Fundamentals of Neural Networks

Seminar Data Mining

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***Abstract*— Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrumCurabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.**

***Index Terms*— test**

I. Introduction

Artificial intelligence systems have been becoming more and more powerful over the last 10 years. We have seen outstanding advances in a variety of fields including computer vision, natural language processing and fraud detection, which power many end-user technologies such as digital assistants or self-driving cars. Much of the recent progress can be attributed to *deep learning*, a powerful set of techniques that enable computers to understand the world by decomposing complex concepts into a hierarchy of simpler abstractions.

While numerous other approaches to machine learning exist, deep learning has shown to outperform other methods in a wide variety of applications. To name a few examples, deep learning models dominate the task of object recognition in images [1], even surpassing human-level performance [2], have been successfully applied to sentiment analysis [3], and have significantly improved speech recognition systems [4]. Deep learning has also been used in problems such as style transfer between images [5], image description generation [6], and learning to play video games [7].

By learning everything required to solve a task purely from raw data, these techniques have alleviated the need for problem-specific expert knowledge. Thus, very similar models building on the same core ideas can be applied to a vast array of different tasks with outstanding success.

One such core idea that is fundamental to deep learning is the *neural network*, a computing model loosely inspired by neuroscience. While neural networks are not new, it was not until recently that enough data and computational resources became available to train them effectively and fully appreciate their power [8, Ch. 1, pp. 18-21].

Since neural networks have become so prevalent in modern machine learning applications, many libraries exist that abstract their concepts and provide simple programming interfaces. However, it does not suffice to be familiar with such libraries to use neural networks effectively; in order to understand which architectures perform well, and why, one must also know their mathematical foundations.

In this paper we thus aim to give a thorough overview of neural networks and the fundamental techniques and algorithms associated with them. We first briefly examine the motivation and history behind neural networks in Section II by introducing the *perceptron* model. Section III then shows how this model has been adjusted and extended to obtain the neural network, focusing in particular on *feedforward neural networks*. In Section IV, we then proceed to explain how these networks can be trained, introducing ideas such as *stochastic gradient descent* and *back-propagation*. Subsequently, Section V discusses the effectiveness of neural networks from a theoretical point of view. In Section VI we examine several extensions to the basic feedforward neural network that are often used in practice, before we conclude our paper in Section VII.

II. The Perceptron

The perceptron [9] is a simple mathematical model of the biological neuron. It accepts n input values $x_1, \dots, x_n$ and computes a corresponding output by computing the weighted sum

$$\sum_{i=1}^{n} w_i x_i, \tag{1}$$

where the weights w_i are the parameters of the model. By representing the input values as a vector $\boldsymbol{x}$ and the weights as a vector $\boldsymbol{w}$, we can rewrite (1) as $\boldsymbol{w}^\top \boldsymbol{x}$. Fig. 1 illustrates the perceptron.

One usecase of the perceptron are binary classification problems. Given a set of m training examples $\mathbb{X} = \{\boldsymbol{x}^{(1)}, \dots, \boldsymbol{x}^{(m)}\}$ and their corresponding binary labels $\mathbb{Y}$, we want to predict whether a given vector $\boldsymbol{x}$ is more likely to have the label $y = 1$ or the label $y = -1$.

For example, our vectors $\boldsymbol{x}^{(i)}$ might describe features of an email using a *bag-of-words* representation. That is, we specify a fixed vocabulary of words and the jth entry in the vector

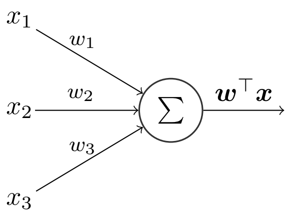

Fig. 1. An illustration of the perceptron model. In this example, the perceptron accepts three inputs x_1, x_2, x_3 and computes the weighted sum $w_1 x_1 + w_2 x_2 + w_3 x_3$.

specifies how often the jth word of the vocabulary occurs in that particular document. The corresponding label $y^{(i)} = 1$ then might signify that the email was a legitimate email, whereas a value of $y^{(i)} = -1$ might label the email as spam.

Prediction with the perceptron model is simple: We predict a value $\hat{y}$ depending on the sign of the weighted sum:

$$\hat{y} = \begin{cases} 1 & \text{if } \boldsymbol{w}^\top \boldsymbol{x} \geq 0, \\ -1 & \text{if } \boldsymbol{w}^\top \boldsymbol{x} < 0. \end{cases} \tag{2}$$

The weights are randomly initialized and the model thus makes arbitrary predictions in the beginning. In the process of *training* the perceptron, we iteratively adjust the weights so that the predictions on the ovserved training data $\mathbb{X}$ improve.

One way to train the model is the perceptron learning algorithm proposed by [10]. The intuition behind the algorithm is that we step through the training data and only update $\boldsymbol{w}$ if the label that the model predicts is different from the actual observed label. However, if the model predicts a positive label and the actual label is negative, we adjust all weights corresponding to the training example a little in the negative direction, and we adjust the weights in the positive direction in the opposite case. Details can be found in ??.

As we have seen, the perceptron is a rather simple model and it is thus no surprise that its performance is rather poor. In particular, it can be shown that the perceptron can only learn to classify linearly separable data [??]. For example, no combination of weights can successfully solve the XOR problem ....

More sophisticated models needed which eventually led to the development of the neural network.

III. Feedforward Neural Networks

IV. Training Feedforward Neural Networks

V. Universal Approximation Capabilities

VI. Extensions and Applications

VII. Conclusion

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