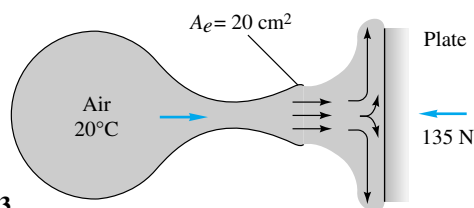


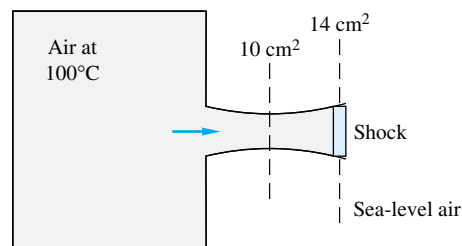
the density is 0.32 kg/m^3 . Assuming isentropic flow, (a) find the mass flow. (b) Is the flow choked? If so, estimate A^* . Also estimate (c) p_1 and (d) Ma_1 .

- P9.52** A converging-diverging nozzle exits smoothly to sea-level standard atmosphere. It is supplied by a 40-m^3 tank initially at 800 kPa and 100°C . Assuming isentropic flow in the nozzle, estimate (a) the throat area and (b) the tank pressure after 10 s of operation. The exit area is 10 cm^2 .
- P9.53** Air flows steadily from a reservoir at 20°C through a nozzle of exit area 20 cm^2 and strikes a vertical plate as in Fig. P9.53. The flow is subsonic throughout. A force of 135 N is required to hold the plate stationary. Compute (a) V_e , (b) Ma_e , and (c) p_0 if $p_a = 101 \text{ kPa}$.



P9.53

- P9.54** For flow of air through a normal shock the upstream conditions are $V_1 = 600 \text{ m/s}$, $T_{01} = 500 \text{ K}$, and $p_{01} = 700 \text{ kPa}$. Compute the downstream conditions Ma_2 , V_2 , T_2 , p_2 , and p_{02} .
- P9.55** Air, supplied by a reservoir at 450 kPa , flows through a converging-diverging nozzle whose throat area is 12 cm^2 . A normal shock stands where $A_1 = 20 \text{ cm}^2$. (a) Compute the pressure just downstream of this shock. Still farther downstream, at $A_3 = 30 \text{ cm}^2$, estimate (b) p_3 , (c) A_3^* , and (d) Ma_3 .
- P9.56** Air from a reservoir at 20°C and 500 kPa flows through a duct and forms a normal shock downstream of a throat of area 10 cm^2 . By an odd coincidence it is found that the stagnation pressure downstream of this shock exactly equals the throat pressure. What is the area where the shock wave stands?
- P9.57** Air flows from a tank through a nozzle into the standard atmosphere, as in Fig. P9.57. A normal shock stands in the exit of the nozzle, as shown. Estimate (a) the pressure in the tank and (b) the mass flow.



P9.57

P9.58 Argon (Table A.4) approaches a normal shock with $V_1 = 700 \text{ m/s}$, $p_1 = 125 \text{ kPa}$, and $T_1 = 350 \text{ K}$. Estimate (a) V_2 and (b) p_2 . (c) What pressure p_2 would result if the same velocity change V_1 to V_2 were accomplished isentropically?

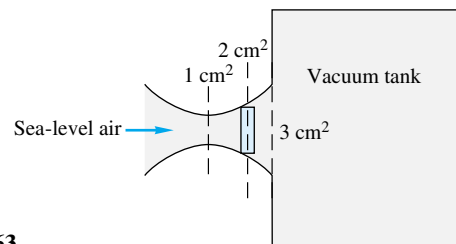
P9.59 Air, at stagnation conditions of 450 K and 250 kPa , flows through a nozzle. At section 1, where the area is 15 cm^2 , there is a normal shock wave. If the mass flow is 0.4 kg/s , estimate (a) the Mach number and (b) the stagnation pressure just downstream of the shock.

P9.60 When a pitot tube such as in Fig. 6.30 is placed in a supersonic flow, a normal shock will stand in front of the probe. Suppose the probe reads $p_0 = 190 \text{ kPa}$ and $p = 150 \text{ kPa}$. If the stagnation temperature is 400 K , estimate the (supersonic) Mach number and velocity upstream of the shock.

P9.61 Repeat Prob. 9.56 except this time let the odd coincidence be that the static pressure downstream of the shock exactly equals the throat pressure. What is the area where the shock stands?

P9.62 An atomic explosion propagates into still air at 14.7 lbf/in^2 absolute and 520°R . The pressure just inside the shock is 5000 lbf/in^2 absolute. Assuming $k = 1.4$, what are the speed C of the shock and the velocity V just inside the shock?

P9.63 Sea-level standard air is sucked into a vacuum tank through a nozzle, as in Fig. P9.63. A normal shock stands where the nozzle area is 2 cm^2 , as shown. Estimate (a) the pressure in the tank and (b) the mass flow.

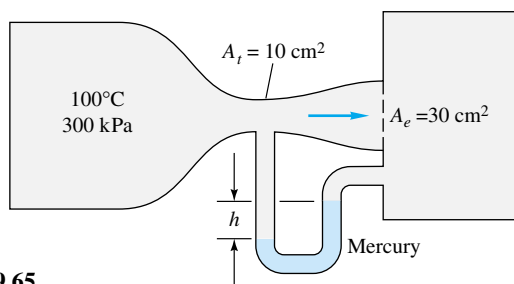


P9.63

P9.64 Air in a large tank at 100°C and 150 kPa exhausts to the atmosphere through a converging nozzle with a 5-cm^2 throat area. Compute the exit mass flow if the atmospheric pressure is (a) 100 kPa , (b) 60 kPa , and (c) 30 kPa .

P9.65 Air flows through a converging-diverging nozzle between two large reservoirs, as shown in Fig. P9.65. A mercury manometer between the throat and the downstream reservoir reads $h = 15 \text{ cm}$. Estimate the downstream reservoir pressure. Is there a normal shock in the flow? If so, does it stand in the exit plane or farther upstream?

P9.66 In Prob. 9.65 what would be the mercury-manometer reading h if the nozzle were operating exactly at supersonic design conditions?

**P9.65**

P9.67 In Prob. 9.65 estimate the complete range of manometer readings h for which the flow through the nozzle is entirely isentropic, except possibly in the exit plane.

P9.68 Air in a tank at 120 kPa and 300 K exhausts to the atmosphere through a 5-cm²-throat converging nozzle at a rate of 0.12 kg/s. What is the atmospheric pressure? What is the maximum mass flow possible at low atmospheric pressure?

P9.69 With reference to Prob. 3.68, show that the thrust of a rocket engine exhausting into a vacuum is given by

$$F = \frac{p_0 A_e (1 + k \text{Ma}_e^2)}{\left(1 + \frac{k-1}{2} \text{Ma}_e^2\right)^{k/(k-1)}}$$

where A_e = exit area

Ma_e = exit Mach number

p_0 = stagnation pressure in combustion chamber

Note that stagnation temperature does not enter into the thrust.

P9.70 Air, at stagnation temperature 100°C, expands isentropically through a nozzle of 6-cm² throat area and 18-cm² exit area. The mass flow is at its maximum value of 0.5 kg/s. Estimate the exit pressure for (a) subsonic and (b) supersonic exit flow.

P9.71 For the nozzle of Prob. 9.70, allowing for nonisentropic flow, what is the range of exit tank pressures p_b for which (a) the diverging nozzle flow is fully supersonic, (b) the exit flow is subsonic, (c) the mass flow is independent of p_b , (d) the exit plane pressure p_e is independent of p_b , and (e) $p_e < p_b$?

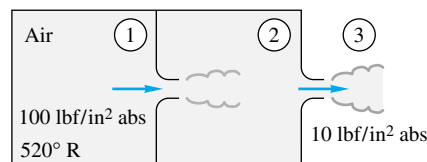
P9.72 Suppose the nozzle flow of Prob. 9.70 is not isentropic but instead has a normal shock at the position where area is 15 cm². Compute the resulting mass flow, exit pressure, and exit Mach number.

P9.73 Air flows isentropically in a converging-diverging nozzle with a throat area of 3 cm². At section 1, the pressure is 101 kPa, the temperature is 300 K, and the velocity is 868 m/s. (a) Is the nozzle choked? Determine (b) A_1 and (c) the mass flow. Suppose, without changing stagnation conditions or A_1 , the (flexible) throat is reduced to 2 cm². As-

suming shock-free flow, will there be any change in the gas properties at section 1? If so, compute new p_1 , V_1 , and T_1 and explain.

P9.74 The perfect-gas assumption leads smoothly to Mach-number relations which are very convenient (and tabulated). This is not so for a real gas such as steam. To illustrate, let steam at $T_0 = 500^\circ\text{C}$ and $p_0 = 2$ MPa expand isentropically through a converging nozzle whose exit area is 10 cm². Using the steam tables, find (a) the exit pressure and (b) the mass flow when the flow is sonic, or choked. What complicates the analysis?

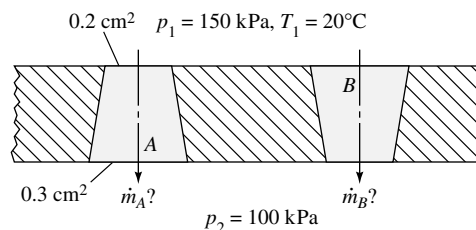
***P9.75** A double-tank system in Fig. P9.75 has two identical converging nozzles of 1-in² throat area. Tank 1 is very large, and tank 2 is small enough to be in steady-flow equilibrium with the jet from tank 1. Nozzle flow is isentropic, but entropy changes between 1 and 3 due to jet dissipation in tank 2. Compute the mass flow. (If you give up, Ref. 14, pp. 288–290, has a good discussion.)

**P9.75**

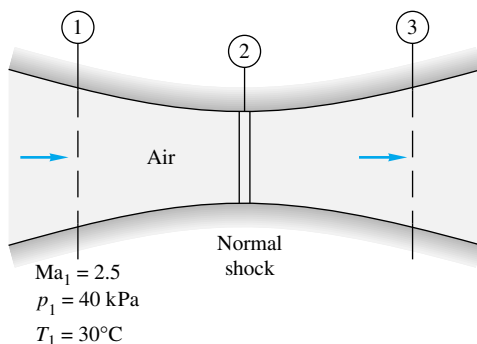
P9.76 A large reservoir at 20°C and 800 kPa is used to fill a small insulated tank through a converging-diverging nozzle with 1-cm² throat area and 1.66-cm² exit area. The small tank has a volume of 1 m³ and is initially at 20°C and 100 kPa. Estimate the elapsed time when (a) shock waves begin to appear inside the nozzle and (b) the mass flow begins to drop below its maximum value.

P9.77 A perfect gas (not air) expands isentropically through a supersonic nozzle with an exit area 5 times its throat area. The exit Mach number is 3.8. What is the specific-heat ratio of the gas? What might this gas be? If $p_0 = 300$ kPa, what is the exit pressure of the gas?

P9.78 The orientation of a hole can make a difference. Consider holes A and B in Fig. P9.78, which are identical but reversed. For the given air properties on either side, compute the mass flow through each hole and explain why they are different.

**P9.78**

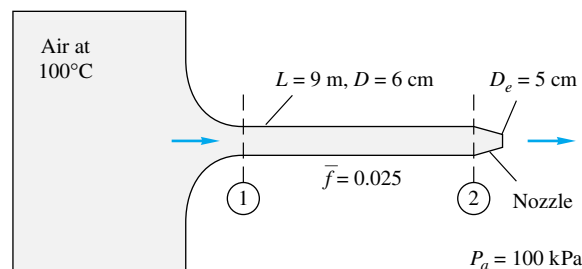
- P9.79** Air with $p_0 = 300$ kPa and $T_0 = 500$ K flows through a converging-diverging nozzle with throat area of 1 cm^2 and exit area of 3 cm^2 into a receiver tank. The mass flow is 195.2 kg/h . For what range of receiver pressure is this mass flow possible?
- P9.80** A sea-level automobile tire is initially at 32 lbf/in^2 gage pressure and 75°F . When it is punctured with a hole which resembles a converging nozzle, its pressure drops to 15 lbf/in^2 gage in 12 min. Estimate the size of the hole, in thousandths of an inch. The tire volume is 2.5 ft^3 .
- P9.81** Helium, in a large tank at 100°C and 400 kPa , discharges to a receiver through a converging-diverging nozzle designed to exit at $\text{Ma} = 2.5$ with exit area 1.2 cm^2 . Compute (a) the receiver pressure and (b) the mass flow at design conditions. (c) Also estimate the range of receiver pressures for which mass flow will be a maximum.
- P9.82** Air at 500 K flows through a converging-diverging nozzle with throat area of 1 cm^2 and exit area of 2.7 cm^2 . When the mass flow is 182.2 kg/h , a pitot-static probe placed in the exit plane reads $p_0 = 250.6\text{ kPa}$ and $p = 240.1\text{ kPa}$. Estimate the exit velocity. Is there a normal shock wave in the duct? If so, compute the Mach number just downstream of this shock.
- P9.83** When operating at design conditions (smooth exit to sea-level pressure), a rocket engine has a thrust of 1 million lbf. The chamber pressure and temperature are 600 lbf/in^2 absolute and 4000°R , respectively. The exhaust gases approximate $k = 1.38$ with a molecular weight of 26. Estimate (a) the exit Mach number and (b) the throat diameter.
- P9.84** Air flows through a duct as in Fig. P9.84, where $A_1 = 24\text{ cm}^2$, $A_2 = 18\text{ cm}^2$, and $A_3 = 32\text{ cm}^2$. A normal shock stands at section 2. Compute (a) the mass flow, (b) the Mach number, and (c) the stagnation pressure at section 3.

**P9.84**

- P9.85** A large tank delivers air through a nozzle of 1-cm^2 throat area and 2.7-cm^2 exit area. When the receiver pressure is 125 kPa , a normal shock stands in the exit plane. Estimate

(a) the throat pressure and (b) the stagnation pressure in the upstream supply tank.

- P9.86** Air enters a 3-cm-diameter pipe 15 m long at $V_1 = 73\text{ m/s}$, $p_1 = 550\text{ kPa}$, and $T_1 = 60^\circ\text{C}$. The friction factor is 0.018 . Compute V_2 , p_2 , T_2 , and p_{02} at the end of the pipe. How much additional pipe length would cause the exit flow to be sonic?
- P9.87** Air enters a duct of $L/D = 40$ at $V_1 = 170\text{ m/s}$ and $T_1 = 300\text{ K}$. The flow at the exit is choked. What is the average friction factor in the duct for adiabatic flow?
- P9.88** Air enters a 5- by 5-cm square duct at $V_1 = 900\text{ m/s}$ and $T_1 = 300\text{ K}$. The friction factor is 0.02 . For what length duct will the flow exactly decelerate to $\text{Ma} = 1.0$? If the duct length is 2 m , will there be a normal shock in the duct? If so, at what Mach number will it occur?
- P9.89** Air flows adiabatically in a 5-cm-diameter tube with $f \approx 0.025$. At section 1, $V_1 = 75\text{ m/s}$, $T_1 = 350\text{ K}$, and $p_1 = 300\text{ kPa}$. How much further down the tube will (a) the pressure be 156 kPa , (b) the temperature be 343 K , and (c) the flow reach the choking point?
- P9.90** Air, supplied at $p_0 = 700\text{ kPa}$ and $T_0 = 330\text{ K}$, flows through a converging nozzle into a pipe of 2.5-cm diameter which exits to a near vacuum. If $\bar{f} = 0.022$, what will be the mass flow through the pipe if its length is (a) 0 m , (b) 1 m , and (c) 10 m ?
- P9.91** Air flows steadily from a tank through the pipe in Fig. P9.91. There is a converging nozzle on the end. If the mass flow is 3 kg/s and the nozzle is choked, estimate (a) the Mach number at section 1 and (b) the pressure inside the tank.

**P9.91**

- P9.92** Modify Prob. 9.91 as follows. Let the pressure in the tank be 700 kPa , and let the nozzle be choked. Determine (a) Ma_2 and (b) the mass flow.
- P9.93** Air flows adiabatically in a 3-cm-diameter duct. The average friction factor is 0.015 . If, at the entrance, $V = 950\text{ m/s}$ and $T = 250\text{ K}$, how far down the tube will (a) the Mach number be 1.8 or (b) the flow be choked?
- P9.94** Compressible pipe flow with friction, Sec. 9.7, assumes constant stagnation enthalpy and mass flow but variable

momentum. Such a flow is often called *Fanno flow*, and a line representing all possible property changes on a temperature-entropy chart is called a *Fanno line*. Assuming a perfect gas with $k = 1.4$ and the data of Prob. 9.86, draw a Fanno curve of the flow for a range of velocities from very low ($Ma \ll 1$) to very high ($Ma \gg 1$). Comment on the meaning of the maximum-entropy point on this curve.

P9.95 Helium (Table A.4) enters a 5-cm-diameter pipe at $p_1 = 550$ kPa, $V_1 = 312$ m/s, and $T_1 = 40^\circ\text{C}$. The friction factor is 0.025. If the flow is choked, determine (a) the length of the duct and (b) the exit pressure.

P9.96 Derive and verify the adiabatic-pipe-flow velocity relation of Eq. (9.74), which is usually written in the form

$$\frac{\bar{f}L}{D} + \frac{k+1}{k} \ln \frac{V_2}{V_1} = \frac{a_0^2}{k} \left(\frac{1}{V_1^2} - \frac{1}{V_2^2} \right)$$

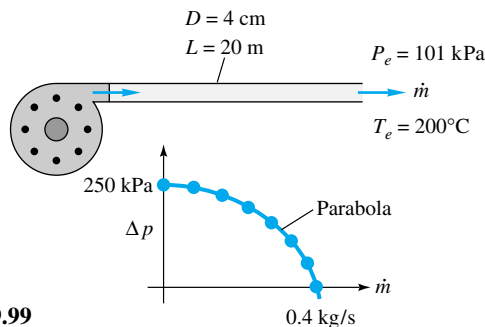
P9.97 By making a few algebraic substitutions, show that Eq. (9.74), or the relation in Prob. 9.96, may be written in the density form

$$\rho_1^2 = \rho_2^2 + \rho^{*2} \left(\frac{2k}{k+1} \frac{\bar{f}L}{D} + 2 \ln \frac{\rho_1}{\rho_2} \right)$$

Why is this formula awkward if one is trying to solve for the mass flow when the pressures are given at sections 1 and 2?

P9.98 Compressible *laminar* flow, $f \approx 64/\text{Re}$, may occur in capillary tubes. Consider air, at stagnation conditions of 100°C and 200 kPa, entering a tube 3 cm long and 0.1 mm in diameter. If the receiver pressure is near vacuum, estimate (a) the average Reynolds number, (b) the Mach number at the entrance, and (c) the mass flow in kg/h.

P9.99 A compressor forces air through a smooth pipe 20 m long and 4 cm in diameter, as in Fig. P9.99. The air leaves at 101 kPa and 200°C . The compressor data for pressure rise versus mass flow are shown in the figure. Using the Moody chart to estimate \bar{f} , compute the resulting mass flow.

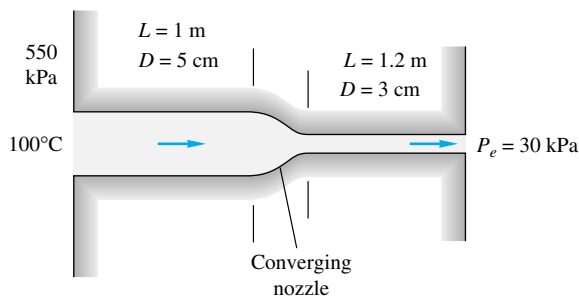


P9.99

P9.100 Modify Prob. 9.99 as follows. Find the length of 4-cm-diameter pipe for which the pump pressure rise will be exactly 200 kPa.

P9.101 How do the compressible-pipe-flow formulas behave for small pressure drops? Let air at 20°C enter a tube of diameter 1 cm and length 3 m. If $\bar{f} = 0.028$ with $p_1 = 102$ kPa and $p_2 = 100$ kPa, estimate the mass flow in kg/h for (a) isothermal flow, (b) adiabatic flow, and (c) incompressible flow (Chap. 6) at the entrance density.

P9.102 Air at 550 kPa and 100°C enters a smooth 1-m-long pipe and then passes through a second smooth pipe to a 30-kPa reservoir, as in Fig. P9.102. Using the Moody chart to compute \bar{f} , estimate the mass flow through this system. Is the flow choked?



P9.102

P9.103 Natural gas, with $k \approx 1.3$ and a molecular weight of 16, is to be pumped through 100 km of 81-cm-diameter pipeline. The downstream pressure is 150 kPa. If the gas enters at 60°C , the mass flow is 20 kg/s, and $\bar{f} = 0.024$, estimate the required entrance pressure for (a) isothermal flow and (b) adiabatic flow.

P9.104 A tank of oxygen (Table A.4) at 20°C is to supply an astronaut through an umbilical tube 12 m long and 2 cm in diameter. The exit pressure in the tube is 40 kPa. If the desired mass flow is 90 kg/h and $\bar{f} = 0.025$, what should be the pressure in the tank?

P9.105 Air enters a 5-cm-diameter pipe at $p_1 = 200$ kPa and $T_1 = 350$ K. The downstream receiver pressure is 74 kPa. The friction factor is 0.02. If the exit is choked, what is (a) the length of the pipe and (b) the mass flow? (c) If p_1 , T_1 , and p_{receiver} stay the same, what pipe length will cause the mass flow to increase by 50 percent over (b)? *Hint:* In part (c) the exit pressure does not equal the receiver pressure.

P9.106 Air at 300 K flows through a duct 50 m long with $\bar{f} = 0.019$. What is the minimum duct diameter which can carry the flow without choking if the entrance velocity is (a) 50 m/s, (b) 150 m/s, and (c) 420 m/s?

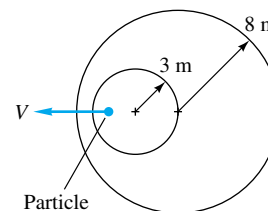
P9.107 A fuel-air mixture, assumed equivalent to air, enters a duct combustion chamber at $V_1 = 104$ m/s and $T_1 = 300$ K. What amount of heat addition in kJ/kg will cause the exit flow to be choked? What will be the exit Mach number and temperature if 504 kJ/kg is added during combustion?

- P9.108** What happens to the inlet flow of Prob. 9.107 if the combustion yields 1500 kJ/kg heat addition and p_{01} and T_{01} remain the same? How much is the mass flow reduced?
- P9.109** A jet engine at 7000-m altitude takes in 45 kg/s of air and adds 550 kJ/kg in the combustion chamber. The chamber cross section is 0.5 m^2 , and the air enters the chamber at 80 kPa and 5°C . After combustion the air expands through an isentropic converging nozzle to exit at atmospheric pressure. Estimate (a) the nozzle throat diameter, (b) the nozzle exit velocity, and (c) the thrust produced by the engine.
- P9.110** Compressible pipe flow with heat addition, Sec. 9.8, assumes constant momentum ($p + \rho V^2$) and constant mass flow but variable stagnation enthalpy. Such a flow is often called *Rayleigh flow*, and a line representing all possible property changes on a temperature-entropy chart is called a *Rayleigh line*. Assuming air passing through the flow state $p_1 = 548 \text{ kPa}$, $T_1 = 588 \text{ K}$, $V_1 = 266 \text{ m/s}$, and $A = 1 \text{ m}^2$, draw a Rayleigh curve of the flow for a range of velocities from very low ($\text{Ma} \ll 1$) to very high ($\text{Ma} \gg 1$). Comment on the meaning of the maximum-entropy point on this curve.
- P9.111** Add to your Rayleigh line of Prob. 9.110 a Fanno line (see Prob. 9.94) for stagnation enthalpy equal to the value associated with state 1 in Prob. 9.110. The two curves will intersect at state 1, which is subsonic, and at a certain state 2, which is supersonic. Interpret these two states vis-à-vis Table B.2.
- P9.112** Air enters a duct subsonically at section 1 at 1.2 kg/s. When 650 kW of heat is added, the flow chokes at the exit at $p_2 = 95 \text{ kPa}$ and $T_2 = 700 \text{ K}$. Assuming frictionless heat addition, estimate (a) the velocity and (b) the stagnation pressure at section 1.
- P9.113** Air enters a constant-area duct at $p_1 = 90 \text{ kPa}$, $V_1 = 520 \text{ m/s}$, and $T_1 = 558^\circ\text{C}$. It is then cooled with negligible friction until it exits at $p_2 = 160 \text{ kPa}$. Estimate (a) V_2 , (b) T_2 , and (c) the total amount of cooling in kJ/kg.
- P9.114** We have simplified things here by separating friction (Sec. 9.7) from heat addition (Sec. 9.8). Actually, they often occur together, and their effects must be evaluated simultaneously. Show that, for flow with friction and heat transfer in a constant-diameter pipe, the continuity, momentum, and energy equations may be combined into the following differential equation for Mach-number changes:

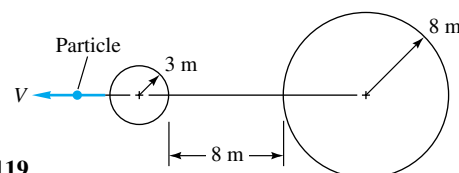
$$\frac{d \text{Ma}^2}{\text{Ma}^2} = \frac{1 + k \text{Ma}^2}{1 - \text{Ma}^2} \frac{dQ}{c_p T} + \frac{k \text{Ma}^2 [2 + (k - 1) \text{Ma}^2]}{2(1 - \text{Ma}^2)} \frac{f dx}{D}$$

where dQ is the heat added. A complete derivation, including many additional combined effects such as area change and mass addition, is given in chap. 8 of Ref. 8.

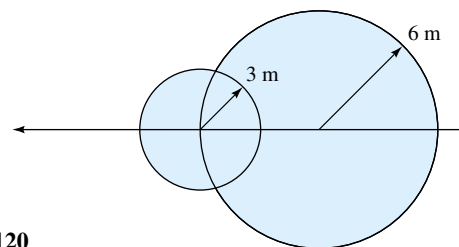
- P9.115** Air flows subsonically in a duct with negligible friction. When heat is added in the amount of 948 kJ/kg, the pressure drops from $p_1 = 200$ to $p_2 = 106 \text{ kPa}$. Estimate (a) Ma_1 , (b) T_1 , and (c) V_1 , assuming $T_{01} = 305 \text{ K}$.
- P9.116** An observer at sea level does not hear an aircraft flying at 12,000-ft standard altitude until it is 5 (statute) mi past her. Estimate the aircraft speed in ft/s.
- P9.117** An observer at sea level does not hear an aircraft flying at 6000-m standard altitude until 15 s after it has passed overhead. Estimate the aircraft speed in m/s.
- P9.118** A particle moving at uniform velocity in sea-level standard air creates the two disturbance spheres shown in Fig. P9.118. Compute the particle velocity and Mach number.



- P9.119** The particle in Fig. P9.119 is moving supersonically in sea-level standard air. From the two given disturbance spheres, compute the particle Mach number, velocity, and Mach angle.



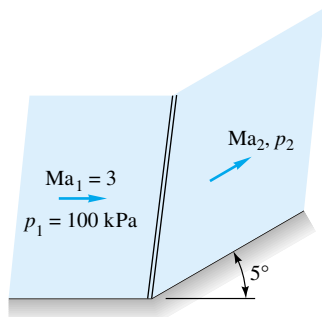
- P9.120** The particle in Fig. P9.120 is moving in sea-level standard air. From the two disturbance spheres shown, estimate (a) the position of the particle at this instant and (b) the temperature in $^\circ\text{C}$ at the front stagnation point of the particle.



- P9.121** A thermistor probe, in the shape of a needle parallel to the flow, reads a static temperature of -25°C when inserted into a supersonic airstream. A conical disturbance cone of half-angle 17° is created. Estimate (a) the Mach number,

(b) the velocity, and (c) the stagnation temperature of the stream.

- P9.122** Supersonic air takes a 5° compression turn, as in Fig. P9.122. Compute the downstream pressure and Mach number and the wave angle, and compare with small-disturbance theory.



P9.122

- P9.123** Modify Prob. 9.122 as follows. Let the 5° total turn be in the form of five separate compression turns of 1° each. Compute the final Mach number and pressure, and compare the pressure with an isentropic expansion to the same final Mach number.

- P9.124** Determine the validity of the following alternate relation for the pressure ratio across an oblique shock wave:

$$\frac{p_2}{p_1} = \frac{\cot \theta \sin 2\beta - \cos 2\beta + k}{\cot \theta \sin 2\beta - \cos 2\beta - k}$$

If necessary, your proof (or disproof) may be somewhat tentative and heuristic.

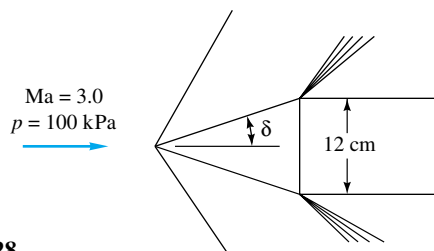
- P9.125** Show that, as the upstream Mach number approaches infinity, the Mach number downstream of an attached oblique-shock wave will have the value

$$Ma_2 \approx \sqrt{\frac{k-1}{2k \sin^2(\beta - \theta)}}$$

- P9.126** Consider airflow at $Ma_1 = 2.2$. Calculate, to two decimal places, (a) the deflection angle for which the downstream flow is sonic and (b) the maximum deflection angle.

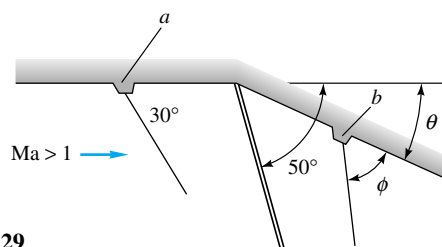
- P9.127** Do the Mach waves upstream of an oblique-shock wave intersect with the shock? Assuming supersonic downstream flow, do the downstream Mach waves intersect the shock? Show that for small deflections the shock-wave angle β lies halfway between μ_1 and $\mu_2 + \theta$ for any Mach number.

- P9.128** Air flows past a two-dimensional wedge-nosed body as in Fig. P9.128. Determine the wedge half-angle δ for which the horizontal component of the total pressure force on the nose is 35 kN/m of depth into the paper.



P9.128

- P9.129** Air flows at supersonic speed toward a compression ramp, as in Fig. P9.129. A scratch on the wall at point *a* creates a wave of 30° angle, while the oblique shock created has a 50° angle. What is (a) the ramp angle θ and (b) the wave angle ϕ caused by a scratch at *b*?

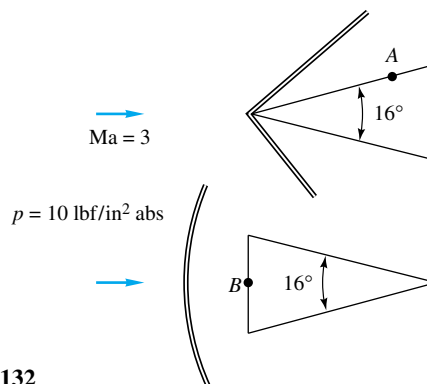


P9.129

- P9.130** Modify Prob. 9.129 as follows. If the wave angle ϕ is 42° , determine (a) the shock-wave angle and (b) the deflection angle.

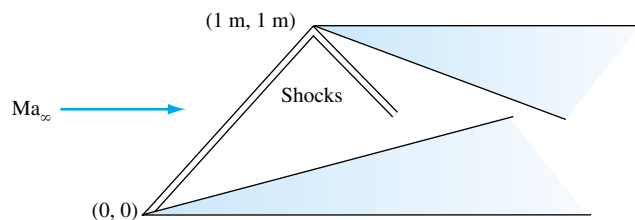
- P9.131** In Fig. P9.128, assume that the approach stream temperature is 20°C . For what wedge half-angle δ will the stream temperature along the wedge surface be 200°C ?

- P9.132** Air flows at $Ma = 3$ and $p = 10 \text{ lbf/in}^2$ absolute toward a wedge of 16° angle at zero incidence in Fig. P9.132. If the pointed edge is forward, what will be the pressure at point *A*? If the blunt edge is forward, what will be the pressure at point *B*?



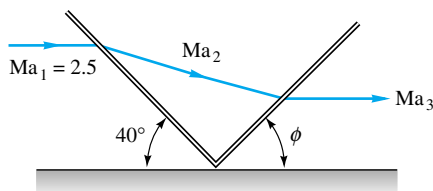
P9.132

- P9.133** Air flows supersonically toward the double-wedge system in Fig. P9.133. The (x, y) coordinates of the tips are given.

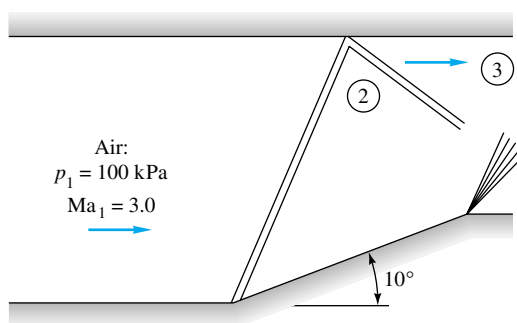
**P9.133**

The shock wave of the forward wedge strikes the tip of the aft wedge. Both wedges have 15° deflection angles. What is the free-stream Mach number?

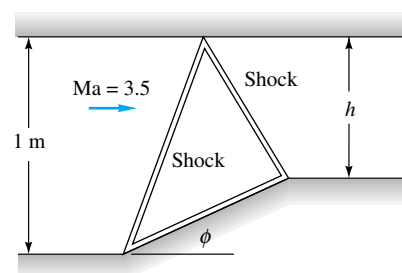
- P9.134** When an oblique shock strikes a solid wall, it reflects as a shock of sufficient strength to cause the exit flow Ma_3 to be parallel to the wall, as in Fig. P9.134. For airflow with $Ma_1 = 2.5$ and $p_1 = 100$ kPa, compute Ma_3 , p_3 , and the angle ϕ .

**P9.134**

- P9.135** A bend in the bottom of a supersonic duct flow induces a shock wave which reflects from the upper wall, as in Fig. P9.135. Compute the Mach number and pressure in region 3.

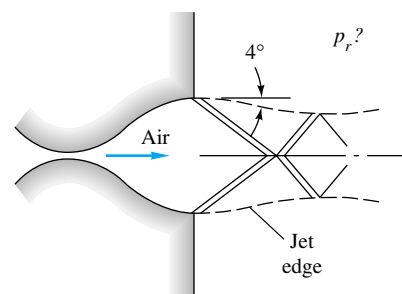
**P9.135**

- P9.136** Figure P9.136 is a special application of Prob. 9.135. With careful design, one can orient the bend on the lower wall so that the reflected wave is exactly canceled by the return bend, as shown. This is a method of reducing the Mach number in a channel (a supersonic diffuser). If the bend angle is $\phi = 10^\circ$, find (a) the downstream width h and (b) the downstream Mach number. Assume a weak shock wave.

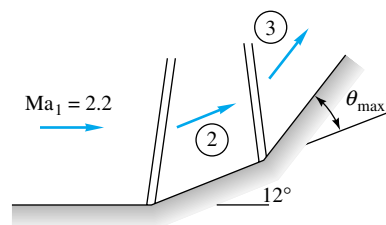
**P9.136**

- P9.137** Generalize Prob. 9.136 into a computer study as follows. Assuming weak shocks, find and plot all combinations of ϕ and h in Fig. P9.136 for which the canceled or “swallowed” reflected shock is possible.

- P9.138** The supersonic nozzle of Fig. P9.138 is overexpanded (case G of Fig. 9.12) with $A_e/A_t = 3.0$ and a stagnation pressure of 350 kPa. If the jet edge makes a 4° angle with the nozzle centerline, what is the back pressure p_r in kPa?

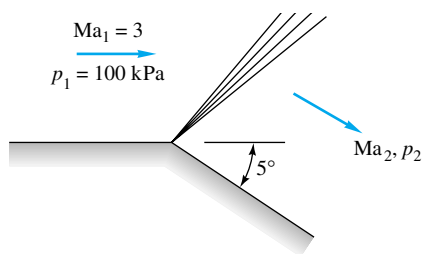
**P9.138**

- P9.139** Airflow at $Ma = 2.2$ takes a compression turn of 12° and then another turn of angle θ in Fig. P9.139. What is the maximum value of θ for the second shock to be attached? Will the two shocks intersect for any θ less than θ_{\max} ?

**P9.139**

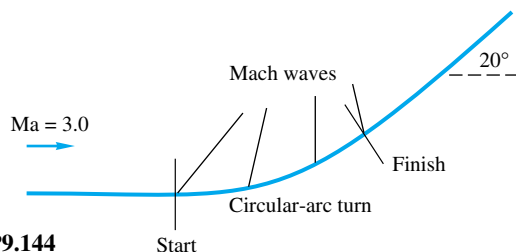
- P9.140** The solution to Prob. 9.122 is $Ma_2 = 2.750$, and $p_2 = 145.5$ kPa. Compare these results with an isentropic compression turn of 5° , using Prandtl-Meyer theory.

- P9.141** Supersonic airflow takes a 5° expansion turn, as in Fig. P9.141. Compute the downstream Mach number and pressure, and compare with small-disturbance theory.



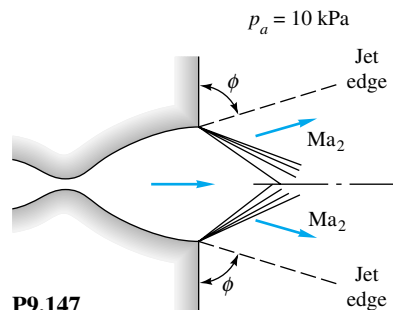
P9.141

- P9.142** A supersonic airflow at $Ma_1 = 3.2$ and $p_1 = 50$ kPa undergoes a compression shock followed by an isentropic expansion turn. The flow deflection is 30° for each turn. Compute Ma_2 and p_2 if (a) the shock is followed by the expansion and (b) the expansion is followed by the shock.
- P9.143** Airflow at $Ma_1 = 3.2$ passes through a 25° oblique-shock deflection. What isentropic expansion turn is required to bring the flow back to (a) Ma_1 and (b) p_1 ?
- P9.144** Consider a smooth isentropic compression turn of 20° , as shown in Fig. P9.144. The Mach waves thus generated will form a converging fan. Sketch this fan as accurately as possible, using at least five equally spaced waves, and demonstrate how the fan indicates the probable formation of an oblique-shock wave.



P9.144

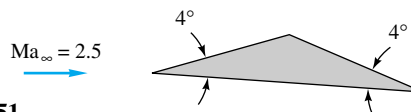
- P9.145** Air at $Ma_1 = 2.0$ and $p_1 = 100$ kPa undergoes an isentropic expansion to a downstream pressure of 50 kPa. What is the desired turn angle in degrees?
- P9.146** Helium, at 20°C and $V_1 = 2010$ m/s, undergoes a Prandtl-Meyer expansion until the temperature is -50°C . Estimate the turn angle in degrees.
- P9.147** A converging-diverging nozzle with a 4:1 exit-area ratio and $p_0 = 500$ kPa operates in an underexpanded condition (case I of Fig. 9.12b) as in Fig. P9.147. The receiver pressure is $p_a = 10$ kPa, which is less than the exit pressure, so that expansion waves form outside the exit. For the given conditions, what will the Mach number Ma_2 and the angle ϕ of the edge of the jet be? Assume $k = 1.4$ as usual.
- P9.148** Repeat Example 9.19 for an angle of attack of 6° . Is the lift coefficient linear with angle α in this range of $0^\circ \leq \alpha \leq 8^\circ$? Is the drag coefficient parabolic with α in this range?
- P9.149** Repeat Example 9.21 for an angle of attack of 2° . Is the lift coefficient linear with angle α in this range of $0^\circ \leq \alpha$



P9.147

$\leq 8^\circ$? Why does the drag coefficient not have the simple parabolic form $C_D \approx K\alpha^2$ in this range?

- P9.150** A flat-plate airfoil with $C = 1.2$ m is to have a lift of 30 kN/m when flying at 5000-m standard altitude with $U_\infty = 641$ m/s. Using Ackeret theory, estimate (a) the angle of attack and (b) the drag force in N/m.
- P9.151** Air flows at $Ma = 2.5$ past a half-wedge airfoil whose angles are 4° , as in Fig. P9.151. Compute the lift and drag coefficient at α equal to (a) 0° and (b) 6° .



P9.151

- P9.152** A supersonic airfoil has a parabolic symmetric shape for upper and lower surfaces

$$y_{u,l} = \pm 2t \left(\frac{x}{C} - \frac{x^2}{C^2} \right)$$

such that the maximum thickness is t at $x = \frac{1}{2}C$. Compute the drag coefficient at zero incidence by Ackeret theory, and compare with a symmetric double wedge of the same thickness.

- P9.153** A supersonic transport has a mass of 65 Mg and cruises at 11-km standard altitude at a Mach number of 2.25. If the angle of attack is 2° and its wings can be approximated by flat plates, estimate (a) the required wing area in m^2 and (b) the thrust required in N.
- P9.154** A symmetric supersonic airfoil has its upper and lower surfaces defined by a sine-wave shape:

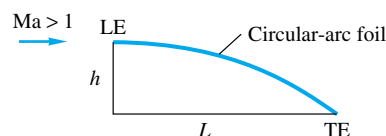
$$y = \frac{t}{2} \sin \frac{\pi x}{C}$$

where t is the maximum thickness, which occurs at $x = C/2$. Use Ackeret theory to derive an expression for the drag coefficient at zero angle of attack. Compare your result with Ackeret theory for a symmetric double-wedge airfoil of the same thickness.

- P9.155** For the sine-wave airfoil shape of Prob. 9.154, with $Ma_\infty = 2.5$, $k = 1.4$, $t/C = 0.1$, and $\alpha = 0^\circ$, plot (without com-

puting the overall forces) the pressure distribution $p(x)/p_\infty$ along the upper surface for (a) Ackeret theory and (b) an oblique shock plus a continuous Prandtl-Meyer expansion.

- P9.156** A thin circular-arc airfoil is shown in Fig. P9.156. The leading edge is parallel to the free stream. Using linearized (small-turning-angle) supersonic-flow theory, derive a formula for the lift and drag coefficient for this orientation, and compare with Ackeret-theory results for an angle of attack $\alpha = \tan^{-1}(h/L)$.



P9.156

- P9.157** Prove from Ackeret theory that for a given supersonic airfoil shape with sharp leading and trailing edges and a given thickness, the minimum-thickness drag occurs for a symmetric double-wedge shape.

Word Problems

- W9.1** Notice from Table 9.1 that (a) water and mercury and (b) aluminum and steel have nearly the same speeds of sound, yet the second of the two materials is much denser. Can you account for this oddity? Can molecular theory explain it?
- W9.2** When an object approaches you at $Ma = 0.8$, you can hear it, according to Fig. 9.18a. But would there be a Doppler shift? For example, would a musical tone seem to you to have a higher or a lower pitch?
- W9.3** The subject of this chapter is commonly called *gas dynamics*. But can liquids not perform in this manner? Using water as an example, make a rule-of-thumb estimate of the pressure level needed to drive a water flow at velocities comparable to the sound speed.
- W9.4** Suppose a gas is driven at compressible subsonic speeds by a large pressure drop, p_1 to p_2 . Describe its behavior on an appropriately labeled Mollier chart for (a) frictionless flow

in a converging nozzle and (b) flow with friction in a long duct.

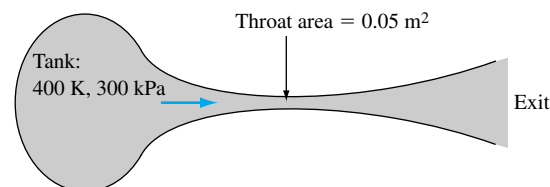
- W9.5** Describe physically what the “speed of sound” represents. What kind of pressure changes occur in air sound waves during ordinary conversation?
- W9.6** Give a physical description of the phenomenon of choking in a converging-nozzle gas flow. Could choking happen even if wall friction were not negligible?
- W9.7** Shock waves are treated as discontinuities here, but they actually have a very small finite thickness. After giving it some thought, sketch your idea of the distribution of gas velocity, pressure, temperature, and entropy through the inside of a shock wave.
- W9.8** Describe how an observer, running along a normal-shock wave at finite speed V , will see what appears to be an oblique-shock wave. Is there any limit to the running speed?

Fundamentals of Engineering Exam Problems

One-dimensional compressible-flow problems have become quite popular on the FE Exam, especially in the afternoon sessions. In the following problems, assume one-dimensional flow of ideal air, $R = 287 \text{ J/(kg}\cdot\text{K)}$ and $k = 1.4$.

- FE9.1** For steady isentropic flow, if the absolute temperature increases 50 percent, by what ratio does the static pressure increase?
(a) 1.12, (b) 1.22, (c) 2.25, (d) 2.76, (e) 4.13
- FE9.2** For steady isentropic flow, if the density doubles, by what ratio does the static pressure increase?
(a) 1.22, (b) 1.32, (c) 1.44, (d) 2.64, (e) 5.66
- FE9.3** A large tank, at 500 K and 200 kPa, supplies isentropic airflow to a nozzle. At section 1, the pressure is only 120 kPa. What is the Mach number at this section?
(a) 0.63, (b) 0.78, (c) 0.89, (d) 1.00, (e) 1.83
- FE9.4** In Prob. FE9.3 what is the temperature at section 1?
(a) 300 K, (b) 408 K, (c) 417 K, (d) 432 K, (e) 500 K

- FE9.5** In Prob. FE9.3, if the area at section 1 is 0.15 m^2 , what is the mass flow?
(a) 38.1 kg/s, (b) 53.6 kg/s, (c) 57.8 kg/s, (d) 67.8 kg/s, (e) 77.2 kg/s
- FE9.6** For steady isentropic flow, what is the maximum possible mass flow through the duct in Fig. FE9.6?
(a) 9.5 kg/s, (b) 15.1 kg/s, (c) 26.2 kg/s, (d) 30.3 kg/s, (e) 52.4 kg/s



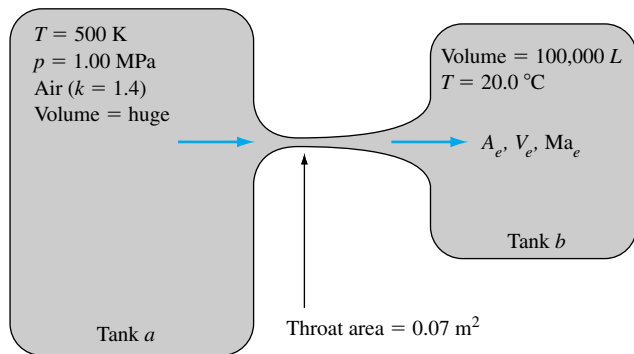
FE9.6

- FE9.7** If the exit Mach number in Fig. FE9.6 is 2.2, what is the exit area?
 (a) 0.10 m^2 , (b) 0.12 m^2 , (c) 0.15 m^2 , (d) 0.18 m^2 ,
 (e) 0.22 m^2
- FE9.8** If there are no shock waves and the pressure at one duct section in Fig. FE9.6 is 55.5 kPa, what is the velocity at that section?
 (a) 166 m/s, (b) 232 m/s, (c) 554 m/s, (d) 706 m/s,
 (e) 774 m/s

- FE9.9** If, in Fig. FE9.6, there is a normal shock wave at a section where the area is 0.07 m^2 , what is the air density just upstream of that shock?
 (a) 0.48 kg/m^3 , (b) 0.78 kg/m^3 , (c) 1.35 kg/m^3 ,
 (d) 1.61 kg/m^3 , (e) 2.61 kg/m^3
- FE9.10** In Prob. FE9.9, what is the Mach number just downstream of the shock wave?
 (a) 0.42, (b) 0.55, (c) 0.63, (d) 1.00, (e) 1.76

Comprehensive Problems

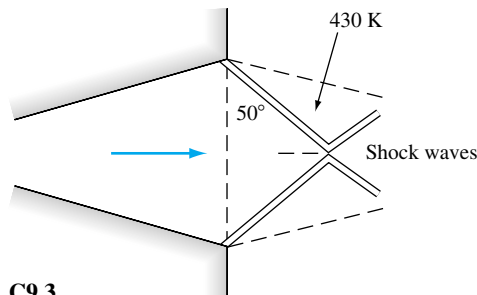
- C9.1** The converging-diverging nozzle sketched in Fig. C9.1 is designed to have a Mach number of 2.00 at the exit plane (assuming the flow remains nearly isentropic). The flow travels from tank *a* to tank *b*, where tank *a* is much larger than tank *b*. (a) Find the area at the exit A_e and the back pressure p_b which will allow the system to operate at design conditions. (b) As time goes on, the back pressure will grow, since the second tank slowly fills up with more air. Since tank *a* is huge, the flow in the nozzle will remain the same, however, until a normal shock wave appears at the exit plane. At what back pressure will this occur? (c) If tank *b* is held at constant temperature, $T = 20^\circ\text{C}$, estimate how long it will take for the flow to go from design conditions



C9.1

to the condition of part (b), i.e., with a shock wave at the exit plane.

- C9.2** Two large air tanks, one at 400 K and 300 kPa and the other at 300 K and 100 kPa, are connected by a straight tube 6 m long and 5 cm in diameter. The average friction factor is 0.0225. Assuming adiabatic flow, estimate the mass flow through the tube.
- *C9.3** Figure C9.3 shows the exit of a converging-diverging nozzle, where an oblique-shock pattern is formed. In the exit plane, which has an area of 15 cm^2 , the air pressure is 16 kPa and the temperature is 250 K. Just outside the exit shock, which makes an angle of 50° with the exit plane, the temperature is 430 K. Estimate (a) the mass flow, (b) the throat area, (c) the turning angle of the exit flow, and, in the tank supplying the air, (d) the pressure and (e) the temperature.

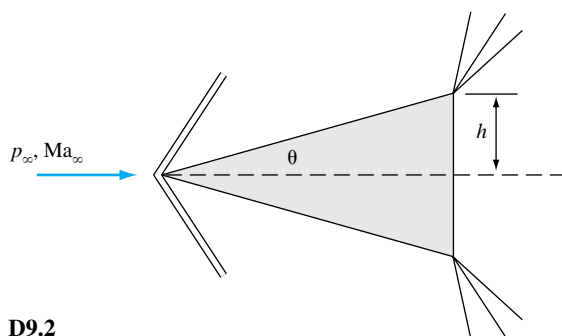


C9.3

Design Projects

- D9.1** It is desired to select a rectangular wing for a fighter aircraft. The plane must be able (a) to take off and land on a 4500-ft-long sea-level runway and (b) to cruise supersonically at $\text{Ma} = 2.3$ at 28,000-ft altitude. For simplicity, assume a wing with zero sweepback. Let the aircraft maximum weight equal $(30 + n)(1000) \text{ lbf}$, where n is the number of letters in your surname. Let the available sea-level maximum thrust be one-third of the maximum weight, decreasing at altitude proportional to ambient density. Mak-

ing suitable assumptions about the effect of finite aspect ratio on wing lift and drag for both subsonic and supersonic flight, select a wing of minimum area sufficient to perform these takeoff/landing and cruise requirements. Some thought should be given to analyzing the wingtips and wing roots in supersonic flight, where Mach cones form and the flow is not two-dimensional. If no satisfactory solution is possible, gradually increase the available thrust to converge to an acceptable design.



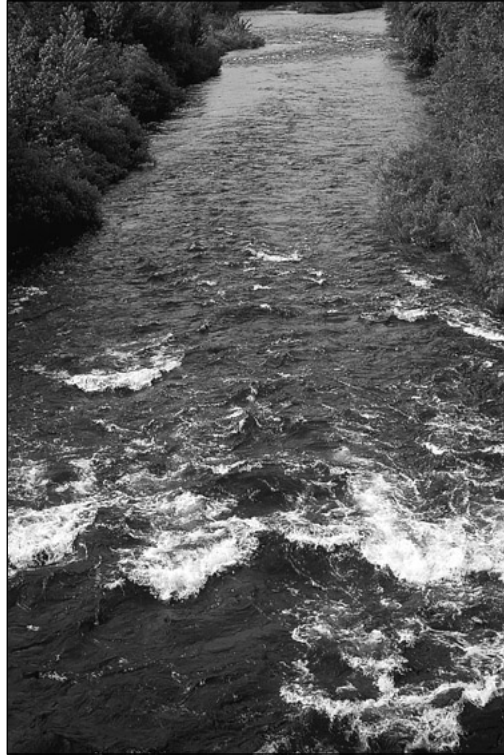
D9.2

D9.2 Consider supersonic flow of air at sea-level conditions past a wedge of half-angle θ , as shown in Fig. D9.2. Assume that the pressure on the back of the wedge equals the fluid pressure as it exits the Prandtl-Meyer fan. (a) Suppose $Ma_\infty = 3.0$. For what angle θ will the supersonic wave-drag coefficient C_D , based on frontal area, be exactly 0.5? (b) Suppose that $\theta = 20^\circ$. Is there a free-stream Mach number for which the wave-drag coefficient C_D , based on frontal area, will be exactly 0.5? (c) Investigate the percent increase in C_D from (a) and (b) due to including boundary-layer friction drag in the calculation.

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The Lehigh River, White Haven, Pennsylvania.
Open channel flows are everywhere, often rough
and turbulent, as in this photo. They are analyzed
by the methods of the present chapter. (*Courtesy of
Dr. E. R. Degginger/Color-Pic Inc.*)

Chapter 10

Open-Channel Flow

Motivation. The duct flows of Chap. 6 were driven by a pressure difference between the ends of the duct. Such channels are closed and full of fluid, either gas or liquid. By contrast, an *open-channel flow* is liquid only and is *not* full; i.e., there is always a free surface exposed to ambient pressure. The basic balance of forces is between gravity (fluid weight) and friction.

Practical open-channel problems almost always concern *water* as the relevant fluid. The flow is generally turbulent, due to its large scale and small kinematic viscosity, and is three-dimensional, sometimes unsteady, and often surprisingly complex due to geometric effects. This chapter presents some simple engineering theories and correlations for steady flow in straight channels of simple geometry. Many of the concepts from steady duct flow—hydraulic diameter, friction factor, head losses—apply also to open channels.

10.1 Introduction

Simply stated, open-channel flow is the flow of a liquid in a conduit with a free surface. There are many practical examples, both artificial (flumes, spillways, canals, weirs, drainage ditches, culverts) and natural (streams, rivers, estuaries, floodplains). This chapter introduces the elementary analysis of such flows, which are dominated by the effects of gravity.

The presence of the free surface, which is essentially at atmospheric pressure, both helps and hurts the analysis. It helps because the pressure can be taken constant along the free surface, which therefore is equivalent to the *hydraulic grade line* (HGL) of the flow. Unlike flow in closed ducts, the pressure gradient is not a direct factor in open-channel flow, where the balance of forces is confined to gravity and friction.¹ But the free surface complicates the analysis because its shape is a priori unknown: The depth profile changes with conditions and must be computed as part of the problem, especially in unsteady problems involving wave motion.

Before proceeding, we remark, as usual, that whole books have been written on open-channel hydraulics [1 to 4]. There are also specialized texts devoted to wave motion [5 to 7] and to engineering aspects of coastal free-surface flows [8, 9]. This chapter is only an introduction to broader and more detailed treatments.

¹Surface tension is rarely important because open channels are normally quite large and have a very large Weber number.

The One-Dimensional Approximation

An open channel always has two sides and a bottom, where the flow satisfies the no-slip condition. Therefore even a straight channel has a three-dimensional velocity distribution. Some measurements of straight-channel velocity contours are shown in Fig. 10.1. The profiles are quite complex, with maximum velocity typically occurring in the midplane about 20 percent below the surface. In very broad shallow channels the maximum velocity is near the surface, and the velocity profile is nearly logarithmic from the bottom to the free surface, as in Eq. (6.84). In noncircular channels there are also secondary motions similar to Fig. 6.16 for closed-duct flows. If the channel curves or meanders, the secondary motion intensifies due to centrifugal effects, with high velocity occurring near the outer radius of the bend. Curved natural channels are subject to strong bottom erosion and deposition effects.

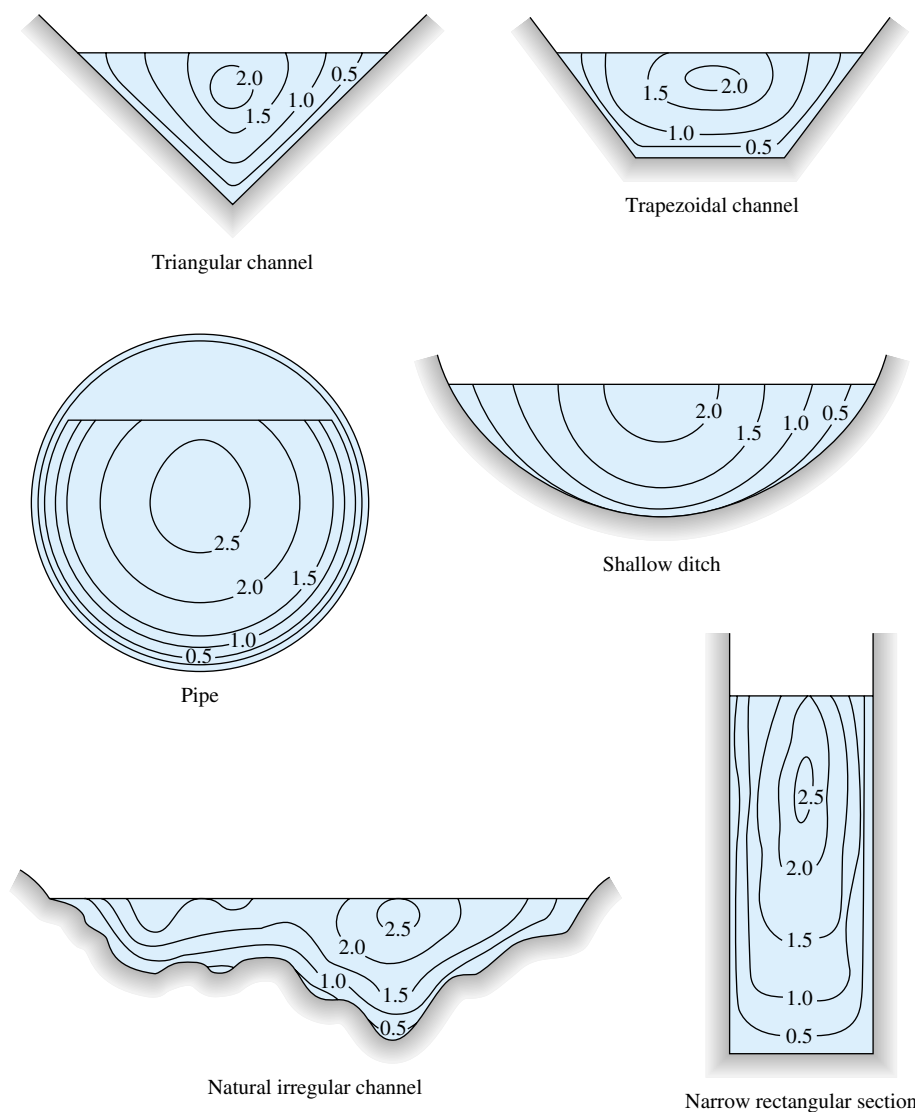
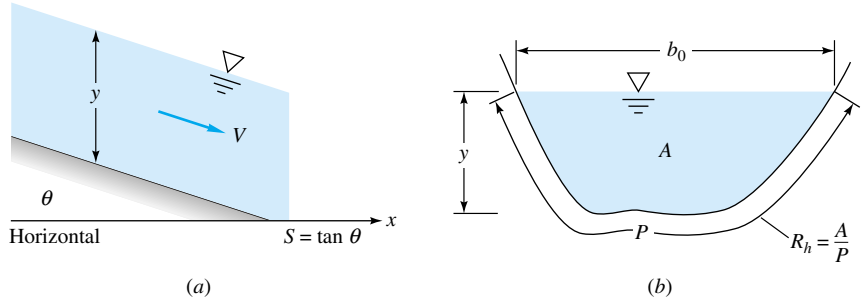


Fig. 10.1 Measured isovelocity contours in typical straight open-channel flows. (From Ref. 3.)

Fig. 10.2 Geometry and notation for open-channel flow: (a) side view; (b) cross section. All these parameters are constant in uniform flow.



With the advent of the supercomputer, it is possible to make numerical simulations of complex flow patterns such as in Fig. 10.1 [23]. However, the practical engineering approach, used here, is to make a one-dimensional-flow approximation, as in Fig. 10.2. Since the liquid density is nearly constant, the steady-flow continuity equation reduces to constant-volume flow Q along the channel

$$Q = V(x)A(x) = \text{const} \quad (10.1)$$

where V is average velocity and A the local cross-sectional area, as sketched in Fig. 10.2.

A second one-dimensional relation between velocity and channel geometry is the energy equation, including friction losses. If points 1 (upstream) and 2 (downstream) are on the free surface, $p_1 = p_2 = p_a$, and we have, for steady flow,

$$\frac{V_1^2}{2g} + z_1 = \frac{V_2^2}{2g} + z_2 + h_f \quad (10.2)$$

where z denotes the total elevation of the free surface, which includes the water depth y (see Fig. 10.2a) plus the height of the (sloping) bottom. The friction head loss h_f is analogous to head loss in duct flow from Eq. (6.30):

$$h_f \approx f \frac{x_2 - x_1}{D_h} \frac{V_{av}^2}{2g} \quad D_h = \text{hydraulic diameter} = \frac{4A}{P} \quad (10.3)$$

where f is the average friction factor (Fig. 6.13) between sections 1 and 2. Since channels are irregular in shape, their “size” is taken to be the hydraulic diameter, with P the *wetted* perimeter—see Fig. 10.2b. Actually, open-channel formulas typically use the hydraulic *radius*

$$R_h = \frac{1}{4} D_h = \frac{A}{P} \quad (10.4)$$

The local Reynolds number of the channel would be $Re = VD_h/\nu$, which is usually highly turbulent ($>10^5$). The only commonly occurring laminar channel flows are the thin sheets which form as rainwater drains from crowned streets and airport runways.

The wetted perimeter P (see Fig. 10.2b) includes the sides and bottom of the channel but not the free surface and, of course, not the parts of the sides above the water level. For example, if a rectangular channel is b wide and h high and contains water to depth y , its wetted perimeter is

$$P = b + 2y \quad (10.5)$$

not $2b + 2h$.