Technische Universität München

Fundamentals of Neural Networks

Fundamentals of Neural Networks

June 12, 2018 Technische Universität München

Zeitung \Rightarrow Künstliche Intelligenz, Gedichte schreiben SZ \Rightarrow Deep Learning

Introduction



Figure 1: A self-driving car.

Credit: Marc van der Chijs / CC BY-ND 2.0

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2018-06-11

Introduction

Figure 1: A salf-debing cor.

Introduction



Figure 2: A digital assistant. Credit: Kārlis Dambrāns / CC BY 2.0

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Figure 2. A fight instant.
Cutt Valid Chrotise / C Et V.

Introduction

Jedes intelligente System benutzt neuronale Netze

The Perceptron

The Perceptron

MNIST Data Sample

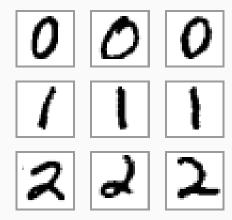


Figure 3: Examples from the MNIST database. Credit: Josef Steppan / CC BY-SA 4.0

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The Perceptron

MNIST Data Sample

With Data Sample

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-Example Task

The Perceptron

Example Task

• Predict whether an input image of a handwritten digit shows a zero or another digit

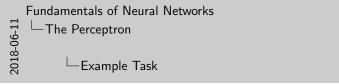
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- \bullet The image is represented as a flattened vector of pixel intensities $\textbf{x} \in \mathbb{R}^{784}$



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Example Task

- Predict whether an input image of a handwritten digit shows a zero or another digit
 The image is represented as a flattened vector of pixel
- intensities $\mathbf{x} \in \mathbb{R}^{284}$
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Example Task

- Predict whether an input image of a handwritten digit shows a zero or another digit
- \bullet The image is represented as a flattened vector of pixel intensities $\textbf{x} \in \mathbb{R}^{784}$
- \bullet The output should be 1 if the image shows a zero, otherwise it should be -1
- Idea: Assign a weight to every input pixel

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The Perceptron

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Model Specification

The perceptron accepts n input values and computes an output value \hat{y} :

$$\hat{y} = \operatorname{sign}\left(\sum_{i=1}^{n} w_i x_i\right)$$

$$\equiv \hat{y} = \operatorname{sign}\left(\mathbf{w}^{\top} \mathbf{x}\right)$$
(1)

Visual Representation

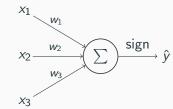


Figure 4: A visual representation of the perceptron model.



1950

Neuron im Gehirn: Nervenzelle Eingaben Ausgaben

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• The perceptron is often used in a modified form

☐ The Perceptron
☐ Generalizations

f: Aktivierungsfunktion

$$\hat{y} = \operatorname{sign}\left(\mathbf{w}^{\top}\mathbf{x} + b\right) \tag{2}$$



f: Aktivierungsfunktion

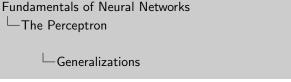
Generalizations

- The perceptron is often used in a modified form
- A scalar bias value can be added to the output computation:

$$\hat{y} = \operatorname{sign}\left(\mathbf{w}^{\top}\mathbf{x} + b\right) \tag{2}$$

• The sign function can be replaced with a generic function f:

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f: Aktivierungsfunktion

2018-06-1

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 $\hat{y} = f\left(\mathbf{w}^{T}\mathbf{x} + b\right)$

Generalizations

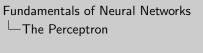
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• These modified perceptrons are often called *neurons* or simply *units*



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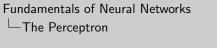
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- **Notation**: We denote the *weighted input* as

$$z = \mathbf{w}^{\top} \mathbf{x} + b \tag{4}$$



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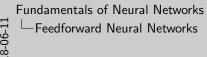
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Feedforward Neural Networks



Feedforward Neural Networks

Perzeptron linear: Viele Funktionen können nicht gelernt werden

Networks of neurons

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└─Networks of neurons

• Idea: A combination of multiple neurons could make much better predictions

Angelehnt an Gehirn

Auch Multilayer Perzeptron

• A feedforward neural network is a layered architecture of neurons

Fundamentals of Neural Networks

Feedforward Neural Networks

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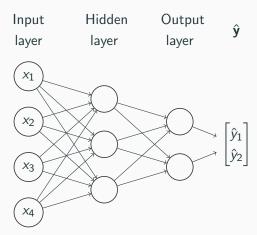
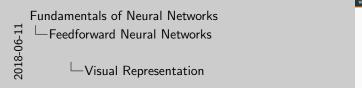


Figure 5: A three-layer feedforward neural network.





Tiefe

Verbindungen nur zwischen Layers

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Fundamentals of Neural Networks -Feedforward Neural Networks

Output Layer

. The design of the output layer depends on the task that we

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10

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- Regression: one single linear neuron
- Binary classification: one single sigmoid neuron

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Feedforward Neural Networks

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The Logistic Sigmoid Function

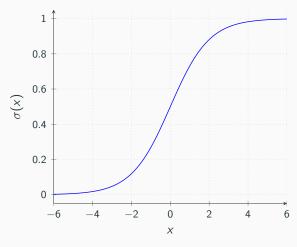
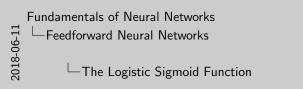
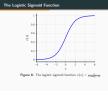


Figure 6: The logistic sigmoid function $\sigma(x) = \frac{1}{1 + \exp(-x)}$





Output Layer

- The design of the output layer depends on the task that we wish to perform
- Regression: one single linear neuron
- Binary classification: one single sigmoid neuron
- *Multiclass classification*: *k* output units with the softmax function

$$softmax(x) = \frac{exp(x)}{\sum_{i=1}^{k} exp(z_i)}$$
 (5)





Output Layer

• The task does not give us any information about how to design the hidden layers

Daher kommt Deep Learning Verschiedene Repräsentationen (Kanten, Objekte,...) \Rightarrow Abstraktionen! Kontinuierlich, nicht linear!

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Feedforward Neural Networks

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The Rectified Linear Function

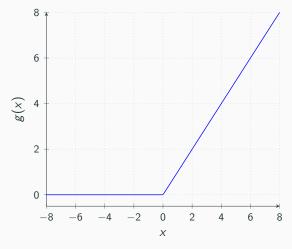
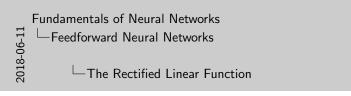
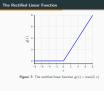


Figure 7: The rectified linear function $g(x) = \max\{0, x\}$





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- Experimentation and trial & error

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Feedforward Neural Networks

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bias value b

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Fundamentals of Neural Networks

-Feedforward Neural Networks Mathematical Formulation f wird komponentenweise angewendet Mathematical Formulation

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Mathematical Formulation

- We can specify a single neuron with a weight vector **w** and a bias value b
- Since a neural network consists of multiple neurons in a layer, we need weight matrices $\mathbf{W}^{(l)}$ and bias vectors $\mathbf{b}^{(l)}$ to specify the parameters of a layer I

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- We can specify a single neuron with a weight vector **w** and a bias value b
- Since a neural network consists of multiple neurons in a layer, we need weight matrices $\mathbf{W}^{(l)}$ and bias vectors $\mathbf{b}^{(l)}$ to specify the parameters of a layer I
- $f^{(I)}$ is the activation function used in the I^{th} layer

f wird komponentenweise angewendet

☐ Mathematical Formulation

• The output at layer *l* is then given by

 $\mathbf{a}^{(I)} = f^{(I)} \left(\mathbf{W}^{(I)\top} \mathbf{a}^{(I-1)} + \mathbf{b}^{(I)} \right)$

Mathematical Formulation

Mathematical Formulation

Neue Perspektive

$$a^0 = x$$

$$\hat{y} = a^L$$

17

(6)

Training Feedforward Neural Networks

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—Training Feedforward Neural Networks

Training Feedforward Neural Networks

Training Scenario

Training Feedforward Neural Networks

Training Scenario

ullet We have training examples $\mathbb{X}=(\mathbf{x}^{(1)},\ldots,\mathbf{x}^{(m)})$ with corresponding labels $\mathbb {Y}$

Zufällig initialisiert

 \bullet We want to learn a mapping from $\mathbb X$ to $\mathbb Y$

Fundamentals of Neural Networks

Training Feedforward Neural Networks

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- \bullet We want to learn a mapping from $\mathbb X$ to $\mathbb Y$
- Idea: Iteratively adjust the parameters of the neural network

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Training Feedforward Neural Networks

Training Scenario

We have training examples X = (x⁽¹⁾,...,x^(m)) with corresponding labels Y
 We want to larm a mapping from X to Y
 Idea: Iteratively adjust the parameters of the neural network

Training Scenario

Zufällig initialisiert

• The cost function $J(\theta)$ is a measure of how good the network performs

Von den Parametern zu einem Skalar Größer als 0 Auch Loss oder Error

-Cost Functions

-Training Feedforward Neural Networks

-Cost Functions

• The cost function $J(\theta)$ is a measure of how good the network performs

• Learning can be framed as minimizing the cost function

Von den Parametern zu einem Skalar Größer als 0 Auch Loss oder Error

- Learning can be framed as minimizing the cost function
- The total cost is a sum over the costs of the individual training examples:

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\mathbf{x}^{(i)}, \mathbf{y}^{(i)}, \theta)$$
 (7)

Cost Functions

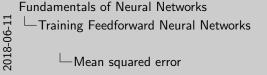
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Von den Parametern zu einem Skalar Größer als 0 Auch Loss oder Error

$$\mathcal{L}(\mathbf{x}, y, \theta) = \frac{1}{2}(\hat{y} - y)^2 \tag{8}$$



In regression, the per-example loss is commonly $\mathcal{L}(\mathbf{x},\mathbf{y},\theta) = \frac{1}{2}(\hat{y}-\mathbf{y})^2$

Mean squared error

Label: Skalar was wir vorhersagen wollen

Distanz

Erfüllt Bedingungen

Cross-entropy

 $L(x, y, \theta) = -y \ln \hat{y} - (1 - y) \ln(1 - \hat{y})$ (9)

Cross-entropy

• In binary classification, we often use the cross-entropy loss

$$\mathcal{L}(\mathbf{x}, y, \boldsymbol{\theta}) = -y \ln \hat{y} - (1 - y) \ln(1 - \hat{y}) \tag{9}$$

MSE schlecht in Klassifikation

Label: 1 oder 0

— In aufzeichnen!

$$\mathcal{L}(\mathbf{x},\mathbf{y},oldsymbol{ heta}) = -\ln \hat{y}_i$$

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Cross-entropy

. In multiclass classification, the cross-entropy becomes $\mathcal{L}(x,y,\theta)=-\ln\hat{y},$

Cross-entropy

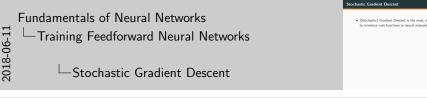
Label: i-te Klasse

Maximum Likelihood Estimation

(10)

Stochastic Gradient Descent

• (Stochastic) Gradient Descent is the most common algorithm to minimize cost functions in neural networks



⇒ Kleine Änderungen in die entgegengesetze Richtung des Gradienten Learning Rate Hyperparameter durch rumexperimentieren Erweiterungen ullet A change $\Delta heta$ in the parameters corresponds roughly to the change

$$\Delta J(\theta) \approx \nabla J(\theta)^{\top} \Delta \theta \tag{11}$$



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• To minimize $J(\theta)$, choose

$$\Delta \theta = -\eta \nabla J(\theta), \tag{12}$$



⇒ Kleine Änderungen in die entgegengesetze Richtung des Gradienten Learning Rate Hyperparameter durch rumexperimentieren Erweiterungen

- (Stochastic) Gradient Descent is the most common algorithm to minimize cost functions in neural networks
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• To minimize $J(\theta)$, choose

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• Stochastic Gradient Descent computes only an approximation of the gradient



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Stochastic Gradient Descent

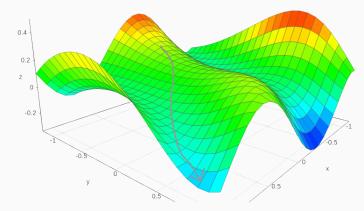
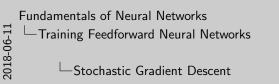
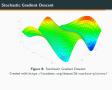


Figure 8: Stochastic Gradient Descent.

Created with https://academo.org/demos/3d-surface-plotter/





Back-propagation

• The back-propagation algorithm efficiently computes the gradient of the cost function

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- It can be derived by recursively applying the chain rule to the layers of the neural network, beginning with the output layer

Fundamentals of Neural Networks

__Training Feedforward Neural Networks

-Back-propagation

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Back-propagation

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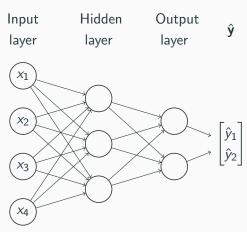
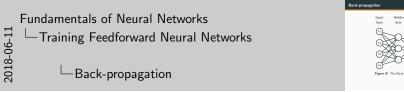


Figure 9: The Back-propagation algorithm.



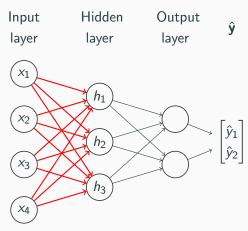
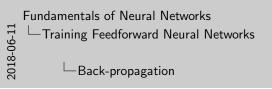


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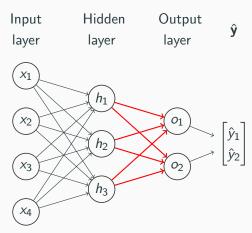
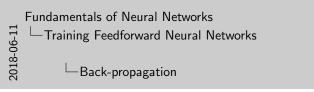


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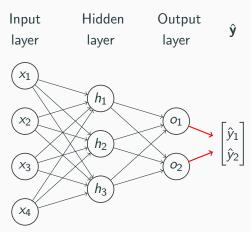
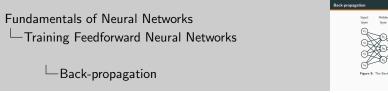


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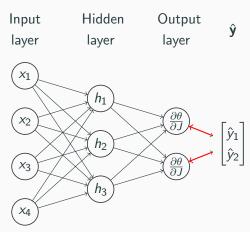
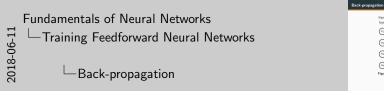


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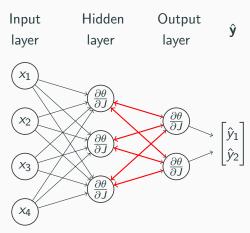
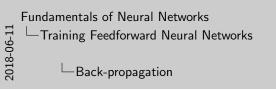


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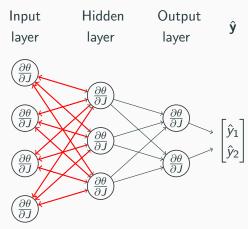
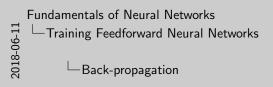


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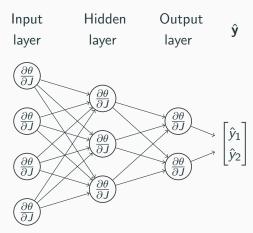
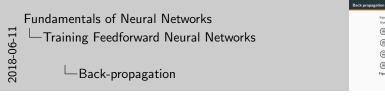


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 Propagate all training examples of a minibatch forward through the network

The Complete Learning Algorithm

1. Propagate all training examples of a minibatch forward through the network

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- 3. Compute all gradients using back-propagation

Fundamentals of Neural Networks -Training Feedforward Neural Networks

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The Complete Learning Algorithm

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- 2. Compute the cost for each training example
- 3. Compute all gradients using back-propagation
- 4. Compute the average gradient
- 5. Update the parameters in the negative direction of the gradient

Fundamentals of Neural Networks

- Training Feedforward Neural Networks

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- 2. Compute the cost for each training example
- 3. Compute all gradients using back-propagation
- 4. Compute the average gradient
- 5. Update the parameters in the negative direction of the gradient
- 6. Repeat until the cost is low enough

Fundamentals of Neural Networks

—Training Feedforward Neural Networks

The Complete Learning Algorithm

The Complete Learning Algorithm

 Propagate all training examples of a minibatch forward through the network

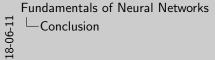
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Conclusion



Conclusion

Komplexe Netzwerke einfacher Einheiten

Abstraktionen Lernen: Kleine Updates der Parameter so dass das Netzwerk besser wird

Überall in Deep Learning

Viele weitere Anwendungen in der Zukunft

