Fundamentals of Neural Networks

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June 12, 2018

Technische Universität München

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Introduction



Figure 1: A self-driving car.

Credit: Marc van der Chijs / CC BY-ND 2.0

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Introduction

Figure 1. A salf-driving car.

Introduction



Figure 2: A digital assistant. Credit: Kārlis Dambrāns / CC BY 2.0

Fundamentals of Neural Networks

Figure 2 Credit: Kirla

Introduction

Jedes intelligente System benutzt neuronale Netze

The Perceptron

Fundamentals of Neural Networks

—The Perceptron

The Perceptron

Fundamentals of Neural Networks

-Example Task

Example Task

• Predict whether an input image of a handwritten digit shows a zero or another digit

MNIST Data Sample

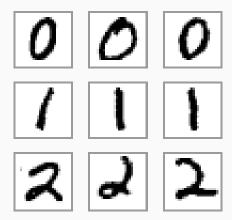


Figure 3: Examples from the MNIST database. Credit: Josef Steppan / CC BY-SA 4.0

Fundamentals of Neural Networks

The Perceptron

MNIST Data Sample



2018-06-11

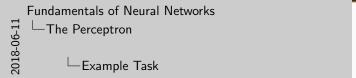
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- Idea: Assign a weight to every input pixel

Fundamentals of Neural Networks The Perceptron 2018-06-1 -Example Task

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☐ Model Specification

The perceptron accepts n input values and computes an output value \hat{y} :

$$\hat{y} = \operatorname{sign}\left(\sum_{i=1}^{n} w_i x_i\right)$$

$$\equiv \hat{y} = \operatorname{sign}\left(\mathbf{w}^{\top} \mathbf{x}\right)$$
(1)

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Visual Representation

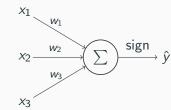


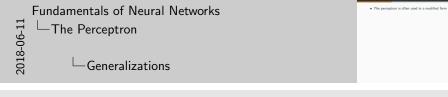
Figure 4: A visual representation of the perceptron model.



1950

Neuron im Gehirn: Nervenzelle Eingaben Ausgaben

• The perceptron is often used in a modified form



 $f:\ Aktivier ungsfunktion$

- The perceptron is often used in a modified form
- A scalar bias value can be added to the output computation:

$$\hat{y} = \operatorname{sign}\left(\mathbf{w}^{\top}\mathbf{x} + b\right) \tag{2}$$



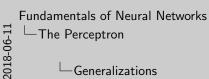
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> $\hat{y} = \text{sign} \left(\mathbf{w}^{\top} \mathbf{x} + b \right)$. The sign function can be replaced with a generic function f $\hat{y} = f\left(\mathbf{w}^{T}\mathbf{x} + b\right)$

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Fundamentals of Neural Networks

—The Perceptron

 \sqsubseteq Generalizations

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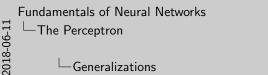
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- **Notation**: We denote the *weighted input* as

$$z = \mathbf{w}^{\top} \mathbf{x} + b \tag{4}$$



-Generalizations

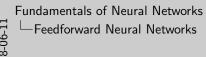
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f: Aktivierungsfunktion

Feedforward Neural Networks



Feedforward Neural Networks

Perzeptron linear: Viele Funktionen können nicht gelernt werden

better predictions

└─Networks of neurons

Networks of neurons

• Idea: A combination of multiple neurons could make much

Angelehnt an Gehirn Auch Multilayer Perzeptron

- Idea: A combination of multiple neurons could make much better predictions
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Angelehnt an Gehirn

Auch Multilayer Perzeptron

Feedforward Neural Networks

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- The input of a layer is the output of the previous layer

Fundamentals of Neural Networks

—Feedforward Neural Networks

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Angelehnt an Gehirn Auch Multilayer Perzeptron

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Visual Representation

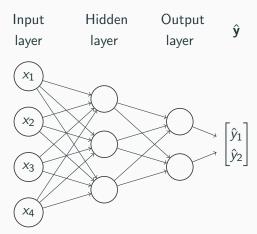


Figure 5: A three-layer feedforward neural network.



Tiefe
Verbindungen nur zwischen Layers

Fundamentals of Neural Networks

-Feedforward Neural Networks

 The design of the output layer depends on the task that we wish to perform

Output Layer

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-Feedforward Neural Networks

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- Regression: one single linear neuron

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The Logistic Sigmoid Function

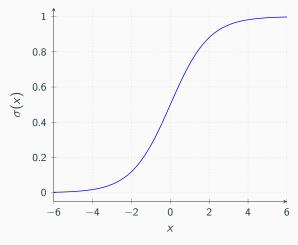
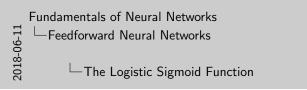
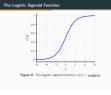


Figure 6: The logistic sigmoid function $\sigma(x) = \frac{1}{1 + \exp(-x)}$

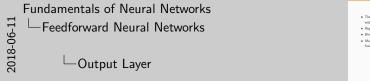




Output Layer

- The design of the output layer depends on the task that we wish to perform
- Regression: one single linear neuron
- Binary classification: one single sigmoid neuron
- *Multiclass classification: k* output units with the softmax function

$$softmax(x) = \frac{exp(x)}{\sum_{i=1}^{k} exp(z_i)}$$
 (5)



• The design of the output layer depends on the task that we wish to perform $P_{\rm efficient} = \frac{1}{N_{\rm efficient}} = \frac{1}{$

Output Layer

softmax: Generalisierung, normalisiert

Viele andere Probleme: Auffälliges Verhalten, Bildgenerierung

• The task does not give us any information about how to design the hidden layers

Daher kommt Deep Learning Verschiedene Repräsentationen (Kanten, Objekte,...) \Rightarrow Abstraktionen! Kontinuierlich, nicht linear!

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- Deep networks perform almost always better in practice
- Activation function: Three common choices are the logistic sigmoid, the tanh, and the rectified linear function

Fundamentals of Neural Networks

Feedforward Neural Networks

Hidden Layers

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The Rectified Linear Function

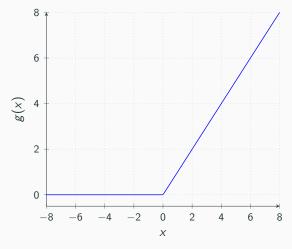
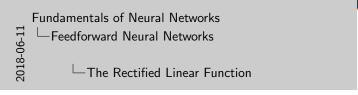
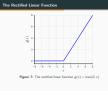


Figure 7: The rectified linear function $g(x) = \max\{0, x\}$





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Fundamentals of Neural Networks

Feedforward Neural Networks

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bias value b

• We can specify a single neuron with a weight vector **w** and a

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Fundamentals of Neural Networks -Feedforward Neural Networks

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Mathematical Formulation







-Mathematical Formulation

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f wird komponentenweise angewendet

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- Since a neural network consists of multiple neurons in a layer, we need weight matrices $\mathbf{W}^{(l)}$ and bias vectors $\mathbf{b}^{(l)}$ to specify the parameters of a layer I
- $f^{(I)}$ is the activation function used in the I^{th} layer

f wird komponentenweise angewendet

☐ Mathematical Formulation

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Mathematical Formulation

• The output at layer *l* is then given by

$$\mathbf{a}^{(l)} = f^{(l)} \left(\mathbf{W}^{(l)\top} \mathbf{a}^{(l-1)} + \mathbf{b}^{(l)} \right)$$
 (6)

 $a^0 = x$ $\hat{y} = a^L$

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Training Feedforward Neural Networks

Fundamentals of Neural Networks

Training Feedforward Neural Networks

Training Feedforward Neural Networks

Training Scenario

ullet We have training examples $\mathbb{X}=(\mathbf{x}^{(1)},\ldots,\mathbf{x}^{(m)})$ with corresponding labels $\mathbb {Y}$

Zufällig initialisiert

Training Scenario

 \bullet We want to learn a mapping from $\mathbb X$ to $\mathbb Y$

Fundamentals of Neural Networks

Training Feedforward Neural Networks

**We have taking campin X = (x⁽¹⁾,...,x⁽ⁿ⁾) with corresponding limits Y

**We must to knot a suppose from X to Y

Training Scenario

Zufällig initialisiert

- \bullet We want to learn a mapping from $\mathbb X$ to $\mathbb Y$
- Idea: Iteratively adjust the parameters of the neural network

Fundamentals of Neural Networks

__Training Feedforward Neural Networks

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Training Scenario

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Zufällig initialisiert

• The cost function $J(\theta)$ is a measure of how good the network performs

Von den Parametern zu einem Skalar Größer als 0 Auch Loss oder Error

-Cost Functions

- -Training Feedforward Neural Networks
 - -Cost Functions

• The cost function $J(\theta)$ is a measure of how good the network performs

• Learning can be framed as minimizing the cost function

Von den Parametern zu einem Skalar Größer als 0

Auch Loss oder Error

- Learning can be framed as minimizing the cost function
- The total cost is a sum over the costs of the individual training examples:

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\mathbf{x}^{(i)}, \mathbf{y}^{(i)}, \theta)$$
 (7)

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Von den Parametern zu einem Skalar Größer als 0 Auch Loss oder Error

Mean squared error

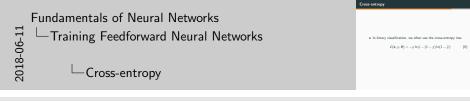
• In regression, the per-example loss is commonly

$$\mathcal{L}(\mathbf{x}, y, \boldsymbol{\theta}) = \frac{1}{2}(\hat{y} - y)^2 \tag{8}$$

Label: Skalar was wir vorhersagen wollen Distanz Erfüllt Bedingungen

└─Mean squared error

$$\mathcal{L}(\mathbf{x}, y, \boldsymbol{\theta}) = -y \ln \hat{y} - (1 - y) \ln(1 - \hat{y}) \tag{9}$$



MSE schlecht in Klassifikation Label: 1 oder 0

$$\mathcal{L}(\mathbf{x},\mathbf{y},oldsymbol{ heta}) = -\ln \hat{y}_i$$

Fundamentals of Neural Networks -Training Feedforward Neural Networks 2018-06-1

Cross-entropy

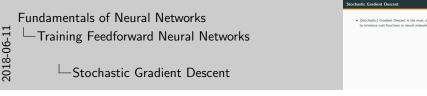
Label: i-te Klasse

Maximum Likelihood Estimation

Cross-entropy $\mathcal{L}(\mathbf{x}, \mathbf{y}, \boldsymbol{\theta}) = -\ln \hat{y}_i$

Stochastic Gradient Descent

• (Stochastic) Gradient Descent is the most common algorithm to minimize cost functions in neural networks



⇒ Kleine Änderungen in die entgegengesetze Richtung des Gradienten Learning Rate Hyperparameter durch rumexperimentieren Erweiterungen ullet A change $\Delta heta$ in the parameters corresponds roughly to the change

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Stochastic Gradient Descent

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• Stochastic Gradient Descent computes only an approximation of the gradient



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Stochastic Gradient Descent

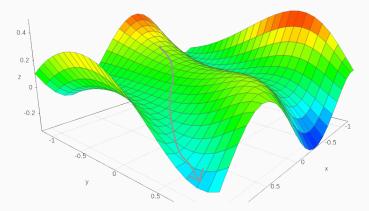


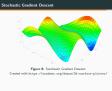
Figure 8: Stochastic Gradient Descent.

Created with https://academo.org/demos/3d-surface-plotter/

Fundamentals of Neural Networks

Training Feedforward Neural Networks

Stochastic Gradient Descent



-Back-propagation

• The back-propagation algorithm efficiently computes the gradient of the cost function

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- It can be derived by recursively applying the chain rule to the layers of the neural network, beginning with the output layer

Fundamentals of Neural Networks

Training Feedforward Neural Networks

Back-propagation

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Back-propagation

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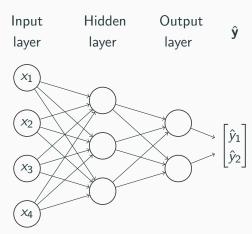


Figure 9: The Back-propagation algorithm.



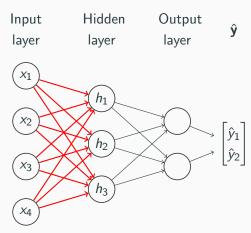
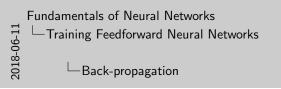


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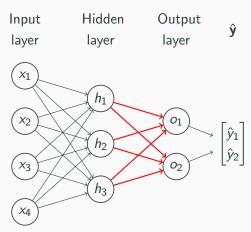
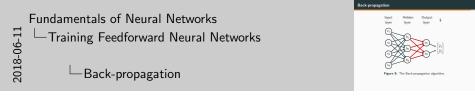


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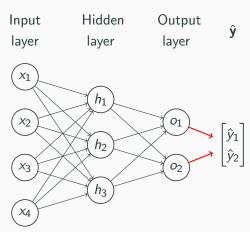
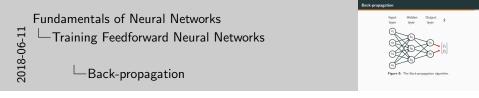


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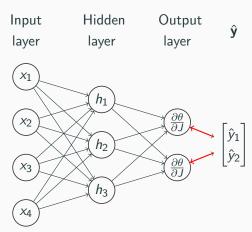


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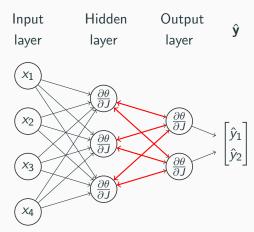
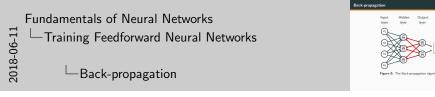


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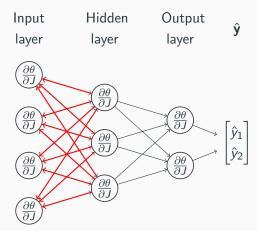
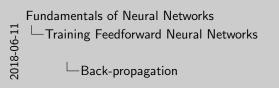


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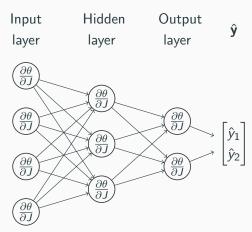
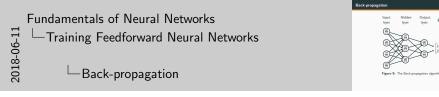
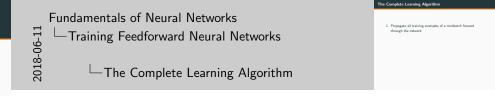


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1. Propagate all training examples of a minibatch forward through the network



The Complete Learning Algorithm

- 1. Propagate all training examples of a minibatch forward through the network
- 2. Compute the cost for each training example

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- 3. Compute all gradients using back-propagation

Fundamentals of Neural Networks -Training Feedforward Neural Networks

1. Propagate all training examples of a minibatch forward through the network 2. Compute the cost for each training example

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The Complete Learning Algorithm

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The Complete Learning Algorithm

- 1. Propagate all training examples of a minibatch forward through the network
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Fundamentals of Neural Networks 1. Propagate all training examples of a minibatch forward -Training Feedforward Neural Networks through the network 4. Compute the average gradient The Complete Learning Algorithm

The Complete Learning Algorithm

2. Compute the cost for each training example

- 1. Propagate all training examples of a minibatch forward through the network
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- 5. Update the parameters in the negative direction of the gradient

Fundamentals of Neural Networks through the network -Training Feedforward Neural Networks 2018-06-1 The Complete Learning Algorithm

The Complete Learning Algorithm

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- 2. Compute the cost for each training example
- 3. Compute all gradients using back-propagation
- 4. Compute the average gradient
- 5. Update the parameters in the negative direction of the gradient
- 6. Repeat until the cost is low enough

Fundamentals of Neural Networks

Training Feedforward Neural Networks

The Complete Learning Algorithm

The Complete Learning Algorithm

 Propagate all training examples of a minibatch forward through the network

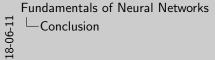
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Conclusion



Conclusion

Komplexe Netzwerke einfacher Einheiten

Abstraktionen Lernen: Kleine Updates der Parameter so dass das Netzwerk besser wird

Überall in Deep Learning

Viele weitere Anwendungen in der Zukunft

Thank you!