Technische Universität München

Fundamentals of Neural Networks

Fundamentals of Neural Networks

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e 12, 2018 toische Universität Mänchen

In den letzten Jahren Fortschritt in Künstlicher Intelligenz

Introduction



Figure 1: A self-driving car.

Credit: Marc van der Chijs / CC BY-ND 2.0

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2018-06-12

Introduction

Figur 1: A salf-driving car.

Introduction



Figure 2: A digital assistant. Credit: Kārlis Dambrāns / CC BY 2.0

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Figure 2: Crudit: Klaria D

Introduction

Jedes intelligente System benutzt neuronale Netze

Outline

Fundamentals of Neural Networks

Outline

Outline

Outline

The Perceptron

Fundamentals of Neural Networks

—The Perceptron

The Perceptron

MNIST Data Sample

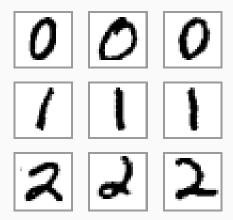


Figure 3: Examples from the MNIST database. Credit: Josef Steppan / CC BY-SA 4.0

Fundamentals of Neural Networks

The Perceptron

MNIST Data Sample

Fig. 1. Example to the MNIST data to the MNIST data

Example Task

 Predict whether an input image of a handwritten digit shows a zero or another digit

• Predict whether an input image of a handwritten digit shows a zero or another digit

Example Task

z.B. negative Gewichte in der Mitte

The Perceptron

 \bullet The image is represented as a flattened vector of pixel intensities $\textbf{x} \in \mathbb{R}^{784}$

Fundamentals of Neural Networks
The Perceptron
Example Task

Example Task

a zero or another digit

The image is represented as a flattened vector of pixel intensities x ∈ ℝ⁷⁸⁴

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4

Example Task

- Predict whether an input image of a handwritten digit shows a zero or another digit
- \bullet The image is represented as a flattened vector of pixel intensities $\textbf{x} \in \mathbb{R}^{784}$
- \bullet The output should be 1 if the image shows a zero, otherwise it should be -1

Fundamentals of Neural Networks

The Perceptron

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- Idea: Assign a weight to every input pixel

Fundamentals of Neural Networks

—The Perceptron

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2018-06-12 The Perceptron

Fundamentals of Neural Networks

 $\hat{y} = sign\left(\sum_{i=1}^{n} w_i x_i\right)$ $\equiv \hat{y} = \text{sign} \left(\mathbf{w}^{\top} \mathbf{x} \right)$

Model Specification

☐ Model Specification

The perceptron accepts n input values and computes an output value \hat{y} :

$$\hat{y} = \operatorname{sign}\left(\sum_{i=1}^{n} w_i x_i\right)$$

$$\equiv \hat{y} = \operatorname{sign}\left(\mathbf{w}^{\top} \mathbf{x}\right)$$
(1)

Visual Representation



Figure 4: A visual representation of the perceptron model.

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The Perceptron

Visual Representation

1950

Neuron im Gehirn: Nervenzelle Eingaben Ausgaben

• The perceptron is often used in a modified form

└─Generalizations

 $f:\ Aktivier ungsfunktion$

• A scalar bias value can be added to the output computation:

$$\hat{y} = \operatorname{sign}\left(\mathbf{w}^{\top}\mathbf{x} + b\right) \tag{2}$$



f: Aktivierungsfunktion

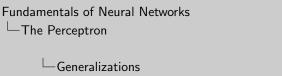
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 $\hat{y} = \text{sign} (\mathbf{w} \times + b)$ (2) The sign function can be replaced with a generic function f: $\hat{y} = f (\mathbf{w}^{T} \times + b)$ (3)

f: Aktivierungsfunktion

Generalizations

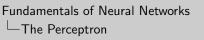
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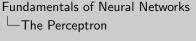
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- **Notation**: We denote the *weighted input* as

$$z = \mathbf{w}^{\mathsf{T}} \mathbf{x} + b \tag{4}$$

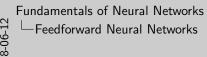


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Feedforward Neural Networks



Feedforward Neural Networks

Perzeptron linear: Viele Funktionen können nicht gelernt werden

Networks of neurons

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└─Networks of neurons

• Idea: A combination of multiple neurons could make much better predictions

Angelehnt an Gehirn Auch Multilayer Perzeptron

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└─Networks of neurons

Networks of neurons

- Idea: A combination of multiple neurons could make much better predictions
- A feedforward neural network is a layered architecture of neurons

Angelehnt an Gehirn Auch Multilayer Perzeptron

Visual Representation

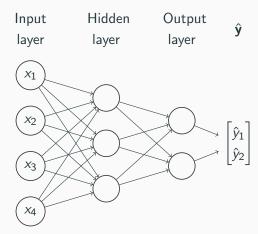
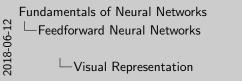


Figure 5: A three-layer feedforward neural network.





Tiefe
Verbindungen nur zwischen Layers
Ein Input wird durchpropagiert

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Fundamentals of Neural Networks -Feedforward Neural Networks

Output Layer

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-Feedforward Neural Networks

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- Regression: one single linear neuron

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Fundamentals of Neural Networks

Feedforward Neural Networks

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Output Layer

Binary classification: one single sigmoid neuron

The Logistic Sigmoid Function

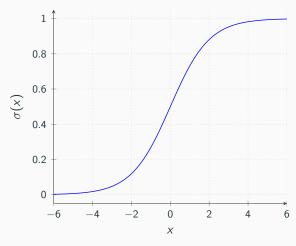
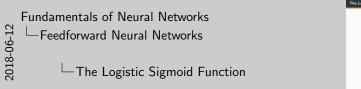
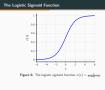


Figure 6: The logistic sigmoid function $\sigma(x) = \frac{1}{1 + \exp(-x)}$.

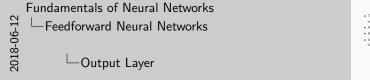




Output Layer

- The design of the output layer depends on the task that we wish to perform
- Regression: one single linear neuron
- Binary classification: one single sigmoid neuron
- *Multiclass classification*: *k* output units with the softmax function

$$\operatorname{softmax}(x) = \frac{\exp(x)}{\sum_{i=1}^{k} \exp(z_i)}$$
 (5)



• The design of the costpact layer depends on the tank that we wish to perform $\bullet \ \ \, \text{Regressions} \ \, \text{one single linear measure} \\ \bullet \ \, \text{Regressions} \ \, \text{consistence one single signed seasons} \\ \bullet \ \, \text{Multiclase classification:} \ \, \text{dought units with the softmax function} \\ \bullet \ \, \text{Multiclase classification:} \ \, \text{dought units with the softmax function} \\ \bullet \ \, \text{softmax}\{x\} = \frac{\exp(x)}{\sum_{i=1}^n \exp(x)} \end{aligned} \eqno(5)$

Output Layer

softmax: Generalisierung, normalisiert

Viele andere Probleme: Auffälliges Verhalten, Bildgenerierung

Hidden Layers

• The task does not give us any information about how to design the hidden layers

Daher kommt Deep Learning Verschiedene Repräsentationen (Kanten, Objekte,...) ⇒ Abstraktionen! Kontinuierlich, nicht linear!

Feedforward Neural Networks

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Feedforward Neural Networks

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The Rectified Linear Function

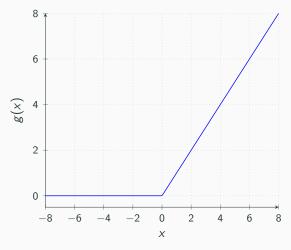
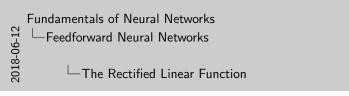


Figure 7: The rectified linear function $g(x) = \max\{0, x\}$.





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- Experimentation and trial & error

Fundamentals of Neural Networks

Feedforward Neural Networks

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-Feedforward Neural Networks Mathematical Formulation

Fundamentals of Neural Networks

f wird komponentenweise angewendet

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Mathematical Formulation

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- Since a neural network consists of multiple neurons in a layer, we need weight matrices $\mathbf{W}^{(l)}$ and bias vectors $\mathbf{b}^{(l)}$ to specify the parameters of a layer I

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☐ Mathematical Formulation

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• Since a neural network consists of multiple neurons in a layer, we need weight matrices $\mathbf{W}^{(l)}$ and bias vectors $\mathbf{b}^{(l)}$ to specify the parameters of a layer I

• $f^{(I)}$ is the activation function used in the I^{th} layer

f wird komponentenweise angewendet

Mathematical Formulation

Mathematical Formulation

• The output at layer *l* is then given by

$$\mathbf{a}^{(l)} = f^{(l)} \left(\mathbf{W}^{(l)\top} \mathbf{a}^{(l-1)} + \mathbf{b}^{(l)} \right)$$
 (6)

Neue Perspektive

$$a^0 = x$$
$$\hat{y} = a^L$$

Training Feedforward Neural Networks

Fundamentals of Neural Networks

Training Feedforward Neural Networks

Training Feedforward Neural Networks

Training Scenario

Training Scenario

ullet We have training examples $\mathbb{X}=(\mathbf{x}^{(1)},\ldots,\mathbf{x}^{(m)})$ with corresponding labels $\mathbb {Y}$

Zufällig initialisiert

 \bullet We want to learn a mapping from $\mathbb X$ to $\mathbb Y$

Fundamentals of Neural Networks

Training Feedforward Neural Networks

We have taking cample X = (x⁽¹⁾,...,x⁽ⁿ⁾) with corresponding links Y

We want to larn a singuing from X to Y

Training Scenario

Zufällig initialisiert

Training Scenario

- -Training Feedforward Neural Networks
 - Training Scenario

- We have training examples $\mathbb{X}=(\mathbf{x}^{(1)},\dots,\mathbf{x}^{(m)})$ with corresponding labels \mathbb{Y}
- \bullet We want to learn a mapping from $\mathbb X$ to $\mathbb Y$
- Idea: Iteratively adjust the parameters of the neural network

Zufällig initialisiert

• The cost function $J(\theta)$ is a measure of how good the network performs

Von den Parametern zu einem Skalar Größer als 0 Auch Loss oder Error

-Cost Functions

Auch Loss oder Error

- Training Feedforward Neural Networks
 - -Cost Functions

- The cost function $J(\theta)$ is a measure of how good the network performs
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Von den Parametern zu einem Skalar Größer als 0

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Cost Functions

-Cost Functions

- The cost function $J(\theta)$ is a measure of how good the network performs
- Learning can be framed as minimizing the cost function
- The total cost is a sum over the costs of the individual training examples:

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\mathbf{x}^{(i)}, \mathbf{y}^{(i)}, \theta)$$
 (7)

Von den Parametern zu einem Skalar Größer als 0 Auch Loss oder Error

Mean squared error

└─Mean squared error

Label: Skalar was wir vorhersagen wollen

Distanz

Erfüllt Bedingungen

• In regression, the per-example loss is commonly

$$\mathcal{L}(\mathbf{x}, y, \boldsymbol{\theta}) = \frac{1}{2}(\hat{y} - y)^2 \tag{8}$$

Cross-entropy

• In binary classification, we often use the cross-entropy loss

$$\mathcal{L}(\mathbf{x}, y, \boldsymbol{\theta}) = -y \ln \hat{y} - (1 - y) \ln(1 - \hat{y}) \tag{9}$$

MSE schlecht in Klassifikation

Cross-entropy

Label: 1 oder 0

— In aufzeichnen!

• In multiclass classification, the cross-entropy becomes

$$\mathcal{L}(\mathbf{x},\mathbf{y},oldsymbol{ heta}) = -\ln \hat{y}_i$$

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Cross-entropy

 $\mathcal{L}(\mathbf{x}, \mathbf{y}, \boldsymbol{\theta}) = -\ln \hat{y}_i$

Cross-entropy

Label: i-te Klasse

Maximum Likelihood Estimation

Stochastic Gradient Descent

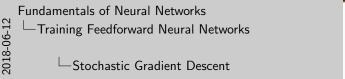
• (Stochastic) Gradient Descent is the most common algorithm to minimize cost functions in neural networks



 \Rightarrow Kleine Änderungen in die entgegengesetze Richtung des Gradienten Learning Rate Hyperparameter durch rumexperimentieren Erweiterungen

• To minimize $J(\theta)$, make small updates in the negative direction of the gradient:

$$\Delta \theta = -\eta \nabla J(\theta) \tag{11}$$





Stochastic Gradient Descent

⇒ Kleine Änderungen in die entgegengesetze Richtung des Gradienten Learning Rate Hyperparameter durch rumexperimentieren Erweiterungen • To minimize $J(\theta)$, make small updates in the negative direction of the gradient:

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• Stochastic Gradient Descent computes only an approximation of the gradient



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Stochastic Gradient Descent

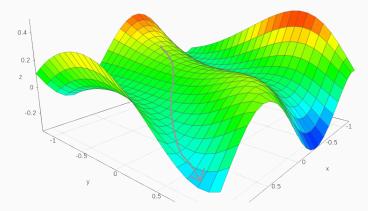
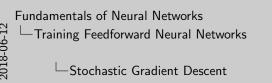
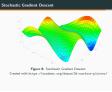


Figure 8: Stochastic Gradient Descent.

Created with https://academo.org/demos/3d-surface-plotter/





Back-propagation

• The back-propagation algorithm efficiently computes the gradient of the cost function

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- It can be derived by recursively applying the chain rule to the layers of the neural network, beginning with the output layer

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-Training Feedforward Neural Networks

-Back-propagation

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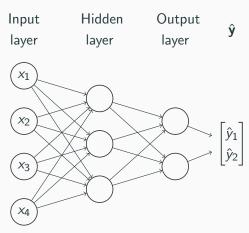


Figure 9: The Back-propagation algorithm.



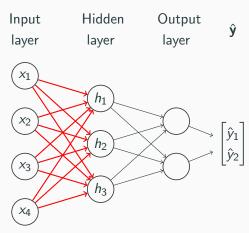
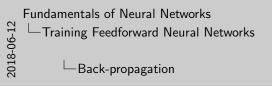


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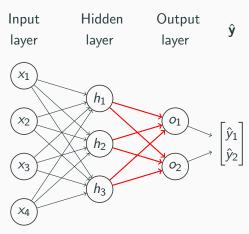
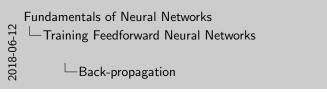


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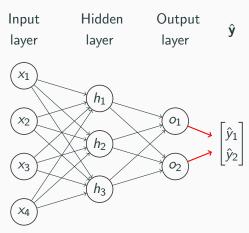


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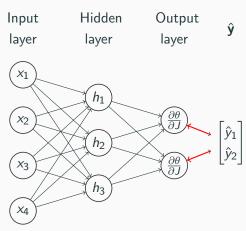
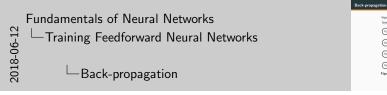


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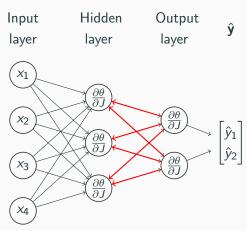
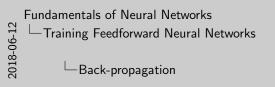


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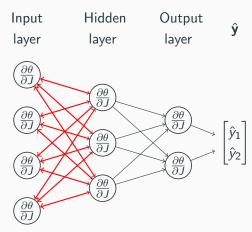
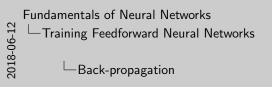


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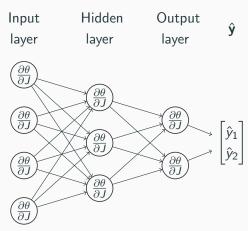
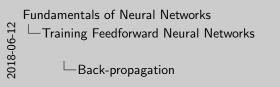


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The Complete Learning Algorithm

through the network

1. Propagate all training examples of a minibatch forward through the network

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Training Feedforward Neural Networks

The Complete Learning Algorithm

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The Complete Learning Algorithm

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- 2. Compute the cost for each training example
- 3. Compute all gradients using back-propagation

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- 4. Compute the average gradient

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- 3. Compute all gradients using back-propagation
- 4. Compute the average gradient
- 5. Update the parameters in the negative direction of the gradient

Fundamentals of Neural Networks

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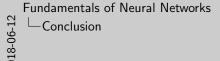
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- 3. Compute all gradients using back-propagation
- 4. Compute the average gradient
- 5. Update the parameters in the negative direction of the gradient
- 6. Repeat until the cost is low enough

The Complete Learning Algorithm

The Complete Learning Algorithm

- Propagate all training examples of a minibatch forward through the network
- 2. Compute the cost for each training example
- Compute the cost for each training example
 Compute all gradients using back-nonnegate
- Compute all gradients using back-propag
 Compute the average gradient
- Update the parameters in the negative direction of the
- 6. Repeat until the cost is low enough

Conclusion



Conclusion

Komplexe Netzwerke einfacher Einheiten

Abstraktionen Lernen: Kleine Updates der Parameter so dass das Netzwerk besser wird

Überall in Deep Learning

Viele weitere Anwendungen in der Zukunft

