Chapter 1

Basic arithmetic

1.1 Introduction

Whole numbers are called **integers**. +3, +5 and +72 are examples of positive integers; −13, −6 and −51 are examples of negative integers. Between positive and negative integers is the number 0 which is neither positive nor negative.

The four basic arithmetic operators are add (+), subtract (−), multiply (×) and divide (÷).

It is assumed that adding, subtracting, multiplying and dividing reasonably small numbers can be achieved without a calculator. However, if revision of this area is needed then some worked problems are included in the following sections.

When **unlike signs** occur together in a calculation, the overall sign is **negative**. For example,

$$3+(-4)=3+-4=3-4=\mathbf{-1}$$

and

$$(+5)\times(-2)=\mathbf{-10}$$

Like signs together give an overall **positive sign**. For example,

$$3-(-4)=3--4=3+4=\mathbf{7}$$

and

$$(-6)\times(-4)=\mathbf{+24}$$

1.2 Revision of addition and subtraction

You can probably already add two or more numbers together and subtract one number from another. However, if you need a revision then the following worked problems should be helpful.

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Problem 1. Determine 735 + 167

$$\begin{array}{rccc} & \mathbf{H} & \mathbf{T} & \mathbf{U} \\ & 7 & 3 & 5 \\ + & 1 & 6 & 7 \\ \hline & 9 & 0 & 2 \\ \hline & 1 & 1 & \end{array}$$

(i) 5 + 7 = 12. Place 2 in units (U) column. Carry 1 in the tens (T) column.

(ii) 3 + 6 + 1 (carried) = 10. Place the 0 in the tens column. Carry the 1 in the hundreds (H) column.

(iii) 7 + 1 + 1 (carried) = 9. Place the 9 in the hundreds column.

Hence, **735 + 167 = 902**

Problem 2. Determine 632 − 369

$$\begin{array}{rccc} & \mathbf{H} & \mathbf{T} & \mathbf{U} \\ & 6 & 3 & 2 \\ - & 3 & 6 & 9 \\ \hline & 2 & 6 & 3 \\ \hline \end{array}$$

(i) 2 − 9 is not possible; therefore ‘borrow’ 1 from the tens column (leaving 2 in the tens column). In the units column, this gives us 12 − 9 = 3.

(ii) Place 3 in the units column.

(iii) 2 − 6 is not possible; therefore ‘borrow’ 1 from the hundreds column (leaving 5 in the hundreds column). In the tens column, this gives us 12 − 6 = 6.

(iv) Place the 6 in the tens column.

(v) $5 - 3 = 2$.

(vi) Place the 2 in the hundreds column.

Hence, **632 − 369 = 263**

Problem 3. Add 27, −74, 81 and −19

This problem is written as $27 - 74 + 81 - 19$.

Adding the positive integers: 27, 81

Sum of positive integers is 108

Adding the negative integers: 74, 19

Sum of negative integers is 93

Taking the sum of the negative integers from the sum of the positive integers gives

$$\begin{array}{r} 108 \\ -93 \\ \hline 15 \\ \hline \end{array}$$

Thus, **27 − 74 + 81 − 19 = 15**

Problem 4. Subtract −74 from 377

This problem is written as $377 - -74$. Like signs together give an overall positive sign, hence

$$377 - -74 = 377 + 74 \qquad \begin{array}{r} 377 \\ +\ 74 \\ \hline 451 \\ \hline \end{array}$$

Thus, **377 − −74 = 451**

Problem 5. Subtract 243 from 126

The problem is $126 - 243$. When the second number is larger than the first, take the smaller number from the larger and make the result negative. Thus,

$$126 - 243 = -(243 - 126) \qquad \begin{array}{r} 243 \\ -126 \\ \hline 117 \\ \hline \end{array}$$

Thus, **126 − 243 = −117**

Problem 6. Subtract 318 from −269

The problem is $-269 - 318$. The sum of the negative integers is

$$\begin{array}{r} 269 \\ +318 \\ \hline 587 \\ \hline \end{array}$$

Thus, **−269 − 318 = −587**

Now try the following Practice Exercise

Practice Exercise 1 Further problems on addition and subtraction (answers on page 340)

In Problems 1 to 15, determine the values of the expressions given, without using a calculator.

1. 67 kg − 82 kg + 34 kg
2. 73 m − 57 m
3. 851 mm − 372 mm
4. 124 − 273 + 481 − 398
5. £927 − £114 + £182 − £183 − £247
6. 647 − 872
7. 2417 − 487 + 2424 − 1778 − 4712
8. −38419 − 2177 + 2440 − 799 + 2834
9. £2715 − £18250 + £11471 − £1509 + £113274
10. 47 + (−74) − (−23)
11. 813 − (−674)
12. 3151 − (−2763)
13. 4872 g − 4683 g
14. −23148 − 47724
15. \$53774 − \$38441
16. Holes are drilled 35.7 mm apart in a metal plate. If a row of 26 holes is drilled, determine the distance, in centimetres, between the centres of the first and last holes.
17. Calculate the diameter d and dimensions A and B for the template shown in Figure 1.1. All dimensions are in millimetres.

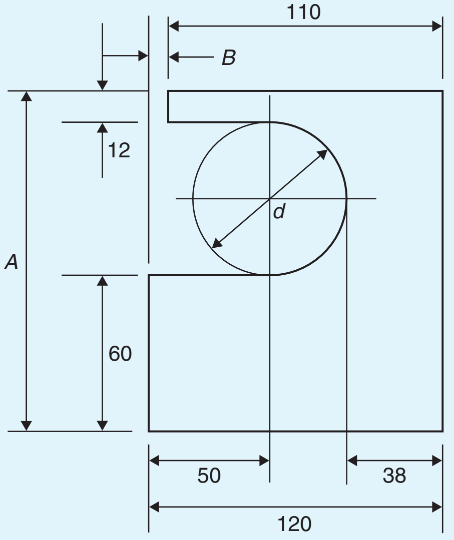

Figure 1.1

1.3 Revision of multiplication and division

You can probably already multiply two numbers together and divide one number by another. However, if you need a revision then the following worked problems should be helpful.

Problem 7. Determine 86×7

$$\begin{array}{rccc} & \mathbf{H} & \mathbf{T} & \mathbf{U} \\ & & 8 & 6 \\ \times & & & 7 \\ \hline & 6 & 0 & 2 \\ \hline & & 4 & \end{array}$$

(i) $7 \times 6 = 42$. Place the 2 in the units (U) column and 'carry' the 4 into the tens (T) column.

(ii) $7 \times 8 = 56$; $56 + 4$ (carried) $= 60$. Place the 0 in the tens column and the 6 in the hundreds (H) column.

Hence, $86 \times 7 = 602$

A good grasp of **multiplication tables** is needed when multiplying such numbers; a reminder of the multiplication table up to 12×12 is shown below. Confidence with handling numbers will be greatly improved if this table is memorized.

Problem 8. Determine 764×38

$$\begin{array}{r} 764 \\ \times \quad 38 \\ \hline 6112 \\ 22920 \\ \hline 29032 \\ \hline \end{array}$$

Multiplication table

×	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

(i) $8 \times 4 = 32$. Place the 2 in the units column and carry 3 into the tens column.

(ii) $8 \times 6 = 48$; $48 + 3$ (carried) $= 51$. Place the 1 in the tens column and carry the 5 into the hundreds column.

(iii) $8 \times 7 = 56$; $56 + 5$ (carried) $= 61$. Place 1 in the hundreds column and 6 in the thousands column.

(iv) Place 0 in the units column under the 2.

(v) $3 \times 4 = 12$. Place the 2 in the tens column and carry 1 into the hundreds column.

(vi) $3 \times 6 = 18$; $18 + 1$ (carried) $= 19$. Place the 9 in the hundreds column and carry the 1 into the thousands column.

(vii) $3 \times 7 = 21$; $21 + 1$ (carried) $= 22$. Place 2 in the thousands column and 2 in the ten thousands column.

(viii) $6112 + 22920 = 29032$

Hence, $\mathbf{764 \times 38 = 29032}$

Again, knowing multiplication tables is rather important when multiplying such numbers.

It is appreciated, of course, that such a multiplication can, and probably will, be performed using a **calculator**. However, there are times when a calculator may not be available and it is then useful to be able to calculate the 'long way'.

Problem 9. Multiply 178 by -46

When the numbers have different signs, the result will be negative. (With this in mind, the problem can now be solved by multiplying 178 by 46). Following the procedure of Problem 8 gives

$$\begin{array}{rr} & 178 \\ \times & 46 \\ \hline & 1068 \\ & 7120 \\ \hline & 8188 \\ \hline \end{array}$$

Thus, $178 \times 46 = 8188$ and $\mathbf{178 \times (-46) = -8188}$

Problem 10. Determine $1834 \div 7$

$$\begin{array}{r} 262 \\ 7\overline{)1834} \end{array}$$

(i) 7 into 18 goes 2, remainder 4. Place the 2 above the 8 of 1834 and carry the 4 remainder to the next digit on the right, making it 43.

(ii) 7 into 43 goes 6, remainder 1. Place the 6 above the 3 of 1834 and carry the 1 remainder to the next digit on the right, making it 14.

(iii) 7 into 14 goes 2, remainder 0. Place 2 above the 4 of 1834.

Hence, $1834 \div 7 = 1834/7 = \dfrac{\mathbf{1834}}{\mathbf{7}} = 262$.

The method shown is called **short division**.

Problem 11. Determine $5796 \div 12$

$$\begin{array}{r} 483 \\ 12\overline{)5796} \\ \underline{48} \\ 99 \\ \underline{96} \\ 36 \\ \underline{36} \\ \underline{00} \end{array}$$

(i) 12 into 5 won't go. 12 into 57 goes 4; place 4 above the 7 of 5796.

(ii) $4 \times 12 = 48$; place the 48 below the 57 of 5796.

(iii) $57 - 48 = 9$.

(iv) Bring down the 9 of 5796 to give 99.

(v) 12 into 99 goes 8; place 8 above the 9 of 5796.

(vi) $8 \times 12 = 96$; place 96 below the 99.

(vii) $99 - 96 = 3$.

(viii) Bring down the 6 of 5796 to give 36.

(ix) 12 into 36 goes 3 exactly.

(x) Place the 3 above the final 6.

(xi) Place the 36 below the 36.

(xii) $36 - 36 = 0$.

Hence, $5796 \div 12 = 5796/12 = \dfrac{\mathbf{5796}}{\mathbf{12}} = 483$.

The method shown is called **long division**.

Now try the following Practice Exercise

Practice Exercise 2 Further problems on multiplication and division (answers on page 340)

Determine the values of the expressions given in problems 1 to 9, without using a calculator.

1. (a) 78×6 (b) 124×7
2. (a) £261×7 (b) £462×9
3. (a) $783\,\text{kg} \times 11$ (b) $73\,\text{kg} \times 8$
4. (a) $27\,\text{mm} \times 13$ (b) $77\,\text{mm} \times 12$
5. (a) 448×23 (b) $143 \times (-31)$
6. (a) $288\,\text{m} \div 6$ (b) $979\,\text{m} \div 11$
7. (a) $\dfrac{1813}{7}$ (b) $\dfrac{896}{16}$
8. (a) $\dfrac{21424}{13}$ (b) $15900 \div 15$
9. (a) $\dfrac{88737}{11}$ (b) $46858 \div 14$
10. A screw has a mass of 15 grams. Calculate, in kilograms, the mass of 1200 such screws (1 kg = 1000 g).

1.4 Highest common factors and lowest common multiples

When two or more numbers are multiplied together, the individual numbers are called **factors**. Thus, a factor is a number which divides into another number exactly. The **highest common factor (HCF)** is the largest number which divides into two or more numbers exactly.
For example, consider the numbers 12 and 15.
The factors of 12 are 1, 2, 3, 4, 6 and 12 (i.e. all the numbers that divide into 12).
The factors of 15 are 1, 3, 5 and 15 (i.e. all the numbers that divide into 15).
1 and 3 are the only **common factors**; i.e., numbers which are factors of **both** 12 and 15.
Hence, **the HCF of 12 and 15 is 3** since 3 is the highest number which divides into **both** 12 and 15.
A **multiple** is a number which contains another number an exact number of times. The smallest number which is exactly divisible by each of two or more numbers is called the **lowest common multiple (LCM)**.
For example, the multiples of 12 are 12, 24, 36, 48, 60, 72,... and the multiples of 15 are 15, 30, 45, 60, 75,...
60 is a common multiple (i.e. a multiple of **both** 12 and 15) and there are no lower common multiples.
Hence, **the LCM of 12 and 15 is 60** since 60 is the lowest number that both 12 and 15 divide into.
Here are some further problems involving the determination of HCFs and LCMs.

Problem 12. Determine the HCF of the numbers 12, 30 and 42

Probably the simplest way of determining an HCF is to express each number in terms of its lowest factors. This is achieved by repeatedly dividing by the prime numbers 2, 3, 5, 7, 11, 13, … (where possible) in turn. Thus,

$$\begin{array}{rcccccccc}
12 = & 2 & \times & 2 & \times & 3 & & \\
30 = & 2 & & & \times & 3 & \times & 5 \\
42 = & 2 & & & \times & 3 & \times & 7
\end{array}$$

The factors which are common to each of the numbers are 2 in column 1 and 3 in column 3, shown by the broken lines. Hence, **the HCF is 2 × 3**; i.e., **6**. That is, 6 is the largest number which will divide into 12, 30 and 42.

Problem 13. Determine the HCF of the numbers 30, 105, 210 and 1155

Using the method shown in Problem 12:

$$\begin{array}{rcccccccccc}
30 = & 2 & \times & 3 & \times & 5 & & & & \\
105 = & & & 3 & \times & 5 & \times & 7 & & \\
210 = & 2 & \times & 3 & \times & 5 & \times & 7 & & \\
1155 = & & & 3 & \times & 5 & \times & 7 & \times & 11
\end{array}$$

The factors which are common to each of the numbers are 3 in column 2 and 5 in column 3. Hence, **the HCF is 3 × 5 = 15**.

Problem 14. Determine the LCM of the numbers 12, 42 and 90

The LCM is obtained by finding the lowest factors of each of the numbers, as shown in Problems 12 and 13 above, and then selecting the largest group of any of the factors present. Thus,

$$12 = \boxed{2 \times 2} \times 3$$
$$42 = 2 \times 3 \times \boxed{7}$$
$$90 = 2 \times \boxed{3 \times 3} \times \boxed{5}$$

The largest group of any of the factors present is shown by the broken lines and are 2×2 in 12, 3×3 in 90, 5 in 90 and 7 in 42.

Hence, **the LCM is $2 \times 2 \times 3 \times 3 \times 5 \times 7 = 1260$** and is the smallest number which 12, 42 and 90 will all divide into exactly.

Problem 15. Determine the LCM of the numbers 150, 210, 735 and 1365

Using the method shown in Problem 14 above:

$$150 = \boxed{2} \times \boxed{3} \times \boxed{5 \times 5}$$
$$210 = 2 \times 3 \times 5 \times 7$$
$$735 = 3 \times 5 \times \boxed{7 \times 7}$$
$$1365 = 3 \times 5 \times 7 \times \boxed{13}$$

Hence, **the LCM is $2 \times 3 \times 5 \times 5 \times 7 \times 7 \times 13 = 95550$**.

Now try the following Practice Exercise

Practice Exercise 3 Further problems on highest common factors and lowest common multiples (answers on page 340)

Find (a) the HCF and (b) the LCM of the following groups of numbers.

1. 8, 12	2. 60, 72
3. 50, 70	4. 270, 900
5. 6, 10, 14	6. 12, 30, 45
7. 10, 15, 70, 105	8. 90, 105, 300
9. 196, 210, 462, 910	10. 196, 350, 770

1.5 Order of precedence and brackets

1.5.1 Order of precedence

Sometimes addition, subtraction, multiplication, division, powers and brackets may all be involved in a calculation. For example,

$$5 - 3 \times 4 + 24 \div (3 + 5) - 3^2$$

This is an extreme example but will demonstrate the order that is necessary when evaluating.

When we read, we read from left to right. However, with mathematics there is a definite order of precedence which we need to adhere to. The order is as follows:

Brackets
Order (or p**O**wer)
Division
Multiplication
Addition
Subtraction

Notice that the first letters of each word spell **BODMAS**, a handy aide-mémoire. **O**rder means p**O**wer. For example, $4^2 = 4 \times 4 = 16$.

$5 - 3 \times 4 + 24 \div (3 + 5) - 3^2$ is evaluated as follows:

$$5 - 3 \times 4 + 24 \div (3 + 5) - 3^2$$
$= 5 - 3 \times 4 + 24 \div 8 - 3^2$ (**B**racket is removed and $3 + 5$ replaced with 8)
$= 5 - 3 \times 4 + 24 \div 8 - 9$ (**O**rder means p**O**wer; in this case, $3^2 = 3 \times 3 = 9$)
$= 5 - 3 \times 4 + 3 - 9$ (**D**ivision: $24 \div 8 = 3$)
$= 5 - 12 + 3 - 9$ (**M**ultiplication: $-3 \times 4 = -12$)
$= 8 - 12 - 9$ (**A**ddition: $5 + 3 = 8$)
$= \mathbf{-13}$ (**S**ubtraction: $8 - 12 - 9 = -13$)

In practice, **it does not matter if multiplication is performed before division or if subtraction is performed before addition**. What is important is that **the process of multiplication and division must be completed before addition and subtraction**.

1.5.2 Brackets and operators

The basic laws governing the **use of brackets and operators** are shown by the following examples.

(a) $2+3=3+2$; i.e., the order of numbers when adding does not matter.

(b) $2\times 3=3\times 2$; i.e., the order of numbers when multiplying does not matter.

(c) $2+(3+4)=(2+3)+4$; i.e., the use of brackets when adding does not affect the result.

(d) $2\times(3\times 4)=(2\times 3)\times 4$; i.e., the use of brackets when multiplying does not affect the result.

(e) $2\times(3+4)=2(3+4)=2\times 3+2\times 4$; i.e., a number placed outside of a bracket indicates that the whole contents of the bracket must be multiplied by that number.

(f) $(2+3)(4+5)=(5)(9)=5\times 9=45$; i.e., adjacent brackets indicate multiplication.

(g) $2[3+(4\times 5)]=2[3+20]=2\times 23=46$; i.e., when an expression contains inner and outer brackets, **the inner brackets are removed first**.

Here are some further problems in which BODMAS needs to be used.

Problem 16. Find the value of $6+4\div(5-3)$

The order of precedence of operations is remembered by the word BODMAS. Thus,

$$
\begin{aligned}
6+4\div(5-3) &= 6+4\div 2 && (\textbf{B}\text{rackets})\\
&= 6+2 && (\textbf{D}\text{ivision})\\
&= \mathbf{8} && (\textbf{A}\text{ddition})
\end{aligned}
$$

Problem 17. Determine the value of $13-2\times 3+14\div(2+5)$

$$
\begin{aligned}
13-2\times 3+14\div(2+5) &= 13-2\times 3+14\div 7 && \text{(B)}\\
&= 13-2\times 3+2 && \text{(D)}\\
&= 13-6+2 && \text{(M)}\\
&= 15-6 && \text{(A)}\\
&= \mathbf{9} && \text{(S)}
\end{aligned}
$$

Problem 18. Evaluate $16\div(2+6)+18[3+(4\times 6)-21]$

$$
\begin{aligned}
&16\div(2+6)+18[3+(4\times 6)-21]\\
&= 16\div(2+6)+18[3+24-21] && \text{(B: inner bracket is determined first)}\\
&= 16\div 8+18\times 6 && \text{(B)}\\
&= 2+18\times 6 && \text{(D)}\\
&= 2+108 && \text{(M)}\\
&= \mathbf{110} && \text{(A)}
\end{aligned}
$$

Note that a number outside of a bracket multiplies all that is inside the brackets. In this case,

$$18[3+24-21]=18[6], \text{ which means } 18\times 6=108$$

Problem 19. Find the value of $23-4(2\times 7)+\dfrac{(144\div 4)}{(14-8)}$

$$
\begin{aligned}
23-4(2\times 7)+\frac{(144\div 4)}{(14-8)} &= 23-4\times 14+\frac{36}{6} && \text{(B)}\\
&= 23-4\times 14+6 && \text{(D)}\\
&= 23-56+6 && \text{(M)}\\
&= 29-56 && \text{(A)}\\
&= \mathbf{-27} && \text{(S)}
\end{aligned}
$$

Problem 20. Evaluate $\dfrac{3+\sqrt{(5^2-3^2)}+2^3}{1+(4\times 6)\div(3\times 4)}+\dfrac{15\div 3+2\times 7-1}{3\times\sqrt{4}+8-3^2+1}$

$$
\begin{aligned}
&\frac{3+\sqrt{(5^2-3^2)}+2^3}{1+(4\times 6)\div(3\times 4)}+\frac{15\div 3+2\times 7-1}{3\times\sqrt{4}+8-3^2+1}\\
&= \frac{3+4+8}{1+24\div 12}+\frac{15\div 3+2\times 7-1}{3\times 2+8-9+1}\\
&= \frac{3+4+8}{1+2}+\frac{5+2\times 7-1}{3\times 2+8-9+1}\\
&= \frac{15}{3}+\frac{5+14-1}{6+8-9+1}\\
&= 5+\frac{18}{6}\\
&= 5+3=\mathbf{8}
\end{aligned}
$$

Now try the following Practice Exercise

Practice Exercise 4 Further problems on order of precedence and brackets (answers on page 340)

Evaluate the following expressions.

1. $14+3\times 15$
2. $17-12\div 4$
3. $86+24\div(14-2)$
4. $7(23-18)\div(12-5)$
5. $63-8(14\div 2)+26$
6. $\dfrac{40}{5}-42\div 6+(3\times 7)$
7. $\dfrac{(50-14)}{3}+7(16-7)-7$
8. $\dfrac{(7-3)(1-6)}{4(11-6)\div(3-8)}$
9. $\dfrac{(3+9\times 6)\div 3-2\div 2}{3\times 6+(4-9)-3^2+5}$
10. $\dfrac{\left(4\times 3^2+24\right)\div 5+9\times 3}{2\times 3^2-15\div 3}+\dfrac{2+27\div 3+12\div 2-3^2}{5+(13-2\times 5)-4}$
11. $\dfrac{1+\sqrt{25}+3\times 2-8\div 2}{3\times 4-\sqrt{\left(3^2+4^2\right)}+1}-\dfrac{(4\times 2+7\times 2)\div 11}{\sqrt{9}+12\div 2-2^3}$

Chapter 2

Fractions

2.1 Introduction

A mark of 9 out of 14 in an examination may be written as $\frac{9}{14}$ or 9/14. $\frac{9}{14}$ is an example of a fraction. The number above the line, i.e. 9, is called the **numerator**. The number below the line, i.e. 14, is called the **denominator**.

When the value of the numerator is less than the value of the denominator, the fraction is called a **proper fraction**. $\frac{9}{14}$ is an example of a proper fraction.

When the value of the numerator is greater than the value of the denominator, the fraction is called an **improper fraction**. $\frac{5}{2}$ is an example of an improper fraction.

A **mixed number** is a combination of a whole number and a fraction. $2\frac{1}{2}$ is an example of a mixed number. In fact, $\frac{5}{2} = 2\frac{1}{2}$.

There are a number of everyday examples in which fractions are readily referred to. For example, three people equally sharing a bar of chocolate would have $\frac{1}{3}$ each. A supermarket advertises $\frac{1}{5}$ off a six-pack of beer; if the beer normally costs £2 then it will now cost £1.60. $\frac{3}{4}$ of the employees of a company are women; if the company has 48 employees, then 36 are women.

Calculators are able to handle calculations with fractions. However, to understand a little more about fractions we will in this chapter show how to add, subtract, multiply and divide with fractions without the use of a calculator.

Problem 1. Change the following improper fractions into mixed numbers:

(a) $\frac{9}{2}$ (b) $\frac{13}{4}$ (c) $\frac{28}{5}$

(a) $\frac{9}{2}$ means 9 halves and $\frac{9}{2} = 9 \div 2$, and $9 \div 2 = 4$ and 1 half, i.e.

$$\frac{9}{2} = \mathbf{4\frac{1}{2}}$$

(b) $\frac{13}{4}$ means 13 quarters and $\frac{13}{4} = 13 \div 4$, and $13 \div 4 = 3$ and 1 quarter, i.e.

$$\frac{13}{4} = \mathbf{3\frac{1}{4}}$$

(c) $\frac{28}{5}$ means 28 fifths and $\frac{28}{5} = 28 \div 5$, and $28 \div 5 =$ 5 and 3 fifths, i.e.

$$\frac{28}{5} = \mathbf{5\frac{3}{5}}$$

Problem 2. Change the following mixed numbers into improper fractions:

(a) $5\frac{3}{4}$ (b) $1\frac{7}{9}$ (c) $2\frac{3}{7}$

(a) $5\frac{3}{4}$ means $5 + \frac{3}{4}$. 5 contains $5 \times 4 = 20$ quarters. Thus, $5\frac{3}{4}$ contains $20 + 3 = 23$ quarters, i.e.

$$5\frac{3}{4} = \mathbf{\frac{23}{4}}$$

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The quick way to change $5\frac{3}{4}$ into an improper fraction is $\frac{4 \times 5 + 3}{4} = \mathbf{\frac{23}{4}}$.

(b) $1\frac{7}{9} = \frac{9 \times 1 + 7}{9} = \mathbf{\frac{16}{9}}$.

(c) $2\frac{3}{7} = \frac{7 \times 2 + 3}{7} = \mathbf{\frac{17}{7}}$.

Problem 3. In a school there are 180 students of which 72 are girls. Express this as a fraction in its simplest form

The fraction of girls is $\frac{72}{180}$.
Dividing both the numerator and denominator by the lowest prime number, i.e. 2, gives

$$\frac{72}{180} = \frac{36}{90}$$

Dividing both the numerator and denominator again by 2 gives

$$\frac{72}{180} = \frac{36}{90} = \frac{18}{45}$$

2 will not divide into both 18 and 45, so dividing both the numerator and denominator by the next prime number, i.e. 3, gives

$$\frac{72}{180} = \frac{36}{90} = \frac{18}{45} = \frac{6}{15}$$

Dividing both the numerator and denominator again by 3 gives

$$\frac{72}{180} = \frac{36}{90} = \frac{18}{45} = \frac{6}{15} = \frac{2}{5}$$

So $\mathbf{\frac{72}{180} = \frac{2}{5}}$ **in its simplest form**.

Thus, $\mathbf{\frac{2}{5}}$ **of the students are girls**.

2.2 Adding and subtracting fractions

When the denominators of two (or more) fractions to be added are the same, the fractions can be added 'on sight'.

For example, $\frac{2}{9} + \frac{5}{9} = \frac{7}{9}$ and $\frac{3}{8} + \frac{1}{8} = \frac{4}{8}$.

In the latter example, dividing both the 4 and the 8 by 4 gives $\frac{4}{8} = \frac{1}{2}$, which is the simplified answer. This is called **cancelling**.

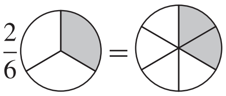
Addition and subtraction of fractions is demonstrated in the following worked examples.

Problem 4. Simplify $\frac{1}{3} + \frac{1}{2}$

(i) Make the denominators the same for each fraction. The lowest number that both denominators divide into is called the **lowest common multiple** or **LCM** (see Chapter 1, page 5). In this example, the LCM of 3 and 2 is 6.

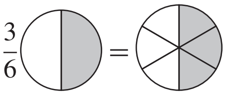
(ii) 3 divides into 6 twice. Multiplying both numerator and denominator of $\frac{1}{3}$ by 2 gives

$$\frac{1}{3} = \frac{2}{6} =$$



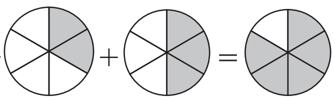
(iii) 2 divides into 6, 3 times. Multiplying both numerator and denominator of $\frac{1}{2}$ by 3 gives

$$\frac{1}{2} = \frac{3}{6} =$$



(iv) Hence,

$$\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \mathbf{\frac{5}{6}} =$$

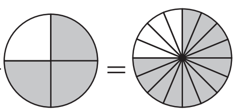


Problem 5. Simplify $\frac{3}{4} - \frac{7}{16}$

(i) Make the denominators the same for each fraction. The lowest common multiple (LCM) of 4 and 16 is 16.

(ii) 4 divides into 16, 4 times. Multiplying both numerator and denominator of $\frac{3}{4}$ by 4 gives

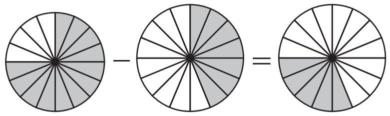
$$\frac{3}{4} = \frac{12}{16} =$$



(iii) $\frac{7}{16}$ already has a denominator of 16.

(iv) Hence,

$$\frac{3}{4}-\frac{7}{16}=\frac{12}{16}-\frac{7}{16}=\mathbf{\frac{5}{16}}$$



Problem 6. Simplify $4\frac{2}{3}-1\frac{1}{6}$

$4\frac{2}{3}-1\frac{1}{6}$ is the same as $\left(4\frac{2}{3}\right)-\left(1\frac{1}{6}\right)$ which is the same as $\left(4+\frac{2}{3}\right)-\left(1+\frac{1}{6}\right)$ which is the same as $4+\frac{2}{3}-1-\frac{1}{6}$ which is the same as $3+\frac{2}{3}-\frac{1}{6}$ which is the same as $3+\frac{4}{6}-\frac{1}{6}=3+\frac{3}{6}=3+\frac{1}{2}$

Thus, $4\frac{2}{3}-1\frac{1}{6}=\mathbf{3\frac{1}{2}}$

Problem 7. Evaluate $7\frac{1}{8}-5\frac{3}{7}$

$$7\frac{1}{8}-5\frac{3}{7}=\left(7+\frac{1}{8}\right)-\left(5+\frac{3}{7}\right)=7+\frac{1}{8}-5-\frac{3}{7}$$
$$=2+\frac{1}{8}-\frac{3}{7}=2+\frac{7\times1-8\times3}{56}$$
$$=2+\frac{7-24}{56}=2+\frac{-17}{56}=2-\frac{17}{56}$$
$$=\frac{112}{56}-\frac{17}{56}=\frac{112-17}{56}=\frac{95}{56}=\mathbf{1\frac{39}{56}}$$

Problem 8. Determine the value of
$$4\frac{5}{8}-3\frac{1}{4}+1\frac{2}{5}$$

$$4\frac{5}{8}-3\frac{1}{4}+1\frac{2}{5}=(4-3+1)+\left(\frac{5}{8}-\frac{1}{4}+\frac{2}{5}\right)$$
$$=2+\frac{5\times5-10\times1+8\times2}{40}$$
$$=2+\frac{25-10+16}{40}$$
$$=2+\frac{31}{40}=\mathbf{2\frac{31}{40}}$$

Now try the following Practice Exercise

Practice Exercise 5 Introduction to fractions (answers on page 340)

1. Change the improper fraction $\frac{15}{7}$ into a mixed number.
2. Change the improper fraction $\frac{37}{5}$ into a mixed number.
3. Change the mixed number $2\frac{4}{9}$ into an improper fraction.
4. Change the mixed number $8\frac{7}{8}$ into an improper fraction.
5. A box contains 165 paper clips. 60 clips are removed from the box. Express this as a fraction in its simplest form.
6. Order the following fractions from the smallest to the largest.
$$\frac{4}{9},\frac{5}{8},\frac{3}{7},\frac{1}{2},\frac{3}{5}$$
7. A training college has 375 students of which 120 are girls. Express this as a fraction in its simplest form.

Evaluate, in fraction form, the expressions given in Problems 8 to 20.

8. $\frac{1}{3}+\frac{2}{5}$
9. $\frac{5}{6}-\frac{4}{15}$
10. $\frac{1}{2}+\frac{2}{5}$
11. $\frac{7}{16}-\frac{1}{4}$
12. $\frac{2}{7}+\frac{3}{11}$
13. $\frac{2}{9}-\frac{1}{7}+\frac{2}{3}$
14. $3\frac{2}{5}-2\frac{1}{3}$
15. $\frac{7}{27}-\frac{2}{3}+\frac{5}{9}$
16. $5\frac{3}{13}+3\frac{3}{4}$
17. $4\frac{5}{8}-3\frac{2}{5}$
18. $10\frac{3}{7}-8\frac{2}{3}$
19. $3\frac{1}{4}-4\frac{4}{5}+1\frac{5}{6}$
20. $5\frac{3}{4}-1\frac{2}{5}-3\frac{1}{2}$