2.3 Multiplication and division of fractions

2.3.1 Multiplication

To multiply two or more fractions together, the numerators are first multiplied to give a single number and this becomes the new numerator of the combined fraction. The denominators are then multiplied together to give the new denominator of the combined fraction.

For example, $\dfrac{2}{3} \times \dfrac{4}{7} = \dfrac{2 \times 4}{3 \times 7} = \mathbf{\dfrac{8}{21}}$

Problem 9. Simplify $7 \times \dfrac{2}{5}$

$$7 \times \frac{2}{5} = \frac{7}{1} \times \frac{2}{5} = \frac{7 \times 2}{1 \times 5} = \frac{14}{5} = \mathbf{2\frac{4}{5}}$$

Problem 10. Find the value of $\dfrac{3}{7} \times \dfrac{14}{15}$

Dividing numerator and denominator by 3 gives

$$\frac{3}{7} \times \frac{14}{15} = \frac{1}{7} \times \frac{14}{5} = \frac{1 \times 14}{7 \times 5}$$

Dividing numerator and denominator by 7 gives

$$\frac{1 \times 14}{7 \times 5} = \frac{1 \times 2}{1 \times 5} = \mathbf{\frac{2}{5}}$$

This process of dividing both the numerator and denominator of a fraction by the same factor(s) is called **cancelling**.

Problem 11. Simplify $\dfrac{3}{5} \times \dfrac{4}{9}$

$$\frac{3}{5} \times \frac{4}{9} = \frac{1}{5} \times \frac{4}{3} \text{ by cancelling}$$
$$= \mathbf{\frac{4}{15}}$$

Problem 12. Evaluate $1\dfrac{3}{5} \times 2\dfrac{1}{3} \times 3\dfrac{3}{7}$

Mixed numbers **must** be expressed as improper fractions before multiplication can be performed. Thus,

$$1\frac{3}{5} \times 2\frac{1}{3} \times 3\frac{3}{7} = \left(\frac{5}{5} + \frac{3}{5}\right) \times \left(\frac{6}{3} + \frac{1}{3}\right) \times \left(\frac{21}{7} + \frac{3}{7}\right)$$
$$= \frac{8}{5} \times \frac{7}{3} \times \frac{24}{7} = \frac{8 \times 1 \times 8}{5 \times 1 \times 1} = \frac{64}{5}$$
$$= \mathbf{12\frac{4}{5}}$$

Problem 13. Simplify $3\dfrac{1}{5} \times 1\dfrac{2}{3} \times 2\dfrac{3}{4}$

The mixed numbers need to be changed to improper fractions before multiplication can be performed.

$$3\frac{1}{5} \times 1\frac{2}{3} \times 2\frac{3}{4} = \frac{16}{5} \times \frac{5}{3} \times \frac{11}{4}$$
$$= \frac{4}{1} \times \frac{1}{3} \times \frac{11}{1} \text{ by cancelling}$$
$$= \frac{4 \times 1 \times 11}{1 \times 3 \times 1} = \frac{44}{3} = \mathbf{14\frac{2}{3}}$$

2.3.2 Division

The simple rule for division is **change the division sign into a multiplication sign and invert the second fraction**.

For example, $\dfrac{2}{3} \div \dfrac{3}{4} = \dfrac{2}{3} \times \dfrac{4}{3} = \mathbf{\dfrac{8}{9}}$

Problem 14. Simplify $\dfrac{3}{7} \div \dfrac{8}{21}$

$$\frac{3}{7} \div \frac{8}{21} = \frac{3}{7} \times \frac{21}{8} = \frac{3}{1} \times \frac{3}{8} \text{ by cancelling}$$
$$= \frac{3 \times 3}{1 \times 8} = \frac{9}{8} = \mathbf{1\frac{1}{8}}$$

Problem 15. Find the value of $5\dfrac{3}{5} \div 7\dfrac{1}{3}$

The mixed numbers must be expressed as improper fractions. Thus,

$$5\frac{3}{5} \div 7\frac{1}{3} = \frac{28}{5} \div \frac{22}{3} = \frac{28}{5} \times \frac{3}{22} = \frac{14}{5} \times \frac{3}{11} = \mathbf{\frac{42}{55}}$$

Problem 16. Simplify $3\dfrac{2}{3} \times 1\dfrac{3}{4} \div 2\dfrac{3}{4}$

Mixed numbers must be expressed as improper fractions before multiplication and division can be performed:

$$3\frac{2}{3} \times 1\frac{3}{4} \div 2\frac{3}{4} = \frac{11}{3} \times \frac{7}{4} \div \frac{11}{4} = \frac{11}{3} \times \frac{7}{4} \times \frac{4}{11}$$

$$= \frac{1 \times 7 \times 1}{3 \times 1 \times 1} \text{ by cancelling}$$

$$= \frac{7}{3} = 2\frac{1}{3}$$

Now try the following Practice Exercise

Practice Exercise 6   Multiplying and dividing fractions (answers on page 340)

Evaluate the following.

1. $\frac{2}{5} \times \frac{4}{7}$
2. $5 \times \frac{4}{9}$
3. $\frac{3}{4} \times \frac{8}{11}$
4. $\frac{3}{4} \times \frac{5}{9}$
5. $\frac{17}{35} \times \frac{15}{68}$
6. $\frac{3}{5} \times \frac{7}{9} \times 1\frac{2}{7}$
7. $\frac{13}{17} \times 4\frac{7}{11} \times 3\frac{4}{39}$
8. $\frac{1}{4} \times \frac{3}{11} \times 1\frac{5}{39}$
9. $\frac{2}{9} \div \frac{4}{27}$
10. $\frac{3}{8} \div \frac{45}{64}$
11. $\frac{3}{8} \div \frac{5}{32}$
12. $\frac{3}{4} \div 1\frac{4}{5}$
13. $2\frac{1}{4} \times 1\frac{2}{3}$
14. $1\frac{1}{3} \div 2\frac{5}{9}$
15. $2\frac{4}{5} \div \frac{7}{10}$
16. $2\frac{3}{4} \div 3\frac{2}{3}$
17. $\frac{1}{9} \times \frac{3}{4} \times 1\frac{1}{3}$
18. $3\frac{1}{4} \times 1\frac{3}{5} \div \frac{2}{5}$

19. A ship's crew numbers 105, of which $\frac{1}{7}$ are women. Of the men, $\frac{1}{6}$ are officers. How many male officers are on board?

20. If a storage tank is holding 450 litres when it is three-quarters full, how much will it contain when it is two-thirds full?

21. Three people, P, Q and R, contribute to a fund. P provides 3/5 of the total, Q provides 2/3 of the remainder and R provides £8. Determine (a) the total of the fund and (b) the contributions of P and Q.

22. A tank contains 24,000 litres of oil. Initially, $\frac{7}{10}$ of the contents are removed, then $\frac{3}{5}$ of the remainder is removed. How much oil is left in the tank?

2.4 Order of precedence with fractions

As stated in Chapter 1, sometimes addition, subtraction, multiplication, division, powers and brackets can all be involved in a calculation. A definite order of precedence must be adhered to. The order is:

Brackets

Order (or p**O**wer)

Division

Multiplication

Addition

Subtraction

This is demonstrated in the following worked problems.

Problem 17. Simplify $\frac{7}{20} - \frac{3}{8} \times \frac{4}{5}$

$$\frac{7}{20} - \frac{3}{8} \times \frac{4}{5} = \frac{7}{20} - \frac{3 \times 1}{2 \times 5} \text{ by cancelling} \quad \text{(M)}$$

$$= \frac{7}{20} - \frac{3}{10} \quad \text{(M)}$$

$$= \frac{7}{20} - \frac{6}{20}$$

$$= \frac{1}{20} \quad \text{(S)}$$

Problem 18. Simplify $\frac{1}{4} - 2\frac{1}{5} \times \frac{5}{8} + \frac{9}{10}$

$$\frac{1}{4} - 2\frac{1}{5} \times \frac{5}{8} + \frac{9}{10} = \frac{1}{4} - \frac{11}{5} \times \frac{5}{8} + \frac{9}{10}$$

$$= \frac{1}{4} - \frac{11}{1} \times \frac{1}{8} + \frac{9}{10} \text{ by cancelling}$$

$$= \frac{1}{4} - \frac{11}{8} + \frac{9}{10} \quad \text{(M)}$$

$$= \frac{1 \times 10}{4 \times 10} - \frac{11 \times 5}{8 \times 5} + \frac{9 \times 4}{10 \times 4}$$

(since the LCM of 4, 8 and 10 is 40)

$$= \frac{10}{40} - \frac{55}{40} + \frac{36}{40}$$

$$= \frac{10 - 55 + 36}{40} \qquad \text{(A/S)}$$

$$= -\frac{9}{40}$$

Problem 19. Simplify

$$2\frac{1}{2} - \left(\frac{2}{5} + \frac{3}{4}\right) \div \left(\frac{5}{8} \times \frac{2}{3}\right)$$

$$2\frac{1}{2} - \left(\frac{2}{5} + \frac{3}{4}\right) \div \left(\frac{5}{8} \times \frac{2}{3}\right)$$

$$= \frac{5}{2} - \left(\frac{2 \times 4}{5 \times 4} + \frac{3 \times 5}{4 \times 5}\right) \div \left(\frac{5}{8} \times \frac{2}{3}\right) \qquad \text{(B)}$$

$$= \frac{5}{2} - \left(\frac{8}{20} + \frac{15}{20}\right) \div \left(\frac{5}{8} \times \frac{2}{3}\right) \qquad \text{(B)}$$

$$= \frac{5}{2} - \frac{23}{20} \div \left(\frac{5}{4} \times \frac{1}{3}\right) \text{ by cancelling} \qquad \text{(B)}$$

$$= \frac{5}{2} - \frac{23}{20} \div \frac{5}{12} \qquad \text{(B)}$$

$$= \frac{5}{2} - \frac{23}{20} \times \frac{12}{5} \qquad \text{(D)}$$

$$= \frac{5}{2} - \frac{23}{5} \times \frac{3}{5} \text{ by cancelling}$$

$$= \frac{5}{2} - \frac{69}{25} \qquad \text{(M)}$$

$$= \frac{5 \times 25}{2 \times 25} - \frac{69 \times 2}{25 \times 2} \qquad \text{(S)}$$

$$= \frac{125}{50} - \frac{138}{50} \qquad \text{(S)}$$

$$= -\frac{13}{50}$$

Problem 20. Evaluate

$$\frac{1}{3} \text{ of } \left(5\frac{1}{2} - 3\frac{3}{4}\right) + 3\frac{1}{5} \div \frac{4}{5} - \frac{1}{2}$$

$$\frac{1}{3} \text{ of } \left(5\frac{1}{2} - 3\frac{3}{4}\right) + 3\frac{1}{5} \div \frac{4}{5} - \frac{1}{2}$$

$$= \frac{1}{3} \text{ of } 1\frac{3}{4} + 3\frac{1}{5} \div \frac{4}{5} - \frac{1}{2} \qquad \text{(B)}$$

$$= \frac{1}{3} \times \frac{7}{4} + \frac{16}{5} \div \frac{4}{5} - \frac{1}{2} \qquad \text{(O)}$$

(Note that the 'of ' is replaced with a multiplication sign.)

$$= \frac{1}{3} \times \frac{7}{4} + \frac{16}{5} \times \frac{5}{4} - \frac{1}{2} \qquad \text{(D)}$$

$$= \frac{1}{3} \times \frac{7}{4} + \frac{4}{1} \times \frac{1}{1} - \frac{1}{2} \text{ by cancelling}$$

$$= \frac{7}{12} + \frac{4}{1} - \frac{1}{2} \qquad \text{(M)}$$

$$= \frac{7}{12} + \frac{48}{12} - \frac{6}{12} \qquad \text{(A/S)}$$

$$= \frac{49}{12}$$

$$= 4\frac{1}{12}$$

Now try the following Practice Exercise

Practice Exercise 7   Order of precedence with fractions (answers on page 340)

Evaluate the following.

1. $2\frac{1}{2} - \frac{3}{5} \times \frac{20}{27}$

2. $\frac{1}{3} - \frac{3}{4} \times \frac{16}{27}$

3. $\frac{1}{2} + \frac{3}{5} \div \frac{9}{15} - \frac{1}{3}$

4. $\frac{1}{5} + 2\frac{2}{3} \div \frac{5}{9} - \frac{1}{4}$

5. $\frac{4}{5} \times \frac{1}{2} - \frac{1}{6} \div \frac{2}{5} + \frac{2}{3}$

6. $\frac{3}{5} - \left(\frac{2}{3} - \frac{1}{2}\right) \div \left(\frac{5}{6} \times \frac{3}{2}\right)$

7. $\frac{1}{2} \text{ of } \left(4\frac{2}{5} - 3\frac{7}{10}\right) + \left(3\frac{1}{3} \div \frac{2}{3}\right) - \frac{2}{5}$

8. $\dfrac{6\frac{2}{3} \times 1\frac{2}{5} - \frac{1}{3}}{6\frac{3}{4} \div 1\frac{1}{2}}$

9. $1\frac{1}{3} \times 2\frac{1}{5} \div \frac{2}{5}$

10. $\frac{1}{4} \times \frac{2}{5} - \frac{1}{5} \div \frac{2}{3} + \frac{4}{15}$

11. $\dfrac{\frac{2}{3} + 3\frac{1}{5} \times 2\frac{1}{2} + 1\frac{1}{3}}{8\frac{1}{3} \div 3\frac{1}{3}}$

12. $\frac{1}{13} \text{ of } \left(2\frac{9}{10} - 1\frac{3}{5}\right) + \left(2\frac{1}{3} \div \frac{2}{3}\right) - \frac{3}{4}$

Revision Test 1 : Basic arithmetic and fractions

This assignment covers the material contained in Chapters 1 and 2. *The marks available are shown in brackets at the end of each question.*

1. Evaluate
   $1009 \, \text{cm} - 356 \, \text{cm} - 742 \, \text{cm} + 94 \, \text{cm}$. (3)

2. Determine £284 × 9. (3)

3. Evaluate
   (a) $-11239 - (-4732) + 9639$
   (b) -164×-12
   (c) 367×-19 (6)

4. Calculate (a) $153 ÷ 9 (b) 1397 g ÷ 11 (4)

5. A small component has a mass of 27 grams. Calculate the mass, in kilograms, of 750 such components. (3)

6. Find (a) the highest common factor and (b) the lowest common multiple of the following numbers: 15 40 75 120. (7)

Evaluate the expressions in questions 7 to 12.

7. $7 + 20 ÷ (9 - 5)$ (3)

8. $147 - 21(24 ÷ 3) + 31$ (3)

9. $40 ÷ (1 + 4) + 7[8 + (3 \times 8) - 27]$ (5)

10. $\dfrac{(7 - 3)(2 - 5)}{3(9 - 5) ÷ (2 - 6)}$ (3)

11. $\dfrac{(7 + 4 \times 5) ÷ 3 + 6 ÷ 2}{2 \times 4 + (5 - 8) - 2^2 + 3}$ (5)

12. $\dfrac{(4^2 \times 5 - 8) ÷ 3 + 9 \times 8}{4 \times 3^2 - 20 ÷ 5}$ (5)

13. Simplify
   (a) $\dfrac{3}{4} - \dfrac{7}{15}$
   (b) $1\dfrac{5}{8} - 2\dfrac{1}{3} + 3\dfrac{5}{6}$ (8)

14. A training college has 480 students of which 150 are girls. Express this as a fraction in its simplest form. (2)

15. A tank contains 18 000 litres of oil. Initially, $\dfrac{7}{10}$ of the contents are removed, then $\dfrac{2}{5}$ of the remainder is removed. How much oil is left in the tank? (4)

16. Evaluate
   (a) $1\dfrac{7}{9} \times \dfrac{3}{8} \times 3\dfrac{3}{5}$
   (b) $6\dfrac{2}{3} ÷ 1\dfrac{1}{3}$
   (c) $1\dfrac{1}{3} \times 2\dfrac{1}{5} ÷ \dfrac{2}{5}$ (10)

17. Calculate
   (a) $\dfrac{1}{4} \times \dfrac{2}{5} - \dfrac{1}{5} ÷ \dfrac{2}{3} + \dfrac{4}{15}$
   (b) $\dfrac{\dfrac{2}{3} + 3\dfrac{1}{5} \times 2\dfrac{1}{2} + 1\dfrac{1}{3}}{8\dfrac{1}{3} ÷ 3\dfrac{1}{3}}$ (8)

18. Simplify $\left\{ \dfrac{1}{13} \text{ of } \left(2\dfrac{9}{10} - 1\dfrac{3}{5} \right) \right\} + \left(2\dfrac{1}{3} ÷ \dfrac{2}{3} \right) - \dfrac{3}{4}$ (8)

Chapter 3

Decimals

3.1 Introduction

The decimal system of numbers is based on the digits 0 to 9.

There are a number of everyday occurrences in which we use decimal numbers. For example, a radio is, say, tuned to 107.5 MHz FM; 107.5 is an example of a decimal number.

In a shop, a pair of trainers cost, say, £57.95; 57.95 is another example of a decimal number. 57.95 is a decimal fraction, where a decimal point separates the integer, i.e. 57, from the fractional part, i.e. 0.95

57.95 actually means $(5 \times 10) + (7 \times 1)$

$$+ \left(9 \times \frac{1}{10}\right) + \left(5 \times \frac{1}{100}\right)$$

3.2 Converting decimals to fractions and vice-versa

Converting decimals to fractions and vice-versa is demonstrated below with worked examples.

Problem 1. Convert 0.375 to a proper fraction in its simplest form

(i) 0.375 may be written as $\dfrac{0.375 \times 1000}{1000}$ i.e.

$0.375 = \dfrac{375}{1000}$

(ii) Dividing both numerator and denominator by 5 gives $\dfrac{375}{1000} = \dfrac{75}{200}$

(iii) Dividing both numerator and denominator by 5 again gives $\dfrac{75}{200} = \dfrac{15}{40}$

(iv) Dividing both numerator and denominator by 5 again gives $\dfrac{15}{40} = \dfrac{\mathbf{3}}{\mathbf{8}}$

Since both 3 and 8 are only divisible by 1, we cannot 'cancel' any further, so $\dfrac{3}{8}$ is the 'simplest form' of the fraction.

Hence, **the decimal fraction $0.375 = \dfrac{3}{8}$ as a proper fraction**.

Problem 2. Convert 3.4375 to a mixed number

(i) 0.4375 may be written as $\dfrac{0.4375 \times 10000}{10000}$ i.e.

$0.4375 = \dfrac{4375}{10000}$

(ii) Dividing both numerator and denominator by 25 gives $\dfrac{4375}{10000} = \dfrac{175}{400}$

(iii) Dividing both numerator and denominator by 5 gives $\dfrac{175}{400} = \dfrac{35}{80}$

(iv) Dividing both numerator and denominator by 5 again gives $\dfrac{35}{80} = \dfrac{7}{16}$

Since both 5 and 16 are only divisible by 1, we cannot 'cancel' any further, so $\dfrac{7}{16}$ is the 'lowest form' of the fraction.

(v) Hence, $0.4375 = \dfrac{7}{16}$

Thus, **the decimal fraction $3.4375 = 3\dfrac{7}{16}$ as a mixed number**.

Problem 3. Express $\dfrac{7}{8}$ as a decimal fraction

To convert a proper fraction to a decimal fraction, the numerator is divided by the denominator.

$$\begin{array}{r} 0.8\ 7\ 5 \\ 8\overline{)7.0\ 0\ 0} \end{array}$$

(i) 8 into 7 will not go. Place the 0 above the 7.

(ii) Place the decimal point above the decimal point of 7.000

(iii) 8 into 70 goes 8, remainder 6. Place the 8 above the first zero after the decimal point and carry the 6 remainder to the next digit on the right, making it 60.

(iv) 8 into 60 goes 7, remainder 4. Place the 7 above the next zero and carry the 4 remainder to the next digit on the right, making it 40.

(v) 8 into 40 goes 5, remainder 0. Place 5 above the next zero.

Hence, **the proper fraction** $\dfrac{7}{8} = 0.875$ **as a decimal fraction**.

Problem 4. Express $5\dfrac{13}{16}$ as a decimal fraction

For mixed numbers it is only necessary to convert the proper fraction part of the mixed number to a decimal fraction.

$$\begin{array}{r} 0.8\ 1\ 2\ 5 \\ 16\overline{)13.0\ 0\ 0\ 0} \end{array}$$

(i) 16 into 13 will not go. Place the 0 above the 3.

(ii) Place the decimal point above the decimal point of 13.0000

(iii) 16 into 130 goes 8, remainder 2. Place the 8 above the first zero after the decimal point and carry the 2 remainder to the next digit on the right, making it 20.

(iv) 16 into 20 goes 1, remainder 4. Place the 1 above the next zero and carry the 4 remainder to the next digit on the right, making it 40.

(v) 16 into 40 goes 2, remainder 8. Place the 2 above the next zero and carry the 8 remainder to the next digit on the right, making it 80.

(vi) 16 into 80 goes 5, remainder 0. Place the 5 above the next zero.

(vii) Hence, $\dfrac{13}{16} = 0.8125$

Thus, **the mixed number** $5\dfrac{13}{16} = 5.8125$ **as a decimal fraction**.

Now try the following Practice Exercise

Practice Exercise 8   Converting decimals to fractions and vice-versa (answers on page 341)

1. Convert 0.65 to a proper fraction.

2. Convert 0.036 to a proper fraction.

3. Convert 0.175 to a proper fraction.

4. Convert 0.048 to a proper fraction.

5. Convert the following to proper fractions.
   (a) 0.65   (b) 0.84   (c) 0.0125
   (d) 0.282   (e) 0.024

6. Convert 4.525 to a mixed number.

7. Convert 23.44 to a mixed number.

8. Convert 10.015 to a mixed number.

9. Convert 6.4375 to a mixed number.

10. Convert the following to mixed numbers.
    (a) 1.82   (b) 4.275   (c) 14.125
    (d) 15.35   (e) 16.2125

11. Express $\dfrac{5}{8}$ as a decimal fraction.

12. Express $6\dfrac{11}{16}$ as a decimal fraction.

13. Express $\dfrac{7}{32}$ as a decimal fraction.

14. Express $11\dfrac{3}{16}$ as a decimal fraction.

15. Express $\dfrac{9}{32}$ as a decimal fraction.

3.3 Significant figures and decimal places

A number which can be expressed exactly as a decimal fraction is called a **terminating decimal**.

For example,

$$3\frac{3}{16} = 3.1825 \text{ is a terminating decimal}$$

A number which cannot be expressed exactly as a decimal fraction is called a **non-terminating decimal**. For example,

$$1\frac{5}{7} = 1.7142857\ldots \text{ is a non-terminating decimal}$$

The answer to a non-terminating decimal may be expressed in two ways, depending on the accuracy required:

(a) correct to a number of **significant figures**, or

(b) correct to a number of **decimal places** i.e. the number of figures after the decimal point.

The last digit in the answer is unaltered if the next digit on the right is in the group of numbers 0, 1, 2, 3 or 4. For example,

$$1.714285\ldots = \mathbf{1.714} \text{ correct to 4 significant figures}$$

$$= \mathbf{1.714} \text{ correct to 3 decimal places}$$

since the next digit on the right in this example is 2. The last digit in the answer is increased by 1 if the next digit on the right is in the group of numbers 5, 6, 7, 8 or 9. For example,

$$1.7142857\ldots = \mathbf{1.7143} \text{ correct to 5 significant figures}$$

$$= \mathbf{1.7143} \text{ correct to 4 decimal places}$$

since the next digit on the right in this example is 8.

Problem 5. Express 15.36815 correct to
(a) 2 decimal places, (b) 3 significant figures,
(c) 3 decimal places, (d) 6 significant figures

(a) $15.36815 = \mathbf{15.37}$ correct to 2 decimal places.

(b) $15.36815 = \mathbf{15.4}$ correct to 3 significant figures.

(c) $15.36815 = \mathbf{15.368}$ correct to 3 decimal places.

(d) $15.36815 = \mathbf{15.3682}$ correct to 6 significant figures.

Problem 6. Express 0.004369 correct to
(a) 4 decimal places, (b) 3 significant figures

(a) $0.004369 = \mathbf{0.0044}$ correct to 4 decimal places.

(b) $0.004369 = \mathbf{0.00437}$ correct to 3 significant figures.

Note that the zeros to the right of the decimal point do not count as significant figures.

Now try the following Practice Exercise

1. Express 14.1794 correct to 2 decimal places.

2. Express 2.7846 correct to 4 significant figures.

3. Express 65.3792 correct to 2 decimal places.

4. Express 43.2746 correct to 4 significant figures.

5. Express 1.2973 correct to 3 decimal places.

6. Express 0.0005279 correct to 3 significant figures.

3.4 Adding and subtracting decimal numbers

When adding or subtracting decimal numbers, care needs to be taken to ensure that the decimal points are beneath each other. This is demonstrated in the following worked examples.

Problem 7. Evaluate $46.8 + 3.06 + 2.4 + 0.09$ and give the answer correct to 3 significant figures

The decimal points are placed under each other as shown. Each column is added, starting from the right.

$$
\begin{array}{r}
46.8 \\
3.06 \\
2.4 \\
+\,0.09 \\
\hline
52.35 \\
\hline
{\scriptstyle 1\,1\,1}
\end{array}
$$

(i) $6 + 9 = 15$. Place 5 in the hundredths column. Carry 1 in the tenths column.

(ii) $8 + 0 + 4 + 0 + 1$ (carried) $= 13$. Place the 3 in the tenths column. Carry the 1 into the units column.

(iii) $6 + 3 + 2 + 0 + 1$ (carried) $= 12$. Place the 2 in the units column. Carry the 1 into the tens column.

(iv) $4 + 1$(carried) $= 5$. Place the 5 in the hundreds column.

Hence,

$$46.8 + 3.06 + 2.4 + 0.09 = 52.35$$

$$= 52.4, \text{ correct to 3 significant figures}$$

> **Problem 8.** Evaluate $64.46 - 28.77$ and give the answer correct to 1 decimal place

As with addition, the decimal points are placed under each other as shown.

$$\begin{array}{r} 64.46 \\ -\underline{28.77} \\ \underline{35.69} \end{array}$$

(i) $6 - 7$ is not possible; therefore 'borrow' 1 from the tenths column. This gives $16 - 7 = 9$. Place the 9 in the hundredths column.

(ii) $3 - 7$ is not possible; therefore 'borrow' 1 from the units column. This gives $13 - 7 = 6$. Place the 6 in the tenths column.

(iii) $3 - 8$ is not possible; therefore 'borrow' from the hundreds column. This gives $13 - 8 = 5$. Place the 5 in the units column.

(iv) $5 - 2 = 3$. Place the 3 in the hundreds column.

Hence,

$$64.46 - 28.77 = 35.69$$

$$= 35.7 \text{ correct to 1 decimal place}$$

> **Problem 9.** Evaluate $312.64 - 59.826 - 79.66 + 38.5$ and give the answer correct to 4 significant figures

The sum of the positive decimal fractions $= 312.64 + 38.5 = 351.14$.
The sum of the negative decimal fractions $= 59.826 + 79.66 = 139.486$.
Taking the sum of the negative decimal fractions from the sum of the positive decimal fractions gives

$$\begin{array}{r} 351.140 \\ -\underline{139.486} \\ \underline{211.654} \end{array}$$

Hence, $351.140 - 139.486 = \mathbf{211.654} = \mathbf{211.7}$, **correct to 4 significant figures**.

Now try the following Practice Exercise

> **Practice Exercise 10   Adding and subtracting decimal numbers (answers on page 341)**
>
> Determine the following without using a calculator.
>
> 1. Evaluate $37.69 + 42.6$, correct to 3 significant figures.
>
> 2. Evaluate $378.1 - 48.85$, correct to 1 decimal place.
>
> 3. Evaluate $68.92 + 34.84 - 31.223$, correct to 4 significant figures.
>
> 4. Evaluate $67.841 - 249.55 + 56.883$, correct to 2 decimal places.
>
> 5. Evaluate $483.24 - 120.44 - 67.49$, correct to 4 significant figures.
>
> 6. Evaluate $738.22 - 349.38 - 427.336 + 56.779$, correct to 1 decimal place.
>
> 7. Determine the dimension marked x in the length of the shaft shown in Figure 3.1. The dimensions are in millimetres.
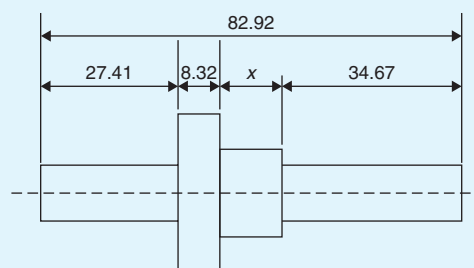>
>
>
> **Figure 3.1**

3.5 Multiplying and dividing decimal numbers

When multiplying decimal fractions:

(a) the numbers are multiplied as if they were integers, and

(b) the position of the decimal point in the answer is such that there are as many digits to the right of it as the sum of the digits to the right of the decimal points of the two numbers being multiplied together.

This is demonstrated in the following worked examples.

Problem 10. Evaluate 37.6×5.4

$$
\begin{array}{r}
376 \\
\times 54 \\
\hline
1504 \\
18800 \\
\hline
20304 \\
\hline
\end{array}
$$

(i) $376 \times 54 = 20304$.

(ii) As there are $1 + 1 = 2$ digits to the right of the decimal points of the two numbers being multiplied together, $37.\underline{6} \times 5.\underline{4}$, then

$$\mathbf{37.6 \times 5.4 = 203.04}$$

Problem 11. Evaluate $44.25 \div 1.2$, correct to (a) 3 significant figures, (b) 2 decimal places

$$44.25 \div 1.2 = \frac{44.25}{1.2}$$

The denominator is multiplied by 10 to change it into an integer. The numerator is also multiplied by 10 to keep the fraction the same. Thus,

$$\frac{44.25}{1.2} = \frac{44.25 \times 10}{1.2 \times 10} = \frac{442.5}{12}$$

The long division is similar to the long division of integers and the steps are as shown.

$$
\begin{array}{r}
36.875 \\
12\overline{)442.500} \\
\underline{36} \\
82 \\
\underline{72} \\
105 \\
\underline{96} \\
90 \\
\underline{84} \\
60 \\
\underline{60} \\
0 \\
\end{array}
$$

(i) 12 into 44 goes 3; place the 3 above the second 4 of 442.500

(ii) $3 \times 12 = 36$; place the 36 below the 44 of 442.500

(iii) $44 - 36 = 8$.

(iv) Bring down the 2 to give 82.

(v) 12 into 82 goes 6; place the 6 above the 2 of 442.500

(vi) $6 \times 12 = 72$; place the 72 below the 82.

(vii) $82 - 72 = 10$.

(viii) Bring down the 5 to give 105.

(ix) 12 into 105 goes 8; place the 8 above the 5 of 442.500

(x) $8 \times 12 = 96$; place the 96 below the 105.

(xi) $105 - 96 = 9$.

(xii) Bring down the 0 to give 90.

(xiii) 12 into 90 goes 7; place the 7 above the first zero of 442.500

(xiv) $7 \times 12 = 84$; place the 84 below the 90.

(xv) $90 - 84 = 6$.

(xvi) Bring down the 0 to give 60.

(xvii) 12 into 60 gives 5 exactly; place the 5 above the second zero of 442.500

(xviii) Hence, $44.25 \div 1.2 = \dfrac{442.5}{12} = \mathbf{36.875}$

So,

(a) $44.25 \div 1.2 = \mathbf{36.9}$, **correct to 3 significant figures**.

(b) $44.25 \div 1.2 = \mathbf{36.88}$, **correct to 2 decimal places**.

Problem 12. Express $7\dfrac{2}{3}$ as a decimal fraction, correct to 4 significant figures

Dividing 2 by 3 gives $\dfrac{2}{3} = 0.666666\ldots$

and $7\dfrac{2}{3} = 7.666666\ldots$

Hence, $\mathbf{7\dfrac{2}{3} = 7.667}$ **correct to 4 significant figures**.

Note that $7.6666\ldots$ is called **7.6 recurring** and is written as **$7.\dot{6}$**

Now try the following Practice Exercise

Practice Exercise 11   Multiplying and dividing decimal numbers (answers on page 341)

In Problems 1 to 8, evaluate without using a calculator.

1. Evaluate 3.57×1.4

2. Evaluate 67.92×0.7

3. Evaluate 167.4×2.3

4. Evaluate 342.6×1.7

5. Evaluate $548.28 \div 1.2$

6. Evaluate $478.3 \div 1.1$, correct to 5 significant figures.

7. Evaluate $563.48 \div 0.9$, correct to 4 significant figures.

8. Evaluate $2387.4 \div 1.5$

In Problems 9 to 14, express as decimal fractions to the accuracy stated.

9. $\dfrac{4}{9}$, correct to 3 significant figures.

10. $\dfrac{17}{27}$, correct to 5 decimal places.

11. $1\dfrac{9}{16}$, correct to 4 significant figures.

12. $53\dfrac{5}{11}$, correct to 3 decimal places.

13. $13\dfrac{31}{37}$, correct to 2 decimal places.

14. $8\dfrac{9}{13}$, correct to 3 significant figures.

15. Evaluate $421.8 \div 17$, (a) correct to 4 significant figures and (b) correct to 3 decimal places.

16. Evaluate $\dfrac{0.0147}{2.3}$, (a) correct to 5 decimal places and (b) correct to 2 significant figures.

17. Evaluate (a) $\dfrac{12.\dot{6}}{1.5}$ (b) $5.\dot{2} \times 12$

18. A tank contains 1800 litres of oil. How many tins containing 0.75 litres can be filled from this tank?

Chapter 4

Using a calculator

4.1 Introduction

In engineering, calculations often need to be performed. For simple numbers it is useful to be able to use mental arithmetic. However, when numbers are larger an electronic calculator needs to be used.

There are several calculators on the market, many of which will be satisfactory for our needs. It is essential to have a **scientific notation calculator** which will have all the necessary functions needed and more.

This chapter assumes you have a **CASIO fx-83ES calculator**, or similar, as shown in Figure 4.1.

Besides straightforward addition, subtraction, multiplication and division, which you will already be able to do, we will check that you can use squares, cubes, powers, reciprocals, roots, fractions and trigonometric functions (the latter in preparation for Chapter 21). There are several other functions on the calculator which we do not need to concern ourselves with at this level.

4.2 Adding, subtracting, multiplying and dividing

Initially, after switching on, press **Mode**.

Of the three possibilities, use **Comp**, which is achieved by pressing **1**.

Next, press **Shift** followed by **Setup** and, of the eight possibilities, use **Mth IO**, which is achieved by pressing **1**.

By all means experiment with the other menu options – refer to your 'User's guide'.

All calculators have **+**, **−**, **× and ÷ functions** and these functions will, no doubt, already have been used in calculations.

> **Problem 1.** Evaluate $364.7 \div 57.5$ correct to 3 decimal places

(i)   Type in 364.7

(ii)   Press ÷.

(iii)   Type in 57.5

(iv)   Press = and the fraction $\dfrac{3647}{575}$ appears.

(v)   Press the $S \Leftrightarrow D$ function and the decimal answer $6.34260869\ldots$ appears.

Alternatively, after step (iii) press Shift and = and the decimal will appear.

Hence, **$364.7 \div 57.5 = 6.343$ correct to 3 decimal places**.

> **Problem 2.** Evaluate $\dfrac{12.47 \times 31.59}{70.45 \times 0.052}$ correct to 4 significant figures

(i)   Type in 12.47

(ii)   Press ×.

(iii)   Type in 31.59

(iv)   Press ÷.

(v)   The denominator must have brackets; i.e. press (.

(vi)   Type in 70.45×0.052 and complete the bracket; i.e.).

(vii)   Press = and the answer $107.530518\ldots$ appears.

Hence, $\dfrac{\mathbf{12.47 \times 31.59}}{\mathbf{70.45 \times 0.052}} = \mathbf{107.5}$ **correct to 4 significant figures**.