Fundamentals of Neural Networks

Mathias Jackermeier

June 10, 2018

Technische Universität München

2018-06-10

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Introduction

Figure 1: A self-driving car.
Credit: Marc van der Chijs / CC BY-ND 2.0

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Figure 2: A digital assistant.
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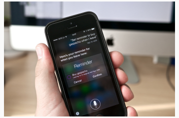

Figure 2: A digital assistant.
Credit: Kārlis Dambrāns / CC BY 2.0

Jedes intelligente System benutzt neuronale Netze

Outline

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Überblick, nicht ins mathematische Detail

The Perceptron

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Example Task

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MNIST Data Sample

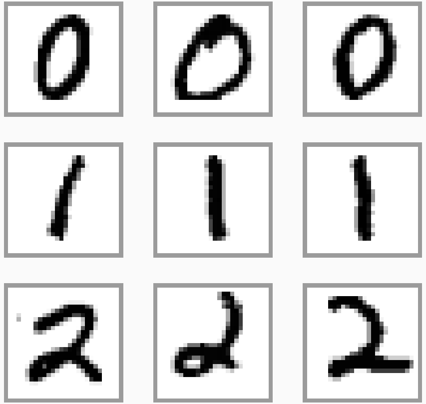
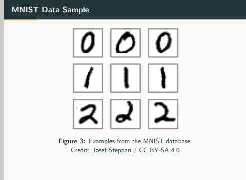


Figure 3: Examples from the MNIST database.
Credit: Josef Steppan / CC BY-SA 4.0



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The perceptron accepts n input values and computes an output value $\hat{y}$:

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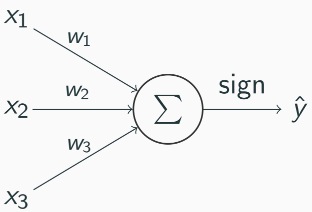
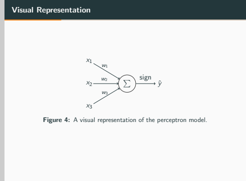


Figure 4: A visual representation of the perceptron model.



1950

Neuron im Gehirn: Nervenzelle Eingaben Ausgaben

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f: Aktivierungsfunktion

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Shortcomings of the Perceptron

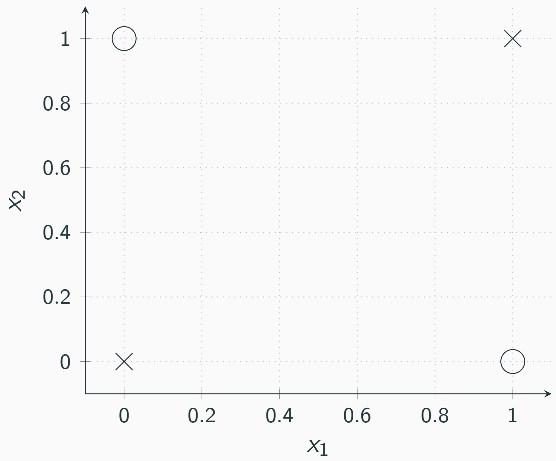
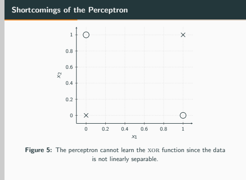


Figure 5: The perceptron cannot learn the XOR function since the data is not linearly separable.



Feedforward Neural Networks

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Networks of neurons

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Angelehnt an Gehirn

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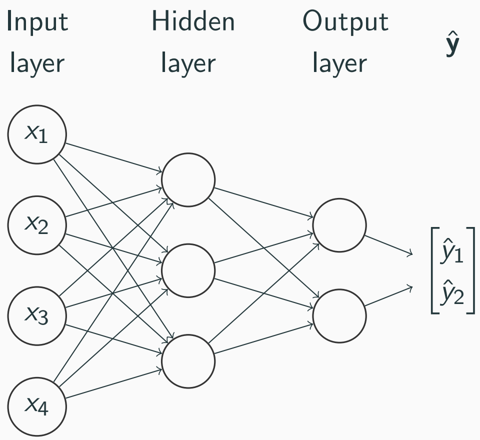
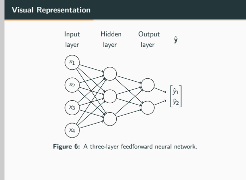


Figure 6: A three-layer feedforward neural network.



Tiefe

Verbindungen nur zwischen Layers

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The Logistic Sigmoid Function

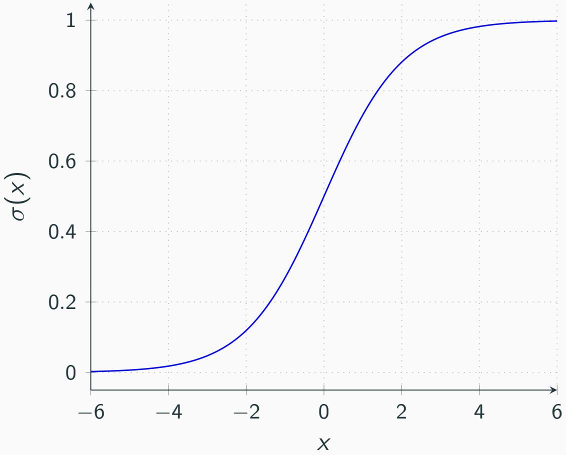
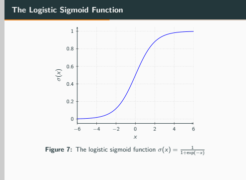


Figure 7: The logistic sigmoid function $\sigma(x) = \frac{1}{1+\exp(-x)}$



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softmax: Generalisierung, normalisiert

Viele andere Probleme: Auffälliges Verhalten, Bildgenerierung

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The Rectified Linear Function

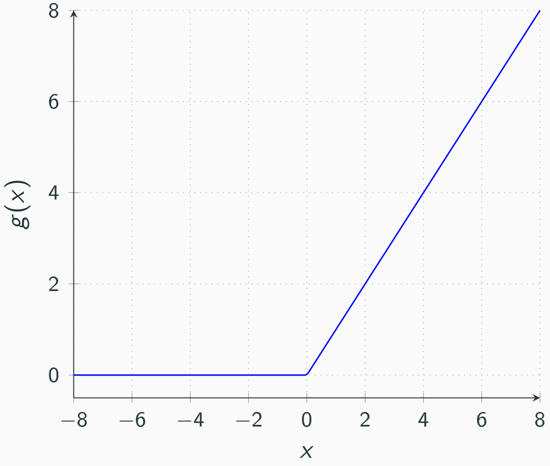
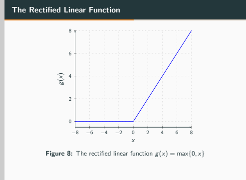


Figure 8: The rectified linear function $g(x) = \max\{0, x\}$



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Label: Skalar was wir vorhersagen wollen

Distanz

Erfüllt Bedingungen

Cross-entropy

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MSE schlecht in Klassifikation

Label: 1 oder 0

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$$\mathcal{L}(\mathbf{x}, \mathbf{y}, \boldsymbol{\theta}) = -\ln \hat{y}_i \tag{11}$$

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Label: one-hot
i-te Klasse
Maximum Likelihood Estimation

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$\Rightarrow$ Kleine Änderungen in die entgegengesetze Richtung des Gradienten
Learning Rate nicht zu klein und nicht zu groß
Erweiterungen

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$$\Delta J(\boldsymbol{\theta}) \approx \nabla J(\boldsymbol{\theta})^\top \Delta\boldsymbol{\theta} \tag{12}$$

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$\Rightarrow$ Kleine Änderungen in die entgegengesetze Richtung des Gradienten

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Erweiterungen

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Stochastic Gradient Descent

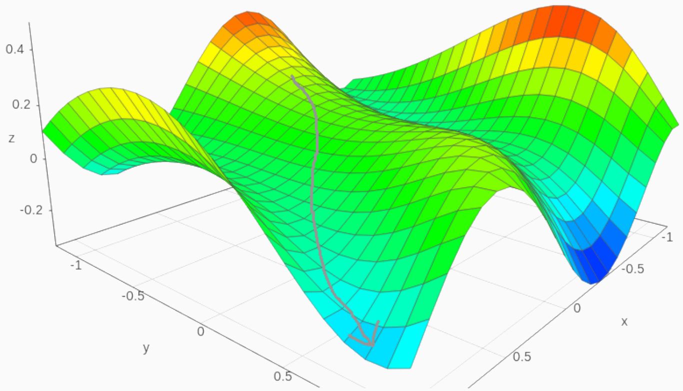
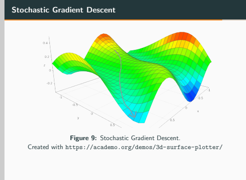


Figure 9: Stochastic Gradient Descent.
Created with `https://academo.org/demos/3d-surface-plotter/`



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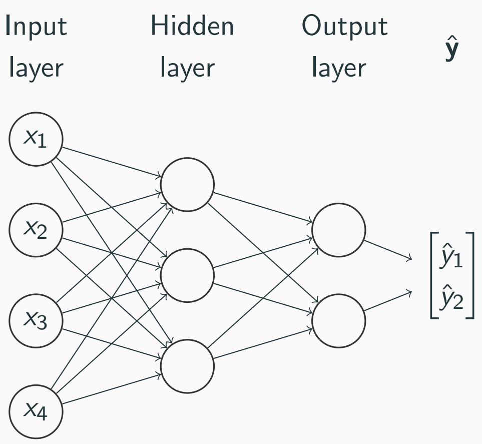
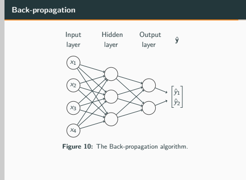


Figure 10: The Back-propagation algorithm.



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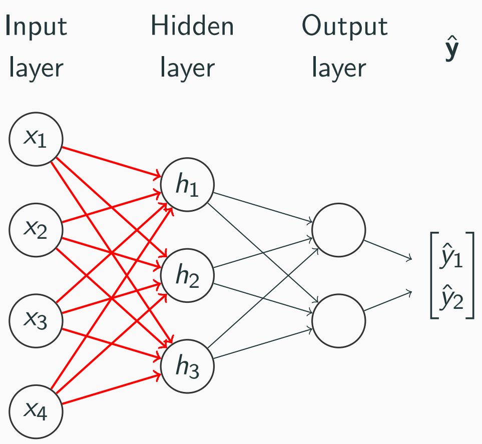

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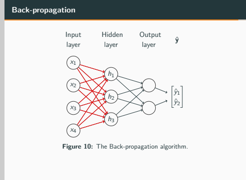

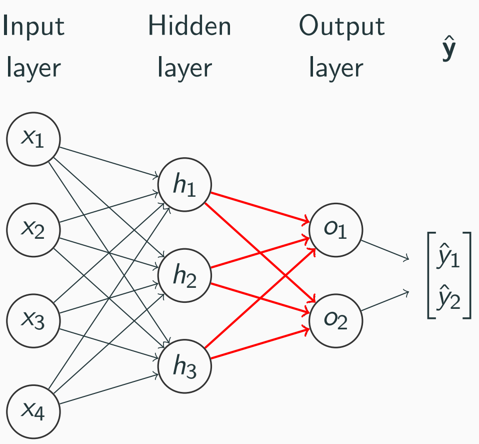

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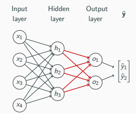

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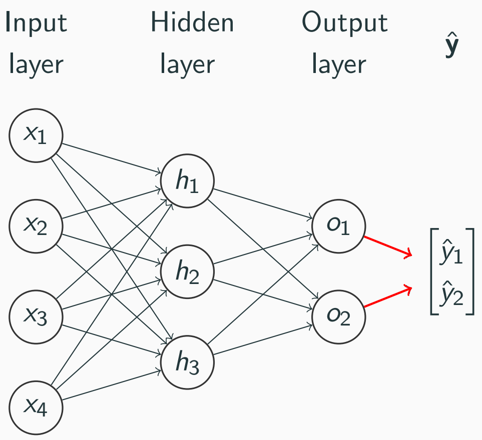

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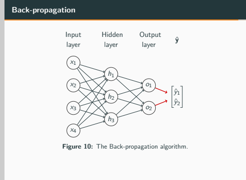

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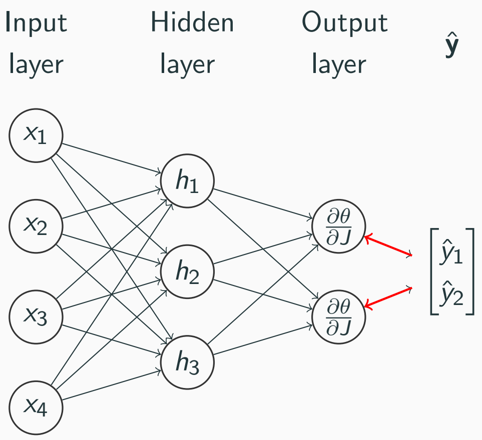

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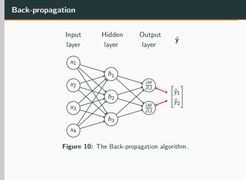

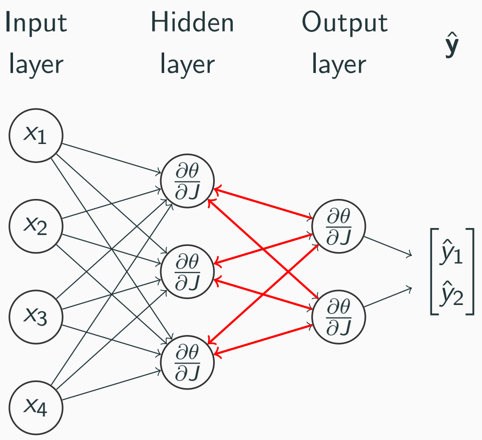

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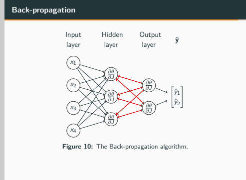

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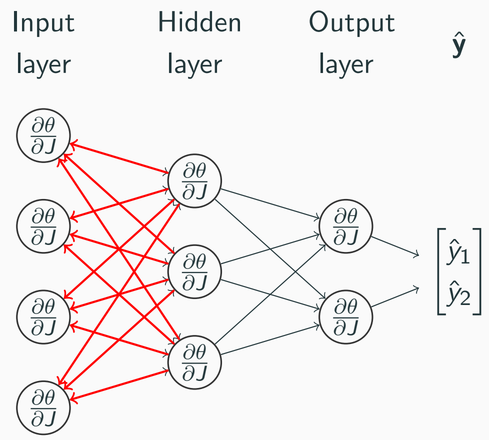

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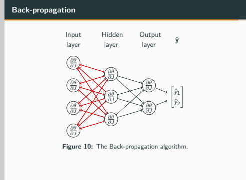

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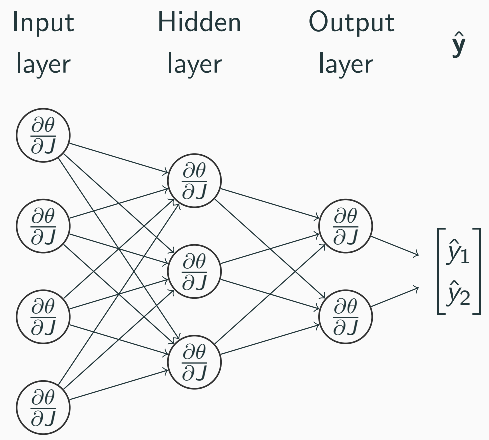

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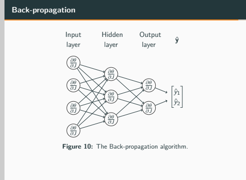

The Complete Learning Algorithm

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Conclusion

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Komplexe Netzwerke einfacher Einheiten
Abstraktionen
Lernen: Kleine Updates der Parameter so dass das Netzwerk besser wird
Überall in Deep Learning
Viele weitere Anwendungen in der Zukunft

Thank you!

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