Chapter 5

Percentages

5.1 Introduction

Percentages are used to give a common standard. The use of percentages is very common in many aspects of commercial life, as well as in engineering. Interest rates, sale reductions, pay rises, exams and VAT are all examples of situations in which percentages are used.
For this chapter you will need to know about decimals and fractions and be able to use a calculator.
We are familiar with the symbol for percentage, i.e. %. Here are some examples.

- Interest rates indicate the cost at which we can borrow money. If you borrow £8000 at a **6.5% interest rate** for a year, it will cost you 6.5% of the amount borrowed to do so, which will need to be repaid along with the original money you borrowed. If you repay the loan in 1 year, how much interest will you have paid?
- A pair of trainers in a shop cost £60. They are advertised in a sale as **20% off**. How much will you pay?
- If you earn £20 000 p.a. and you receive a **2.5% pay rise**, how much extra will you have to spend the following year?
- A book costing £18 can be purchased on the internet for 30% less. What will be its cost?

When we have completed his chapter on percentages you will be able to understand how to perform the above calculations.
Percentages are fractions having 100 as their denominator. For example, the fraction $\frac{40}{100}$ is written as 40% and is read as 'forty per cent'.
The easiest way to understand percentages is to go through some worked examples.

5.2 Percentage calculations

5.2.1 To convert a decimal to a percentage

A decimal number is converted to a percentage by multiplying by 100.

Problem 1. Express 0.015 as a percentage

To express a decimal number as a percentage, merely multiply by 100, i.e.

$$0.015 = 0.015 \times 100\% = \mathbf{1.5\%}$$

Multiplying a decimal number by 100 means moving the decimal point 2 places **to the right**.

Problem 2. Express 0.275 as a percentage

$$0.275 = 0.275 \times 100\% = \mathbf{27.5\%}$$

5.2.2 To convert a percentage to a decimal

A percentage is converted to a decimal number by dividing by 100.

Problem 3. Express 6.5% as a decimal number

$$6.5\% = \frac{6.5}{100} = \mathbf{0.065}$$

Dividing by 100 means moving the decimal point 2 places **to the left**.

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Problem 4. Express 17.5% as a decimal number

$$17.5\% = \frac{17.5}{100} = \mathbf{0.175}$$

5.2.3 To convert a fraction to a percentage

A fraction is converted to a percentage by multiplying by 100.

Problem 5. Express $\frac{5}{8}$ as a percentage

$$\frac{5}{8} = \frac{5}{8} \times 100\% = \frac{500}{8}\% = \mathbf{62.5\%}$$

Problem 6. Express $\frac{5}{19}$ as a percentage, correct to 2 decimal places

$$\begin{aligned}\frac{5}{19} &= \frac{5}{19} \times 100\% \\ &= \frac{500}{19}\% \\ &= 26.3157889\ldots \text{by calculator} \\ &= \mathbf{26.32\%\ correct\ to\ 2\ decimal\ places}\end{aligned}$$

Problem 7. In two successive tests a student gains marks of 57/79 and 49/67. Is the second mark better or worse than the first?

$$57/79 = \frac{57}{79} = \frac{57}{79} \times 100\% = \frac{5700}{79}\% = \mathbf{72.15\%} \text{ correct to 2 decimal places}$$

$$49/67 = \frac{49}{67} = \frac{49}{67} \times 100\% = \frac{4900}{67}\% = \mathbf{73.13\%} \text{ correct to 2 decimal places}$$

Hence, **the second test is marginally better than the first test**. This question demonstrates how much easier it is to compare two fractions when they are expressed as percentages.

5.2.4 To convert a percentage to a fraction

A percentage is converted to a fraction by dividing by 100 and then, by cancelling, reducing it to its simplest form.

Problem 8. Express 75% as a fraction

$$75\% = \frac{75}{100} = \mathbf{\frac{3}{4}}$$

The fraction $\frac{75}{100}$ is reduced to its simplest form by cancelling, i.e. dividing both numerator and denominator by 25.

Problem 9. Express 37.5% as a fraction

$$\begin{aligned}37.5\% &= \frac{37.5}{100} \\ &= \frac{375}{1000} && \text{by multiplying both numerator and denominator by 10} \\ &= \frac{15}{40} && \text{by dividing both numerator and denominator by 25} \\ &= \mathbf{\frac{3}{8}} && \text{by dividing both numerator and denominator by 5}\end{aligned}$$

Now try the following Practice Exercise

Practice Exercise 21 Percentages (answers on page 342)

In problems 1 to 5, express the given numbers as percentages.

1. 0.0032
2. 1.734
3. 0.057
4. 0.374
5. 1.285
6. Express 20% as a decimal number.
7. Express 1.25% as a decimal number.
8. Express $\frac{11}{16}$ as a percentage.
9. Express $\frac{5}{13}$ as a percentage, correct to 3 decimal places.

10. Express as percentages, correct to 3 significant figures,

 (a) $\dfrac{7}{33}$ (b) $\dfrac{19}{24}$ (c) $1\dfrac{11}{16}$

11. Place the following in order of size, the smallest first, expressing each as a percentage correct to 1 decimal place.

 (a) $\dfrac{12}{21}$ (b) $\dfrac{9}{17}$ (c) $\dfrac{5}{9}$ (d) $\dfrac{6}{11}$

12. Express 65% as a fraction in its simplest form.
13. Express 31.25% as a fraction in its simplest form.
14. Express 56.25% as a fraction in its simplest form.
15. Evaluate A to J in the following table.

Decimal number	Fraction	Percentage
0.5	A	B
C	$\frac{1}{4}$	D
E	F	30
G	$\frac{3}{5}$	H
I	J	85

5.3 Further percentage calculations

5.3.1 Finding a percentage of a quantity

To find a percentage of a quantity, convert the percentage to a fraction (by dividing by 100) and remember that 'of' means multiply.

Problem 10. Find 27% of £65

$$27\% \text{ of } £65 = \frac{27}{100} \times 65$$
$$= \mathbf{£17.55} \text{ by calculator}$$

Problem 11. In a machine shop, it takes 32 minutes to machine a certain part. Using a new tool, the time can be reduced by 12.5%. Calculate the new time taken

$$12.5\% \text{ of } 32 \text{ minutes} = \frac{12.5}{100} \times 32$$
$$= 4 \text{ minutes}$$

Hence, **new time taken** = 32 − 4 = **28 minutes**.

Alternatively, if the time is reduced by 12.5%, it now takes 100% − 12.5% = 87.5% of the original time, i.e.

$$87.5\% \text{ of } 32 \text{ minutes} = \frac{87.5}{100} \times 32$$
$$= \mathbf{28\ minutes}$$

Problem 12. A 160 GB iPod is advertised as costing £190 excluding VAT. If VAT is added at 17.5%, what will be the total cost of the iPod?

$$\text{VAT} = 17.5\% \text{ of } £190 = \frac{17.5}{100} \times 190 = £33.25$$

Total cost of iPod = £190 + £33.25 = **£223.25**

A quicker method to determine the total cost is: 1.175 × £190 = **£223.25**

5.3.2 Expressing one quantity as a percentage of another quantity

To express one quantity as a percentage of another quantity, divide the first quantity by the second then multiply by 100.

Problem 13. Express 23 cm as a percentage of 72 cm, correct to the nearest 1%

$$23 \text{ cm as a percentage of } 72 \text{ cm} = \frac{23}{72} \times 100\%$$
$$= 31.94444\ldots\%$$
$$= \mathbf{32\%} \text{ correct to the nearest } 1\%$$

Problem 14. Express 47 minutes as a percentage of 2 hours, correct to 1 decimal place

Note that it is essential that the two quantities are in the **same units**.

Working in minute units, 2 hours = 2 × 60
= 120 minutes

$$47 \text{ minutes as a percentage of } 120 \text{ min} = \frac{47}{120} \times 100\%$$
$$= \mathbf{39.2\%} \text{ correct to 1 decimal place}$$

5.3.3 Percentage change

Percentage change is given by
$$\frac{\text{new value} - \text{original value}}{\text{original value}} \times 100\%.$$

Problem 15. A box of resistors increases in price from £45 to £52. Calculate the percentage change in cost, correct to 3 significant figures

$$\% \text{ change} = \frac{\text{new value} - \text{original value}}{\text{original value}} \times 100\%$$
$$= \frac{52-45}{45} \times 100\% = \frac{7}{45} \times 100$$
$$= \mathbf{15.6\% = percentage\ change\ in\ cost}$$

Problem 16. A drilling speed should be set to 400 rev/min. The nearest speed available on the machine is 412 rev/min. Calculate the percentage overspeed

$$\% \text{ overspeed} = \frac{\text{available speed} - \text{correct speed}}{\text{correct speed}} \times 100\%$$
$$= \frac{412-400}{400} \times 100\% = \frac{12}{400} \times 100\%$$
$$= \mathbf{3\%}$$

Now try the following Practice Exercise

Practice Exercise 22 Further percentages (answers on page 342)

1. Calculate 43.6% of 50 kg.
2. Determine 36% of 27 m.
3. Calculate, correct to 4 significant figures,
   (a) 18% of 2758 tonnes
   (b) 47% of 18.42 grams
   (c) 147% of 14.1 seconds.
4. When 1600 bolts are manufactured, 36 are unsatisfactory. Determine the percentage that is unsatisfactory.
5. Express
   (a) 140 kg as a percentage of 1 t.
   (b) 47 s as a percentage of 5 min.
   (c) 13.4 cm as a percentage of 2.5 m.
6. A block of Monel alloy consists of 70% nickel and 30% copper. If it contains 88.2 g of nickel, determine the mass of copper in the block.
7. An athlete runs 5000 m in 15 minutes 20 seconds. With intense training, he is able to reduce this time by 2.5%. Calculate his new time.
8. A copper alloy comprises 89% copper, 1.5% iron and the remainder aluminium. Find the amount of aluminium, in grams, in a 0.8 kg mass of the alloy.
9. A computer is advertised on the internet at £520, exclusive of VAT. If VAT is payable at 17.5%, what is the total cost of the computer?
10. Express 325 mm as a percentage of 867 mm, correct to 2 decimal places.
11. A child sleeps on average 9 hours 25 minutes per day. Express this as a percentage of the whole day, correct to 1 decimal place.
12. Express 408 g as a percentage of 2.40 kg.
13. When signing a new contract, a Premiership footballer's pay increases from £15 500 to £21 500 per week. Calculate the percentage pay increase, correct to 3 significant figures.
14. A metal rod 1.80 m long is heated and its length expands by 48.6 mm. Calculate the percentage increase in length.
15. 12.5% of a length of wood is 70 cm. What is the full length?
16. A metal rod, 1.20 m long, is heated and its length expands by 42 mm. Calculate the percentage increase in length.

5.4 More percentage calculations

5.4.1 Percentage error

$$\text{Percentage error} = \frac{\text{error}}{\text{correct value}} \times 100\%$$

Problem 17. The length of a component is measured incorrectly as 64.5 mm. The actual length is 63 mm. What is the percentage error in the measurement?

$$\% \text{ error} = \frac{\text{error}}{\text{correct value}} \times 100\%$$
$$= \frac{64.5 - 63}{63} \times 100\%$$
$$= \frac{1.5}{63} \times 100\% = \frac{150}{63}\%$$
$$= \mathbf{2.38\%}$$

The percentage measurement error is **2.38% too high**, which is sometimes written as **+ 2.38% error**.

Problem 18. The voltage across a component in an electrical circuit is calculated as 50 V using Ohm's law. When measured, the actual voltage is 50.4 V. Calculate, correct to 2 decimal places, the percentage error in the calculation

$$\% \text{ error} = \frac{\text{error}}{\text{correct value}} \times 100\%$$
$$= \frac{50.4 - 50}{50.4} \times 100\%$$
$$= \frac{0.4}{50.4} \times 100\% = \frac{40}{50.4}\%$$
$$= \mathbf{0.79\%}$$

The percentage error in the calculation is **0.79% too low**, which is sometimes written as **−0.79% error**.

5.4.2 Original value

$$\text{Original value} = \frac{\text{new value}}{100 \pm \% \text{ change}} \times 100\%$$

Problem 19. A man pays £149.50 in a sale for a DVD player which is labelled '35% off'. What was the original price of the DVD player?

In this case, it is a 35% reduction in price, so we use $\frac{\text{new value}}{100 - \% \text{ change}} \times 100$, i.e. a minus sign in the denominator.

$$\text{Original price} = \frac{\text{new value}}{100 - \% \text{ change}} \times 100$$
$$= \frac{149.5}{100 - 35} \times 100$$
$$= \frac{149.5}{65} \times 100 = \frac{14\,950}{65}$$
$$= \mathbf{£230}$$

Problem 20. A couple buys a flat and make an 18% profit by selling it 3 years later for £153 400. Calculate the original cost of the house

In this case, it is an 18% increase in price, so we use $\frac{\text{new value}}{100 + \% \text{ change}} \times 100$, i.e. a plus sign in the denominator.

$$\text{Original cost} = \frac{\text{new value}}{100 + \% \text{ change}} \times 100$$
$$= \frac{153\,400}{100 + 18} \times 100$$
$$= \frac{153\,400}{118} \times 100 = \frac{15\,340\,000}{118}$$
$$= \mathbf{£130\,000}$$

Problem 21. An electrical store makes 40% profit on each widescreen television it sells. If the selling price of a 32 inch HD television is £630, what was the cost to the dealer?

In this case, it is a 40% mark-up in price, so we use $\frac{\text{new value}}{100 + \% \text{ change}} \times 100$, i.e. a plus sign in the denominator.

$$\text{Dealer cost} = \frac{\text{new value}}{100 + \% \text{ change}} \times 100$$
$$= \frac{630}{100 + 40} \times 100$$
$$= \frac{630}{140} \times 100 = \frac{63\,000}{140}$$
$$= \mathbf{£450}$$

The dealer buys from the manufacturer for £450 and sells to his customers for £630.

5.4.3 Percentage increase/decrease and interest

$$\text{New value} = \frac{100 + \% \text{ increase}}{100} \times \text{original value}$$

Problem 22. £3600 is placed in an ISA account which pays 6.25% interest per annum. How much is the investment worth after 1 year?

$$\text{Value after 1 year} = \frac{100+6.25}{100} \times £3600$$
$$= \frac{106.25}{100} \times £3600$$
$$= 1.0625 \times £3600$$
$$= \mathbf{£3825}$$

Problem 23. The price of a fully installed combination condensing boiler is increased by 6.5%. It originally cost £2400. What is the new price?

$$\text{New price} = \frac{100+6.5}{100} \times £2,400$$
$$= \frac{106.5}{100} \times £2,400 = 1.065 \times £2,400$$
$$= \mathbf{£2,556}$$

Now try the following Practice Exercise

Practice Exercise 23 Further percentages (answers on page 342)

1. A machine part has a length of 36 mm. The length is incorrectly measured as 36.9 mm. Determine the percentage error in the measurement.
2. When a resistor is removed from an electrical circuit the current flowing increases from 450 μA to 531 μA. Determine the percentage increase in the current.
3. In a shoe shop sale, everything is advertised as '40% off'. If a lady pays £186 for a pair of Jimmy Choo shoes, what was their original price?
4. Over a four year period a family home increases in value by 22.5% to £214 375. What was the value of the house 4 years ago?
5. An electrical retailer makes a 35% profit on all its products. What price does the retailer pay for a dishwasher which is sold for £351?
6. The cost of a sports car is £23 500 inclusive of VAT at 17.5%. What is the cost of the car without the VAT added?
7. £8000 is invested in bonds at a building society which is offering a rate of 6.75% per annum. Calculate the value of the investment after 2 years.
8. An electrical contractor earning £36 000 per annum receives a pay rise of 2.5%. He pays 22% of his income as tax and 11% on National Insurance contributions. Calculate the increase he will actually receive per month.
9. Five mates enjoy a meal out. With drinks, the total bill comes to £176. They add a 12.5% tip and divide the amount equally between them. How much does each pay?
10. In December a shop raises the cost of a 40 inch LCD TV costing £920 by 5%. It does not sell and in its January sale it reduces the TV by 5%. What is the sale price of the TV?
11. A man buys a business and makes a 20% profit when he sells it three years later for £222 000. What did he pay originally for the business?
12. A drilling machine should be set to 250 rev/min. The nearest speed available on the machine is 268 rev/min. Calculate the percentage overspeed.
13. Two kilograms of a compound contain 30% of element A, 45% of element B and 25% of element C. Determine the masses of the three elements present.
14. A concrete mixture contains seven parts by volume of ballast, four parts by volume of sand and two parts by volume of cement. Determine the percentage of each of these three constituents correct to the nearest 1% and the mass of cement in a two tonne dry mix, correct to 1 significant figure.
15. In a sample of iron ore, 18% is iron. How much ore is needed to produce 3600 kg of iron?
16. A screw's dimension is $12.5 \pm 8\%$ mm. Calculate the maximum and minimum possible length of the screw.
17. The output power of an engine is 450 kW. If the efficiency of the engine is 75%, determine the power input.

Revision Test 2 : Decimals, calculators and percentages

This assignment covers the material contained in Chapters 3–5. *The marks available are shown in brackets at the end of each question.*

1. Convert 0.048 to a proper fraction. (2)
2. Convert 6.4375 to a mixed number. (3)
3. Express $\frac{9}{32}$ as a decimal fraction. (2)
4. Express 0.0784 correct to 2 decimal places. (2)
5. Express 0.0572953 correct to 4 significant figures. (2)
6. Evaluate
   (a) $46.7 + 2.085 + 6.4 + 0.07$
   (b) $68.51 - 136.34$ (4)
7. Determine 2.37×1.2 (3)
8. Evaluate $250.46 \div 1.1$ correct to 1 decimal place. (3)
9. Evaluate $5.\dot{2} \times 12$ (2)
10. Evaluate the following, correct to 4 significant figures: $3.3^2 - 2.7^3 + 1.8^4$ (3)
11. Evaluate $\sqrt{6.72} - \sqrt[3]{2.54}$ correct to 3 decimal places. (3)
12. Evaluate $\frac{1}{0.0071} - \frac{1}{0.065}$ correct to 4 significant figures. (2)
13. The potential difference, V volts, available at battery terminals is given by $V = E - Ir$. Evaluate V when $E = 7.23$, $I = 1.37$ and $r = 3.60$ (3)
14. Evaluate $\frac{4}{9} + \frac{1}{5} - \frac{3}{8}$ as a decimal, correct to 3 significant figures. (3)
15. Evaluate $\frac{16 \times 10^{-6} \times 5 \times 10^9}{2 \times 10^7}$ in engineering form. (2)
16. Evaluate resistance, R, given $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ when $R_1 = 3.6\,\text{k}\Omega$, $R_2 = 7.2\,\text{k}\Omega$ and $R_3 = 13.6\,\text{k}\Omega$. (3)
17. Evaluate $6\frac{2}{7} - 4\frac{5}{9}$ as a mixed number and as a decimal, correct to 3 decimal places. (3)
18. Evaluate, correct to 3 decimal places: $\sqrt{\left[\frac{2e^{1.7} \times 3.67^3}{4.61 \times \sqrt{3\pi}}\right]}$ (3)
19. If $a = 0.270$, $b = 15.85$, $c = 0.038$, $d = 28.7$ and $e = 0.680$, evaluate v correct to 3 significant figures, given that $v = \sqrt{\left(\frac{ab}{c} - \frac{d}{e}\right)}$ (4)
20. Evaluate the following, each correct to 2 decimal places.
    (a) $\left(\frac{36.2^2 \times 0.561}{27.8 \times 12.83}\right)^3$
    (b) $\sqrt{\left(\frac{14.69^2}{\sqrt{17.42} \times 37.98}\right)}$ (4)
21. If 1.6 km = 1 mile, determine the speed of 45 miles/hour in kilometres per hour. (2)
22. The area A of a circle is given by $A = \pi r^2$. Find the area of a circle of radius $r = 3.73$ cm, correct to 2 decimal places. (3)
23. Evaluate B, correct to 3 significant figures, when $W = 7.20$, $v = 10.0$ and $g = 9.81$, given that $B = \frac{Wv^2}{2g}$ (3)
24. Express 56.25% as a fraction in its simplest form. (3)
25. 12.5% of a length of wood is 70 cm. What is the full length? (3)
26. A metal rod, 1.20 m long, is heated and its length expands by 42 mm. Calculate the percentage increase in length. (2)
27. A man buys a house and makes a 20% profit when he sells it three years later for £312 000. What did he pay for it originally? (3)

Chapter 6

Ratio and proportion

6.1 Introduction

Ratio is a way of comparing amounts of something; it shows how much bigger one thing is than the other. Some practical examples include mixing paint, sand and cement, or screen wash. Gears, map scales, food recipes, scale drawings and metal alloy constituents all use ratios.

Two quantities are in **direct proportion** when they increase or decrease in the **same ratio**. There are several practical engineering laws which rely on direct proportion. Also, calculating currency exchange rates and converting imperial to metric units rely on direct proportion.

Sometimes, as one quantity increases at a particular rate, another quantity decreases at the same rate; this is called **inverse proportion**. For example, the time taken to do a job is inversely proportional to the number of people in a team: double the people, half the time.

When we have completed this chapter on ratio and proportion you will be able to understand, and confidently perform, calculations on the above topics.

For this chapter you will need to know about decimals and fractions and to be able to use a calculator.

6.2 Ratios

Ratios are generally shown as numbers separated by a colon (:) so the ratio of 2 and 7 is written as 2:7 and we read it as a ratio of 'two to seven.'

Some practical examples which are familiar include:

- Mixing 1 measure of screen wash to 6 measures of water; i.e., the ratio of screen wash to water is 1:6
- Mixing 1 shovel of cement to 4 shovels of sand; i.e., the ratio of cement to sand is 1:4
- Mixing 3 parts of red paint to 1 part white, i.e., the ratio of red to white paint is 3:1

Ratio is the number of parts to a mix. The paint mix is 4 parts total, with 3 parts red and 1 part white. 3 parts red paint to 1 part white paint means there is

$$\frac{3}{4} \text{ red paint to } \frac{1}{4} \text{ white paint}$$

Here are some worked examples to help us understand more about ratios.

Problem 1. In a class, the ratio of female to male students is 6:27. Reduce the ratio to its simplest form

(i) Both 6 and 27 can be divided by 3.

(ii) Thus, 6:27 is the same as **2:9**.

6:27 and 2:9 are called **equivalent ratios**.

It is normal to express ratios in their lowest, or simplest, form. In this example, the simplest form is **2:9** which means for every 2 females in the class there are 9 male students.

Problem 2. A gear wheel having 128 teeth is in mesh with a 48-tooth gear. What is the gear ratio?

$$\text{Gear ratio} = 128:48$$

A ratio can be simplified by finding common factors.

(i) 128 and 48 can both be divided by 2, i.e. 128:48 is the same as 64:24

(ii) 64 and 24 can both be divided by 8, i.e. 64:24 is the same as 8:3

(iii) There is no number that divides completely into both 8 and 3 so 8:3 is the simplest ratio, i.e. **the gear ratio is 8:3**

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Thus, 128 : 48 is equivalent to 64 : 24 which is equivalent to 8 : 3 and **8 : 3 is the simplest form**.

Problem 3. A wooden pole is 2.08 m long. Divide it in the ratio of 7 to 19

(i) Since the ratio is 7 : 19, the total number of parts is $7 + 19 = 26$ parts.

(ii) 26 parts corresponds to 2.08 m = 208 cm, hence, 1 part corresponds to $\frac{208}{26} = 8$.

(iii) Thus, 7 parts corresponds to $7 \times 8 =$ **56 cm** and 19 parts corresponds to $19 \times 8 =$ **152 cm**.

Hence, **2.08 m divides in the ratio of 7 : 19 as 56 cm to 152 cm**.
(Check: 56 + 152 must add up to 208, otherwise an error would have been made.)

Problem 4. In a competition, prize money of £828 is to be shared among the first three in the ratio 5 : 3 : 1

(i) Since the ratio is 5 : 3 : 1 the total number of parts is $5 + 3 + 1 = 9$ parts.

(ii) 9 parts corresponds to £828.

(iii) 1 part corresponds to $\frac{828}{9} =$ **£92**, 3 parts corresponds to $3 \times$ £92 = **£276** and 5 parts corresponds to $5 \times$ £92 = **£460**.

Hence, **£828 divides in the ratio of 5 : 3 : 1 as £460 to £276 to £92**. (Check: 460 + 276 + 92 must add up to 828, otherwise an error would have been made.)

Problem 5. A map scale is 1 : 30 000. On the map the distance between two schools is 6 cm. Determine the actual distance between the schools, giving the answer in kilometres

Actual distance between schools

$$= 6 \times 30\,000\,\text{cm} = 180\,000\,\text{cm}$$

$$= \frac{180{,}000}{100}\,\text{m} = 1800\,\text{m}$$

$$= \frac{1800}{1000}\,\text{m} = \mathbf{1.80\,km}$$

(1 mile ≈ 1.6 km, hence the schools are just over 1 mile apart.)

Now try the following Practice Exercise

Practice Exercise 24 Ratios (answers on page 342)

1. In a box of 333 paper clips, 9 are defective. Express the number of non-defective paper clips as a ratio of the number of defective paper clips, in its simplest form.
2. A gear wheel having 84 teeth is in mesh with a 24-tooth gear. Determine the gear ratio in its simplest form.
3. In a box of 2000 nails, 120 are defective. Express the number of non-defective nails as a ratio of the number of defective ones, in its simplest form.
4. A metal pipe 3.36 m long is to be cut into two in the ratio 6 to 15. Calculate the length of each piece.
5. The instructions for cooking a turkey say that it needs to be cooked 45 minutes for every kilogram. How long will it take to cook a 7 kg turkey?
6. In a will, £6440 is to be divided among three beneficiaries in the ratio 4 : 2 : 1. Calculate the amount each receives.
7. A local map has a scale of 1 : 22 500. The distance between two motorways is 2.7 km. How far are they apart on the map?
8. Prize money in a lottery totals £3801 and is shared among three winners in the ratio 4 : 2 : 1. How much does the first prize winner receive?

Here are some further worked examples on ratios.

Problem 6. Express 45 p as a ratio of £7.65 in its simplest form

(i) Changing both quantities to the same units, i.e. to pence, gives a ratio of 45 : 765

(ii) Dividing both quantities by 5 gives
45 : 765 ≡ 9 : 153

(iii) Dividing both quantities by 3 gives
9 : 153 ≡ 3 : 51

(iv) Dividing both quantities by 3 again gives
3 : 51 ≡ 1 : 17

Thus, **45 p as a ratio of £7.65 is 1 : 17**
45 : 765, 9 : 153, 3 : 51 and 1 : 17 are **equivalent ratios** and **1 : 17 is the simplest ratio**.

Problem 7. A glass contains 30 ml of whisky which is 40% alcohol. If 45 ml of water is added and the mixture stirred, what is now the alcohol content?

(i) The 30 ml of whisky contains 40% alcohol $= \frac{40}{100} \times 30 = 12$ ml.

(ii) After 45 ml of water is added we have $30 + 45 = 75$ ml of fluid, of which alcohol is 12 ml.

(iii) Fraction of alcohol present $= \frac{12}{75}$

(iv) Percentage of alcohol present $= \frac{12}{75} \times 100\%$

$= \mathbf{16\%}$.

Problem 8. 20 tonnes of a mixture of sand and gravel is 30% sand. How many tonnes of sand must be added to produce a mixture which is 40% gravel?

(i) Amount of sand in 20 tonnes = 30% of 20 t $= \frac{30}{100} \times 20 = 6$ t.

(ii) If the mixture has 6 t of sand then amount of gravel $= 20 - 6 = 14$ t.

(iii) We want this 14 t of gravel to be 40% of the new mixture. 1% would be $\frac{14}{40}$ t and 100% of the mixture would be $\frac{14}{40} \times 100\,\text{t} = 35$ t.

(iv) If there is 14 t of gravel then amount of sand $= 35 - 14 = 21$ t.

(v) We already have 6 t of sand, so **amount of sand to be added to produce a mixture with 40% gravel** $= 21 - 6 = \mathbf{15}$ **t**.

(Note 1 tonne = 1000 kg.)

Now try the following Practice Exercise

Practice Exercise 25 Further ratios (answers on page 342)

1. Express 130 g as a ratio of 1.95 kg.
2. In a laboratory, acid and water are mixed in the ratio 2 : 5. How much acid is needed to make 266 ml of the mixture?
3. A glass contains 30 ml of gin which is 40% alcohol. If 18 ml of water is added and the mixture stirred, determine the new percentage alcoholic content.
4. A wooden beam 4 m long weighs 84 kg. Determine the mass of a similar beam that is 60 cm long.
5. An alloy is made up of metals P and Q in the ratio 3.25 : 1 by mass. How much of P has to be added to 4.4 kg of Q to make the alloy?
6. 15 000 kg of a mixture of sand and gravel is 20% sand. Determine the amount of sand that must be added to produce a mixture with 30% gravel.

6.3 Direct proportion

Two quantities are in **direct proportion** when they increase or decrease in the **same ratio**. For example, if 12 cans of lager have a mass of 4 kg, then 24 cans of lager will have a mass of 8 kg; i.e., if the quantity of cans doubles then so does the mass. This is direct proportion.

In the previous section we had an example of mixing 1 shovel of cement to 4 shovels of sand; i.e., the ratio of cement to sand was 1 : 4. So, if we have a mix of 10 shovels of cement and 40 shovels of sand and we wanted to double the amount of the mix then we would need to double both the cement and sand, i.e. 20 shovels of cement and 80 shovels of sand. This is another example of direct proportion.

Here are three laws in engineering which involve direct proportion:

(a) **Hooke's law** states that, within the elastic limit of a material, the strain ε produced is directly proportional to the stress σ producing it, i.e. $\varepsilon \propto \sigma$ (note than '$\propto$' means 'is proportional to').

(b) **Charles's law** states that, for a given mass of gas at constant pressure, the volume V is directly proportional to its thermodynamic temperature T, i.e. $V \propto T$.

(c) **Ohm's law** states that the current I flowing through a fixed resistance is directly proportional to the applied voltage V, i.e. $I \propto V$.