Table 8.2 Some quantities and their units that are common in engineering

Quantity	Unit	Symbol
Length	metre	m
Area	square metre	m^2
Volume	cubic metre	m^3
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Speed, velocity	metre per second	m/s
Acceleration	metre per second squared	m/s^2
Density	kilogram per cubic metre	kg/m^3
Temperature	kelvin or Celsius	K or °C
Angle	radian or degree	rad or °
Angular velocity	radian per second	rad/s
Frequency	hertz	Hz
Force	newton	N
Pressure	pascal	Pa
Energy, work	joule	J
Power	watt	W
Charge, quantity of electricity	coulomb	C
Electric potential	volt	V
Capacitance	farad	F
Electrical resistance	ohm	Ω
Inductance	henry	H
Moment of force	newton metre	Nm

The most common multiples are listed in Table 8.3. A knowledge of indices is needed since all of the prefixes are powers of 10 with indices that are a multiple of 3.

Here are some examples of prefixes used with engineering units.

A **frequency of 15 GHz** means 15×10^9 Hz, which is 15 000 000 000 hertz,

i.e. 15 gigahertz is written as 15 GHz and is equal to 15 thousand million hertz.

(Instead of writing 15 000 000 000 hertz, it is much neater, takes up less space and prevents errors caused by having so many zeros, to write the frequency as 15 GHz.)

A **voltage of 40 MV** means 40×10^6 V, which is 40 000 000 volts,

i.e. 40 megavolts is written as 40 MV and is equal to 40 million volts.

An **inductance of 12 mH** means 12×10^{-3} H or $\frac{12}{10^3}$ H or $\frac{12}{1000}$ H, which is 0.012 H,

i.e. 12 millihenrys is written as 12 mH and is equal to 12 thousandths of a henry.

Table 8.3 Common SI multiples

Prefix	Name	Meaning	
G	giga	multiply by 10^9	i.e. $\times$ 1 000 000 000
M	mega	multiply by 10^6	i.e. $\times$ 1 000 000
k	kilo	multiply by 10^3	i.e. $\times$ 1 000
m	milli	multiply by 10^{-3}	i.e. $\times \frac{1}{10^3} = \frac{1}{1000} = 0.001$
μ	micro	multiply by 10^{-6}	i.e. $\times \frac{1}{10^6} = \frac{1}{1\,000\,000} = 0.000001$
n	nano	multiply by 10^{-9}	i.e. $\times \frac{1}{10^9} = \frac{1}{1\,000\,000\,000} = 0.000\,000\,001$
p	pico	multiply by 10^{-12}	i.e. $\times \frac{1}{10^{12}} = \frac{1}{1\,000\,000\,000\,000} =$ 0.000 000 000 001

A **time of 150 ns** means 150×10^{-9} s or $\frac{150}{10^9}$ s, which is 0.000 000 150 s,
i.e. 150 nanoseconds is written as 150 ns and is equal to 150 thousand millionths of a second.
A **force of 20 kN** means 20×10^3 N, which is 20 000 newtons,
i.e. 20 kilonewtons is written as 20 kN and is equal to 20 thousand newtons.
A **charge of 30 μC** means 30×10^{-6} C or $\frac{30}{10^6}$ C, which is 0.000 030 C,
i.e. 30 microcoulombs is written as 30 μC and is equal to 30 millionths of a coulomb.
A **capacitance of 45 pF** means 45×10^{-12} F or $\frac{45}{10^{12}}$ F, which is 0.000 000 000 045 F,
i.e. 45 picofarads is written as 45 pF and is equal to 45 million millionths of a farad.
In engineering it is important to understand what such quantities as 15 GHz, 40 MV, 12 mH, 150 ns, 20 kN, 30 μC and 45 pF mean.

Now try the following Practice Exercise

Practice Exercise 32 SI units and common prefixes (answers on page 343)

1. State the SI unit of volume.
2. State the SI unit of capacitance.
3. State the SI unit of area.
4. State the SI unit of velocity.
5. State the SI unit of density.
6. State the SI unit of energy.
7. State the SI unit of charge.
8. State the SI unit of power.
9. State the SI unit of angle.
10. State the SI unit of electric potential.
11. State which quantity has the unit kg.
12. State which quantity has the unit symbol Ω.
13. State which quantity has the unit Hz.
14. State which quantity has the unit m/s^2.
15. State which quantity has the unit symbol A.
16. State which quantity has the unit symbol H.
17. State which quantity has the unit symbol m.
18. State which quantity has the unit symbol K.
19. State which quantity has the unit Pa.
20. State which quantity has the unit rad/s.
21. What does the prefix G mean?
22. What is the symbol and meaning of the prefix milli?

23. What does the prefix p mean?
24. What is the symbol and meaning of the prefix mega?

8.4 Standard form

A number written with one digit to the left of the decimal point and multiplied by 10 raised to some power is said to be written in **standard form**.

For example, $43\,645 = 4.3645 \times 10^4$ in standard form

and $0.0534 = 5.34 \times 10^{-2}$ in standard form

Problem 1. Express in standard form (a) 38.71 (b) 3746 (c) 0.0124

For a number to be in standard form, it is expressed with only one digit to the left of the decimal point. Thus,

(a) 38.71 must be divided by 10 to achieve one digit to the left of the decimal point and it must also be multiplied by 10 to maintain the equality, i.e.

$$38.71 = \frac{38.71}{10} \times 10 = \mathbf{3.871 \times 10} \text{ in standard form}$$

(b) $3746 = \frac{3746}{1000} \times 1000 = \mathbf{3.746 \times 10^3}$ in standard form.

(c) $0.0124 = 0.0124 \times \frac{100}{100} = \frac{1.24}{100} = \mathbf{1.24 \times 10^{-2}}$ in standard form.

Problem 2. Express the following numbers, which are in standard form, as decimal numbers:

(a) 1.725×10^{-2} (b) 5.491×10^4 (c) 9.84×10^0

(a) $1.725 \times 10^{-2} = \frac{1.725}{100} = \mathbf{0.01725}$ (i.e. move the decimal point 2 places to the left).

(b) $5.491 \times 10^4 = 5.491 \times 10000 = \mathbf{54910}$ (i.e. move the decimal point 4 places to the right).

(c) $9.84 \times 10^0 = 9.84 \times 1 = \mathbf{9.84}$ (since $10^0 = 1$).

Problem 3. Express in standard form, correct to 3 significant figures, (a) $\frac{3}{8}$ (b) $19\frac{2}{3}$ (c) $741\frac{9}{16}$

(a) $\frac{3}{8} = 0.375$, and expressing it in standard form gives

$$0.375 = \mathbf{3.75 \times 10^{-1}}$$

(b) $19\frac{2}{3} = 19.\dot{6} = \mathbf{1.97 \times 10}$ in standard form, correct to 3 significant figures.

(c) $741\frac{9}{16} = 741.5625 = \mathbf{7.42 \times 10^2}$ in standard form, correct to 3 significant figures.

Problem 4. Express the following numbers, given in standard form, as fractions or mixed numbers, (a) 2.5×10^{-1} (b) 6.25×10^{-2} (c) 1.354×10^2

(a) $2.5 \times 10^{-1} = \frac{2.5}{10} = \frac{25}{100} = \mathbf{\frac{1}{4}}$

(b) $6.25 \times 10^{-2} = \frac{6.25}{100} = \frac{625}{10000} = \mathbf{\frac{1}{16}}$

(c) $1.354 \times 10^2 = 135.4 = 135\frac{4}{10} = \mathbf{135\frac{2}{5}}$

Problem 5. Evaluate (a) $(3.75 \times 10^3)(6 \times 10^4)$ (b) $\frac{3.5 \times 10^5}{7 \times 10^2}$, expressing the answers in standard form

(a) $(3.75 \times 10^3)(6 \times 10^4) = (3.75 \times 6)(10^{3+4})$
$= 22.50 \times 10^7$
$= \mathbf{2.25 \times 10^8}$

(b) $\frac{3.5 \times 10^5}{7 \times 10^2} = \frac{3.5}{7} \times 10^{5-2} = 0.5 \times 10^3 = \mathbf{5 \times 10^2}$

Now try the following Practice Exercise

Practice Exercise 33 Standard form (answers on page 343)

In problems 1 to 5, express in standard form.

1. (a) 73.9 (b) 28.4 (c) 197.62
2. (a) 2748 (b) 33170 (c) 274218
3. (a) 0.2401 (b) 0.0174 (c) 0.00923
4. (a) 1702.3 (b) 10.04 (c) 0.0109
5. (a) $\frac{1}{2}$ (b) $11\frac{7}{8}$
   (c) $\frac{1}{32}$ (d) $130\frac{3}{5}$

In problems 6 and 7, express the numbers given as integers or decimal fractions.

6. (a) 1.01×10^3 (b) 9.327×10^2
   (c) 5.41×10^4 (d) 7×10^0

7. (a) 3.89×10^{-2} (b) 6.741×10^{-1}
   (c) 8×10^{-3}

In problems 8 and 9, evaluate the given expressions, stating the answers in standard form.

8. (a) $(4.5 \times 10^{-2})(3 \times 10^3)$
   (b) $2 \times (5.5 \times 10^4)$

9. (a) $\dfrac{6 \times 10^{-3}}{3 \times 10^{-5}}$
   (b) $\dfrac{(2.4 \times 10^3)(3 \times 10^{-2})}{(4.8 \times 10^4)}$

10. Write the following statements in standard form.
    (a) The density of aluminium is 2710 $\mathrm{kg\,m^{-3}}$.
    (b) Poisson's ratio for gold is 0.44
    (c) The impedance of free space is 376.73 Ω.
    (d) The electron rest energy is 0.511 MeV.
    (e) Proton charge–mass ratio is 95789700 $\mathrm{C\,kg^{-1}}$.
    (f) The normal volume of a perfect gas is 0.02241 $\mathrm{m^3\,mol^{-1}}$.

8.5 Engineering notation

In engineering, standard form is not as important as engineering notation. **Engineering notation** is similar to standard form except that the power of 10 **is always a multiple of 3**.

For example, $43\,645 = 43.645 \times 10^3$ in engineering notation

and $0.0534 = 53.4 \times 10^{-3}$ in engineering notation

From the list of engineering prefixes on page 55 it is apparent that all prefixes involve powers of 10 that are multiples of 3.

For example, a force of 43 645 N can rewritten as 43.645×10^3 N and from the list of prefixes can then be expressed as 43.645 kN.

Thus, **43 645 N ≡ 43.645 kN**

To help further, on your calculator is an 'ENG' button. Enter the number 43 645 into your calculator and then press =. Now press the ENG button and the answer is 43.645×10^3. We then have to appreciate that 10^3 is the prefix 'kilo', giving **43 645 N ≡ 43.645 kN**.
In another example, let a current be 0.0745 A. Enter 0.0745 into your calculator. Press =. Now press ENG and the answer is 74.5×10^{-3}. We then have to appreciate that 10^{-3} is the prefix 'milli', giving **0.0745 A ≡ 74.5 mA**.

Problem 6. Express the following in engineering notation and in prefix form:

(a) 300 000 W (b) 0.000068 H

(a) Enter 300 000 into the calculator. Press =

Now press ENG and the answer is 300×10^3.

From the table of prefixes on page 55, 10^3 corresponds to kilo.

Hence, 300 000 W = $\mathbf{300 \times 10^3}$ **W** in engineering notation

= **300 kW** in prefix form.

(b) Enter 0.000068 into the calculator. Press =

Now press ENG and the answer is 68×10^{-6}.

From the table of prefixes on page 55, 10^{-6} corresponds to micro.

Hence, 0.000068 H = $\mathbf{68 \times 10^{-6}}$ **H** in engineering notation

= **68 μH** in prefix form.

Problem 7. Express the following in engineering notation and in prefix form:

(a) 42×10^5 Ω (b) 4.7×10^{-10} F

(a) Enter 42×10^5 into the calculator. Press =

Now press ENG and the answer is 4.2×10^6.

From the table of prefixes on page 55, 10^6 corresponds to mega.

Hence, $42 \times 10^5\ \Omega = \mathbf{4.2 \times 10^6}\ \Omega$ in engineering notation

$= \mathbf{4.2\ M\Omega}$ in prefix form.

(b) Enter $47 \div 10^{10} = \dfrac{47}{10\,000\,000\,000}$ into the calculator. Press =

Now press ENG and the answer is 4.7×10^{-9}.

From the table of prefixes on page 55, 10^{-9} corresponds to nano.

Hence, $47 \div 10^{10}\ \text{F} = \mathbf{4.7 \times 10^{-9}\ F}$ in engineering notation

$= \mathbf{4.7\ nF}$ in prefix form.

Problem 8. Rewrite (a) 0.056 mA in μA (b) 16 700 kHz as MHz

(a) Enter $0.056 \div 1000$ into the calculator (since milli means $\div 1000$). Press =

Now press ENG and the answer is 56×10^{-6}.

From the table of prefixes on page 55, 10^{-6} corresponds to micro.

Hence, $0.056\ \text{mA} = \dfrac{0.056}{1000}\ \text{A} = 56 \times 10^{-6}\ \text{A}$

$= \mathbf{56\ \mu A}$.

(b) Enter $16\,700 \times 1000$ into the calculator (since kilo means $\times 1000$). Press =

Now press ENG and the answer is 16.7×10^6.

From the table of prefixes on page 55, 10^6 corresponds to mega.

Hence, $16\,700\ \text{kHz} = 16\,700 \times 1000\ \text{Hz}$

$= 16.7 \times 10^6\ \text{Hz}$

$= \mathbf{16.7\ MHz}$

Problem 9. Rewrite (a) 63×10^4 V in kV (b) 3100 pF in nF

(a) Enter 63×10^4 into the calculator. Press =

Now press ENG and the answer is 630×10^3.

From the table of prefixes on page 55, 10^3 corresponds to kilo.

Hence, $63 \times 10^4\ \text{V} = 630 \times 10^3\ \text{V} = \mathbf{630\ kV}$.

(b) Enter 3100×10^{-12} into the calculator. Press =

Now press ENG and the answer is 3.1×10^{-9}.

From the table of prefixes on page 55, 10^{-9} corresponds to nano.

Hence, $3100\ \text{pF} = 31 \times 10^{-12}\ \text{F} = 3.1 \times 10^{-9}\ \text{F}$

$= \mathbf{3.1\ nF}$

Problem 10. Rewrite (a) 14 700 mm in metres (b) 276 cm in metres (c) 3.375 kg in grams

(a) 1 m = 1000 mm, hence

$1\ \text{mm} = \dfrac{1}{1000} = \dfrac{1}{10^3} = 10^{-3}\ \text{m}$.

Hence, $14\,700\ \text{mm} = 14\,700 \times 10^{-3}\ \text{m} = \mathbf{14.7\ m}$.

(b) 1 m = 100 cm, hence $1\ \text{cm} = \dfrac{1}{100} = \dfrac{1}{10^2} = 10^{-2}\ \text{m}$.

Hence, $276\ \text{cm} = 276 \times 10^{-2}\ \text{m} = \mathbf{2.76\ m}$.

(c) $1\ \text{kg} = 1000\ \text{g} = 10^3\ \text{g}$

Hence, $3.375\ \text{kg} = 3.375 \times 10^3\ \text{g} = \mathbf{3375\ g}$.

Now try the following Practice Exercise

Practice Exercise 34 Engineering notation (answers on page 343)

In problems 1 to 12, express in engineering notation in prefix form.

1. 60 000 Pa
2. 0.00015 W
3. 5×10^7 V
4. 5.5×10^{-8} F
5. 100 000 W
6. 0.00054 A
7. $15 \times 10^5\ \Omega$
8. 225×10^{-4} V
9. 35 000 000 000 Hz
10. 1.5×10^{-11} F
11. 0.000017 A
12. 46 200 Ω

13. Rewrite 0.003 mA in μA

14. Rewrite 2025 kHz as MHz

15. Rewrite 5×10^4 N in kN

16. Rewrite 300 pF in nF

17. Rewrite 6250 cm in metres

18. Rewrite 34.6 g in kg

In problems 19 and 20, use a calculator to evaluate in engineering notation.

19. $4.5 \times 10^{-7} \times 3 \times 10^4$

20. $\dfrac{(1.6 \times 10^{-5})(25 \times 10^3)}{(100 \times 10^{-6})}$

Revision Test 3 : Ratio, proportion, powers, roots, indices and units

This assignment covers the material contained in Chapters 6–8. *The marks available are shown in brackets at the end of each question.*

1. In a box of 1500 nails, 125 are defective. Express the non-defective nails as a ratio of the defective ones, in its simplest form. (3)

2. Prize money in a lottery totals £4500 and is shared among three winners in the ratio 5:3:1. How much does the first prize winner receive? (3)

3. A simple machine has an effort:load ratio of 3:41. Determine the effort, in newtons, to lift a load of 6.15 kN. (3)

4. If 15 cans of lager weigh 7.8 kg, what will 24 cans weigh? (3)

5. Hooke's law states that stress is directly proportional to strain within the elastic limit of a material. When for brass the stress is 21 MPa, the strain is 250×10^{-6}. Determine the stress when the strain is 350×10^{-6}. (3)

6. If 12 inches = 30.48 cm, find the number of millimetres in 17 inches. (3)

7. If x is inversely proportional to y and $x = 12$ when $y = 0.4$, determine
   (a) the value of x when y is 3.
   (b) the value of y when $x = 2$. (3)

8. Evaluate
   (a) $3 \times 2^3 \times 2^2$
   (b) $49^{\frac{1}{2}}$ (4)

9. Evaluate $\dfrac{3^2 \times \sqrt{36} \times 2^2}{3 \times 81^{\frac{1}{2}}}$ taking positive square roots only. (3)

10. Evaluate $6^4 \times 6 \times 6^2$ in index form. (3)

11. Evaluate
    (a) $\dfrac{2^7}{2^2}$ (b) $\dfrac{10^4 \times 10 \times 10^5}{10^6 \times 10^2}$ (4)

12. Evaluate
    (a) $\dfrac{2^3 \times 2 \times 2^2}{2^4}$
    (b) $\dfrac{(2^3 \times 16)^2}{(8 \times 2)^3}$
    (c) $\left(\dfrac{1}{4^2}\right)^{-1}$ (7)

13. Evaluate
    (a) $(27)^{-\frac{1}{3}}$ (b) $\dfrac{\left(\dfrac{3}{2}\right)^{-2} - \dfrac{2}{9}}{\left(\dfrac{2}{3}\right)^2}$ (5)

14. State the SI unit of (a) capacitance (b) electrical potential (c) work (3)

15. State the quantity that has an SI unit of (a) kilograms (b) henrys (c) hertz (d) m^3 (4)

16. Express the following in engineering notation in prefix form.
    (a) 250 000 J (b) 0.05 H
    (c) 2×10^8 W (d) 750×10^{-8} F (4)

17. Rewrite (a) 0.0067 mA in μA (b) 40×10^4 kV as MV (2)

Chapter 9

Basic algebra

9.1 Introduction

We are already familiar with evaluating formulae using a calculator from Chapter 4.

For example, if the length of a football pitch is L and its width is b, then the formula for the area A is given by

$$A = L \times b$$

This is an **algebraic equation**.

If $L = 120$ m and $b = 60$ m, then the area $A = 120 \times 60 = 7200\,\text{m}^2$.

The total resistance, R_T, of resistors R_1, R_2 and R_3 connected in series is given by

$$R_T = R_1 + R_2 + R_3$$

This is an **algebraic equation**.

If $R_1 = 6.3\,\text{k}\Omega$, $R_2 = 2.4\,\text{k}\Omega$ and $R_3 = 8.5\,\text{k}\Omega$, then

$$R_T = 6.3 + 2.4 + 8.5 = 17.2\,\text{k}\Omega$$

The temperature in Fahrenheit, F, is given by

$$F = \frac{9}{5}C + 32$$

where C is the temperature in Celsius. This is an **algebraic equation**.

If $C = 100°\text{C}$, then $F = \frac{9}{5} \times 100 + 32$
$= 180 + 32 = 212°\text{F}$.

If you can cope with evaluating formulae then you will be able to cope with algebra.

9.2 Basic operations

Algebra merely uses letters to represent numbers.

If, say, a, b, c and d represent any four numbers then in algebra:

(a) $\boldsymbol{a + a + a + a = 4a}$. For example, if $a = 2$, then $2 + 2 + 2 + 2 = 4 \times 2 = 8$.

(b) $\boldsymbol{5b}$ **means** $\boldsymbol{5 \times b}$. For example, if $b = 4$, then $5b = 5 \times 4 = 20$.

(c) $\boldsymbol{2a + 3b + a - 2b} = 2a + a + 3b - 2b = \boldsymbol{3a + b}$

Only similar terms can be combined in algebra. The $2a$ and the $+a$ can be combined to give $3a$ and the $3b$ and $-2b$ can be combined to give $1b$, which is written as b.

In addition, with terms separated by $+$ and $-$ signs, the order in which they are written does not matter. In this example, $2a + 3b + a - 2b$ is the same as $2a + a + 3b - 2b$, which is the same as $3b + a + 2a - 2b$, and so on. (Note that the first term, i.e. $2a$, means $+2a$.)

(d) $\boldsymbol{4abcd = 4 \times a \times b \times c \times d}$

For example, if $a = 3$, $b = -2$, $c = 1$ and $d = -5$, then $4abcd = 4 \times 3 \times -2 \times 1 \times -5 = 120$. (Note that $- \times - = +$)

(e) $\boldsymbol{(a)(c)(d)}$ **means** $\boldsymbol{a \times c \times d}$

Brackets are often used instead of multiplication signs. For example, $(2)(5)(3)$ means $2 \times 5 \times 3 = 30$.

(f) $\boldsymbol{ab = ba}$

If $a = 2$ and $b = 3$ then 2×3 is exactly the same as 3×2, i.e. 6.

(g) $\boldsymbol{b^2 = b \times b}$. For example, if $b = 3$, then $3^2 = 3 \times 3 = 9$.

(h) $\boldsymbol{a^3 = a \times a \times a}$ For example, if $a = 2$, then $2^3 = 2 \times 2 \times 2 = 8$.

Here are some worked examples to help get a feel for basic operations in this introduction to algebra.

DOI: 10.1016/B978-1-85617-697-2.00009-0

9.2.1 Addition and subtraction

Problem 1. Find the sum of $4x, 3x, -2x$ and $-x$

$$4x + 3x + -2x + -x = 4x + 3x - 2x - x$$
(Note that $+ \times - = -$)
$$= \mathbf{4x}$$

Problem 2. Find the sum of $5x, 3y, z, -3x, -4y$ and $6z$

$$\begin{aligned} 5x + 3y + z + -3x + -4y + 6z &= 5x + 3y + z - 3x - 4y + 6z \\ &= 5x - 3x + 3y - 4y + z + 6z \\ &= \mathbf{2x - y + 7z} \end{aligned}$$

Note that the order can be changed when terms are separated by + and − signs. Only similar terms can be combined.

Problem 3. Simplify $4x^2 - x - 2y + 5x + 3y$

$$\begin{aligned} 4x^2 - x - 2y + 5x + 3y &= 4x^2 + 5x - x + 3y - 2y \\ &= \mathbf{4x^2 + 4x + y} \end{aligned}$$

Problem 4. Simplify $3xy - 7x + 4xy + 2x$

$$\begin{aligned} 3xy - 7x + 4xy + 2x &= 3xy + 4xy + 2x - 7x \\ &= \mathbf{7xy - 5x} \end{aligned}$$

Now try the following Practice Exercise

Practice Exercise 35 Addition and subtraction in algebra (answers on page 343)

1. Find the sum of $4a, -2a, 3a$ and $-8a$.
2. Find the sum of $2a, 5b, -3c, -a, -3b$ and $7c$.
3. Simplify $2x - 3x^2 - 7y + x + 4y - 2y^2$.
4. Simplify $5ab - 4a + ab + a$.
5. Simplify $2x - 3y + 5z - x - 2y + 3z + 5x$.
6. Simplify $3 + x + 5x - 2 - 4x$.
7. Add $x - 2y + 3$ to $3x + 4y - 1$.
8. Subtract $a - 2b$ from $4a + 3b$.
9. From $a + b - 2c$ take $3a + 2b - 4c$.
10. From $x^2 + xy - y^2$ take $xy - 2x^2$.

9.2.2 Multiplication and division

Problem 5. Simplify $bc \times abc$

$$\begin{aligned} bc \times abc &= a \times b \times b \times c \times c \\ &= a \times b^2 \times c^2 \\ &= \boldsymbol{ab^2c^2} \end{aligned}$$

Problem 6. Simplify $-2p \times -3p$

$- \times - = +$ hence, $-2p \times -3p = \mathbf{6p^2}$

Problem 7. Simplify $ab \times b^2c \times a$

$$\begin{aligned} ab \times b^2c \times a &= a \times a \times b \times b \times b \times c \\ &= a^2 \times b^3 \times c \\ &= \boldsymbol{a^2b^3c} \end{aligned}$$

Problem 8. Evaluate $3ab + 4bc - abc$ when $a = 3, b = 2$ and $c = 5$

$$\begin{aligned} 3ab + 4bc - abc &= 3 \times a \times b + 4 \times b \times c - a \times b \times c \\ &= 3 \times 3 \times 2 + 4 \times 2 \times 5 - 3 \times 2 \times 5 \\ &= 18 + 40 - 30 \\ &= \mathbf{28} \end{aligned}$$

Problem 9. Determine the value of $5pq^2r^3$, given that $p = 2, q = \frac{2}{5}$ and $r = 2\frac{1}{2}$

$$\begin{aligned} 5pq^2r^3 &= 5 \times p \times q^2 \times r^3 \\ &= 5 \times 2 \times \left(\frac{2}{5}\right)^2 \times \left(2\frac{1}{2}\right)^3 \end{aligned}$$

$$= 5 \times 2 \times \left(\frac{2}{5}\right)^2 \times \left(\frac{5}{2}\right)^3 \qquad \text{since } 2\frac{1}{2} = \frac{5}{2}$$

$$= \frac{5}{1} \times \frac{2}{1} \times \frac{2}{5} \times \frac{2}{5} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2}$$

$$= \frac{1}{1} \times \frac{1}{1} \times \frac{1}{1} \times \frac{1}{1} \times \frac{1}{1} \times \frac{5}{1} \times \frac{5}{1} \qquad \text{by cancelling}$$

$$= 5 \times 5$$

$$= \mathbf{25}$$

Problem 10. Multiply $2a + 3b$ by $a + b$

Each term in the first expression is multiplied by a, then each term in the first expression is multiplied by b and the two results are added. The usual layout is shown below.

$$\begin{array}{lr} & 2a + 3b \\ & \underline{a + b} \\ \text{Multiplying by } a \text{ gives} & 2a^2 + 3ab \qquad \\ \text{Multiplying by } b \text{ gives} & \underline{\qquad 2ab + 3b^2} \\ \text{Adding gives} & \underline{\mathbf{2a^2 + 5ab + 3b^2}} \end{array}$$

Thus, $\mathbf{(2a + 3b)(a + b) = 2a^2 + 5ab + 3b^2}$

Problem 11. Multiply $3x - 2y^2 + 4xy$ by $2x - 5y$

$$\begin{array}{lllll} & 3x & -2y^2 & +4xy & \\ & \underline{2x} & \underline{-5y} & & \\ \text{Multiplying by } 2x \rightarrow & 6x^2 & -4xy^2 & +8x^2y & \\ \text{Multiplying by } -5y \rightarrow & & -20xy^2 & & -15xy + 10y^3 \\ \text{Adding gives} & \multicolumn{4}{l}{\mathbf{6x^2 - 24xy^2 + 8x^2y - 15xy + 10y^3}} \end{array}$$

Thus, $\mathbf{(3x - 2y^2 + 4xy)(2x - 5y)}$
$\mathbf{= 6x^2 - 24xy^2 + 8x^2y - 15xy + 10y^3}$

Problem 12. Simplify $2x \div 8xy$

$2x \div 8xy$ means $\dfrac{2x}{8xy}$

$$\frac{2x}{8xy} = \frac{2 \times x}{8 \times x \times y}$$

$$= \frac{1 \times 1}{4 \times 1 \times y} \qquad \text{by cancelling}$$

$$= \mathbf{\frac{1}{4y}}$$

Problem 13. Simplify $\dfrac{9a^2bc}{3ac}$

$$\frac{9a^2bc}{3ac} = \frac{9 \times a \times a \times b \times c}{3 \times a \times c}$$

$$= 3 \times a \times b$$

$$= \mathbf{3ab}$$

Problem 14. Divide $2x^2 + x - 3$ by $x - 1$

(i) $2x^2 + x - 3$ is called the **dividend** and $x - 1$ the **divisor**. The usual layout is shown below with the dividend and divisor both arranged in descending powers of the symbols.

$$\begin{array}{r} 2x + 3 \\ x - 1 \overline{)\, 2x^2 + x - 3} \\ \underline{2x^2 - 2x} \qquad \\ 3x - 3 \\ \underline{3x - 3} \\ \underline{\cdot \qquad \cdot} \end{array}$$

(ii) Dividing the first term of the dividend by the first term of the divisor, i.e. $\dfrac{2x^2}{x}$ gives $2x$, which is put above the first term of the dividend as shown.

(iii) The divisor is then multiplied by $2x$, i.e. $2x(x - 1) = 2x^2 - 2x$, which is placed under the dividend as shown. Subtracting gives $3x - 3$.

(iv) The process is then repeated, i.e. the first term of the divisor, x, is divided into $3x$, giving $+3$, which is placed above the dividend as shown.

(v) Then $3(x - 1) = 3x - 3$, which is placed under the $3x - 3$. The remainder, on subtraction, is zero, which completes the process.

Thus, $\mathbf{(2x^2 + x - 3) \div (x - 1) = (2x + 3)}$.

(A check can be made on this answer by multiplying $(2x + 3)$ by $(x - 1)$, which equals $2x^2 + x - 3$.)

Problem 15. Simplify $\dfrac{x^3 + y^3}{x + y}$

(i) (iv) (vii)

$$
\begin{array}{r|l}
 & x^2 - xy + y^2 \\
\hline
x+y & x^3 + 0 \quad + 0 \quad + y^3 \\
 & x^3 + x^2y \\
\hline
 & -x^2y \qquad + y^3 \\
 & -x^2y - xy^2 \\
\hline
 & xy^2 + y^3 \\
 & xy^2 + y^3 \\
\hline
 & \cdot \quad \cdot \\
\hline
\end{array}
$$

(i) x into x^3 goes x^2. Put x^2 above x^3.

(ii) $x^2(x+y) = x^3 + x^2y$

(iii) Subtract.

(iv) x into $-x^2y$ goes $-xy$. Put $-xy$ above the dividend.

(v) $-xy(x+y) = -x^2y - xy^2$

(vi) Subtract.

(vii) x into xy^2 goes y^2. Put y^2 above the dividend.

(viii) $y^2(x+y) = xy^2 + y^3$

(ix) Subtract.

Thus, $\boldsymbol{\dfrac{x^3+y^3}{x+y} = x^2 - xy + y^2}$.

The zeros shown in the dividend are not normally shown, but are included to clarify the subtraction process and to keep similar terms in their respective columns.

Problem 16. Divide $4a^3 - 6a^2b + 5b^3$ by $2a - b$

$$
\begin{array}{r|l}
 & 2a^2 - 2ab - b^2 \\
\hline
2a-b & 4a^3 - 6a^2b \qquad + 5b^3 \\
 & 4a^3 - 2a^2b \\
\hline
 & -4a^2b \qquad + 5b^3 \\
 & -4a^2b + 2ab^2 \\
\hline
 & -2ab^2 + 5b^3 \\
 & -2ab^2 + \ \ b^3 \\
\hline
 & 4b^3 \\
\hline
\end{array}
$$

Thus, $\boldsymbol{\dfrac{4a^3 - 6a^2b + 5b^3}{2a-b} = 2a^2 - 2ab - b^2}$, **remainder $4b^3$**.

Alternatively, the answer may be expressed as

$$\boldsymbol{\frac{4a^3 - 6a^2b + 5b^3}{2a-b} = 2a^2 - 2ab - b^2 + \frac{4b^3}{2a-b}}$$

Now try the following Practice Exercise

Practice Exercise 36 Basic operations in algebra (answers on page 343)

1. Simplify $pq \times pq^2r$.
2. Simplify $-4a \times -2a$.
3. Simplify $3 \times -2q \times -q$.
4. Evaluate $3pq - 5qr - pqr$ when $p = 3$, $q = -2$ and $r = 4$.
5. Determine the value of $3x^2yz^3$, given that $x = 2$, $y = 1\frac{1}{2}$ and $z = \frac{2}{3}$
6. If $x = 5$ and $y = 6$, evaluate $\dfrac{23(x-y)}{y + xy + 2x}$
7. If $a = 4, b = 3, c = 5$ and $d = 6$, evaluate $\dfrac{3a+2b}{3c-2d}$
8. Simplify $2x \div 14xy$.
9. Simplify $\dfrac{25x^2yz^3}{5xyz}$
10. Multiply $3a - b$ by $a + b$.
11. Multiply $2a - 5b + c$ by $3a + b$.
12. Simplify $3a \div 9ab$.
13. Simplify $4a^2b \div 2a$.
14. Divide $6x^2y$ by $2xy$.
15. Divide $2x^2 + xy - y^2$ by $x + y$.
16. Divide $3p^2 - pq - 2q^2$ by $p - q$.
17. Simplify $(a+b)^2 + (a-b)^2$.

9.3 Laws of indices

The laws of indices with numbers were covered in Chapter 7; the laws of indices in algebraic terms are as follows:

(1) $\boldsymbol{a^m \times a^n = a^{m+n}}$

For example, $a^3 \times a^4 = a^{3+4} = a^7$