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ABSTRACT *As a strategic factor for a country to survive in the global competition, transportation systems have attracted extensive attention from different disciplines for a long time. Since the introduction of complex network theory in the last decade, however, studies on transport systems have witnessed dramatic progress. Most roads, streets, and rails are organized as a network pattern, while link flows, travel time, or geographical distance are regarded as weights. In this article, the authors will present the current state of topological research on transportation systems under a complex network framework, as well as the efforts and challenges that have been made in the last decade. First, different kinds of transportation systems should be generalized as networks in different ways, which will be explained in the first part of this paper. We follow this by summarizing network measures that describe topological characteristics of transportation networks. Then we discuss the empirical observations from the last decade on real transportation systems at a variety of spatial scales. This paper concludes with some important challenges and open research frontiers in this field.*

Keywords: complex network; transportation; review; topology; urban systems

1. Introduction

Complex network theory is a science that studies the connection and interaction between components in a system. It is an emerging field, developing at a rapid pace. Particularly in the last decade, the proposing of small-world and scale-free models has brought about comprehensive research and interest on this subject from different disciplines, which shows that the structures of networks existing in various domains are more similar than one would have expected. Transportation systems, which take the form of networks, have been a hot research topic in complex network studies. A great deal of progress has been made in various disciplines, including physics, sociology, transport geography — not to mention the research objects ranging from grounded public transportation to maritime systems to aviation systems and so forth. In this case, bridging the gaps among different perspectives and constructing an overall understanding of complex transport networks, especially with the availability of gigantic data sets on large transport systems, becomes the foremost challenge for transport researchers, and the motivation for this article. To these ends, the authors systematically review the results mainly from the last decade, covering most research on

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transport systems with a complex network perspective from various fields. The detailed structure of this article will be explained in Section 1.2.

1.1 *Network Analysis in Early Transport Studies*

Network analysis is a common and effective approach, considering its elegance in reflecting spatial structures, to study river systems, channel patterns, and road networks (Haggett & Chorley, 1969). Many elementary topological measures are familiar to researchers, who divided networks, in terms of the topology, into branching networks, circuit networks, barrier networks, and the like. The historical notable Königsberg's Bridges Problem, solved by Euler in 1735, can be regarded as a simplified and the earliest transportation optimization. It not only laid a foundation for graph theory, but also introduced preliminary network topology into transportation studies. With the development of graph theory in the last century, network analysis also became a core concept in transportation studies (Black, 2003). It developed a topological and mathematic explanation for transport systems. Furthermore, the network data model is considered the most efficient way to extract and reference transport systems in a computer (geographic information system) environment. A set of indicators is developed to quantify transport networks, for example, connectivity, cyclomatic number, diameter, and accessibility (Rodrigue, Comtois, & Slack, 2009). Some of them are analogical to measures in complex network theory. Although many efforts have been made in network analysis in early transport research, the absence of a systematic theoretical scheme is still a prominent problem to be addressed, for which the emerging of complex network theory could offer some enlightened insights.

1.2 *Aim and Structure of this Article*

The primary aim of this article is to offer a comprehensive review of topological representations, measures, and case studies of transport systems from a complex network perspective. However, it is worth mentioning that the authors not only summarize these works simply, but also compare various definitions in a wide range of references, classify them according to their mutual relationships, and delineate a clear scheme of this subject for readers. On the other hand, emphasis should be shed on the practical significance of this review work. Network measures included in this article can be applied widely in transportation practice to identify critical nodes and links in a system, which can provide necessary warning for potential attacks or faults. Moreover, human behaviors are too complicated to be fully captured. Fortunately, they are affected to a large extent by the structure of underlying networks topology from an interconnected perspective is the first step to identify human movement patterns, and will contribute greatly to solving modern traffic problems, including congestion, navigation, searching, etc.

The remainder of this paper is arranged as follows. Section 2 introduces various representations for different systems and research aims. In Section 3, the local and global measures are compared; the cell properties of planar networks also are discussed. Section 4 summarizes the empirical analysis in this field in terms of five kinds of real systems. Section 5 concludes the paper and points to future work.

2. Representation of Transport Systems

2.1 *Basic Network Representation*

According to graph theory (Diestel, 2005), the basic representation of a network can be generalized as $G = (N, E)$, which means that it is composed of a node (vertex) set N and an edge (link) set E (Figure 1(a)). Besides, in order to store and calculate a network in a computer environment much more easily, the adjacent matrix is introduced (Figure 1(b)). For an n-node network, its adjacent matrix would be an $n*n$ matrix. If there is an edge between two nodes, then the entry element in the matrix equals to 1; otherwise it is 0.

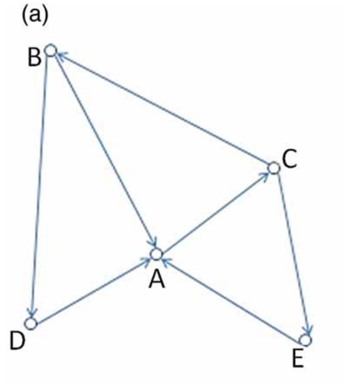
A directed network can be defined by differentiating the direction of a link (Figure 1(a)). Moreover, if the different importances of edges in the network are considered, a weighted network will be generated. In this sense, an unweighted network can be regarded as a special case when every link presents the same importance.

2.2 *Representation of Non-Planar Transportation Networks*

First, we shall clarify two concepts: planar and non-planar graphs. As their names imply, planar graphs are embedded in the Euclidean space, and their edges never intersect with each other if there is no channel or bridge (Wilson, 2010). On the contrary, the edges of non-planar graphs can meet each other without forming a node within a plane.

In contrast to other ground transportation systems, aviation networks and maritime transport networks present more non-planar characteristics. They possess a relatively straightforward representation by taking stops as nodes and the routes between them as links. The directed and weighted networks can be obtained by considering the directions and differences of routes. It is worth noting that because of the existence of intersections between links, the degrees in non-planar networks could be much bigger than those in planar networks.

(a)



(b)

	A	B	C	D	E
A	0	0	1	0	0
B	1	0	0	1	0
C	0	1	0	0	1
D	1	0	0	0	0
E	1	0	0	0	0

Figure 1. (a) Network representation of a system with five nodes and seven directed edges; (b) its corresponding adjacent matrix.

2.3 *Representation of Urban Road Networks*

Urban streets and roads constitute the skeleton of a city, and therefore play a crucial role in shaping the configuration of urban forms. Street networks are typical spatial planar graphs and also are a crucial subject to researchers on space syntax. Typically, there are two common ways to extract planar networks: the primal approach and the dual representation. The primal representation is a natural and intuitional approach, taking route segments as links, and intersections or ends of them as nodes (Crucitti, Latora, & Porta, 2006; Porta, Crucitti, & Latora, 2006b; Scellato, Cardillo, Latora, & Porta, 2006). This method retains the geometric patterns and geographical properties of transportation systems. It is this reason that makes the primal approach the acknowledged data model for geo-spatial data sets and computer simulation.

After Hiller and Hanson (1984) introduced the topological perspective into space syntax in the mid-1980s, the dual representation of spaces came into our sights (Batty, 2004; Jiang & Claramunt, 2002). This approach focuses on the topology of transportation networks by taking roads or routes as vertices and intersections as links (Jiang, 2007; Porta, Crucitti, & Latora, 2006a). In this sense, it reflects the functional structure of networks, whereas the Euclidean coordinates of nodes and spatial distances of edges are neglected. This representation offers a higher degree of abstraction and presents a better picture of road hierarchy. Therefore, it is conducive to navigation in street networks. Walkers can get a local connectivity graph even in a very long street, but unfortunately it could not guarantee the optimal option because of the absence of distance information. To avoid this shortcoming, researchers (Hu, Jiang, Wang, & Wu, 2009; Hu, Jiang, Wu, Wang, & Wu, 2008; Zeng, Guo, Zhu, Mitrovic, & Tadic, 2009) introduced the holding capacity and turning ability to combine the motion and navigation in a dual graph. It is found that a well-planned lattice road network is much more efficient than a self-organized one when there is a crisis or congestion; this discovery offers a meaningful reference for transport planners.

In the process of constructing a dual network, the implication of nodes — that is, the alignment of road segments — is a key factor to subsequent analysis. Jiang and Claramunt (2004) considered named streets as vertices (Kalapala, Sanwalani, Clauset, & Moore, 2006) (Figure 2(b)). However, Porta et al. (2006a) argued that this method would introduce some subjectivity into the analysis; instead they used the intersection continuity negotiation (ICN) model by examining the good continuation among all pairs of incident edges in the street networks (Figure 2). Some further improvements are made based on these two approaches by considering the deflecting angle (Masucci, Smith, Crooks, & Batty, 2009) and different join principles in terms of every-best-fit, self-best-fit, and self-fit to connect neighboring segments (Jiang, Yin, & Zhao, 2009). Different methods may generate different strategies for searching for the shortest path length.

There are intrinsic correlations between primal and dual representations (Batty, 2004), although they seem contradictory because of opposite generalization rules. Note that the dual model here is not the dual of planar graphs, which takes faces as vertices, but shares some similarities with the linear dual of planar graphs (Anez, de la Barra, & Perez, 1996; Barthelemy, 2011). In transport practice, a dual graph can easily represent various operators, for example, turning restrictions and multiple transit routes. Therefore it can reduce the complexity and time for transport computation, including shortest path search, capacity,

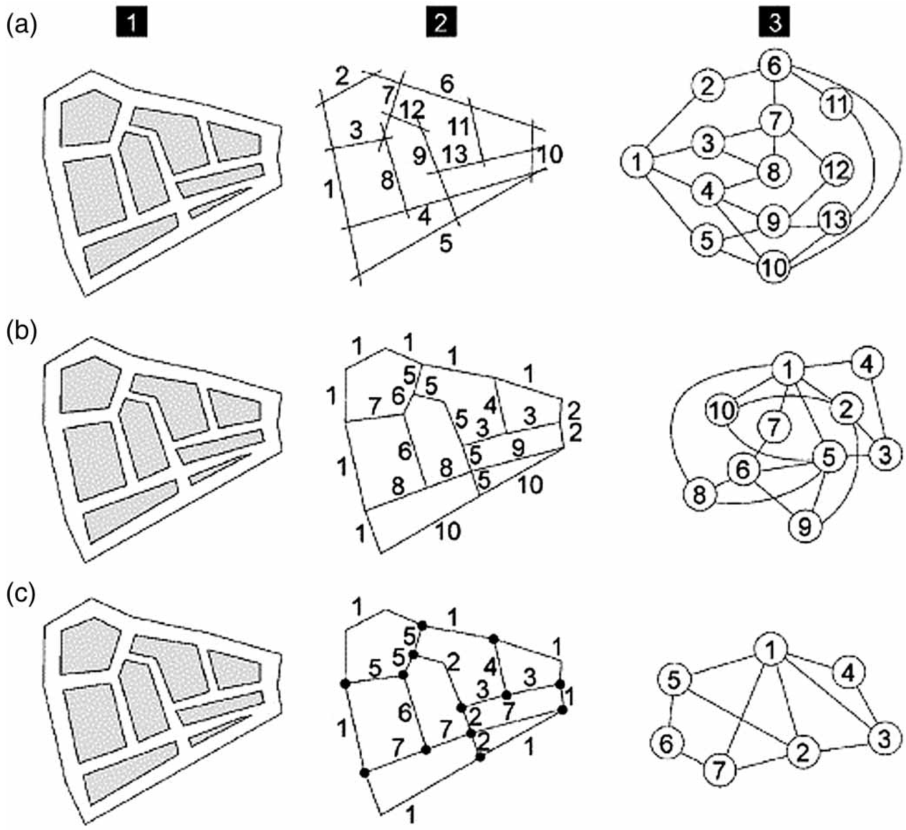


Figure 2. Different representations in network analysis. Column 1 indicates the fiction urban street network, column 2 is the primal graphs based on different nomination mechanisms, and column 3 represents the corresponding dual connectivity graph. Row A represents the axial map, row B is the result based on the named street representation (Jiang & Claramunt, 2004); and row C is obtained from the ICN process. *Source*: Porta et al. (2006a).

assignment algorithms, etc. We cannot elaborate the applications because of space limitations, but interested readers can refer to research by Mozes (2013).

2.4 *Representation of Public Transport Systems*

In general, public transportation systems consist of metro, bus, or railway networks and so on. These systems comprise a set of routes with distributed stops or stations. With the availability of databases of real networks and rapidly growing computer capacity, more and more efforts have been made to understand public transport systems because of their importance to our daily lives. Progress has been obtained in representing them in distinct L-space and P-space (Ferber, Holovatch, Holovatch, & Palchykov, 2007; Sienkiewicz & Holyst, 2005; Xu, Hu, Liu, & Liu, 2007b), and different topological characteristics generated by representations also are discussed.

L-space depicts the original configuration of real-life transport networks, in which stops or stations are vertices. Two vertices are connected if they are consecutive on an arbitrary route (Figure 3(b)) (Sienkiewicz & Holyst, 2005; Wang,

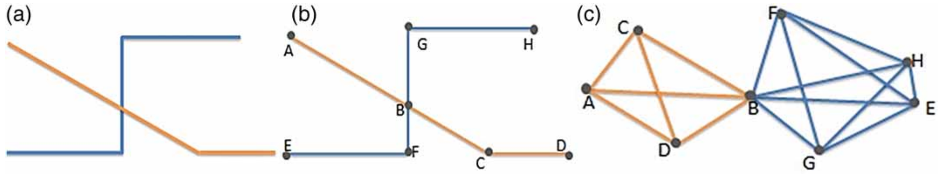


Figure 3. (a) Two simplified public bus routes and the representations in (b) L-space and (c) P-space.

Jiang, Wu, Wang, & Wu, 2008). Sometimes it could be referred to as the space of stops or space of stations (Kurant & Thiran, 2006).

In comparison, Sen et al. (2003) introduced P-space as a new representation pattern. All of the stops of a route can be connected because a directed line can be detected between one another; that is to say, if there is at least one line connecting two nodes, then a link will be constructed between them. Consequently, a fully connected graph can be established for a route by taking all the stops on the route as nodes (Figure 3(c)). In this sense, the path length between two nodes indicates the required transfer times during travel, so that the P-space is also called the space of transfer (Kurant & Thiran, 2006). For example, in Figure 3(c), passengers need to take only one transfer from node H to node D, although there are, in fact, many stops between them.

Additionally, the bipartite viewpoint also is introduced by mapping vehicles and stations as two sets of vertices, and a vehicle can be connected to all stations it passes through. This method is useful in analyzing cooperation transport networks (Ferber, Holovatch, Holovatch, & Palchykov, 2009; Seaton & Hackett, 2004). With such a representation, the underlying physical infrastructure and traffic flows are studied separately, while both routes and stations are considered as nodes (Latora & Marchiori, 2002; Wang, Hu, Wu, & Niu, 2009). Derrible and Kennedy (2009) recently introduced another approach by considering several types of vertices and planarity. More details can be found in their series of studies (Derrible & Kennedy, 2010a, 2010b).

3. Basic Measures

3.1 *Node Centrality*

Centrality is a fundamental concept for network topological analysis. It was first proposed in sociology (Freeman, 1977, 1979) and quickly adopted by a wide range of other disciplines.

3.1.1 *Degree.* Node degree is the most straightforward index to quantify the individual centrality. It is believed that the most important node must be the most active one, so degree indicates the number of edges connected to a node, which is defined based on an adjacent matrix as

$$k_i = \sum_{j \in V} a_{ij}, \tag{1}$$

V represents the neighbor set of node i, and a_{ij} is the entry value in the adjacent matrix. In essence, node degree equals the connectivity in space syntax. If a directed network is considered, then the degree can be extended to in-degree and out-degree, which respectively calculate the number of links ending in or starting from the node. They are written as

$$k^i_{in} = \sum_{j \in V} a_{ji}, \tag{2}$$

$$k^i_{\text{out}} = \sum_{j \in V} a_{\text{ij}}. \tag{3}$$

The well-known scale-free network is based on the degree definitions. If the degree of a graph follows a power-law distribution — that is, $p(k) \propto 1/k^{\mu}$ — then it can be generalized as scale-free (Barabási & Albert, 1999). It indicates that there is a huge heterogeneity in the network; a few nodes are highly connected hubs, whereas most of the rest are very poorly connected. This pattern imposes important consequences for dynamic processes on transport networks; therefore, it has attracted much attention since it was proposed.

3.1.2 *Betweenness centrality.* Betweenness quantifies the level of intermediate importance of a node in the interaction between other nodes (Freeman, 1977, 1979) (Figure 4(1)). The node betweenness can be defined as

$$C_i^B = \sum_{j \neq g} \frac{n_{jg}(i)}{n_{jg}}, \tag{4}$$

where $n_{jg}(i)$ is the number of the shortest paths between node j and g, which are passing through node i, while n_{jg} is the number of all shortest paths between them. Analogically, edge betweenness can be obtained, which is useful to detect the community structure of networks (Scellato et al., 2006). It is worth noting that in real transportation systems, these nodes with a high betweenness would impose critical impacts on network security (Barthelemy, 2004)

3.1.3 *Closeness centrality.* As the name shows, closeness centrality calculates how far it is from a given node to all other nodes in a network (Figure 4(2)).

$$C_i^C = \frac{1}{\sum_{j \neq i} d_{ij}}. \tag{5}$$

Here, d_{ij} is the shortest path length between i and j. For a non-valued graph, it becomes the geodesic distance. However, it works only in a connected graph. When the distance between two nodes equals 0, the formula would become infinite.

3.1.4 *Straightness centrality.* Straightness centrality is an important measure for navigating transport networks. It introduces the spatial distance and presents to what extent the shortest path between node pairs in the network deviates

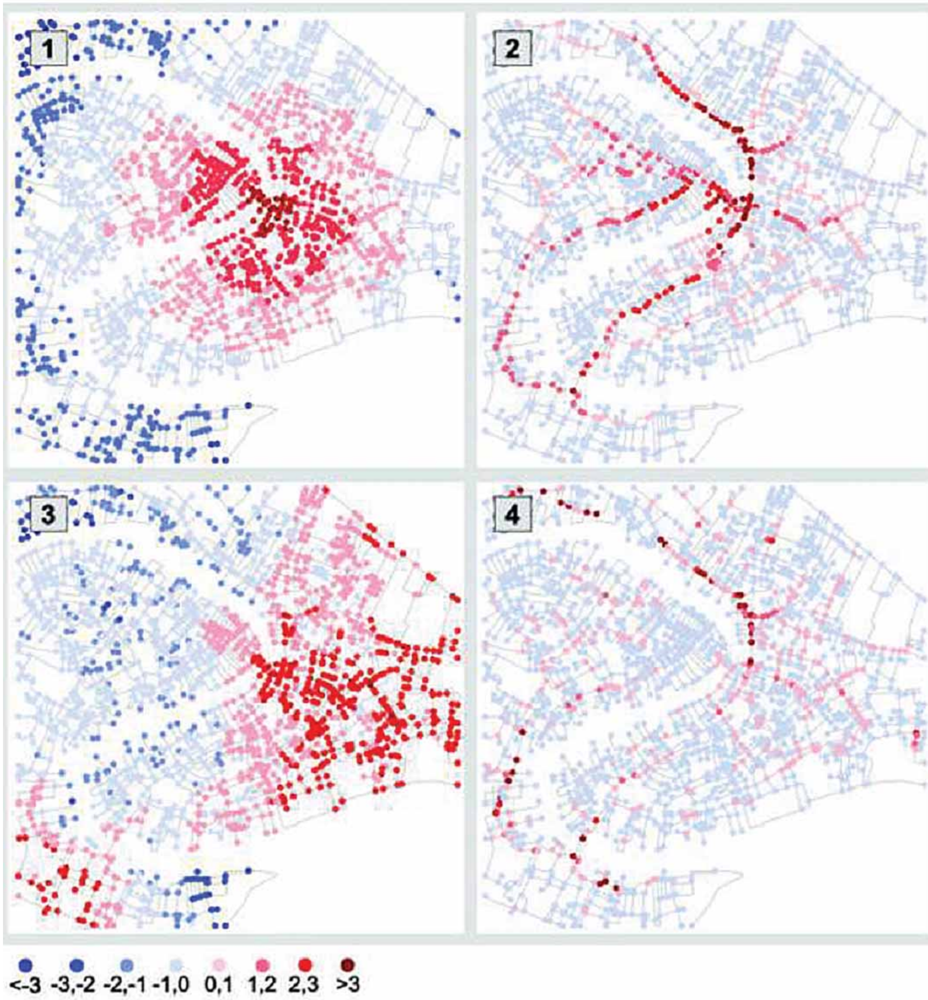


Figure 4. Four centrality measures (1) closeness, (2) betweenness, (3) straightness and (4) information centrality in the primal representation of Venice. *Source*: Crucitti et al. (2006).

from their geographical distance (Crucitti et al., 2006; Porta et al., 2009) (Figure 4(3)).

$$C_i^S = \sum_{j \neq i} \frac{d_{ij}^{\mathrm{Eucl}}}{d_{ij}}, \tag{6}$$

where d_{ij}^{Eucl} represents the Euclidean distance between i and j, and d_{ij} is the shortest path length in the network.

3.1.5 *Weight and Strength*. Most real networks present a high degree of heterogeneity in the intensity of communications (Boccaletti, Latora, Moreno, Chavez, & Hwang, 2006). As a supplement, edge weight was proposed to measure the centrality of connections between node pairs in networks. For real transport systems, the weight can be the traffic flow, travel time, geographic distance, or the like, of the link. Therefore, the node strength s_i can be generalized in terms of the definition of node degree.

$$s_i = \sum_{j \in V} w_{ij}. \tag{7}$$

Similarly, V represents the neighbor set of node i, and w_{ij} describes the weight of the edge between i and j. For a directed network, in-strength and out-strength of a node respectively estimate the weights of edges that go in or depart from it. They can be calculated analogous to the in-degree and out-degree.

$$s^i_{in} = \sum_{j \in V} w_{ji}, \tag{8}$$

$$s^i_{in} = \sum_{j \in V} w_{ji}. \tag{9}$$

Weighted networks provide a good description and explanation for the rich dynamics observed in real transportation.

3.2 *Clustering*

3.2.1 *Clustering coefficient*. The clustering coefficient is used to quantify the degree of clustering of a graph. It is calculated as the probability that two neighbors of a node are likely to be connected themselves, which can be further defined as

$$c_i = \frac{2m_i}{k_i(k_i - 1)}, \tag{10}$$

where m_i indicates the number of edges between the first neighbors of node i, and k_i denotes the node degree of i. This definition considers only the immediate neighbors of nodes; sometimes it is also necessary to consider the n-neighborhood for navigation and optimization in transport systems (Jiang, 2004). To this aim, the n-clustering coefficient is proposed as

$$c_i^{(n)} = \frac{2m_i^{(n)}}{k_i^{(n)}(k_i^{(n)} - 1)}. \tag{11}$$

Then Equation (6) can be regarded as a special case of Equation (7) when $n = 1$.

Given a weighted network, the clustering coefficient should be rewritten as (Barrat, Barthelemy, & Vespignani, 2004)

$$c_i^w = \frac{1}{s_i(k_i - 1)} \sum_{j,h \in V} \frac{w_{ij} + w_{ih}}{2} a_{ij} a_{ih} a_{jh}. \tag{12}$$

Here, a_{ij} is the entry element in the adjacent matrix for node i and j. If $c_i^w > c_i$ holds, then these edges with larger weights in the graph are more likely to form interconnected triplets. This phenomenon is called rich-club. In the opposite, $c_i^w < c_i$ indicates these larger-weight edges are not included in interconnected triplets.

3.2.2 *Degree-degree correlation*. Another quantity used to probe the networks' local architecture is the average degree of nearest neighbors, $k_{nn}(i)$:

$$k_{nn}(i) = \frac{1}{k_i} \sum_{j \in V} k_j, \tag{13}$$

where k_i is the degree of node i. The degree correlation reflecting a node's connection preference can be probed. If high-degree nodes tend to link with each other, this tendency is regarded as assortativity; otherwise, disassortativity is detected.

3.3 *Global Network Measures*

3.3.1 *Characteristic path length*. Characteristic path length is a global property important to the communication in networks. It reflects the internal structure of a network by containing the internal separations of all node pairs.

$$L = \frac{1}{N(N-1)} \sum_{i \neq j} d_{ij}, \tag{14}$$

where N is the number of vertices, and d_{ij} indicates the distance between two arbitrary vertices in the network (Boccaletti et al., 2006). For a non-valued graph, the distance equals the geodesic length. However, it confronts the divergence problem for non-connected graphs.

In particular, a small-world network can be characterized by a big clustering coefficient and short average path length (Watts & Strogatz, 1998). That is to say, any randomly selected nodes can be reachable by a small number of hops despite the large size of the network. A small-world network bridges the random network, which presents a short path length, and the regular network with a high clustering extent. Therefore, it has been acknowledged as one of the most important properties of networks and has been verified by many empirical studies, including social, technological, information, and biological networks (Newman, 2003).

3.3.2 *Efficiency*. Latora and Marchiori (2001) proposed an index to measure the performance of the information exchange of networks by assuming that the communication efficiency inside a network depends on the shortest path length between node pairs. In this case, the efficiency is estimated as

$$E = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{d_{ij}}. \tag{15}$$

It is calculated from the inverse of path distance between nodes, therefore avoiding the meaningless definition in non-connected graphs. Furthermore, it can be divided into global efficiency when all nodes in the network are considered and local efficiency when taking the average of efficiency values of subgraphs.

However, for congested transport networks, passengers will select their routes in terms of real-time flows and cost; therefore, another efficiency measure was proposed in the context of network equilibrium (Nagurney & Qiang, 2007). Given the set of origin/destination (O/D) pairs R, including the N_{r} O/D routes in the network, and the real-time demand on an O/D route f_{r}, the efficiency is

$$E' = \frac{\sum_{r \in R} f_{\mathrm{r}}/\eta_{\mathrm{r}}}{N_{\mathrm{r}}}, \tag{16}$$

where η_{r} is the equilibrium disutility, representing the minimum equilibrium travel cost of the route. This measure does not count routes without associated demand and it can be extended in continuous time and discrete time versions (Nagurney & Qiang, 2008). It offers meaningful implications to evaluate the robustness of congested networks. On the other hand, it has a high requirement for evolutionary traffic data, so has not been discussed widely in empirical analysis.

3.3.3 *Information centrality.* Based on their network efficiency definition, Latora and Marchiori (2007) proposed information centrality, which also occurred in a series of subsequent analyses conducted by their research group (Crucitti et al., 2006; Porta et al., 2006a, 2006b). It is quantified by the relative difference of network efficiency caused by removing node i from graph g.

$$C_i^I = \frac{E(g) - E(g')}{E(g)}. \tag{17}$$

Here, g' corresponds to the network after deleting node i and its incident edges. Analogously, it also can be applied to a group of points or an edge (Scellato et al., 2006). Information centrality can help us distinguish those critical intersections or roads in transport systems at a global level, thus improving the response to emergency attacks (Figure 4(4)).

3.4 *Cell Properties*

3.4.1 *Cell and connectivity.* In planar networks, a cell is defined as a closed circuit that contains at least three vertices and three edges (Haggett & Chorley, 1969). In this case, for a planar graph including N vertices and E edges, Euler's formula gives the number of its cells F as

$$F = E - N + 1. \tag{18}$$

Actually, early in the 1960s, a series of indices for transport network analysis was developed (Garrisoon & Marble, 1962; Kansky, 1963). The alpha index (α), which is also called the meshedness coefficient, is a measure of the structure of cycles in planar graphs.

$$\alpha = \frac{F}{F_{\max}} = \frac{E - N + 1}{2N - 5}. \tag{19}$$

The value of α can vary from 0 to 1, corresponding to the tree structure to the complete graph.

The Beta index (β) is the most straightforward measure of the connectivity of a network, and a bigger value indicates a greater connectivity.

$$\beta = \frac{E}{N}. \tag{20}$$

The gamma index (γ) calculates the ratio of the actual number of edges to the number of all possible edges, which is given as

$$\gamma = \frac{E}{3(N-2)}. \tag{21}$$

The value of the gamma index is between 0 and 1, and 1 indicates a fully connected graph.

3.4.2 *Planarity*. Furthermore, researchers have also developed many other measures to reveal thoroughly topological structures of real planar networks. For example, Xie and Levinson (2007, 2009) proposed treeness and circuitness in terms of two different connection patterns of road networks. The treeness is defined as

$$\phi_{\mathrm{tree}} = \frac{L_{\mathrm{t}}}{L_{\mathrm{g}}}, \tag{22}$$

where L_{t} is the length of links in the part of the tree structure and L_{g} is the total length of all segments in the network. Then the circuitness is equal to $\phi_{\mathrm{circuit}} = 1 - \phi_{\mathrm{tree}}$. More detailed works about eta index, theta index, ringness, webness, beltness, etc. can be found in studies by Levinson (Levinson, 2012; Xie & Levinson, 2007, 2009, 2011).

Note that the topological measures we introduced in the above sections also can be extended for cell properties, such as cell degree, cell area, and so on (Barthelemy & Flammini, 2008; Chan, Donner, & Lämmer, 2011). Furthermore, considering the two-dimensional characteristics of transportation systems, the form factor, which describes the shape of cellular structures, is also useful. It is defined as the ratio of the area of a real cell to the area of its circumscribed circle,

$$\phi_{\mathrm{f}} = \frac{S}{S_{\mathrm{cir}}} = \frac{4S}{\pi l_{\max}^2}, \tag{23}$$

where S represents the real area of the given cell, and $l_{\max}$ is the maximum cell diameter. The form factor of cells in transport systems should be the consequence of interactions between urban land use, population density, economy inequality, and other important urban elements.

4. Empirical Analysis

4.1 *Railway Networks*

From this section, the focus will be shifted from theoretical foundations to empirical analyses. Railway systems are an important part of national infrastructure and play a critical role in inter-city commuting. However, a railway system usually covers a huge area and involves a large number of stations and routes, so that it is very difficult to collect high-quality research data. It is for this reason that there have not been many studies on real railway networks.

Sen et al. (2003) first explored the structure of the Indian railway network based on a complex network perspective. They construct the network in P-space by identifying stations as nodes and a train that stops at any two stations as links. Based on this, the small-world effect and an exponential degree distribution are detected. The same result is also observed from the Japanese railway network around Tokyo Metropolitan, based on the P-space representation (Majima, Katuhara, & Takadama, 2007).

In comparison, the Chinese railway network, a much larger data set including 3915 stations and 22,259 railways, exhibits power-law distributions on both degree and strength (Li & Cai, 2007), and a small-world property. Disassortativity is also detected when the correlations between different measures are explored (Figure 5(a)). From a totally different perspective, Guo and Cai (2008) extracted China's railway network in the L-space and considered the utilized efficiency of stations as a new weight measurement. Consequently, they concluded that China's railway network is an assortative scale-free network (Figure 5(b)). Furthermore, Wang et al. (2008) merged stations in the same city into a node according to the geographic coarse-graining and, therefore, got a G-space representation by considering cities as nodes and train routes between them as links. They fit the cumulative degree distribution of in-degree and out-degree by a shifted power law as follows:

$$p(x) = a(x+b)^{\alpha}. \tag{24}$$

It is worth noting that in the three works about the weighted Chinese railway network, they all detected the correlation between node strength and degree as a scaling relation (Table 1).

To separate the underlying infrastructure and surface traffic flows in railway systems, a multi-layer viewpoint is also introduced. Kurant and Thiran (2006) created two layers of undirected physical graph G^{ϕ} and the logic graph G^{λ} for three railway systems, and they found a very weak correlation between degree k^{ϕ} and real load l or the betweenness b^{ϕ} and l. In the studies of Wang, Zhou, Shi, Wang, He (2009), stations and trains are considered separate layers, and a train is connected to all stations where it stops. Comparatively, Zou, Zhou, Liu, Xu, and He (2010) take the L-space and P-space representations of China's railway system as two layers. They insist that the two layers reflect different aspects of transportation systems and that combining them is helpful to understand the "network of networks". Finally, they conclude that degree or clustering coefficient in two layers both display high correlations.

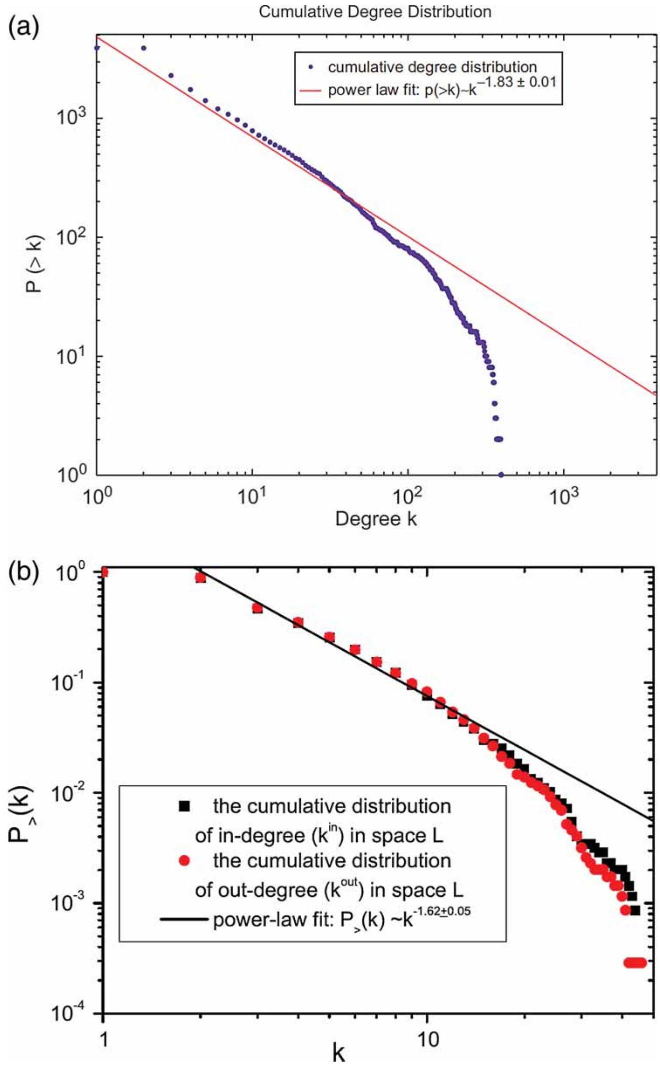


Figure 5. Degree distributions of China railway network respectively in (a) P-space (Li & Cai, 2007) and (b) L-space (Guo & Cai, 2008).

4.2 *Urban Public Transport Networks*

4.2.1 *Centrality.* In general, the scale-free structure is very common for urban public transport systems in the space of L (Ferber et al., 2007), but almost ruled out for the space of P. The reason for this phenomenon is that connections between stations are highly correlated in L-space, whereas the routes are linked nearly randomly in P-space. Only Ferber et al. (2007, 2009) reported three exceptions — London, Paris, and Los Angeles — of which the routes are evolved in a correlated way during their evolution process.

Betweenness could be considered another indication of the topological centrality for transport systems. Researchers (Sienkiewicz & Holyst, 2005) compared the betweenness of 22 Polish public transport networks in different spaces. The scaling relation between betweenness and degree can be detected in L-space,

Table 1. A summary of empirical studies on railway networks

Name	N	D	<C>	L	P(k)	Space
Indian railway network (Sen et al., 2003)	587	5	0.69	2.16	Exponential	P
China railway network (Li & Cai, 2007)	3915	5	0.835	3.5	Scale-free	P
Japan railway network (Majima et al., 2007)	371	–	–	–	Scale-free	L
		6	0.92	2.6	Exponential	P
China railway network (Wang et al., 2008)	3110	–	0.2798	11.18	Shifted power law	L
	298	–	0.3778	5.21	Shifted power law	G
China railway network (Guo & Cai, 2008)	3467	–	–	–	Scale-free	L
Polish railway network (Kurant & Thiran, 2006)	1533	76	0.1681	19	Unclear	L
		4	0.6829	2.3	Exponential	P
Swiss railway network (Kurant & Thiran, 2006)	1613	61	0.0949	16.3	Unclear	L
		8	0.9095	3.6	Exponential	P
Central Europe railway network (Kurant & Thiran, 2006)	4853	48	0.3401	12.6	Unclear	L
		8	0.7347	3.7	Exponential	P

Notes: N denotes the number of nodes; d is the diameter; $<C>$ is the average clustering coefficient while L is the characteristic path length of the network. $P(k)$ describes the node degree distribution and "Space" indicates in which kind of space the network is represented. The same definitions apply to Tables 2–4.

but only for larger degrees in P-space. From a larger scale, betweenness centrality was discussed by studying 28 worldwide metro transit systems (Derrible, 2012); a process of democratization was detected, which means the centrality becomes more evenly distributed with network size.

4.2.2 *Small-world property*. From Table 2, we can summarize that the small-world property is more visible in P-space than in L-space for all urban public transport systems. It is reasonable if we take different representation approaches explained in Section 2.3 into considerations. It is also consistent with the results of Roth, Kang, Batty, and Barthelemy (2012), in which they studied the 14 largest metro systems in the world. But they take it a further step forward by mixing spatial and topological properties in an evolution perspective.

A rich-club phenomenon, which means edges with a higher weight are more likely to be clustered, can be detected by comparing the topological clustering coefficient and the weighted clustering coefficient (Montis, Barthelemy, Chessa, & Vespignani, 2007; Soh et al., 2010). That is to say, if the weighted clustering coefficient is larger than the topological one, the interconnected groups in the traffic network are formed more likely by routes with high traffics, and vice versa.

4.2.3 *Weight and strength*. In addition to the above-mentioned topological characteristics, dynamic traffic flows usually are considered the weight between two stations in weighted public transport systems. Soh et al. (2010) even noticed the divergences of traffic flows between weekdays and the weekend by adopting a more detailed database.

In L-space, Lee, Jung, Park, and Chio (2008) found that distributions of the weight and strength of Seoul subway system exhibit totally different trends. The strength distribution follows a log-normal trend with a peak. They discovered that the number of passengers arriving at or departing from single stations has a characteristic size, whereas the passenger flows themselves do not (Soh et al.,

Table 2. A summary of empirical studies on public transport networks

Name	N	$<k>$	$<C>$	L	$P(k)$	Space
Boston subway network (Latora & Marchiori, 2002)	124	–	–	15.55	–	L
Boston subway network (Seaton & Hackett, 2004)	124	27.60	0.927	1.811	–	P
Vienna subway network (Seaton & Hackett, 2004)	76	20.66	0.945	1.865	–	P
22 Polish public transport network (Sienkiewicz & Holyst, 2005)	152–2811	2.48–3.08	0.032–0.161	6.83–21.52	Power-law	L
		32.94–90.93	0.682–0.847	1.71–2.90	Exponential	P
3 Chinese public transport networks (Lu & Shi, 2007)	192–495	2.26–3.06	0.01–0.16	10.41–11.38	No universal pattern	L
		23.59–45.54	0.73–0.89	2.00–2.18	No universal pattern	P
4 Chinese bus transport networks (Chen, Li, & He, 2007)	827–4374	44.46–84.01	0.73–0.78	2.53–2.89	Exponential	P
3 Chinese bus-transport networks (Xu et al., 2007b)	1150–3938	2.88–3.22	0.09–0.21	7.13–12.56	Power-law	L
		41.06–94.19	0.73–0.78	2.54–2.66	Exponential	P
14 urban public transport networks (Ferber et al., 2007)	1544–46244	–	0.01–0.13	7.08–85.84	Heavy-tail	L
		–	0.69–0.95	2.26–4.79	Heavy-tail	P
Seoul subway system (Lee et al., 2008)	380	–	0.0064	20	–	L
Wuhan public transport network (Wang et al., 2009)	1229	82.31	0.76	1.94	Exponential	P
Qingdao public transport network (Liu, Li, Shao, & Sun, 2009)	864	80	0.433	2.135	Exponential	P
Singapore public transportation system (Soh et al., 2010)	4130/4139	103.172/87.03	0.562/0.534	2.54/2.57	Exponential	P

Note: $<k>$ is the average degree of the network.

Table 3. A summary of empirical studies on aviation networks

Name	N	E	$<C>$	L	$P(k)$
US flight network (Chi et al., 2003)	215	116 725	0.618	2.403	Two-regime power-law
World-wide airport network (Guimera & Amaral, 2004, 2005)	3883	531 574	0.62	4.4	Truncated power-law
China airport network (Li & Cai, 2004)	128	1165	0.733	2.067	Two-regime power law
World-wide airport network (Barrat et al., 2004)	3880	18 810	–	4.37	Scale-free with an exponential cut-off
Italian airport network (Guida & Maria, 2007)	42	310	0.07–0.10	1.98–2.14	Two-regime power-law
Indian airport network (Bagler, 2008)	79	228	0.733	2.067	Scale-free
US air transportation network (Xu & Harriss, 2008)	272	6566	0.73–0.78	1.84–1.93	Truncated power-law
China airport network (Liu et al., 2009)	121	1378	0.748	2.263	Two-regime power-law
US airport network (Paleari et al., 2010)	657	5488	0.45	3.38	Two-regime power-law
Europe airport network (Paleari et al., 2010)	467	5544	0.38	2.80	Two-regime power-law
China airport network (Paleari et al., 2010)	144	1329	0.49	2.34	Two-regime power-law
Indian airport network (Sapre & Parekh, 2011)	84	13 909	0.645	2.17	Scale-free
China air transport network (Wang et al., 2011)	144	1018	0.69	2.23	Exponential
China's aviation network (Lin, 2012)	140	1044	0.737	2.108	Two-regime power-law

Note: E denotes the number of edges in the network.

2010). In contrast, if each route is considered as a node, two nodes have an intersection when the two routes intersect each other; then the weight fits a power-law

Table 4. A summary of empirical studies on maritime networks

Name	N	$<k>$	$<C>$	L	$<w>$	$P(k)$	Space
China's ship-transport network (Xu et al., 2007b)	162	3.11	0.54	5.86	25.06	Scale-free	L
	162	8.27	0.83	3.87	47.01	Truncated power-law	P
Worldwide maritime transportation network (Hu & Zhu, 2009)	878	9.04	0.40	3.60	–	Truncated power-law	L
	878	28.44	0.71	2.66	–	Exponential	P
Global cargo-ship network (Kaluza et al., 2010)	951	76.4	0.49	2.5	–	Right-skewed	L
Container ships network (Kaluza et al., 2010)	378	32.4	0,52	2.76	–	Right-skewed	L
Bulk dry carriers network (Kaluza et al., 2010)	616	44.6	0.43	2.57	–	Right-skewed	L
Oil tankers network (Kaluza et al., 2010)	505	33.3	0.44	2.74	–	Right-skewed	L

Note: $<k>$is the average degree; $<w>$ represents the average weight of the network.

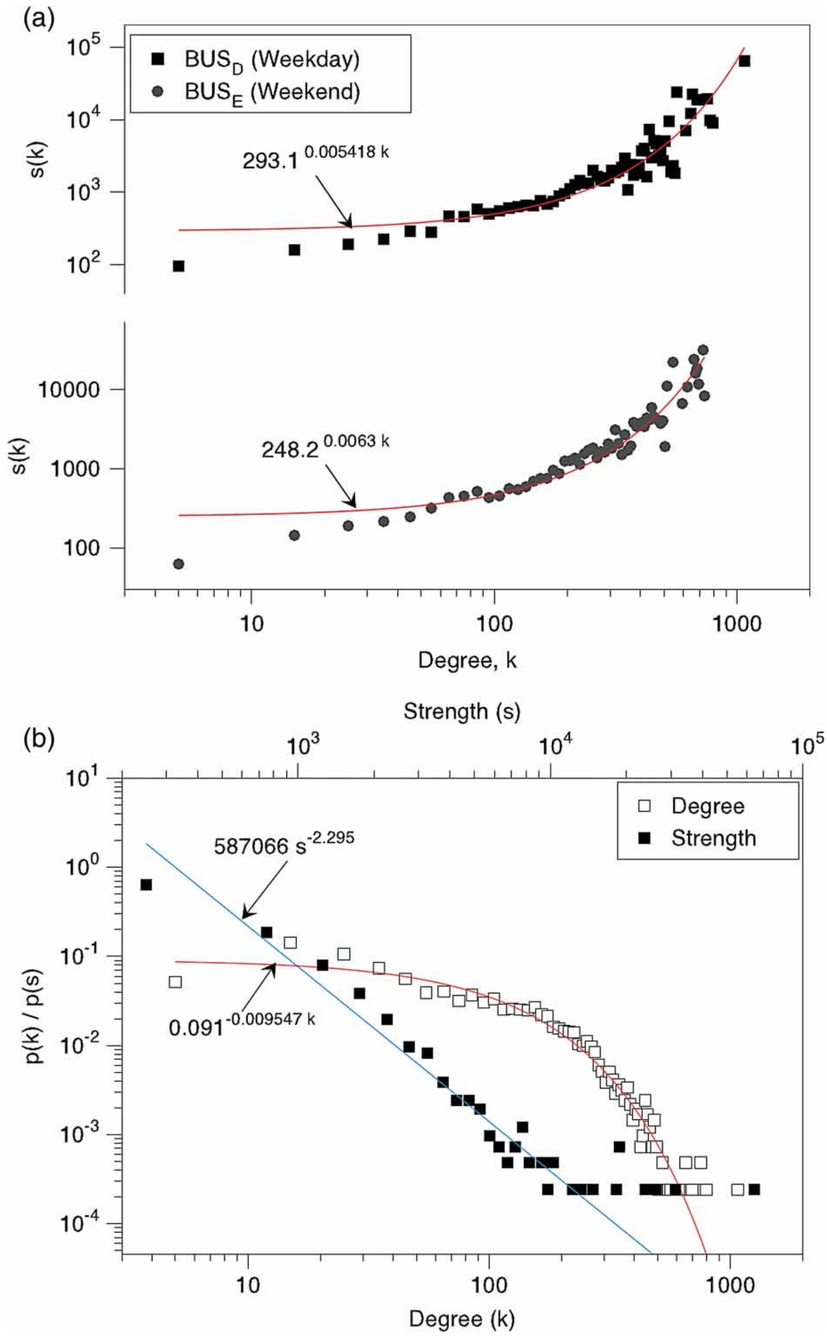


Figure 6. (a) The scaling correlation between the degree k and strength $s(k)$ and (b) probability distributions of degree $p(k)$ and strength $p(s)$, for Singapore bus transportation network in P-space. *Source*: Soh et al. (2010).

distribution, whereas the strength might display two-regime power laws with different exponents, which is also called the double Pareto law (Xu et al., 2007b).

Although there is a positive correlation between the degree and the strength (Figure 6(a)), they, respectively, possess different distribution patterns

(Figure 6(b)). The strength distribution in P-space always conforms to a power-law distribution, in contrast to the exponential trend of degree. Therefore, although the connection between nodes may be constructed randomly, the magnitudes of the travel flows are not.

4.2.4 *Efficiency.* From the initial stage, researchers have realized that the ultimate aim of studying public transport systems is to improve the efficiency of travels and optimize underlying structures. The Boston subway system is considered to achieve 63% of the global efficiency of the ideal subway, whereas the local efficiency is only 3%. This result implies that the network has a low degree of fault tolerance (Latora & Marchiori, 2002). However, Lee et al. (2008) held that network efficiency should be calculated based on physical distance rather than on network distance because realistic networks could not have all-to-all connections between stations or stops. In this case, the number of real links should be far fewer than the full-connected value of $N(N-1)$.

4.3 *Aviation Networks*

4.3.1 *Small-world effects*. Composed of airports and routes, aviation networks are relatively simple examples of transportation systems because of their non-planar properties. Many real airport systems present distinct topological properties, although there are some common architectures. Either for national aviation systems, such as Italian (Guida & Maria, 2007), Indian (Bagler, 2008), Chinese (Li & Cai, 2004; Lin, 2012; Wang, Mo, Wang, & Jin, 2011), USA (Paleari, Redondi, & Malighetti, 2010), and so on, or for some regional (Paleari et al., 2010; Sapre & Parekh, 2011) and worldwide aviation systems (Guimerà & Amaral, 2004, 2005), the small-world structure is detected (Table 3), although different results emerged as to the degree distributions. To some extent, the small-world structure is acknowledged to be efficient for aviation networks (Chi et al., 2003; Liu & Zhou, 2007).

4.3.2 *Degree distribution and correlation*. In aviation networks, the highly connected nodes are not necessarily the most central ones in traffic because of the multi-community structure of networks, resulting in some cities with high degrees presenting low betweenness centrality (Guida & Maria, 2007; Guimerà, Mossa, Turtschi, & Amaral, 2005). Most airport networks do not display a strict power-law degree distribution; instead, the truncated or two-regime power-laws (double Pareto law) are found (Table 3). Clauset, Shalizi, and Newman (2009) explained in detail about the detection and characterization of power-law distributions in empirical analysis. Readers who are interested in the two-regime power law could refer Reed's work about the Pareto law (Reed, 2003; Reed & Jorgensen, 2004).

If an aviation network is considered as directed, there will be a strong correlation between its in-degree and out-degree (Li & Cai, 2004; Rocha, 2009). It is natural for aviation networks because each airport should maintain the balance of its traffic flow. Besides, the heterogeneous degree correlations also are discussed widely. For the worldwide aviation network, assortativity emerges only for cities whose degree is smaller than 10 (Barrat et al., 2004). Comparatively, for some national airport networks, disassortativity is detected after certain threshold values of degree (Li & Cai, 2004; Wang et al., 2011).

4.3.3 *Efficiency*. In aviation networks, the path length represents the number of transfer times a passenger needs to take from origin to destination. Li and Cai (2004) defined the diameter and efficiency of a sub-cluster and proved that a shorter diameter indicates a more efficient network (Figure 7). In the Indian airport network, around 99% of paths are reachable by changing a maximum of two flights (Bagler, 2008). Besides the path length, travel time also needs to be considered to evaluate the performance of aviation networks. Through a comparative study of accessibility and shortest travel time, which includes the waiting time at intermediate airports, China's network was found to provide the highest speeds, while the US network is considered as the most coordinated, if we consider indirect flights (Paleari et al, 2010; Wang et al., 2011). Unfortunately, because it is difficult to collect data on the waiting time between different flights, there is not much progress on this topic.

4.3.4 *Evolution of aviation networks*. It is necessary and interesting to study the evolution of the topological structures in terms of the time scale; unfortunately, only a few works exist in this direction because there are little data available. The Brazilian airport network experienced overall growth during the time period of 1995–2006, when some routes shrank (Rocha, 2009). However, there was no clear fluctuation for the clustering coefficient or the average path length during those years. Similar results also were concluded from the Chinese airport network (Zhang, Cao, Du, & Cai, 2010).

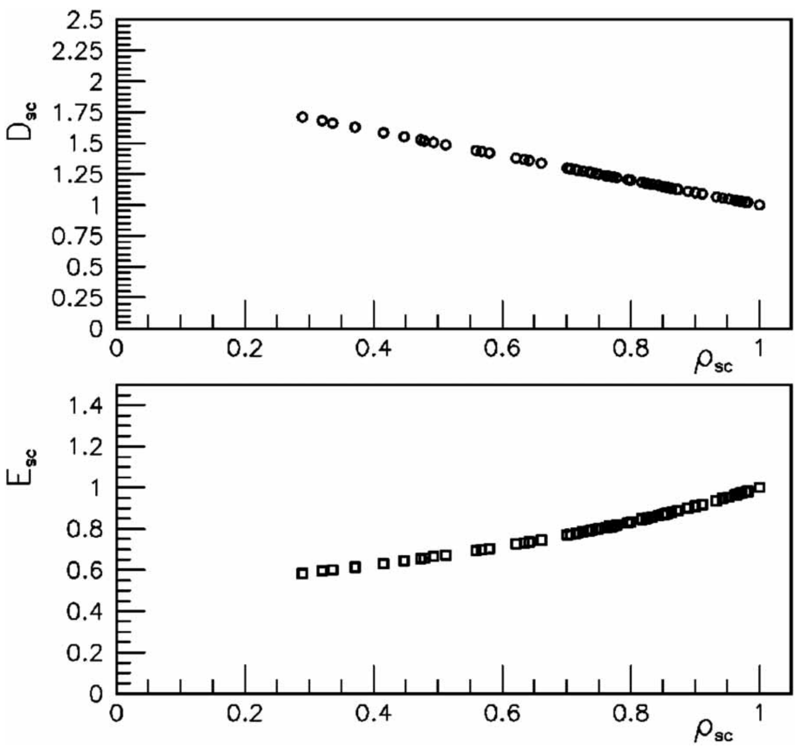


Figure 7. The overall relation between diameter D_{sc} and efficiency E_{sc} of sub-clusters of the airport network of China. ρ_{dc} is the density of connectivity of a sub-cluster. *Source*: Li and Cai (2004).

In addition, the evolution of a country's aviation network could reflect the evolution of its important cities to some extent. Ma and Timerlake (2008) studied some centrality measures of China's inter-city airport network to identify and explain the change process of leading cities between 1990 and 2005 in China.

4.4 *Maritime Transportation Networks*

Another important part of the transport systems is ship-transport networks, which carry 90% of the world's trade (Kaluza, Kolzsch, Gastner, & Blasius, 2010). Strictly speaking, maritime transportation networks belong to the class of non-planar graphs; however, they are still constrained by geographical embedding. Xu, Hu, and Liu (2007a) first analyzed China's ship-transport network, which comprises 42 seaports, 120 river ports, and a large number of passenger liners. They found the disassortative mixing and a power-law degree distribution in L-space, but assortative mixing and truncated power law in P-space. However, the small-world property is applicable in both representations, although it is much more visible in P-space. Besides, a hierarchical structure also is detected by confirming a scaling relation between the clustering coefficient and node degree of the network. If considering the traffic flow between two ports as edge weight, the weighted ship-transport network is symmetric, but the weight distribution indicates a high level of heterogeneity, which is similar to aviation networks (Guimerà & Amaral, 2004; Li & Cai, 2004).

Subsequently, a global perspective has been adopted. Hu and Zhu (2009) probed the worldwide maritime transport network with links being container liners, and found a small-world network with power-law behavior (Table 4). It is not surprising that there are strong correlations between degree and degree, strength and degree, or betweenness and degree, no matter whether they were in the space L or P (Figure 8). Besides, they also calculated the weighted clustering coefficient and the normalized rich-club coefficient of the network. Like the railway networks in Table 1, the clustering coefficient in P-space is much larger than the one in L-space. What is interesting is that these weighted clustering coefficients are also larger than unweighted ones in both spaces, implying high traffic edges between interconnected vertices.

Shortly after this work, Kaluza et al. (2010) enlarged the dataset to take into account the bulk dry carriers, container ships, and oil tankers of global cargo ships. They did not care about different representations but compared movement patterns of different ship types and group ports into sub-clusters by optimizing the modularity.

Majima et al. (2007) considered the combination of railway, subway, and waterbus networks in Tokyo. They proved that local efficiency has increased with the combination. More importantly, the robustness and centrality around stations to which the waterbus connects have increased.

4.5 *Urban Street Networks*

4.5.1 *Self-organized and planned evolution patterns*. Cities could be developed in distinct ways, including self-organized (bottom-up) and planned (top-down) evolutions. Differentiating and comparing the resultant street systems under different growing mechanisms is of utmost importance for city planners and policy-makers (Jiang, Duan, Lu, Yang, & Zhao, 2013). Scellato et al. (2006) calcu-

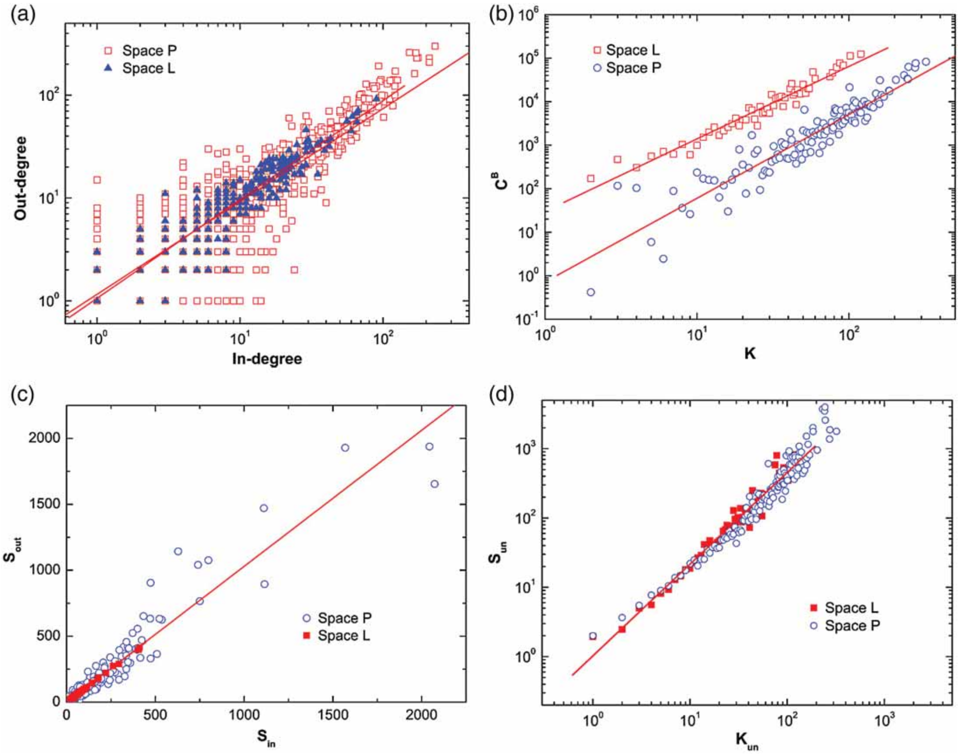


Figure 8. Correlation between (a) in-degree and out-degree, (b) betweenness C^B and degree k, (c) in-strength S_{in} and out-strength S_{out}, (d) undirected strength S_{un} and undirected degree k_{un} in L-space and P-space for the worldwide maritime network. *Source*: Hu and Zhu (2009).

lated the edge betweenness and node information centrality of urban street systems for Bologna and San Francisco, which are respectively representative of self-organized and planned cities. They built the betweenness-based and information-based maximum centrality spanning trees and the minimum length spanning trees for two cities. Results showed that the shortest length performs more effectively for simulating planned urban fabrics than self-organized ones, whereas gathering people or resources is typical of the latter patterns. In terms of some larger databases, the universal power law could be regarded as a signature of self-organized cities in most cases, although some exceptions still exist (Crucitti et al., 2006; Scellato et al., 2006), while the cities under strict planning are supposed to present an exponential (Crucitti et al., 2006) or two-regime centrality distribution (Jiang, 2007).

4.5.2 *Heterogeneity of urban street networks*. Almost in the same period, some researchers noticed that in an urban street network, only a very few streets intersect with a large number of other streets, while most streets possess very few intersections. Taking Venezia as an example; just 4 out of 783 streets occupy the upper 20% interval of the degree range of values, while the 674 streets fall into the lower 20% (Porta et al., 2006a). Jiang (2009) even believed that less than 1% of streets, out of the upper 20%, could form a backbone of the total street system. Lammer, Gehlsen, and Helbing (2006) revealed an extreme concentration of traffic on most important intersections, and introduced the Gini index to quantify the betweenness centrality distribution of road networks. In Figure 9, the Gini index of Dresden equals 0.870, signifying a high inequality. Almost 80% of the traffic volume is concentrated on fewer than 10% of the streets. It is worth mentioning that such a broad heterogeneity may lead to traffic congestions on the systems.

4.5.3 *Planarity and cellular structures*. Urban street networks, as the most fundamental part of infrastructures in cities, present strict planarity. If we build a street network in a primal approach, it would show no or little clustering because roads tend to be parallel more than forming triangles.

The meshedness values of real systems are considerably variable, from 0.009 for the El Agustino district of Peru (Buhl et al., 2006) to 0.348 for New York (Cardillo, Scellato, Latora, & Porta, 2006). Undoubtedly, the higher a city's meshedness coefficient is, the more complexity its urban form presents. It also coincides with the results of studies based on the 50 largest metropolitan areas in the USA (Levinson, 2012), that α, β, γ indexes increase with the city size. In comparison, the circuitness is relatively stable, while the treeness decreases with the city size. It is easy to understand this conclusion. The bigger a city is, the less dendritic it looks. With economic and industrial developments, a large city or a large metropolitan will evolve into a polycentric structure in the real world; it is also a critical issue that needs to be considered during the transport planning process.

Moreover, the length and the area of cells are also interesting to researchers. The former refers to the number of edges or vertices of a cell. Masucci et al. (2009) found that in the London street network, cells of length 4 and 5 are more numerous than ones of length 3. This conclusion perfectly coincides with the low clustering in street networks. Studies on area distribution of cells show that power law is

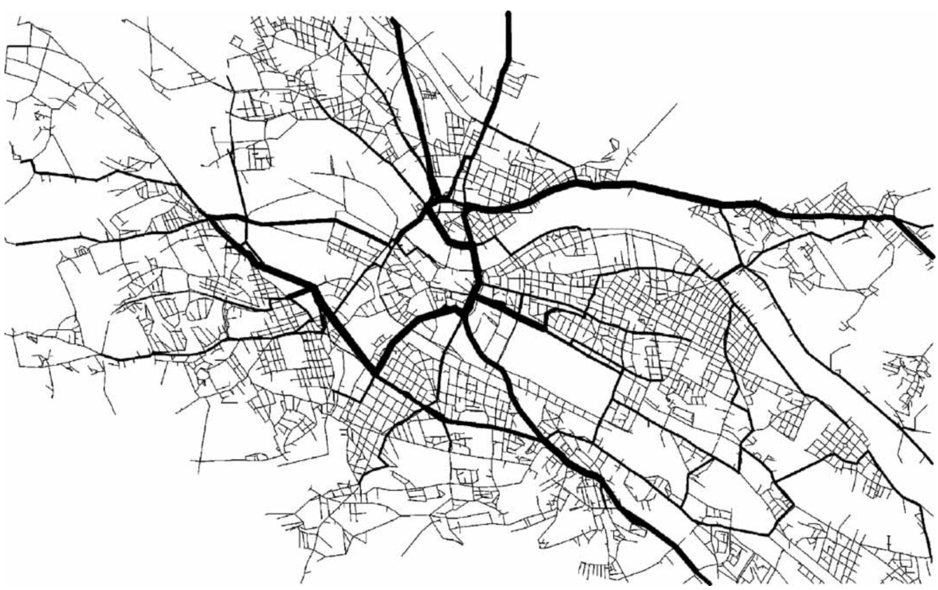


Figure 9. Heterogeneity of the street networks of Dresden, with the width of streets indicating the corresponding betweenness values. *Source*: Lammer et al. (2006).

universal for real systems and random graphs. (Lammer et al., 2006; Masucci et al., 2009).

Analogical with node degree, the neighborhood degree (k_c) of a cell for planar street networks, which indicated the number of adjacent cells, is also investigated. It is found that most of the cells have three to six neighbors, and the degree of four is the most frequent. Then we can conjecture that the corresponding form factors of urban street networks, which estimate the diversity of each cell shape, might vary from 0.4 to 0.6 based on the following deduction:

$$\phi_{\mathrm{f}}^{\min}(k_{\mathrm{c}}=3)=\frac{3\sqrt{3}}{4\pi}\approx 0.41, \tag{25}$$

$$\phi_{\mathrm{f}}^{\max}(k_{\mathrm{c}}=4)=\frac{8}{4\pi}\approx 0.64. \tag{26}$$

Indeed, form factors of urban street networks from the 20 largest Germany cities fall mostly in the range from 0.3 to 0.6 (Chan et al., 2011; Lammer et al., 2006), in good agreement with the above theoretical conjecture. Recently, Chan et al. (2011) have made a big step forward in understanding the interrelations between cell areas, cell degrees, and form factors by adopting the same database of Lammer et al. (2006).

5. Summary and Conclusions

This paper has reviewed research works that were produced during the last decade on the topological structure of transport networks from a complex network perspective. Generally, the representations for non-planar networks are much more straightforward than those of planar networks. For planar networks, the primal representation maintains the geometric patterns and geographical properties of transportation systems, while the dual approach focuses on their functional structures. As for urban public transportation systems, slight differences between representation approaches in L-space and P-space to extract characteristics of stops, stations, and routes can cause distinct results on the same system.

With significant progress in complex network theory in the last decade, a huge number of measures have been presented to estimate the degree of importance of nodes, edges, and communities in networks. In this work, the authors classified measures by comparing them among different scales. For a single node or an edge, degree, betweenness, closeness, straightness, and strength centrality can reveal respectively distinct features of a real transport network. Besides, from a larger scale, the regional clustering and global measures are further discussed. The last part in this section is allocated to cellular structure.

Following the theoretical research, empirical analyses are definitely an indispensable supplement. Here we summarized a huge number of empirical studies from various subjects in terms of different kinds of transportation systems, including railway systems, urban public transportation networks, aviation networks, maritime networks, and urban street networks. Results show that the complex network theory discloses many more similar properties in real transport systems than anyone could expect, such as the widely present small-world effect and scale-free distribution. On the other hand, different functions, sizes, representation spaces, and geographical scales all impose impacts on the results. For example, the scale-free structure is very common for urban public transport systems in the L-space representation, but almost ruled out for those in the P-space. These comparisons and summarizations provide meaningful inspiration for planners on the transportation system design and optimization.

Complex network theory has undergone rapid developments in the last decade; much work lies ahead in this field. There is still a long way to go to apply all achievements in transport practice. First, exploring network cellular structures is of utmost importance to understanding the planarity of grounded transportation systems and coping with modern traffic dilemmas. More geographic constraints and urban attributes, such as population density distribution and land-use structures, should be considered for general contexts. Second, with the availability of GPS and various mobile devices, recognizing and tracking human mobility patterns is becoming a hot issue in many research areas. Undoubtedly, exploring the relationship between the underlying topological structure of transportation systems and human movements on them is much more challenging, demanding that transport researchers have a good capacity to handle gigantic amounts of data. Admittedly, many studies reviewed in this article are still in the theoretical stage. More efforts should be made to explore potential applications and meanings for practical transport planning, especially when different characteristics for different transport systems are considered. Much progress can be expected with more collaboration among different subject areas in the future.

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