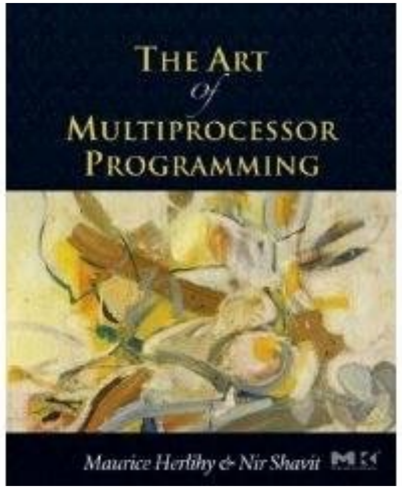
Concurrent Objects

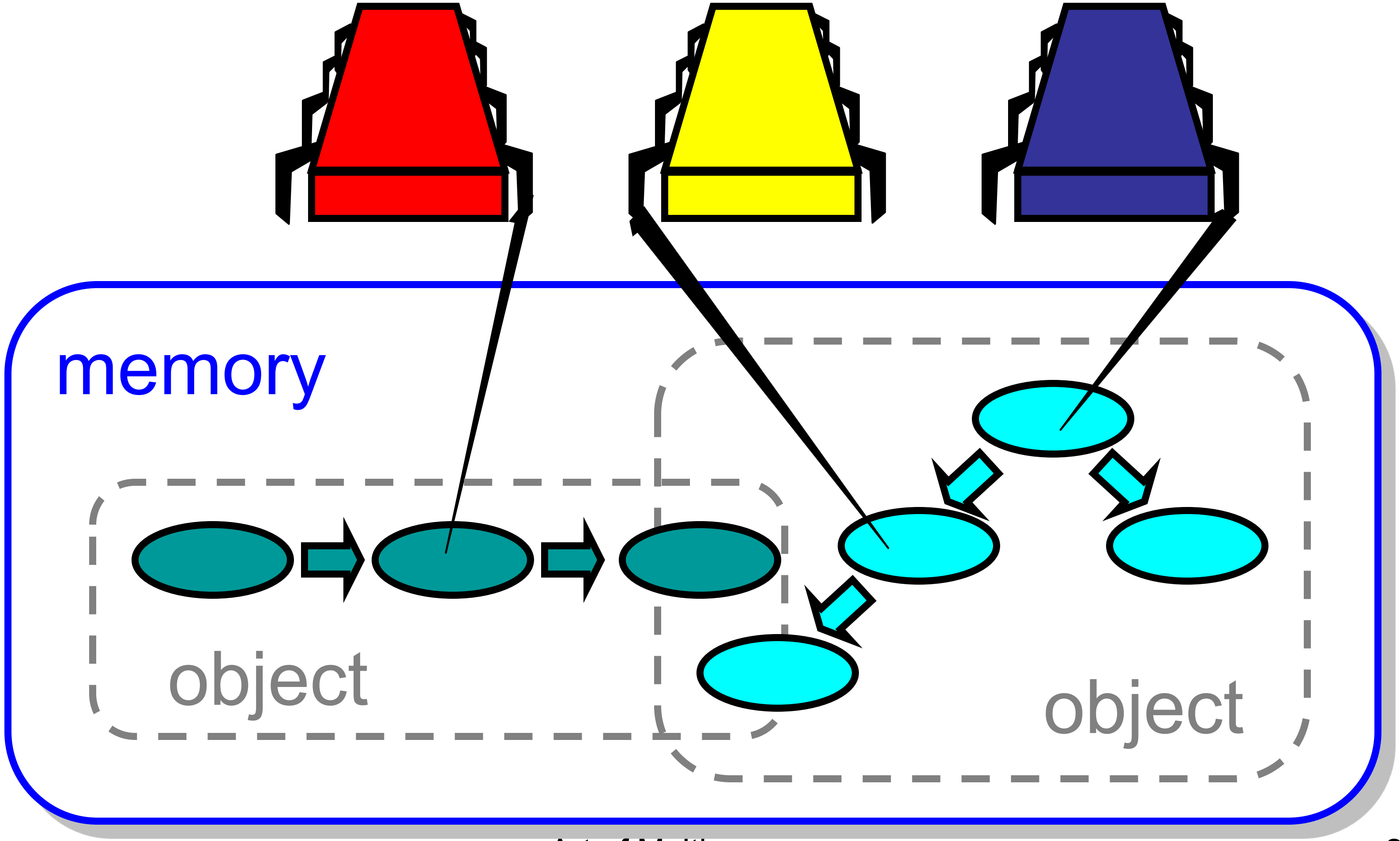


Companion slides for

The Art of Multiprocessor Programming

by Maurice Herlihy & Nir Shavit

Concurrent Computation



Objectivism

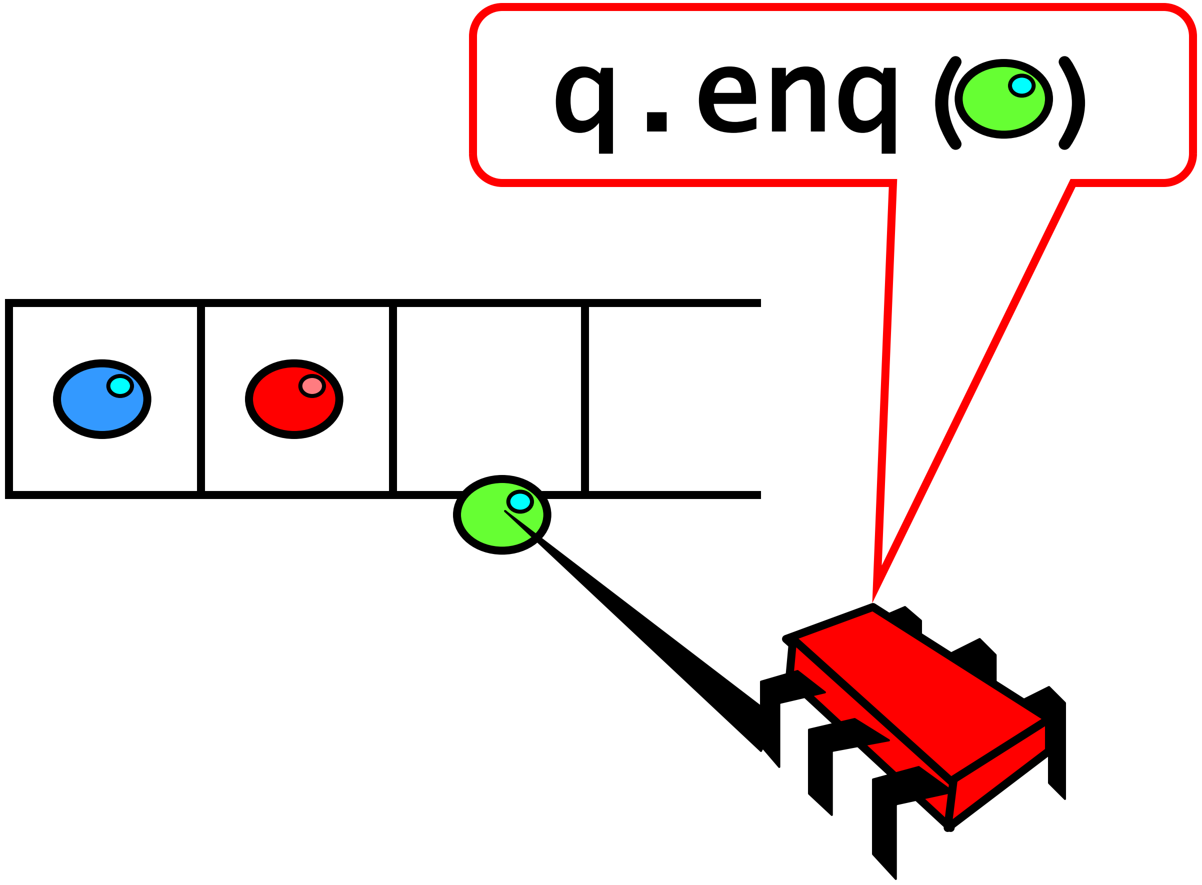
- What is a concurrent object?
  - How do we **describe** one?
  - How do we **implement** one?
  - How do we **tell if we're right**?

Objectivism

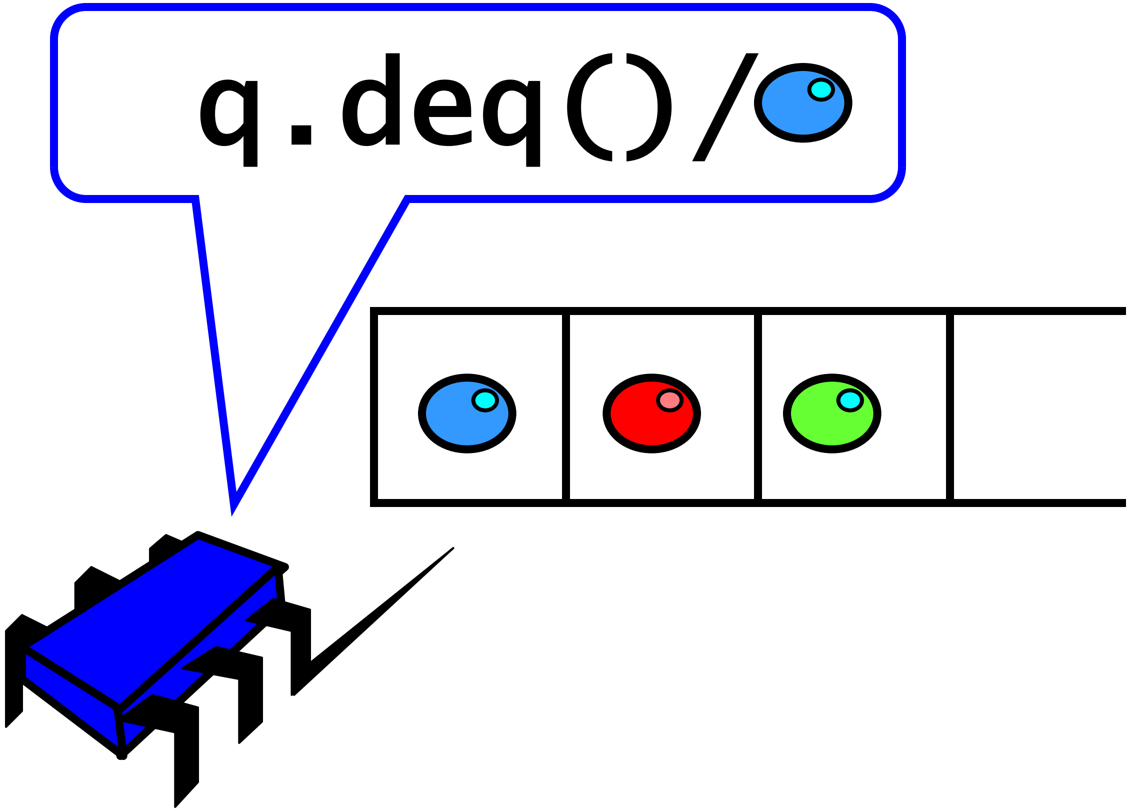
- What is a concurrent object?
  - How do we **describe** one?

  - How do we **tell if we're right**?

FIFO Queue: Enqueue Method



FIFO Queue: Dequeue Method



A Lock-Based Queue

```
class LockBasedQueue<T> {
  int head, tail;
  T[] items;
  Lock lock;
  public LockBasedQueue(int capacity) {
    head = 0; tail = 0;
    lock = new ReentrantLock();
    items = (T[]) new Object[capacity];
}
```

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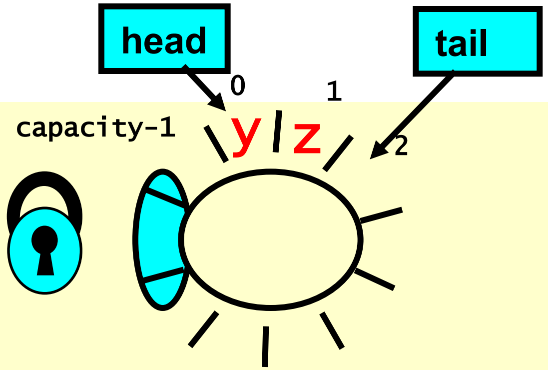
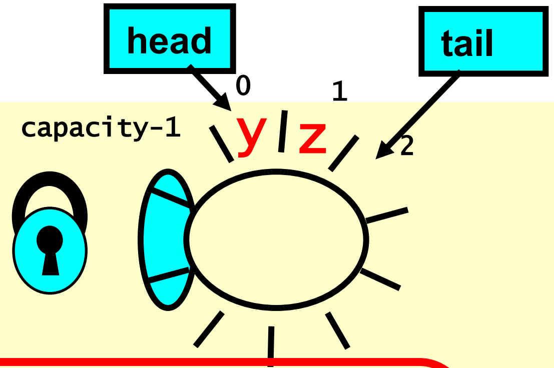


Queue fields protected by single shared lock

A Lock-Based Queue

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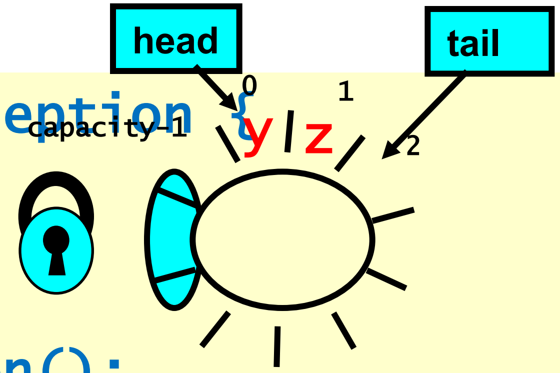
```

Initially head = tail

Implementation: Deq




```
public T deq() throws EmptyException {
  lock.lock();
  try {
    if (tail == head)
       throw new EmptyException();
    T x = items[head % items.length];
    head++;
    return x;
  } finally {
    lock.unlock();
  }
}
```

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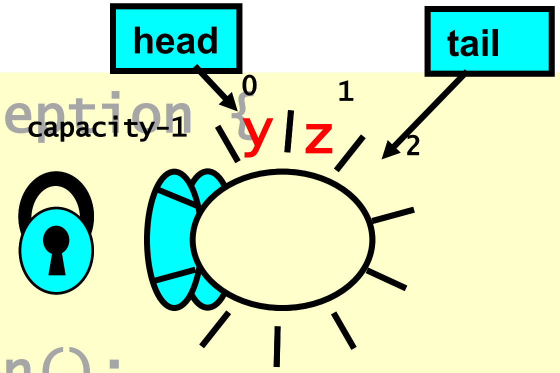

Method calls mutually exclusive

Implementation: Deq

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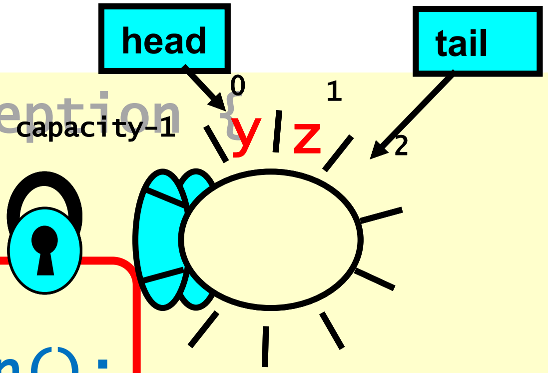

If queue empty throw exception

Implementation: Deq

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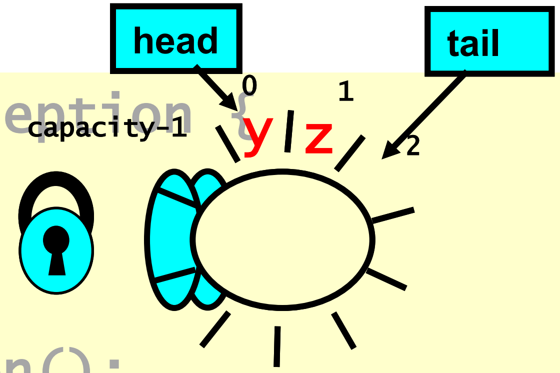

Queue not empty: remove item and update head

Implementation: Deq

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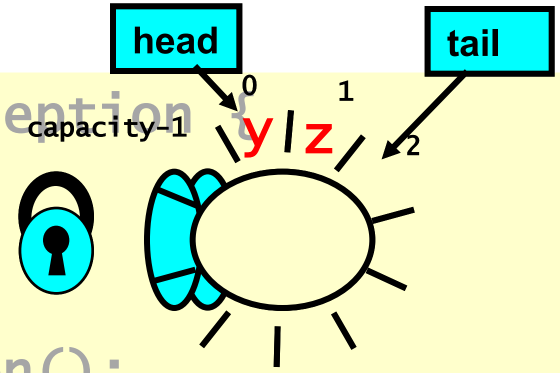

Return result

Implementation: Deq

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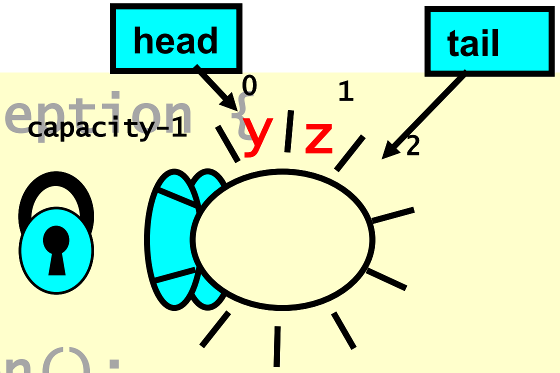

Release lock no matter what!

Implementation: Deq

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  }
}
```

Should be correct because modifications are mutually exclusive…

Now consider the following implementation

- The same thing without mutual exclusion
- For simplicity, only two threads
  - One thread enq only
  - The other deq only

Wait-free 2-Thread Queue

```
public class WaitFreeQueue {

  int head = 0, tail = 0;
  items = (T[]) new Object[capacity];

  public void enq(Item x) {
    if (tail-head == capacity) throw
         new FullException();
    items[tail % capacity] = x; tail++;
  }
  public Item deq() {
     if (tail == head) throw
         new EmptyException();
     Item item = items[head % capacity]; head++;
     return item;
}}
```

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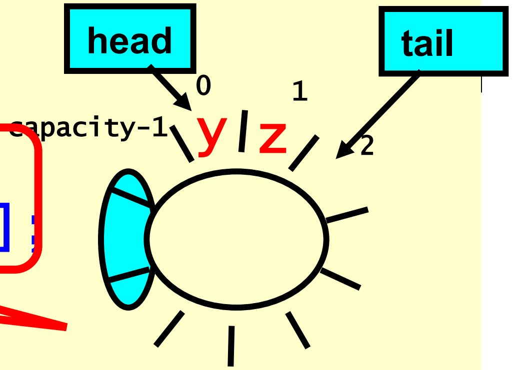
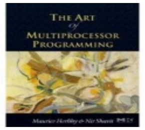
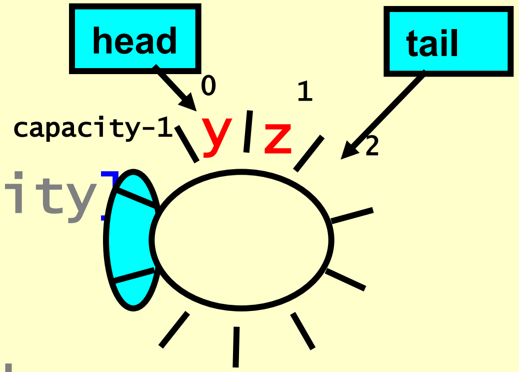




Wait-free 2-Thread Queue

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     if (tail == head) thr
        new EmptyEx
    Queue is updated                    ++;
     return item;
}}
```

How do we define “correct” when modifications are not mutually exclusive?

Defining concurrent queue implementations

- Need a way to specify a concurrent queue object
- Need a way to prove that an algorithm implements the object's specification
- Lets talk about object specifications …

Correctness and Progress

- In a concurrent setting, we need to specify both the safety and the liveness properties of an object
- Need a way to define
  - when an implementation is correct
  - the conditions under which it guarantees progress

Let's begin with correctness

Sequential Objects

- Each object has a ***state***
  - Usually given by a set of ***fields***
  - Queue example: sequence of items
- Each object has a set of ***methods***
  - Only way to manipulate state
  - Queue example: **enq** and **deq** methods

Sequential Specifications

- If (precondition)
  - the object is in such-and-such a state
  - before you call the method,
- Then (postcondition)
  - the method will return a particular value
  - or throw a particular exception.
- and (postcondition, con't)
  - the object will be in some other state
  - when the method returns,

Pre and PostConditions for Dequeue

- Precondition:
  - Queue is non-empty
- Postcondition:
  - Returns first item in queue
- Postcondition:
  - Removes first item in queue

Pre and PostConditions for Dequeue

- Precondition:
  - Queue is empty
- Postcondition:
  - Throws Empty exception
- Postcondition:
  - Queue state unchanged

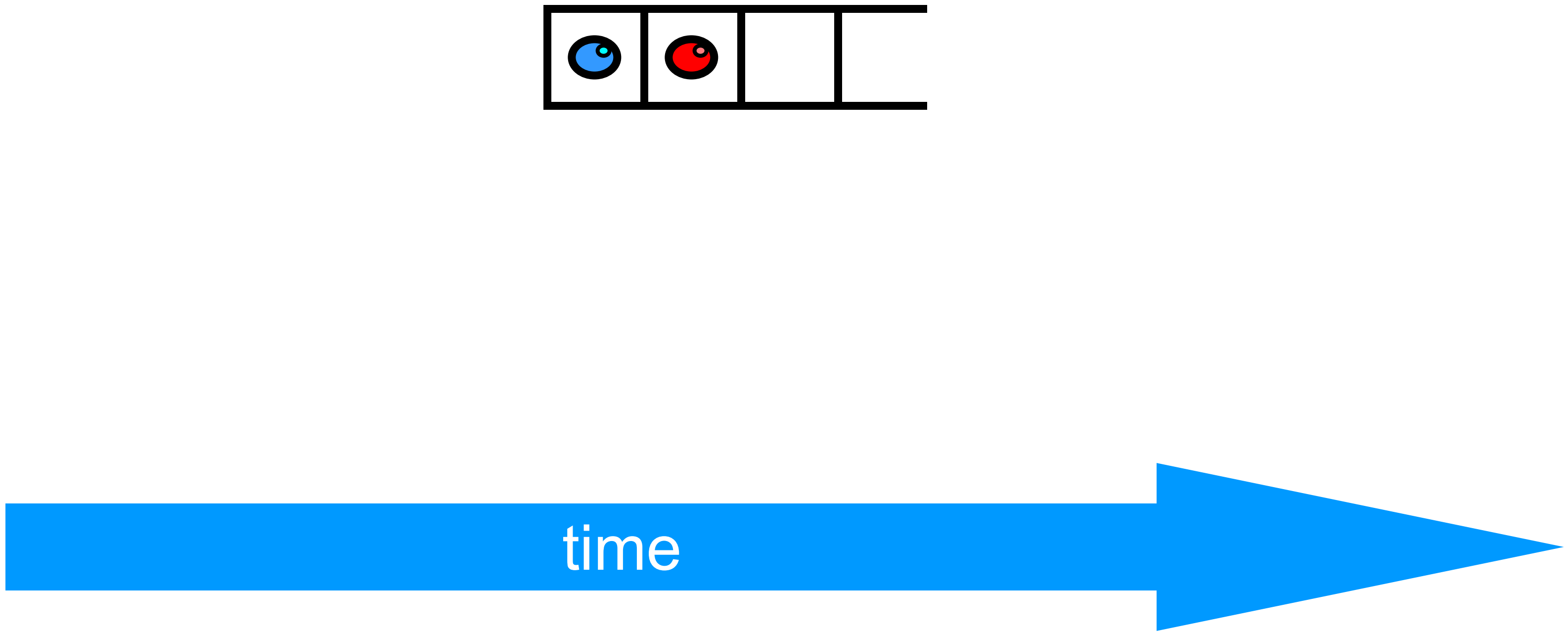
Why Sequential Specifications Totally Rock

- Interactions among methods captured by side-effects on object state
  - State meaningful between method calls
- Documentation size linear in number of methods
  - Each method described in isolation
- Can add new methods
  - Without changing descriptions of old methods

What About Concurrent Specifications ?

- Methods?
- Documentation?
- Adding new methods?

Methods Take Time



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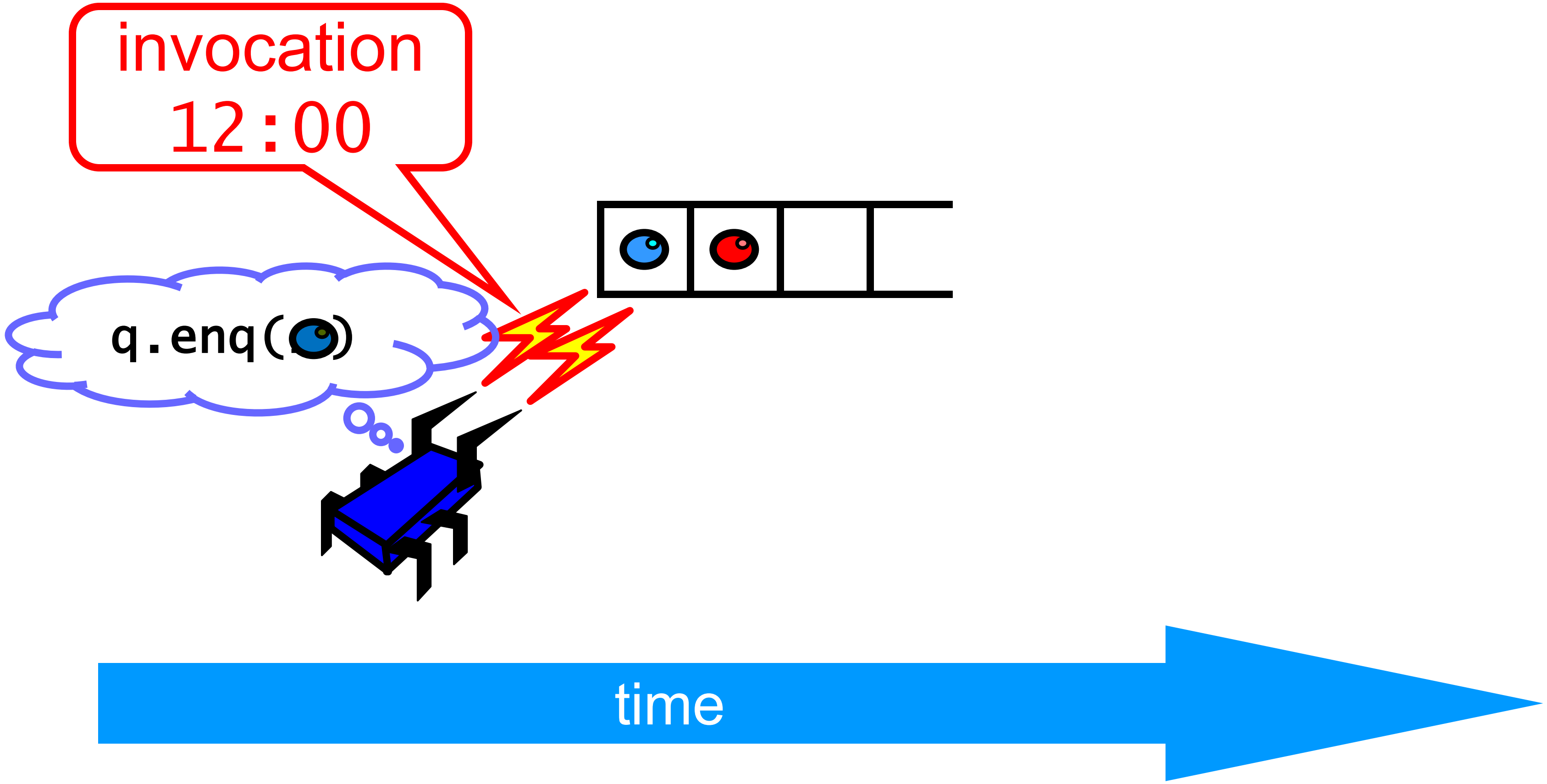

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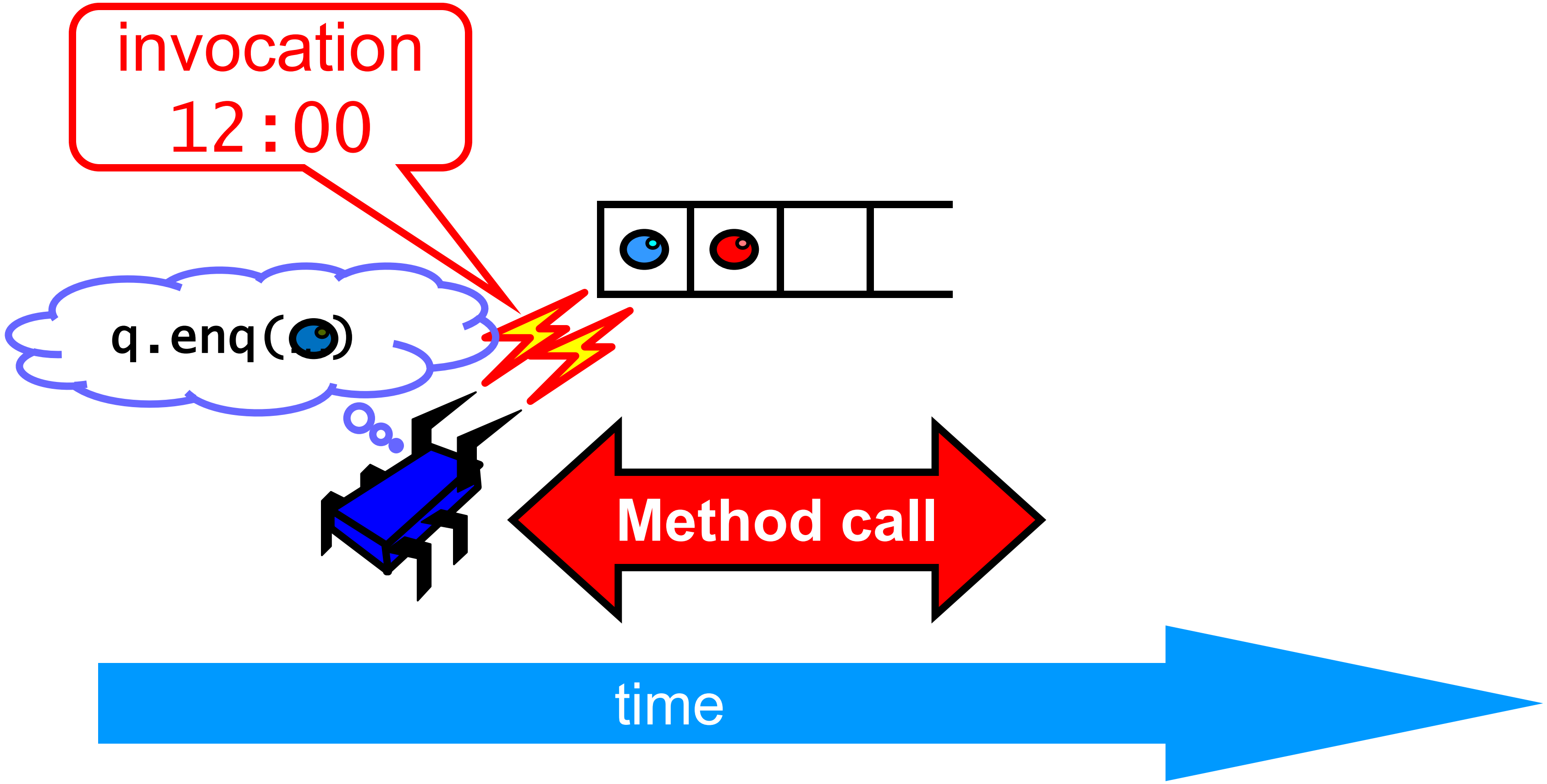

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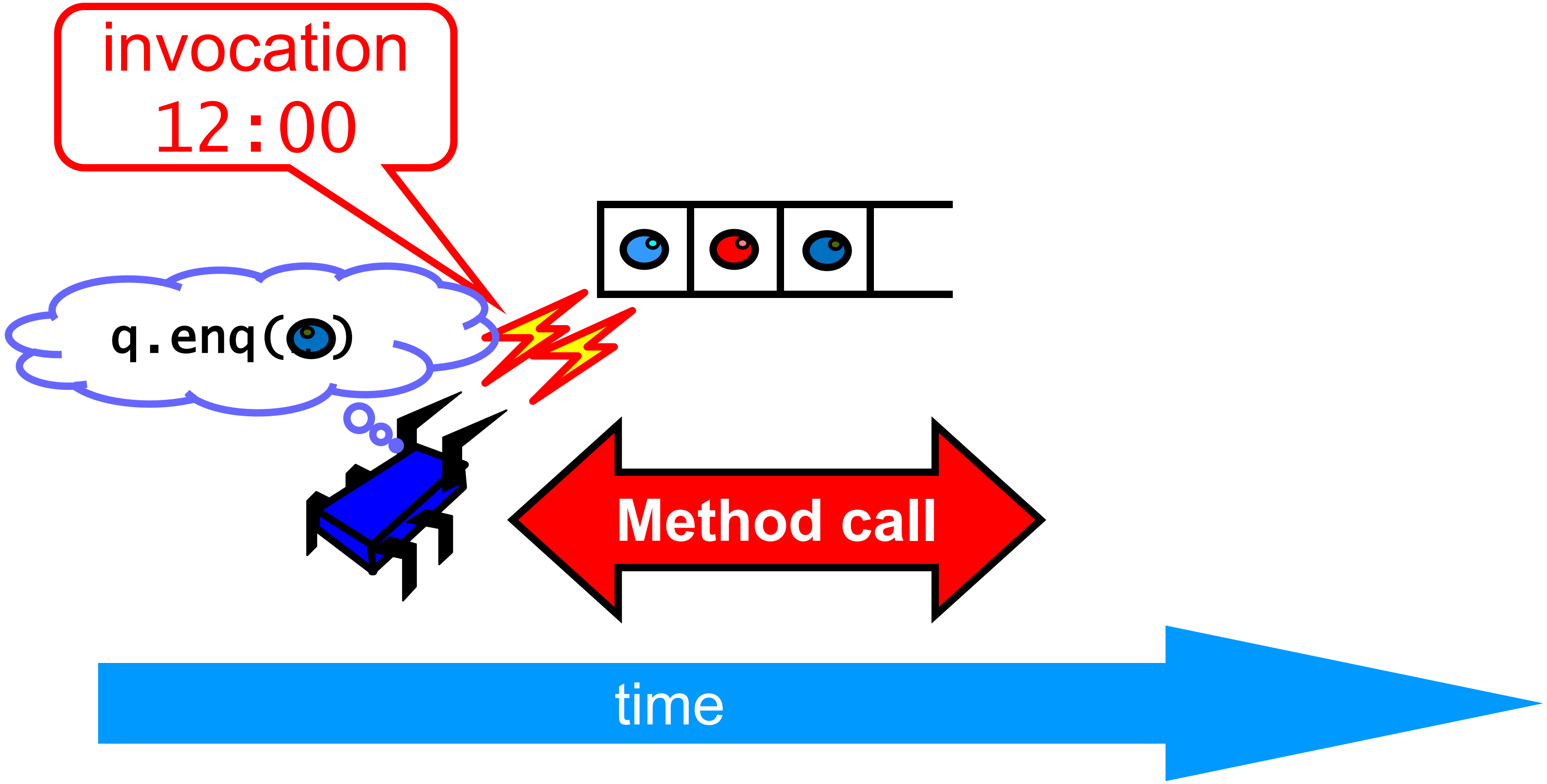

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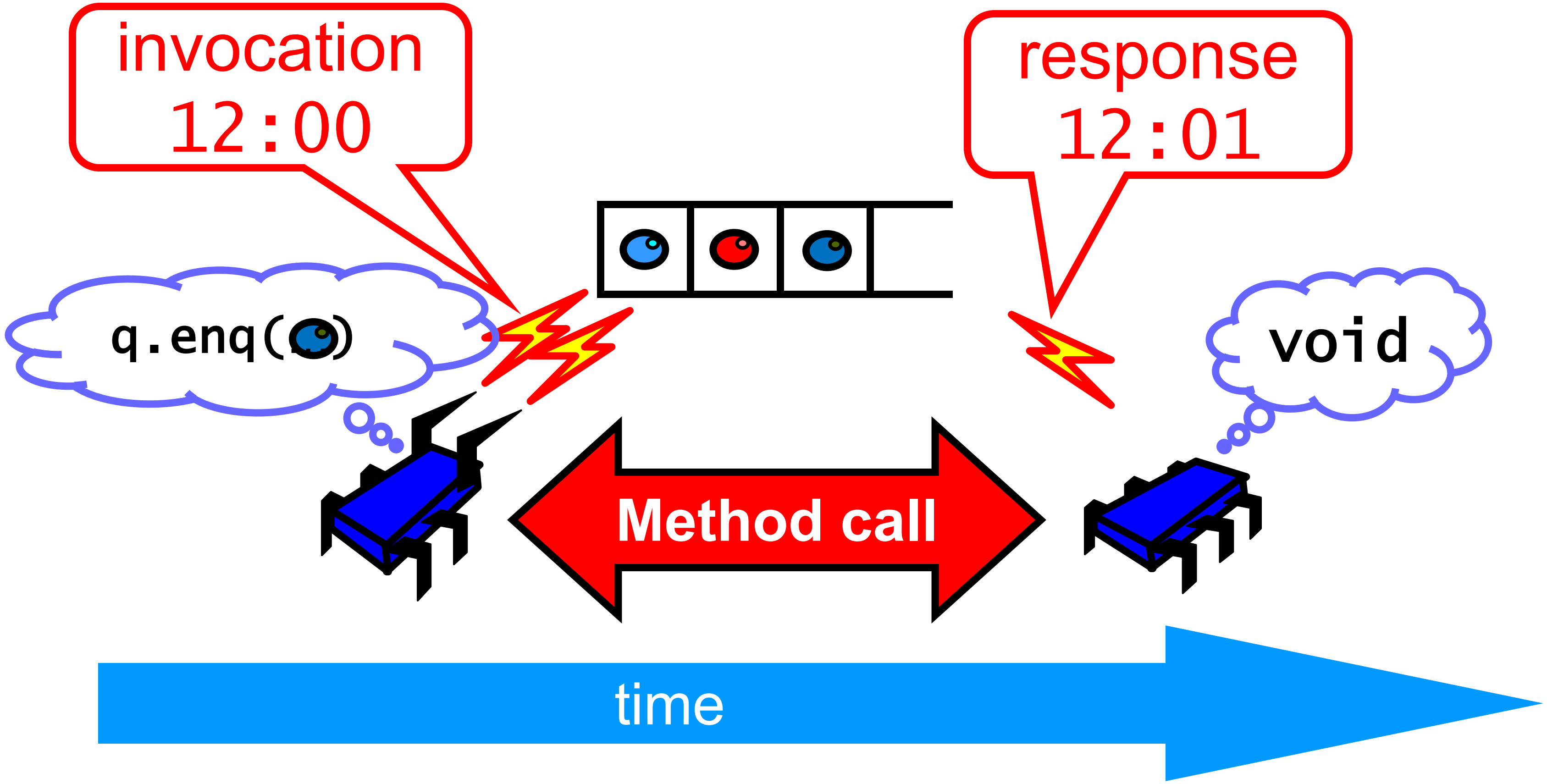
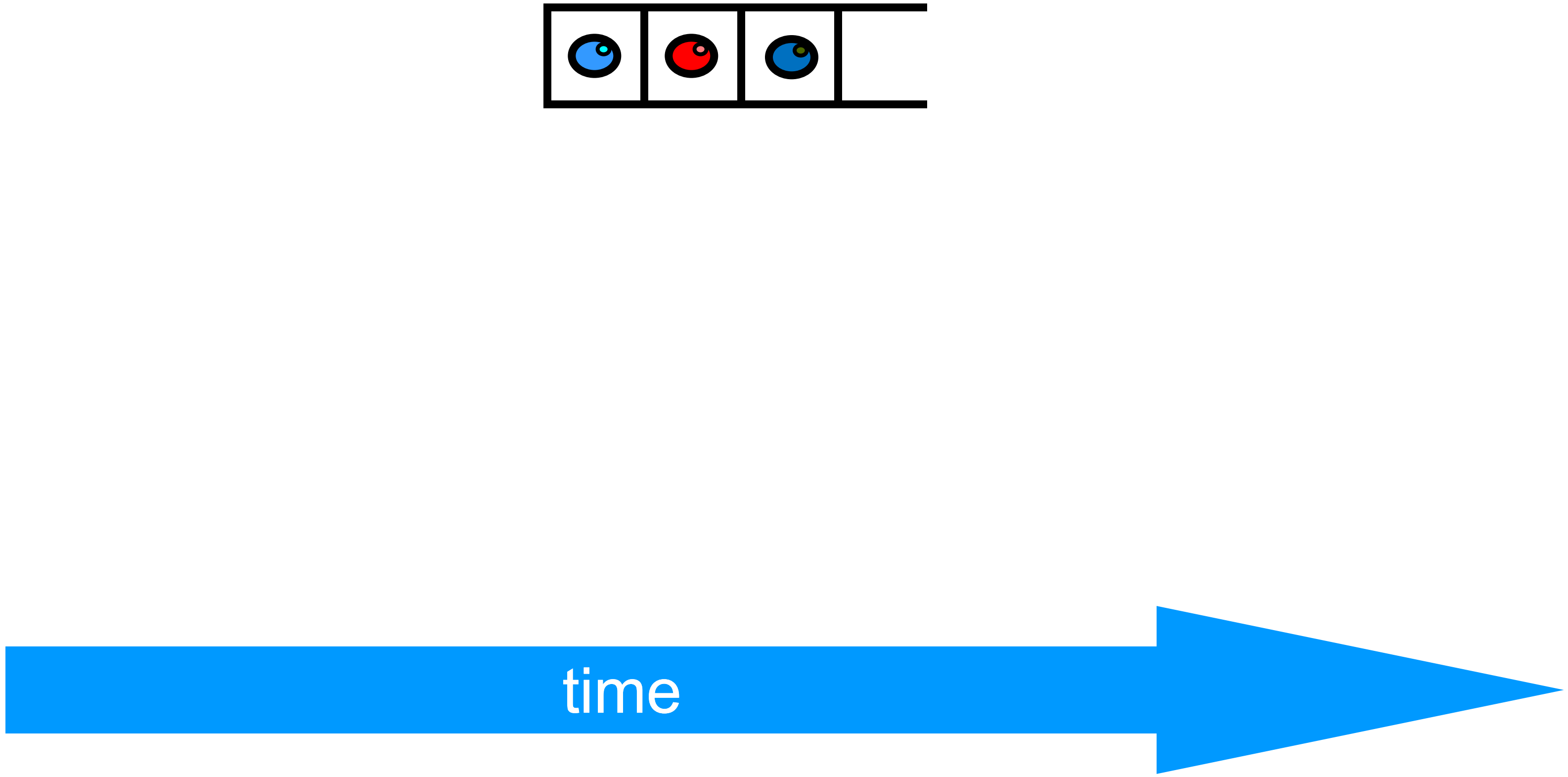


Sequential vs Concurrent

- Sequential
  - Methods take time? Who knew?
- Concurrent
  - Method call is not an event
  - Method call is an interval.

Concurrent Methods Take Overlapping Time



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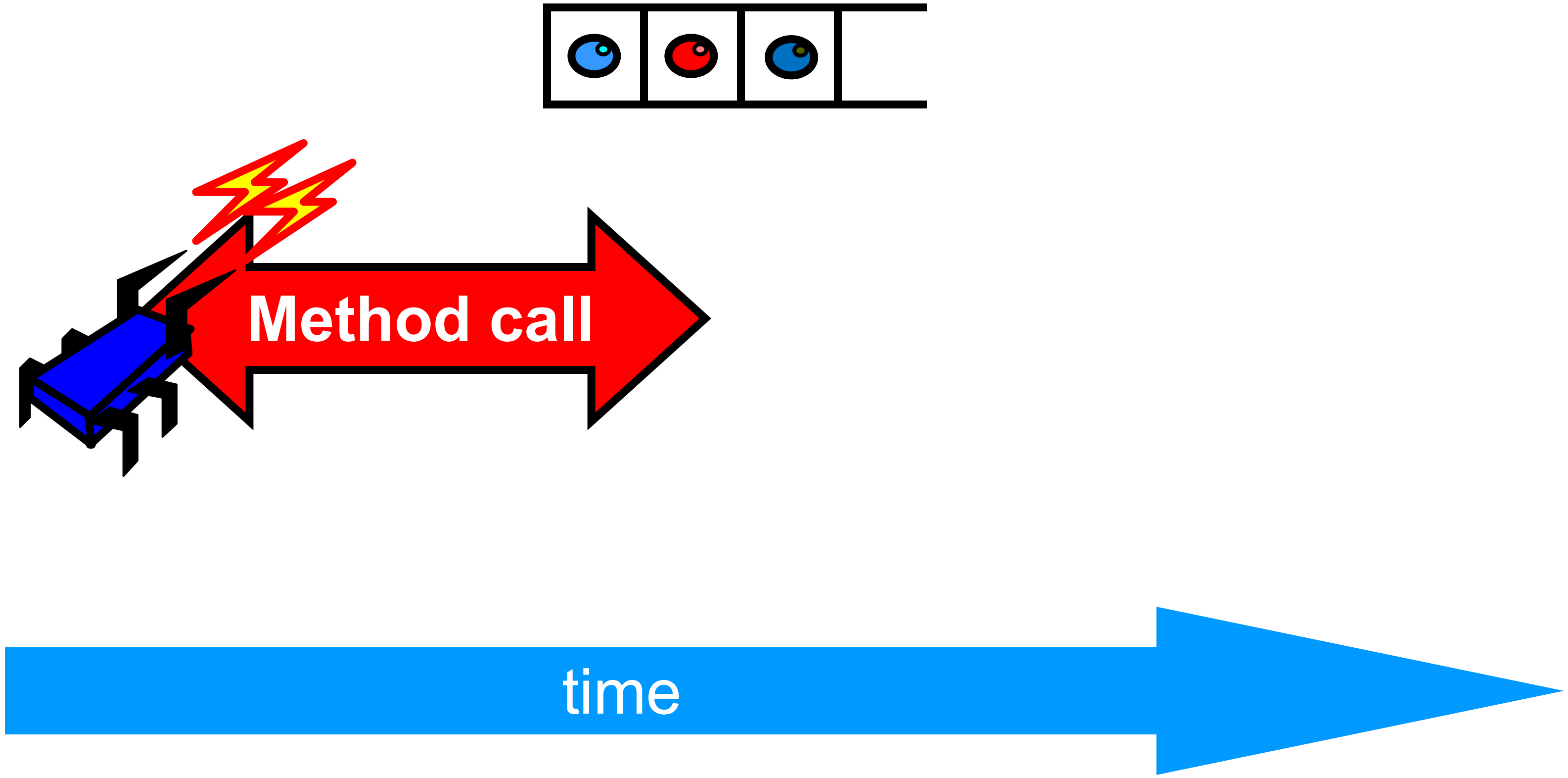

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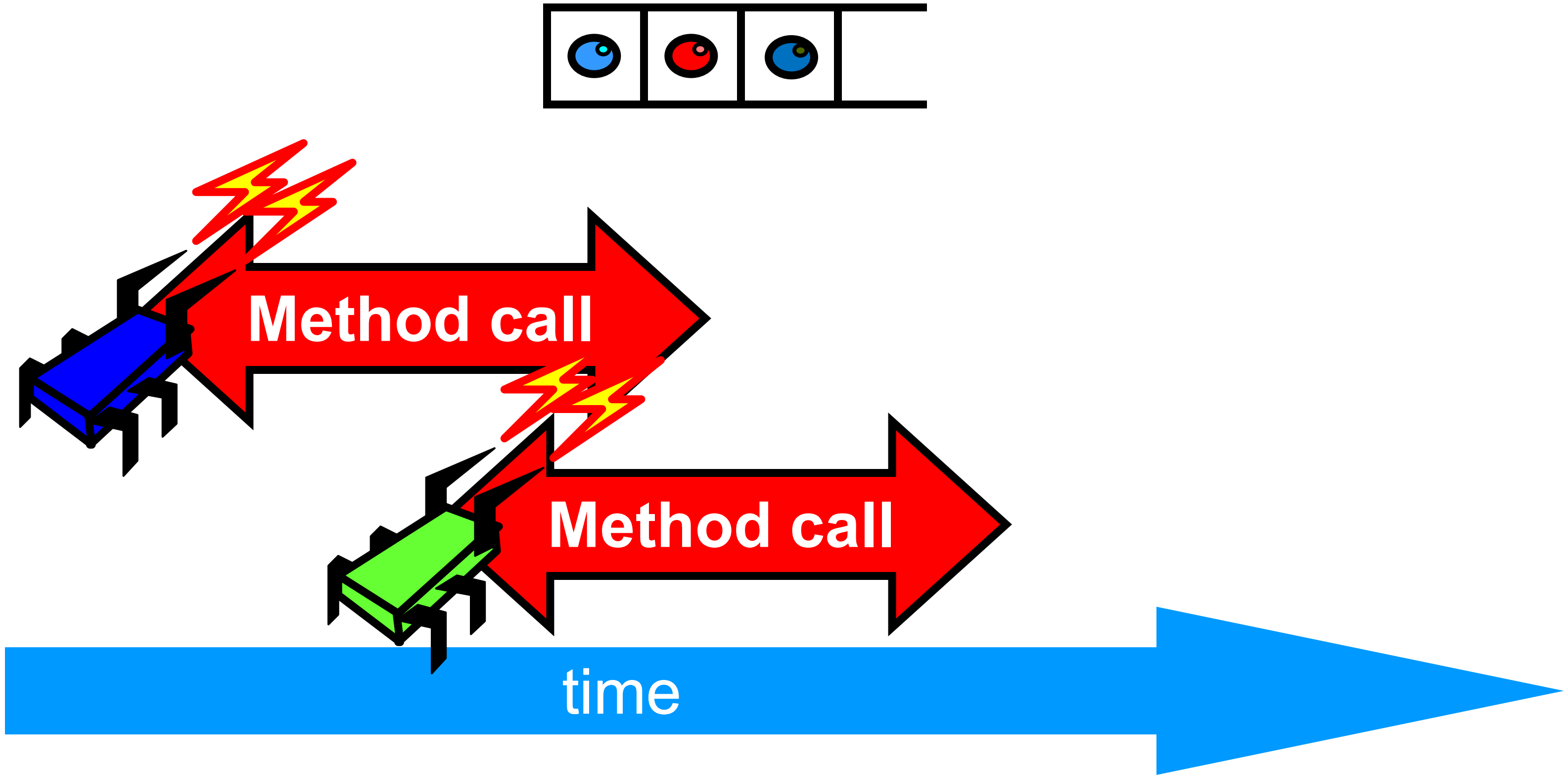

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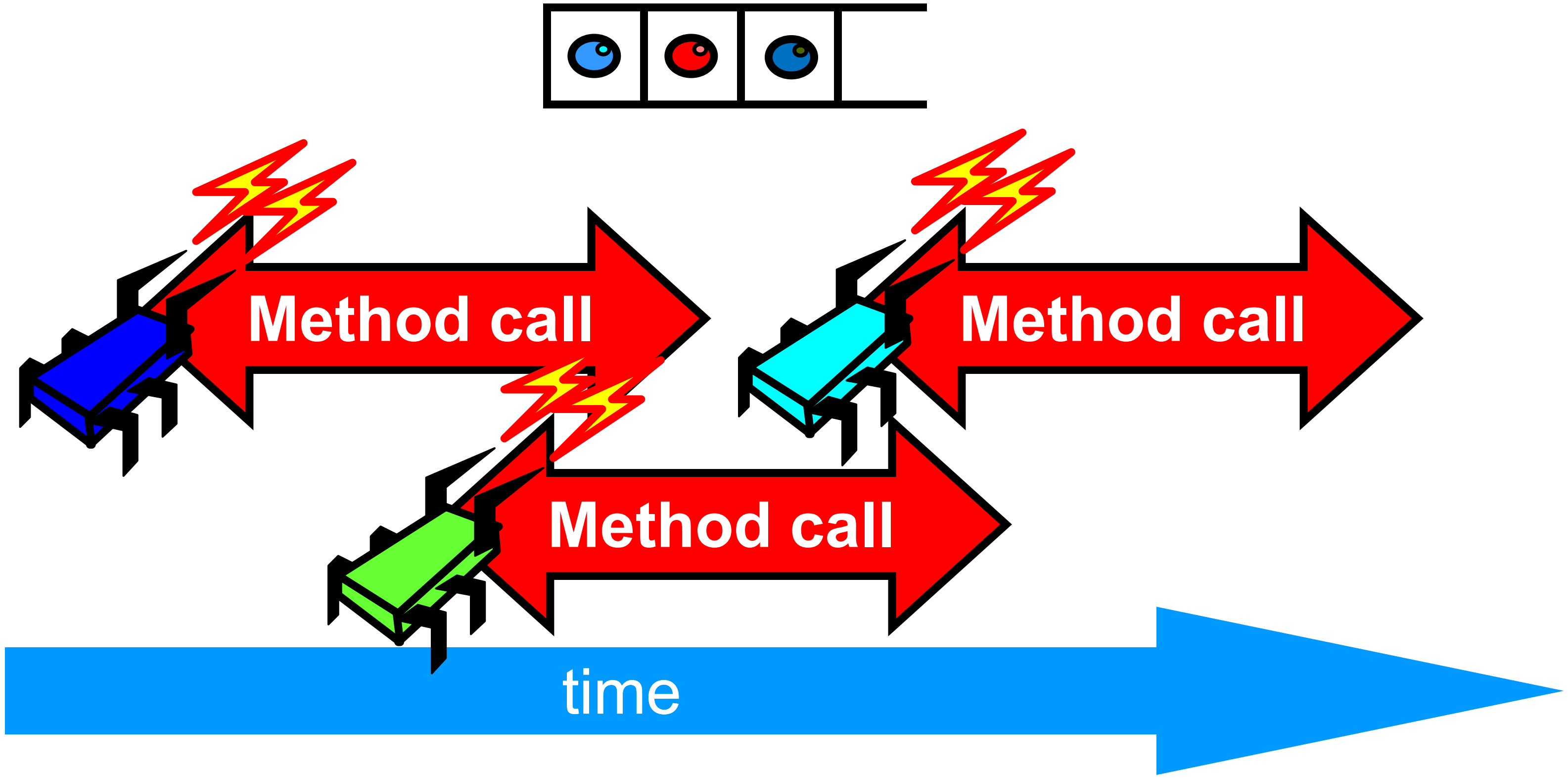

Sequential vs Concurrent

- Sequential:
  - Object needs meaningful state only ***between*** method calls
- Concurrent
  - Because method calls overlap, object might ***never*** be between method calls

Sequential vs Concurrent

- Sequential:
  - Each method described in isolation
- Concurrent
  - Must characterize ***all*** possible interactions with concurrent calls
    - What if two `enq`s overlap?
    - Two `deq`s? `enq` and `deq`? …

Sequential vs Concurrent

- Sequential:
  - Can add new methods without affecting older methods
- Concurrent:
  - Everything can potentially interact with everything else

Sequential vs Concurrent

- Sequential:
  - Can add new methods without affecting older methods
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  - Everything can potentially interact with everything else

Panic!

The Big Question

- What does it mean for a *concurrent* object to be correct?
  - What *is* a concurrent FIFO queue?
  - FIFO means strict temporal order
  - Concurrent means ambiguous temporal order

Intuitively…

```
public T deq() throws EmptyException {
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  try {
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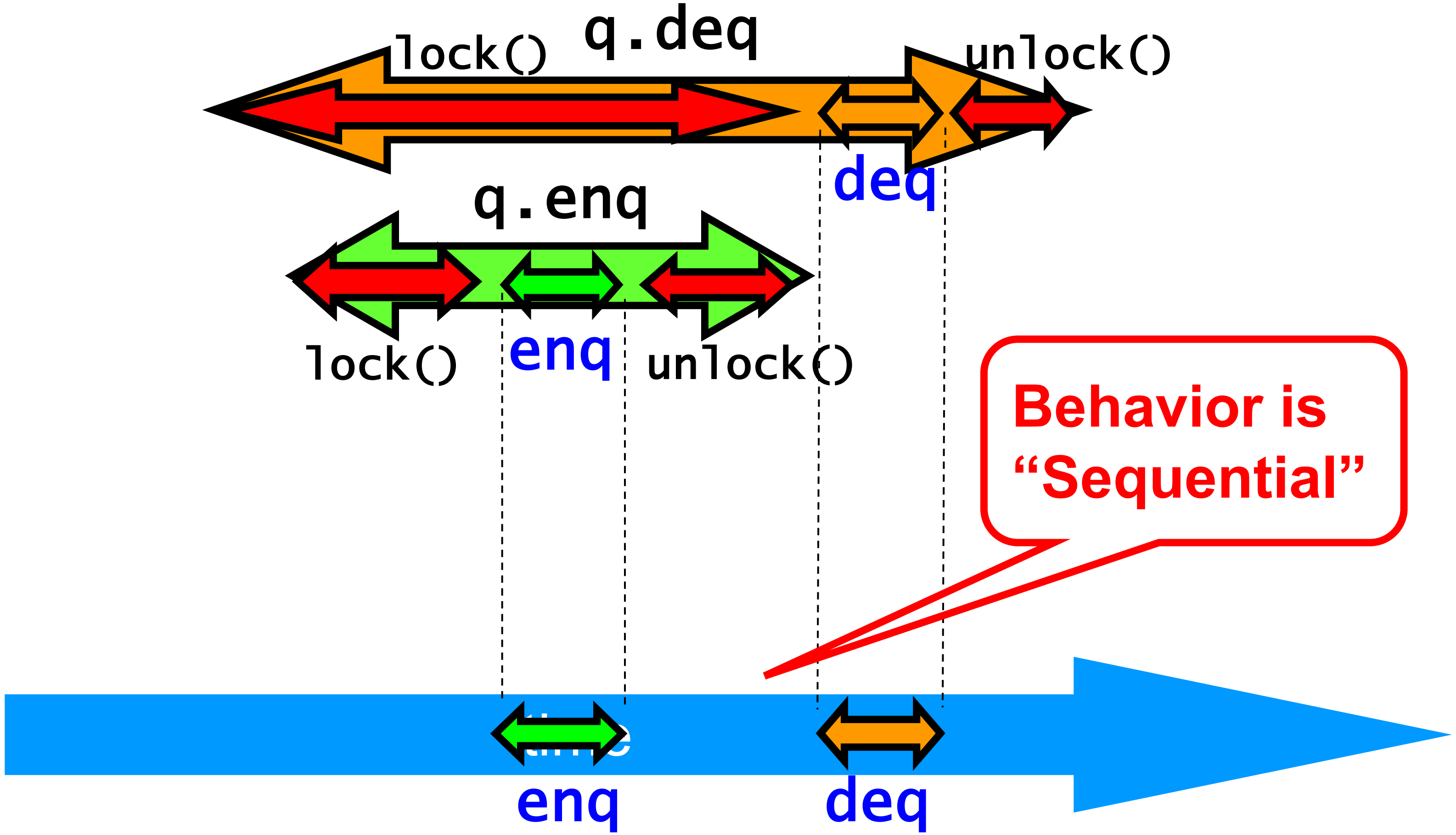
```

All modifications of queue are done mutually exclusive

Intuitively

Let's capture the idea of describing the concurrent via the sequential



Behavior is "Sequential"

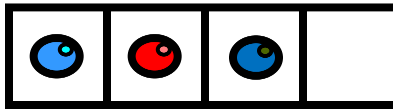
Linearizability

- Each method should
  - "take effect"
  - Instantaneously
  - Between invocation and response events
- Object is correct if this "sequential" behavior is correct
- Any such concurrent object is
  - **Linearizable™**

Is it really about the object?

- Each method should
  - "take effect"
  - Instantaneously
  - Between invocation and response events
- Sounds like a property of an execution…
- A linearizable object: one all of whose possible executions are linearizable

Example



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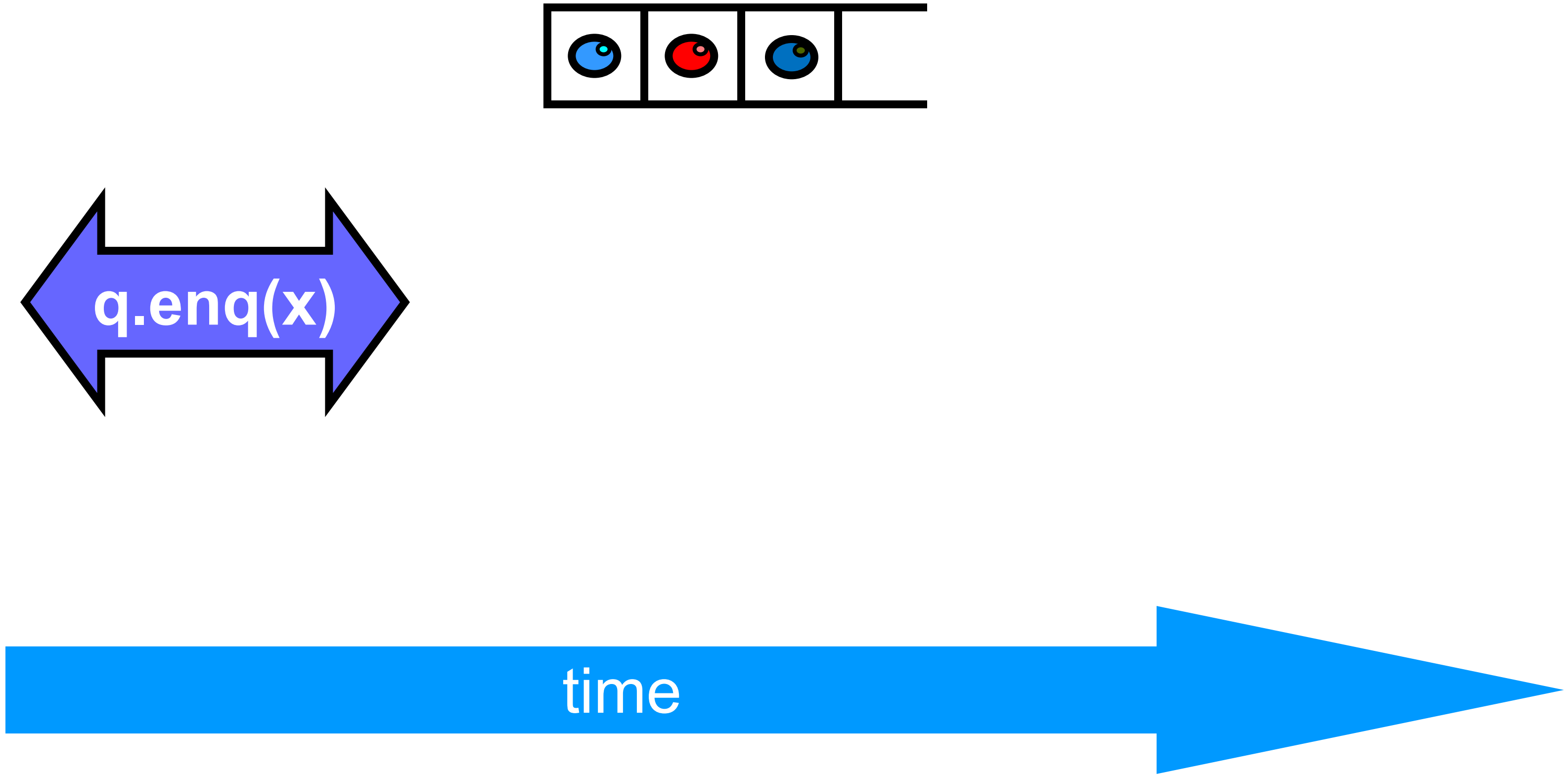

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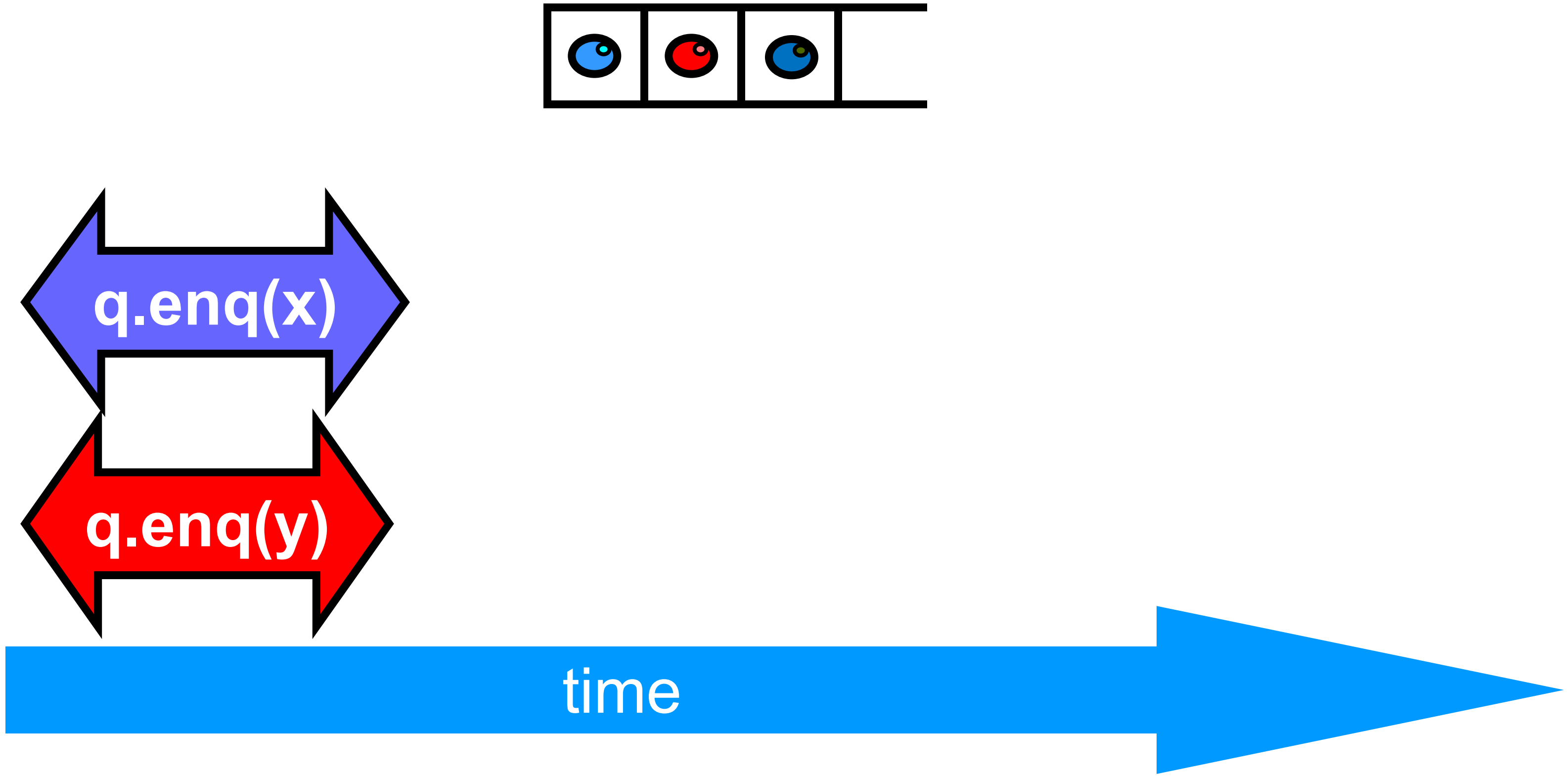

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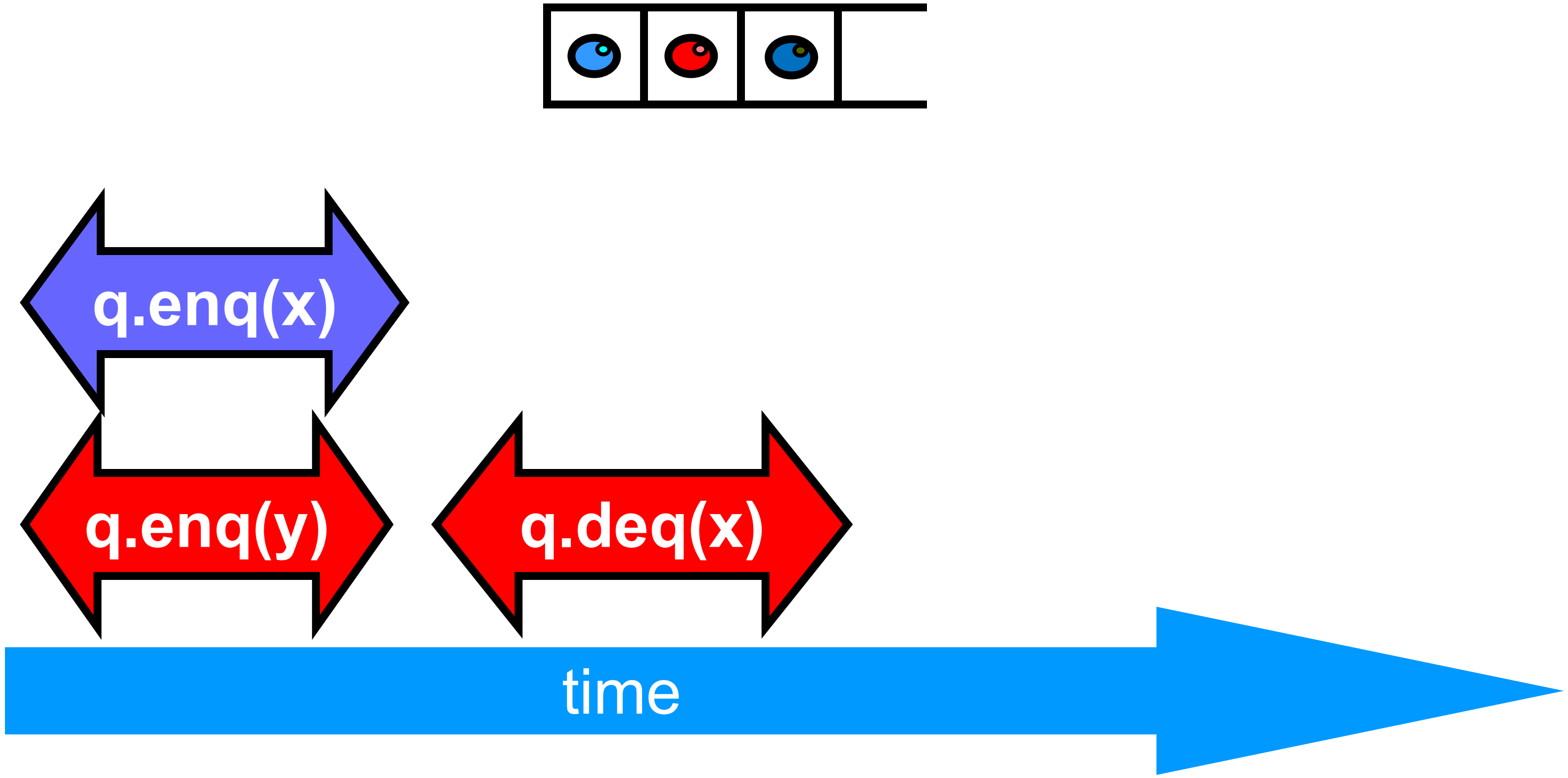

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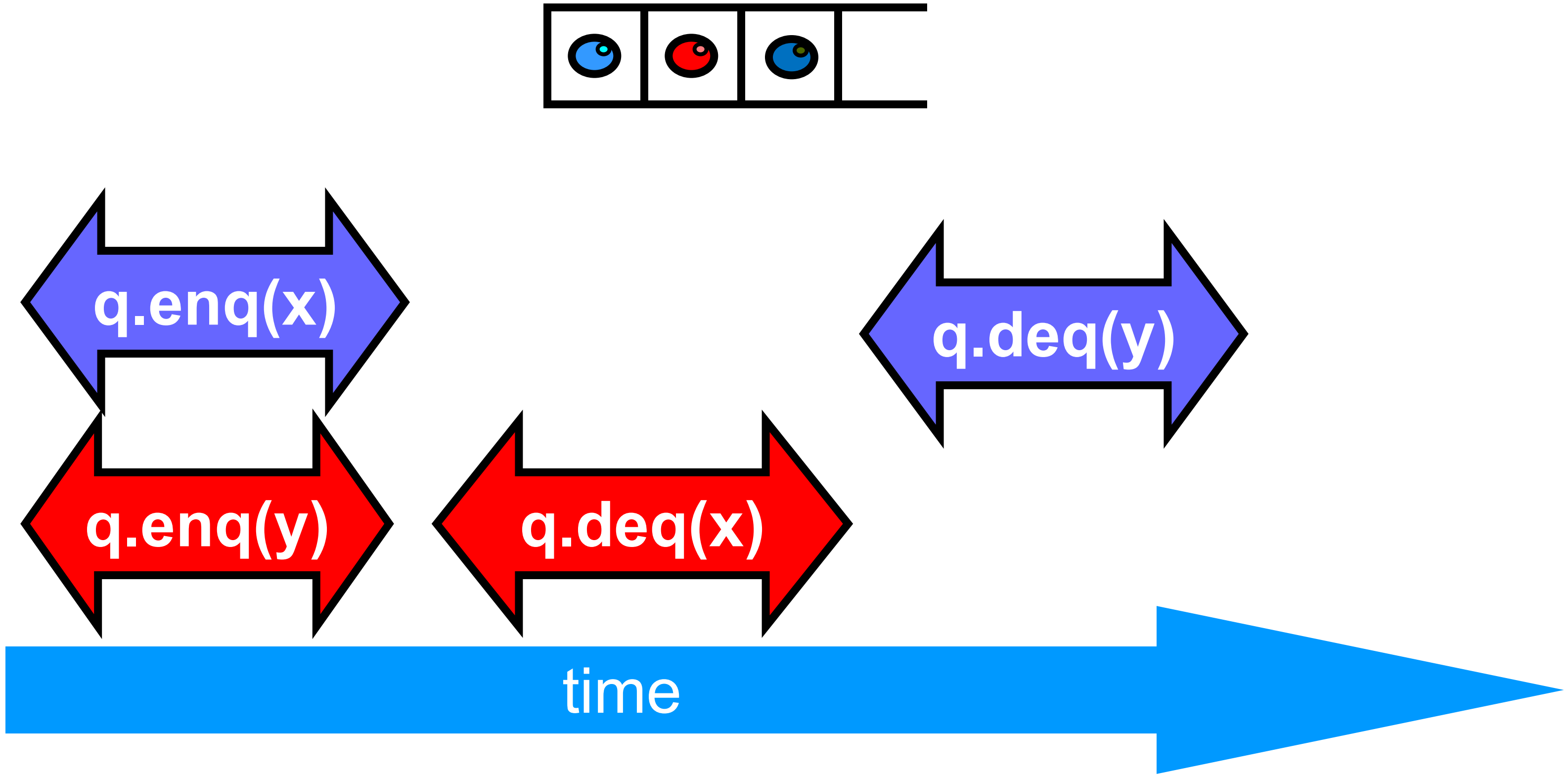

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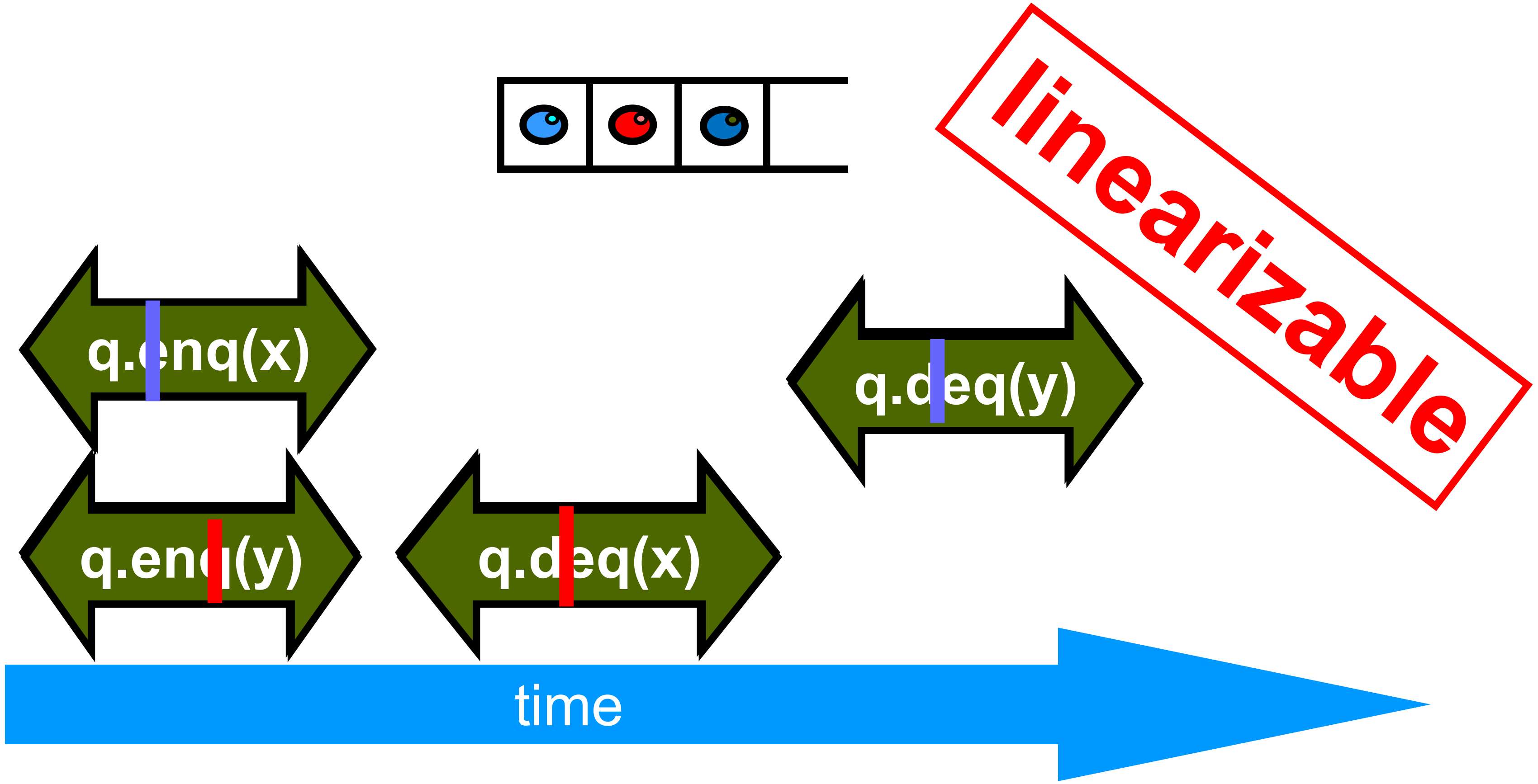

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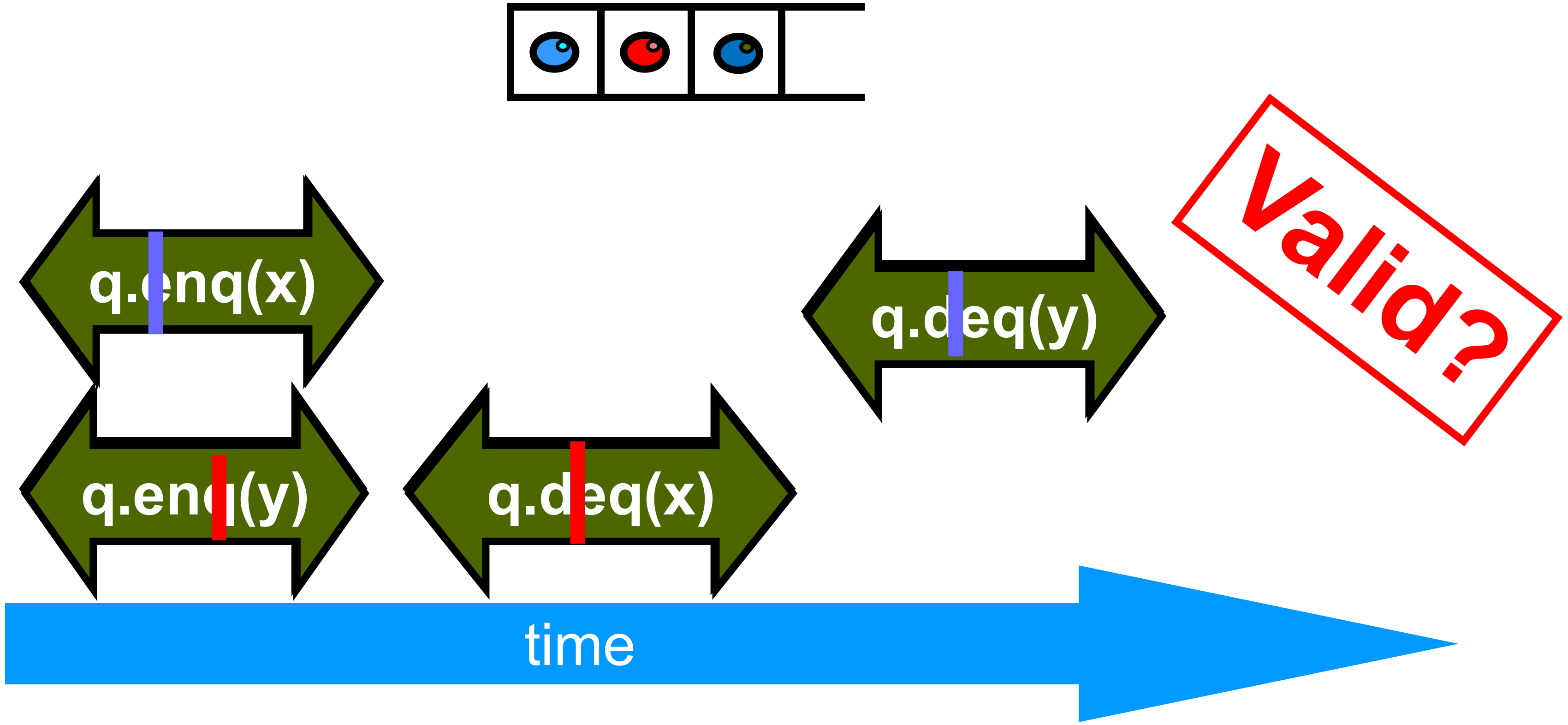

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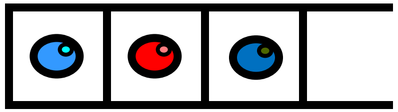

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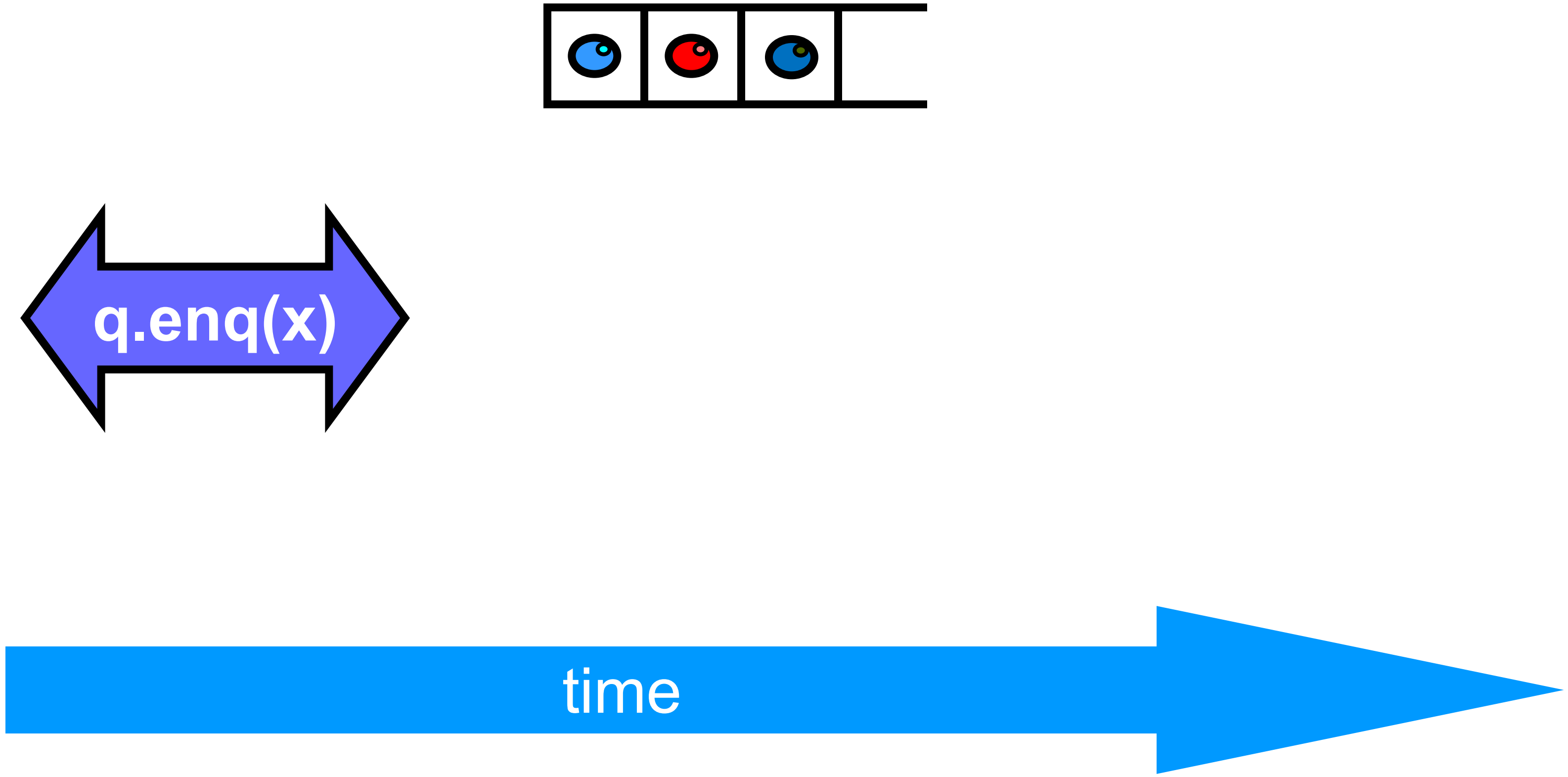

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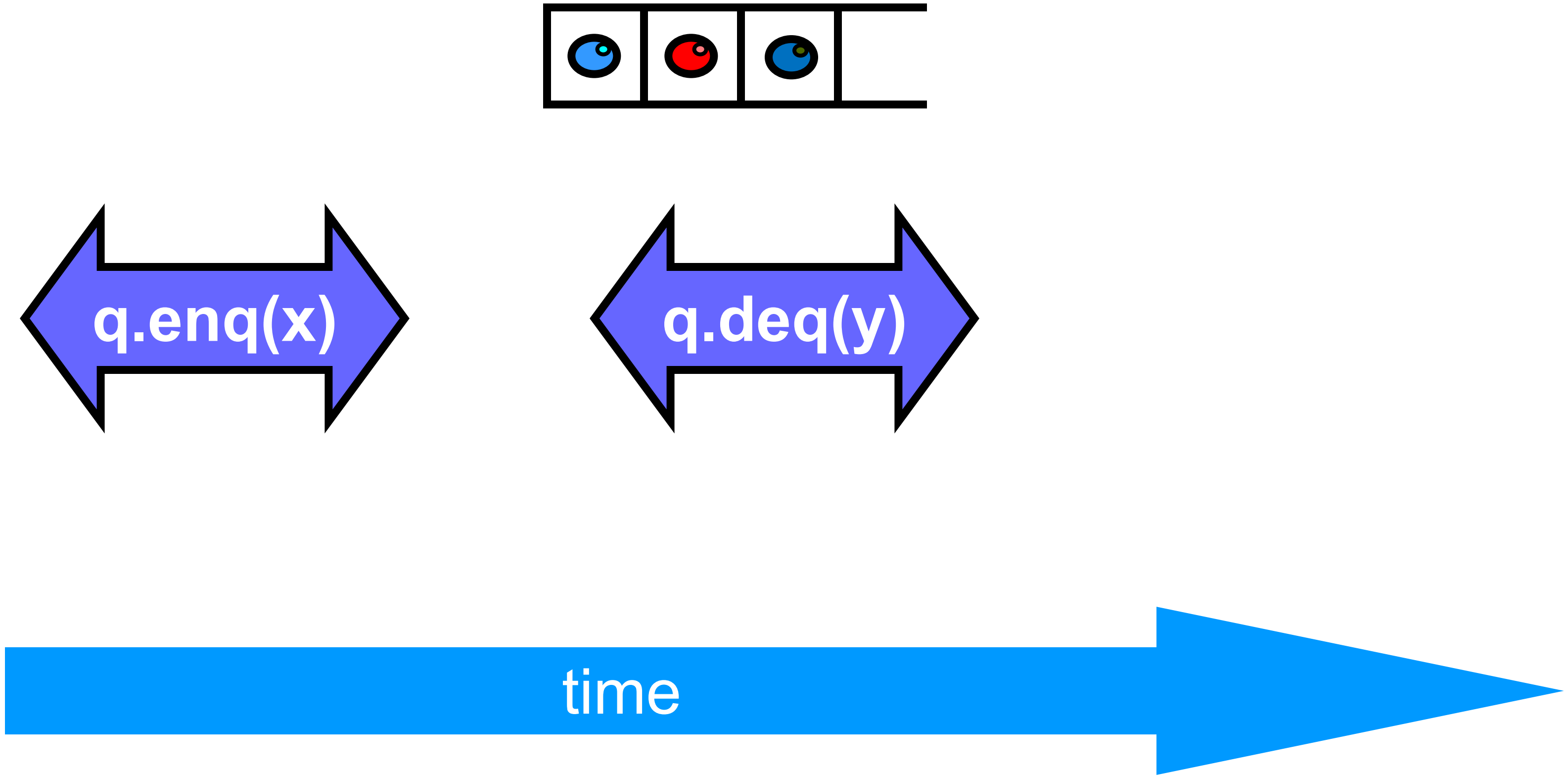

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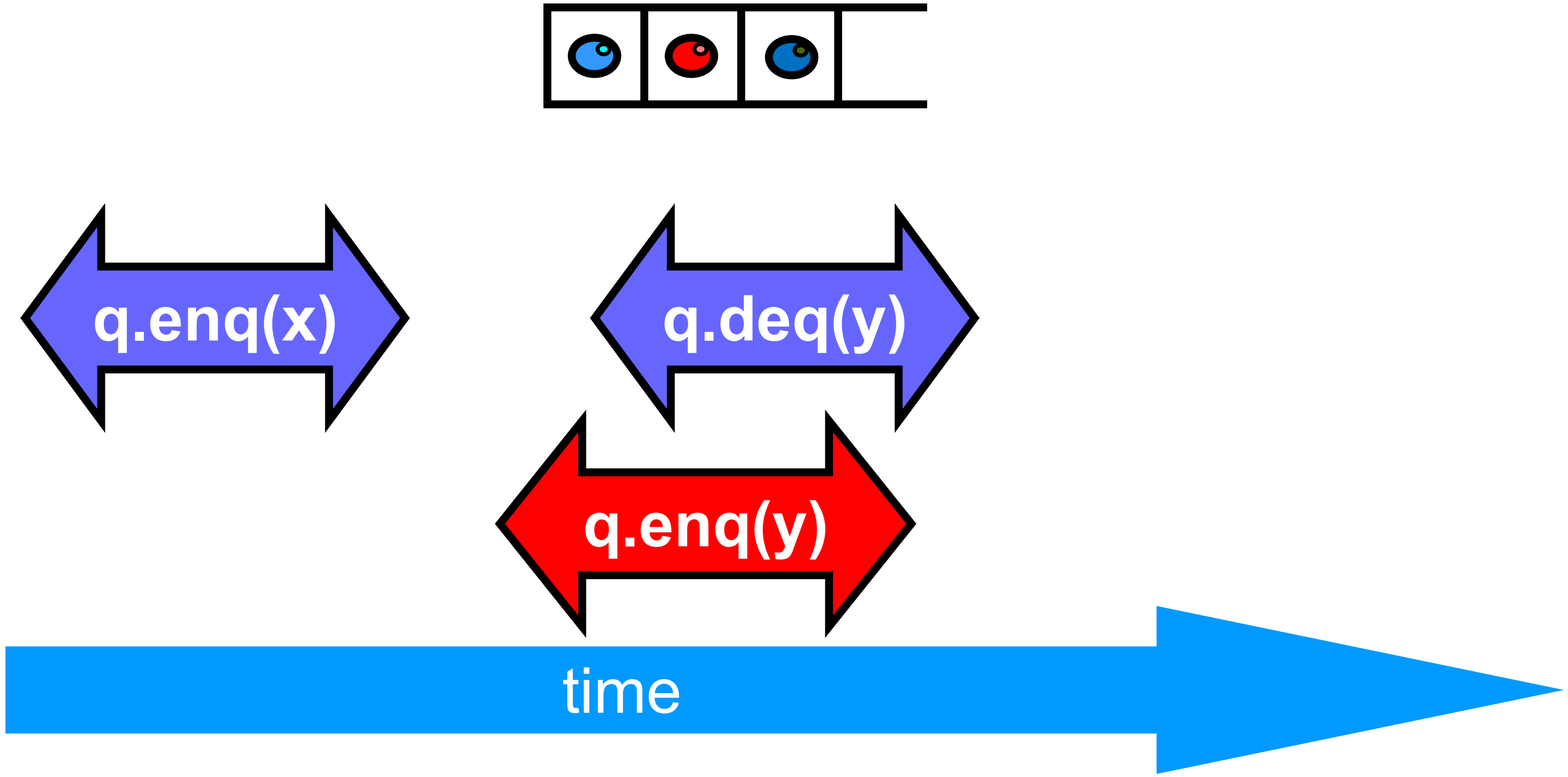

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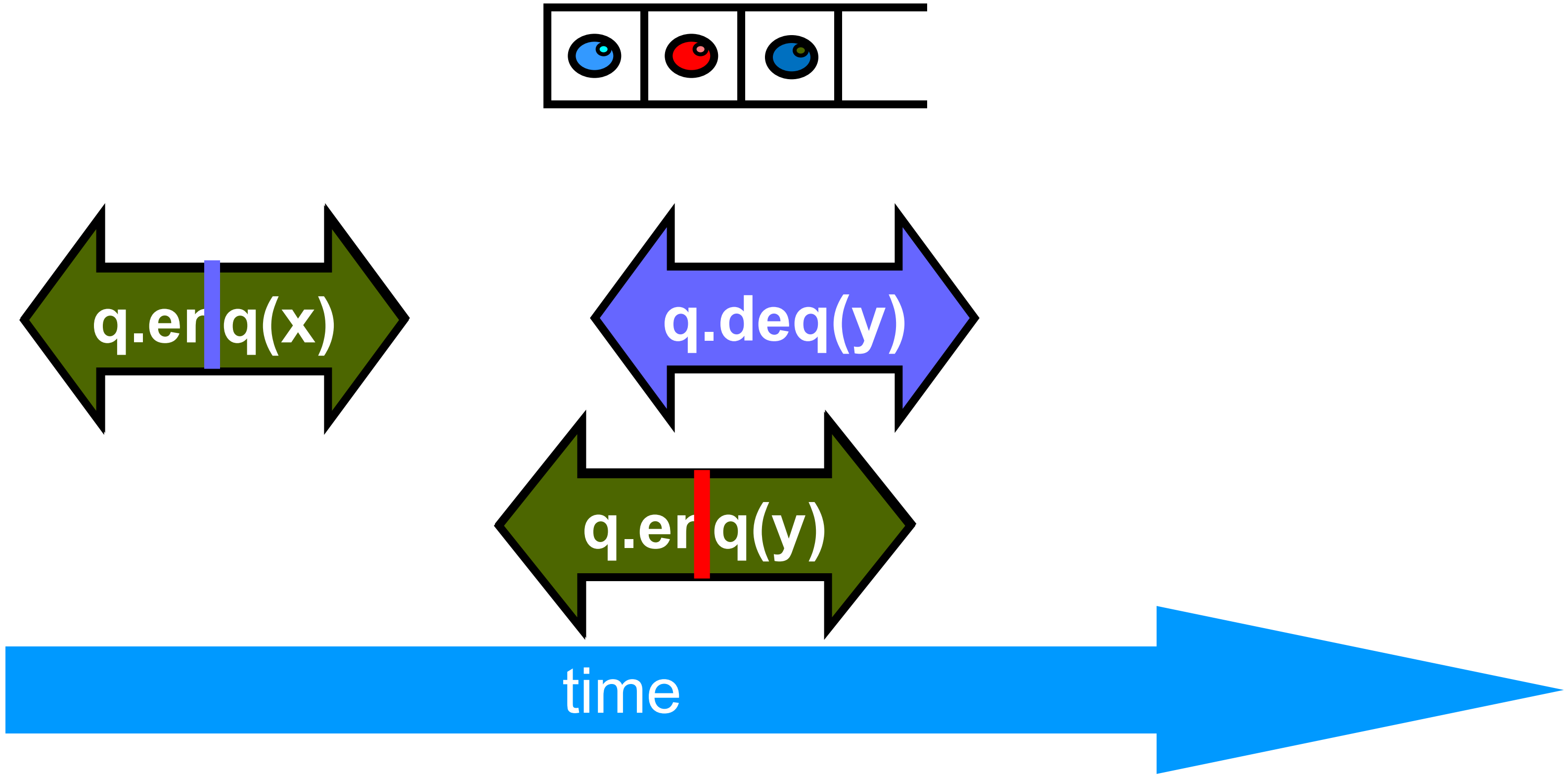

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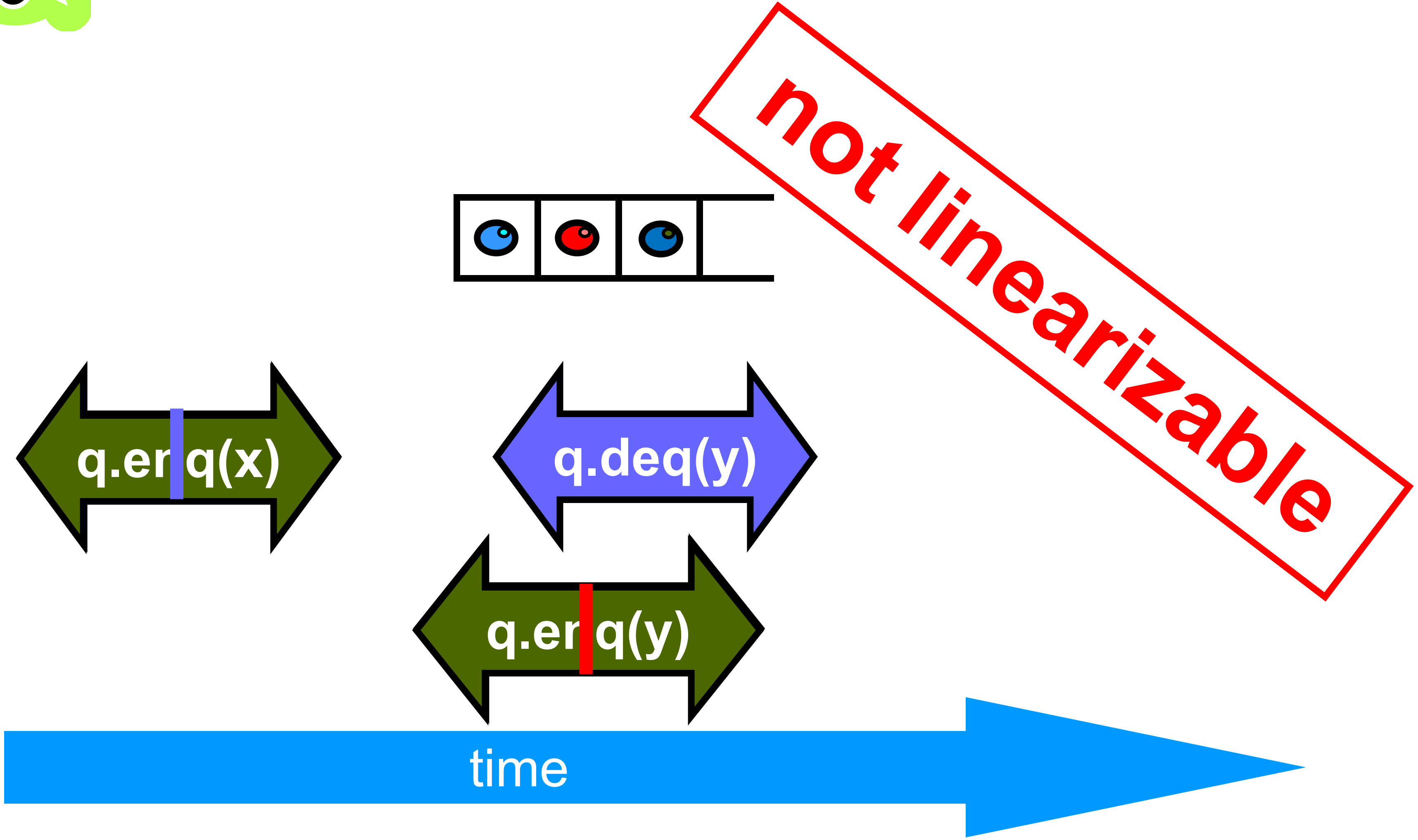

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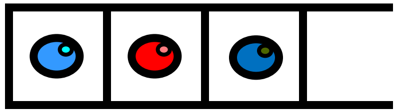

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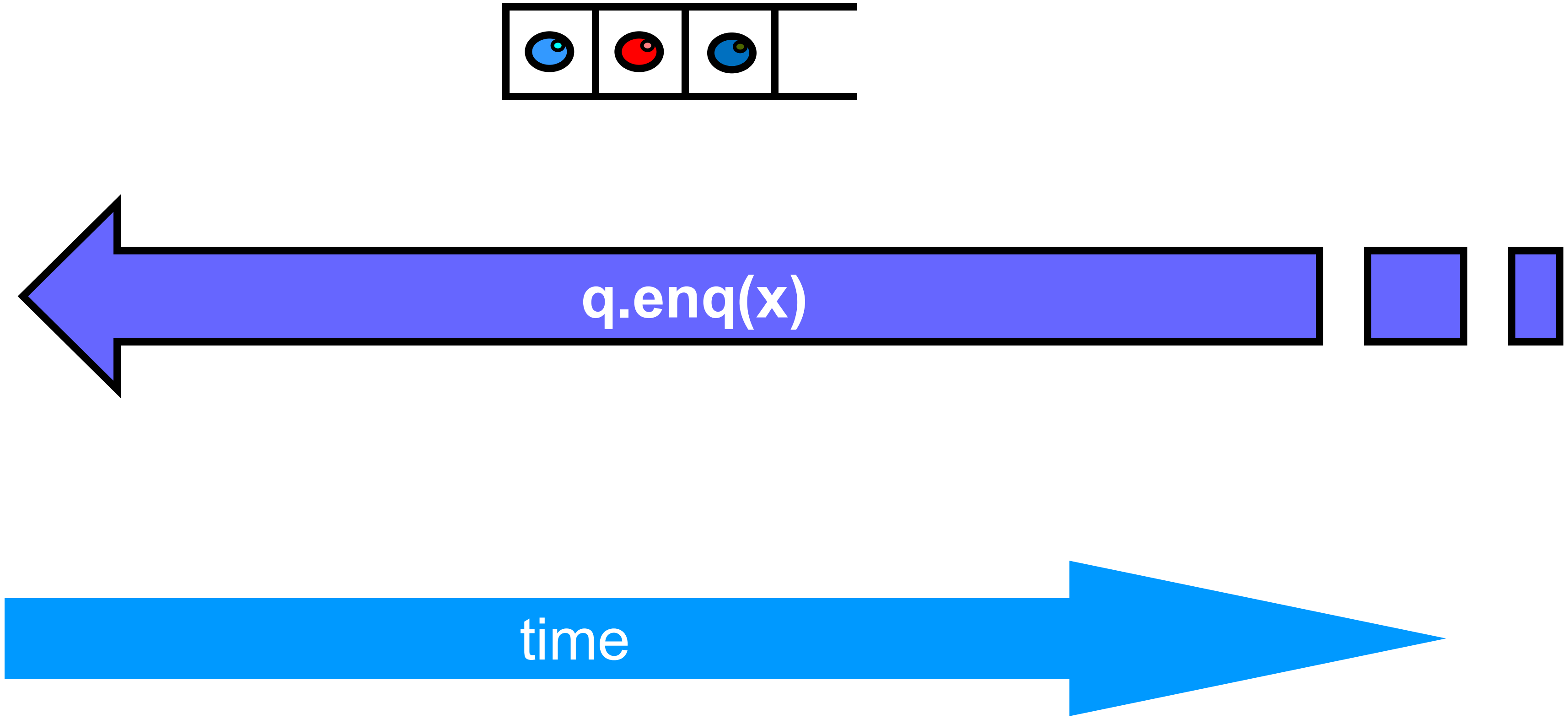

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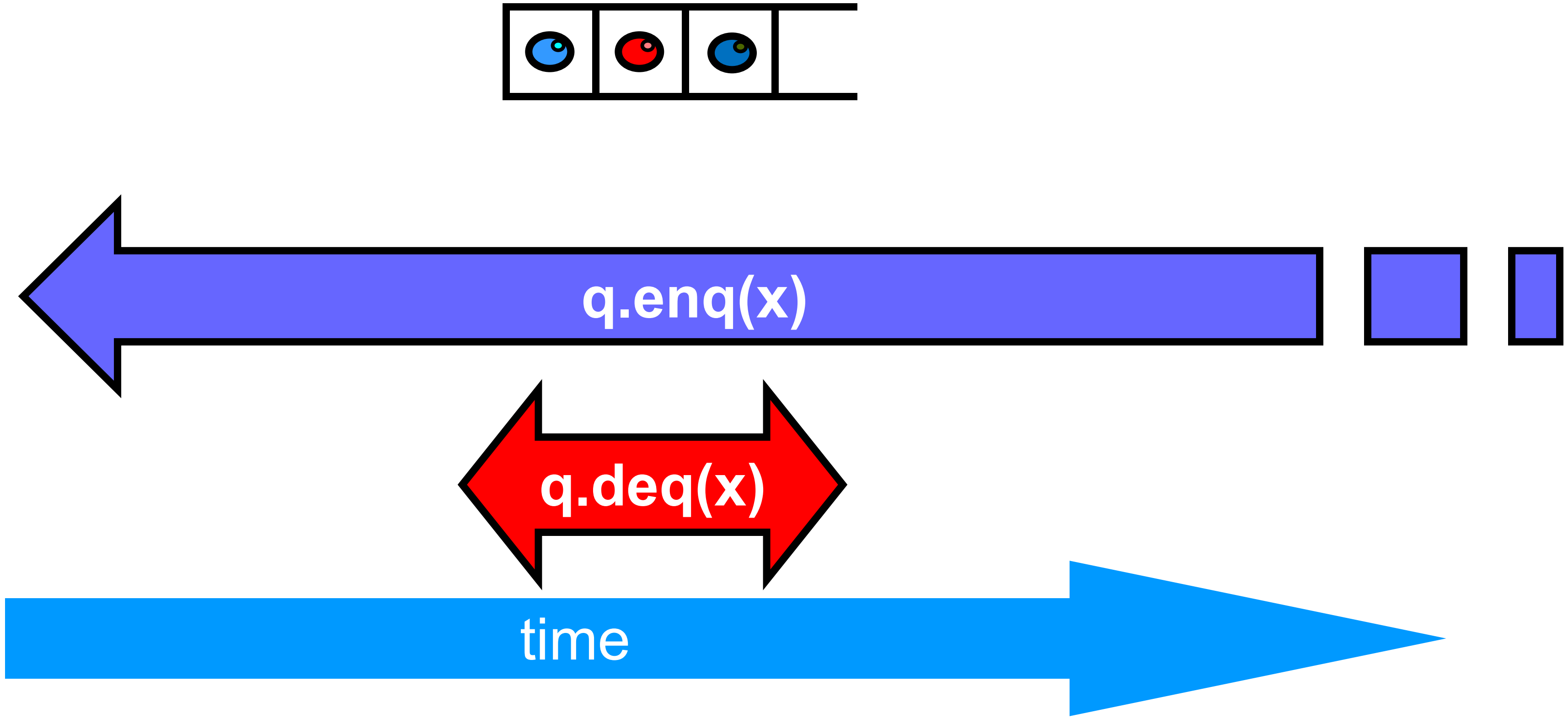

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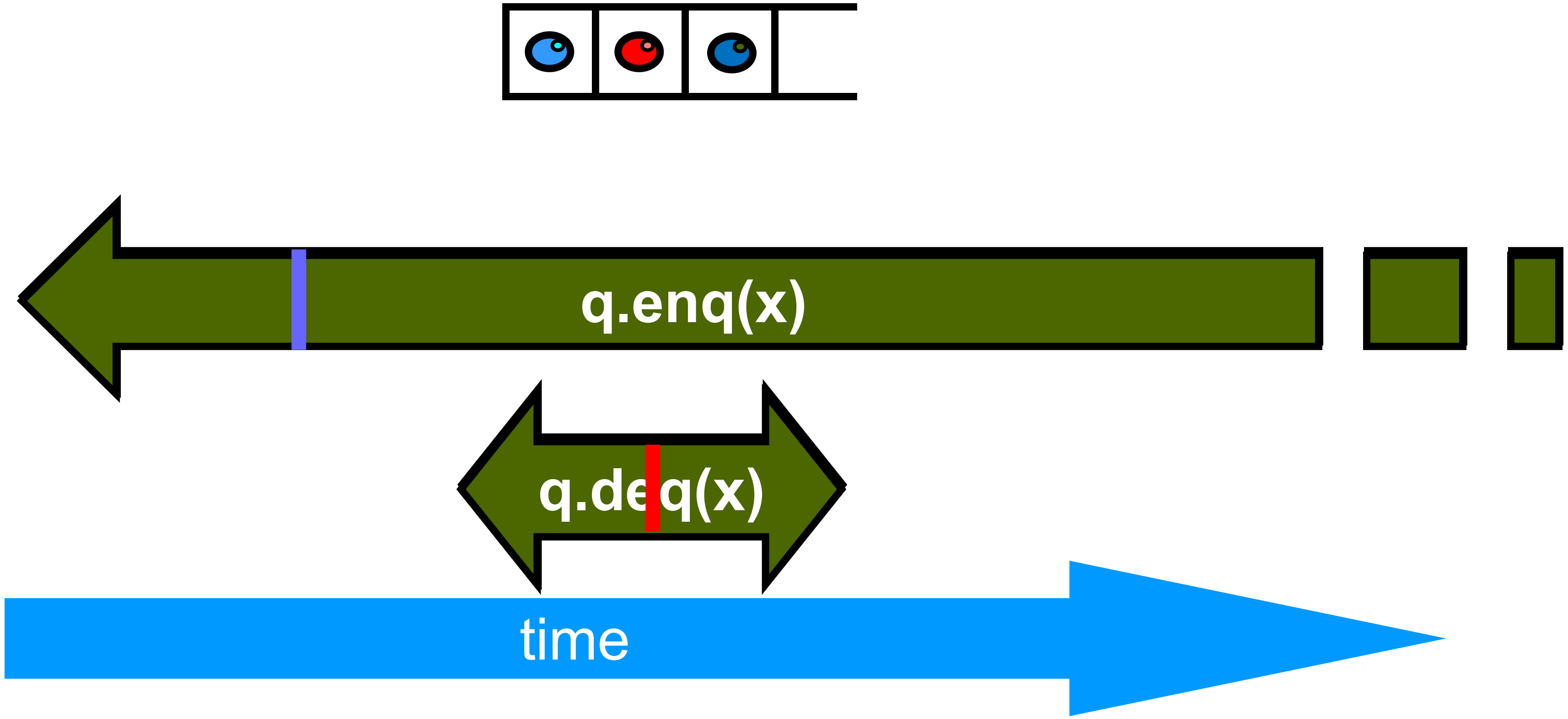

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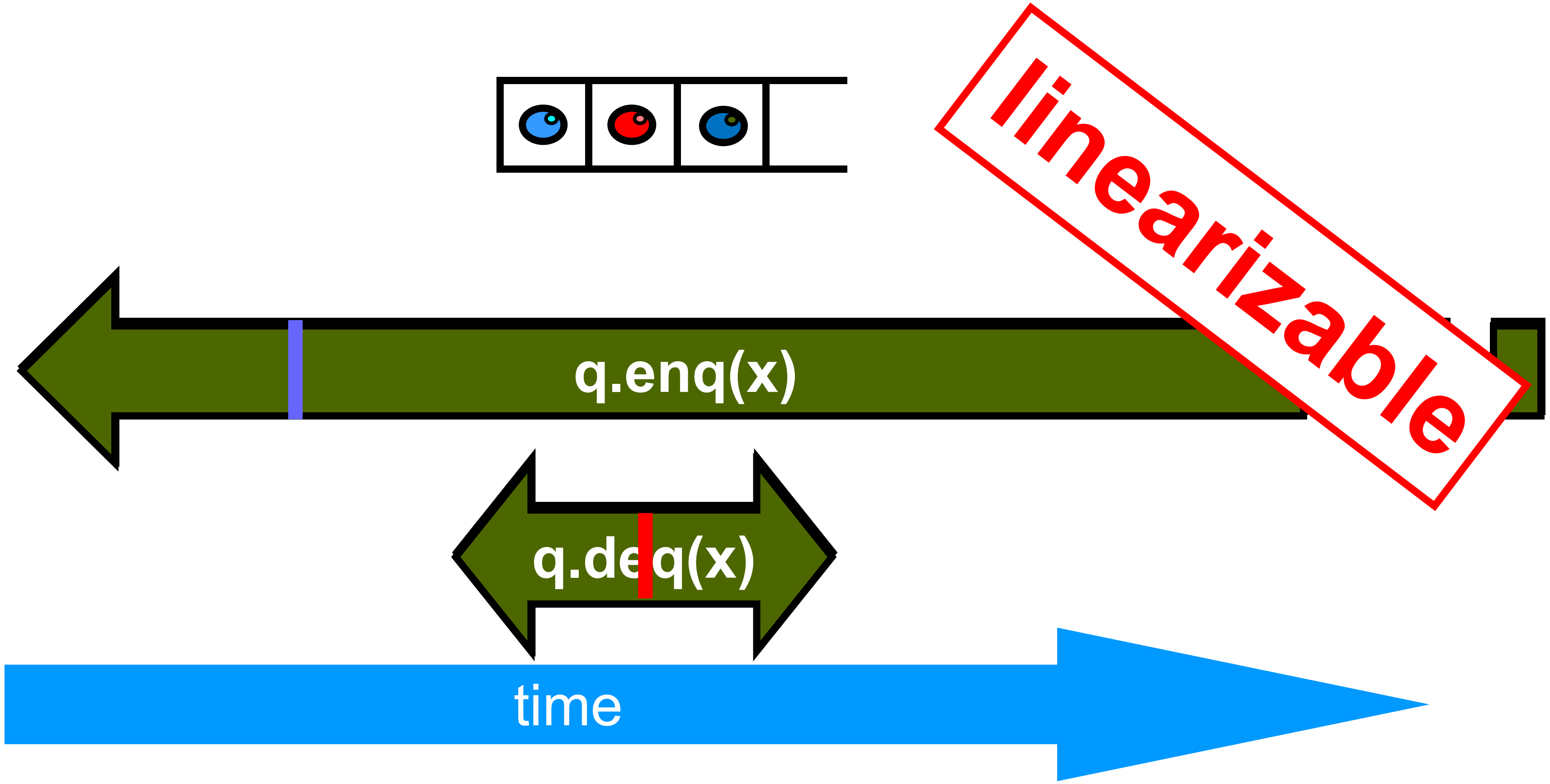

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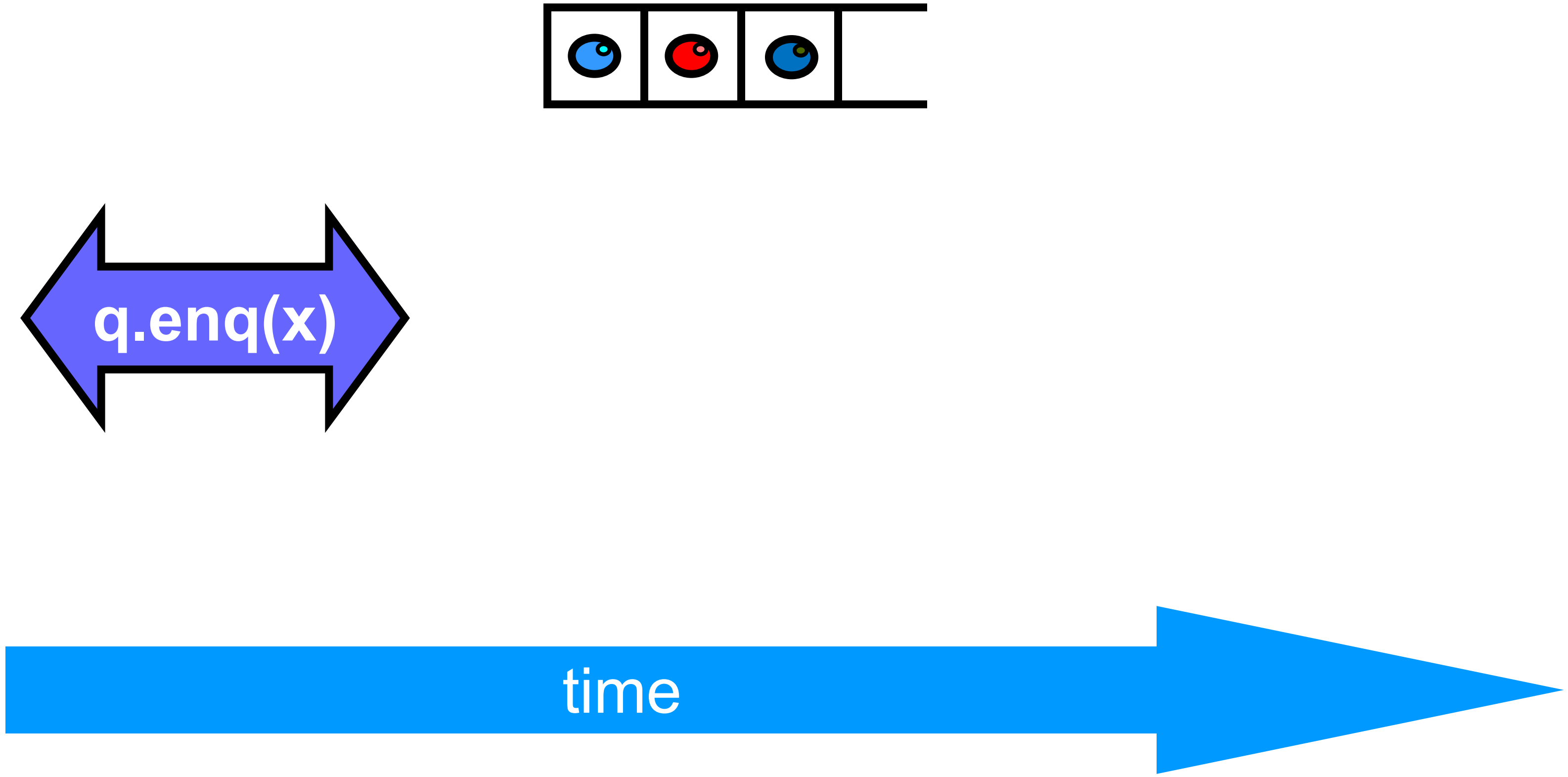

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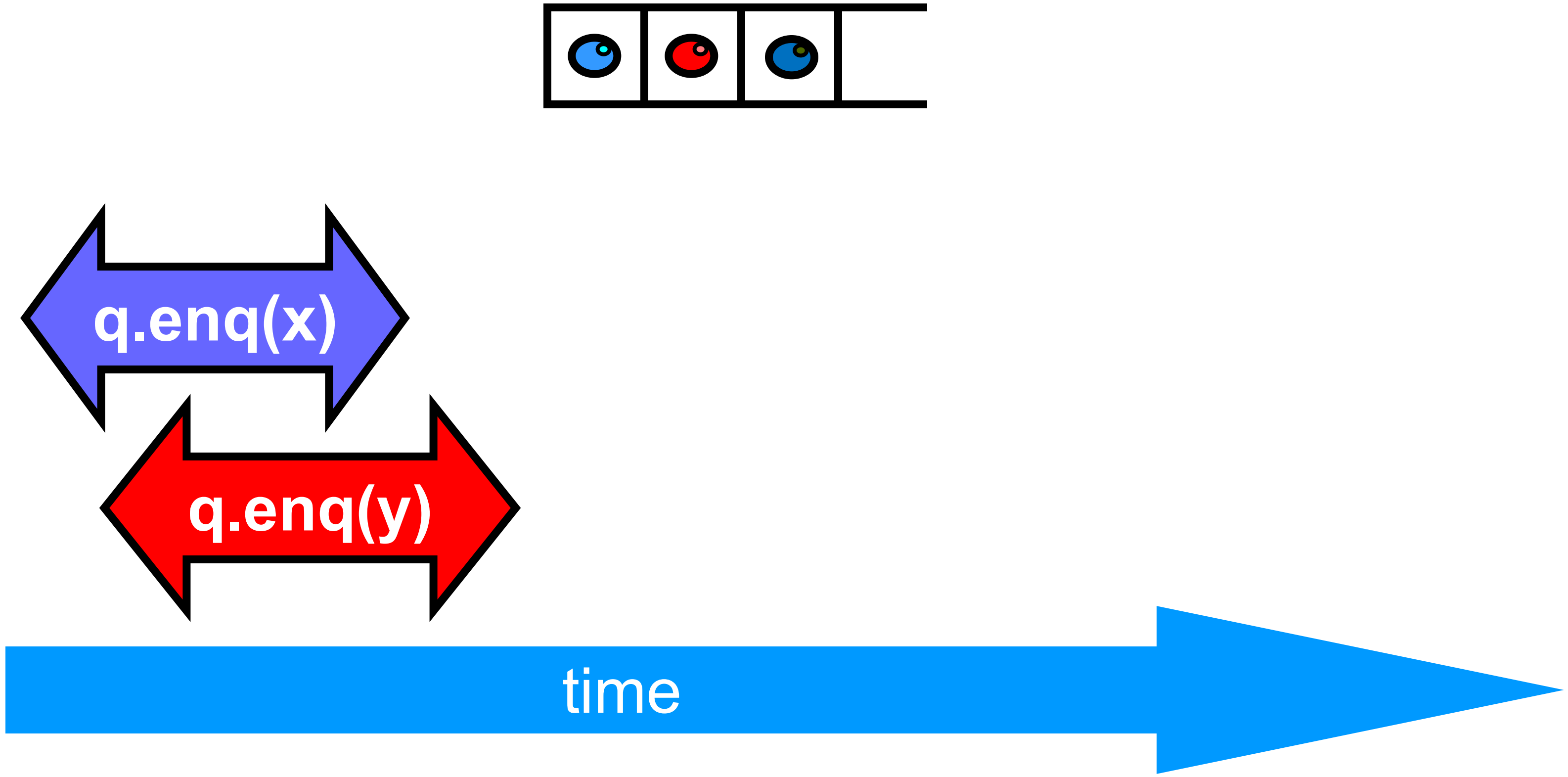

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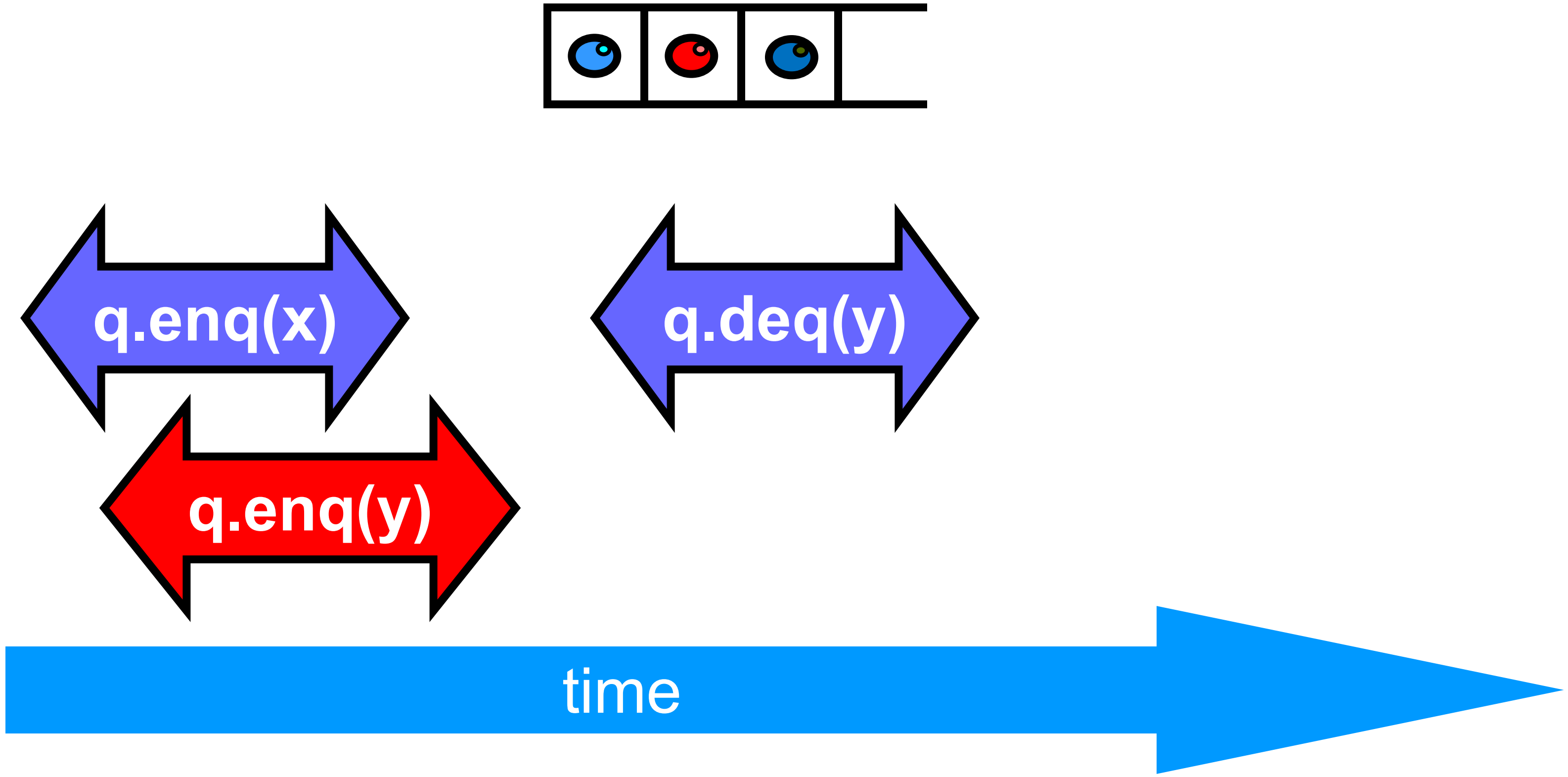

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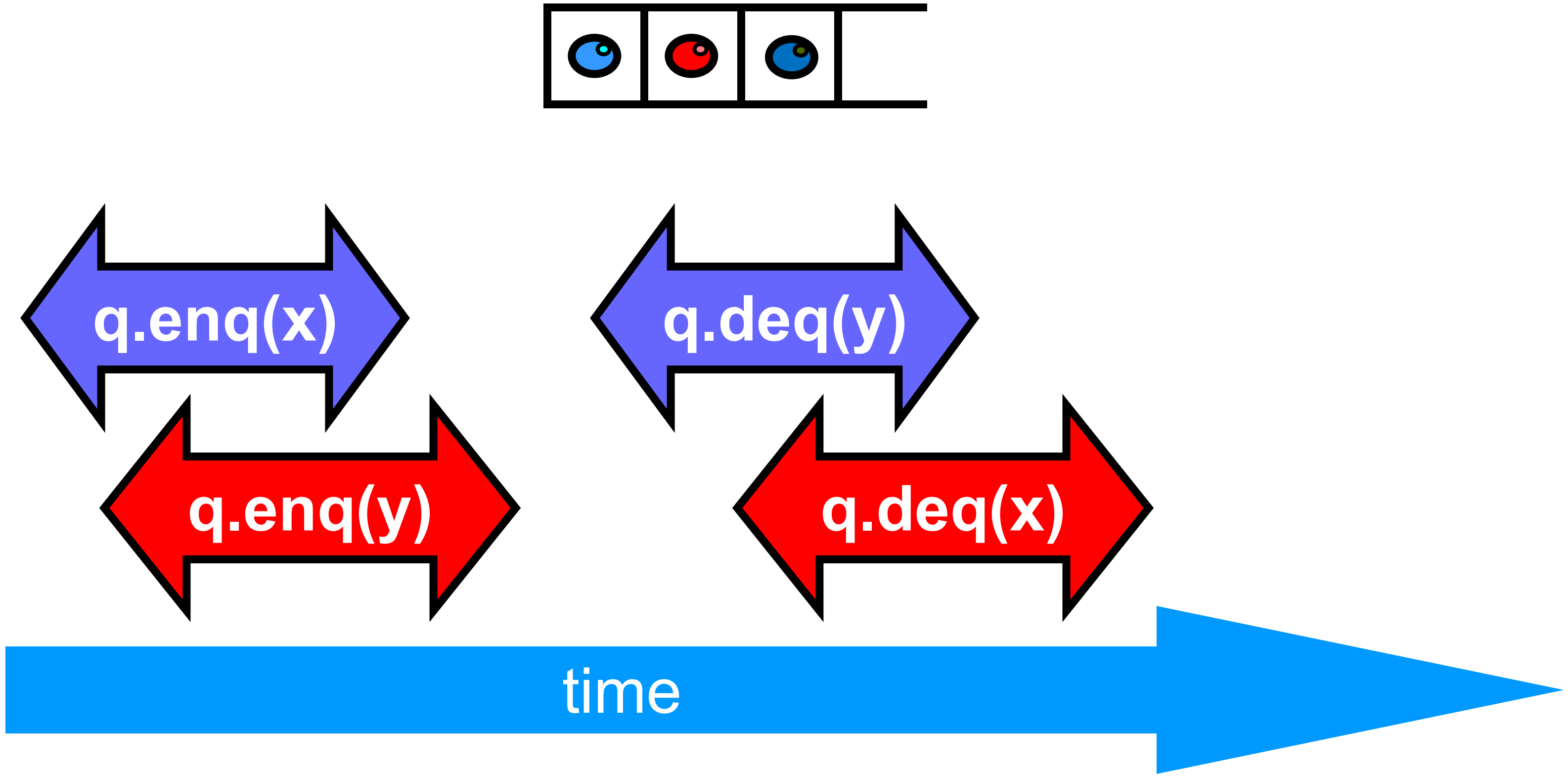

Example

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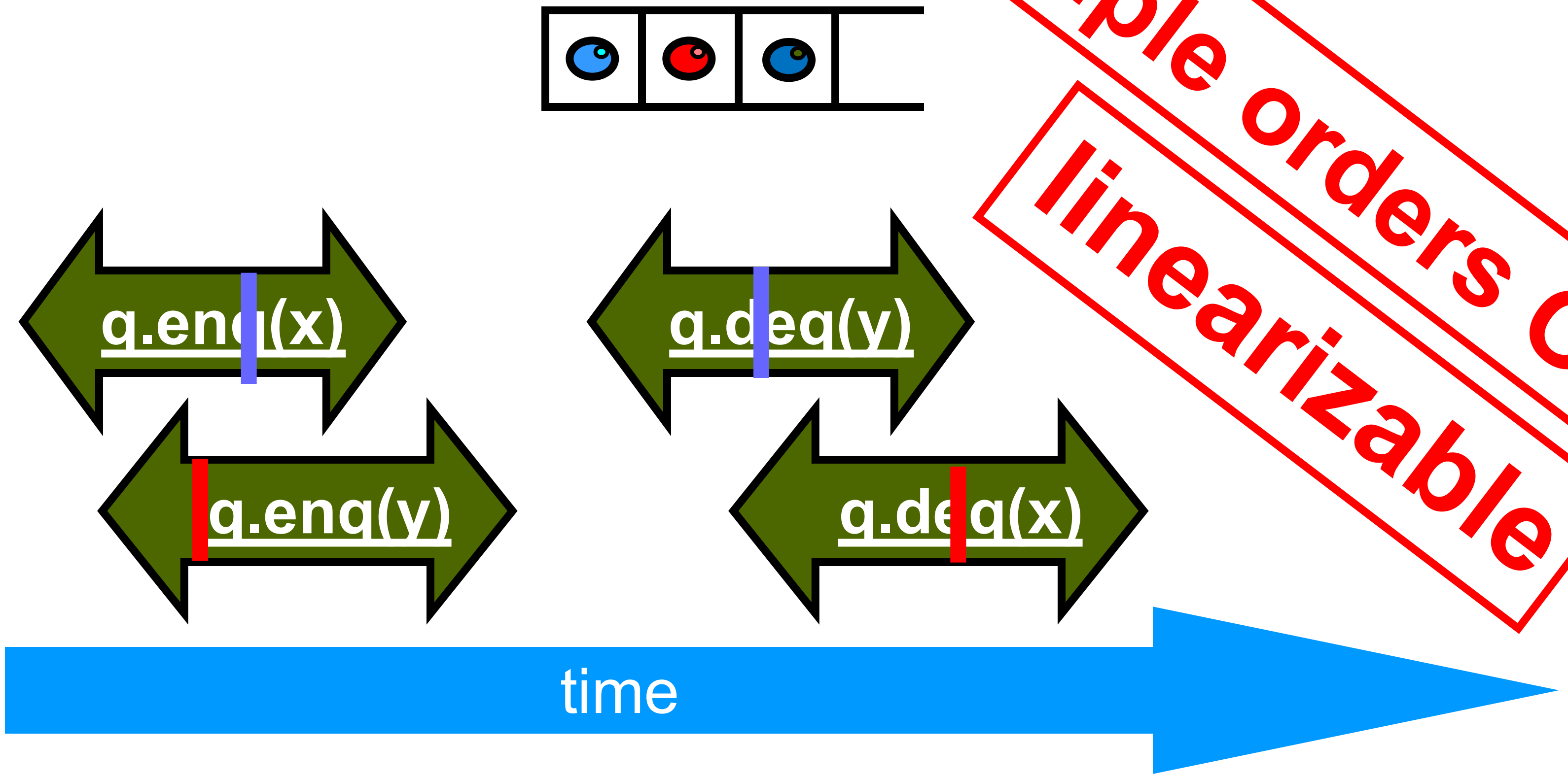
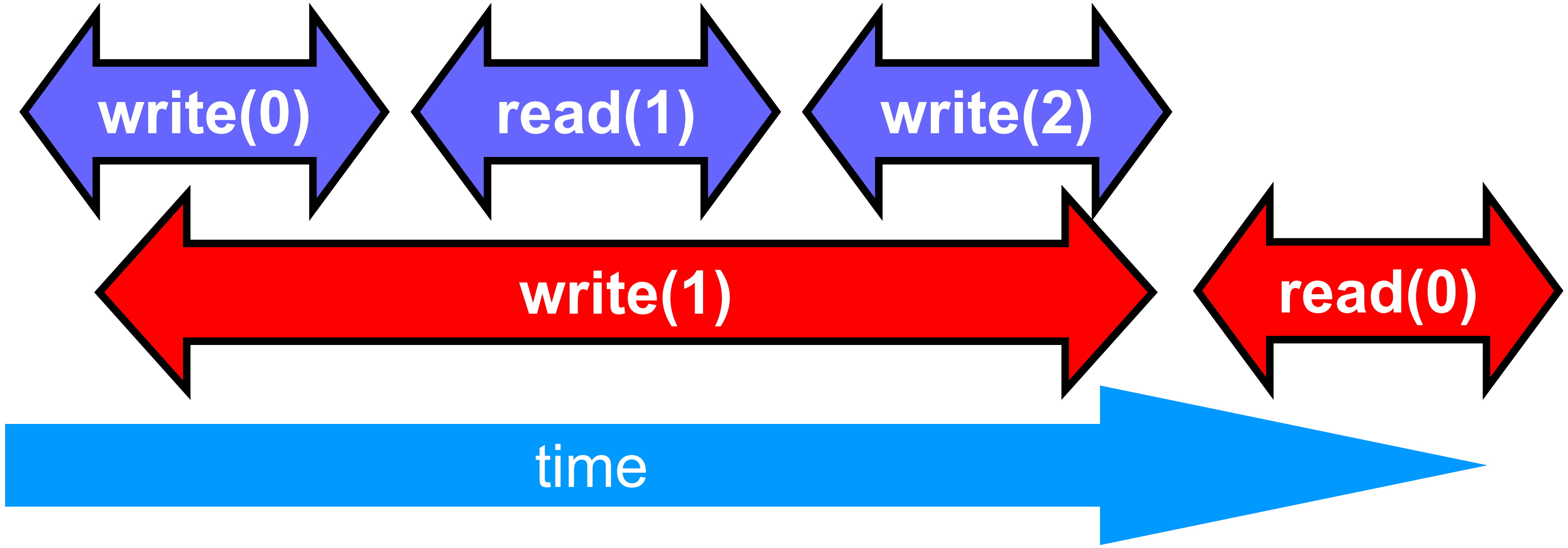


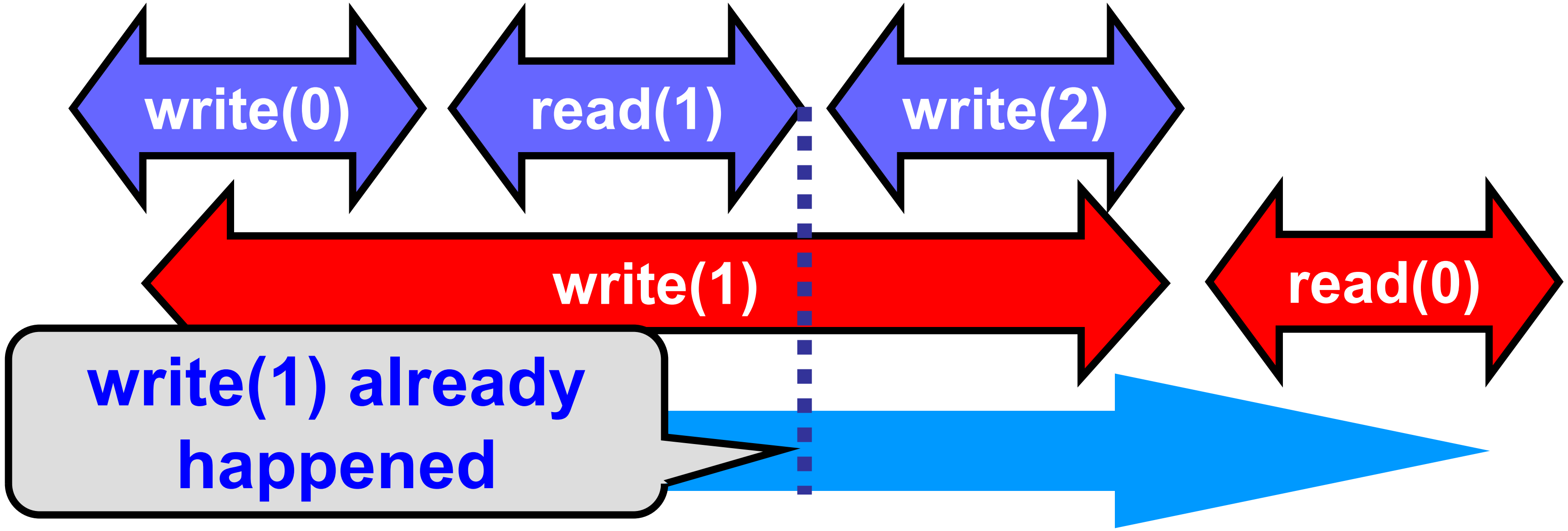
multiple orders OK

linearizable

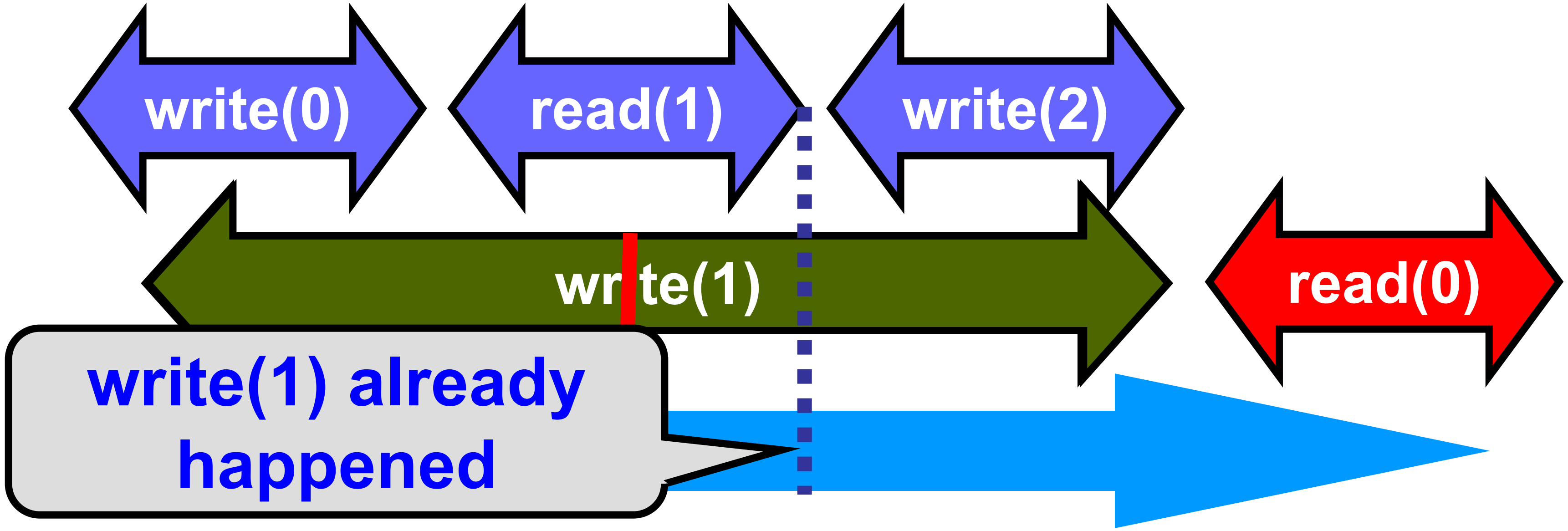
Read/Write Register Example



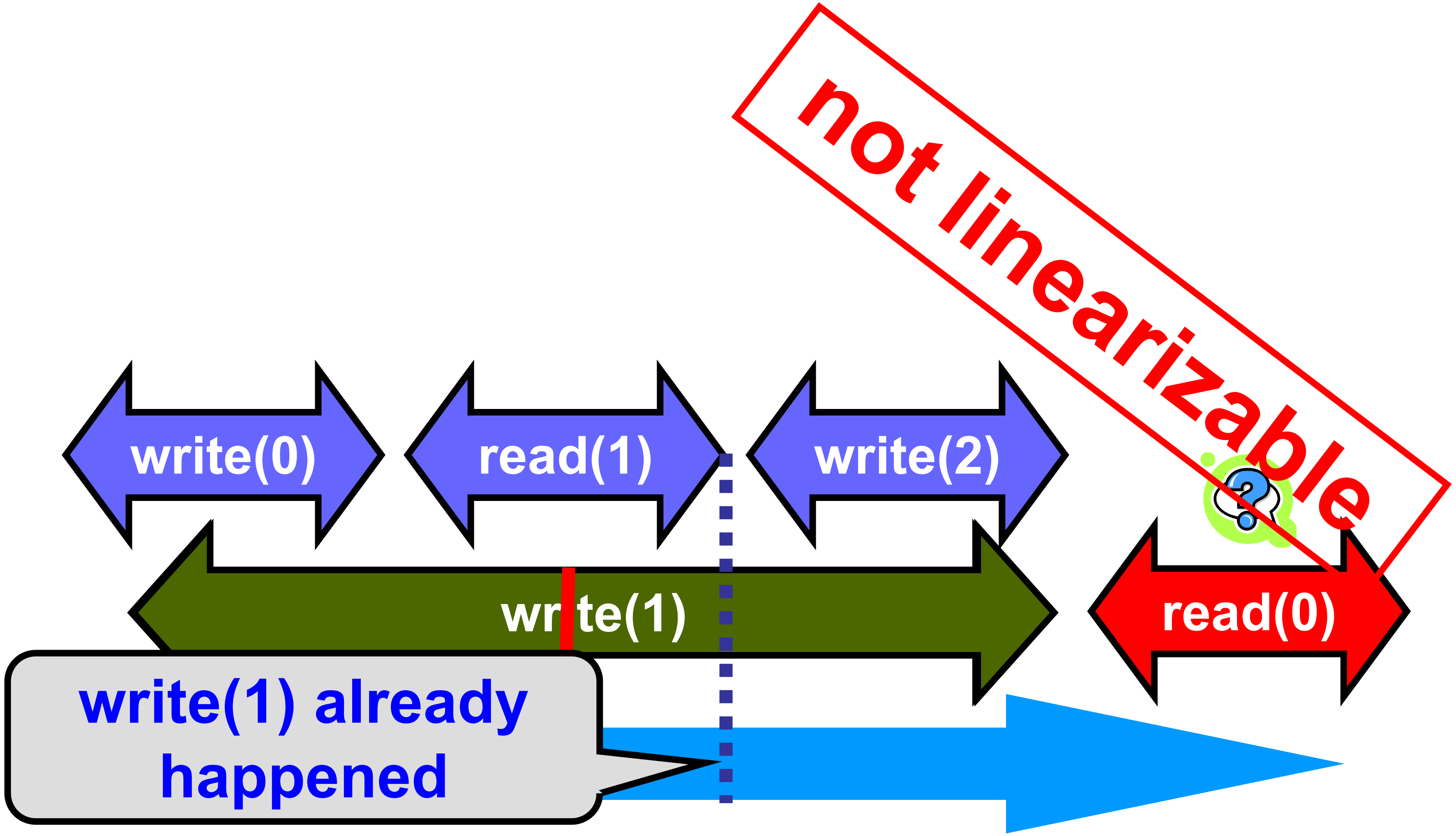
Read/Write Register Example



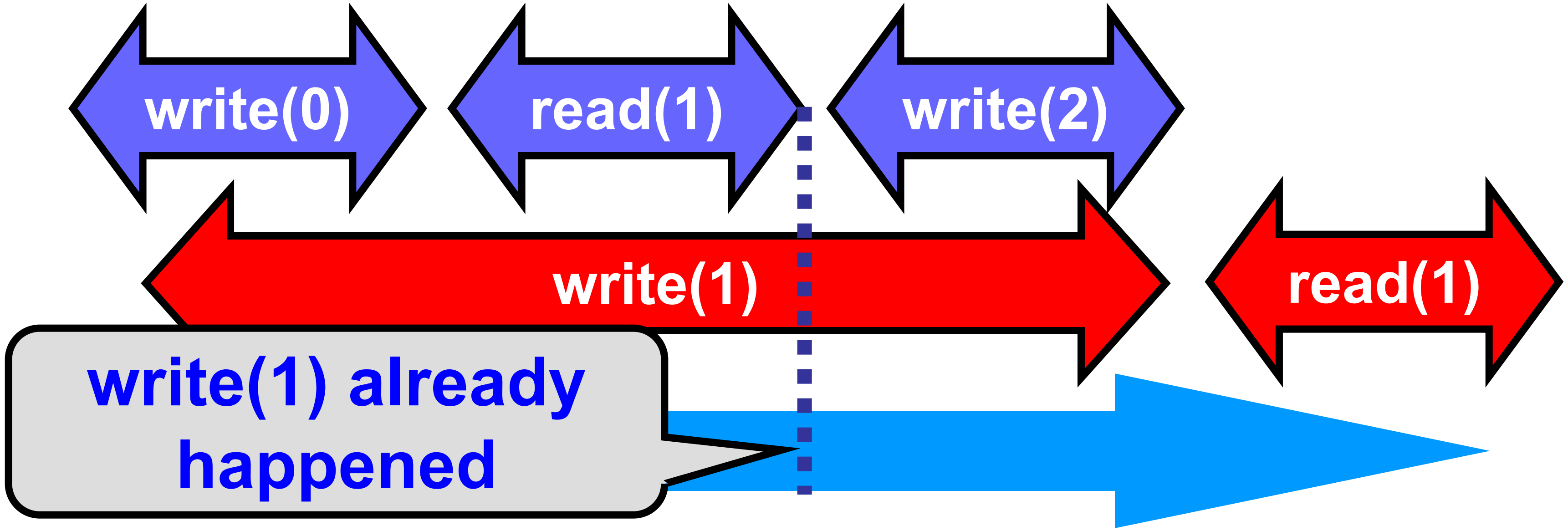
Read/Write Register Example



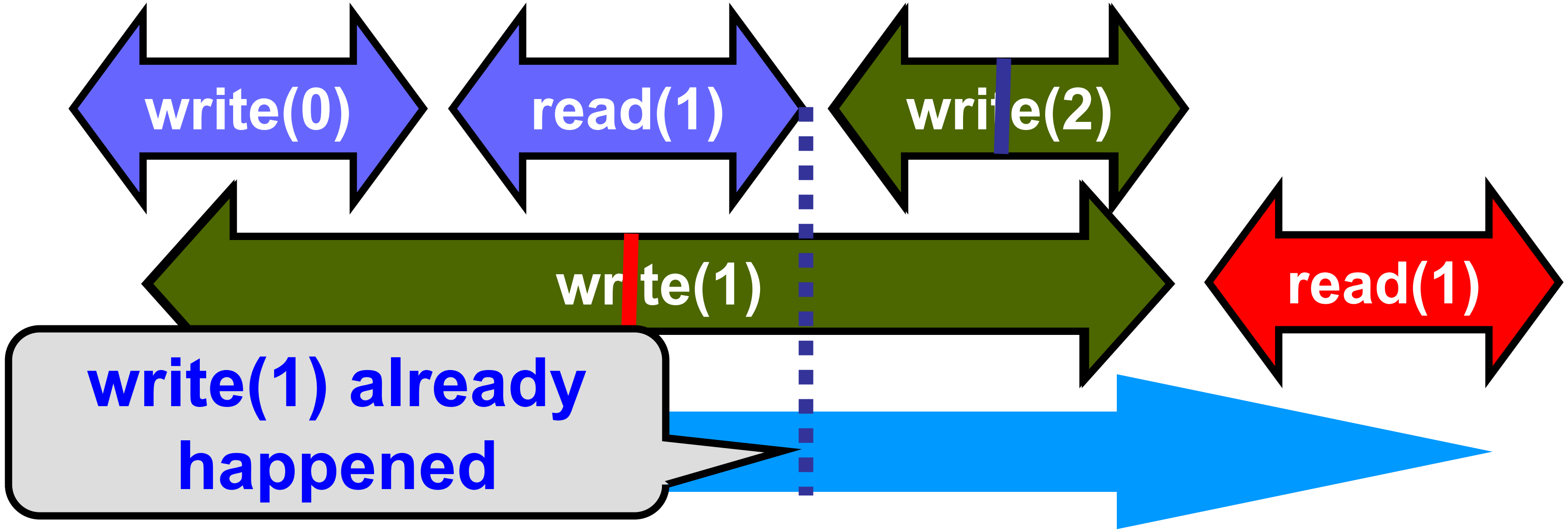
Read/Write Register Example



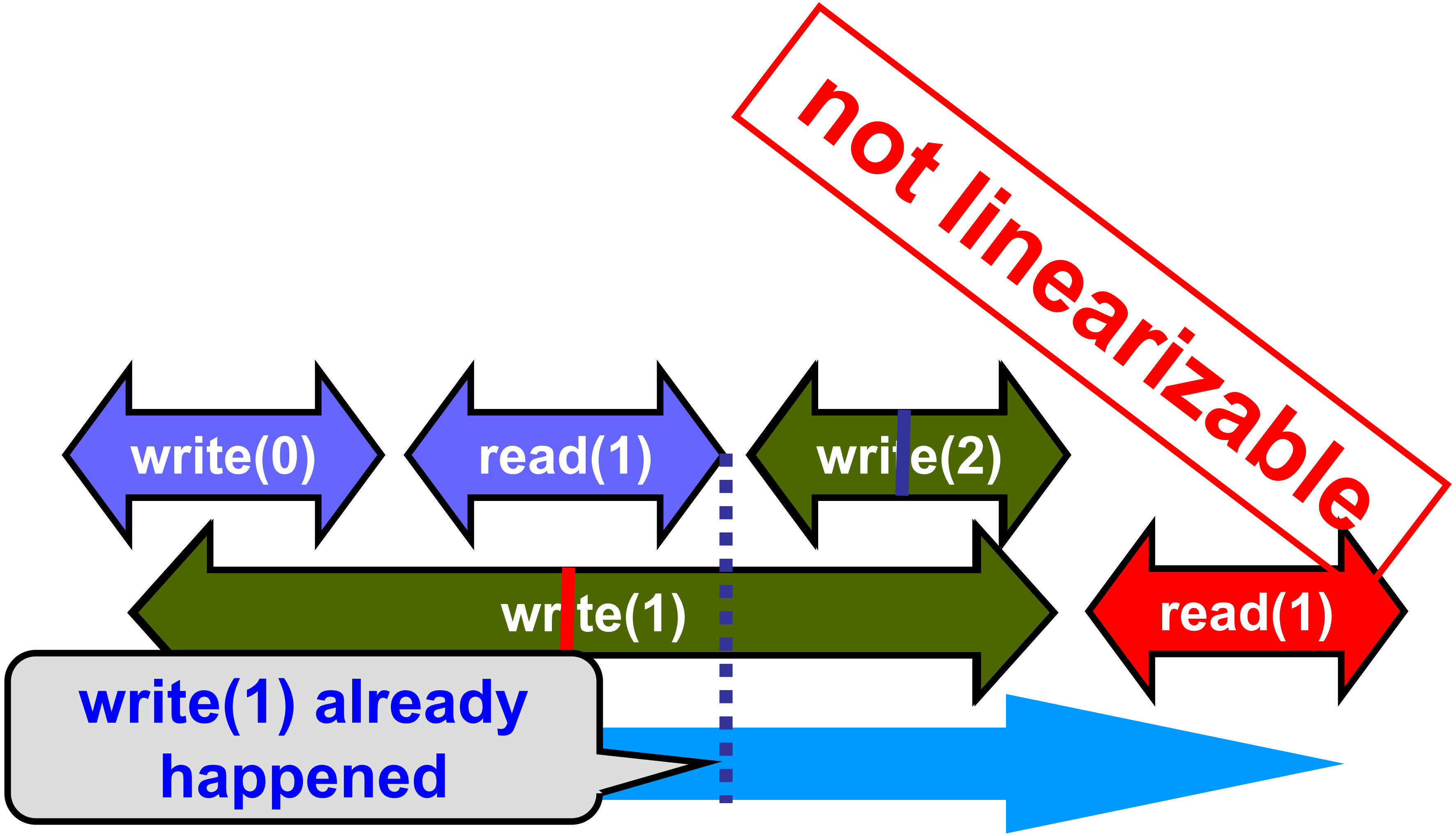
Read/Write Register Example



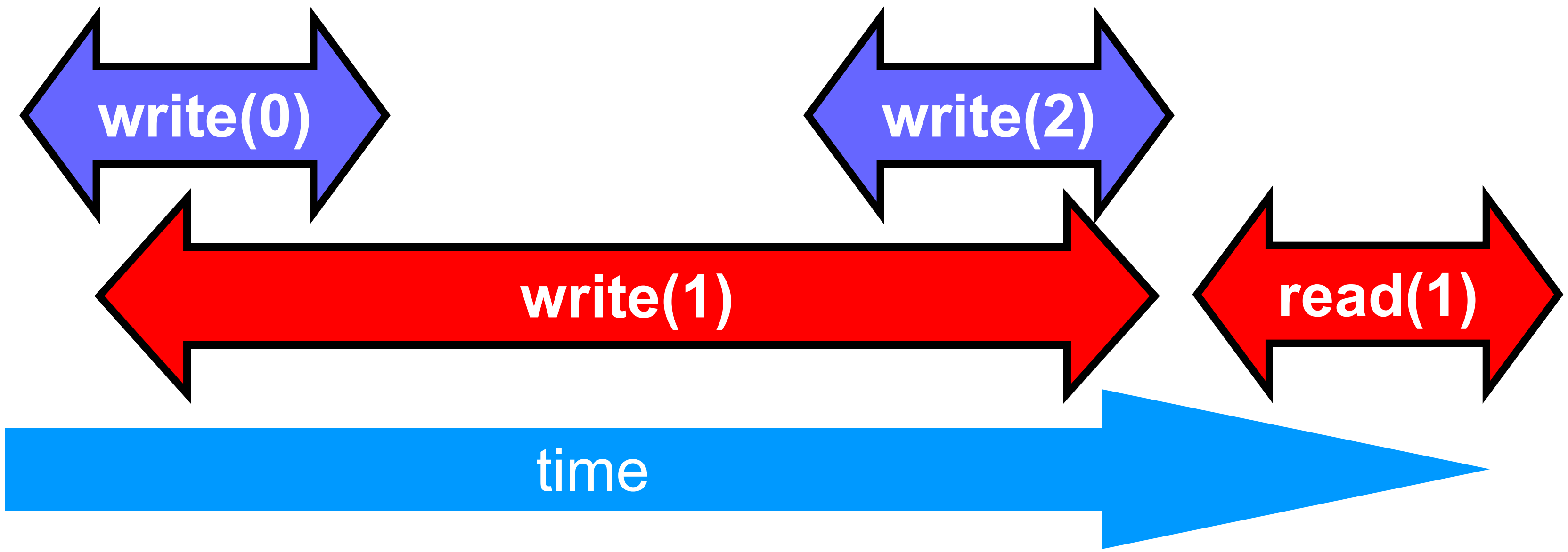
Read/Write Register Example



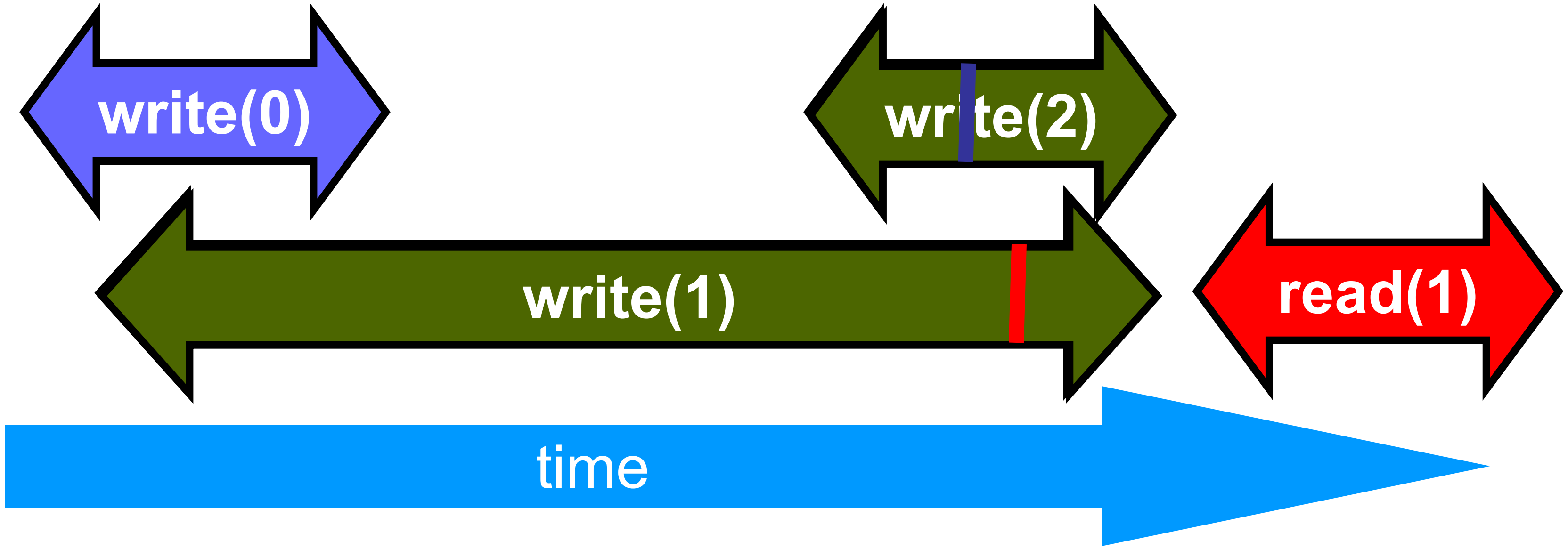
Read/Write Register Example



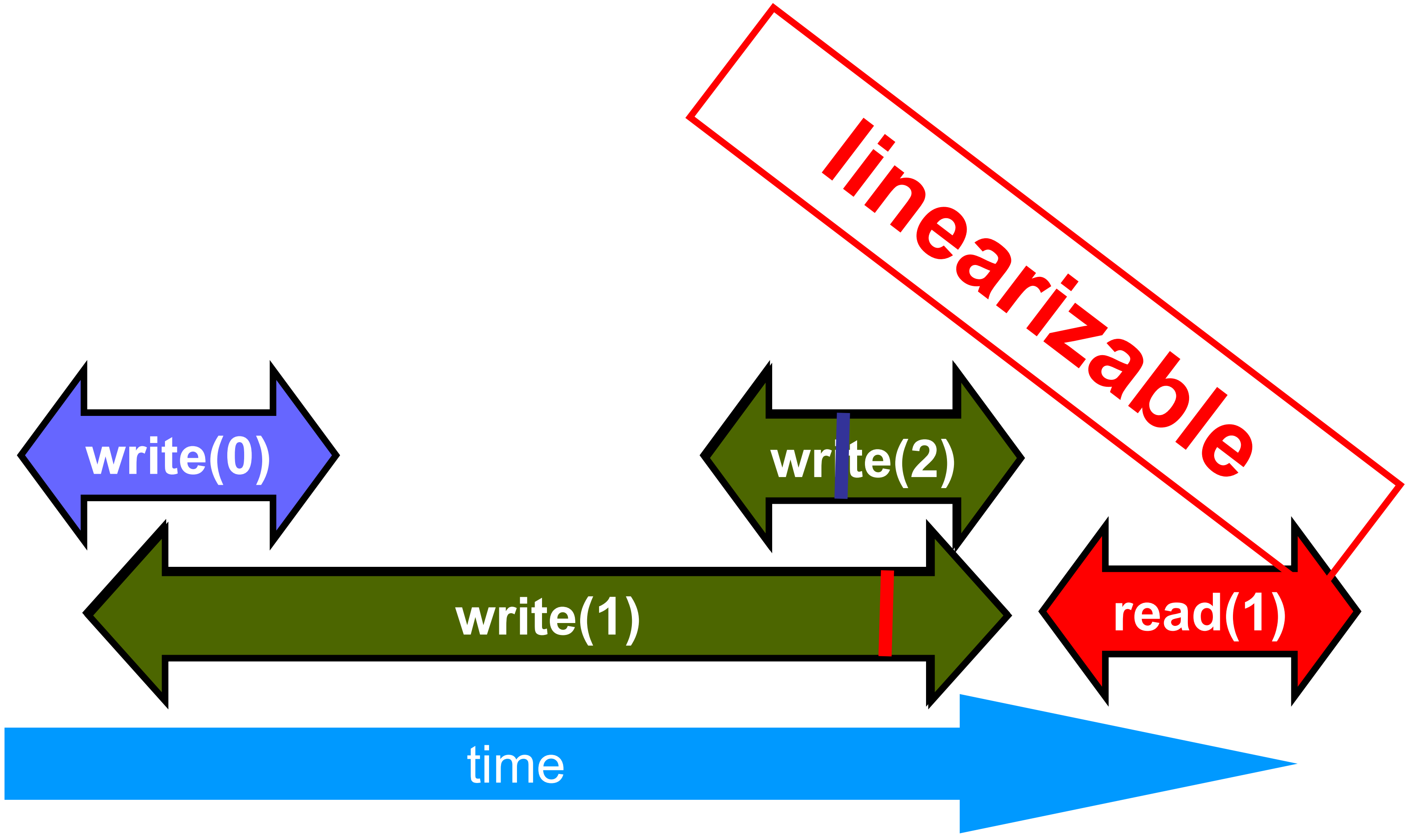
Read/Write Register Example



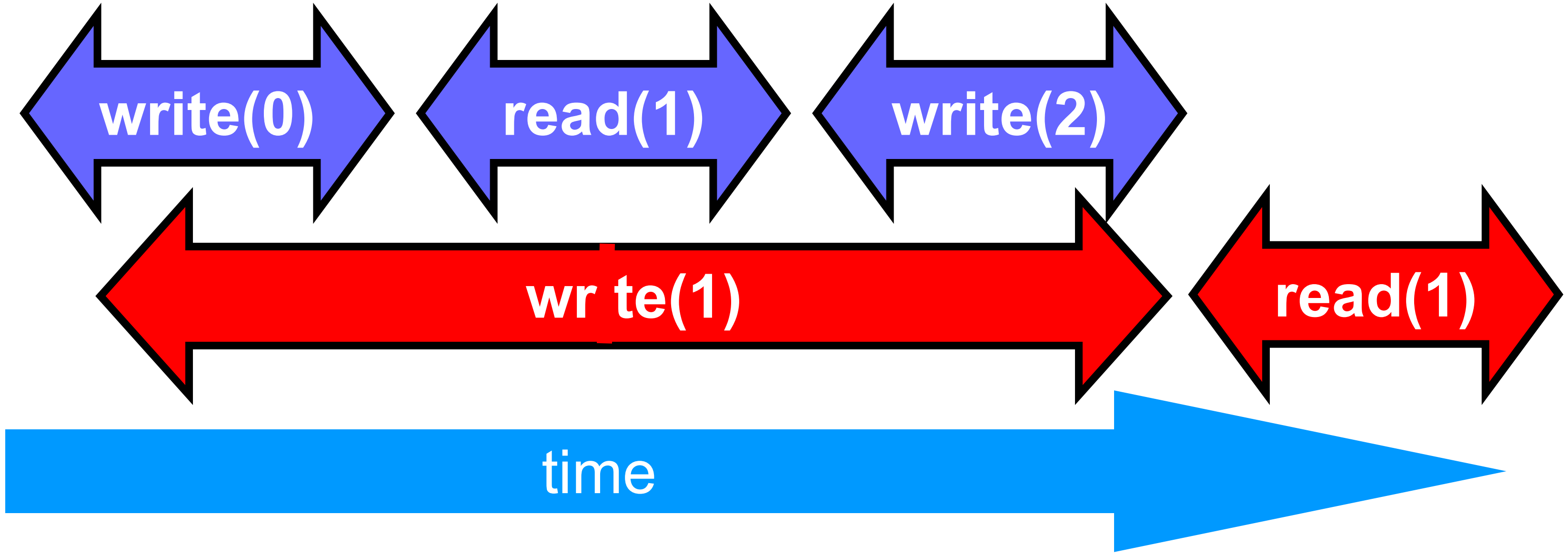
Read/Write Register Example



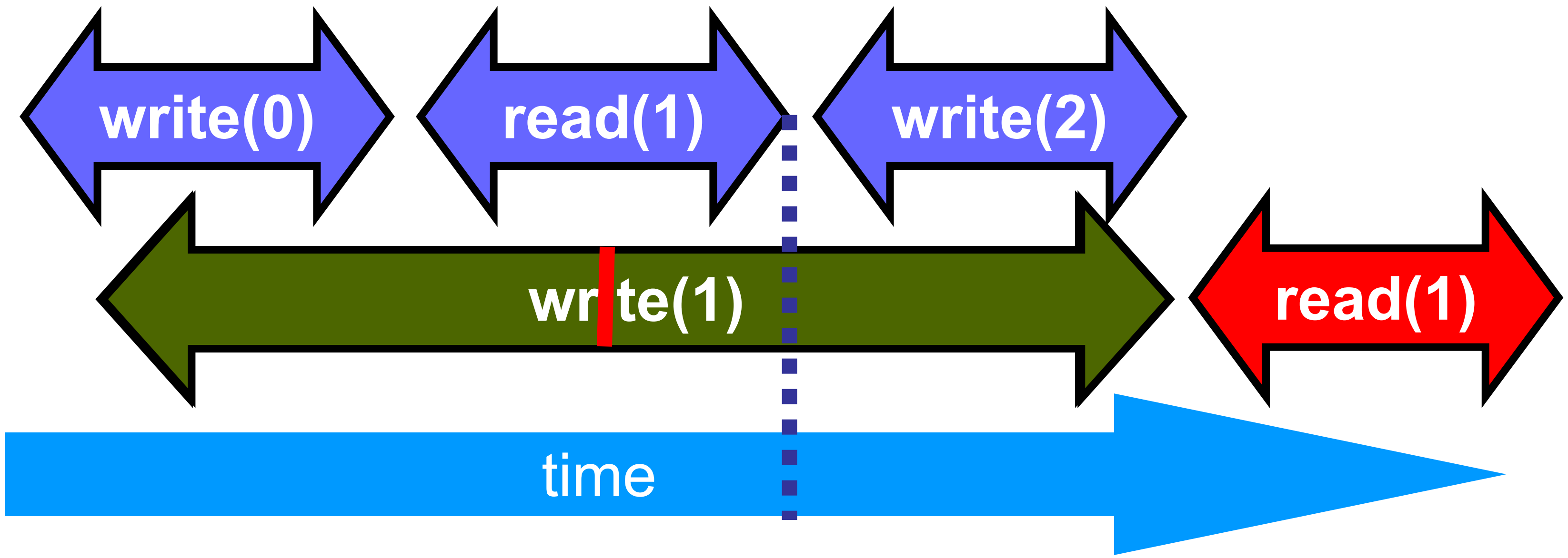
Read/Write Register Example



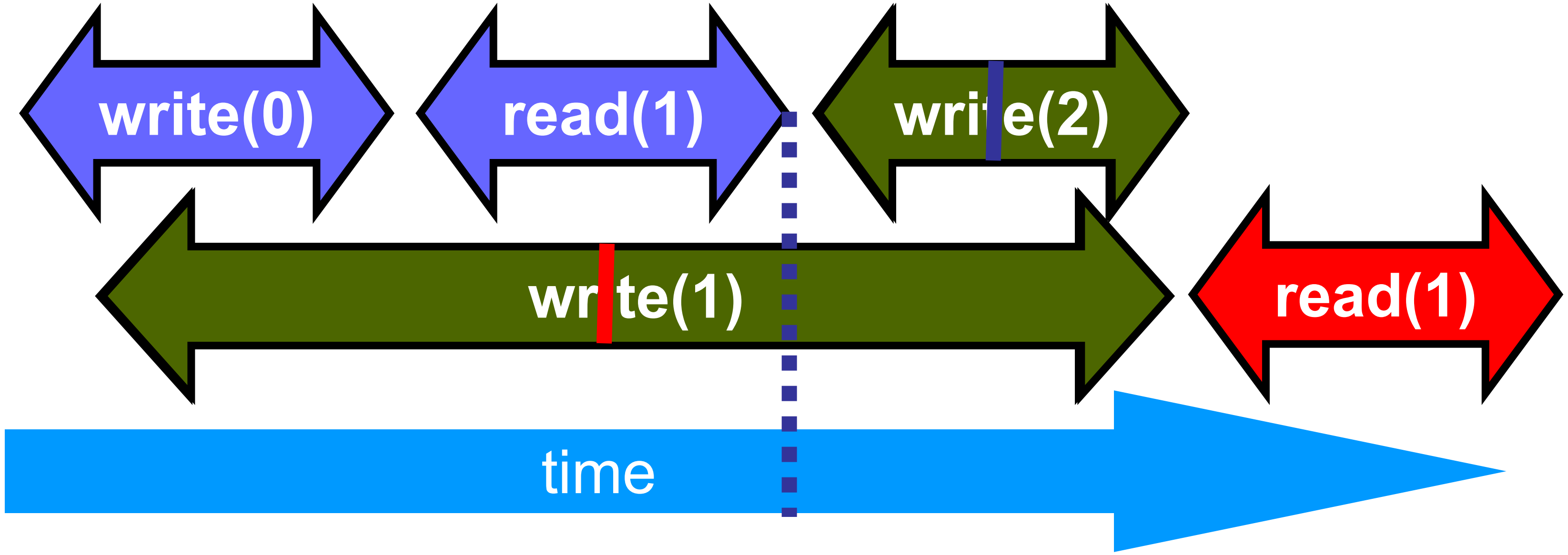
Read/Write Register Example



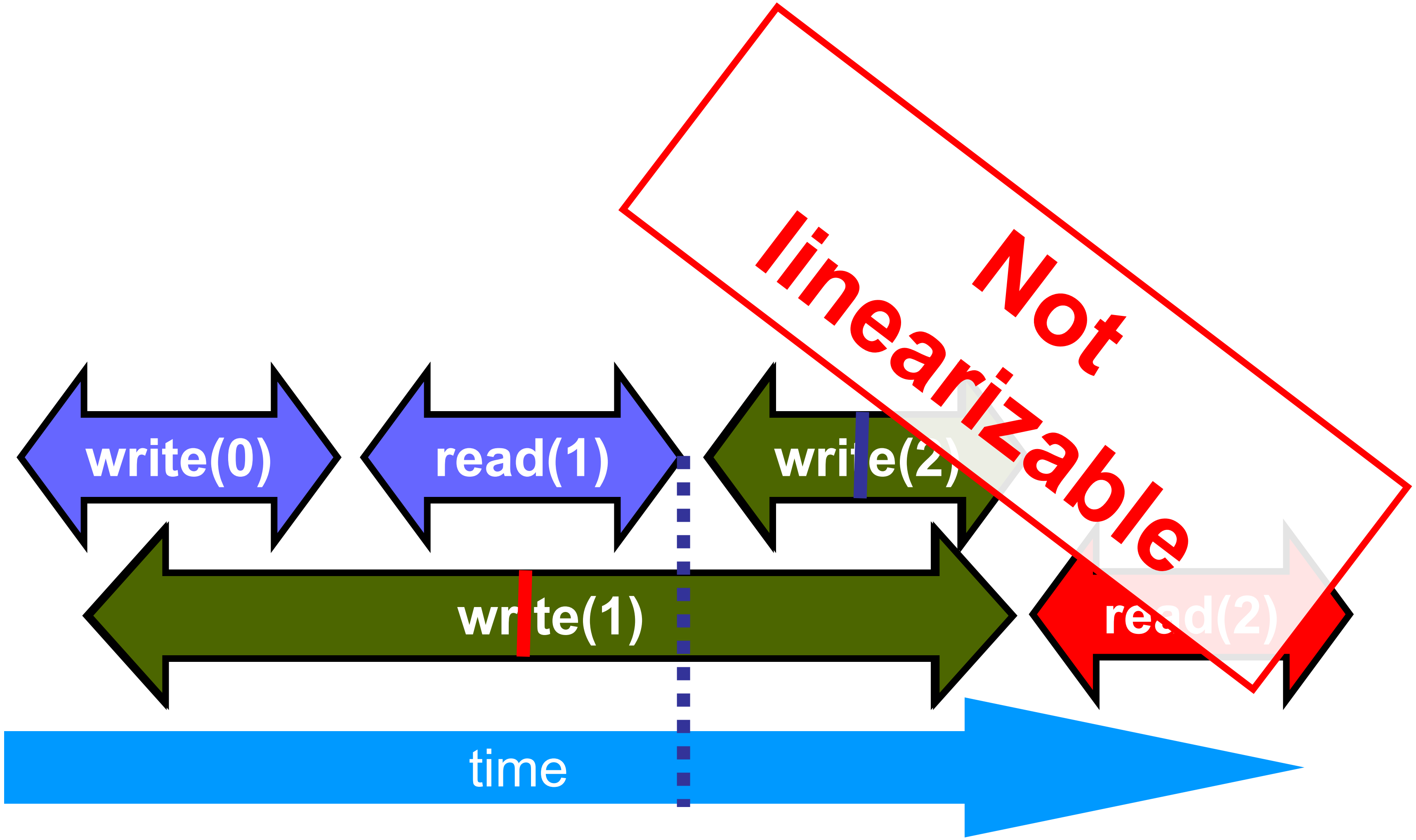
Read/Write Register Example



Read/Write Register Example



Read/Write Register Example



Talking About Executions

- Why?
  – Can’t we specify the linearization point of each operation without describing an execution?
- Not Always
  – In some cases, linearization point depends on the execution

Formal Model of Executions

- Define precisely what we mean
  - Ambiguity is bad when intuition is weak
- Allow reasoning
  - Formal
  - But mostly informal
    - In the long run, actually more important

Split Method Calls into Two Events

- Invocation
  - method name & args
  - `q.enq(x)`
- Response
  - result or exception
  - `q.enq(x)` returns `void`
  - `q.deq()` returns `x`
  - `q.deq()` throws `empty`

Invocation Notation

A q.enq(x)

Invocation Notation

A q.enq(x)

thread

Invocation Notation

A q.enq(x)

thread

method

Invocation Notation

A q.enq(x)

thread

method

object

Invocation Notation

A q.enq(x)

thread

method

object

arguments

Response Notation

A q: void

Response Notation

A q: void

thread

Response Notation

A q: void

thread

result

Response Notation

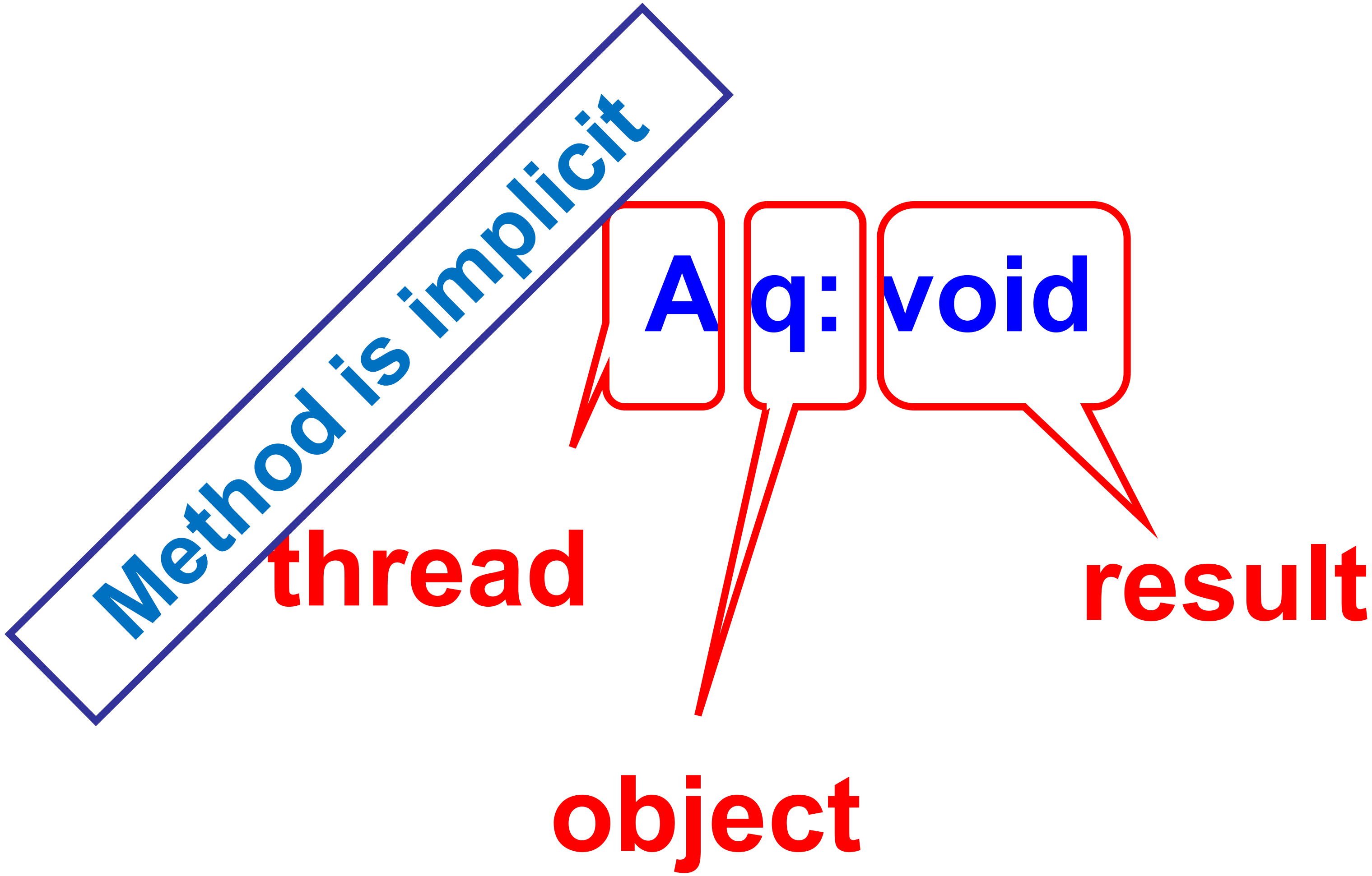
A q: void

thread

result

object

Response Notation



Response Notation

Method is implicit

A q: empty()

thread

object

exception

History - Describing an Execution

```
      A q.enq(3)
      A q:void
      A q.enq(5)
H =   B p.enq(4)
      B p:void
      B q.deq()
      B q:3
```

Sequence of invocations and responses

Definition

- Invocation & response *match* if

Thread names agree

Object names agree

```
A q.enq(3)
A q:void
```

Method call

Object Projections

```
      A q.enq(3)
      A q:void
H =   B p.enq(4)
      B p:void
      B q.deq()
      B q:3
```

Object Projections

```
A q.enq(3)
A q:void
```

H|q =

```
B q.deq()
B q:3
```

Thread Projections

H =
```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
```

Thread Projections

$H|B =$

```
B p.enq(4)
B p:void
B q.deq()
B q:3
```

Complete Subhistory

```
      A q.enq(3)
      A q:void
      A q.enq(5)
H =   B p.enq(4)
      B p:void
      B q.deq()
      B q:3
```

An invocation is *pending* if it has no matching respnse

Complete Subhistory

```
      A q.enq(3)
      A q:void
      A q.enq(5)
H =   B p.enq(4)
      B p:void
      B q.deq()
      B q:3
```

May or may not have taken effect

Complete Subhistory

```
      A q.enq(3)
      A q:void
      A q.enq(5)
H =   B p.enq(4)
      B p:void
      B q.deq()
      B q:3
```

discard pending invocations

Complete Subhistory

```
A q.enq(3)
A q:void
```

Complete(H) =

```
B p.enq(4)
B p:void
B q.deq()
B q:3
```

Sequential Histories

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
A q:enq(5)
```

Sequential Histories

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
A q:enq(5)
```

match

(4)

Sequential Histories

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
A q:enq(5)
```

match

match

Sequential Histories

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
A q:enq(5)
```

match

match

match

Sequential Histories

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
A q:enq(5)
```

match

match

match

Final pending invocation OK

Sequential Histories

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
A q:enq(5)
```

match

match

match

Final pending invocation OK

Method calls of different threads do not interleave

Well-Formed Histories

```
    A q.enq(3)
    B p.enq(4)
    B p:void
H=  B q.deq()
    A q:void
    B q:3
```

Well-Formed Histories

Per-thread projections sequential

$H|B$=

```
B p.enq(4)
B p:void
B q.deq()
B q:3
```

H=

```
A q.enq(3)
B p.enq(4)
B p:void
B q.deq()
A q:void
B q:3
```

Well-Formed Histories

Per-thread projections sequential

```
H=   A q.enq(3)
     B p.enq(4)
     B p:void
     B q.deq()
     A q:void
     B q:3
```

```
H|B= B p.enq(4)
     B p:void
     B q.deq()
     B q:3
```

```
H|A= A q.enq(3)
     A q:void
```

Equivalent Histories

Threads see the same thing in both

$$\begin{cases} H|A = G|A \\ H|B = G|B \end{cases}$$

H=

```
A q.enq(3)
B p.enq(4)
B p:void
B q.deq()
A q:void
B q:3
```

G=

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
```

Sequential Specifications

- A sequential specification is some way of telling whether a
  - Single-thread, single-object history
  - Is legal
- For example:
  - Pre and post-conditions
  - But plenty of other techniques exist …

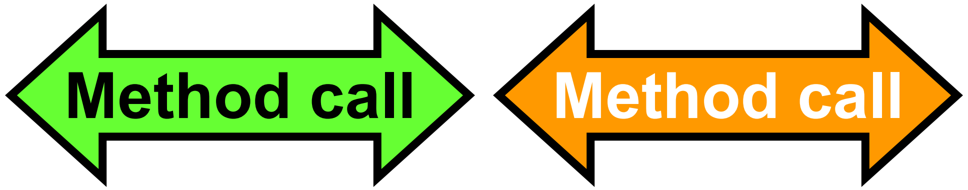
Legal Histories

- A sequential (multi-object) history H is legal if
  - For every object **x**
  - **H|x** is in the sequential spec for **x**

Precedence

```
A q.enq(3)
B p.enq(4)
B p.void
A q:void
B q.deq()
B q:3
```

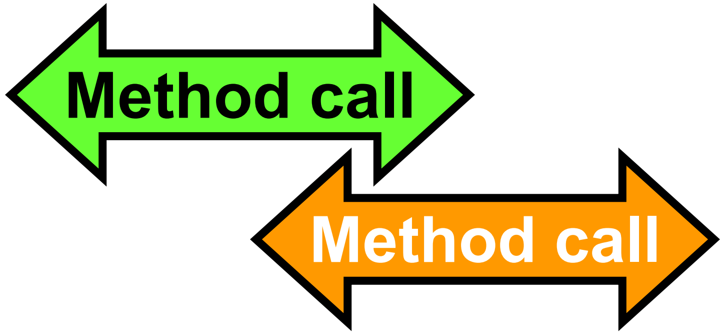
A method call precedes another if response event precedes invocation event



Non-Precedence

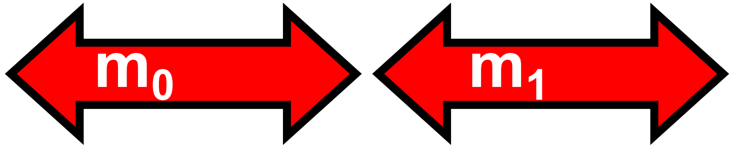
```
A q.enq(3)
B p.enq(4)
B p.void
B q.deq()
A q:void
B q:3
```

Some method calls overlap one another



Notation

- Given
  - History **H**
  - method executions $\mathbf{m_0}$ and $\mathbf{m_1}$ in **H**
- We say $m_0 \rightarrow_{\mathbf{H}} m_1$, if
  - $\mathbf{m_0}$ precedes $\mathbf{m_1}$
- Relation $\mathbf{m_0} \rightarrow_{\mathbf{H}} \mathbf{m_1}$ is a
  - Partial order
  - Total order if **H** is sequential



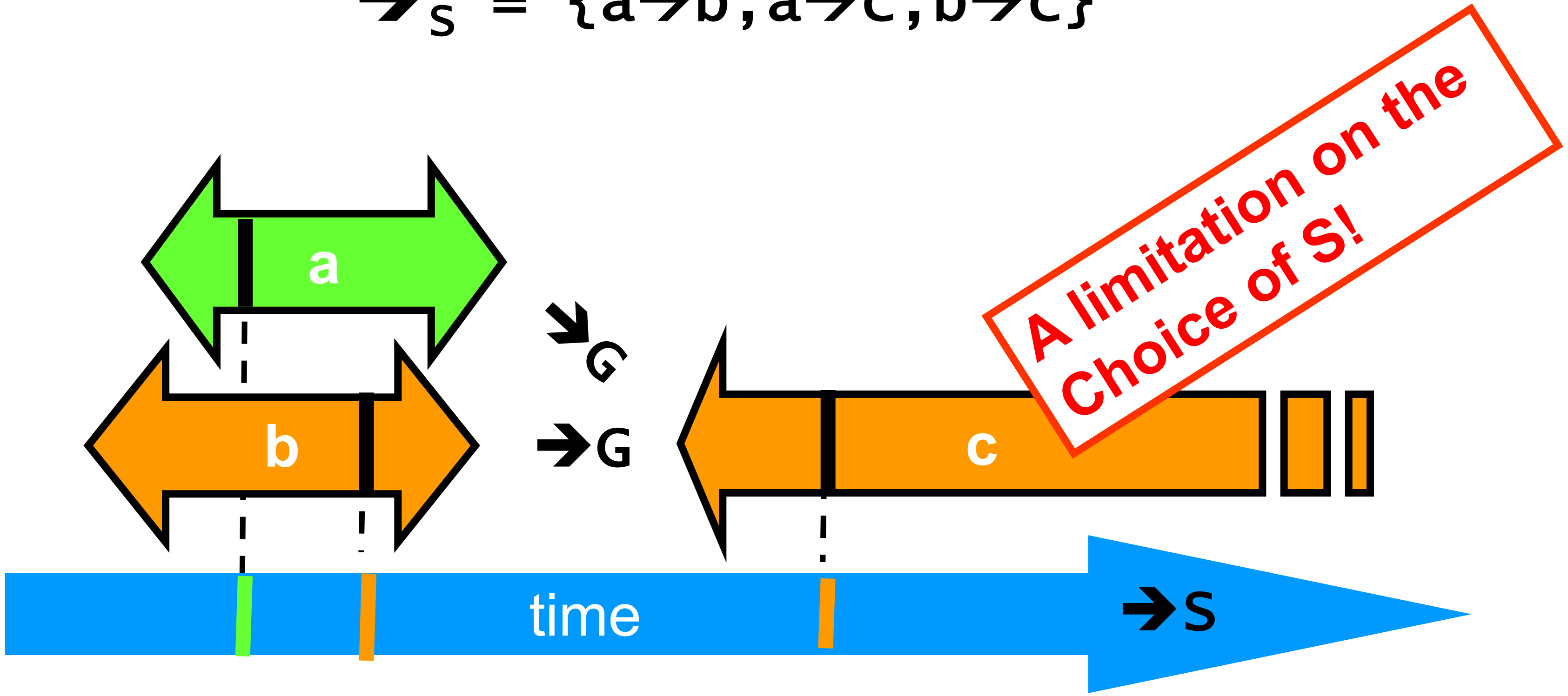
Linearizability

- History H is ***linearizable*** if it can be extended to **G** by
  - Appending zero or more responses to pending invocations
  - Discarding other pending invocations
- So that G is equivalent to
  - Legal sequential history **S**
  - where $\rightarrow_{\mathbf{G}} \subset \rightarrow_{\mathbf{S}}$

Ensuring $\rightarrow_G \subset \rightarrow_S$

$\rightarrow_G$ = {a→c,b→c}

$\rightarrow_S$ = {a→b,a→c,b→c}

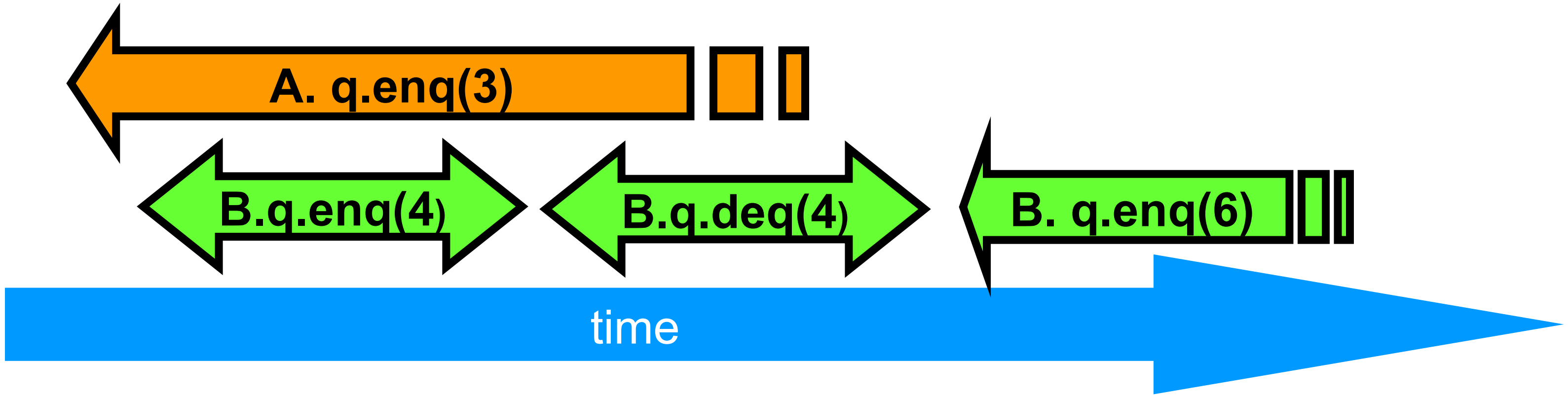


Remarks

- Some pending invocations
  - Took effect, so keep them
  - Discard the rest
- Condition $\rightarrow_{\mathbf{G}} \subset \rightarrow_{\mathbf{S}}$
  - Means that **S** respects "real-time order" of **G**

Example

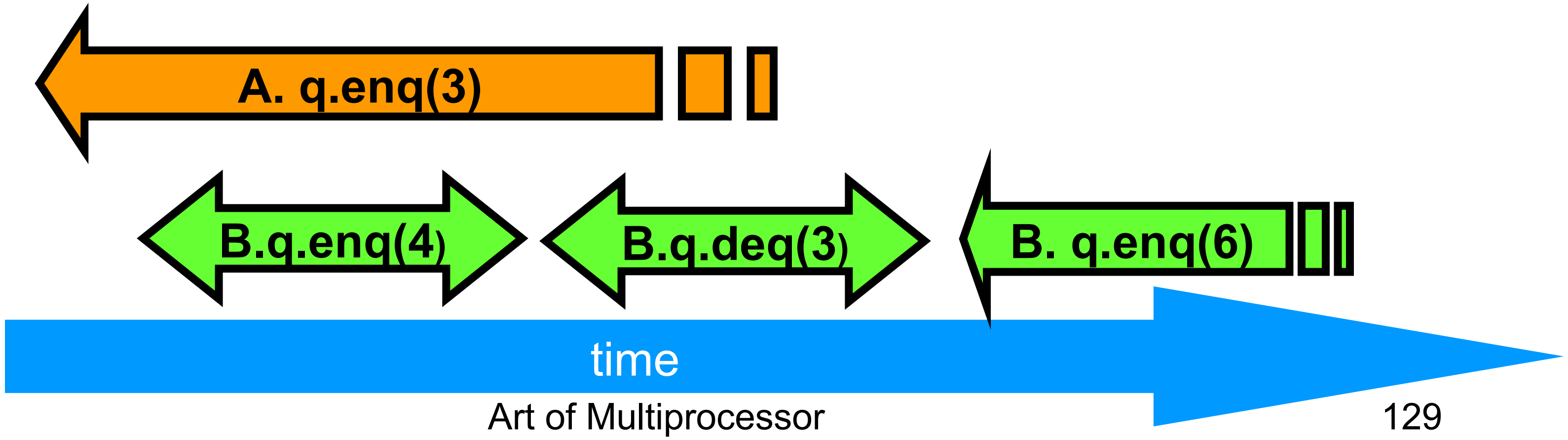
```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
```



Example

```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
```

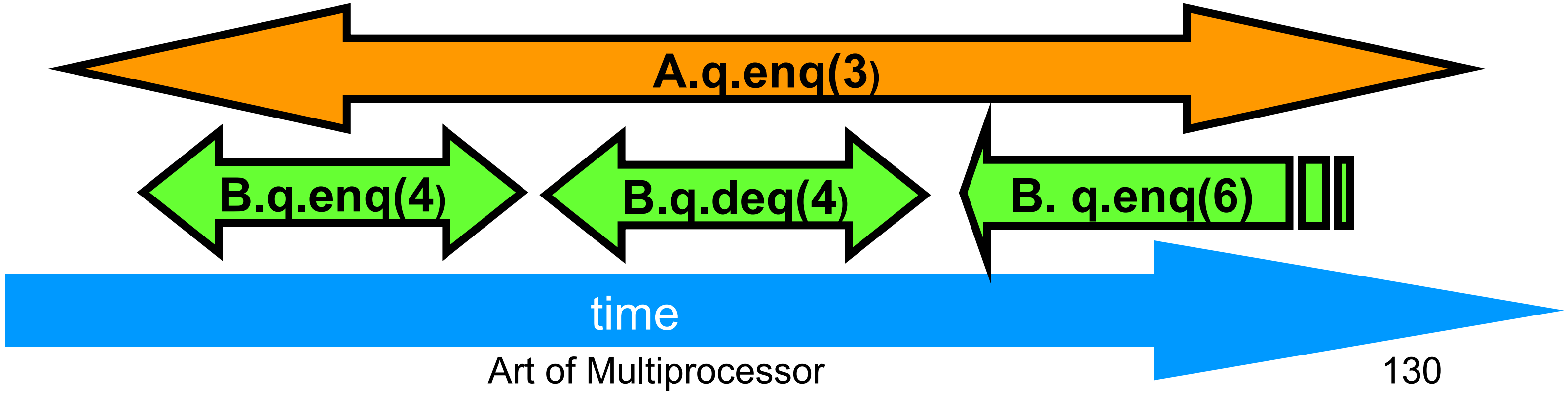
Complete this pending invocation



Example

```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
A q:void
```

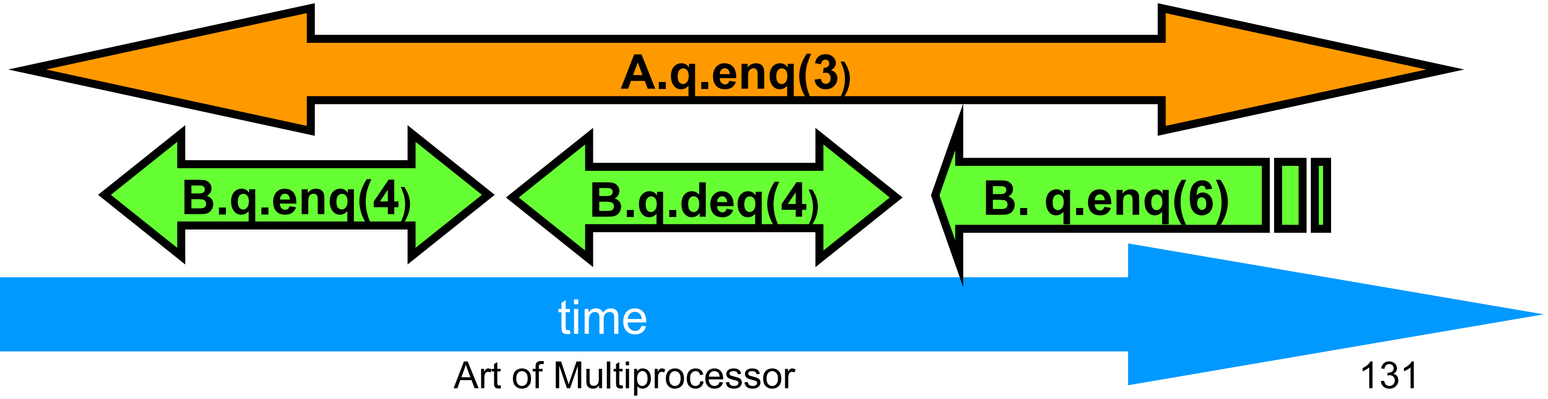
Complete this pending invocation



Example

discard this one

```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
A q:void
```

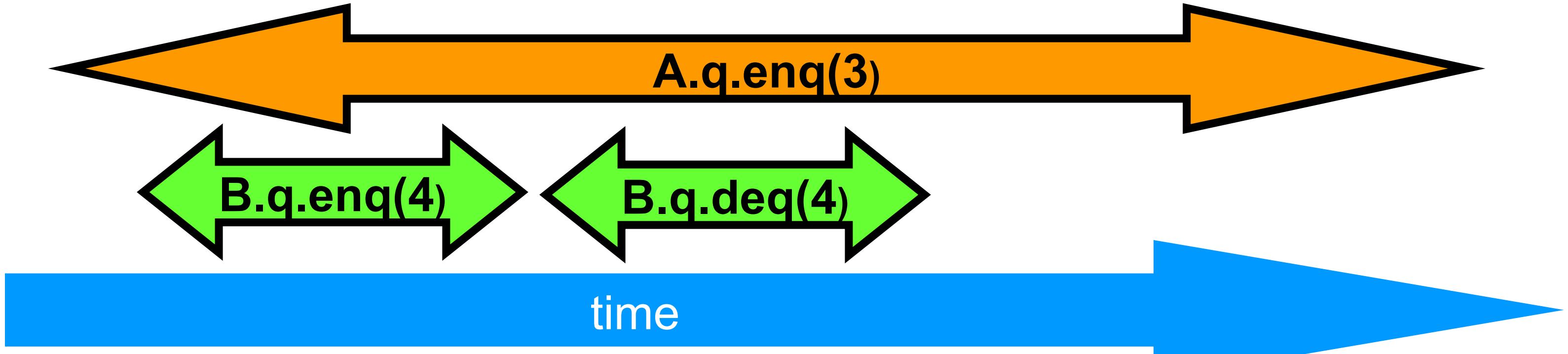


Example

```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4

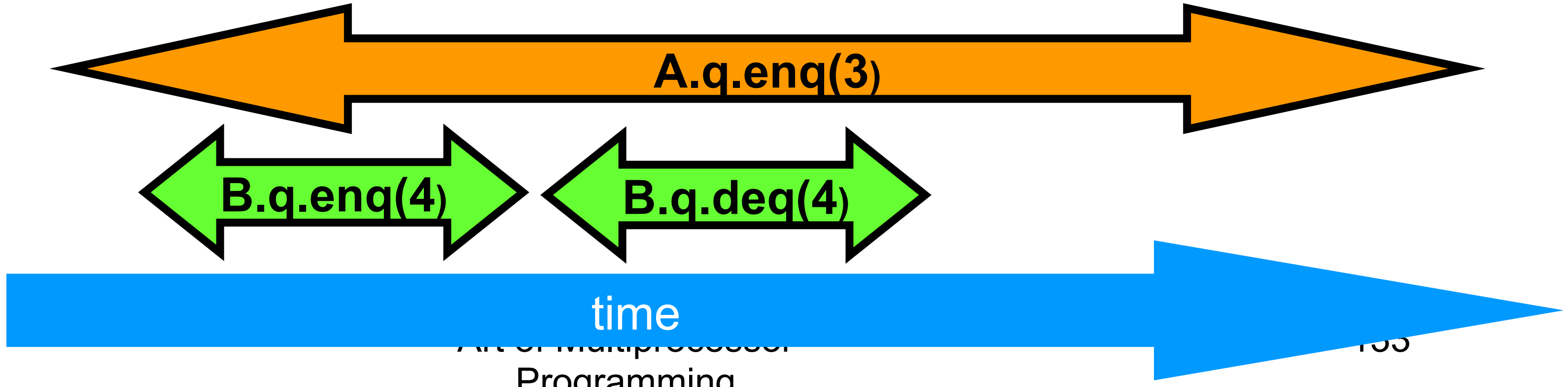
A q:void
```

discard this one



Example

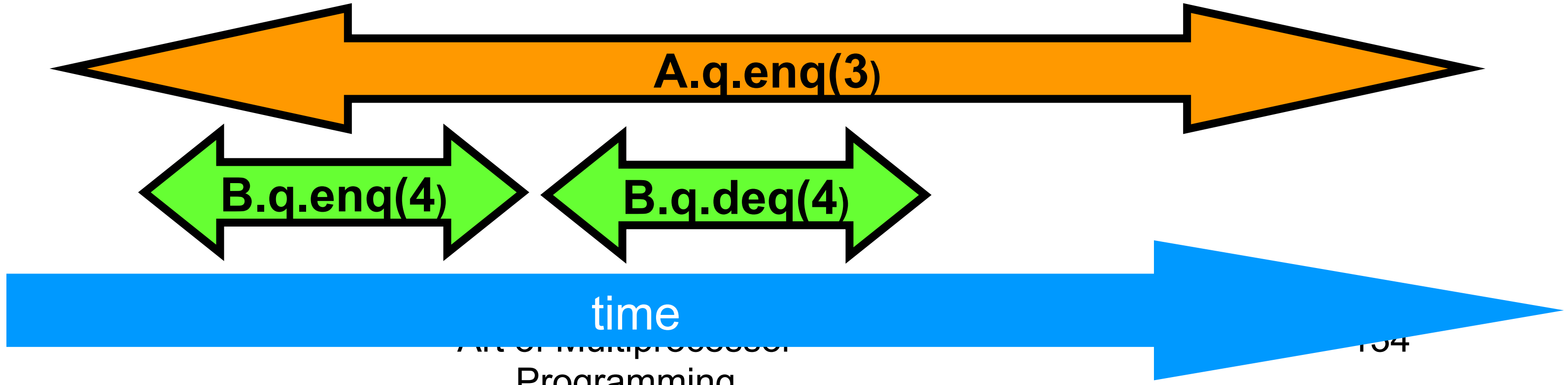
```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void
```



Example

```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void
```

```
B q.enq(4)
B q:void
A q.enq(3)
A q:void
B q.deq()
B q:4
```

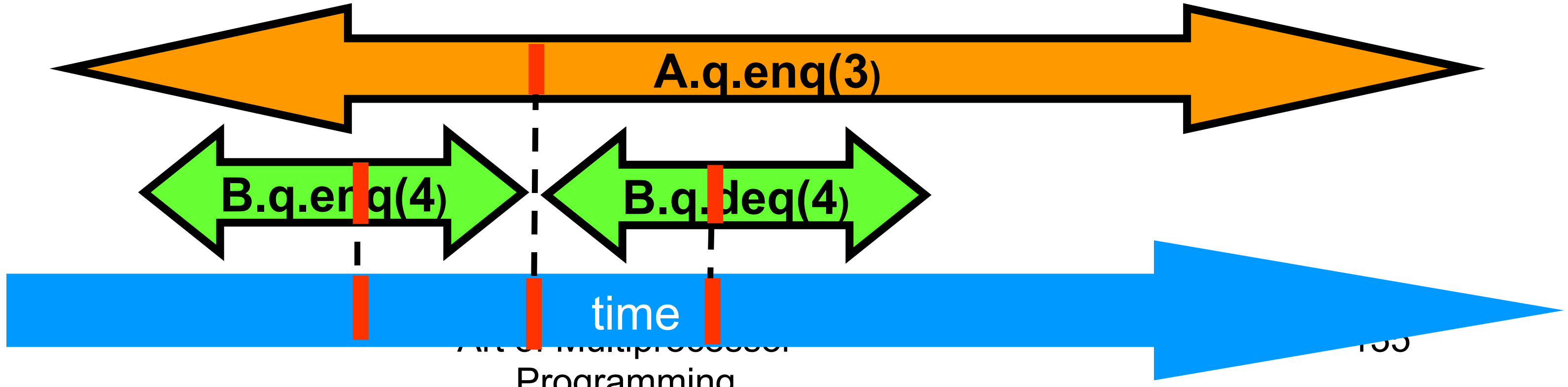


Example

```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void
```

Equivalent sequential history

```
B q.enq(4)
B q:void
A q.enq(3)
A q:void
B q.deq()
B q:4
```



Composability Theorem

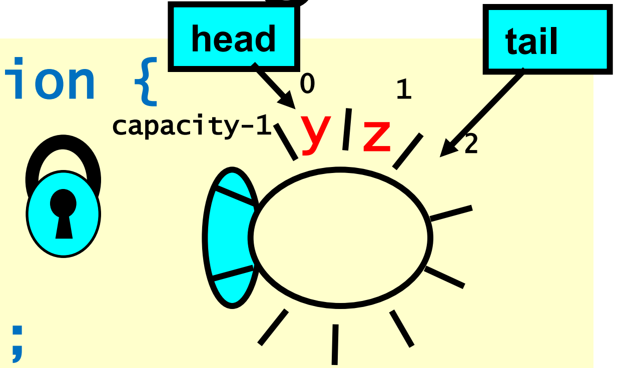
- History H is linearizable if and only if
  - For every object x
  - H|x is linearizable
- We care about objects only!
  - (Materialism?)

Why Does Composability Matter?

- Modularity
- Can prove linearizability of objects in isolation
- Can compose independently-implemented objects

Reasoning About Linearizability: Locking

```
public T deq() throws EmptyException {
  lock.lock();
  try {
    if (tail == head)
       throw new EmptyException();
    T x = items[head % items.length];
    head++;
    return x;
  } finally {
    lock.unlock();
  }
}
```



Reasoning About Linearizability: Locking

```
public T deq() throws EmptyException {
  lock.lock();
  try {
    if (tail == head)
       throw new EmptyException();
    T x = items[head % items.length];
    head++;
    return x;
  } finally {
    lock.unlock();
  }
}
```

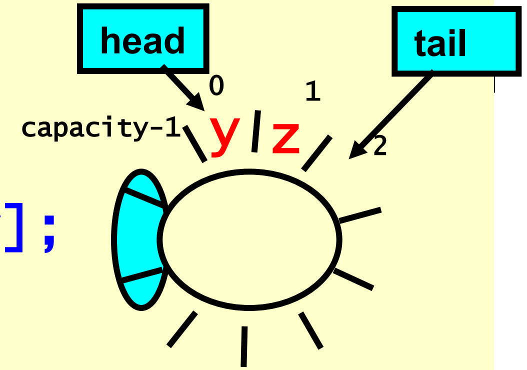
Linearization points are when locks are released

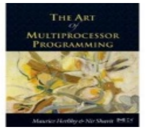
More Reasoning: Wait-free

```
public class WaitFreeQueue {

  int head = 0, tail = 0;
  items = (T[]) new Object[capacity];

  public void enq(Item x) {
    if (tail-head == capacity) throw
         new FullException();
    items[tail % capacity] = x; tail++;
  }
  public Item deq() {
     if (tail == head) throw
         new EmptyException();
     Item item = items[head % capacity]; head++;
     return item;
}}
```





More Reasoning: Wait-free

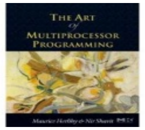
```
public clas   itFreeQueue {

  int        l =
  it       new Ob

       id enq(Item x) {
       ail-head == capacity) throw
       new FullException();
     items[tail % capacity] = x; tail++;
  }
  public Item deq() {
     if (tail == head) throw
         new EmptyException();
     Item item = items[head % capacity]; head++;
     return item;
}}
```

Remember that there is only one enqueuer and only one dequeuer

Linearization order is order head and tail fields modified



Strategy

- Identify one atomic step where method “happens”
  - Critical section
  - Machine instruction
- Doesn’t always work
  - Might need to define several different steps for a given method

Linearizability: Summary

- Powerful specification tool for shared objects
- Allows us to capture the notion of objects being “atomic”
- Don’t leave home without it