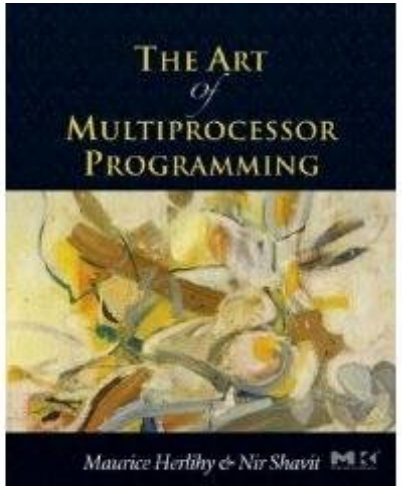
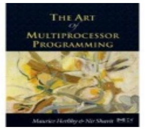
Mutual Exclusion



Hyungsoo Jung



Mutual Exclusion

- We will clarify our understanding of mutual exclusion
- We will also show you how to reason about various properties in an asynchronous concurrent setting

Mutual Exclusion

In his 1965 paper E. W. Dijkstra wrote:

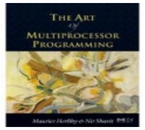
"Given in this paper is a solution to a problem which, to the knowledge of the author, has been an open question since at least 1962, irrespective of the solvability. [...] Although the setting of the problem might seem somewhat academic at first, the author trusts that anyone familiar with the logical problems that arise in computer coupling will appreciate the significance of the fact that this problem indeed can be solved."

Mutual Exclusion

- Formal problem definitions
- Solutions for 2 threads
- Solutions for *n* threads
- Fair solutions
- Inherent costs

Warning

- You will never use these protocols
  - Get over it
- You are advised to understand them
  - The same issues show up everywhere
  - Except hidden and more complex



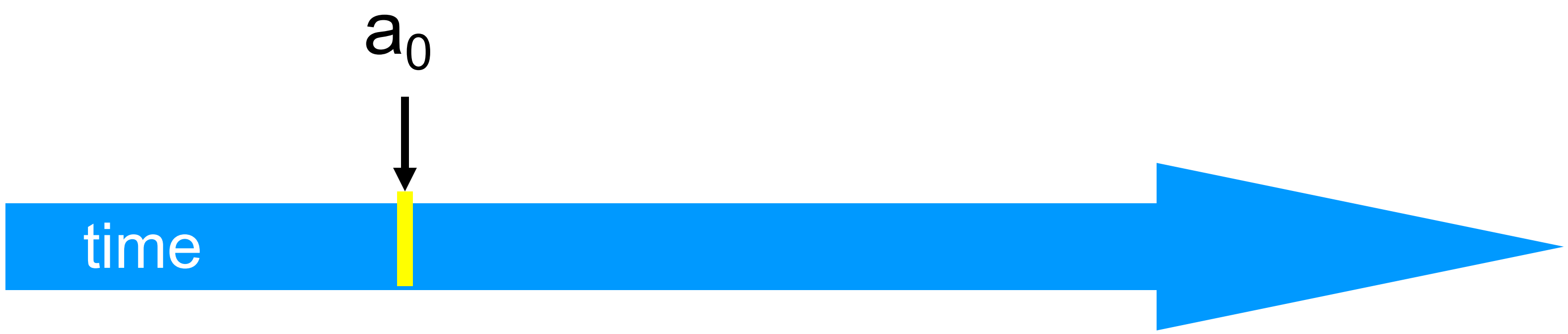
Time

- “Absolute, true and mathematical time, of itself and from its own nature, flows equably without relation to anything external.” (I. Newton, 1689)

- “Time is, like, Nature’s way of making sure that everything doesn’t happen all at once.” (Anonymous, circa 1968)

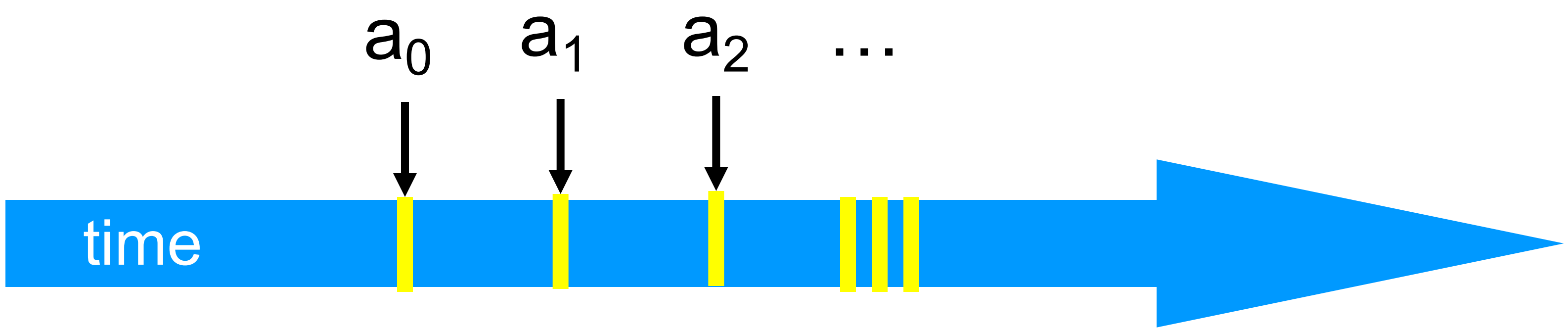
Events

- An *event* a_0 of thread A is
  - Instantaneous
  - No simultaneous events (break ties)



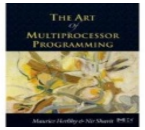
Threads

- A *thread* A is (formally) a sequence a_0, a_1, … of events
  - "Trace" model
  - Notation: $a_0 \rightarrow a_1$ indicates order

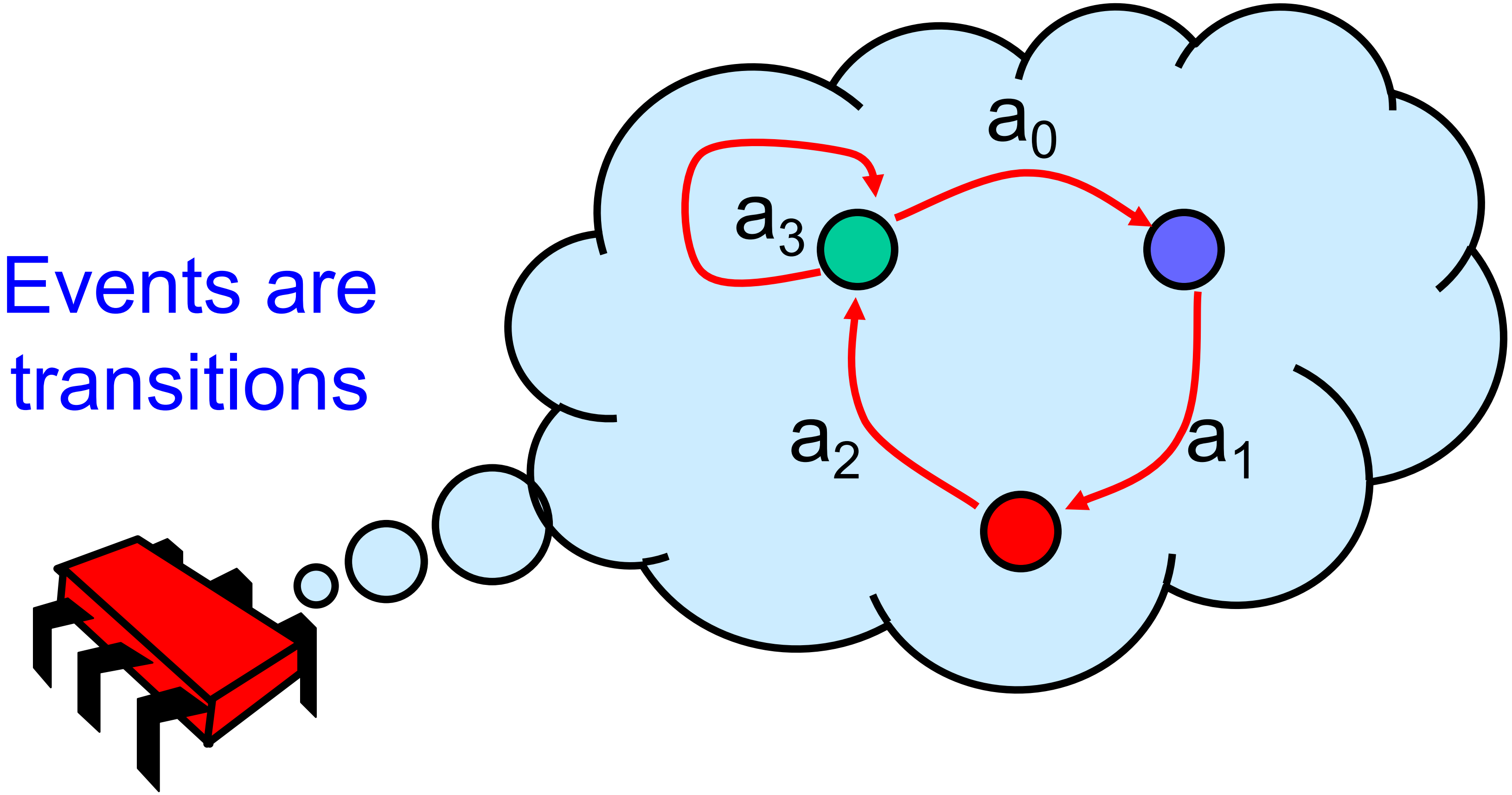


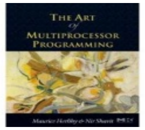
Example Thread Events

- Assign to shared variable
- Assign to local variable
- Invoke method
- Return from method
- Lots of other things …



Threads are State Machines





States

- Thread State
  - Program counter
  - Local variables
- System state
  - Object fields (shared variables)
  - Union of thread states

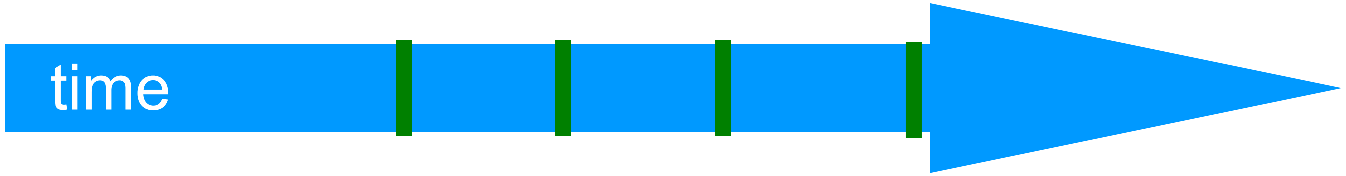
Concurrency

- Thread A

Concurrency

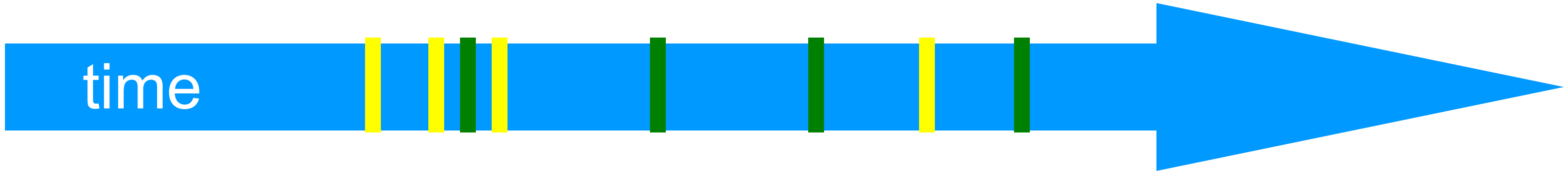
- Thread A

- Thread B



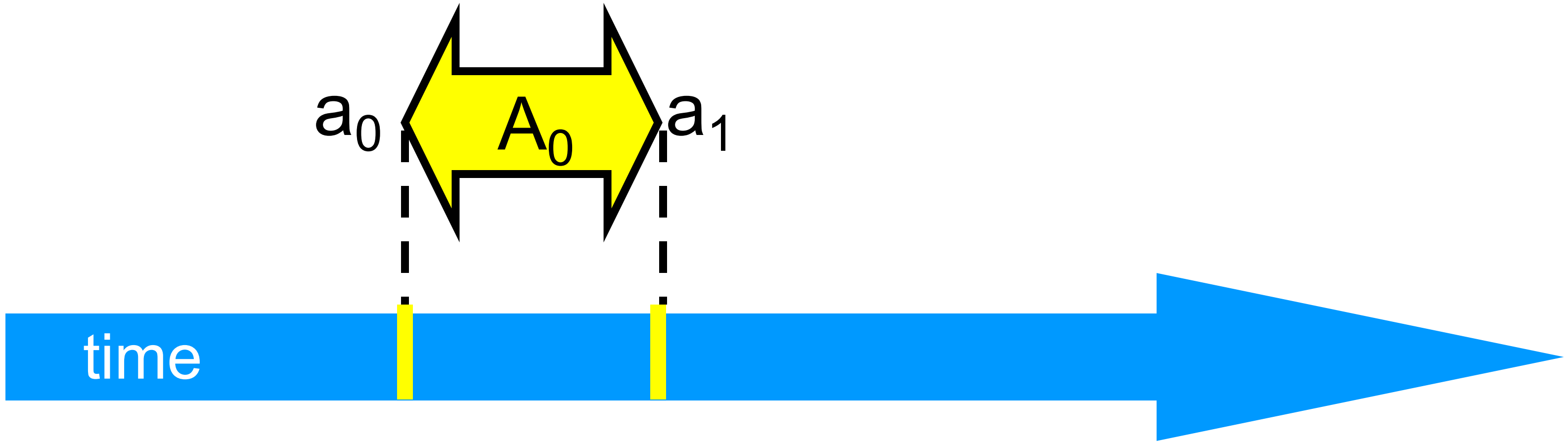
Interleavings

- Events of two or more threads
  - Interleaved
  - Not necessarily independent (why?)

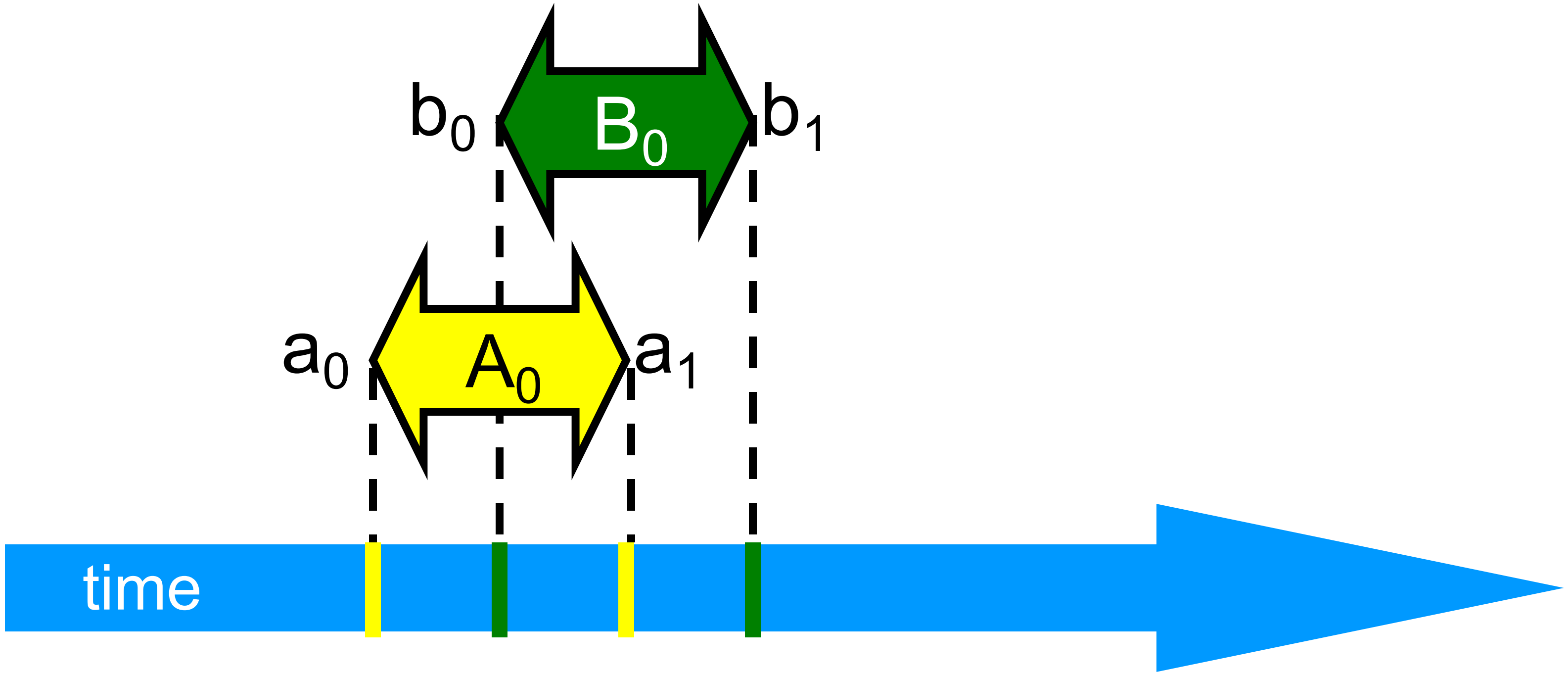


Intervals

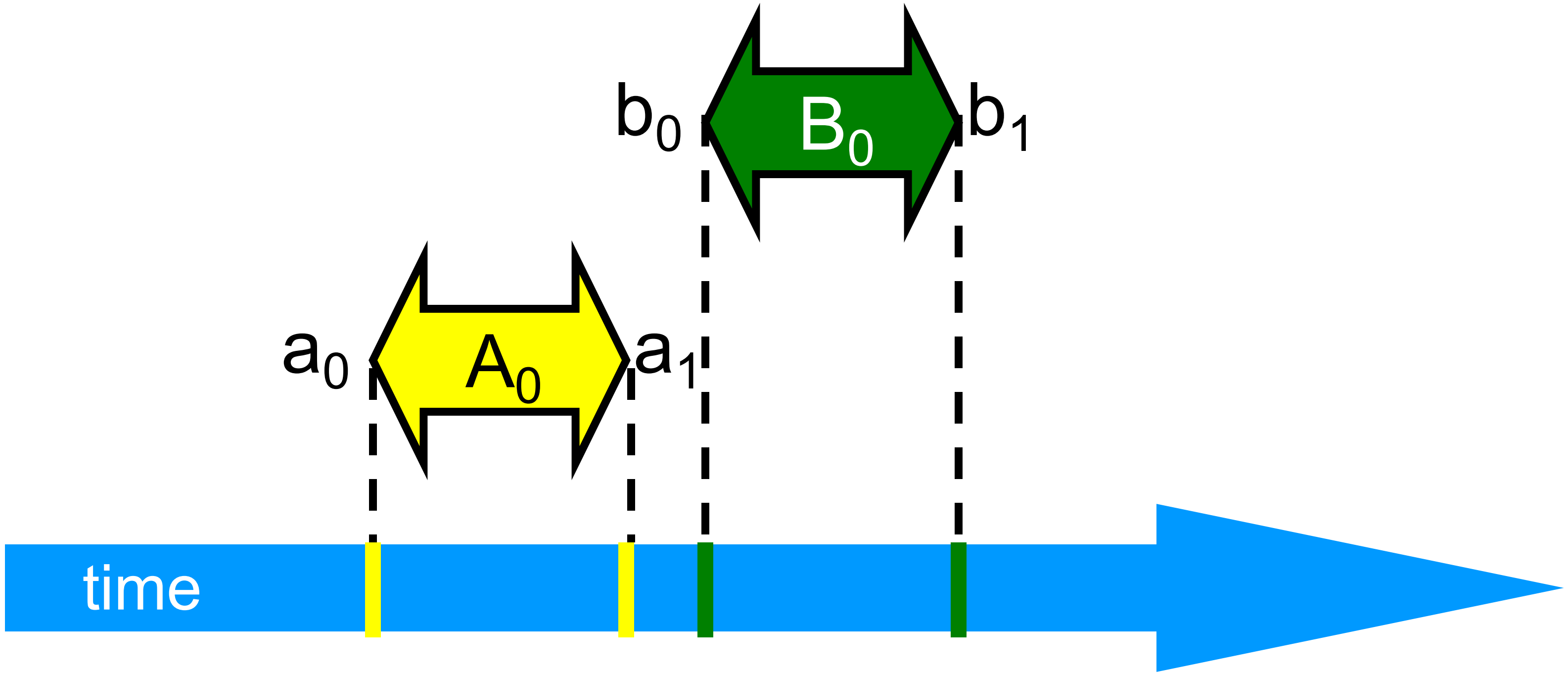
- An *interval* $A_0 = (a_0, a_1)$ is
  - Time between events a_0 and a_1

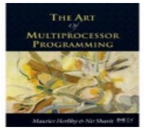


Intervals may Overlap



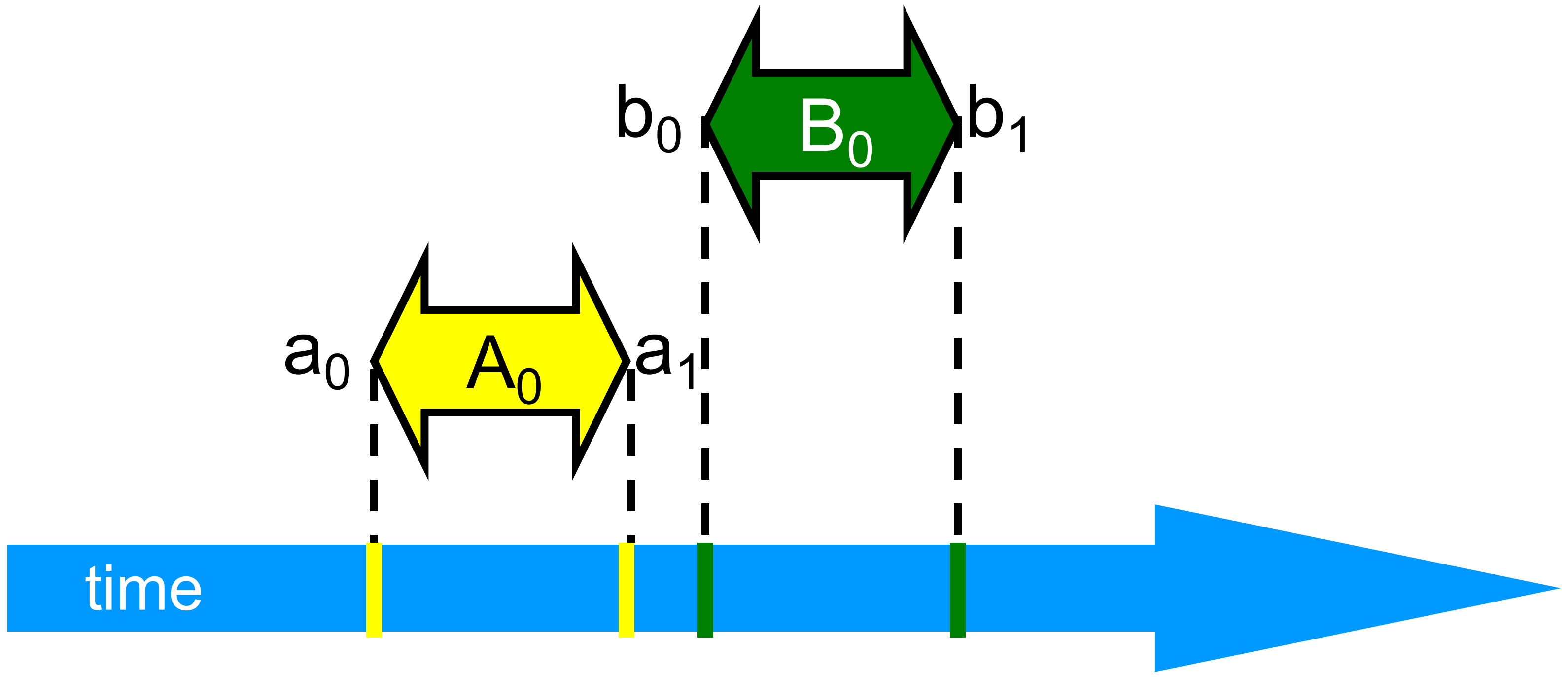
Intervals may be Disjoint



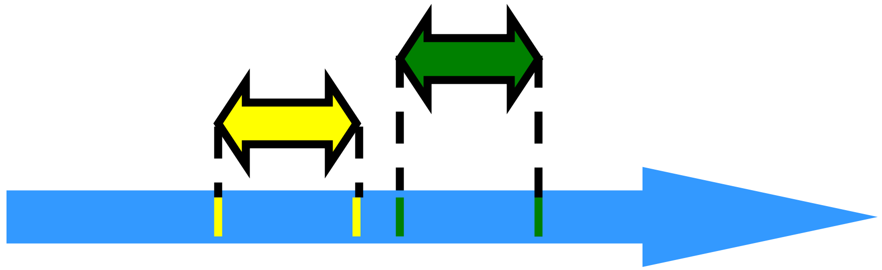


Precedence

Interval A_0 precedes interval B_0

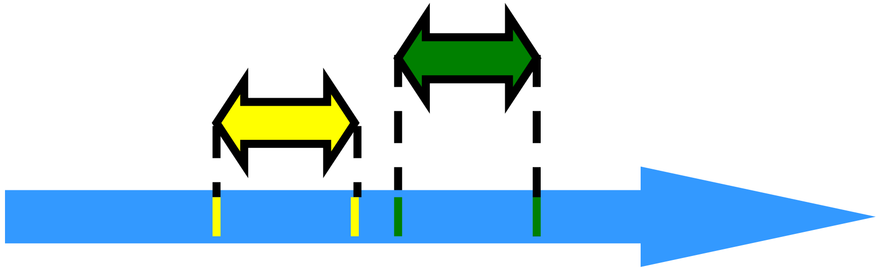


Precedence

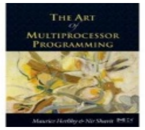


- Notation: $A_0 \rightarrow B_0$
- Formally,
  - End event of A_0 before start event of B_0
  - Also called “happens before” or “precedes”

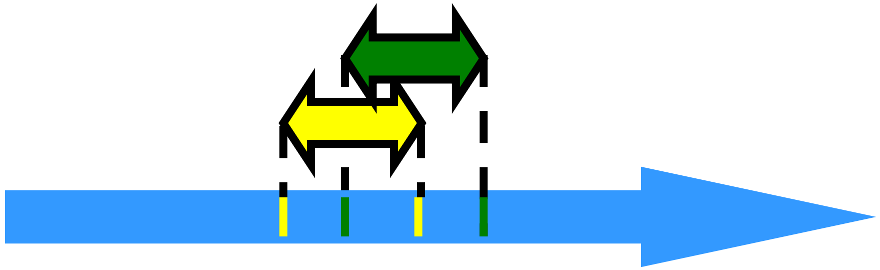
Precedence Ordering



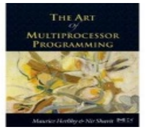
- Remark: $A_0 \rightarrow B_0$ is just like saying
  - 1066 AD → 1492 AD,
  - Middle Ages → Renaissance,
- Oh wait,
  - what about this week vs this month?



Precedence Ordering



- Never true that A → A
- If A →B then not true that B →A
- If A →B & B →C then A →C
- Funny thing: A →B & B →A might both be false!



Partial Orders

(review)

- Irreflexive:
  - Never true that A $\rightarrow$ A
- Antisymmetric:
  - If A $\rightarrow$ B then not true that B $\rightarrow$ A
- Transitive:
  - If A $\rightarrow$ B & B $\rightarrow$ C then A $\rightarrow$ C

Total Orders

(review)

- Also
  - Irreflexive
  - Antisymmetric
  - Transitive
- Except that for every distinct A, B,
  - Either $A \rightarrow B$ or $B \rightarrow A$

Repeated Events

```
while (mumble) {
   a0; a1;
}
```

k-th occurrence of event a_0

a_0^k

k-th occurrence of interval $A_0 = (a_0, a_1)$

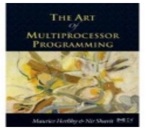
A_0^k

Implementing a Counter

```
public class Counter {
  private long value;

  public long getAndIncrement() {
    temp  = value;
    value = temp + 1;
    return temp;
  }
}
```

Make these steps *indivisible* using locks

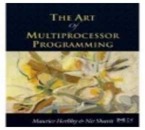


Locks (Mutual Exclusion)

```
public interface Lock {

 public void lock();

 public void unlock();
}
```



Locks (Mutual Exclusion)

```
public interface Lock {

 public void lock();

 public void unlock();
}
```

acquire lock

Locks (Mutual Exclusion)

```
public interface Lock {

 public void lock();

 public void unlock();
}
```

acquire lock

release lock

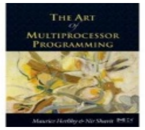
Using Locks

```
public class Counter {
  private long value;
  private Lock lock;
  public long getAndIncrement() {
   lock.lock();
   try {
    int temp = value;
    value = value + 1;
   } finally {
     lock.unlock();
   }
   return temp;
  }}
```

Using Locks

```
public class Counter {
  private long value;
  private Lock lock;
  public long getAndIncrement() {
   lock.lock();
   try {
    int temp = value;
    value = value + 1;
   } finally {
     lock.unlock();
   }
   return temp;
  }}
```

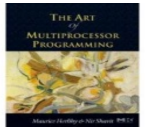
acquire Lock



Using Locks

```
public class Counter {
  private long value;
  private Lock lock;
  public long getAndIncrement() {
   lock.lock();
   try {
    int temp = value;
    value = value + 1;
   } finally {
     lock.unlock();
   }
   return temp;
  }}
```

Release lock (no matter what)



Using Locks

```
public class Counter {
  private long value;
  private Lock lock;
  public long getAndIncrement() {
   lock.lock();
   try {
    int temp = value;
    value = value + 1;
   } finally {
     lock.unlock();
   }
   return temp;
  }}
```

critical section

Mutual Exclusion

- Let $CS_i^k \Leftrightarrow$ be thread i's k-th critical section execution

Mutual Exclusion

- Let $CS_i^k \Leftrightarrow$ be thread i's k-th critical section execution
- And $CS_j^m \Leftrightarrow$ be thread j's m-th critical section execution

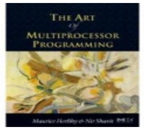
Mutual Exclusion

- Let CS_i^k be thread i's k-th critical section execution
- And CS_j^m be j's m-th execution
- Then either
  - or

Mutual Exclusion

- Let CS_i^k ⟺ be thread i's k-th critical section execution
- And CS_j^m ⟺ be j's m-th execution
- Then either
  - ⟺ ⟺ or ⟺ ⟺

$CS_i^k \rightarrow CS_j^m$



Mutual Exclusion

- Let CS_i^k ⇔ be thread i's k-th critical section execution
- And CS_j^m ⇔ be j's m-th execution
- Then either
  - ⇔ ⇔ or ⇔ ⇔

$CS_i^k \rightarrow CS_j^m$

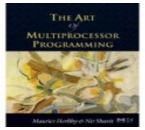
$CS_j^m \rightarrow CS_i^k$

Deadlock-Free

- If some thread calls **lock()**
  - And never returns
  - Then other threads must complete **lock()** and **unlock()** calls infinitely often
- System as a whole makes progress
  - Even if individuals starve

Starvation-Free

- If some thread calls lock()
  - It will eventually return
- Individual threads make progress

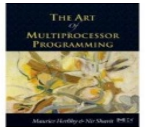


Two-Thread vs *n*-Thread Solutions

- 2-thread solutions first
  - Illustrate most basic ideas
  - Fits on one slide
- Then *n*-thread solutions

Two-Thread Conventions

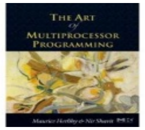
```
class … implements Lock {
  …
  // thread-local index, 0 or 1
  public void lock() {
    int i = ThreadID.get();
    int j = 1 - i;
  …
  }
}
```



Two-Thread Conventions

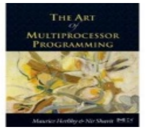
```
class … implements Lock {
  …
  // thread-local index, 0 or 1
  public void lock() {
    int i = ThreadID.get();
    int j = 1 - i;
    …
  }
}
```

Henceforth: i is current thread, j is other thread



LockOne

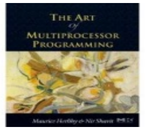
```
class LockOne implements Lock {
private boolean[] flag = new boolean[2];
public void lock() {
   flag[i] = true;
   while (flag[j]) {}
 }
```



LockOne

```
class LockOne implements Lock {
private boolean[] flag = new boolean[2];
public void lock() {
  flag[i] = true;
  while (flag[j]) {}
 }
```

Each thread has flag



LockOne

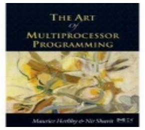
```
class LockOne implements Lock {
private boolean[] flag = new boolean[2];
public void lock() {
  flag[i] = true;
  while (flag[j]) {}
 }
```

Set my flag

LockOne

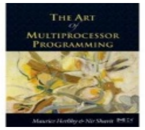
```
class LockOne implements Lock {
private boolean[] flag = new boolean[2];
public void lock() {
  flag[i] = true;
  while (flag[j]) {}
 }
```

Wait for other flag to become false



LockOne Satisfies Mutual Exclusion

- Assume CS_A^j overlaps CS_B^k
- Consider each thread's last (j-th and k-th) read and write in the lock() method before entering
- Derive a contradiction



From the Code

- $write_A(flag[A]{=}true) \rightarrow read_A(flag[B]{=}{=}false) \rightarrow CS_A$

- $write_B(flag[B]{=}true) \rightarrow read_B(flag[A]{=}{=}false) \rightarrow CS_B$

```
class LockOne implements Lock {
…
public void lock() {
  flag[i] = true;
  while (flag[j]) {}
 }
```

From the Assumption

- **read$_A$(flag[B]==false) $\rightarrow$ write$_B$(flag[B]=true)**
- **read$_B$(flag[A]==false) $\rightarrow$ write$_A$(flag[A]=true)**

Combining

- Assumptions:
  - $\text{read}_A(\text{flag[B]==false}) \rightarrow \text{write}_B(\text{flag[B]=true})$
  - $\text{read}_B(\text{flag[A]==false}) \rightarrow \text{write}_A(\text{flag[A]=true})$
- From the code
  - $\text{write}_A(\text{flag[A]=true}) \rightarrow \text{read}_A(\text{flag[B]==false})$
  - $\text{write}_B(\text{flag[B]=true}) \rightarrow \text{read}_B(\text{flag[A]==false})$

Combining

- Assumptions:
  - read$_A$(flag[B]==false) → write$_B$(flag[B]=true)
  - read$_B$(flag[A]==false) → write$_A$(flag[A]=true)
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  - write$_A$(flag[A]=true) → read$_A$(flag[B]==false)
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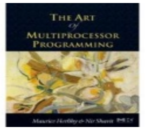

Combining

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  - $read_A(flag[B]==false) \rightarrow write_B(flag[B]=true)$
  - $read_B(flag[A]==false) \rightarrow write_A(flag[A]=true)$
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Combining

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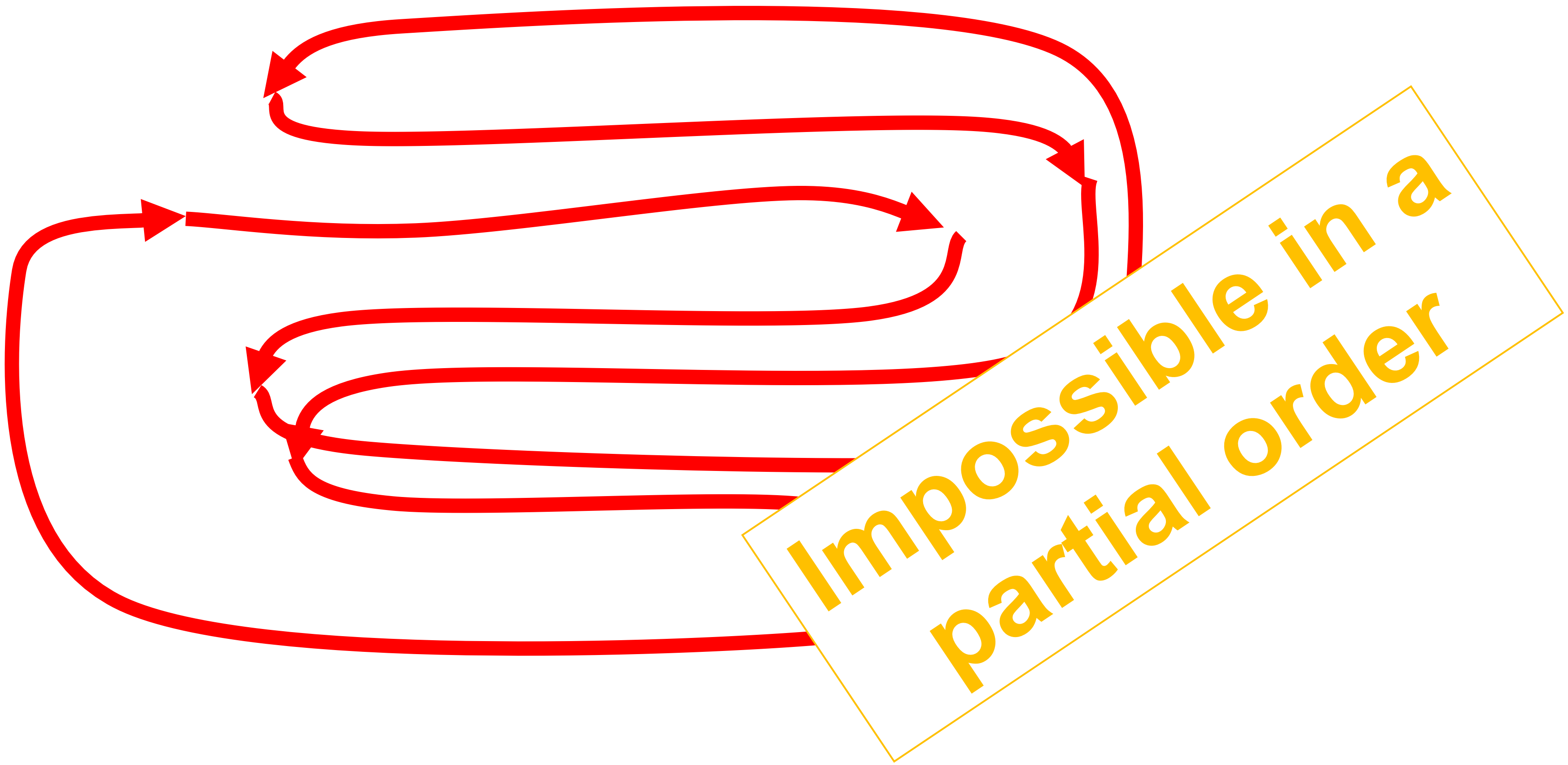
  - $\text{write}_B(\text{flag[B]=true}) \rightarrow \text{read}_B(\text{flag[A]==false})$

Cycle!



Deadlock Freedom

- LockOne Fails deadlock-freedom
  - Concurrent execution can deadlock

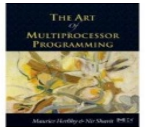
```
flag[i] = true;       flag[j] = true;
while (flag[j]){}     while (flag[i]){}
```

  - Sequential executions OK

LockTwo

```
public class LockTwo implements Lock {
 private int victim;
 public void lock() {
  victim = i;
  while (victim == i) {};
 }

 public void unlock() {}
}
```



LockTwo

```
public class LockTwo implements Lock {
 private int victim;
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 }

 public void unlock() {}
}
```
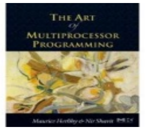

Let other go first



LockTwo

```
public class LockTwo implements Lock {
 private int victim;
 public void lock() {
 victim = i;
  while (victim == i) {};
 }

 public void unlock() {}
}
```

Wait for permission

LockTwo

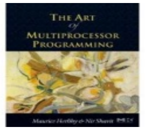
```
public class Lock2 implements Lock {
 private int victim;
 public void lock() {
  victim = i;
  while (victim == i) {};
 }

 public void unlock() {}
}
```

Nothing to do

LockTwo Claims

- Satisfies mutual exclusion
  - If thread `i` in CS
  - Then `victim == j`
  - Cannot be both 0 and 1
- Not deadlock free
  - Sequential execution deadlocks
  - Concurrent execution does not

```
public void LockTwo() {
  victim = i;
  while (victim == i) {};
 }
```



Peterson's Algorithm

```
public void lock() {
 flag[i] = true;
 victim  = i;
 while (flag[j] && victim == i) {};
}
public void unlock() {
 flag[i] = false;
}
```

Peterson's Algorithm

Announce I'm interested

```
public void lock() {
 flag[i] = true;
 victim  = i;
 while (flag[j] && victim == i) {};
}
public void unlock() {
 flag[i] = false;
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```

Peterson's Algorithm

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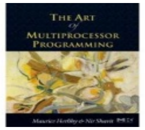
}
```

Announce I'm interested

Defer to other



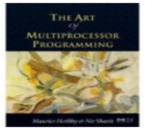
Peterson's Algorithm

```
public void lock() {
 flag[i] = true;
 victim  = i;
 while (flag[j] && victim == i) {};
}
public void unlock() {
 flag[i] = false;
}
```

Announce I'm interested

Defer to other

Wait while other interested & I'm the victim



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public void lock() {
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 while (flag[j] && victim == i) {};
 }
public void unlock() {
 flag[i] = false;
}
```

Announce I'm interested

Defer to other

Wait while other interested & I'm the victim

No longer interested

Mutual Exclusion

(1) $\text{write}_B(\text{Flag}[B]=\text{true}) \rightarrow \text{write}_B(\text{victim}=B)$

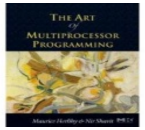
```
public void lock() {
 flag[i] = true;
 victim  = i;
 while (flag[j] && victim == i) {};
}
```

From the Code

Also from the Code

(2) $\text{write}_A(\text{victim}=A) \rightarrow \text{read}_A(\text{flag}[B])$
$\rightarrow \text{read}_A(\text{victim})$

```
public void lock() {
 flag[i] = true;
 victim  = i;
 while (flag[j] && victim == i) {};
}
```



Assumption

(3) $\text{write}_B(\text{victim=B}) \rightarrow \text{write}_A(\text{victim=A})$

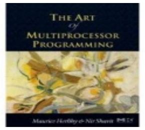
W.L.O.G. assume A is the last thread to write **victim**

Combining Observations

(1) $\text{write}_B(\text{flag[B]=true}) \rightarrow \text{write}_B(\text{victim=B})$

(3) $\text{write}_B(\text{victim=B}) \rightarrow \text{write}_A(\text{victim=A})$

(2) $\text{write}_A(\text{victim=A}) \rightarrow \text{read}_A(\text{flag[B]})$
$\rightarrow \text{read}_A(\text{victim})$



Combining Observations

(1) $\text{write}_B(\text{flag}[B]=\text{true}) \rightarrow$

(3) $\text{write}_B(\text{victim}=B) \rightarrow$

(2) $\text{write}_A(\text{victim}=A) \rightarrow \text{read}_A(\text{flag}[B])$
$\rightarrow \text{read}_A(\text{victim})$

Combining Observations

(1) $\text{write}_B(\text{flag}[B]=\text{true}) \rightarrow$

(3) $\text{write}_B(\text{victim}=B) \rightarrow$

(2) $\text{write}_A(\text{victim}=A) \rightarrow \text{read}_A(\text{flag}[B])$

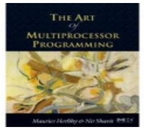
$\rightarrow \text{read}_A(\text{victim})$

A read flag[B] == true and victim == A, so it could not have entered the CS **(QED)**

Deadlock Free

```
public void lock() {
  …
  while (flag[j] && victim == i) {};
```

- Thread blocked
  - only at `while` loop
  - only if other’s flag is true
  - only if it is the **victim**
- Solo: other’s flag is false
- Both: one or the other not the victim

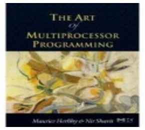


Starvation Free

- Thread `i` blocked only if `j` repeatedly re-enters so that

  `flag[j] == true` and `victim == i`
- When `j` re-enters
  - it sets `victim` to `j`.
  - So `i` gets in

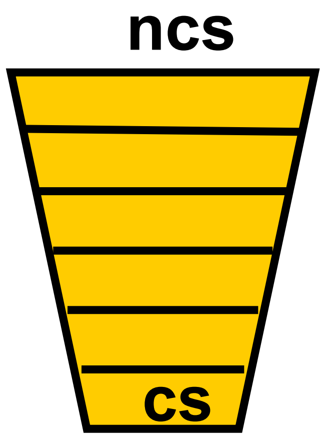
```
public void lock() {
  flag[i] = true;
  victim  = i;
  while (flag[j] && victim == i) {};
}

public void unlock() {
  flag[i] = false;
}
```

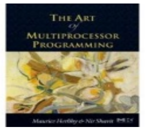


The Filter Algorithm for *n* Threads

There are n-1 “waiting rooms” called levels

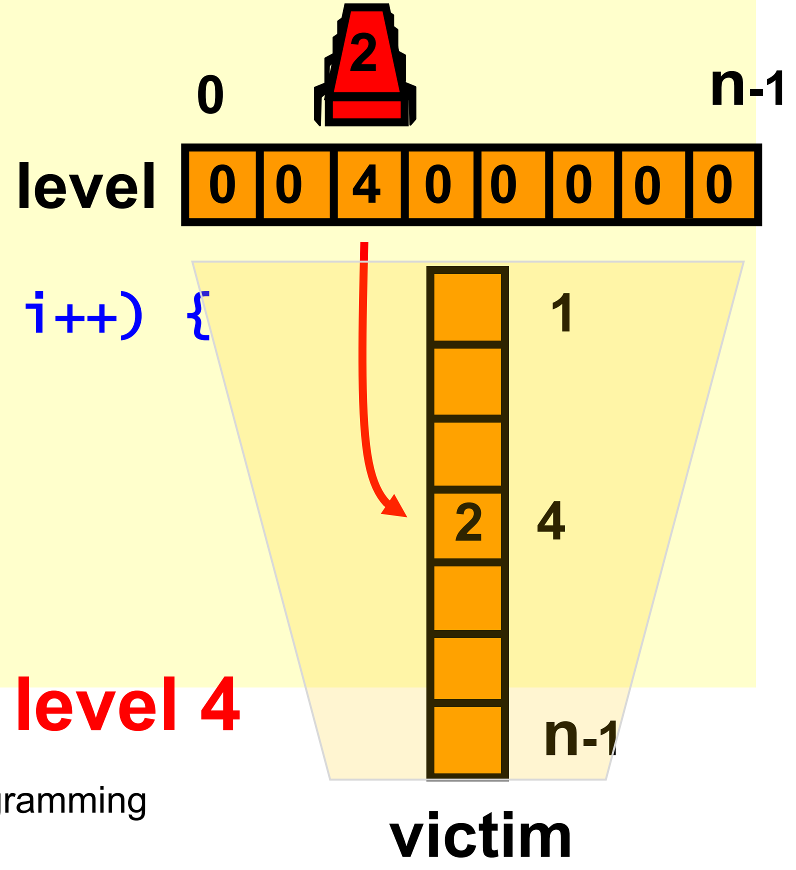
- At each level
  - At least one enters level
  - At least one blocked if many try
- Only one thread makes it through



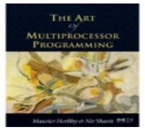


Filter

```
class Filter implements Lock {
   int[] level;  // level[i] for thread i
   int[] victim; // victim[L] for level L

  public Filter(int n) {
     level  = new int[n];
     victim = new int[n];
     for (int i = 1; i < n; i++) {
         level[i] = 0;
     }}
   …
}
```

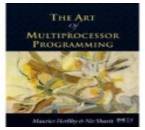


Thread 2 at level 4



Filter

```
class Filter implements Lock {
  …

  public void lock(){
    for (int L = 1; L < n; L++) {
      level[i]  = L;
      victim[L] = i;

      while ((∃ k != i level[k] >= L) &&
             victim[L] == i ) {};
    }}
  public void unlock() {
    level[i] = 0;
  }}
```



Filter

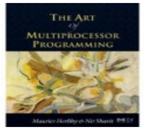
```
class Filter implements Lock {
  …

  public void lock() {
    for (int L = 1; L < n; L++) {
      level[i]  = L;
      victim[L] = i;

      while ((∃ k != i) level[k] >= L) &&
             victim[L] == i) {};
    }}
  public void release(int i) {
    level[i] = 0;
  }}
```

One level at a time

Filter

```
class Filter implements Lock {
  …

  public void lock() {
    for (int L = 1; L < n; L++) {
      level[i]  = L;
      victim[L] = i;

      while ((∃ k != i) level[k] >= L) &&
             victim[L] == i)
    }}
  public void release(int i) {
    level[i] = 0;
  }}
```

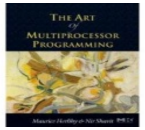
Announce intention to enter level L



Filter

```
class Filter implements Lock {
  int level[n];
  int victim[n];
  public void lock() {
    for (int L = 1; L < n; L++) {
      level[i]  = L;
      victim[L] = i;

      while ((∃ k != i) level[k] >= L) &&
             victim[L] == i) {};
    }}
  public void release(int i)
    level[i] = 0;
  }}
```

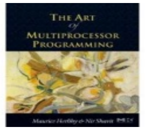
Give priority to anyone but me



Filter

Wait as long as someone else is at same or higher level, and I'm designated victim

```
public void lock() {
  for (int L = 1; L < n; L++) {
    level[i]  = L;
    victim[L] = i;

    while ((∃ k != i) level[k] >= L) &&
           victim[L] == i) {};
  }}
public void release(int i) {
  level[i] = 0;
}}
```



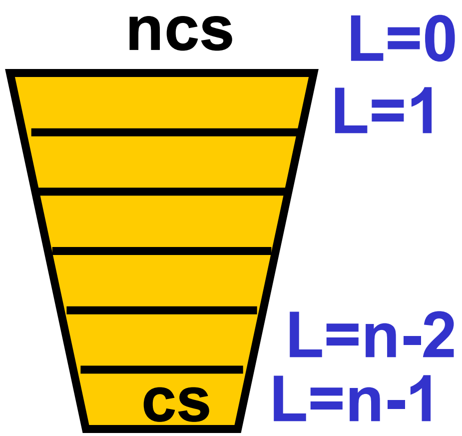
Filter

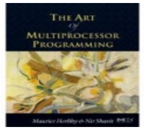
```
class Filter implements Lock {
  int level[n];
  int victim[n];
  public void lock() {
    for (int L = 1; L < n; L++) {
      level[i]  = L;
      victim[L] = i;
      while ((∃ k != i) level[k] >= L) &&
             victim[L] == i) {};
    }}
```

Thread *enters* level L when it completes the loop

Claim

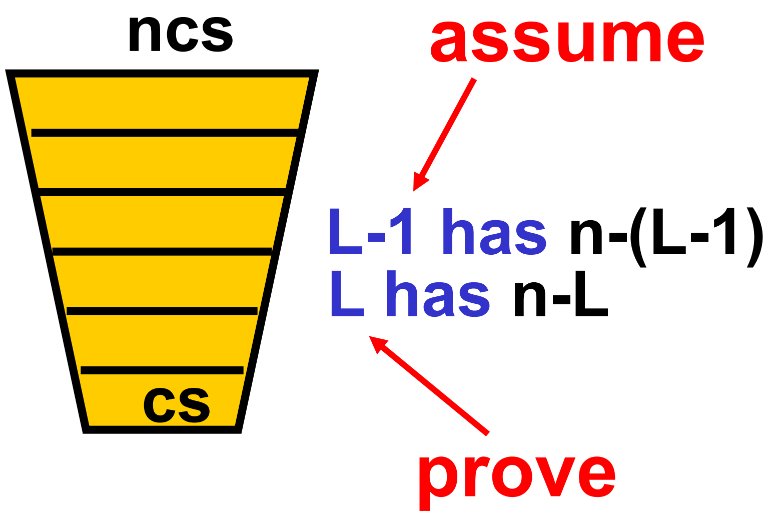
- Start at level L=0
- At most n-L threads enter level L
- Mutual exclusion at level L=n-1



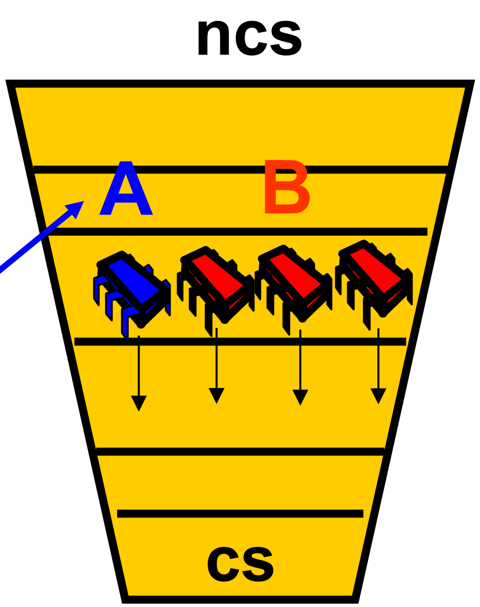


Induction Hypothesis

- No more than n-(L-1) at level L-1
- Induction step: by contradiction
- Assume all at level L-1 enter level L
- A last to write victim[L]
- B is any other thread at level L



Proof Structure



Assumed to enter L-1

n-L+1 = 4

n-L+1 = 4

By way of contradiction all enter L

Last to write victim[L]

Show that A must have seen B in level L and since victim[L] == A could not have entered

Just Like Peterson

(1) $\text{write}_B(\text{level}[B]=L) \rightarrow \text{write}_B(\text{victim}[L]=B)$

```
public void lock() {
 for (int L = 1; L < n; L++) {
   level[i] = L;
   victim[L]  = i;
   while ((∃ k != i) level[k] >= L)
          && victim[L] == i) {};
   }}
```

From the Code

From the Code

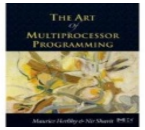
(2) $\text{write}_A(\text{victim}[L]=A) \rightarrow \text{read}_A(\text{level}[B])$
$\rightarrow \text{read}_A(\text{victim}[L])$

```
public void lock() {
 for (int L = 1; L < n; L++) {
   level[i] = L;
   victim[L]  = i;
   while ((∃ k != i) level[k] >= L)
           && victim[L] == i) {};
   }}
```

By Assumption

(3) $\text{write}_B(\text{victim}[L]=B) \rightarrow \text{write}_A(\text{victim}[L]=A)$

By assumption, A is the last thread to write **victim[L]**



Combining Observations

(1) $\text{write}_B(\text{level[B]=L}) \rightarrow \text{write}_B(\text{victim[L]=B})$

(3) $\text{write}_B(\text{victim[L]=B}) \rightarrow \text{write}_A(\text{victim[L]=A})$

(2) $\text{write}_A(\text{victim[L]=A}) \rightarrow \text{read}_A(\text{level[B]})$
$\rightarrow \text{read}_A(\text{victim[L]})$

Combining Observations

(1) $\text{write}_B(\text{level}[B]=L) \rightarrow$

(3) $\text{write}_B(\text{victim}[L]=B) \rightarrow \text{write}_A(\text{victim}[L]=A)$

(2) $\rightarrow \text{read}_A(\text{level}[B])$

$\rightarrow \text{read}_A(\text{victim}[L])$

Combining Observations

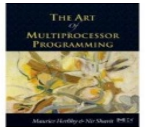
(1) $write_B(level[B]=L) \rightarrow$

(3) $write_B(victim[L]=B) \rightarrow write_A(victim[L]=A)$

(2) $\rightarrow read_A(level[B])$

$\rightarrow read_A(victim[L])$

A read level[B] ≥ L, and victim[L] = A, so it could not have entered level L!

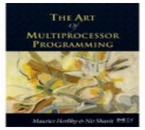


No Starvation

- Filter Lock satisfies properties:
  - Just like Peterson Alg at any level
  - So no one starves
- But what about fairness?
  - Threads can be overtaken by others

Bounded Waiting

- Want stronger fairness guarantees
- Thread not “overtaken” too much
- If A starts before B, then A enters before B?
- But what does “start” mean?
- Need to adjust definitions ….

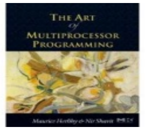


Bounded Waiting

- Divide `lock()` method into 2 parts:
  - Doorway interval:
    - Written $\mathbf{D_A}$
    - always finishes in finite steps
  - Waiting interval:
    - Written $\mathbf{W_A}$
    - may take unbounded steps

First-Come-First-Served

- For threads A and B:
  - If $D_A^k \rightarrow D_B^j$
    - A's k-th doorway precedes B's j-th doorway
  - Then $CS_A^k \rightarrow CS_B^j$
    - A's k-th critical section precedes B's j-th critical section
    - B cannot overtake A



Fairness Again

- Filter Lock satisfies properties:
  - No one starves
  - But very weak fairness
    - Can be overtaken **arbitrary** # of times
  - That's pretty lame…

Bakery Algorithm

- Provides First-Come-First-Served
- How?
  - Take a “number”
  - Wait until lower numbers have been served
- Lexicographic order
  - (a,i) > (b,j)
    - If a > b, or a = b and i > j

Bakery Algorithm

```
class Bakery implements Lock {
   boolean[] flag;
   Label[] label;
  public Bakery (int n) {
    flag  = new boolean[n];
    label = new Label[n];
    for (int i = 0; i < n; i++) {
       flag[i] = false; label[i] = 0;
    }
  }
 …
```

Bakery Algorithm

```
class Bakery implements Lock {
   boolean[] flag;
   Label[] label;
  public Bakery (int n) {
    flag  = new boolean[n];
    label = new Label[n];
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    }
  }
 …
```
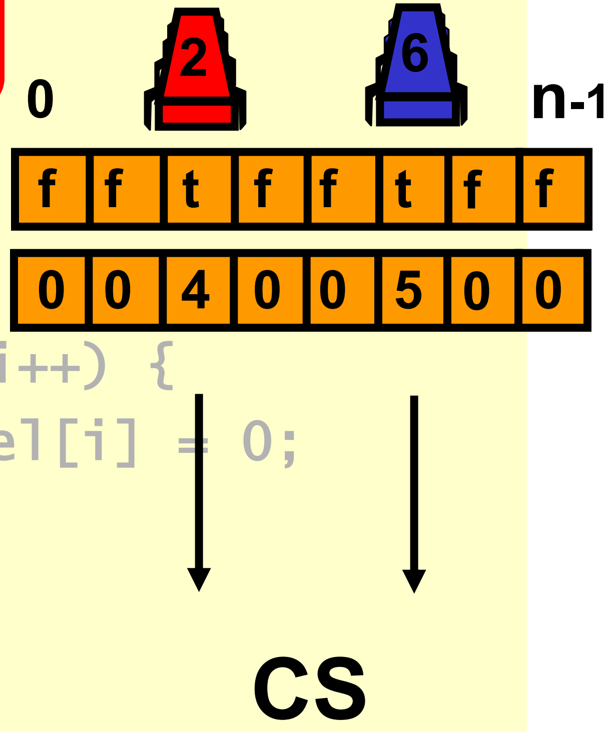
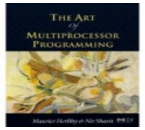



Bakery Algorithm

```
class Bakery implements Lock {
  …
 public void lock() {
  flag[i]  = true;
  label[i] = max(label[0], …,label[n-1])+1;
  while (∃k flag[k]
           && (label[i],i) > (label[k],k));
 }
```



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  label[i] = max(label[0], …,label[n-1])+1;
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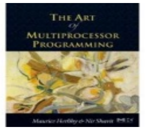
           && (label[i],i) > (label[k],k));
 }
```

Doorway



Bakery Algorithm

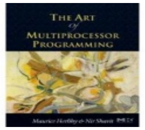
```
class Bakery implements Lock {
  …
 public void lock() {
  flag[i]  = true;
  label[i] = max(label[0], …,label[n-1])+1;
  while (∃k flag[k]
          && (label[i],i) > (label[k],k));
 }
```

I’m interested

Bakery Algorithm

```
class Bakery implements Lock {
  …
 public void lock() {
  flag[i]  = true;
  label[i] = max(label[0], …,label[n-1])+1;
  while (∃k flag[k]
           && (label[i],i) > (label[k],k));
 }
```

Take increasing label (read labels in some arbitrary order)



Bakery Algorithm

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  flag[i]  = true;
  label[i] = max(label[0], …,label[n-1])+1;
  while (∃k flag[k]
          && (label[i],i) > (label[k],k));
 }
```
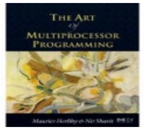

Someone is interested

Bakery Algorithm

```
class Bakery implements Lock {
  boolean flag[n];
  int label[n];

 public void lock() {
  flag[i]  = true;
  label[i] = max(label[0], …,label[n-1])+1;
  while (∃k flag[k]
          && (label[i],i) > (label[k],k));
 }
```

Someone is interested …

… whose (label,i) in lexicographic order is lower



Bakery Algorithm

```
class Bakery implements Lock {

    …

 public void unlock() {
   flag[i] = false;
 }
}
```

Bakery Algorithm

```
class Bakery implements Lock {

    …

 public void unlock() {
   flag[i] = false;
 }
}
```

No longer interested

labels are always increasing

No Deadlock

- There is always one thread with earliest label
- Ties are impossible (why?)

First-Come-First-Served

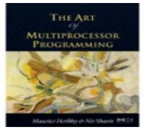
- If $D_A \rightarrow D_B$ then
  - A's label is smaller
- And:
  - $write_A(label[A]) \rightarrow$
  - $read_B(label[A]) \rightarrow$
  - $write_B(label[B]) \rightarrow read_B(flag[A])$
- So B sees
  - smaller label for A
  - locked out while flag[A] is true

```
class Bakery implements Lock {

public void lock() {
  flag[i]  = true;
  label[i] = max(label[0],
                 …,label[n-1])+1;
  while (∃k flag[k]
           && (label[i],i) >
  (label[k],k));
 }
```

Mutual Exclusion

- Suppose A and B in CS together
- Suppose A has earlier label
- When B entered, it must have seen
  - flag[A] is *false*, or
  - label[A] > label[B]

```
class Bakery implements Lock {

public void lock() {
  flag[i]  = true;
  label[i] = max(label[0],
                 …,label[n-1])+1;
  while (∃k flag[k]
           && (label[i],i) >
  (label[k],k));
 }
```

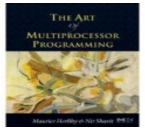


Mutual Exclusion

- Labels are strictly increasing so
- B must have seen flag[A] == false

Mutual Exclusion

- Labels are strictly increasing so
- B must have seen flag[A] == false
- $\text{Labeling}_B \rightarrow \text{read}_B(\text{flag[A]}) \rightarrow \text{write}_A(\text{flag[A]}) \rightarrow \text{Labeling}_A$

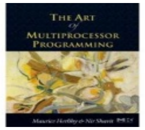


Mutual Exclusion

- Labels are strictly increasing so
- B must have seen flag[A] == false
- $\text{Labeling}_B \rightarrow \text{read}_B(\text{flag[A]}) \rightarrow \text{write}_A(\text{flag[A]}) \rightarrow \text{Labeling}_A$
- Which contradicts the assumption that A has an earlier label

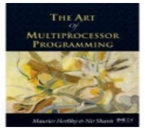
Deep Philosophical Question

- The Bakery Algorithm is
  - Succinct,
  - Elegant, and
  - Fair.
- Q: So why isn't it practical?
- A: Well, you have to read N distinct variables



Shared Memory

- Shared read/write memory locations called Registers (historical reasons)
- Come in different flavors
  - Multi-Reader-Single-Writer (`Flag[]`)
  - Multi-Reader-Multi-Writer (**Victim**`[]`)
  - Not that interesting: SRMW and SRSW



Summary of Lecture

- In the 1960's several incorrect solutions to starvation-free mutual exclusion using RW-registers were published…
- Today we know how to solve FIFO N thread mutual exclusion using 2N RW-Registers