8. $\frac{2}{a}=\frac{3}{8}$

9. $\frac{1}{3n}+\frac{1}{4n}=\frac{7}{24}$

10. $\frac{x+3}{4}=\frac{x-3}{5}+2$

11. $\frac{3t}{20}=\frac{6-t}{12}+\frac{2t}{15}-\frac{3}{2}$

12. $\frac{y}{5}+\frac{7}{20}=\frac{5-y}{4}$

13. $\frac{v-2}{2v-3}=\frac{1}{3}$

14. $\frac{2}{a-3}=\frac{3}{2a+1}$

15. $\frac{x}{4}-\frac{x+6}{5}=\frac{x+3}{2}$

16. $3\sqrt{t}=9$

17. $2\sqrt{y}=5$

18. $4=\sqrt{\left(\frac{3}{a}\right)}+3$

19. $\frac{3\sqrt{x}}{1-\sqrt{x}}=-6$

20. $10=5\sqrt{\left(\frac{x}{2}-1\right)}$

21. $16=\frac{t^2}{9}$

22. $\sqrt{\left(\frac{y+2}{y-2}\right)}=\frac{1}{2}$

23. $\frac{6}{a}=\frac{2a}{3}$

24. $\frac{11}{2}=5+\frac{8}{x^2}$

## 11.3 Practical problems involving simple equations

There are many practical situations in engineering in which solving equations is needed. Here are some worked examples to demonstrate typical practical situations

**Problem 17.** Applying the principle of moments to a beam results in the equation

$$F\times 3=(7.5-F)\times 2$$

where $F$ is the force in newtons. Determine the value of $F$

Removing brackets gives $\quad 3F=15-2F$

Rearranging gives $\quad 3F+2F=15$

i.e. $\quad 5F=15$

Dividing both sides by 5 gives $\quad \frac{5F}{5}=\frac{15}{5}$

from which, force, $\boldsymbol{F=3N}$.

**Problem 18.** A copper wire has a length $L$ of 1.5 km, a resistance $R$ of $5\Omega$ and a resistivity of $17.2\times10^{-6}\,\Omega\,\text{mm}$. Find the cross-sectional area, $a$, of the wire, given that $R=\frac{\rho L}{a}$

Since $R=\frac{\rho L}{a}$ then

$$5\Omega=\frac{(17.2\times10^{-6}\Omega\,\text{mm})(1500\times10^{3}\,\text{mm})}{a}.$$

From the units given, $a$ is measured in $\text{mm}^2$.

Thus, $\quad 5a=17.2\times10^{-6}\times1500\times10^{3}$

and $\quad a=\frac{17.2\times10^{-6}\times1500\times10^{3}}{5}$

$$=\frac{17.2\times1500\times10^{3}}{10^{6}\times5}=\frac{17.2\times15}{10\times5}=5.16$$

Hence, the cross-sectional area of the wire is $\mathbf{5.16\,mm^2}$.

**Problem 19.** $PV=mRT$ is the characteristic gas equation. Find the value of gas constant $R$ when pressure $P=3\times10^{6}\,\text{Pa}$, volume $V=0.90\,\text{m}^3$, mass $m=2.81\,\text{kg}$ and temperature $T=231\,\text{K}$

Dividing both sides of $PV=mRT$ by $mT$ gives

$$\frac{PV}{mT}=\frac{mRT}{mT}$$

Cancelling gives $\quad \frac{PV}{mT}=R$

Substituting values gives $\quad R=\frac{(3\times10^{6})\,(0.90)}{(2.81)\,(231)}$

Using a calculator, **gas constant**, $\boldsymbol{R = 4160\,\mathrm{J/(kg\,K)}}$, correct to 4 significant figures.

**Problem 20.** A rectangular box with square ends has its length 15 cm greater than its breadth and the total length of its edges is 2.04 m. Find the width of the box and its volume

Let $x$ cm = width = height of box. Then the length of the box is $(x+15)$ cm, as shown in Figure 11.1.

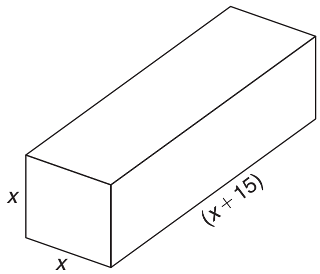


**Figure 11.1**

The length of the edges of the box is $2(4x) + 4(x+15)$ cm, which equals 2.04 m or 204 cm.

Hence, $204 = 2(4x) + 4(x+15)$

$$204 = 8x + 4x + 60$$

$$204 - 60 = 12x$$

i.e. $144 = 12x$

and $x = 12\,\text{cm}$

Hence, **the width of the box is 12 cm**.

$$\textbf{Volume of box} = \text{length} \times \text{width} \times \text{height}$$

$$= (x+15)(x)(x) = (12+15)(12)(12)$$

$$= (27)(12)(12)$$

$$= \mathbf{3888\,cm^3}$$

**Problem 21.** The temperature coefficient of resistance $\alpha$ may be calculated from the formula $R_t = R_0(1+\alpha t)$. Find $\alpha$, given $R_t = 0.928$, $R_0 = 0.80$ and $t = 40$

Since $R_t = R_0(1+\alpha t)$, then

$$0.928 = 0.80[1+\alpha(40)]$$

$$0.928 = 0.80 + (0.8)(\alpha)(40)$$

$$0.928 - 0.80 = 32\alpha$$

$$0.128 = 32\alpha$$

Hence, $\boldsymbol{\alpha} = \dfrac{0.128}{32} = \mathbf{0.004}$

**Problem 22.** The distance $s$ metres travelled in time $t$ seconds is given by the formula $s = ut + \frac{1}{2}at^2$, where $u$ is the initial velocity in m/s and $a$ is the acceleration in m/s$^2$. Find the acceleration of the body if it travels 168 m in 6 s, with an initial velocity of 10 m/s

$$s = ut + \frac{1}{2}at^2, \text{ and } s = 168, u = 10 \text{ and } t = 6$$

Hence, $168 = (10)(6) + \dfrac{1}{2}a(6)^2$

$$168 = 60 + 18a$$

$$168 - 60 = 18a$$

$$108 = 18a$$

$$a = \frac{108}{18} = 6$$

Hence, **the acceleration of the body is 6 m/s$^2$**.

**Problem 23.** When three resistors in an electrical circuit are connected in parallel the total resistance $R_T$ is given by $\dfrac{1}{R_T} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$. Find the total resistance when $R_1 = 5\,\Omega$, $R_2 = 10\,\Omega$ and $R_3 = 30\,\Omega$

$$\frac{1}{R_T} = \frac{1}{5} + \frac{1}{10} + \frac{1}{30} = \frac{6+3+1}{30} = \frac{10}{30} = \frac{1}{3}$$

Taking the reciprocal of both sides gives $\boldsymbol{R_T = 3\,\Omega}$

Alternatively, if $\dfrac{1}{R_T} = \dfrac{1}{5} + \dfrac{1}{10} + \dfrac{1}{30}$, the LCM of the denominators is $30R_T$.

Hence, $30R_T\left(\dfrac{1}{R_T}\right) = 30R_T\left(\dfrac{1}{5}\right)$

$$+ 30R_T\left(\frac{1}{10}\right) + 30R_T\left(\frac{1}{30}\right).$$

Cancelling gives $30 = 6R_T + 3R_T + R_T$

i.e. $30 = 10R_T$

and $\boldsymbol{R_T} = \dfrac{30}{10} = \mathbf{3\,\Omega}$, as above.

**Now try the following Practice Exercise**

**Practice Exercise 44 Practical problems involving simple equations (answers on page 344)**

1. A formula used for calculating resistance of a cable is $R = \dfrac{\rho L}{a}$. Given $R = 1.25$, $L = 2500$ and $a = 2 \times 10^{-4}$, find the value of $\rho$.

2. Force $F$ newtons is given by $F = ma$, where $m$ is the mass in kilograms and $a$ is the acceleration in metres per second squared. Find the acceleration when a force of 4 kN is applied to a mass of 500 kg.

3. $PV = mRT$ is the characteristic gas equation. Find the value of $m$ when $P = 100 \times 10^3$, $V = 3.00$, $R = 288$ and $T = 300$.

4. When three resistors $R_1$, $R_2$ and $R_3$ are connected in parallel, the total resistance $R_T$ is determined from $\dfrac{1}{R_T} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$
   (a) Find the total resistance when $R_1 = 3\,\Omega$, $R_2 = 6\,\Omega$ and $R_3 = 18\,\Omega$.
   (b) Find the value of $R_3$ given that $R_T = 3\,\Omega$, $R_1 = 5\,\Omega$ and $R_2 = 10\,\Omega$.

5. Six digital camera batteries and 3 camcorder batteries cost £96. If a camcorder battery costs £5 more than a digital camera battery, find the cost of each.

6. Ohm's law may be represented by $I = V/R$, where $I$ is the current in amperes, $V$ is the voltage in volts and $R$ is the resistance in ohms. A soldering iron takes a current of 0.30 A from a 240 V supply. Find the resistance of the element.

7. The distance, $s$, travelled in time $t$ seconds is given by the formula $s = ut + \frac{1}{2}at^2$ where $u$ is the initial velocity in m/s and $a$ is the acceleration in m/s$^2$. Calculate the acceleration of the body if it travels 165 m in 3 s, with an initial velocity of 10 m/s.

Here are some further worked examples on solving simple equations in practical situations.

**Problem 24.** The extension $x$ m of an aluminium tie bar of length $l$ m and cross-sectional area $A$ m$^2$ when carrying a load of $F$ Newtons is given by the modulus of elasticity $E = Fl/Ax$. Find the extension of the tie bar (in mm) if $E = 70 \times 10^9$ N/m$^2$, $F = 20 \times 10^6$ N, $A = 0.1\text{m}^2$ and $l = 1.4$ m

$E = Fl/Ax$, hence

$$70 \times 10^9 \frac{\text{N}}{\text{m}^2} = \frac{(20 \times 10^6\,\text{N})(1.4\,\text{m})}{(0.1\text{m}^2)(x)}$$

(the unit of $x$ is thus metres)

$$70 \times 10^9 \times 0.1 \times x = 20 \times 10^6 \times 1.4$$

$$x = \frac{20 \times 10^6 \times 1.4}{70 \times 10^9 \times 0.1}$$

Cancelling gives $\quad x = \dfrac{2 \times 1.4}{7 \times 100}\,\text{m}$

$$= \frac{2 \times 1.4}{7 \times 100} \times 1000\,\text{mm}$$

$$= 4\,\text{mm}$$

Hence, **the extension of the tie bar, $x = 4$ mm**.

**Problem 25.** Power in a d.c. circuit is given by $P = \dfrac{V^2}{R}$ where $V$ is the supply voltage and $R$ is the circuit resistance. Find the supply voltage if the circuit resistance is 1.25 Ω and the power measured is 320 W

Since $P = \dfrac{V^2}{R}$, then $320 = \dfrac{V^2}{1.25}$

$$(320)(1.25) = V^2$$

i.e. $\quad V^2 = 400$

**Supply voltage,** $\quad \mathbf{V = \sqrt{400} = \pm 20\,V}$

**Problem 26.** A painter is paid £6.30 per hour for a basic 36 hour week and overtime is paid at one and a third times this rate. Determine how many hours the painter has to work in a week to earn £319.20

Basic rate per hour = £6.30 and overtime rate per hour $= 1\frac{1}{3} \times £6.30 = £8.40$

Let the number of overtime hours worked $= x$

Then, $\quad (36)(6.30) + (x)(8.40) = 319.20$

$$226.80 + 8.40x = 319.20$$

$$8.40x = 319.20 - 226.80 = 92.40$$
$$x = \frac{92.40}{8.40} = 11$$

Thus, 11 hours overtime would have to be worked to earn £319.20 per week. Hence, **the total number of hours worked** is $36 + 11$, i.e. **47 hours**.

**Problem 27.** A formula relating initial and final states of pressures, $P_1$ and $P_2$, volumes, $V_1$ and $V_2$, and absolute temperatures, $T_1$ and $T_2$, of an ideal gas is $\dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2}$. Find the value of $P_2$ given $P_1 = 100 \times 10^3$, $V_1 = 1.0$, $V_2 = 0.266$, $T_1 = 423$ and $T_2 = 293$

Since $\dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2}$

then $\dfrac{(100 \times 10^3)(1.0)}{423} = \dfrac{P_2(0.266)}{293}$

Cross-multiplying gives

$$(100 \times 10^3)(1.0)(293) = P_2(0.266)(423)$$
$$P_2 = \frac{(100 \times 10^3)(1.0)(293)}{(0.266)(423)}$$

Hence, $\mathbf{P_2 = 260 \times 10^3}$ or $\mathbf{2.6 \times 10^5}$.

**Problem 28.** The stress, $f$, in a material of a thick cylinder can be obtained from $\dfrac{D}{d} = \sqrt{\left(\dfrac{f+p}{f-p}\right)}$. Calculate the stress, given that $D = 21.5$, $d = 10.75$ and $p = 1800$

Since $\dfrac{D}{d} = \sqrt{\left(\dfrac{f+p}{f-p}\right)}$ then $\dfrac{21.5}{10.75} = \sqrt{\left(\dfrac{f+1800}{f-1800}\right)}$

i.e. $$2 = \sqrt{\left(\frac{f+1800}{f-1800}\right)}$$

Squaring both sides gives $$4 = \frac{f+1800}{f-1800}$$

Cross-multiplying gives

$$4(f - 1800) = f + 1800$$
$$4f - 7200 = f + 1800$$
$$4f - f = 1800 + 7200$$
$$3f = 9000$$
$$f = \frac{9000}{3} = 3000$$

Hence, **stress, $\mathbf{f = 3000}$**

**Problem 29.** 12 workmen employed on a building site earn between them a total of £4035 per week. Labourers are paid £275 per week and craftsmen are paid £380 per week. How many craftsmen and how many labourers are employed?

Let the number of craftsmen be $c$. The number of labourers is therefore $(12 - c)$.

The wage bill equation is

$$380c + 275(12 - c) = 4035$$
$$380c + 3300 - 275c = 4035$$
$$380c - 275c = 4035 - 3300$$
$$105c = 735$$
$$c = \frac{735}{105} = 7$$

Hence, there are **7 craftsmen** and $(12 - 7)$, i.e. **5 labourers** on the site.

**Now try the following Practice Exercise**

### Practice Exercise 45 Practical problems involving simple equations (answers on page 344)

1. A rectangle has a length of 20 cm and a width $b$ cm. When its width is reduced by 4 cm its area becomes 160 cm$^2$. Find the original width and area of the rectangle.
2. Given $R_2 = R_1(1 + \alpha t)$, find $\alpha$ given $R_1 = 5.0$, $R_2 = 6.03$ and $t = 51.5$
3. If $v^2 = u^2 + 2as$, find $u$ given $v = 24$, $a = -40$ and $s = 4.05$
4. The relationship between the temperature on a Fahrenheit scale and that on a Celsius scale is given by $F = \frac{9}{5}C + 32$. Express 113°F in degrees Celsius.
5. If $t = 2\pi\sqrt{\dfrac{w}{Sg}}$, find the value of $S$ given $w = 1.219$, $g = 9.81$ and $t = 0.3132$
6. Two joiners and five mates earn £1824 between them for a particular job. If a joiner earns £72 more than a mate, calculate the earnings for a joiner and for a mate.

7. An alloy contains 60% by weight of copper, the remainder being zinc. How much copper must be mixed with 50 kg of this alloy to give an alloy containing 75% copper?

8. A rectangular laboratory has a length equal to one and a half times its width and a perimeter of 40 m. Find its length and width.

9. Applying the principle of moments to a beam results in the following equation:

$$F \times 3 = (5 - F) \times 7$$

where $F$ is the force in newtons. Determine the value of $F$.

## *Revision Test 4: Algebra and simple equations*

This assignment covers the material contained in Chapters 9–11. *The marks available are shown in brackets at the end of each question.*

1. Evaluate $3pqr^3 - 2p^2qr + pqr$ when $p = \frac{1}{2}$, $q = -2$ and $r = 1$. (3)

In problems 2 to 7, simplify the expressions.

2. $\dfrac{9p^2qr^3}{3pq^2r}$ (3)
3. $2(3x - 2y) - (4y - 3x)$ (3)
4. $(x - 2y)(2x + y)$ (3)
5. $p^2q^{-3}r^4 \times pq^2r^{-3}$ (3)
6. $(3a - 2b)^2$ (3)
7. $\dfrac{a^4b^2c}{ab^3c^2}$ (3)
8. Factorize

   (a) $2x^2y^3 - 10xy^2$

   (b) $21ab^2c^3 - 7a^2bc^2 + 28a^3bc^4$ (5)
9. Factorize and simplify

   $$\frac{2x^2y + 6xy^2}{x + 3y} - \frac{x^3y^2}{x^2y}$$ (5)
10. Remove the brackets and simplify $10a - [3(2a - b) - 4(b - a) + 5b]$ (4)
11. Simplify $x \div 5x - x + (2x - 3x)x$ (4)
12. Simplify $3a + 2a \times 5a + 4a \div 2a - 6a$ (4)
13. Solve the equations

   (a) $3a = 39$

   (b) $2x - 4 = 9$ (3)
14. Solve the equations

   (a) $\frac{4}{9}y = 8$

   (b) $6x - 1 = 4x + 5$ (4)
15. Solve the equation $5(t - 2) - 3(4 - t) = 2(t + 3) - 40$ (4)
16. Solve the equations:

   (a) $\dfrac{3}{2x + 1} = \dfrac{1}{4x - 3}$

   (b) $2x^2 = 162$ (7)
17. Kinetic energy is given by the formula, $E_k = \frac{1}{2}mv^2$ joules, where $m$ is the mass in kilograms and $v$ is the velocity in metres per second. Evaluate the velocity when $E_k = 576 \times 10^{-3}$ J and the mass is 5 kg. (4)
18. An approximate relationship between the number of teeth $T$ on a milling cutter, the diameter of the cutter $D$ and the depth of cut $d$ is given by $T = \dfrac{12.5D}{D + 4d}$. Evaluate $d$ when $T = 10$ and $D = 32$. (5)
19. The modulus of elasticity $E$ is given by the formula $E = \dfrac{FL}{xA}$ where $F$ is force in newtons, $L$ is the length in metres, $x$ is the extension in metres and $A$ the cross-sectional area in square metres. Evaluate $A$, in square centimetres, when $E = 80 \times 10^9$ N/m$^2$, $x = 2$ mm, $F = 100 \times 10^3$ N and $L = 2.0$ m. (5)

# Chapter 12

# Transposing formulae

## 12.1 Introduction

In the formula $I = \frac{V}{R}$, $I$ is called the **subject of the formula**.

Similarly, in the formula $y = mx + c$, $y$ is the subject of the formula.

When a symbol other than the subject is required to be the subject, the formula needs to be rearranged to make a new subject. This rearranging process is called **transposing the formula** or **transposition**.

For example, in the above formulae,

$$\text{if } I = \frac{V}{R} \text{ then } V = IR$$

$$\text{and if } y = mx + c \text{ then } x = \frac{y - c}{m}$$

How did we arrive at these transpositions? This is the purpose of this chapter — to show how to transpose formulae. A great many equations occur in engineering and it is essential that we can transpose them when needed.

## 12.2 Transposing formulae

There are no new rules for transposing formulae. The same rules as were used for simple equations in Chapter 11 are used; i.e., **the balance of an equation must be maintained**: whatever is done to one side of an equation must be done to the other.

It is best that you cover simple equations before trying this chapter.

Here are some worked examples to help understanding of transposing formulae.

**Problem 1.** Transpose $p = q + r + s$ to make $r$ the subject

The object is to obtain $r$ on its own on the LHS of the equation. Changing the equation around so that $r$ is on the LHS gives

$$q + r + s = p \qquad (1)$$

From Chapter 11 on simple equations, a term can be moved from one side of an equation to the other side as long as the sign is changed.

Rearranging gives $\boldsymbol{r = p - q - s}$.

Mathematically, we have subtracted $q + s$ from both sides of equation (1).

**Problem 2.** If $a + b = w - x + y$, express $x$ as the subject

As stated in Problem 1, a term can be moved from one side of an equation to the other side but with a change of sign.

Hence, rearranging gives $\boldsymbol{x = w + y - a - b}$

**Problem 3.** Transpose $v = f\lambda$ to make $\lambda$ the subject

$v = f\lambda$ relates velocity $v$, frequency $f$ and wavelength $\lambda$

Rearranging gives $\quad f\lambda = v$

Dividing both sides by $f$ gives $\quad \frac{f\lambda}{f} = \frac{v}{f}$

Cancelling gives $\quad \boldsymbol{\lambda = \frac{v}{f}}$

**Problem 4.** When a body falls freely through a height $h$, the velocity $v$ is given by $v^2 = 2gh$. Express this formula with $h$ as the subject

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Rearranging gives $2gh = v^2$

Dividing both sides by $2g$ gives $\frac{2gh}{2g} = \frac{v^2}{2g}$

Cancelling gives $\boldsymbol{h = \frac{v^2}{2g}}$

**Problem 5.** If $I = \frac{V}{R}$, rearrange to make $V$ the subject

$I = \frac{V}{R}$ is Ohm's law, where $I$ is the current, $V$ is the voltage and $R$ is the resistance.

Rearranging gives $\frac{V}{R} = I$

Multiplying both sides by $R$ gives $R\left(\frac{V}{R}\right) = R(I)$

Cancelling gives $\boldsymbol{V = IR}$

**Problem 6.** Transpose $a = \frac{F}{m}$ for $m$

$a = \frac{F}{m}$ relates acceleration $a$, force $F$ and mass $m$.

Rearranging gives $\frac{F}{m} = a$

Multiplying both sides by $m$ gives $m\left(\frac{F}{m}\right) = m(a)$

Cancelling gives $F = ma$

Rearranging gives $ma = F$

Dividing both sides by $a$ gives $\frac{ma}{a} = \frac{F}{a}$

i.e. $\boldsymbol{m = \frac{F}{a}}$

**Problem 7.** Rearrange the formula $R = \frac{\rho L}{A}$ to make (a) $A$ the subject and (b) $L$ the subject

$R = \frac{\rho L}{A}$ relates resistance $R$ of a conductor, resistivity $\rho$, conductor length $L$ and conductor cross-sectional area $A$.

(a) Rearranging gives $\frac{\rho L}{A} = R$

Multiplying both sides by $A$ gives

$$A\left(\frac{\rho L}{A}\right) = A(R)$$

Cancelling gives $\rho L = AR$

Rearranging gives $AR = \rho l$

Dividing both sides by $R$ gives $\frac{AR}{R} = \frac{\rho L}{R}$

Cancelling gives $\boldsymbol{A = \frac{\rho L}{R}}$

(b) Multiplying both sides of $\frac{\rho L}{A} = R$ by $A$ gives

$$\rho L = AR$$

Dividing both sides by $\rho$ gives $\frac{\rho L}{\rho} = \frac{AR}{\rho}$

Cancelling gives $\boldsymbol{L = \frac{AR}{\rho}}$

**Problem 8.** Transpose $y = mx + c$ to make $m$ the subject

$y = mx + c$ is the equation of a straight line graph, where $y$ is the vertical axis variable, $x$ is the horizontal axis variable, $m$ is the gradient of the graph and $c$ is the $y$-axis intercept.

Subtracting $c$ from both sides gives $y - c = mx$

or $mx = y - c$

Dividing both sides by $x$ gives $\boldsymbol{m = \frac{y - c}{x}}$

**Now try the following Practice Exercise**

## Practice Exercise 46 Transposing formulae (answers on page 344)

Make the symbol indicated the subject of each of the formulae shown and express each in its simplest form.

1. $a + b = c - d - e$ $(d)$
2. $y = 7x$ $(x)$
3. $pv = c$ $(v)$
4. $v = u + at$ $(a)$
5. $V = IR$ $(R)$
6. $x + 3y = t$ $(y)$
7. $c = 2\pi r$ $(r)$
8. $y = mx + c$ $(x)$

9. $I = PRT$ $(T)$
10. $X_L = 2\pi fL$ $(L)$
11. $I = \dfrac{E}{R}$ $(R)$
12. $y = \dfrac{x}{a} + 3$ $(x)$
13. $F = \dfrac{9}{5}C + 32$ $(C)$
14. $X_C = \dfrac{1}{2\pi fC}$ $(f)$

## 12.3 Further transposing of formulae

Here are some more transposition examples to help us further understand how more difficult formulae are transposed.

**Problem 9.** Transpose the formula $v = u + \dfrac{Ft}{m}$ to make $F$ the subject

$v = u + \dfrac{Ft}{m}$ relates final velocity $v$, initial velocity $u$, force $F$, mass $m$ and time $t$. $\left(\dfrac{F}{m}\text{ is acceleration } a.\right)$

Rearranging gives $\quad u + \dfrac{Ft}{m} = v$

and $\quad \dfrac{Ft}{m} = v - u$

Multiplying each side by $m$ gives

$$m\left(\frac{Ft}{m}\right) = m(v-u)$$

Cancelling gives $\quad Ft = m(v-u)$

Dividing both sides by $t$ gives $\quad \dfrac{Ft}{t} = \dfrac{m(v-u)}{t}$

Cancelling gives $\boldsymbol{F = \dfrac{m(v-u)}{t}}$ or $\boldsymbol{F = \dfrac{m}{t}(v-u)}$

This shows two ways of expressing the answer. There is often more than one way of expressing a transposed answer. In this case, these equations for $F$ are equivalent; neither one is more correct than the other.

**Problem 10.** The final length $L_2$ of a piece of wire heated through $\theta$°C is given by the formula $L_2 = L_1(1+\alpha\theta)$ where $L_1$ is the original length. Make the coefficient of expansion $\alpha$ the subject

Rearranging gives $\quad L_1(1+\alpha\theta) = L_2$

Removing the bracket gives $\quad L_1 + L_1\alpha\theta = L_2$

Rearranging gives $\quad L_1\alpha\theta = L_2 - L_1$

Dividing both sides by $L_1\theta$ gives $\quad \dfrac{L_1\alpha\theta}{L_1\theta} = \dfrac{L_2 - L_1}{L_1\theta}$

Cancelling gives $\quad \boldsymbol{\alpha = \dfrac{L_2 - L_1}{L_1\theta}}$

An alternative method of transposing $L_2 = L_1(1+\alpha\theta)$ for $\alpha$ is:

Dividing both sides by $L_1$ gives $\quad \dfrac{L_2}{L_1} = 1 + \alpha\theta$

Subtracting 1 from both sides gives $\quad \dfrac{L_2}{L_1} - 1 = \alpha\theta$

or $\quad \alpha\theta = \dfrac{L_2}{L_1} - 1$

Dividing both sides by $\theta$ gives $\quad \boldsymbol{\alpha = \dfrac{\dfrac{L_2}{L_1} - 1}{\theta}}$

The two answers $\alpha = \dfrac{L_2 - L_1}{L_1\theta}$ and $\alpha = \dfrac{\dfrac{L_2}{L_1} - 1}{\theta}$ look quite different. They are, however, equivalent. The first answer looks tidier but is no more correct than the second answer.

**Problem 11.** A formula for the distance $s$ moved by a body is given by $s = \dfrac{1}{2}(v+u)t$. Rearrange the formula to make $u$ the subject

Rearranging gives $\quad \dfrac{1}{2}(v+u)t = s$

Multiplying both sides by 2 gives $\quad (v+u)t = 2s$

Dividing both sides by $t$ gives $\quad \dfrac{(v+u)t}{t} = \dfrac{2s}{t}$

Cancelling gives $\quad v + u = \dfrac{2s}{t}$

Rearranging gives $\quad \boldsymbol{u = \dfrac{2s}{t} - v}$ or $\boldsymbol{u = \dfrac{2s - vt}{t}}$

**Problem 12.** A formula for kinetic energy is $k = \dfrac{1}{2}mv^2$. Transpose the formula to make $v$ the subject

Rearranging gives $\quad \dfrac{1}{2}mv^2 = k$

Whenever the prospective new subject is a squared term, that term is isolated on the LHS and then the square root of both sides of the equation is taken.

Multiplying both sides by 2 gives $mv^2 = 2k$

Dividing both sides by $m$ gives $\frac{mv^2}{m} = \frac{2k}{m}$

Cancelling gives $v^2 = \frac{2k}{m}$

Taking the square root of both sides gives

$$\sqrt{v^2} = \sqrt{\left(\frac{2k}{m}\right)}$$

i.e. $$\boldsymbol{v = \sqrt{\left(\frac{2k}{m}\right)}}$$

**Problem 13.** In a right-angled triangle having sides $x$, $y$ and hypotenuse $z$, Pythagoras' theorem states $z^2 = x^2 + y^2$. Transpose the formula to find $x$

Rearranging gives $x^2 + y^2 = z^2$

and $x^2 = z^2 - y^2$

Taking the square root of both sides gives

$$\boldsymbol{x = \sqrt{z^2 - y^2}}$$

**Problem 14.** Transpose $y = \frac{ML^2}{8EI}$ to make $L$ the subject

Multiplying both sides by $8EI$ gives $8EIy = ML^2$

Dividing both sides by $M$ gives $\frac{8EIy}{M} = L^2$

or $L^2 = \frac{8EIy}{M}$

Taking the square root of both sides gives

$$\sqrt{L^2} = \sqrt{\frac{8EIy}{M}}$$

i.e. $$\boldsymbol{L = \sqrt{\frac{8EIy}{M}}}$$

**Problem 15.** Given $t = 2\pi\sqrt{\frac{l}{g}}$, find $g$ in terms of $t$, $l$ and $\pi$

Whenever the prospective new subject is within a square root sign, it is best to isolate that term on the LHS and then to square both sides of the equation.

Rearranging gives $2\pi\sqrt{\frac{l}{g}} = t$

Dividing both sides by $2\pi$ gives $\sqrt{\frac{l}{g}} = \frac{t}{2\pi}$

Squaring both sides gives $\frac{l}{g} = \left(\frac{t}{2\pi}\right)^2 = \frac{t^2}{4\pi^2}$

Cross-multiplying, (i.e. multiplying each term by $4\pi^2 g$), gives $4\pi^2 l = gt^2$

or $gt^2 = 4\pi^2 l$

Dividing both sides by $t^2$ gives $\frac{gt^2}{t^2} = \frac{4\pi^2 l}{t^2}$

Cancelling gives $$\boldsymbol{g = \frac{4\pi^2 l}{t^2}}$$

**Problem 16.** The impedance $Z$ of an a.c. circuit is given by $Z = \sqrt{R^2 + X^2}$ where $R$ is the resistance. Make the reactance, $X$, the subject

Rearranging gives $\sqrt{R^2 + X^2} = Z$

Squaring both sides gives $R^2 + X^2 = Z^2$

Rearranging gives $X^2 = Z^2 - R^2$

Taking the square root of both sides gives

$$\boldsymbol{X = \sqrt{Z^2 - R^2}}$$

**Problem 17.** The volume $V$ of a hemisphere of radius $r$ is given by $V = \frac{2}{3}\pi r^3$. (a) Find $r$ in terms of $V$. (b) Evaluate the radius when $V = 32\,\text{cm}^3$

(a) Rearranging gives $\frac{2}{3}\pi r^3 = V$

Multiplying both sides by 3 gives $2\pi r^3 = 3V$

Dividing both sides by $2\pi$ gives $\frac{2\pi r^3}{2\pi} = \frac{3V}{2\pi}$

Cancelling gives $r^3 = \frac{3V}{2\pi}$

Taking the cube root of both sides gives

$$\sqrt[3]{r^3} = \sqrt[3]{\left(\frac{3V}{2\pi}\right)}$$

i.e. $$\boldsymbol{r = \sqrt[3]{\left(\frac{3V}{2\pi}\right)}}$$

(b) When $V = 32\text{cm}^3$,

$$\textbf{radius } \boldsymbol{r} = \sqrt[3]{\left(\frac{3V}{2\pi}\right)} = \sqrt[3]{\left(\frac{3\times 32}{2\pi}\right)} = \mathbf{2.48\,cm}.$$

**Now try the following Practice Exercise**

### Practice Exercise 47 Further transposing formulae (answers on page 345)

Make the symbol indicated the subject of each of the formulae shown in problems 1 to 13 and express each in its simplest form.

1. $S = \dfrac{a}{1-r}$ $(r)$
2. $y = \dfrac{\lambda(x-d)}{d}$ $(x)$
3. $A = \dfrac{3(F-f)}{L}$ $(f)$
4. $y = \dfrac{AB^2}{5CD}$ $(D)$
5. $R = R_0(1+\alpha t)$ $(t)$
6. $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ $(R_2)$
7. $I = \dfrac{E-e}{R+r}$ $(R)$
8. $y = 4ab^2c^2$ $(b)$
9. $\dfrac{a^2}{x^2} + \dfrac{b^2}{y^2} = 1$ $(x)$
10. $t = 2\pi\sqrt{\dfrac{L}{g}}$ $(L)$
11. $v^2 = u^2 + 2as$ $(u)$
12. $A = \dfrac{\pi R^2\theta}{360}$ $(R)$
13. $N = \sqrt{\left(\dfrac{a+x}{y}\right)}$ $(a)$
14. Transpose $Z = \sqrt{R^2 + (2\pi f L)^2}$ for $L$ and evaluate $L$ when $Z = 27.82$, $R = 11.76$ and $f = 50$.

## 12.4 More difficult transposing of formulae

Here are some more transposition examples to help us further understand how more difficult formulae are transposed.

**Problem 18.** (a) Transpose $S = \sqrt{\dfrac{3d\,(L-d)}{8}}$ to make $l$ the subject. (b) Evaluate $L$ when $d = 1.65$ and $S = 0.82$

The formula $S = \sqrt{\dfrac{3d\,(L-d)}{8}}$ represents the sag $S$ at the centre of a wire.

(a) Squaring both sides gives $$S^2 = \frac{3d(L-d)}{8}$$

Multiplying both sides by 8 gives

$$8S^2 = 3d(L-d)$$

Dividing both sides by $3d$ gives $$\frac{8S^2}{3d} = L - d$$

Rearranging gives $$\boldsymbol{L = d + \frac{8S^2}{3d}}$$

(b) When $d = 1.65$ and $S = 0.82$,

$$\boldsymbol{L} = d + \frac{8\,S^2}{3d} = 1.65 + \frac{8\times 0.82^2}{3\times 1.65} = \mathbf{2.737}$$

**Problem 19.** Transpose the formula $p = \dfrac{a^2x^2 + a^2y}{r}$ to make $a$ the subject

Rearranging gives $$\frac{a^2x^2 + a^2y}{r} = p$$

Multiplying both sides by $r$ gives

$$a^2x + a^2y = rp$$

Factorizing the LHS gives $$a^2(x+y) = rp$$

Dividing both sides by $(x+y)$ gives

$$\frac{a^2(x+y)}{(x+y)} = \frac{rp}{(x+y)}$$