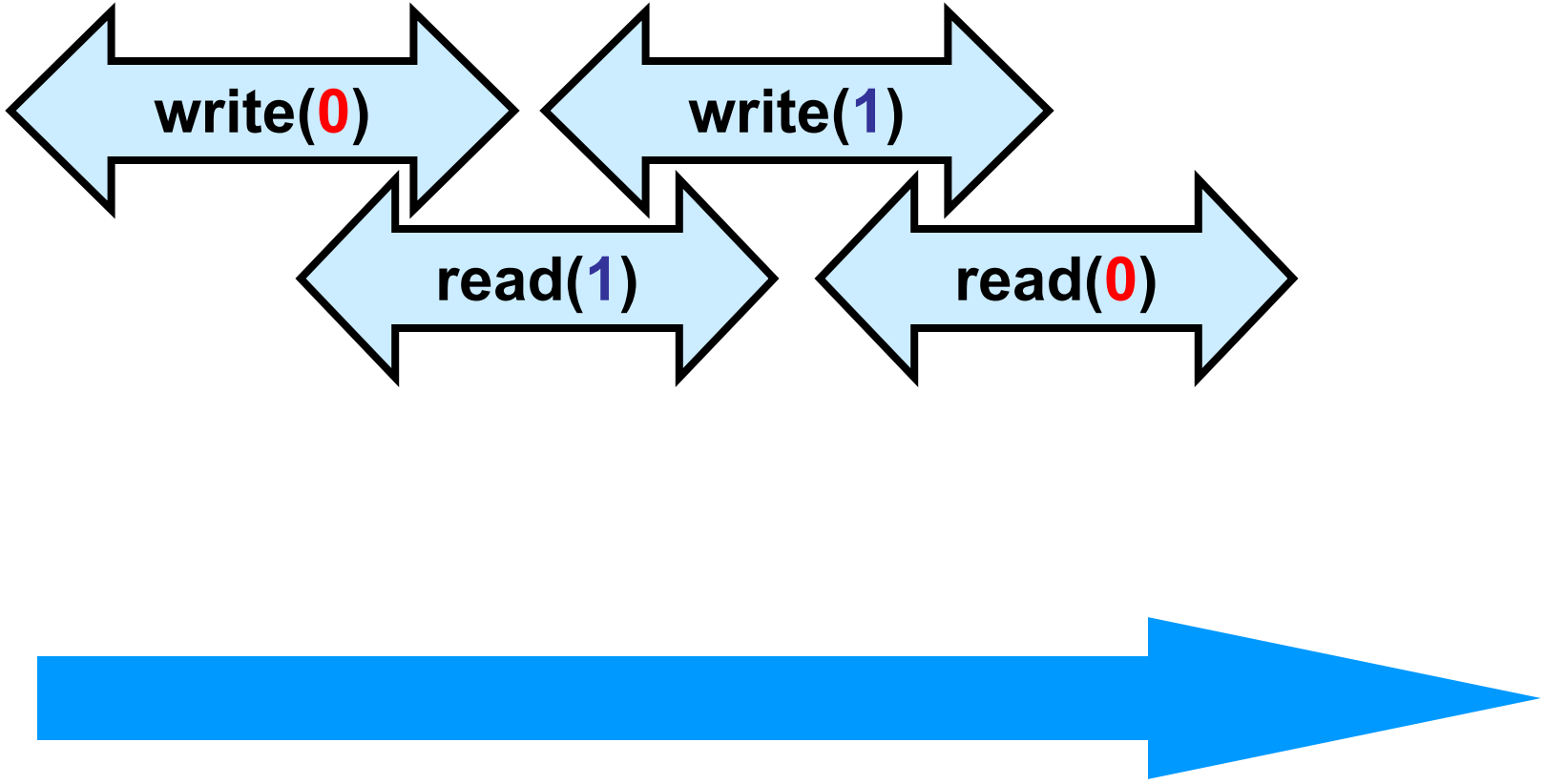
Regular Register

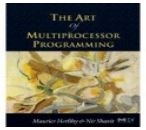


- Single Writer
- Readers return:
  - Old value if no overlap (safe)
  - Old or one of new values if overlap

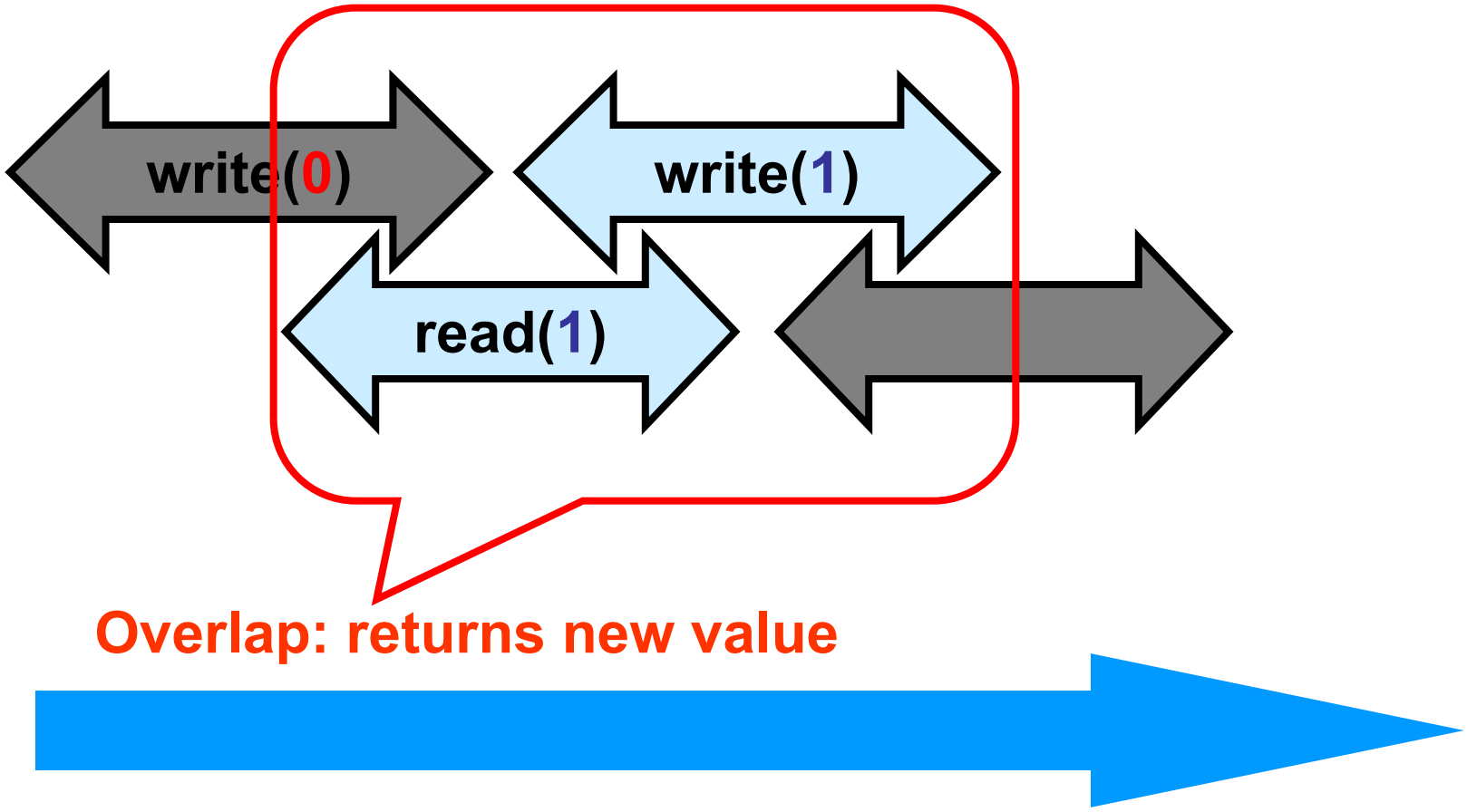


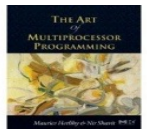
Regular or Not?



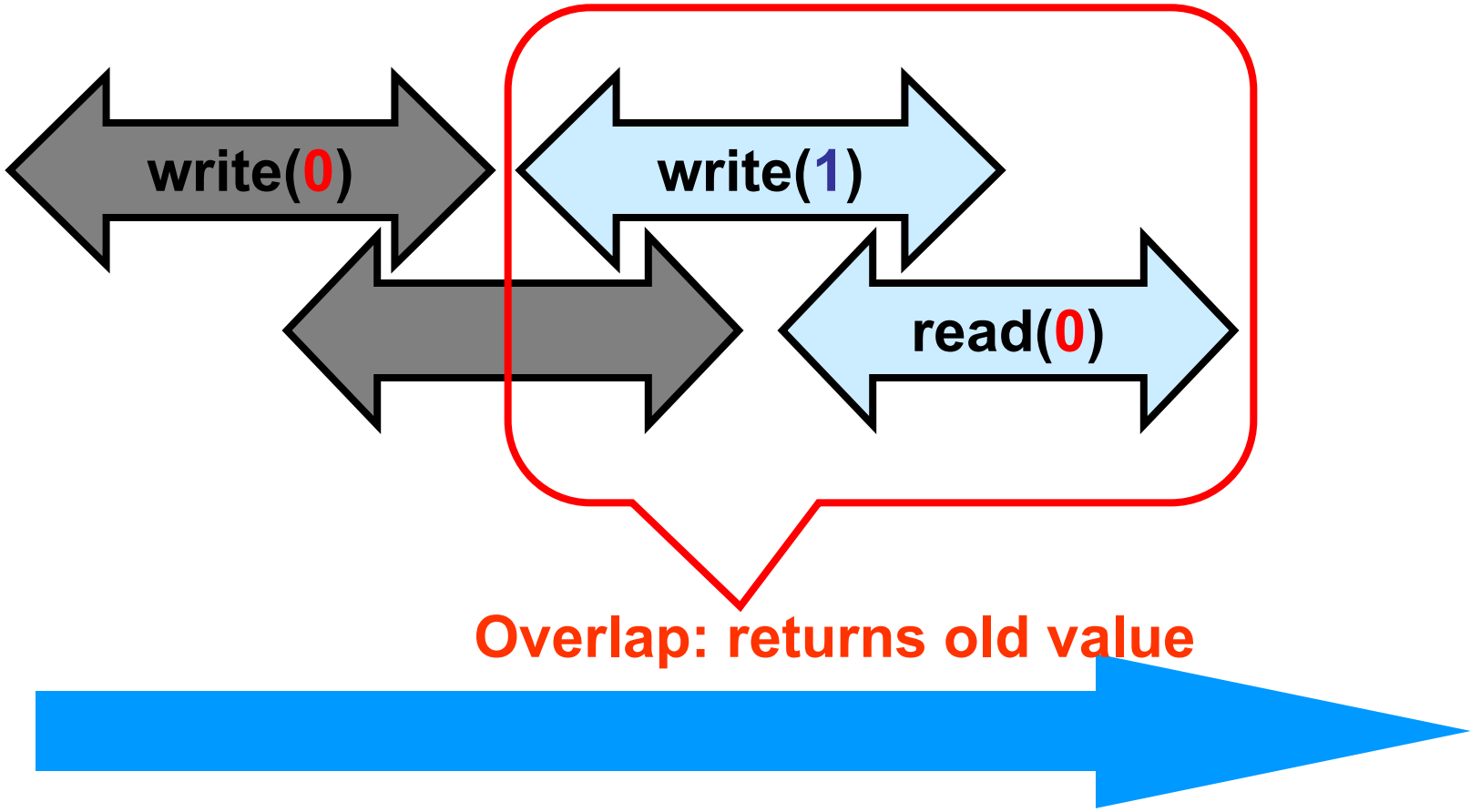


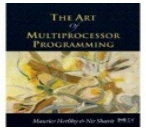
Regular or Not?



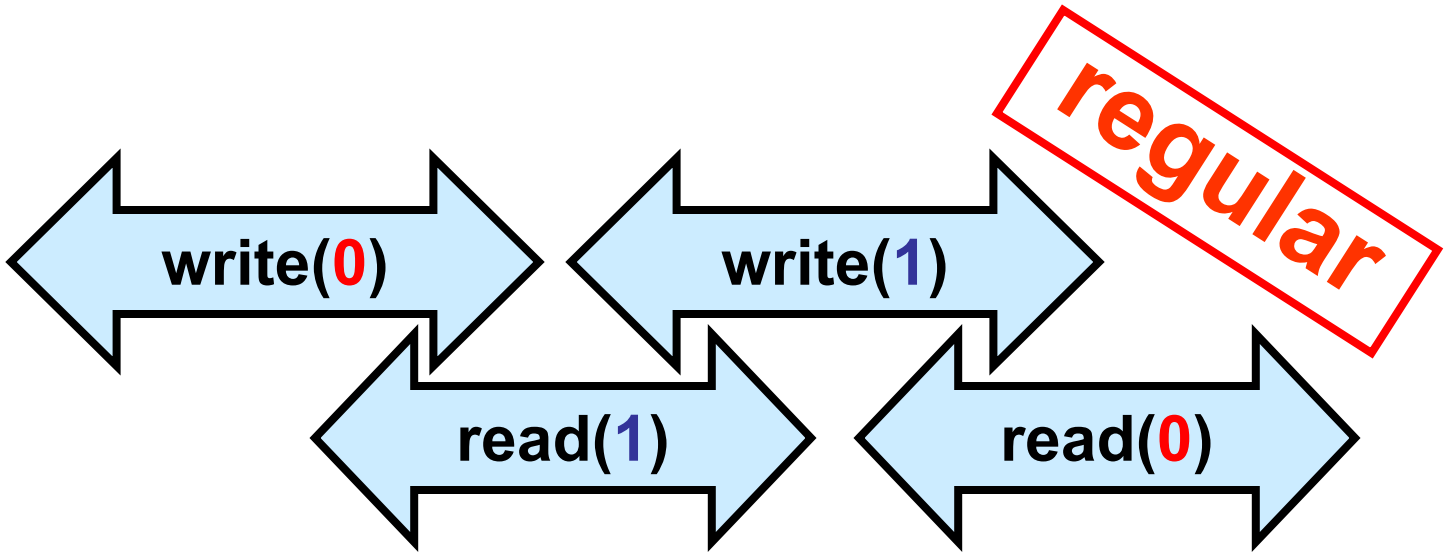


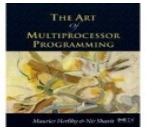
Regular or Not?



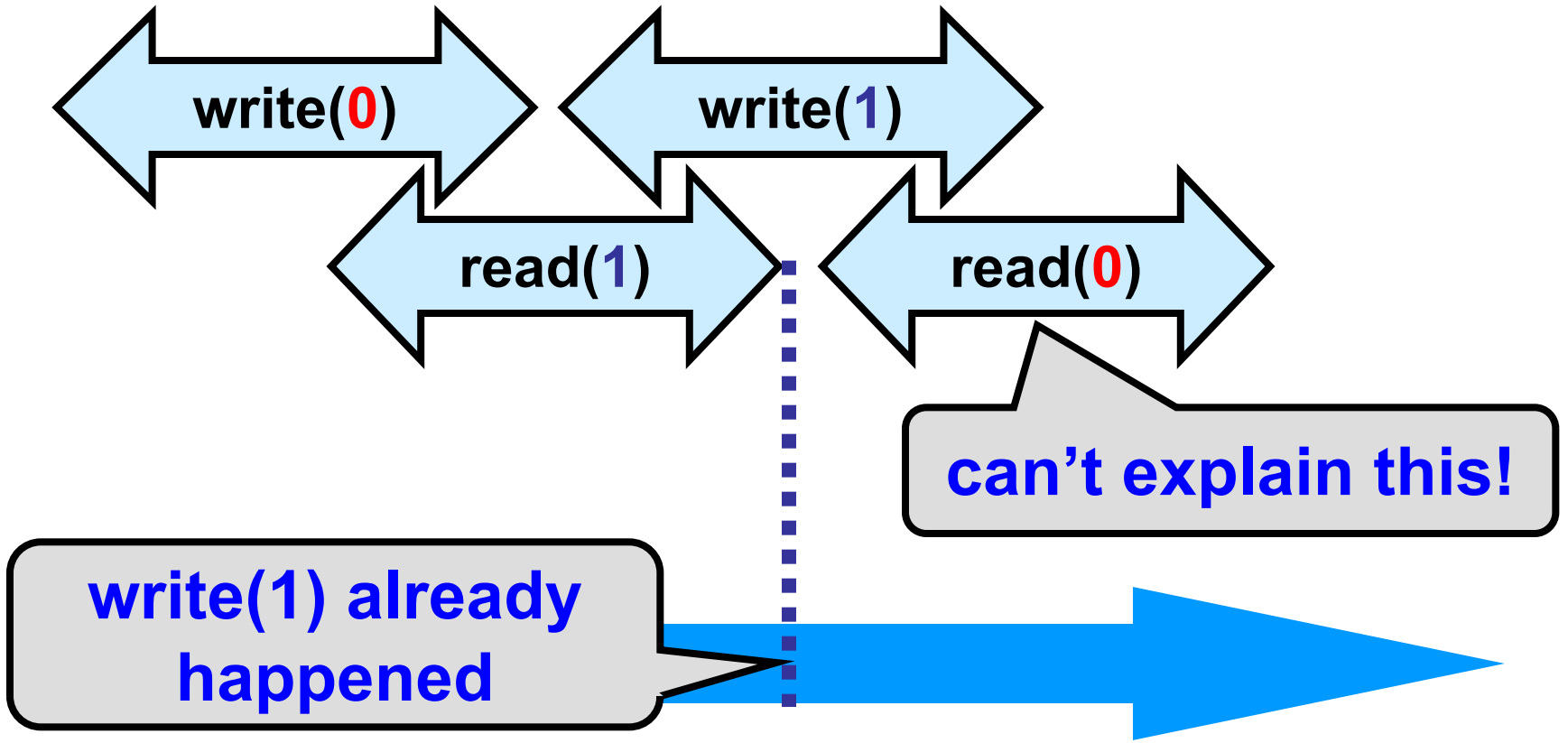


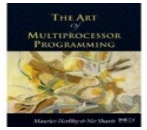
Regular or Not?



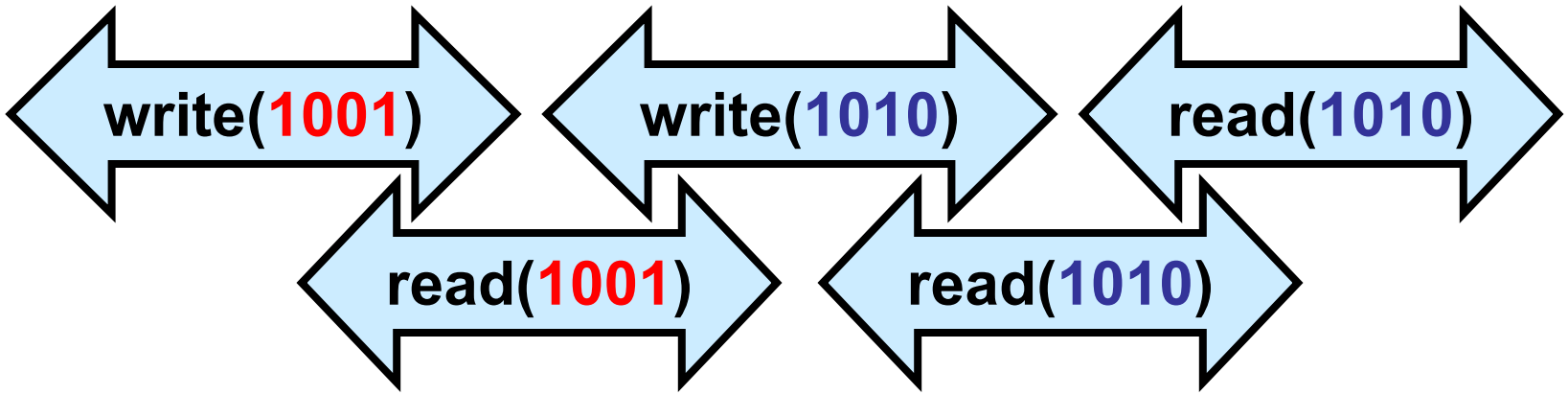


Regular ≠ Linearizable

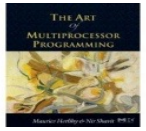




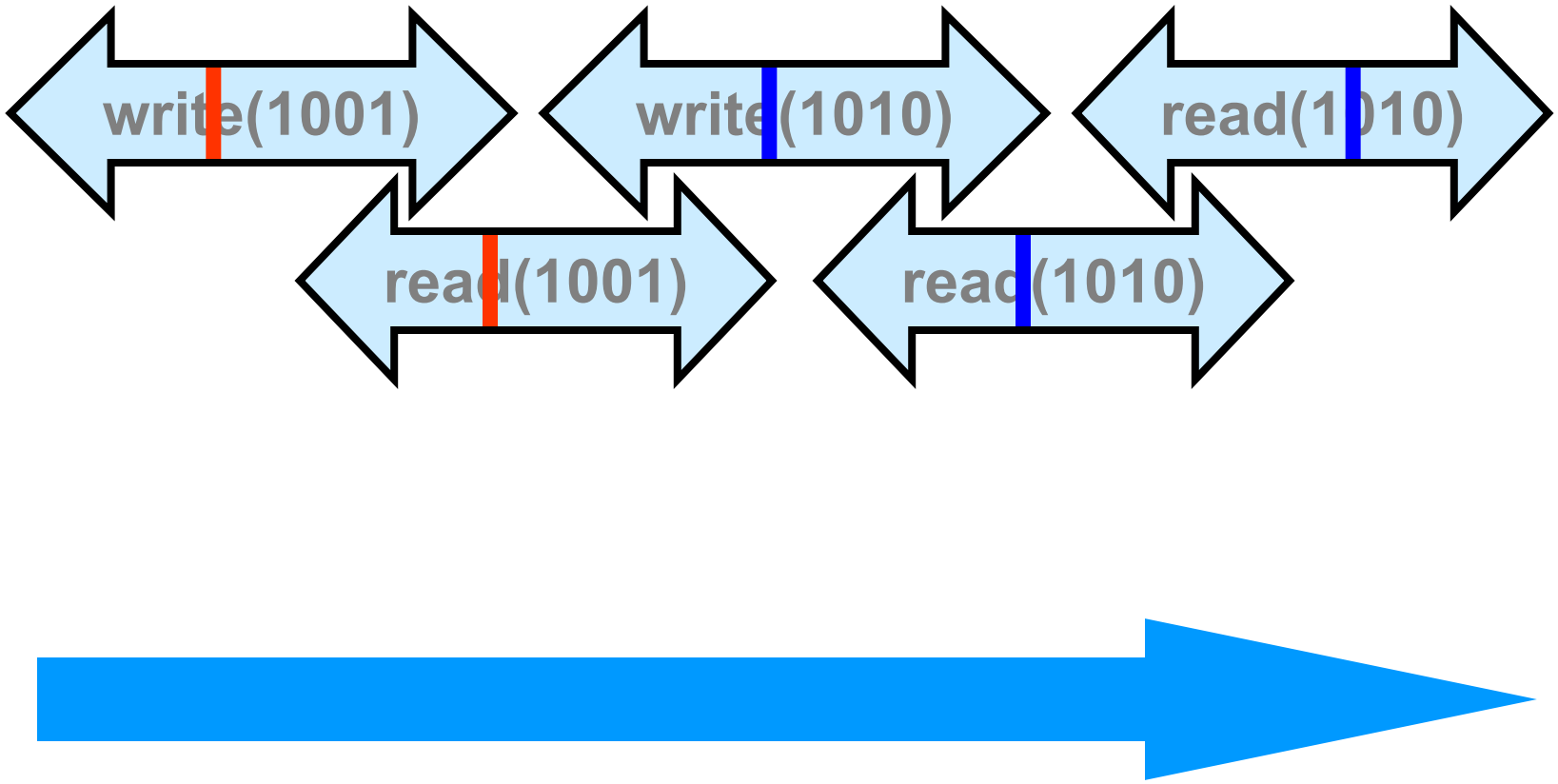
Atomic Register

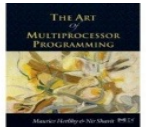


Linearizable to sequential safe register

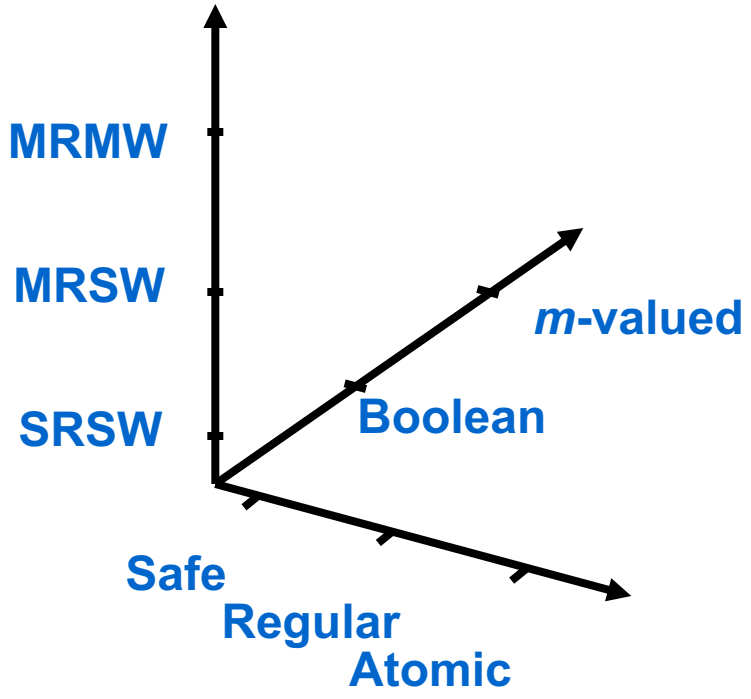


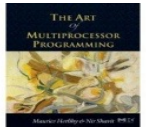
Atomic Register



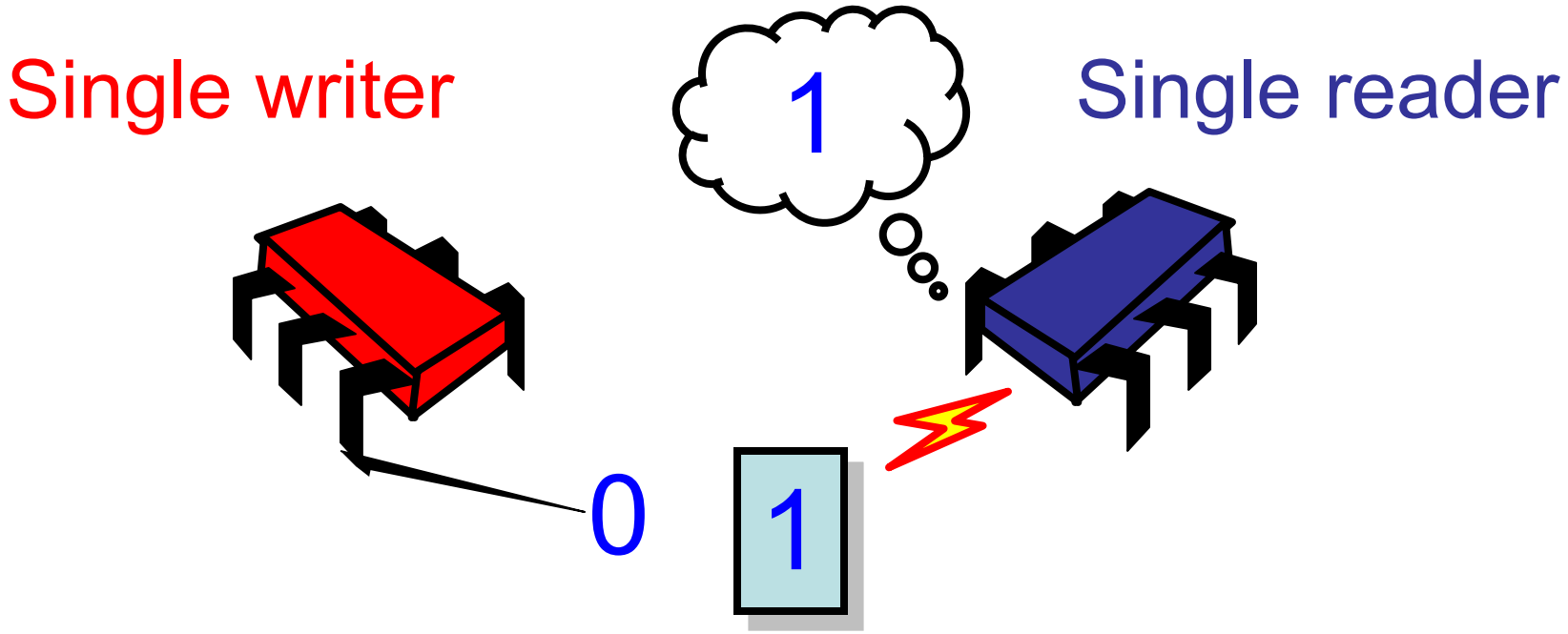


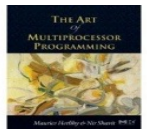
Register Space





Weakest Register

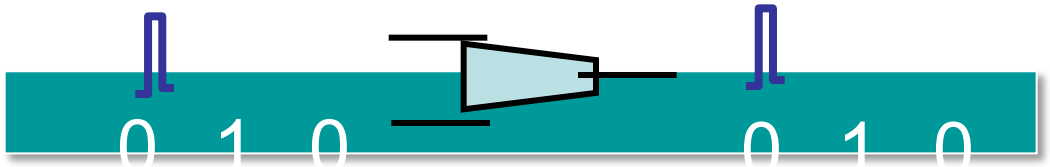




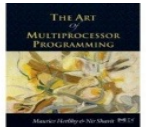
Weakest Register

Single writer

Single reader



Get correct reading if not during state transition

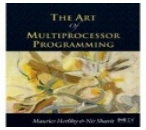


Results

- From SRSW safe Boolean register
  - All the other registers
  - Mutual exclusion
- But not everything!
  - Consensus hierarchy

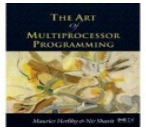
Foundations of the field

The really cool stuff …



Locking within Registers

- Not interesting to rely on mutual exclusion in register constructions
- We want registers to implement mutual exclusion!
- It's cheating to use mutual exclusion to implement itself!

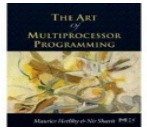


Definition

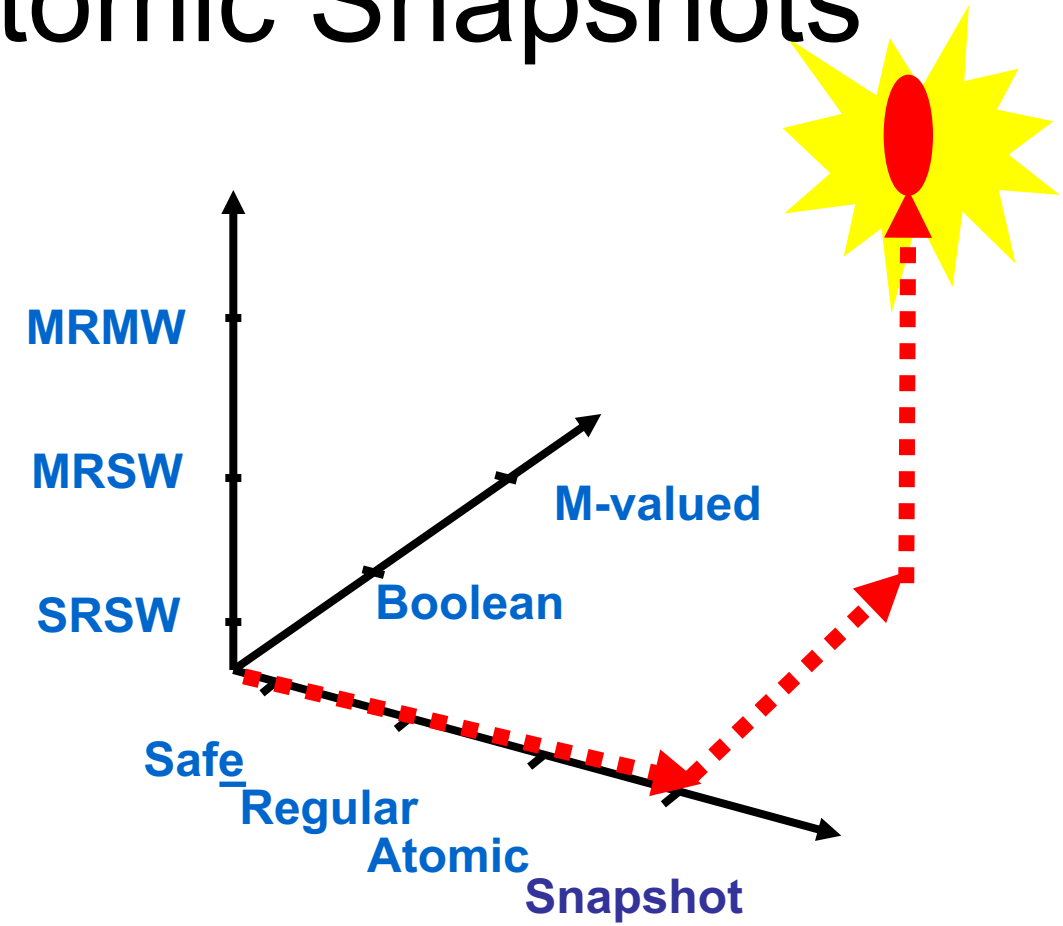
An object implementation is ***wait-free*** if every method call completes in a finite number of steps

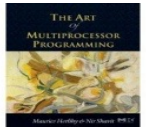
No mutual exclusion

- Thread could halt in critical section
- Build mutual exclusion from registers



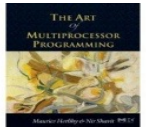
From Safe SRSW Boolean to Atomic Snapshots





Road Map

- SRSW safe Boolean
- MRSW safe Boolean
- MRSW regular Boolean
- MRSW regular
- MRSW atomic
- MRMW atomic
- Atomic snapshot



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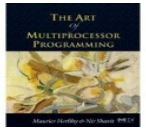

Register Names

```
public class SafeBoolMRSWRegister
 implements Register<Boolean> {
  public boolean read() { … }
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}
```

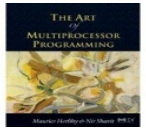

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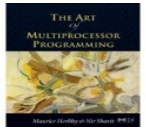
property



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property

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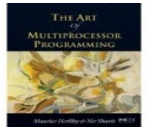

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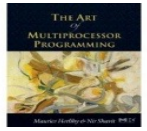
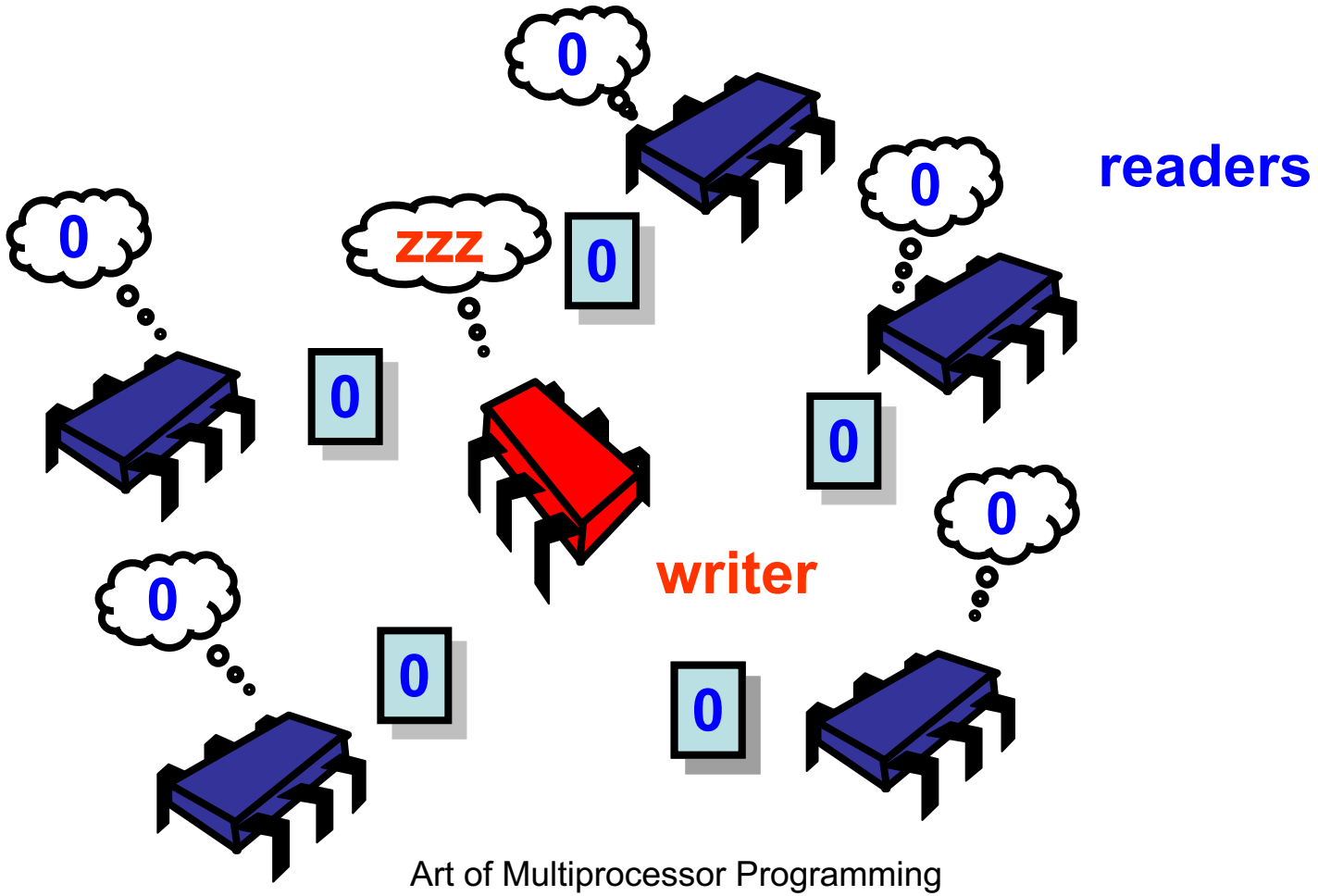
property

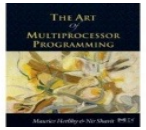
type

how many readers & writers?



Safe Boolean MRSW from Safe Boolean SRSW





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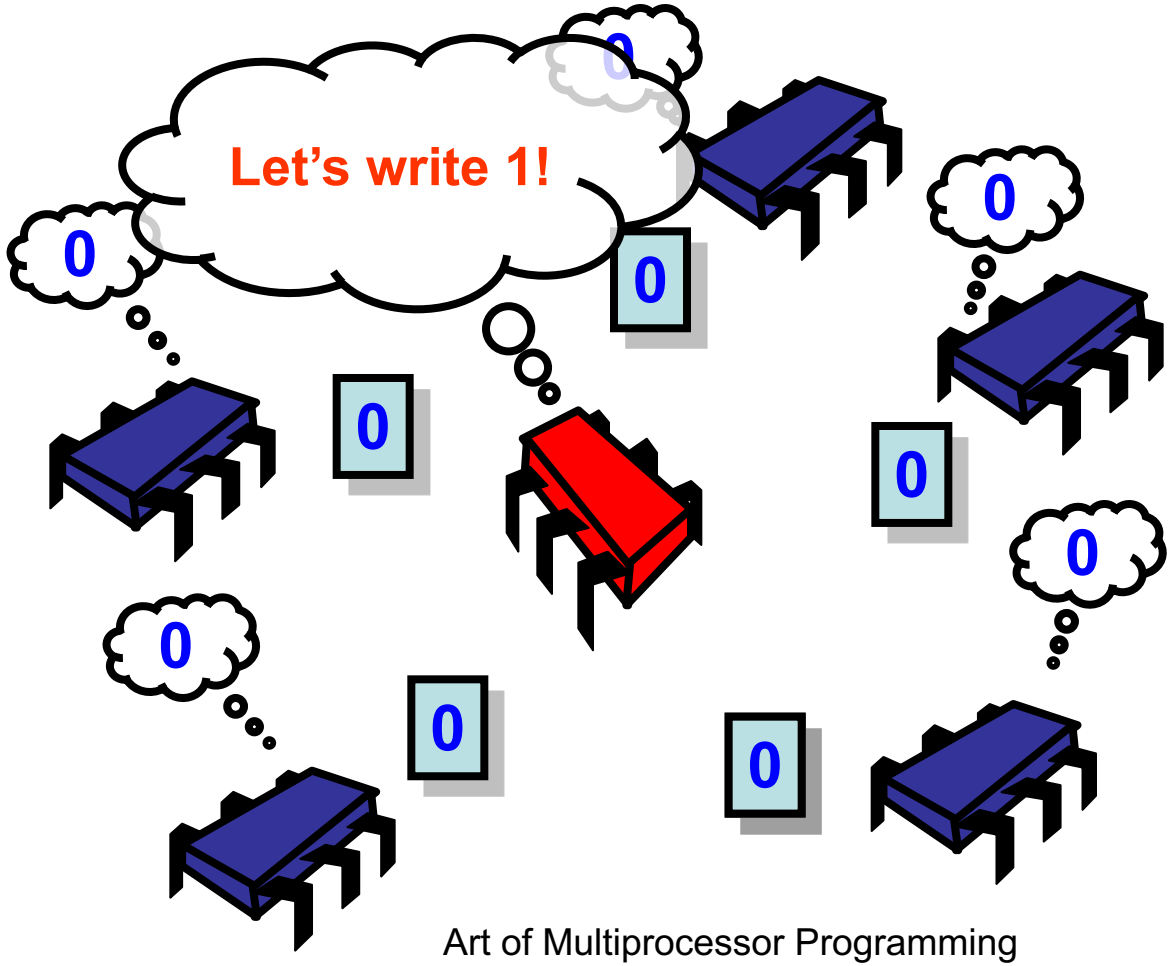

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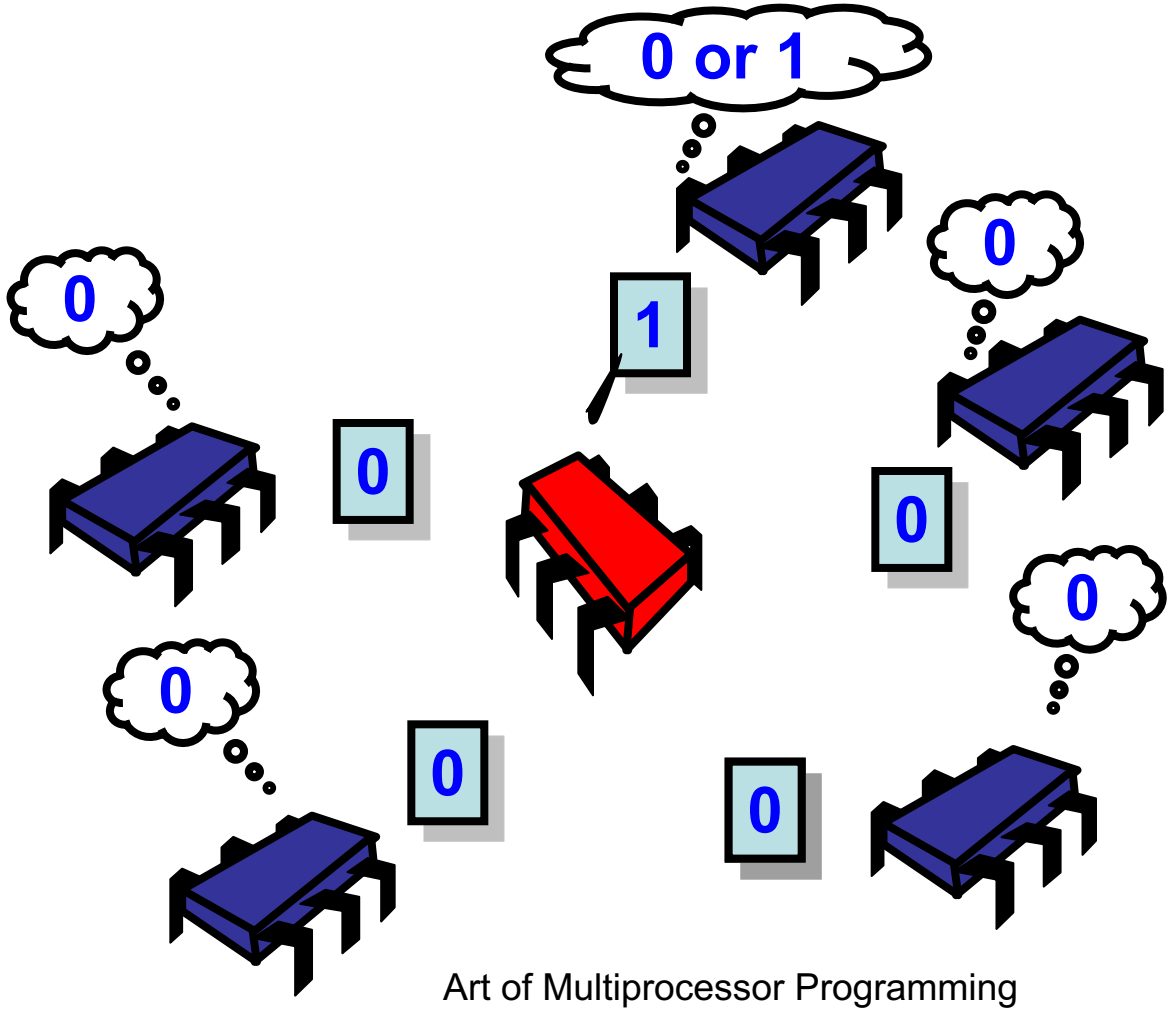

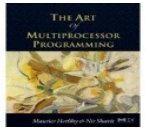

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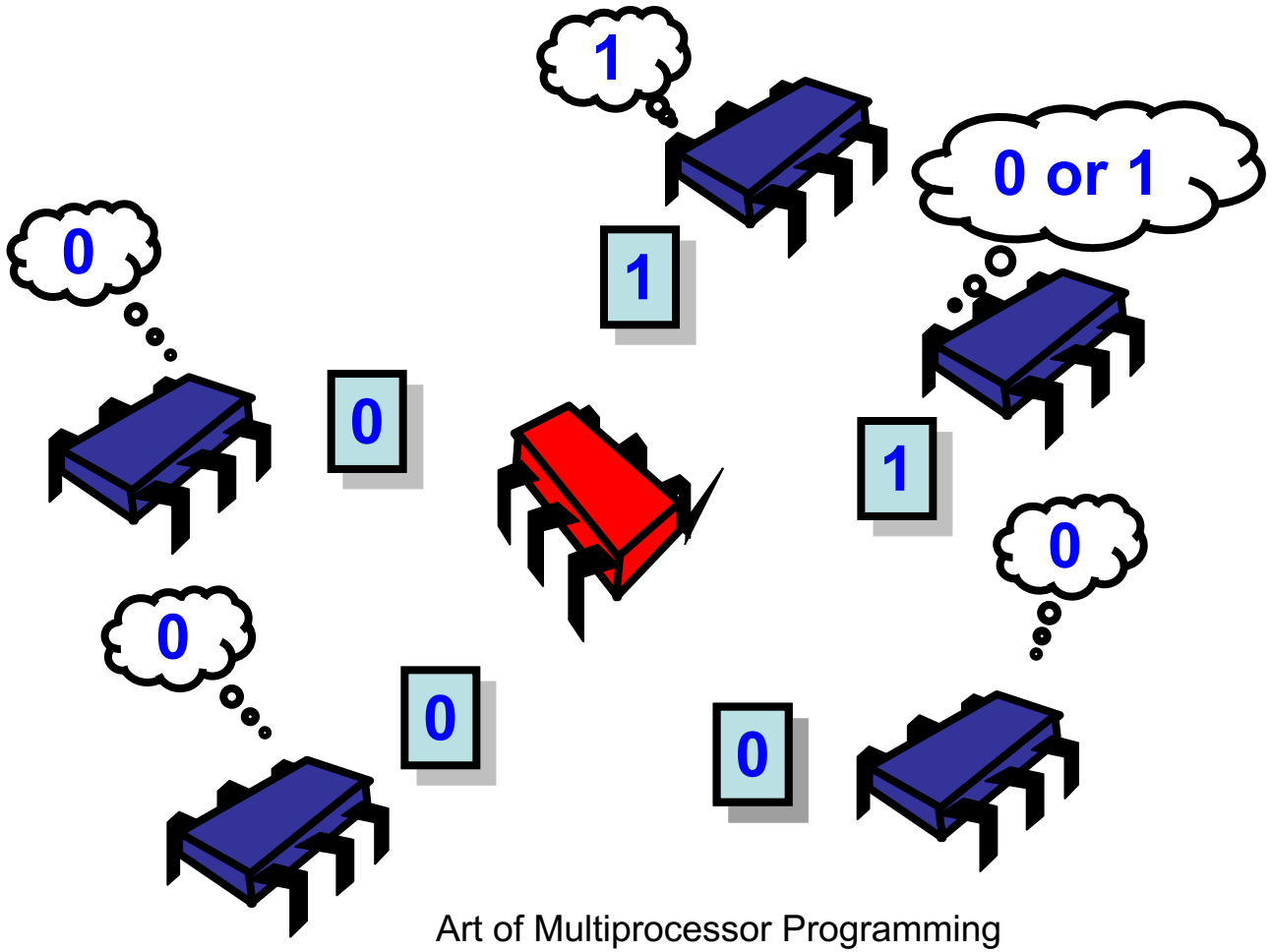

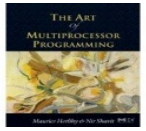

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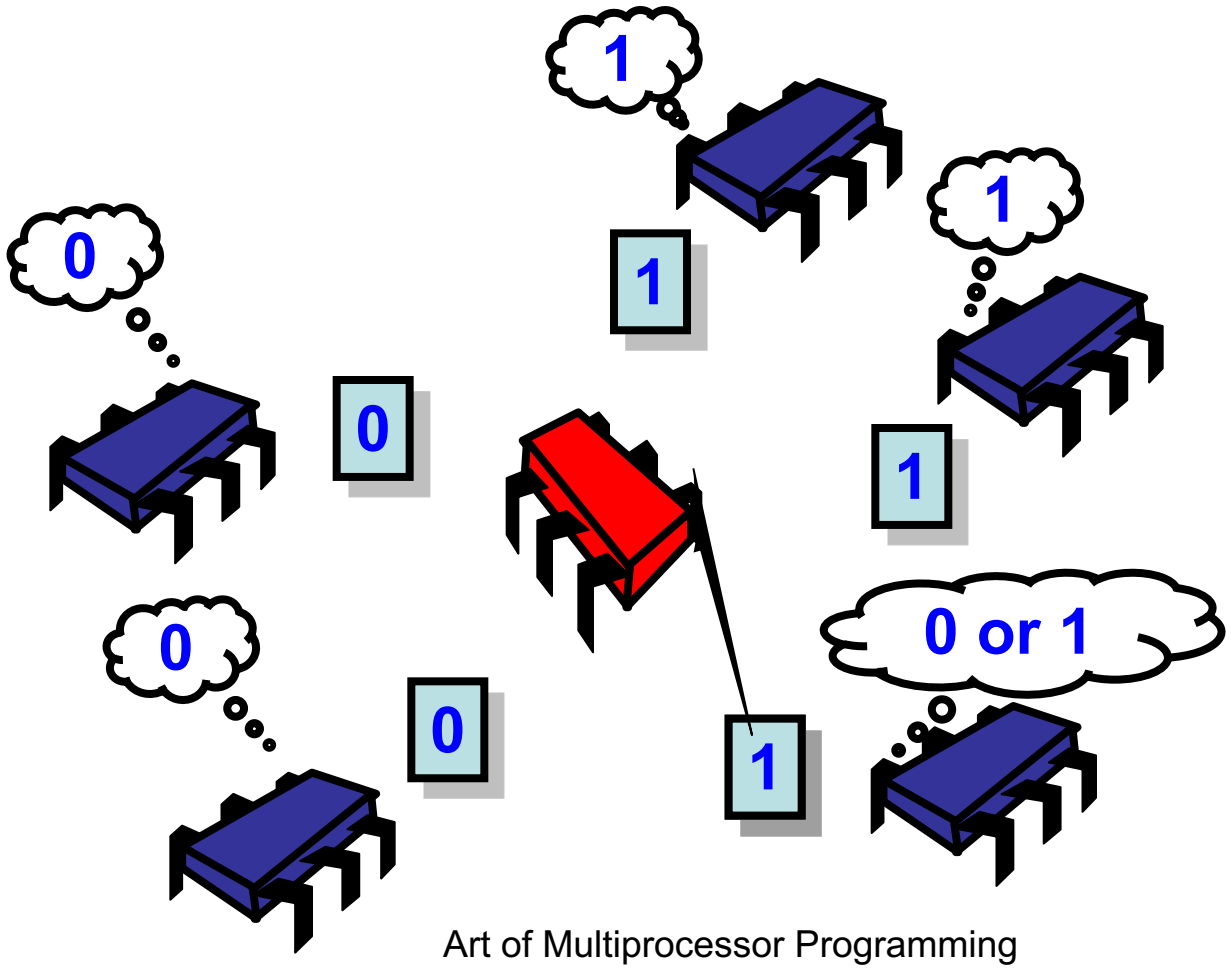

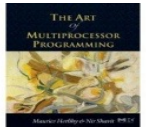

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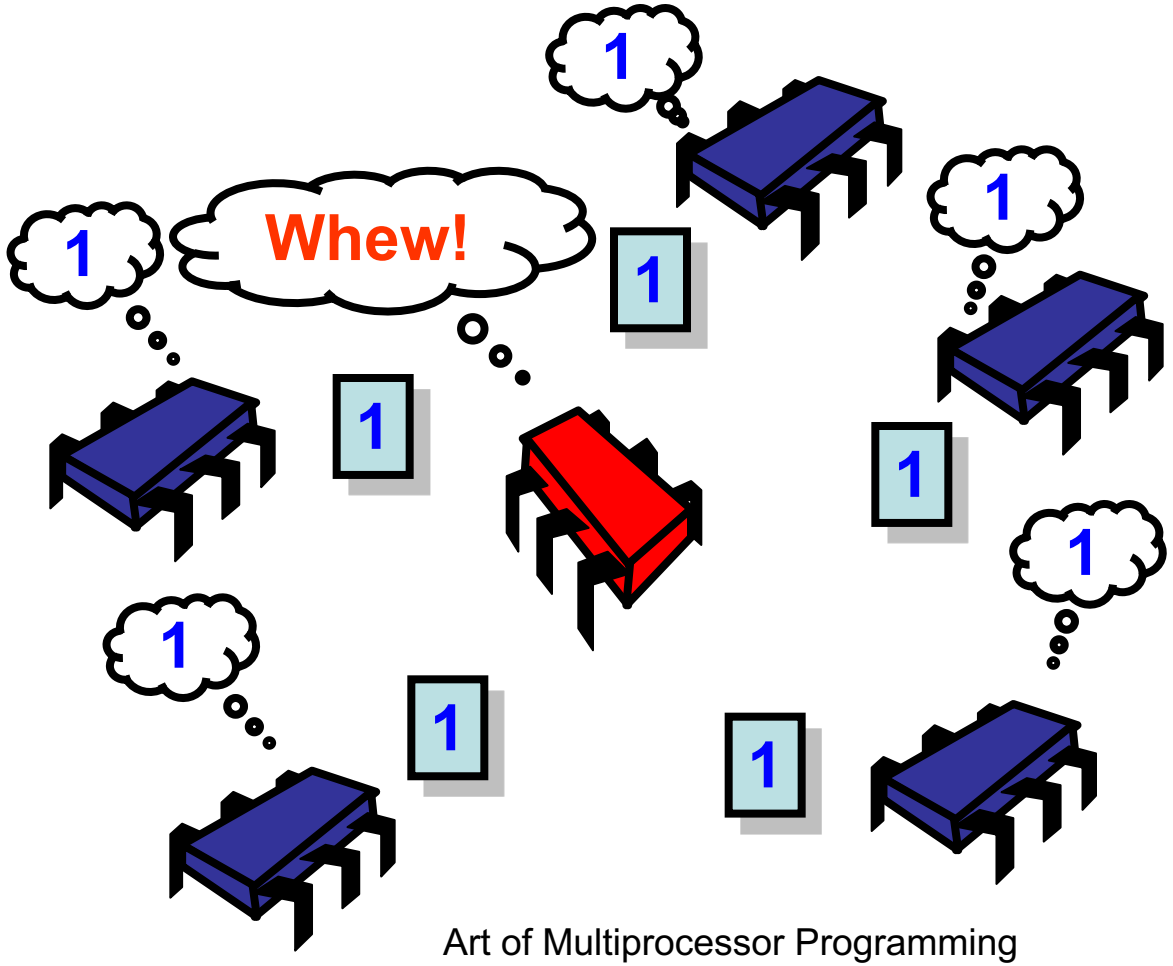

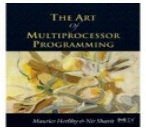

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   for (int j = 0; j < N; j++)
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  }
  public boolean read() {
   int i = ThreadID.get();
   return r[i].read();
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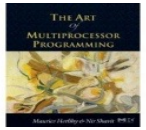

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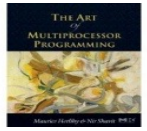
   return r[i].read();
  }}
```

Each thread has own safe SRSW register



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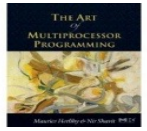
write method



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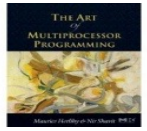
Write each thread's register one at a time



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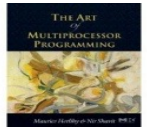
read method



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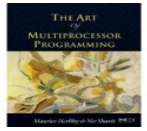
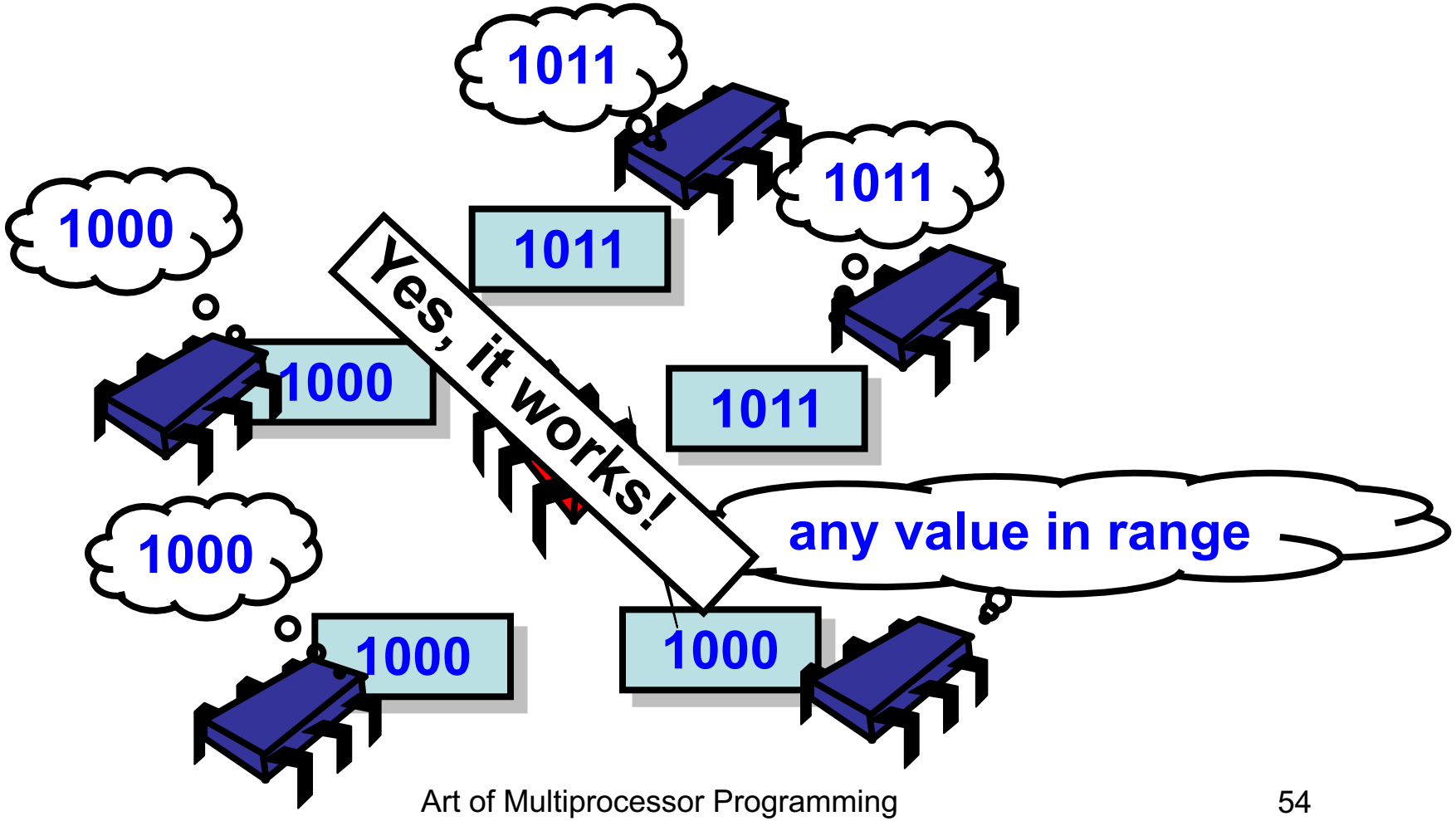
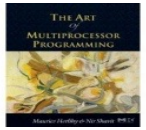
Read my own register



Safe Multi-Valued MRSW from Safe Multi-Valued SRSW?





Road Map

- SRSW safe Boolean
- MRSW safe Boolean
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- MRSW regular
- MRSW atomic
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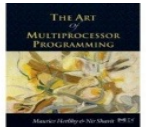
Questions?



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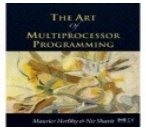
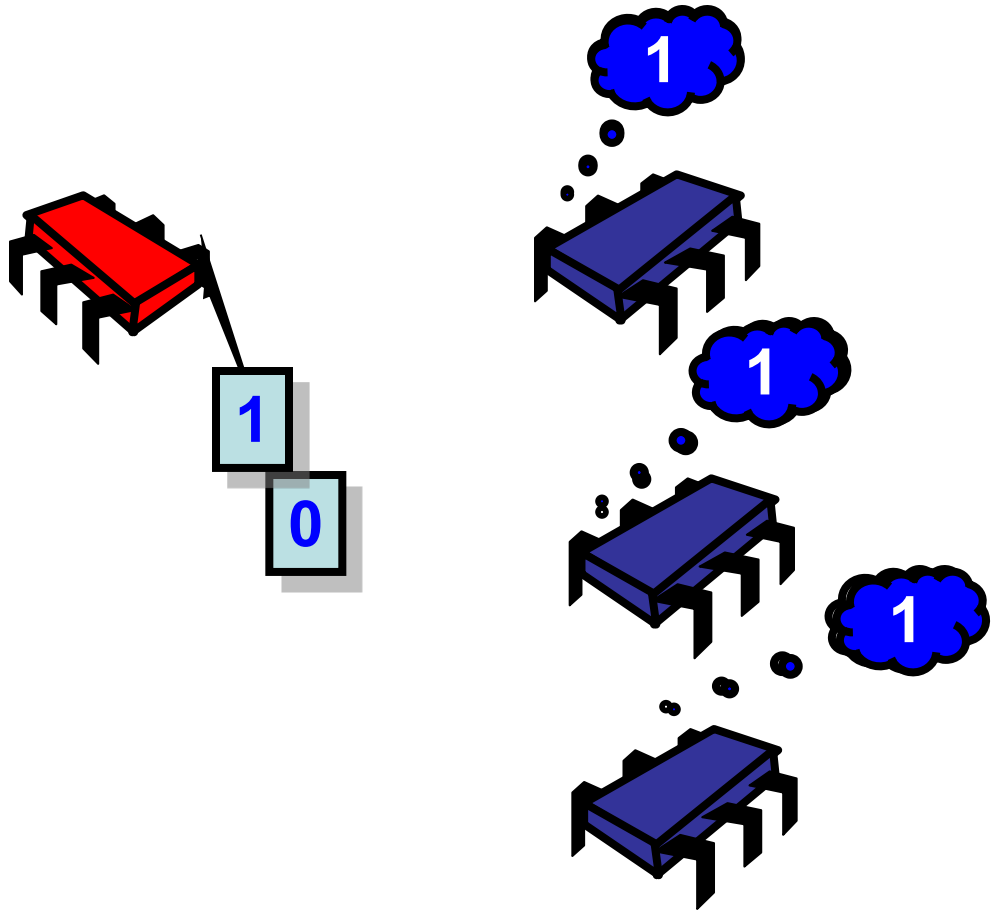
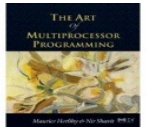


Regular Boolean MRSW from Safe Boolean MRSW





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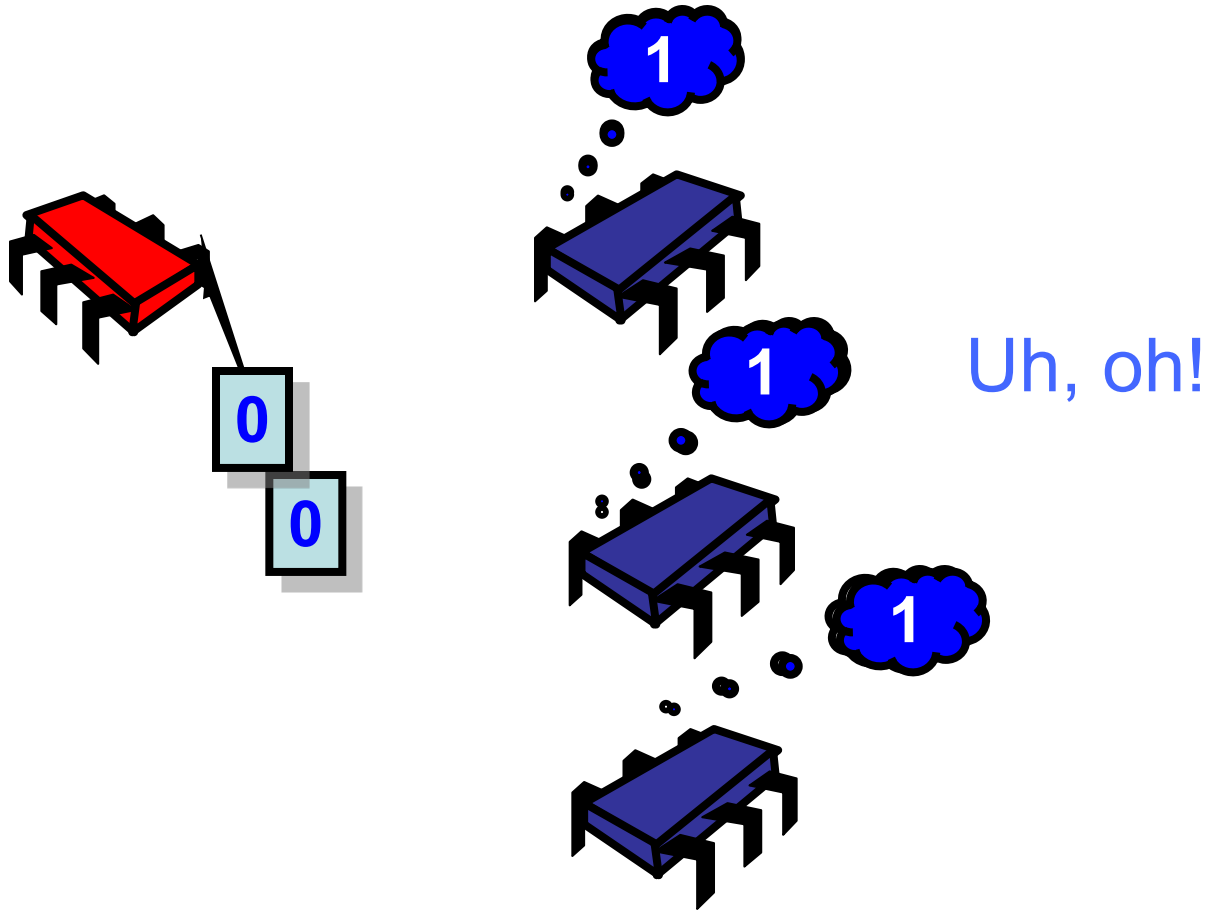
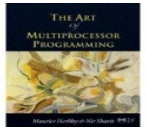
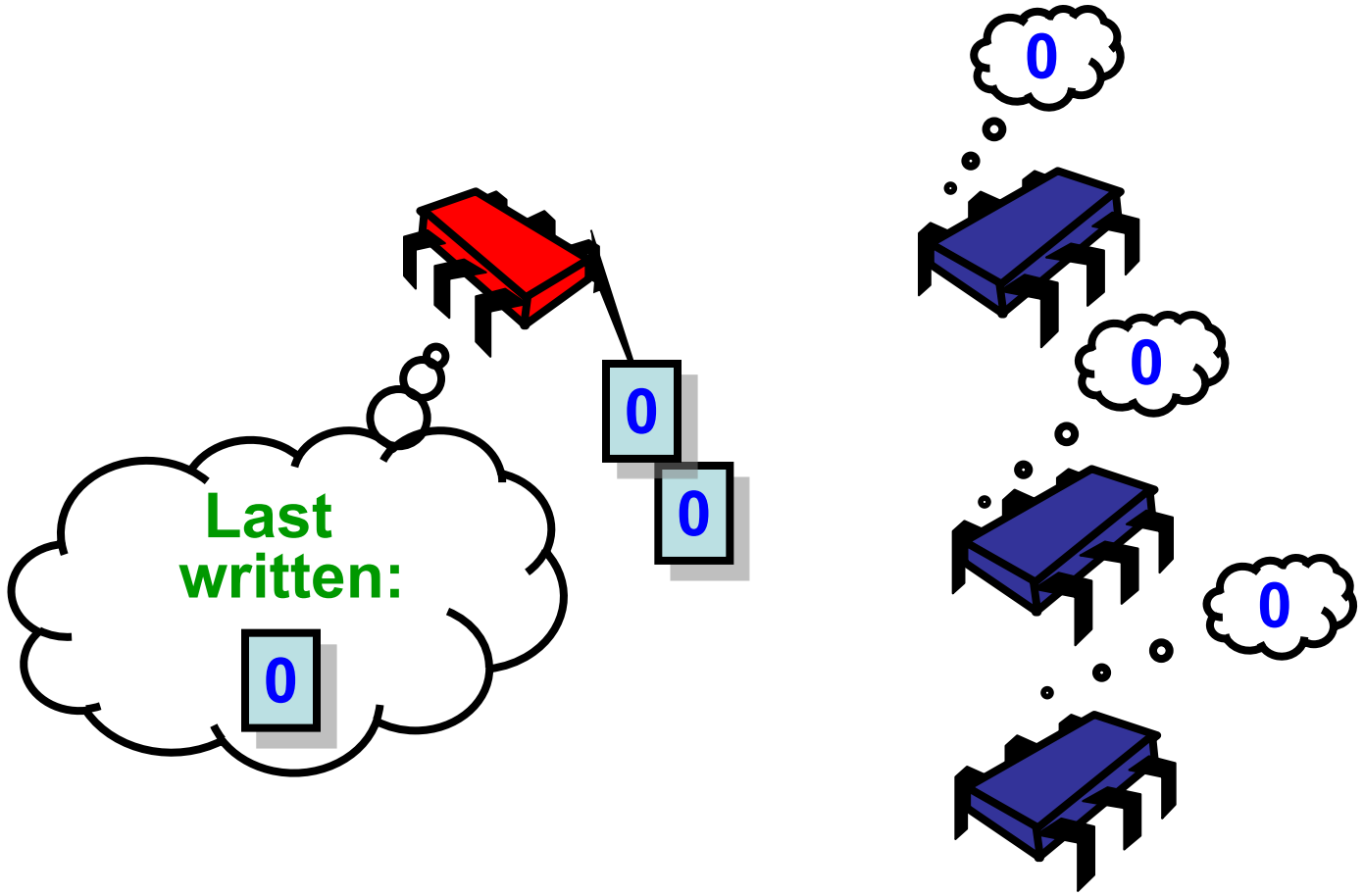


Uh, oh!



Regular Boolean MRSW from Safe Boolean MRSW



Regular Boolean MRSW from Safe Boolean MRSW

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  public void write(boolean x) {
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    old = x;
   }}
  public boolean read() {
   return value.read();
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```

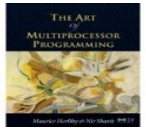

Regular Boolean MRSW from Safe Boolean MRSW

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  public void write(boolean x) {
   if (old != x) {
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  public boolean read() {
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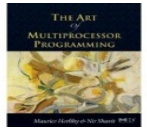
  }}
```

Last bit this thread wrote

(made-up syntax)



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```
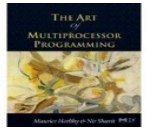

Actual value



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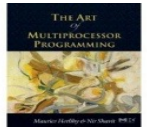
```

Is new value different from last value I wrote?



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```
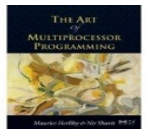

If so, change it (otherwise don't!)



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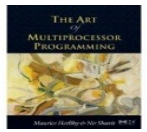
```

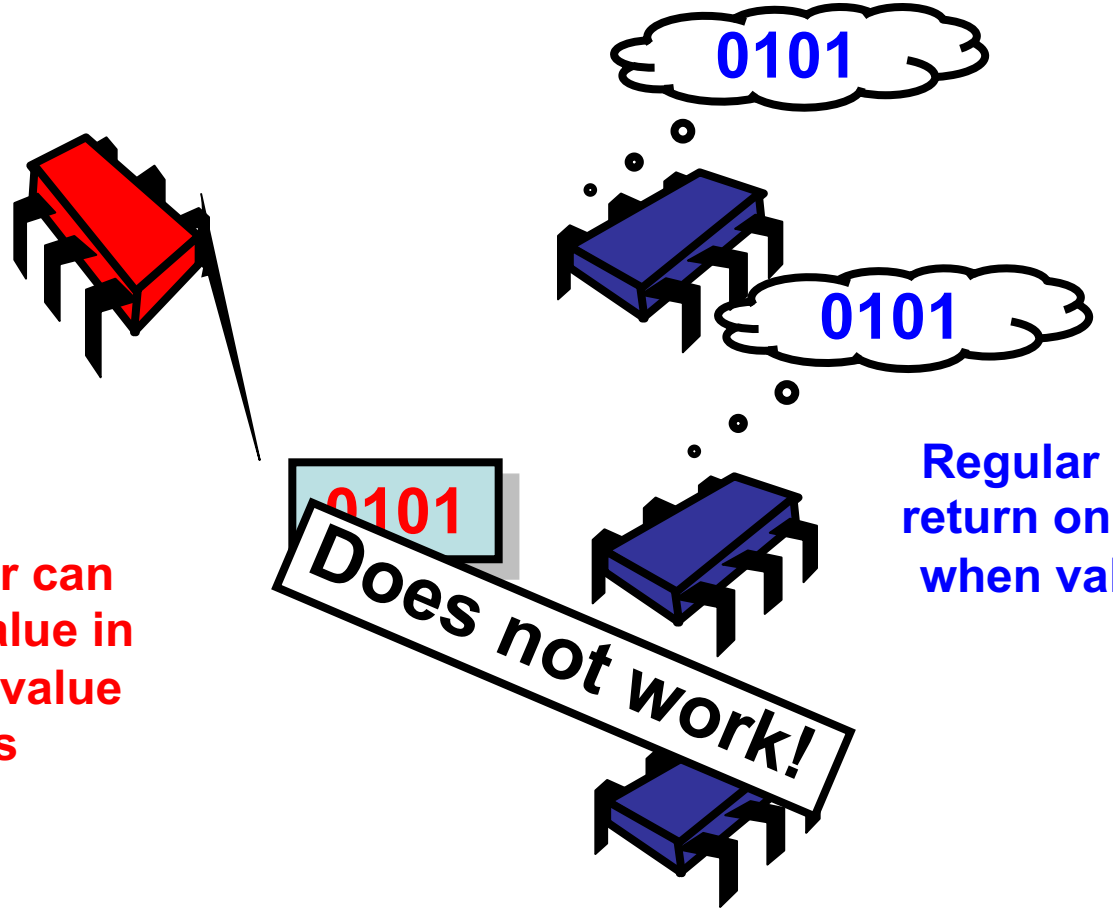
Overlap? What overlap?

No problem

either Boolean value works

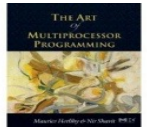


Regular Multi-Valued MRSW from Safe Multi-Valued MRSW?



Safe register can return any value in range when value changes

Regular register can return only old or new when value changes



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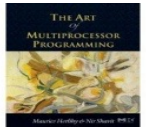

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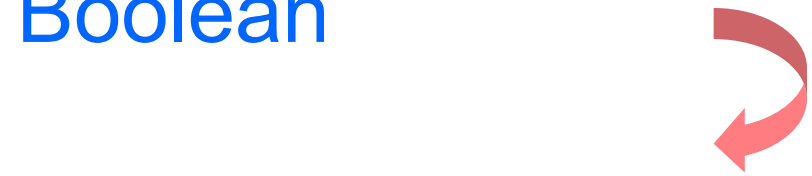
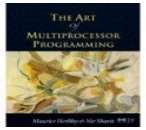
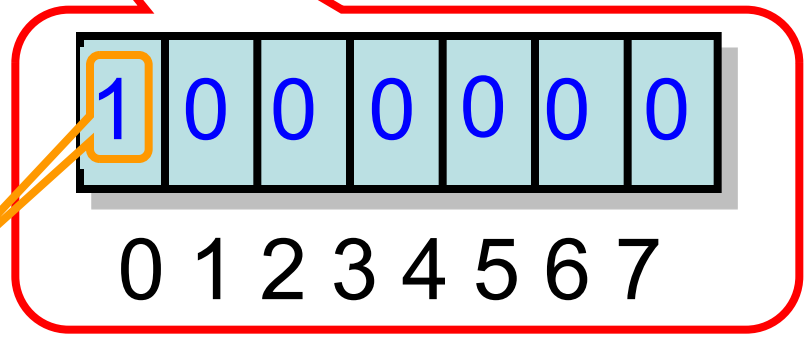




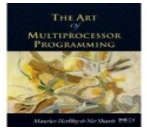
Representing *m* Values

Unary representation:

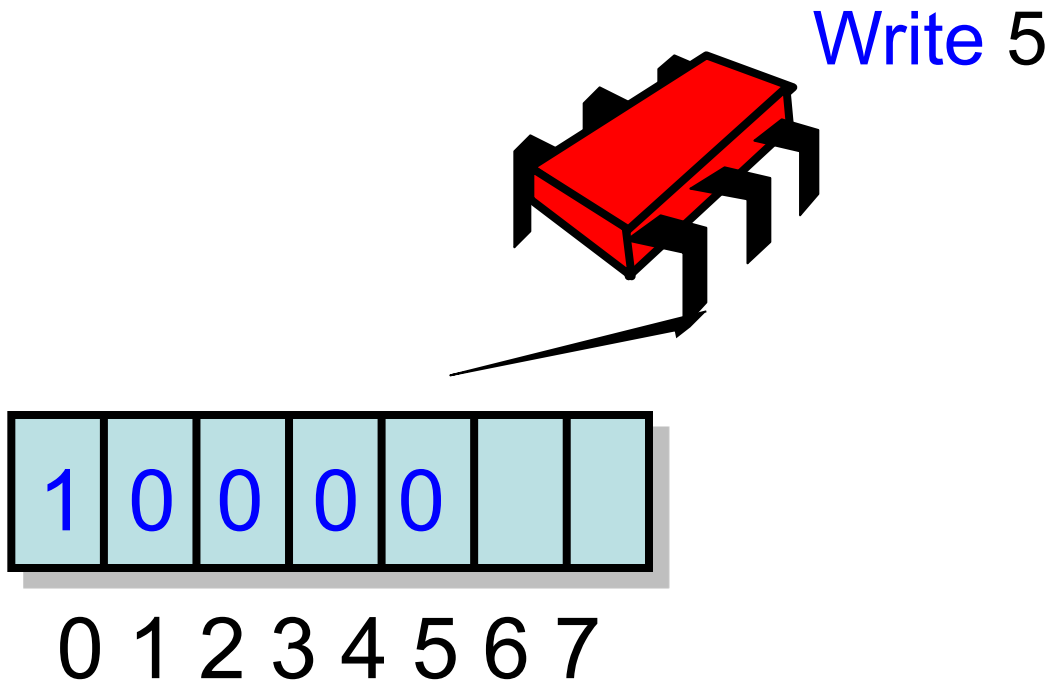
bit[i] means value i

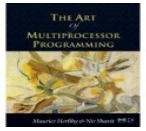


Initially 0

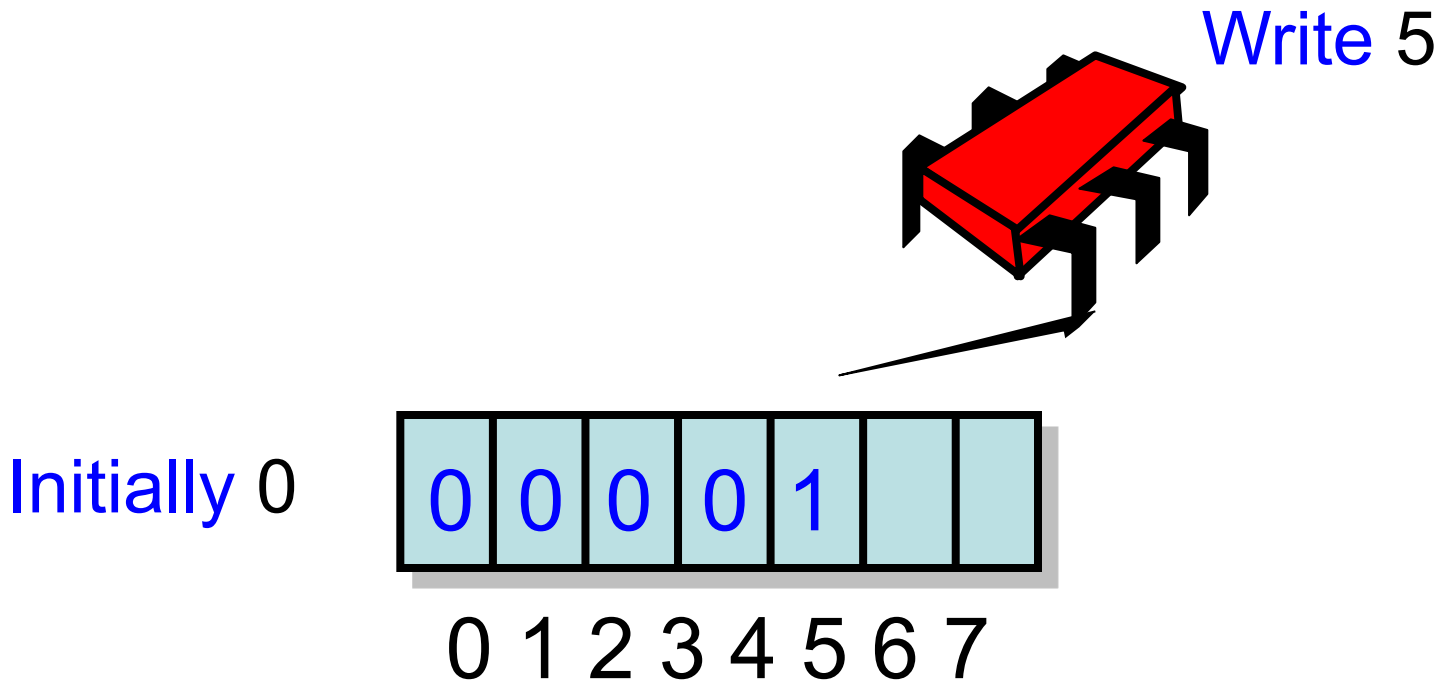


Writing *m*-Valued Register

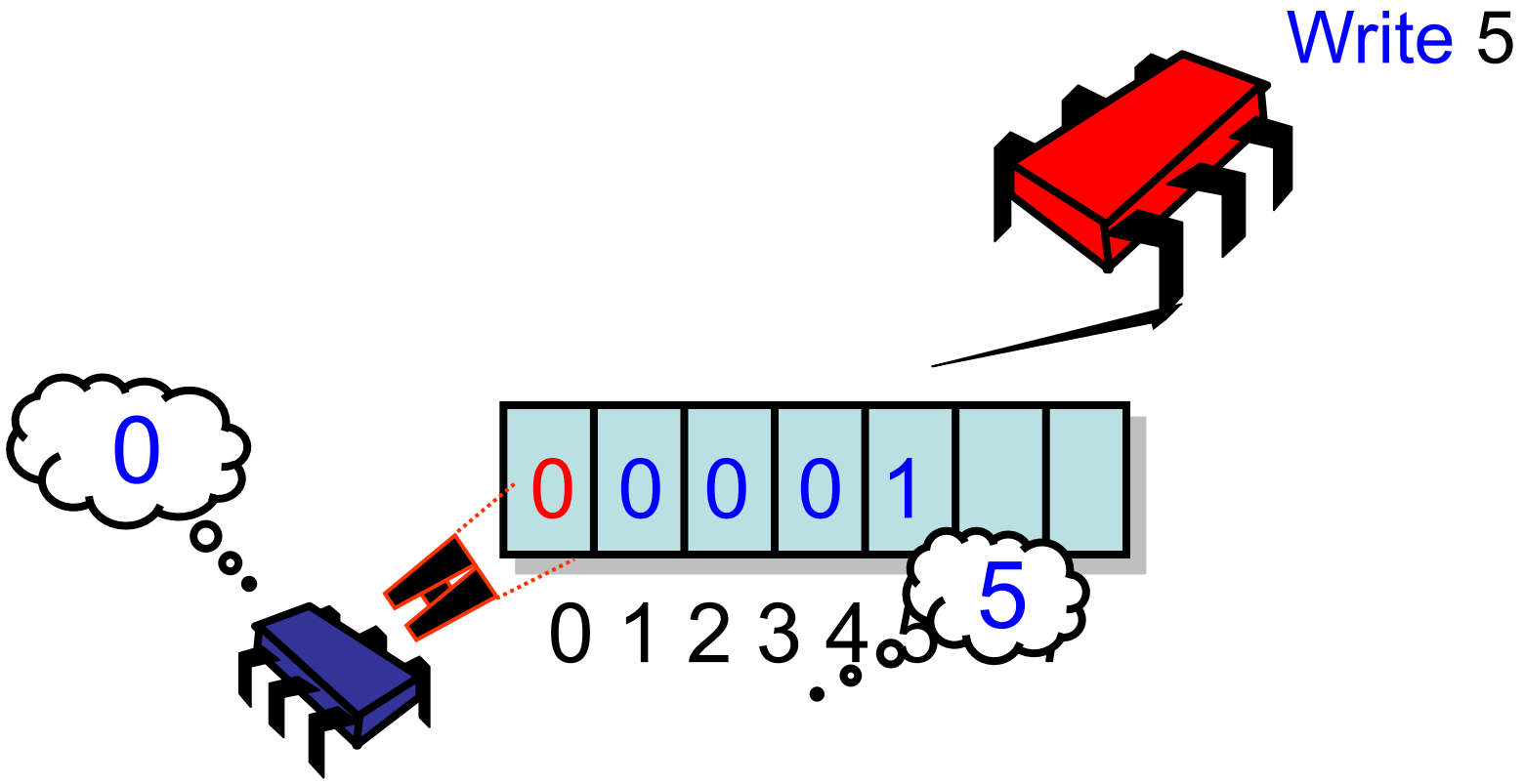


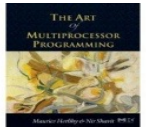


Writing *m*-Valued Register



Writing *m*-Valued Register



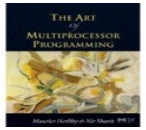


MRSW Regular *m*-valued from MRSW Regular Boolean

```
public class RegMRSWRegister implements Register{
  RegBoolMRSWRegister[M] bit;

  public void write(int x) {
    this.bit[x].write(true);
    for (int i=x-1; i>=0; i--)
      this.bit[i].write(false);
  }

  public int read() {
    for (int i=0; i < M; i++)
      if (this.bit[i].read())
        return i;
   }}
```



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        return i;
   }}
```
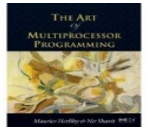

Unary representation:
bit[i] means value i



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  }

  public int read() {
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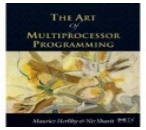
      if (bit[i].read())
        return i;
   }}
```

set bit x



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  public int read() {
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      if (bit[i].read())
        return i;
   }}
```
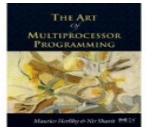

Clear bits from higher to lower



MRSW Regular *m*-valued from MRSW Regular Boolean

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  public void write(int x) {
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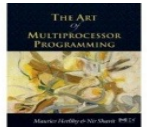
      bit[i].write(false);
  }

  public int read() {
    for (int i=0; i < M; i++)
      if (bit[i].read())
        return i;
    }}
```

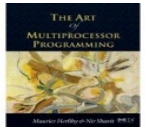
Scan from lower to higher & return first bit set



Road Map

- SRSW safe Boolean
- MRSW safe Boolean
- MRSW regular Boolean
- MRSW regular
- MRSW atomic
- MRMW atomic
- Atomic snapshot

Questions?



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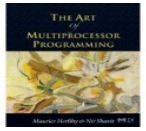
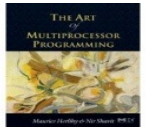


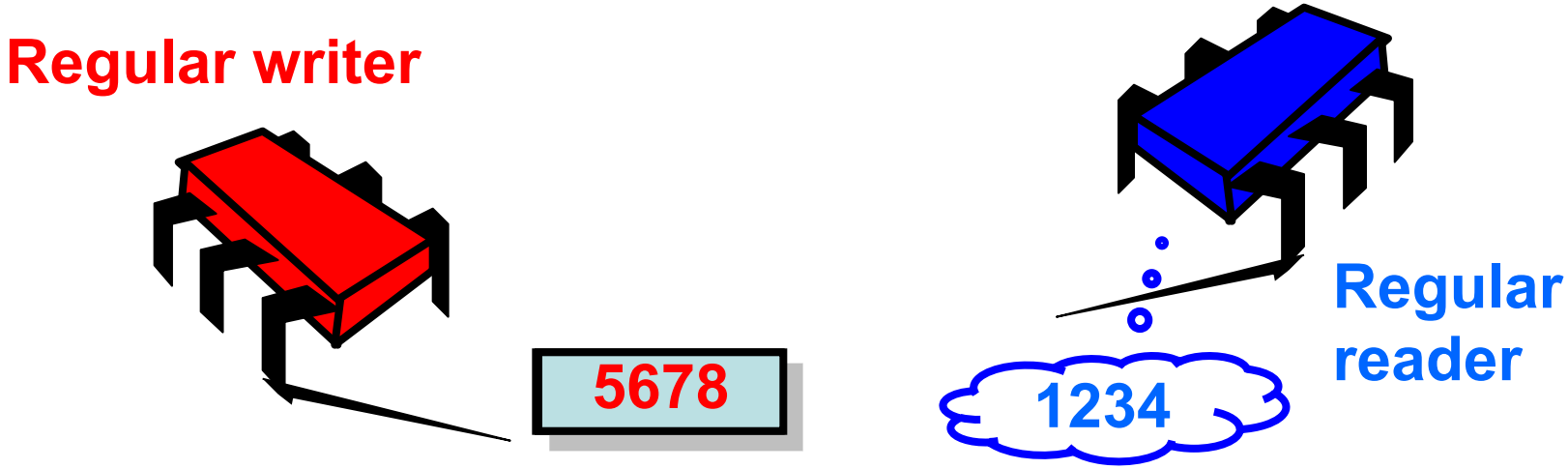
Road Map (Slight Detour)

- SRSW safe Boolean
- MRSW safe Boolean
- MRSW regular Boolean
- MRSW regular
- MRSW atomic
- MRMW atomic
- Atomic snapshot

SRSW Atomic

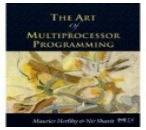


SRSW Atomic From SRSW Regular

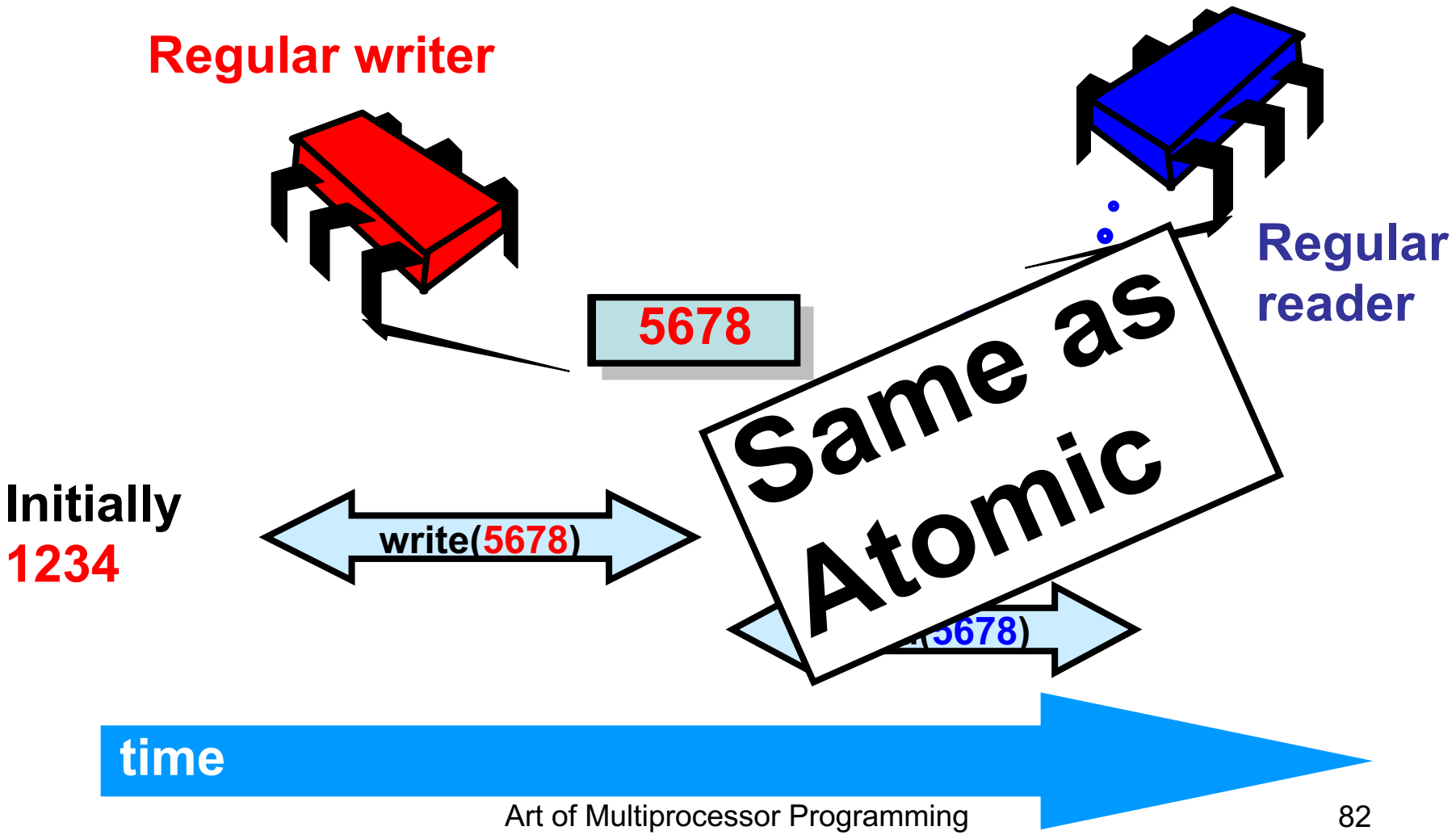


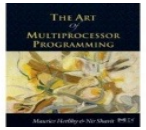
Concurrent Reading

When is this a problem?

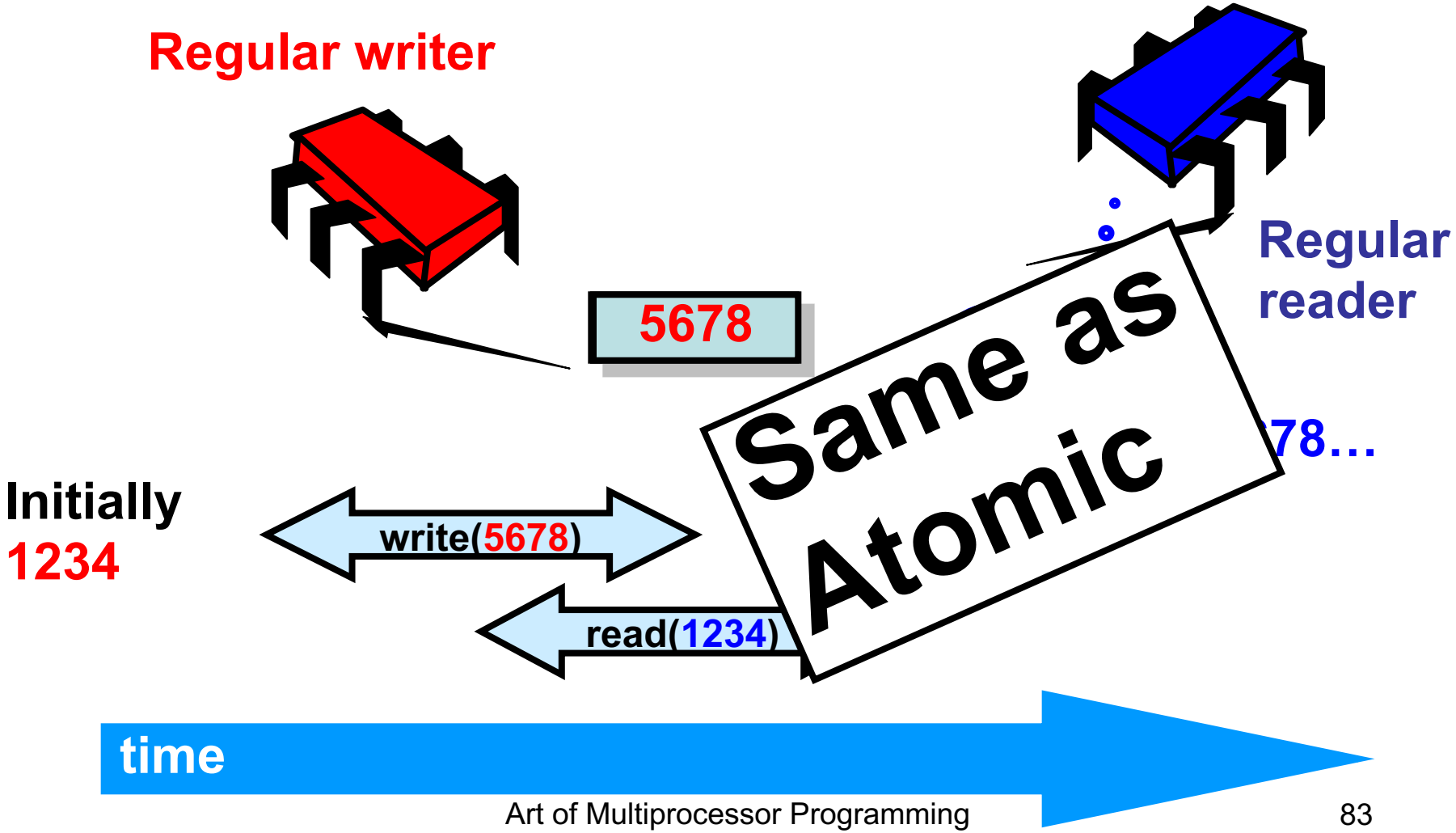


SRSW Atomic From SRSW Regular

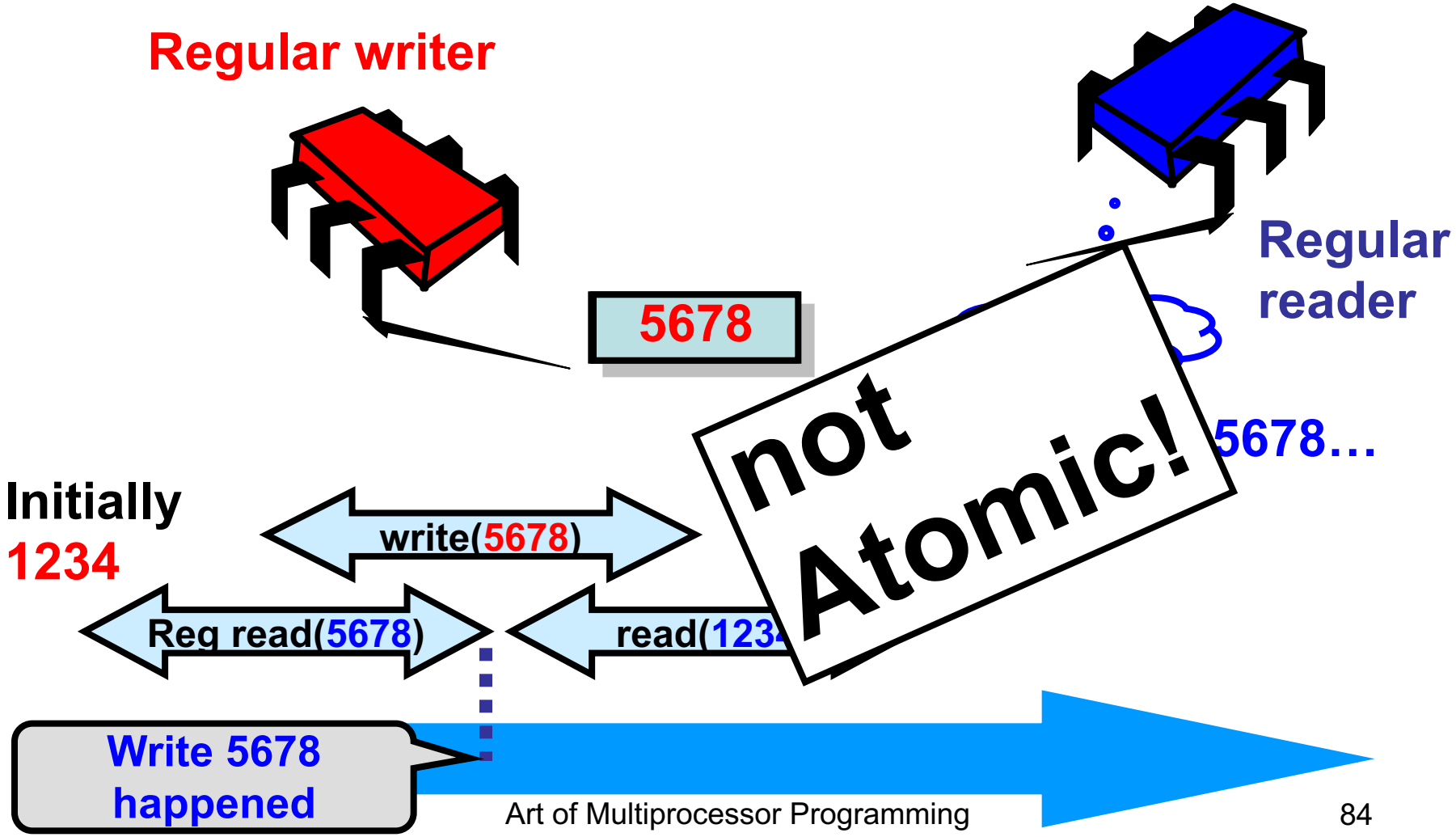




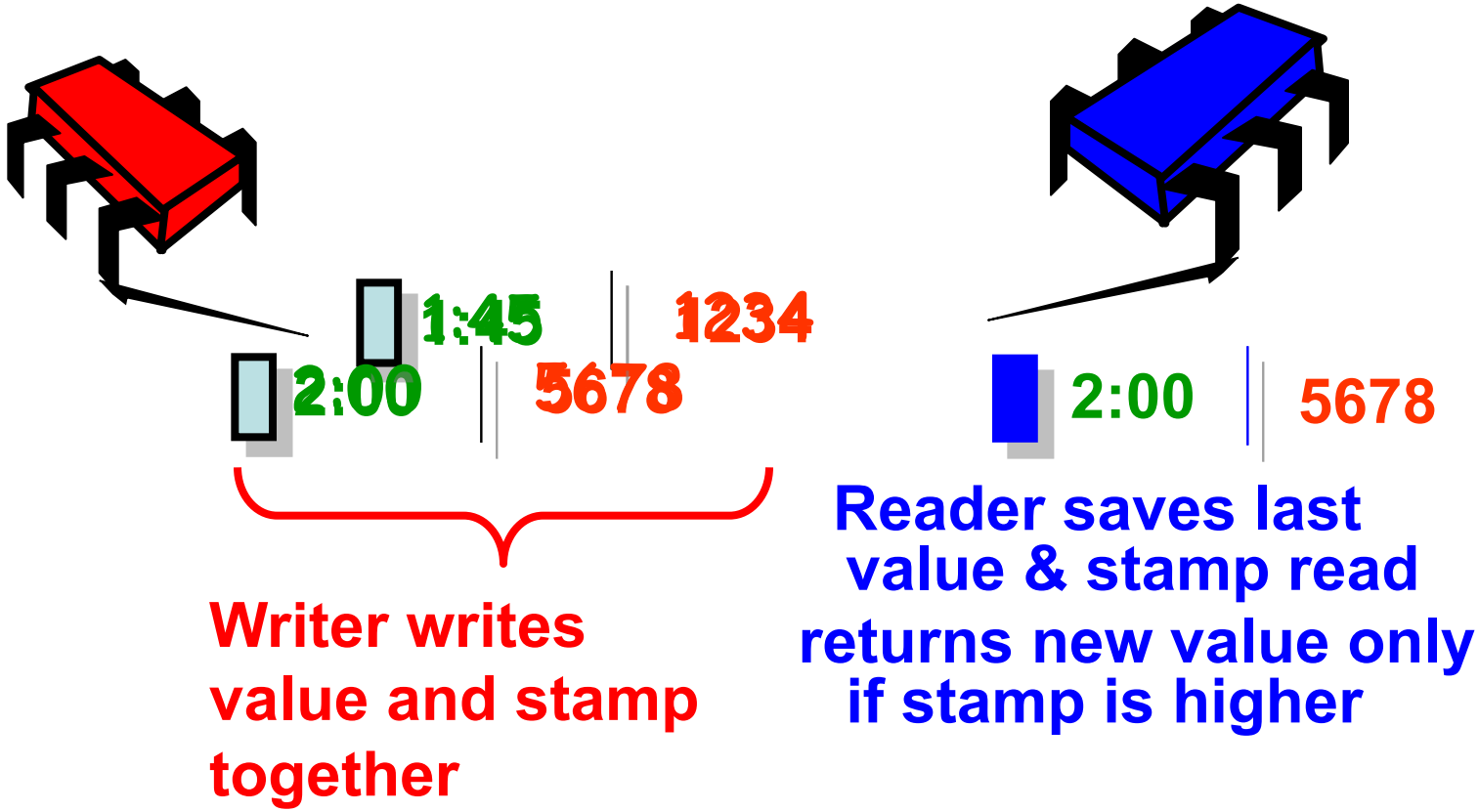
SRSW Atomic From SRSW Regular

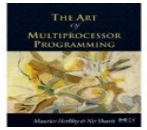


SRSW Atomic From SRSW Regular

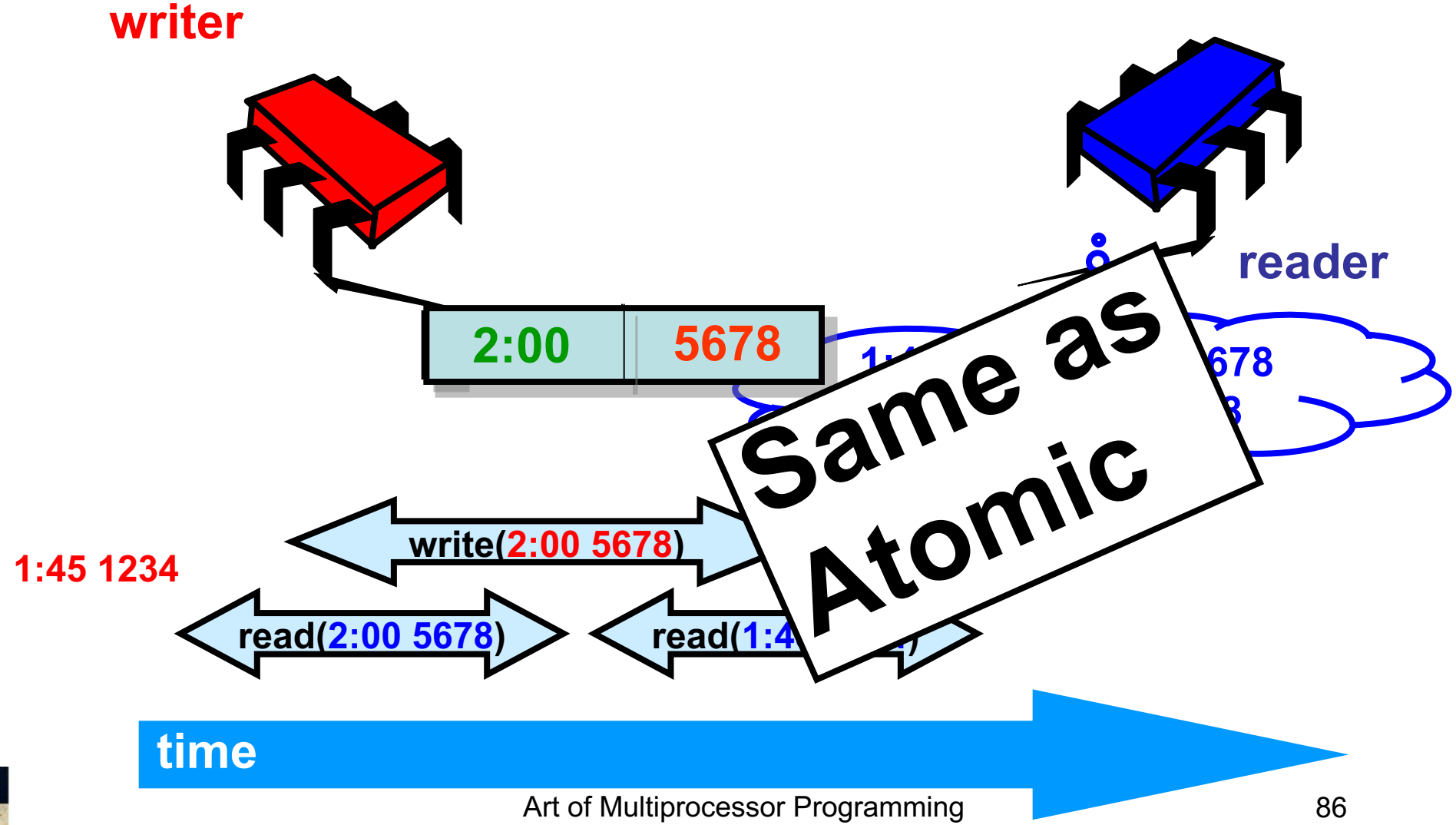


Timestamped Values

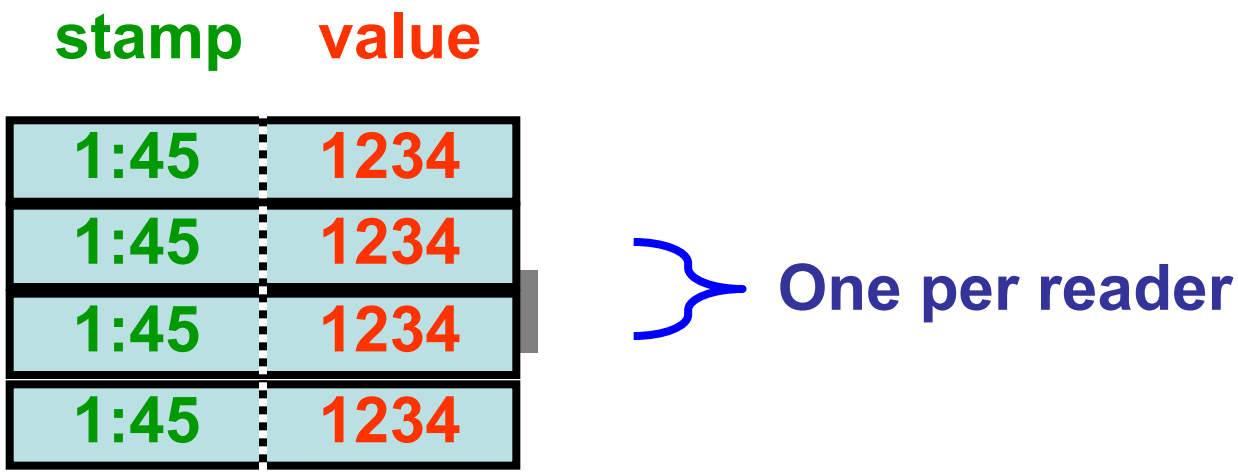


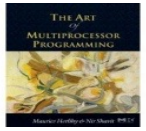


SRSW Atomic From SRSW Regular

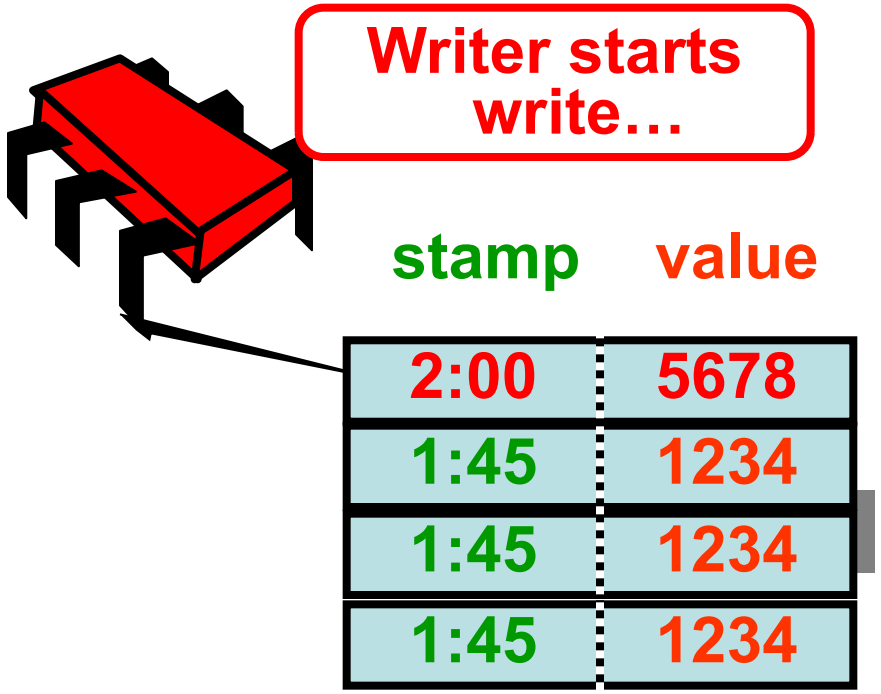


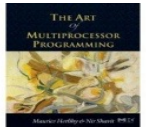
Atomic Single-Reader to Atomic Multi-Reader



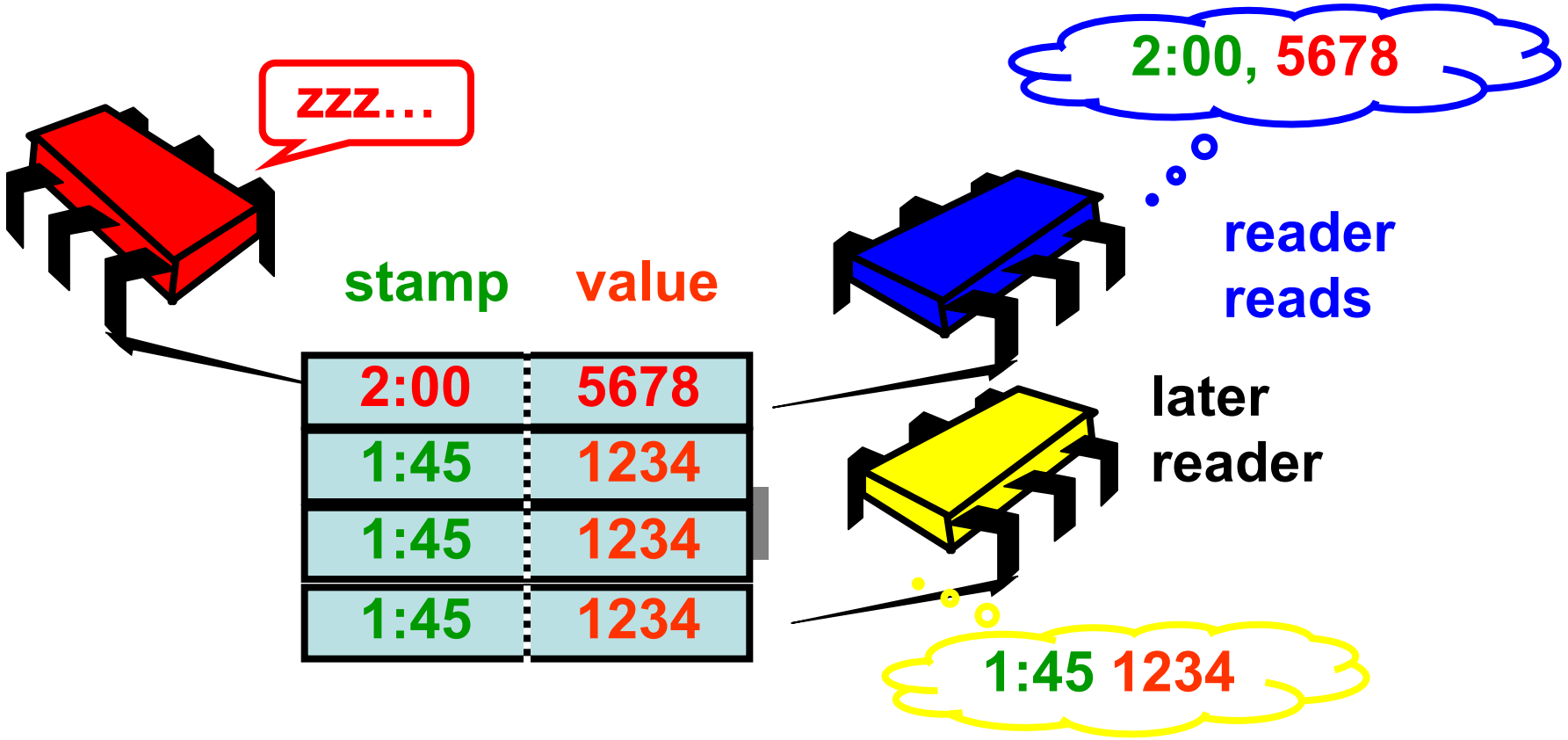


Another Scenario

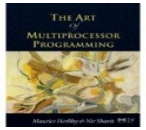




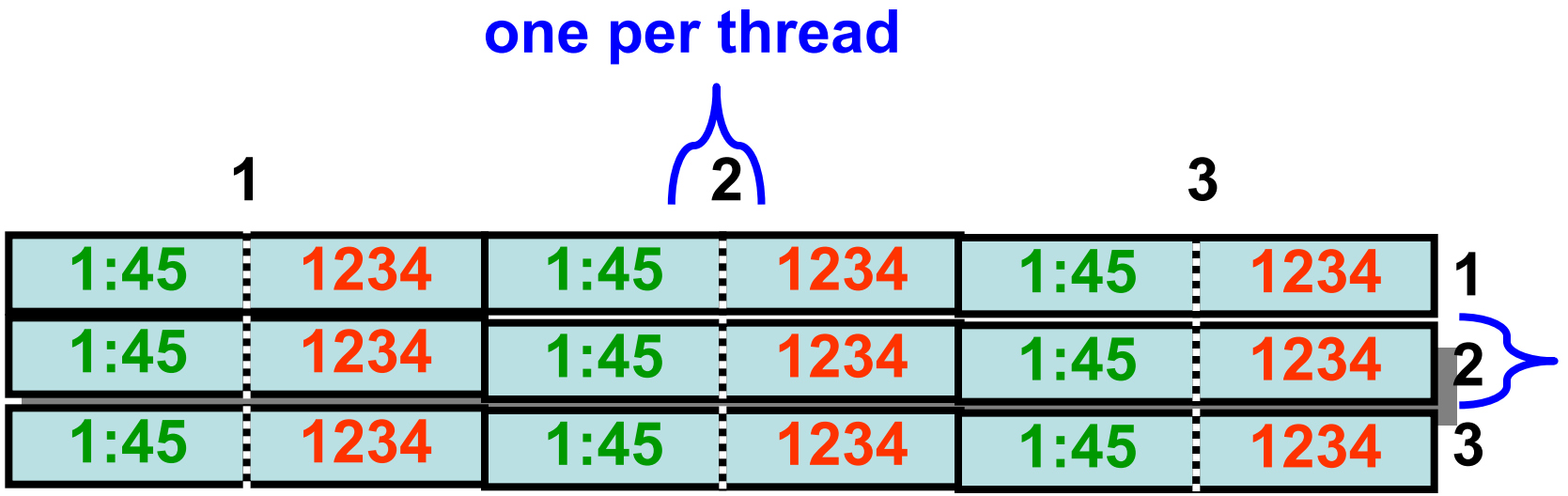
Another Scenario

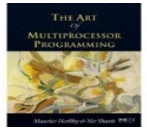


Yellow was completely after Blue but read earlier value…not linearizable!

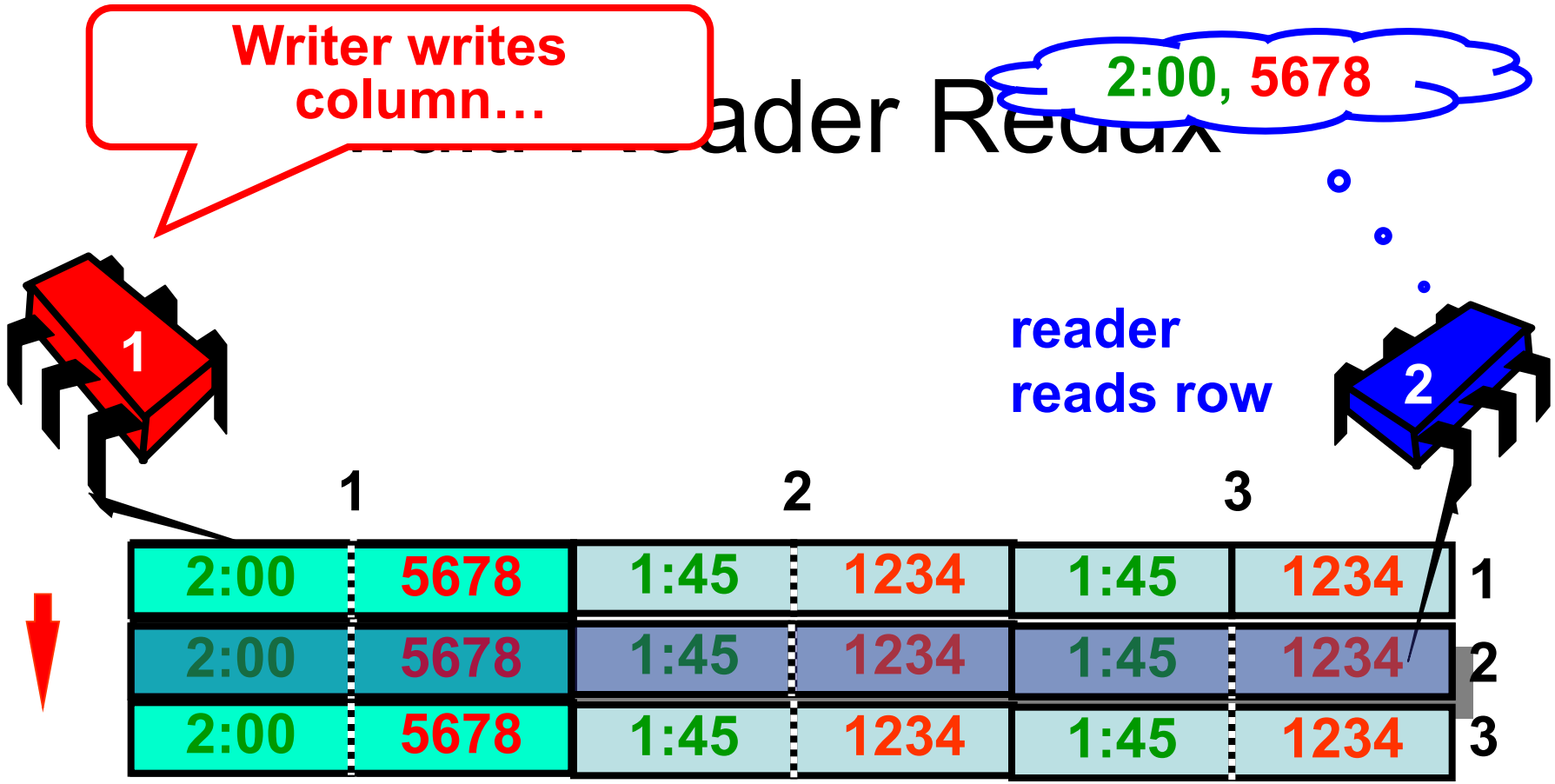


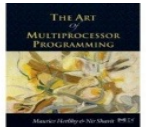
Multi-Reader Redux





Reader Redux





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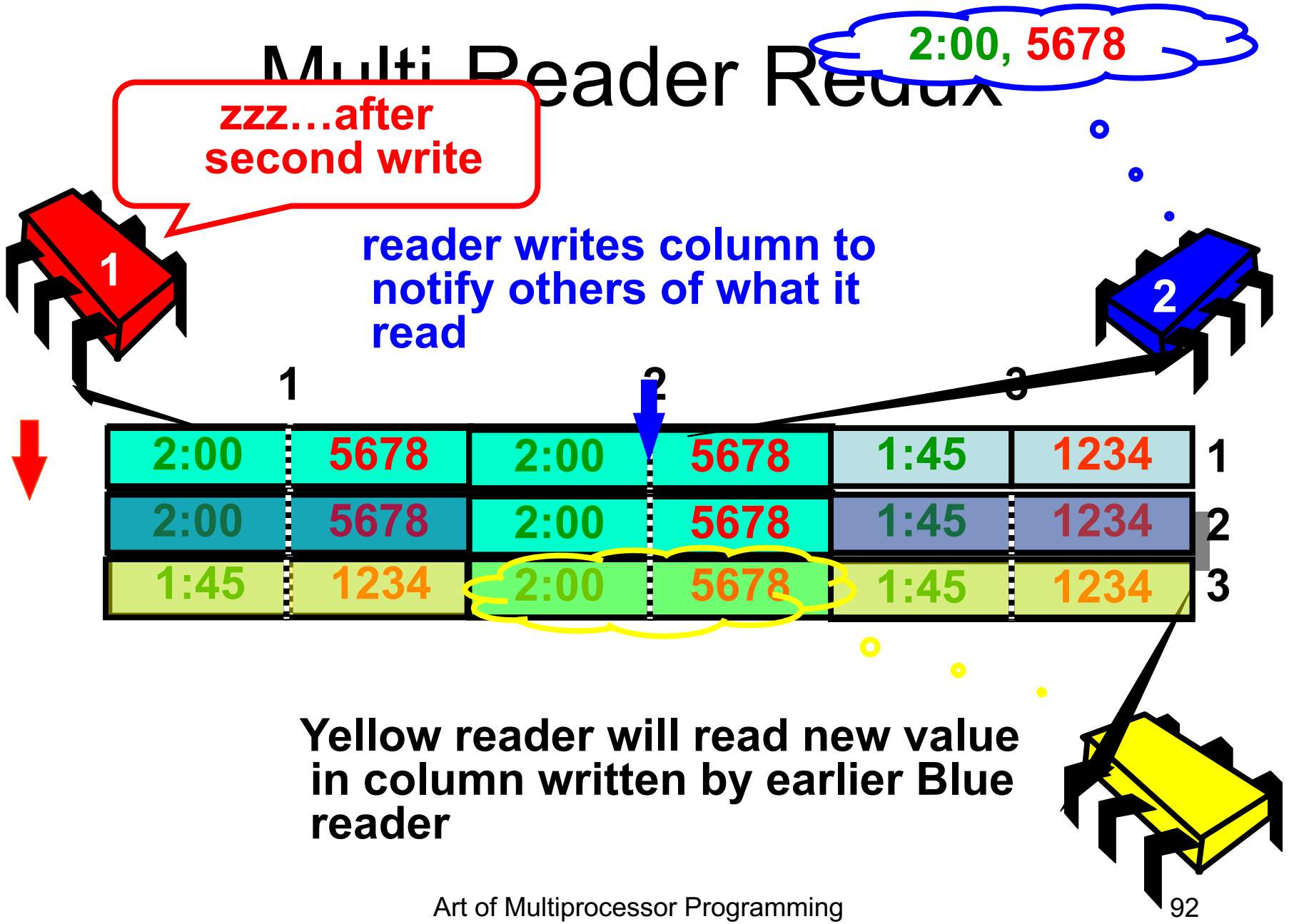
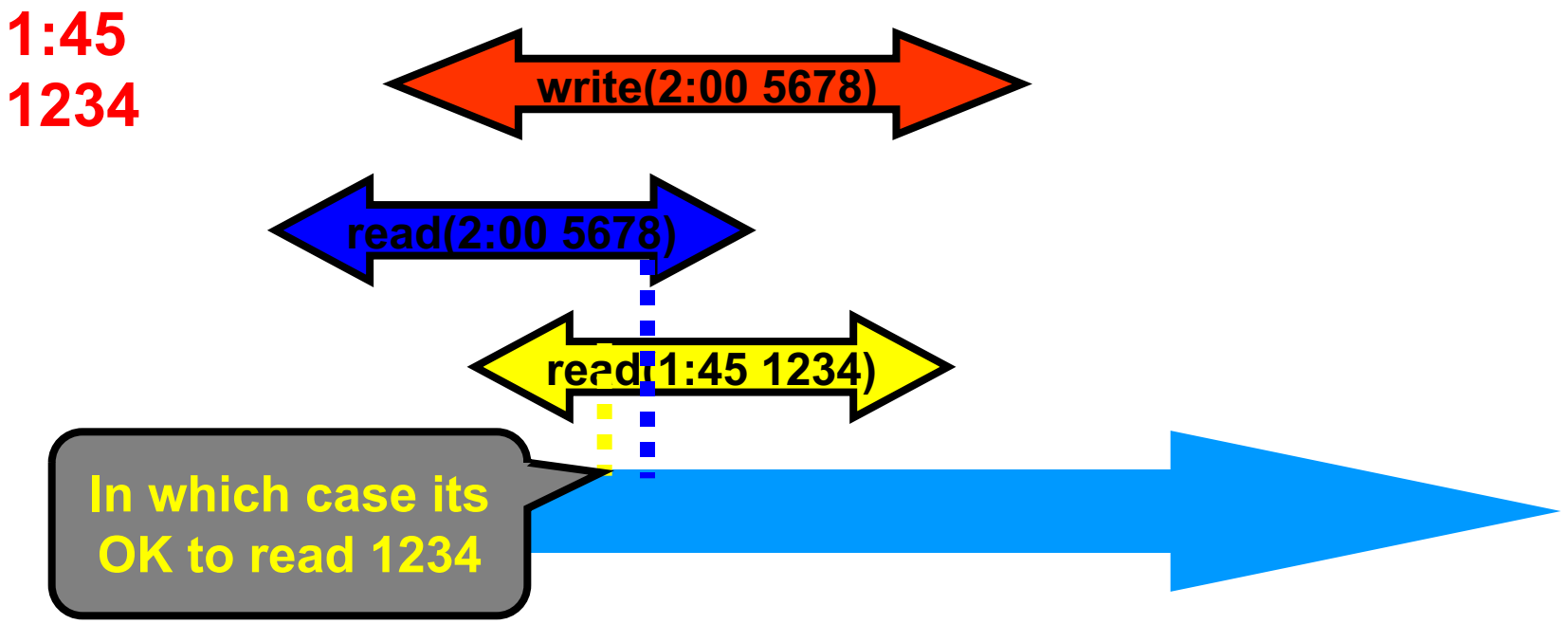
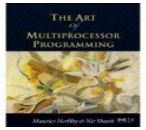
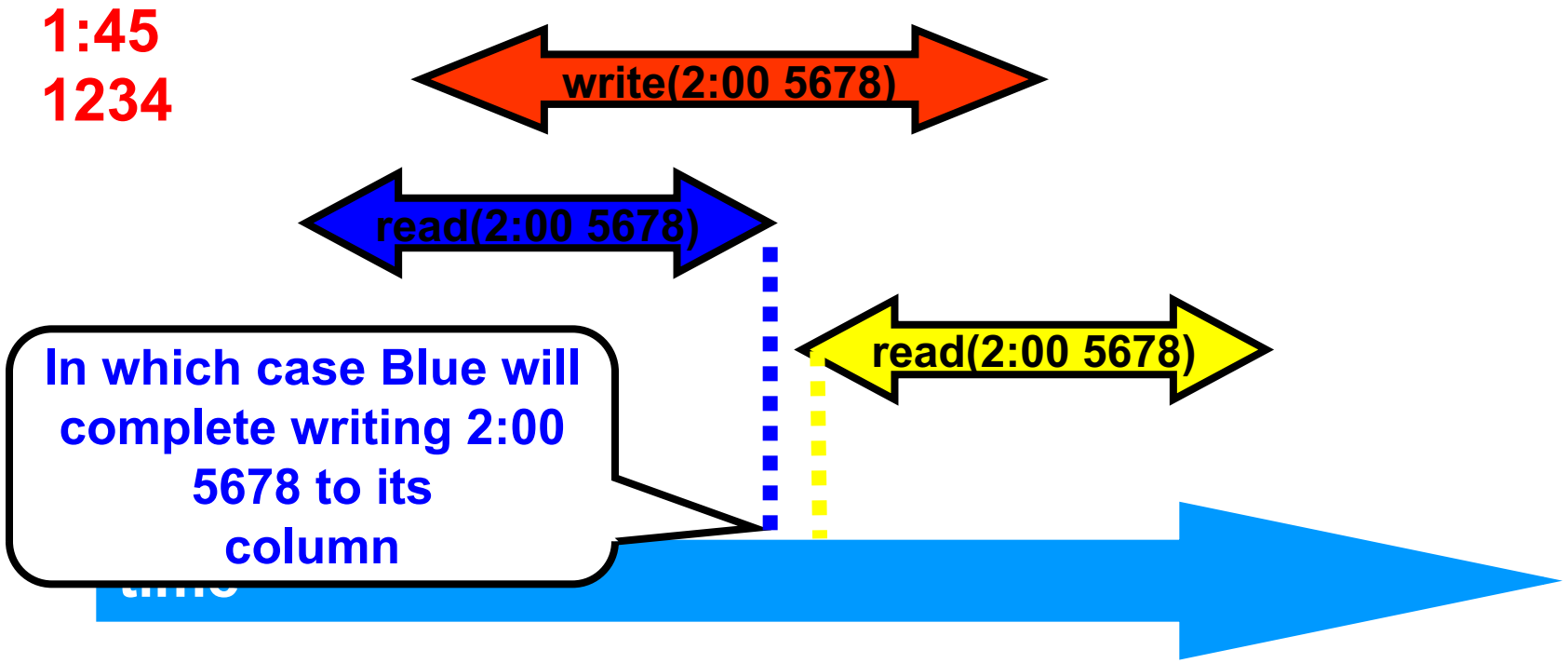


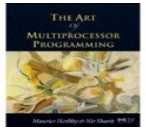
Can't Yellow Miss Blue's Update? … Only if Readers Overlap…





Bad Case Only When Readers Don't Overlap





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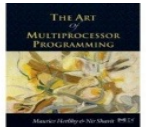
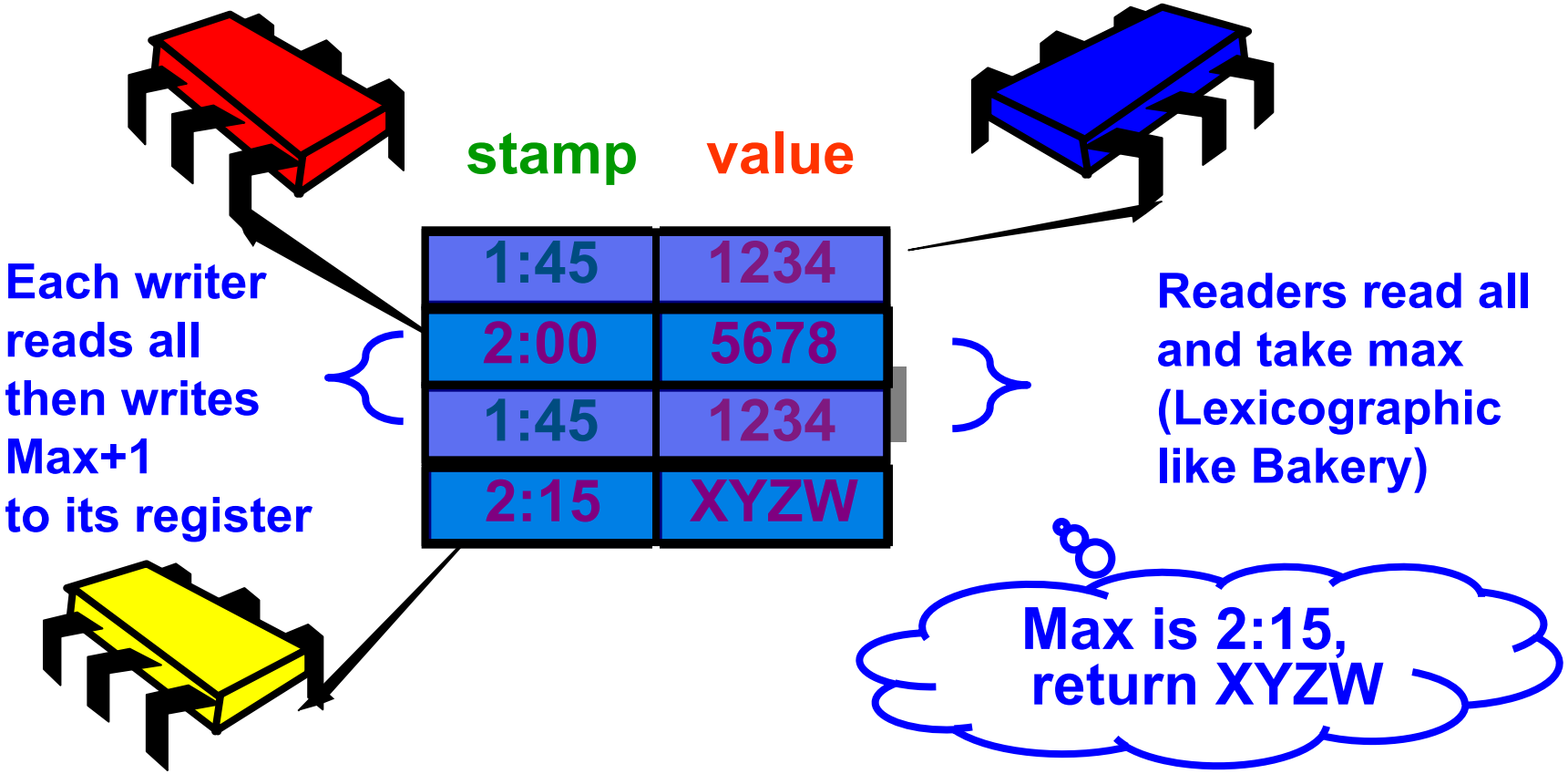
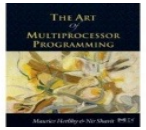
Next

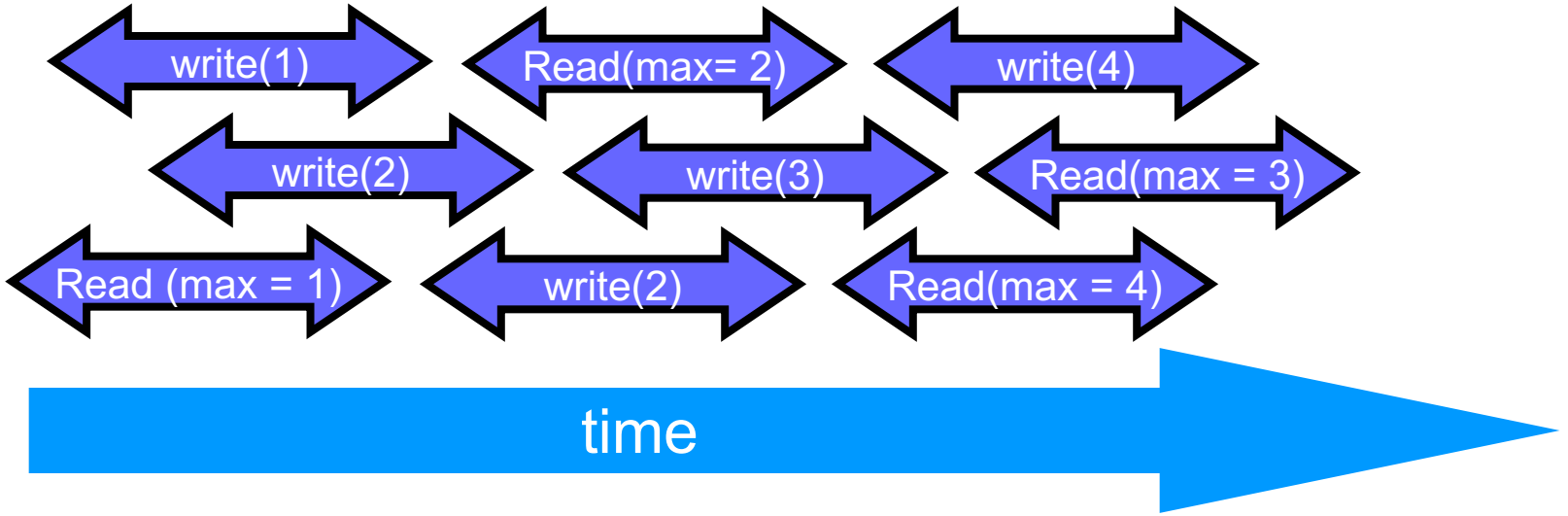


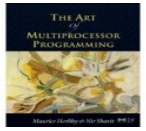
Multi-Writer Atomic From Multi-Reader Atomic



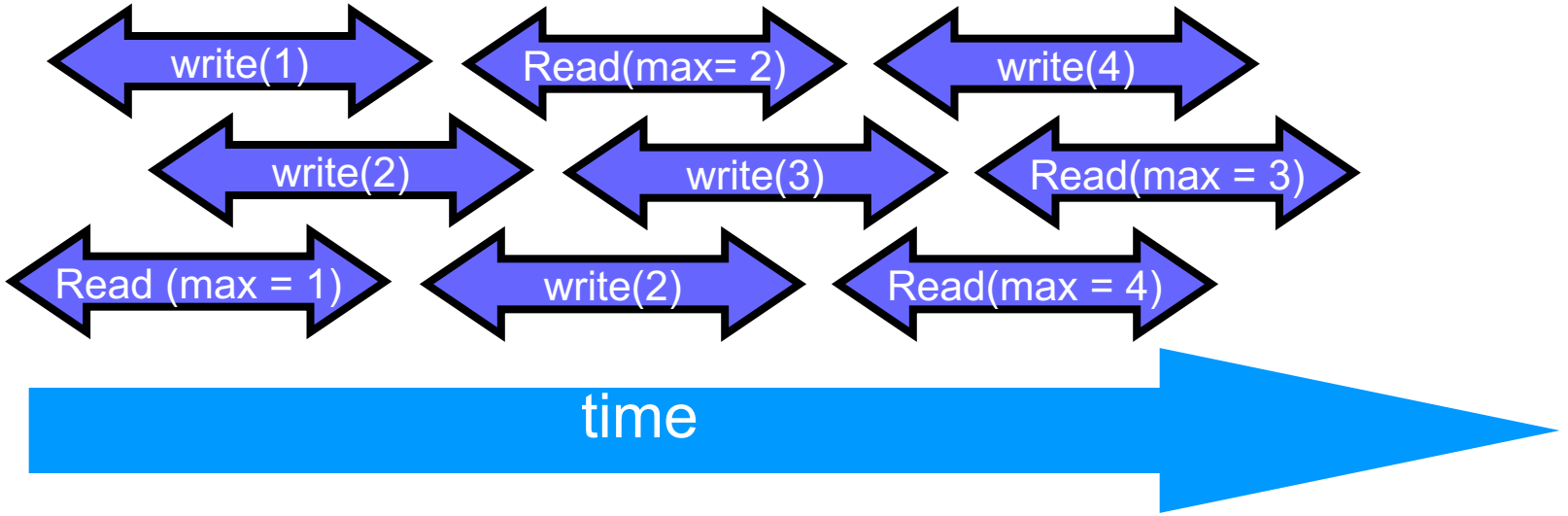


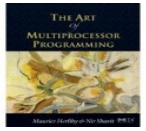
Atomic Execution
Means it is Linearizable



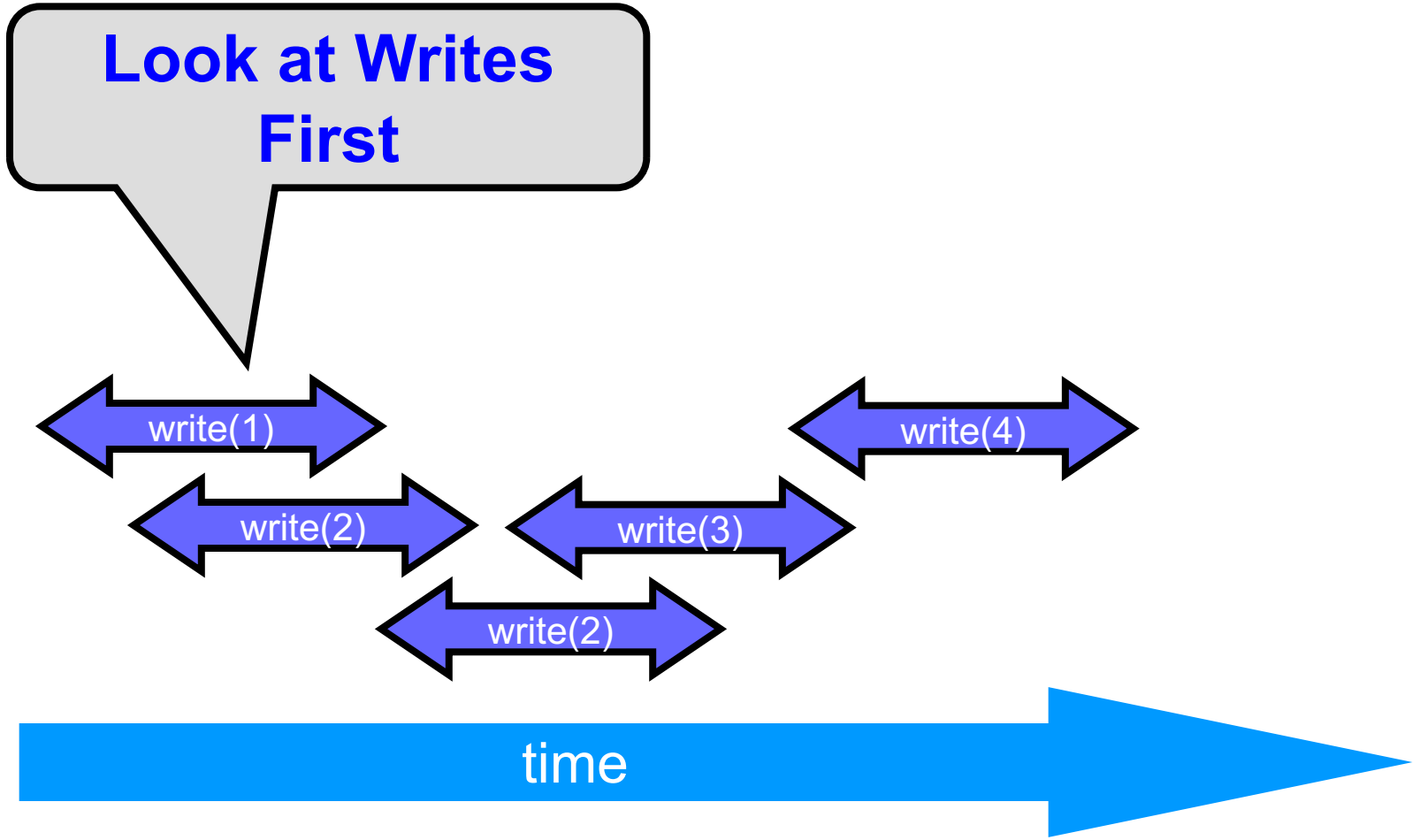


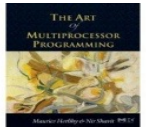
Linearization Points



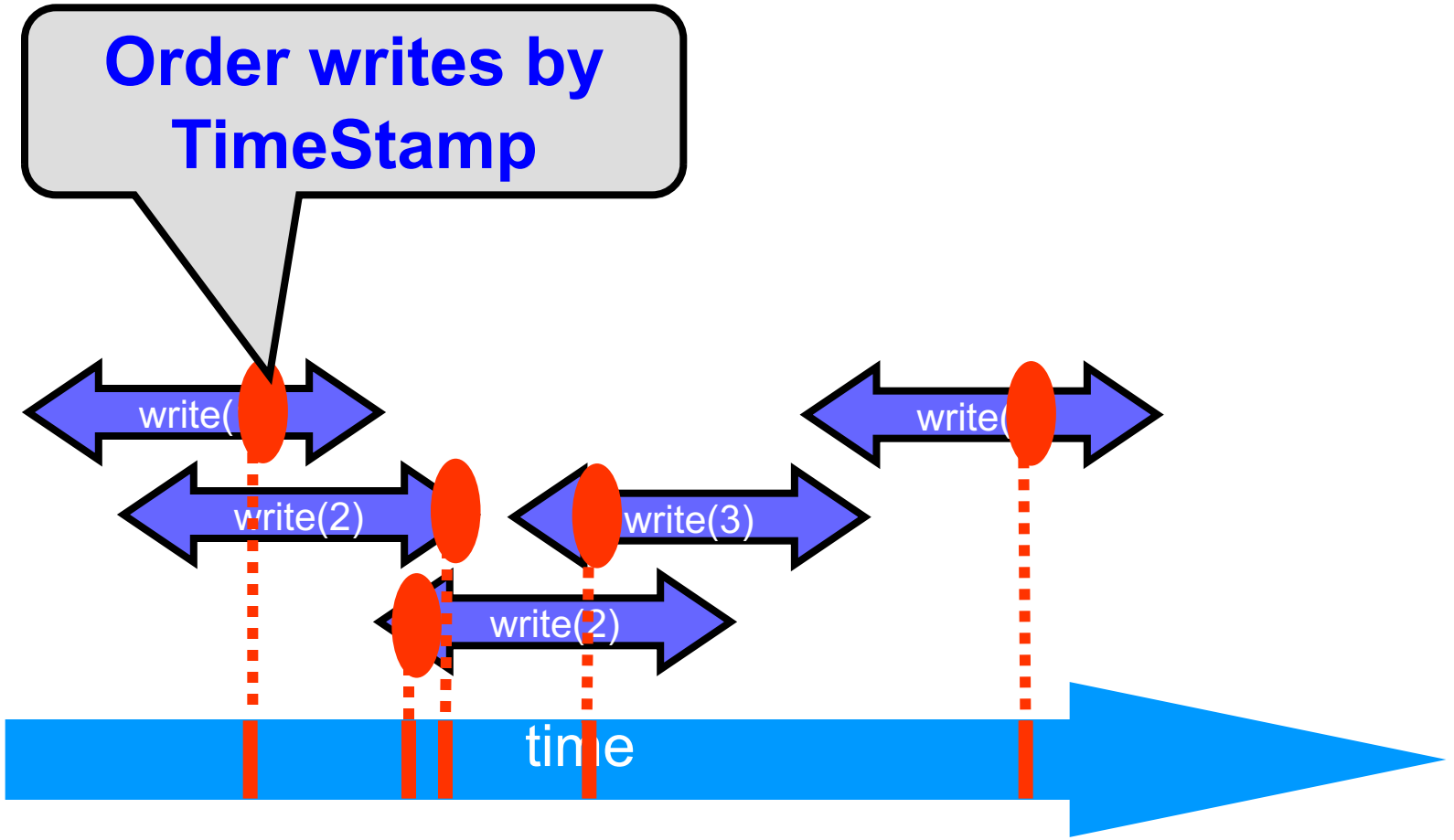


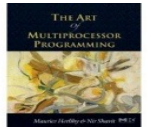
Linearization Points



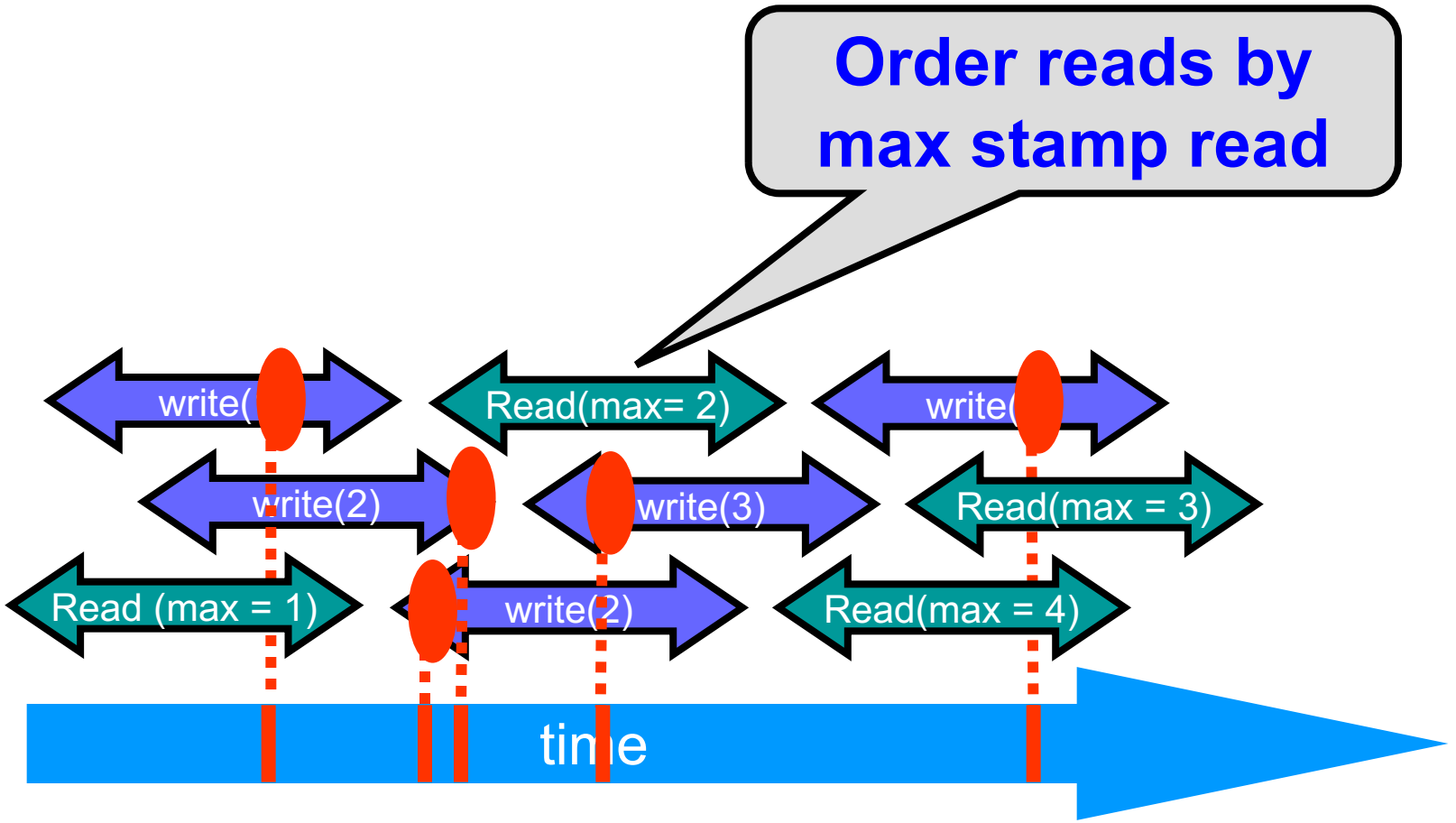


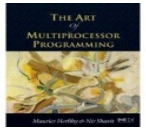
Linearization Points



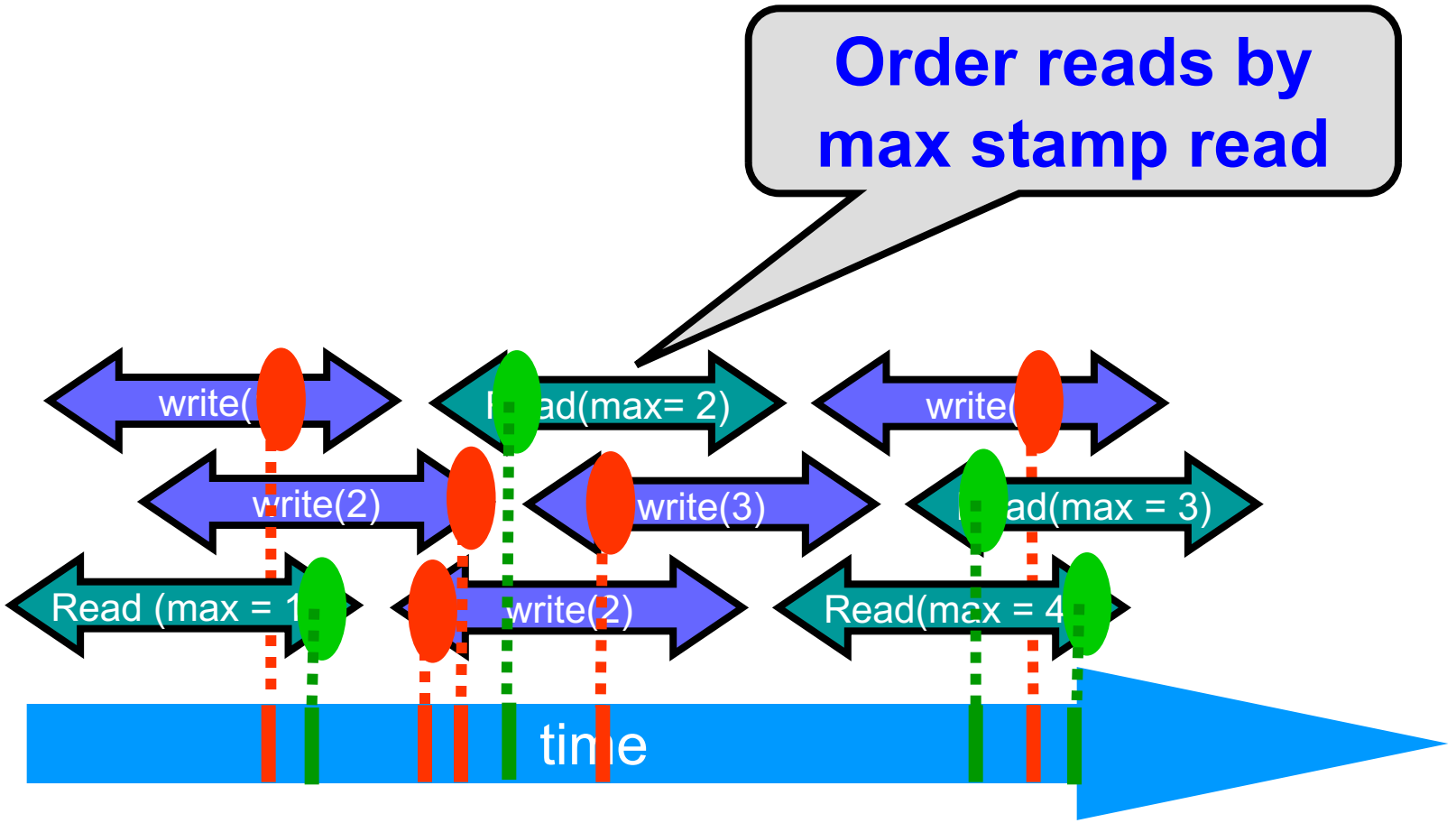


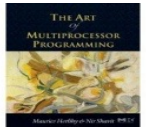
Linearization Points



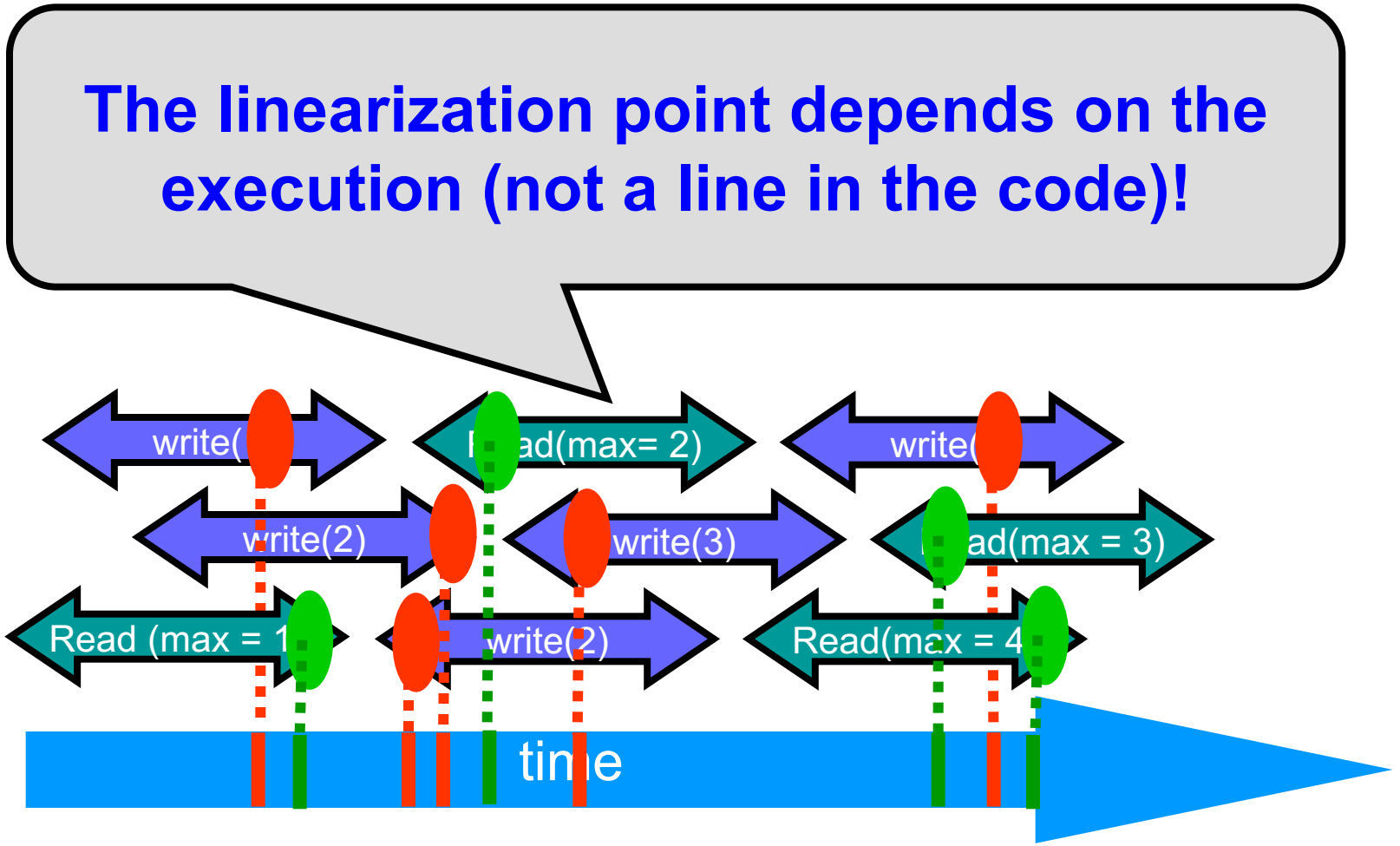


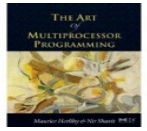
Linearization Points





Linearization Points

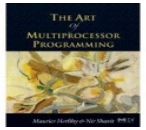




Road Map

- SRSW safe Boolean
- MRSW safe Boolean
- MRSW regular Boolean
- MRSW regular
- MRSW atomic
- MRMW atomic
- Atomic snapshot

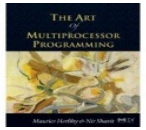
Questions?



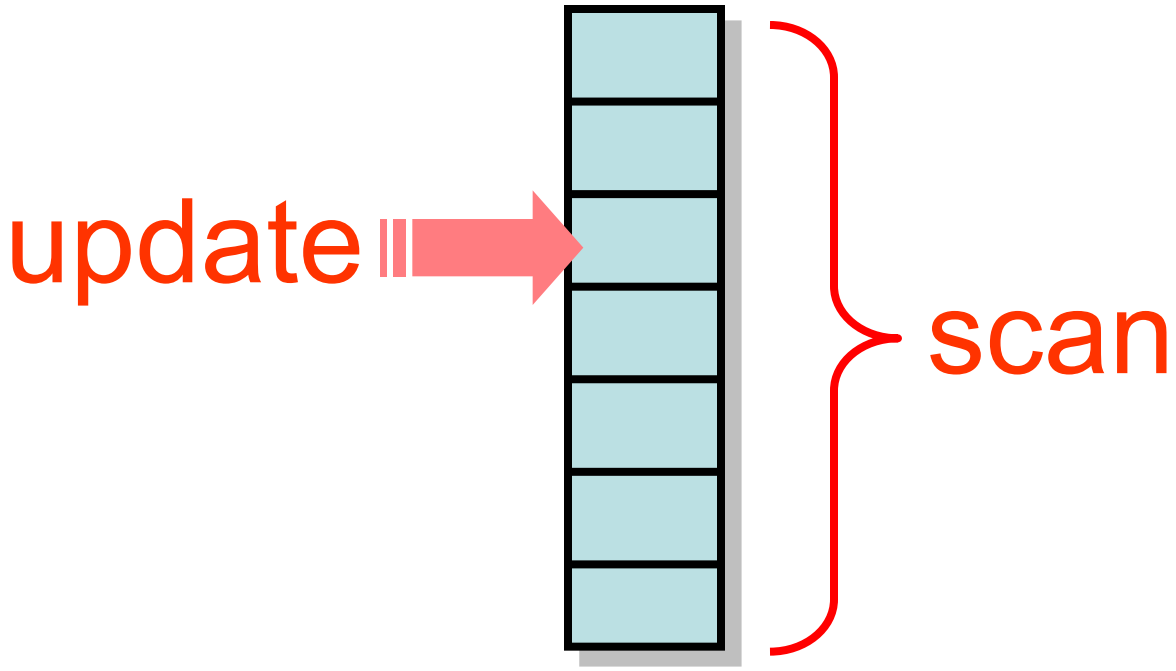
Road Map

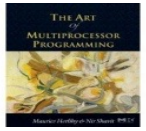
- SRSW safe Boolean
- MRSW safe Boolean
- MRSW regular Boolean
- MRSW regular
- MRSW atomic
- MRMW atomic
- Atomic snapshot

Next



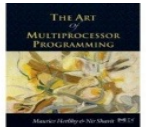
Atomic Snapshot





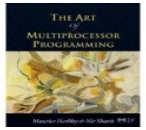
Atomic Snapshot

- Array of SWMR atomic registers
- Take instantaneous snapshot of all
- Generalizes to MRMW registers …



Snapshot Interface

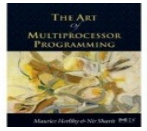
```
public interface Snapshot {
  public int update(int v);
  public int[] scan();
}
```



Snapshot Interface

Thread i writes v to its register

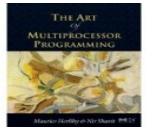
```
public interface Snapshot {
  public int update(int v);
   public int[] scan();
}
```



Snapshot Interface

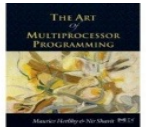
Instantaneous snapshot of all theads' registers

```
public interface Snapshot {
  public int update(int v);
  public int[] scan();
}
```



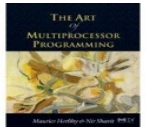
Atomic Snapshot

- Collect
  - Read values one at a time
- Problem
  - Incompatible concurrent collects
  - Result not linearizable



Clean Collects

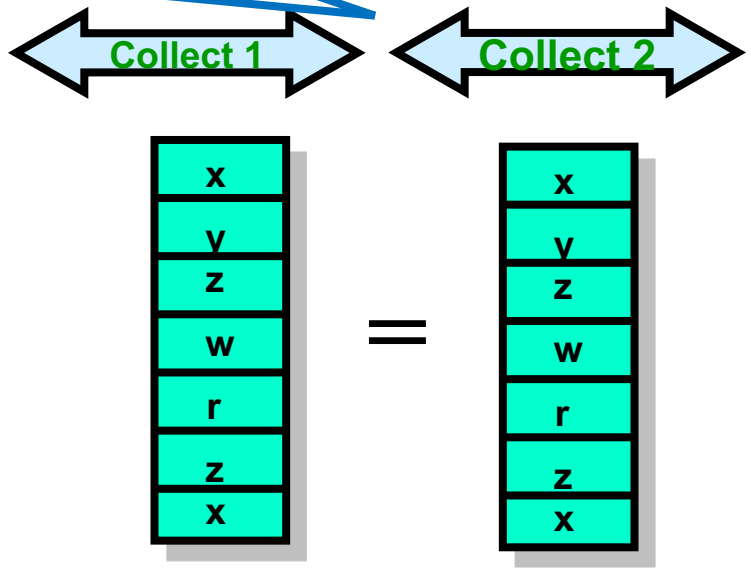
- Clean Collect
  - Collect during which nothing changed
  - Can we make it happen?
  - Can we detect it?

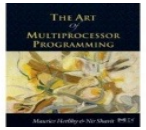


Simple Snapshot

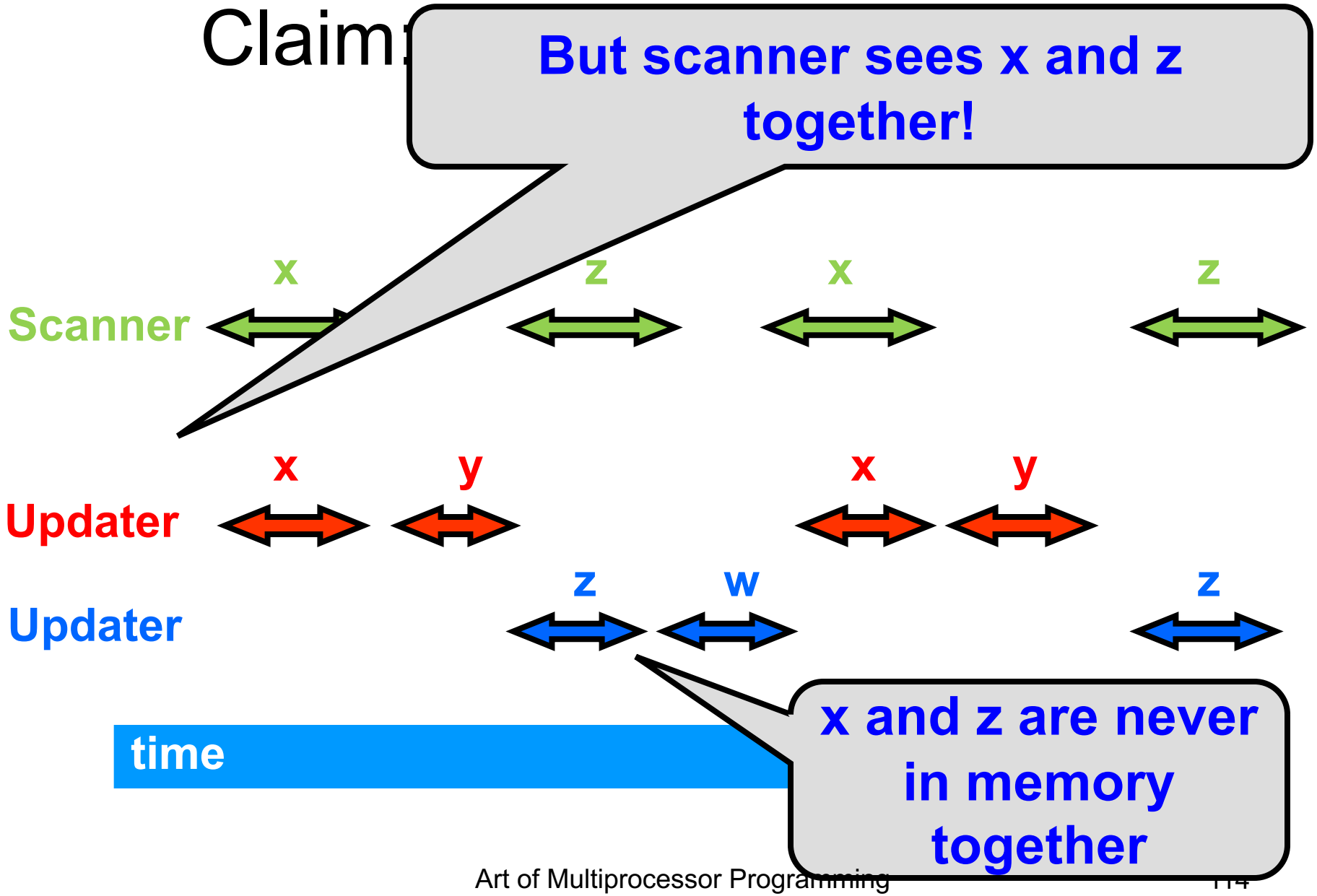
- Put increasing labels on each entry
- Collec
- If both
  - We're done
- Otherwise,
  - Try again

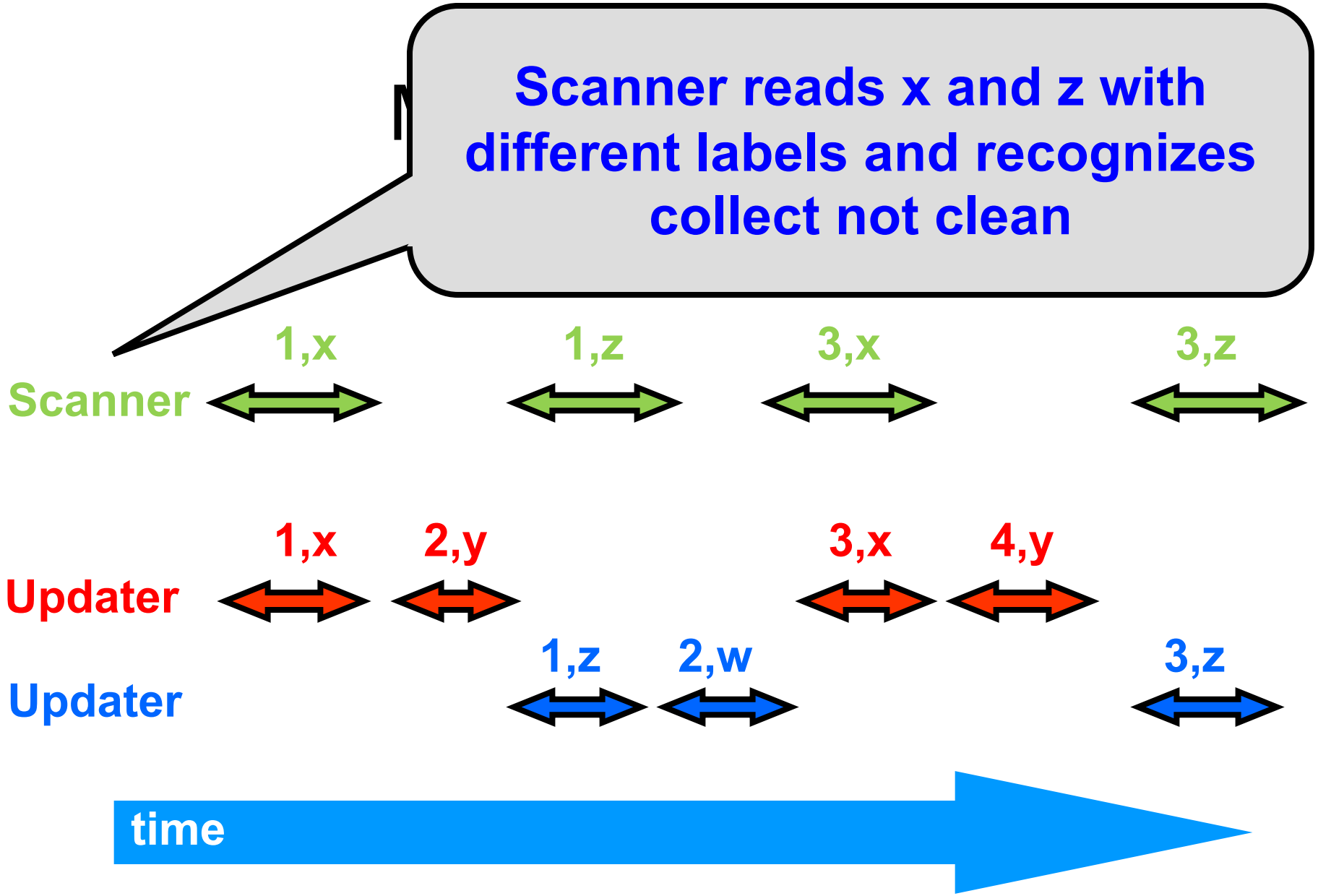
Problem: Scanner might not be collecting a snapshot!





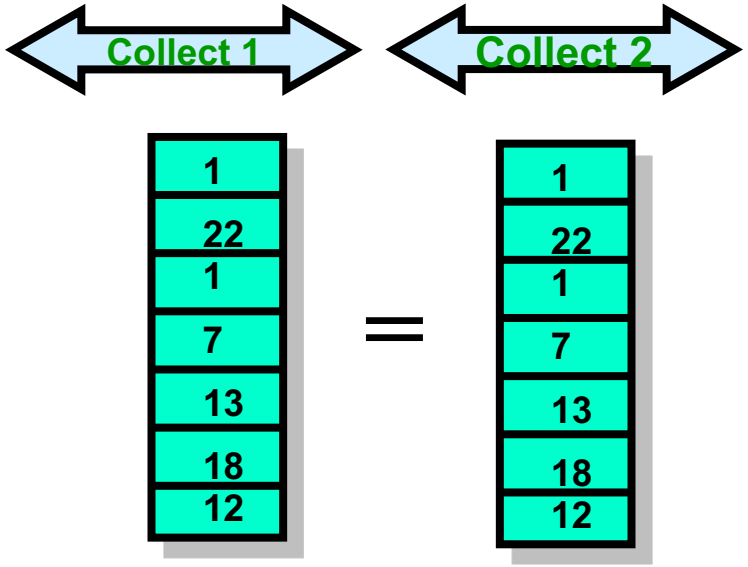
Claim:

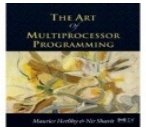

But scanner sees x and z together!
Scanner
x
z
x
z
Updater
x
y
x
y
Updater
z
w
z
time
x and z are never in memory together


Scanner reads x and z with different labels and recognizes collect not clean
Scanner
1,x
1,z
3,x
3,z
Updater
1,x
2,y
3,x
4,y
Updater
1,z
2,w
3,z
time

Simple Snapshot

- Collect twice
- If both agree,
  - We're done
- Otherwise,
  - Try again

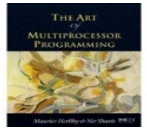




Simple Snapshot: Update

```
public class SimpleSnapshot implements Snapshot {
  private AtomicMRSWRegister[] register;

  public void update(int value) {
    int i = Thread.myIndex();
       LabeledValue oldValue = register[i].read();
    LabeledValue newValue =
     new LabeledValue(oldValue.label+1, value);
    register[i].write(newValue);
  }
```



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    int i = Thread.myIndex();
    LabeledValue oldValue = register[i].read();
    LabeledValue newValue =
     new LabeledValue(oldValue.label+1, value);
    register[i].write(newValue);
  }
```
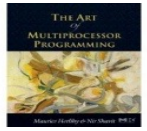

One single-writer register per thread



Simple Snapshot: Update

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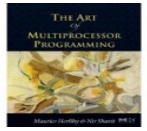
  private AtomicMRSWRegister[] register;

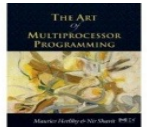
  public void update(int value) {
    int i = Thread.myIndex();
    LabeledValue oldValue = register[i].read();
    LabeledValue newValue =
     new LabeledValue(oldValue.label+1, value);
    register[i].write(newValue);
  }
```

Write each time with higher label



Simple Snapshot: Collect

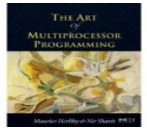
```
private LabeledValue[] collect() {
 LabeledValue[] copy =
  new LabeledValue[n];
 for (int j = 0; j < n; j++)
  copy[j] = this.register[j].read();
 return copy;
}
```



Simple Snapshot

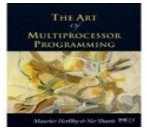
```
private LabeledValue[] collect() {
 LabeledValue[] copy =
  new LabeledValue[n];
 for (int j = 0; j < n; j++)
  copy[j] = this.register[j].read();
 return copy;
}
```

Just read each register into array



Simple Snapshot: Scan

```
public int[] scan() {
 LabeledValue[] oldCopy, newCopy;
 oldCopy = collect();
 collect: while (true) {
  newCopy = collect();
  if (!equals(oldCopy, newCopy)) {
     oldCopy = newCopy;
     continue collect;
  }
  return getValues(newCopy);
}}
```



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 oldCopy = collect();
 collect: while (true) {
  newCopy = collect();
  if (!equals(oldCopy, newCopy)) {
     oldCopy = newCopy;
     continue collect;
  }
  return getValues(newCopy);
}}
```
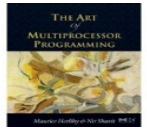

Collect once



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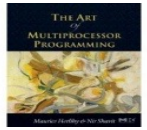
 oldCopy = collect();
 collect: while (true) {
  newCopy = collect();
   if (!equals(oldCopy, newCopy)) {
      oldCopy = newCopy;
      continue collect;
   }
   return getValues(newCopy);
}}
```

Collect once

Collect twice



Simple Snapshot: Scan

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 oldCopy = collect();
 collect: while (true) {
  newCopy = collect();
  if (!equals(oldCopy, newCopy)) {
    oldCopy = newCopy;
    continue collect;
  }
   return getValues(newCopy);
}}
```
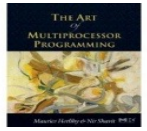

Collect once

Collect twice

On mismatch, try again



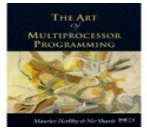
Simple Snapshot: Scan

```
public int[] scan() {
 LabeledValue[] oldCopy, newCopy;
 oldCopy = collect();
 collect: while (true) {
  newCopy = collect();
   if (!equals(oldCopy, newCopy)) {
    oldCopy = newCopy;
     continue collect;
   }
  return getValues(newCopy);
}}
```

Collect once

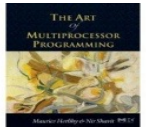
Collect twice

On match, return values



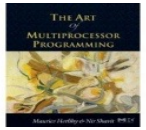
Simple Snapshot

- Linearizable
- Update is wait-free
  - No unbounded loops
- But Scan can starve
  - If interrupted by concurrent update



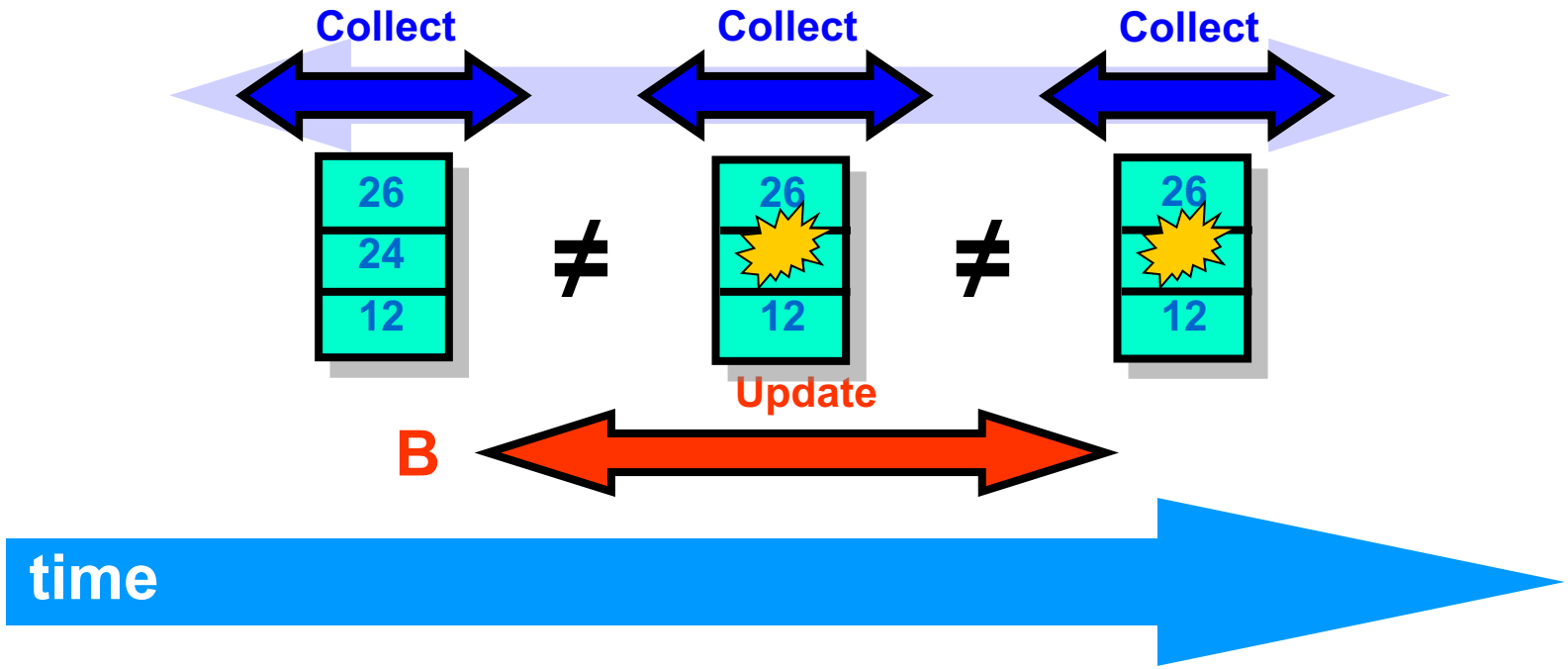
Wait-Free Snapshot

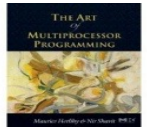
- Add a scan before every update
- Write resulting snapshot together with update value
- If scan is continuously interrupted by updates, scan can take the update's snapshot



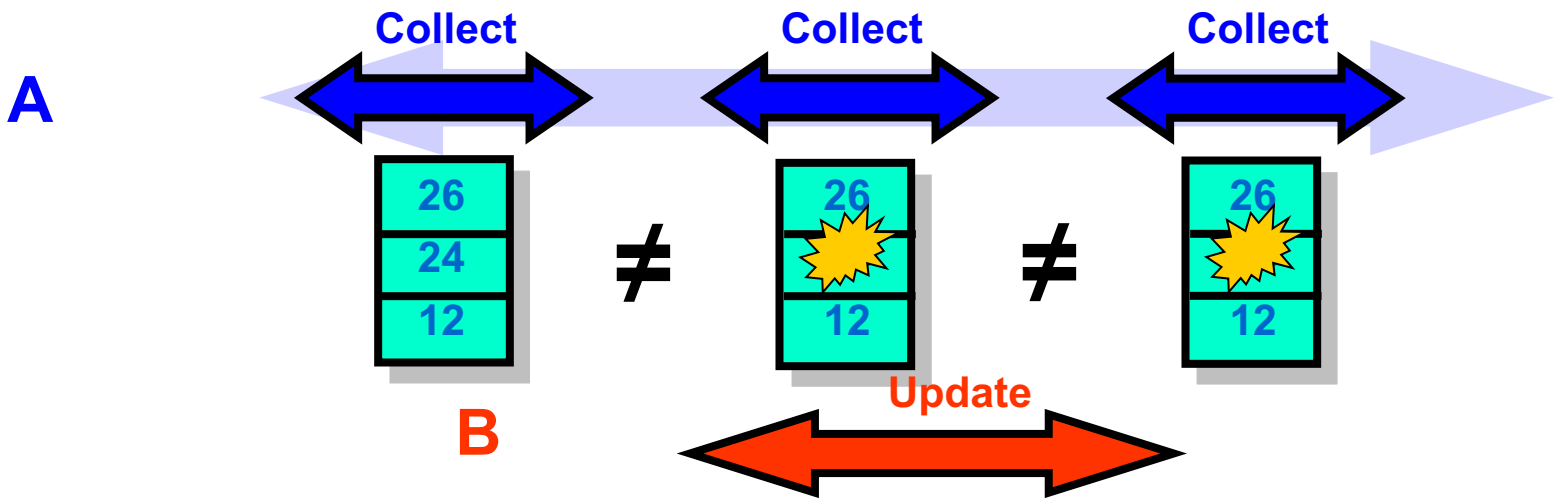
Wait-free Snapshot

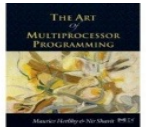
If A's scan observes that B moved ***twice***, then B completed an update while A's scan was in progress



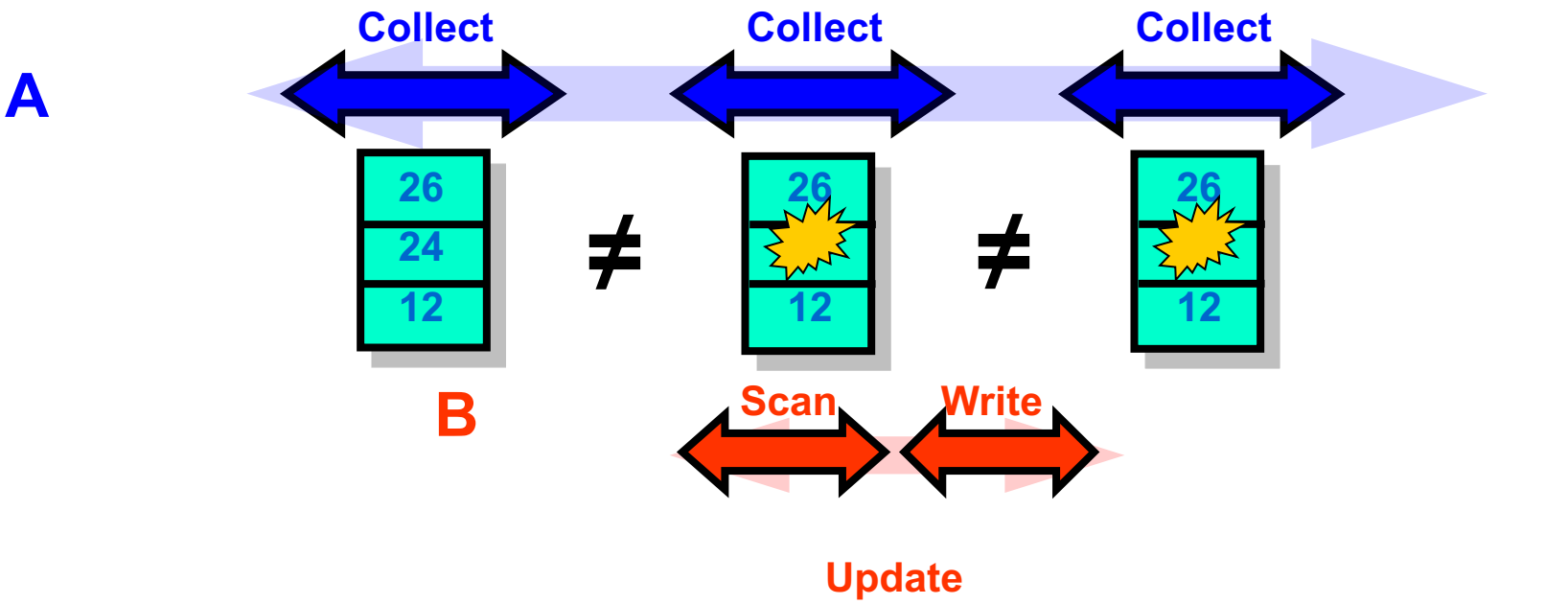


Wait-free Snapshot



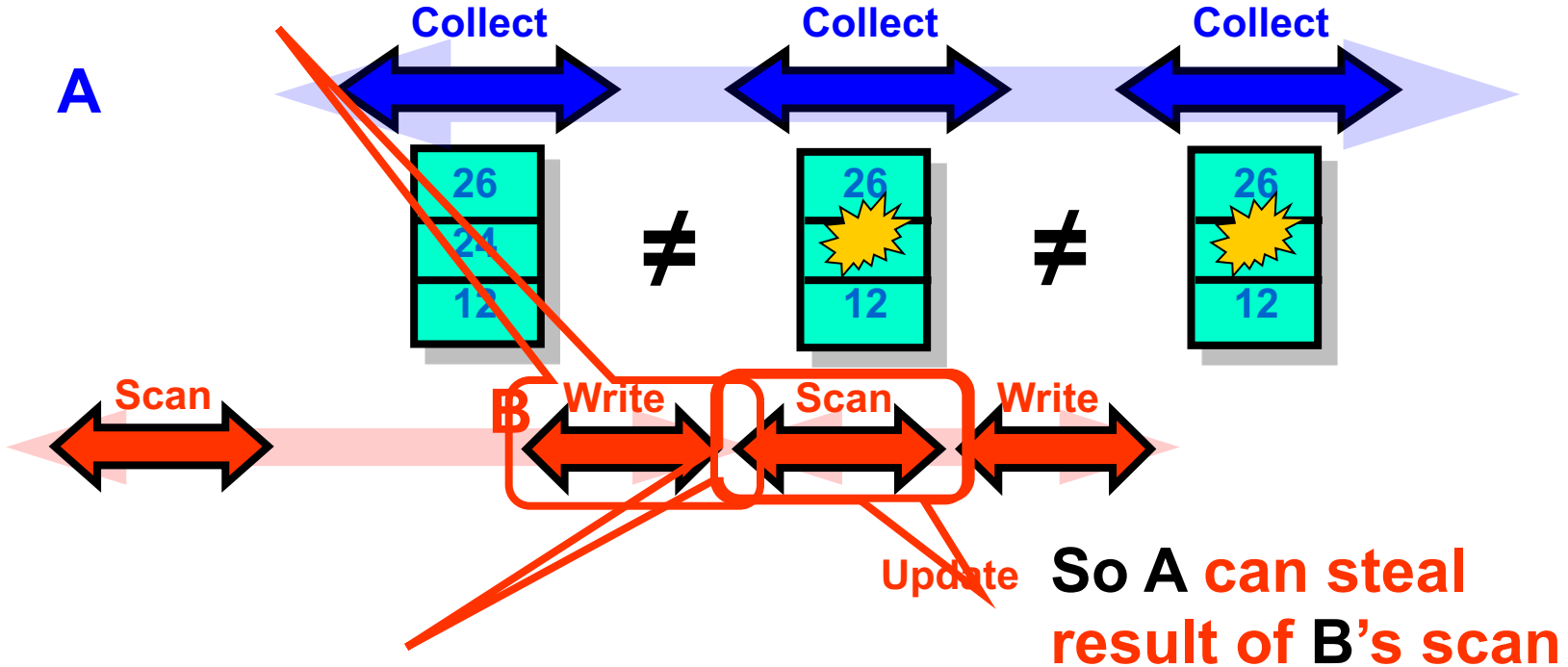


Wait-free Snapshot



Wait-free Snapshot

B's 1st update must have written during 1st collect

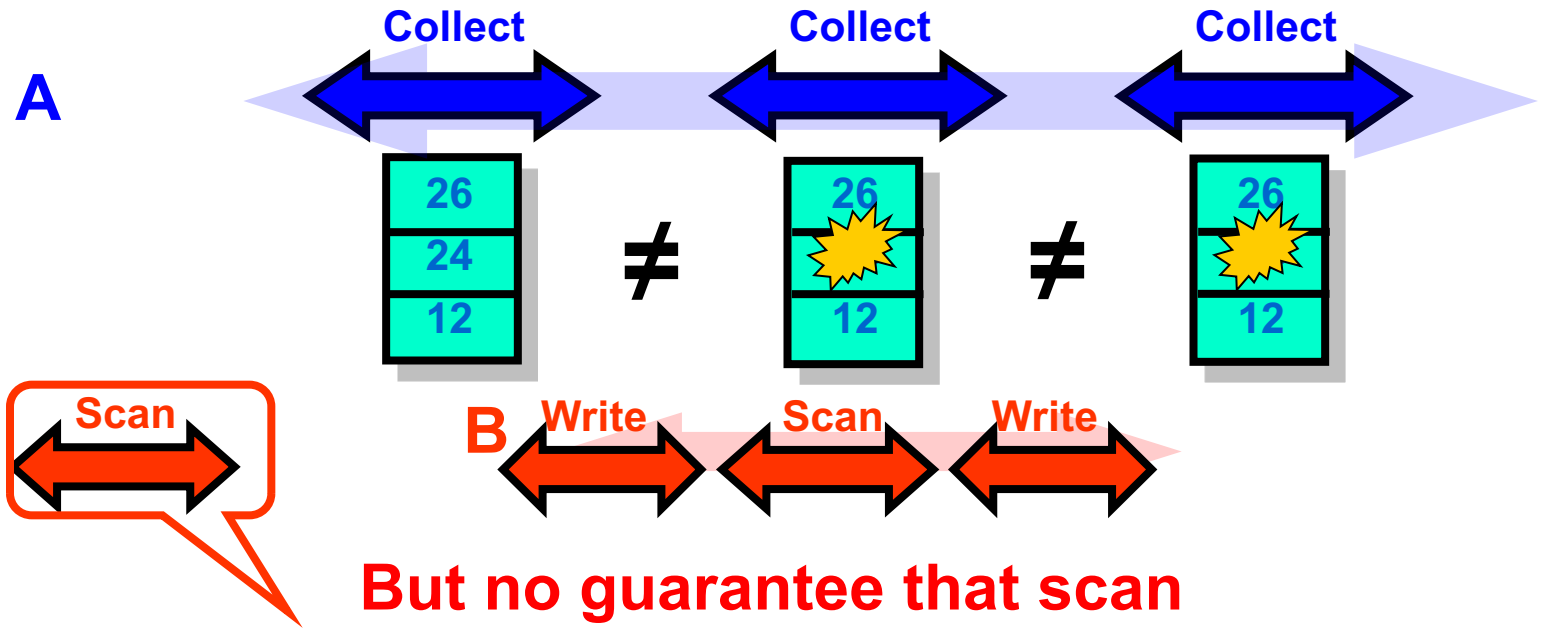


So A can steal result of B's scan

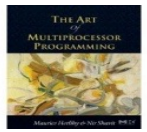
So scan of B's second update must be within interval of A's scan

time

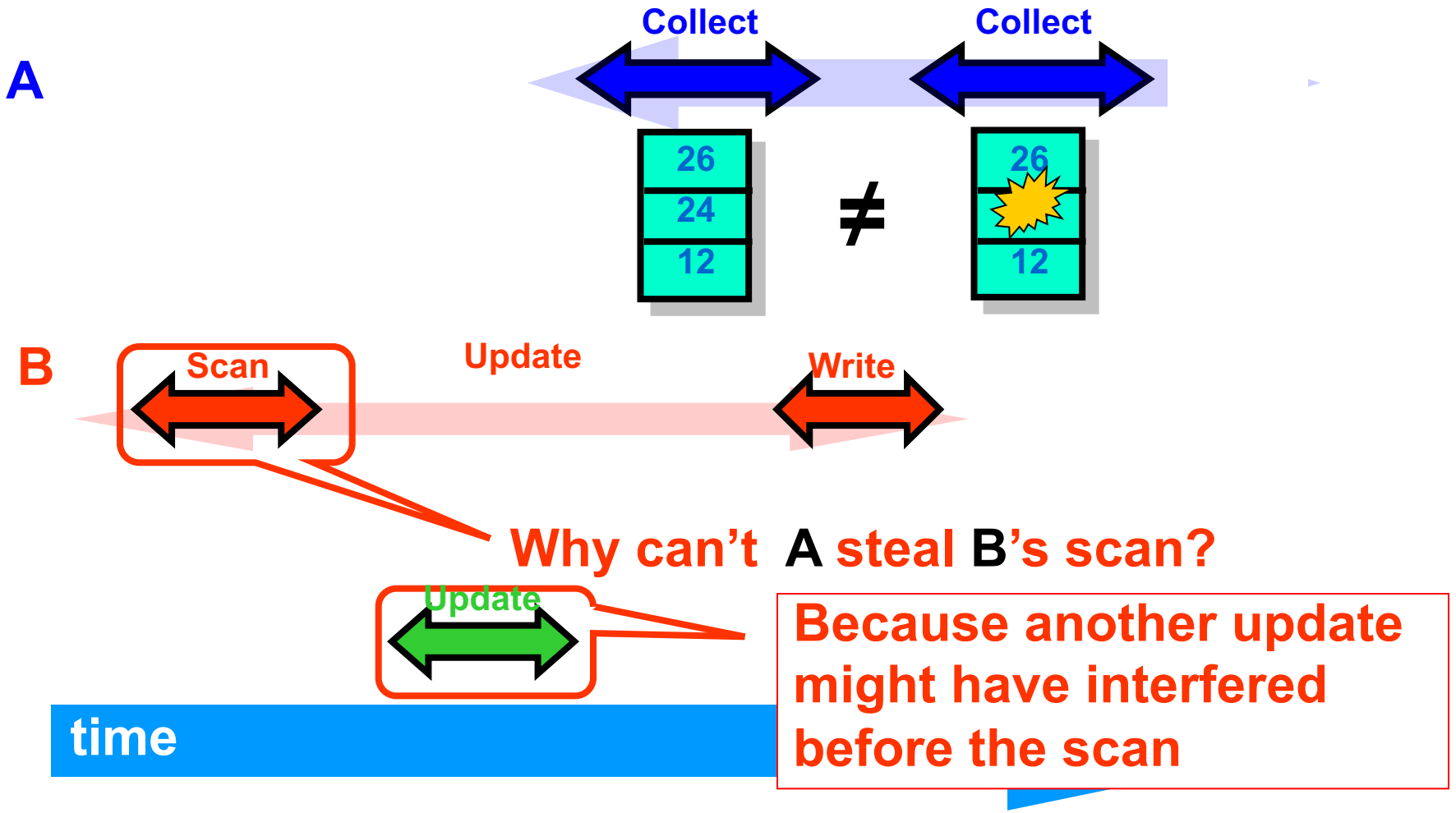
Wait-free Snapshot

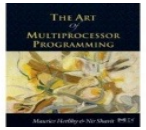


But no guarantee that scan of B's 1st update can be used… Why?

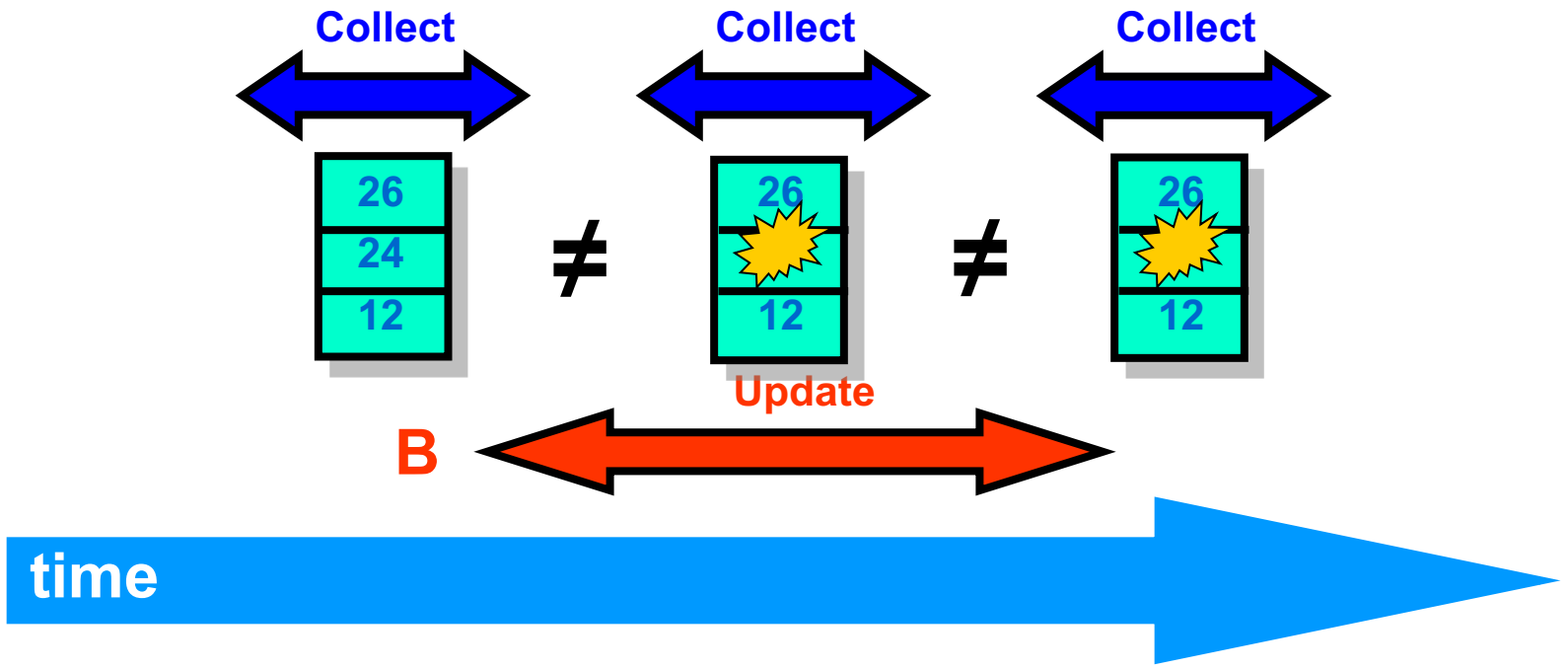


Once is not Enough

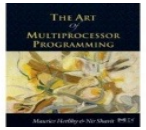




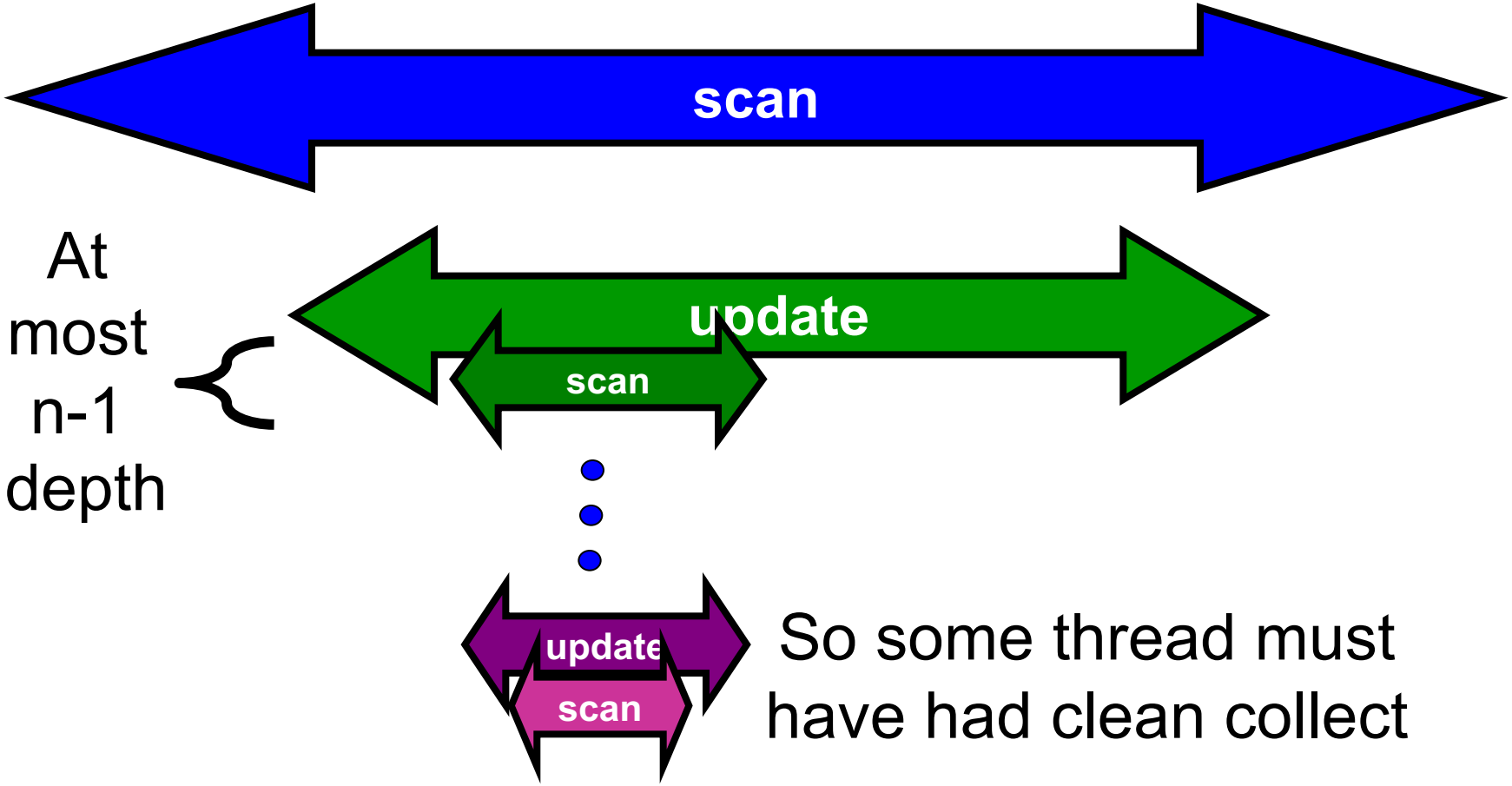
Someone Must Move Twice



If we collect *n* times…some thread must move twice (pigeonhole principle)

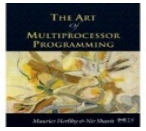


Scan is Wait-free



Wait-Free Snapshot Label

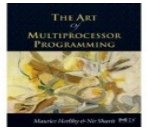
```
public class SnapValue {
 public int   label;
 public int   value;
 public int[] snap;
}
```



Wait-Free Snapshot Label

```
public class SnapValue {
 public int   label;
 public int   value;
 public int[] snap;
}
```

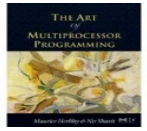
Counter incremented with each snapshot



Wait-Free Snapshot Label

```
public class SnapValue {
 public int   label;
 public int   value;
 public int[] snap;
}
```

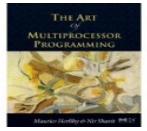
Actual value



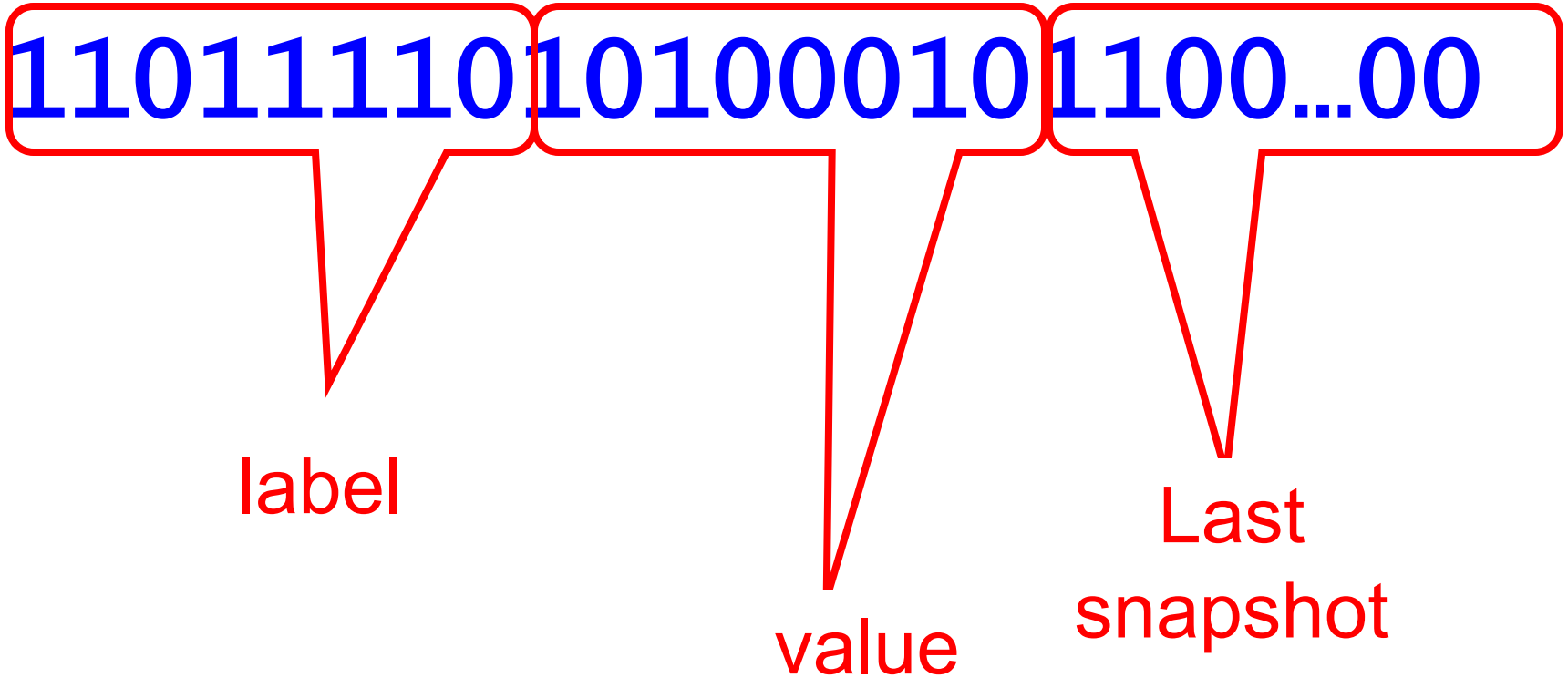
Wait-Free Snapshot Label

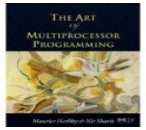
```
public class SnapValue {
 public int   label;
 public int   value;
 public int[] snap;
}
```

most recent snapshot



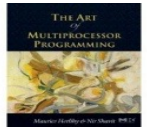
Wait-Free Snapshot Label





Wait-free Update

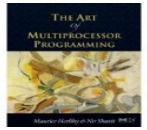
```
public void update(int value) {
 int i = Thread.myIndex();
 int[] snap = this.scan();
 SnapValue oldValue = r[i].read();
 SnapValue newValue =
  new SnapValue(oldValue.label+1,
                value, snap);
 r[i].write(newValue);
 }
```



Wait-free Scan

```
public void update(int value) {
 int i = Thread.myIndex();
 int[] snap = this.scan();
 SnapValue oldValue = r[i].read();
 SnapValue newValue =
  new SnapValue(oldValue.label+1,
                value, snap);
 r[i].write(newValue);
 }
```

Take scan

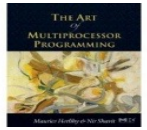


Wait-free Scan

```
public void update(int value) {
 int i = Thread.myIndex();
 int[] snap = this.scan();
 SnapValue oldValue = r[i].read();
 SnapValue newValue =
  new SnapValue(oldValue.label+1,
                value, snap);
 r[i].write(newValue);
 }
```

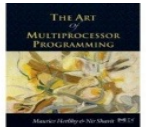
Take scan

Label value with scan



Wait-free Scan

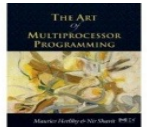
```
 public int[] scan() {
  SnapValue[] oldCopy, newCopy;
  boolean[] moved = new boolean[n];
  oldCopy = collect();
  collect: while (true) {
  newCopy = collect();
  for (int j = 0; j < n; j++) {
    if (oldCopy[j].label != newCopy[j].label) {
       …
  }}
  return getValues(newCopy);
}}}
```



Wait-free Scan

```
 public int[] scan() {
  SnapValue[] oldCopy, newCopy;
  boolean[] moved = new boolean[n];
  oldCopy = collect();
  collect: while (true) {
  newCopy = collect();
  for (int j = 0; j < n; j++) {
    if (oldCopy[j].label != newCopy[j].label) {
      …
  }}
  return getValues(newCopy);
}}}
```

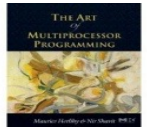
Keep track of who moved



Wait-free Scan

```
 public int[] scan() {
  SnapValue[] oldCopy, newCopy;
  boolean[] moved = new boolean[n];
  oldCopy = collect();
  collect: while (true) {
  newCopy = collect();
  for (int j = 0; j < n; j++) {
    if (oldCopy[j].label != newCopy[j].label) {
       …
  }}
  return getValues(newCopy);
}}}
```

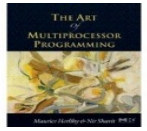
Repeated double collect



Wait-free Scan

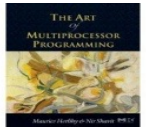
```
 public int[] scan() {
  SnapValue[] oldCopy, newCopy;
  boolean[] moved = new boolean[n];
  oldCopy = collect();
  collect: while (true) {
  newCopy = collect();
  for (int j = 0; j < n; j++) {
    if (oldCopy[j].label != newCopy[j].label) {
      …
  }}
  return getValues(newCopy);
}}}
```

If mismatch detected…



Mismatch Detected

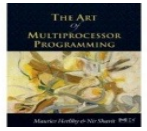
```
 if (oldCopy[j].label != newCopy[j].label) {
  if (moved[j]) {       // second move
   return newCopy[j].snap;
  } else {
   moved[j] = true;
   oldCopy = newCopy;
   continue collect;
 }}}
 return getValues(newCopy);
}}}
```



Mismatch Detected

```
 if (oldCopy[j].label != newCopy[j].label) {
  if (moved[j]) {
   return newCopy[j].snap;
  } else {
   moved[j] = true;
   oldCopy = newCopy;
   continue collect;
 }}}
 return getValues(newCopy);
}}}
```

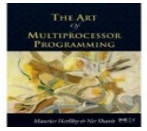
If thread moved twice, just steal its second snapshot



Mismatch Detected

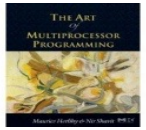
```
 if (oldCopy[j].label != newCopy[j].label) {
  if (moved[j]) {       // second move
   return newCopy[j].snap;
  } else {
   moved[j] = true;
   oldCopy = newCopy;
   continue collect;
 }}}
 return getValues(newCopy);
}}}
```

Remember that thread moved



Observations

- Uses unbounded counters
  - can be replaced with 2 bits
- Assumes SWMR registers
  - for labels
  - can be extended to MRMW



Summary

- We saw we could implement MRMW multi valued snapshot objects
- From SRSW binary safe registers (simple flipflops)
- But what is the next step to attempt with read-write registers?