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Integrated Exploration using Time-based Potential Rails

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Abstract— **Integrated exploration is the most complete task in mobile robotics, and corresponds to the union of mapping, localization and motion planning. A powerful integrated exploration solution must take into account decisions that improve the quality of the map construction, such as closing loops, at the same time that the environment is explored. Potential fields and boundary value problems (BVP) have been used with success in tasks of planning, localization and exploration, but not yet in integrated strategies. In this paper, we present an integrated exploration strategy using a time varying BVP-based exploration. Our strategy consists of creating potential rails that guide the robot to regions that are either unexplored or were visited a long time ago. We also apply local distortions in the potential field to generate a loop closure strategy. Experimental results demonstrate that our method improves the quality of the map construction, keeping the balance between revisiting and exploratory activities.**

I. Introduction

The ability of building the map of the environment is important for most of the tasks performed by autonomous mobile robots [1]. An integrated exploration technique must perform an efficient exploration of the environment while minimizing the final map degradation due to localization errors. The design of integrated exploration techniques must balance efficiency and map construction effectiveness.

By using an integrated exploration which re-enters known regions the robot reduces localization errors, while in an exploration strategy that only favors unknown regions, the probability of making the correct associations is reduced [2]. Furthermore, localization problems tend to increase as the robot moves, since odometry errors are cumulative. Thus, each map associated to a small traversed region is generally only locally consistent. It was shown that by going back to loops we can correct the local error. However, by continuously re-entering unknown regions, strategies based on probabilistic filtering will loose diversity in terms of global solutions due to resampling problems. The result is a trade-off between revisiting and the compromise with previously obtained solutions [2], [3].

In the same way, once the robot finishes exploration, the transition between map construction and map usage should be smooth. That is, we want the robot, e.g. a patrolling or exposition robot, to make use of the map as soon as possible. This also implies that once the first part of the job – i.e. integrated exploration – is finished, the robot must know where to go back in the environment to be effective in its job. Therefore, the robot must return to known areas, but giving priority to those which were not visited for the longest period of time.

Potential fields and boundary value problems (BVP) for the Laplace Equation were applied to exploration in the work of Prestes et al. [4], [5]. Prestes et al. [6] also developed a strategy to distort the potential field. This can change the robot movement while it is performing the gradient descent, without loosing the qualities of the Laplace Equation (e.g. the absence of local minima). BVP exploration has additional advantages, including: smooth movements; easy understanding and implementation; and use of local windows to improve the speed of convergence of the potential field. However, it is in essence a greedy exploration what is known to be ineffective for integrated exploration. Furthermore, once the whole map is explored the potential quickly converges to the potential of the walls. Thus, we can no longer use the potential field to move the robot again.

In this paper we present an integrated exploration in terms of a time varying BVP problem. We use an online constructed Voronoi Skeleton as a complimentary time varying boundary. The Skeleton cells keep track of time of visit information, guiding the potential field to unknown regions and also to those not visited for the longest time. The global skeleton information enables the use of a local potential window, which reduces the computational cost of BVP and maintains the effectiveness of the method. We propose the use of potential distortions [6] to direct the robot to close loops (making turns). Moreover, our method never converges to a flattened potential field – even if there are no more unexplored boundary cells – since there will always be an "oldest cell". When the integrated exploration is finished, the robot is immediately guided to those regions not visited for the longest time in an allegedly perpetual cycle – what can be interesting for patrol and exposition robots.

This paper is presented as follows. Section II presents the related work. Section III introduces essential background information on potential fields. Sections IV and V discuss our time-based exploration and our loop closure strategies, respectively. Section VI shows the experiments where we compare two strategies using potential fields: the greedy strategy and the proposed method. Finally, in Section VII we discuss our method, draw our conclusions, and present the future work.

II. Related Work

In this Section we review some relevant work in the context of our work.

Yamauchi's [7] introductory greedy strategy explores the environment by continuously driving the robot towards the

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closest frontier using laser and sonar sensors along with a reactive obstacle avoidance technique. Makarenko et al. [8] does it by using a weighted function which considers different factors such as map construction coverage, accuracy, and exploration speed. The localization quality is measured applying a Kalman filter at a few selected positions during planning. The best positions are used as points of interest.

Prestes et al. [4], [6] reformulate the exploration problem as a Dirichlet BVP for the Laplace equation. The unexplored regions have low potential, while the walls have high potential. The numerical solution is obtained through the finite-differences method [6]. The robot is always attracted to the larger or closer frontiers. When there is no unexplored frontiers left, the solution converges to the potential of the walls and the robot stops. The algorithm is fairly simple, however it has a few drawbacks: it uses a greedy strategy, which is not appropriate when there are SLAM errors; it is not possible to control the revisit of explored regions; and it can be expensive for large maps. Later [5], the method was improved by using a local map with size varying according to the smallest laser reading to reduce the computational cost of the method. Unfortunately, this is only possible when there are nearby unexplored frontiers inside the local window, otherwise the global map is used instead. Shade and Newman [9] adapt BVP to 3-dimensional (3D) environments. They developed a parameter free strategy using BVP in an octree map to reduce the computational cost of the method. The use of irregular grids makes it run in real-time for 2D applications such as ours, but unfortunately not enough for a real-time application in 3D environments.

Stachniss et al. [2] strategy merges a grid-based Rao-Blackwellized Particle Filter (RBPF) and an active loop-closing technique, using a topological map. In another work, they devised a heuristic that considers information gain (i.e. changes in the particles entropy) in loop closures, revisits and exploration of unknown regions. The idea is to keep the robot inside a loop only while the diversity is not degraded [3]. These strategies enable behavior switching using a few parameters but they are environment dependent and unstable during the loop closure. Freda et al. [10] adapts a Sensor-based Random Tree (SRT) for the integrated exploration problem. They also use information gain and localizability to evaluate candidate configurations for exploration. The strategy naturally merges sensor information and map correction procedures. Amigoni [1] presents an exploration test framework. He compares different exploration strategies, evaluating different parameters. Exploration strategies which balance utility and cost show better results than those using only utility. Later, Amigoni et al. [11] blends information gain and traveling cost to construct maps, using point maps as safe zones for the robot. Blanco et al. [12] state that loop closing is key to reduce the robot path uncertainty and to improve the resulting maps. They rely on the entropy of the expected map of the RBPF – an integration of the map hypotheses of all particles – to detect opportunities to close loops and also to guide the robot during the exploration. Juliá et al. [13] present a hybrid reactive/deliberative method to multi-robot integrated exploration. They rely on the concepts of expected safe zone and gateway cell to avoid the presence of local minima and to decide between exploring the current zone or changing to other zones. They also take into account localizability measures to decide if it is necessary to return to previously explored areas. Carlone et al. [14] propose the use of the Kullback-Leibler divergence to analyze the particle-based SLAM approximation. The divergence is adapted to the notion of expected information gain, which enables the decision between exploration and revisiting actions. The technique is compared to naive gain, entropy-based gain, and the expected map, showing better results to detect and close loops. In all these works, there is an active change between exploratory and revisiting behaviors.

In our approach, we take advantage of the theory behind harmonic fields and the recently developed strategy to distort potentials [6] to force loop closing by favoring turns on bifurcations, which is a powerful strategy when the environment is not known a priori. This is done without the explicit need to switch the robot behavior. The equations we employ and the underlying boundary conditions, along with the preferences in front of the robot, naturally mimic these behaviors.

III. Exploration based on BVP Applying Local Distortions

The exploration based on Boundary Value Problem (BVP) [6] uses the gradient descent of a potential field to smoothly guide the robot to a goal position in the environment. Local distortions are applied in the field, to increase the preferences for specific regions. This potential information is computed from the numeric solution of

$$\nabla^2 \, p(\mathbf{r}) = \epsilon(\mathbf{r}) \left(\left| \frac{\partial p(\mathbf{r})}{\partial x} \right| + \left| \frac{\partial p(\mathbf{r})}{\partial y} \right| \right) \tag{1}$$

with Dirichlet boundary conditions, where $p(\mathbf{r})$ is the potential at position $\mathbf{r} \in \Re^2$ and $\epsilon \in \Re$ is the intensity of the perturbation produced in the region defined by r.

To compute Eq. 1 numerically, the environment is divided into a regular grid, where each cell stores its potential value and is also associated to a square region. By using Dirichlet boundary conditions, the cells representing obstacles have a fixed high potential value (p=1), whereas the cells representing the unexplored areas (goals or sinks) have low potential value (p=0). The remaining cells have their potential computed iteratively through the finite difference method [6].

The intensity of a perturbation applied over the potential field is defined by ϵ. If $\epsilon > 0$, the concavity of the potential field increases, while $\epsilon < 0$ the convexity of the field increases. These effects can be observed in the 1-D example shown in Fig. 1. We set the potentials of the two boundary points as $p(0) = 0$ and $p(30) = 1$, and compute the intermediate potentials using three different values of epsilon in the central region of the environment (highlighted). When $\epsilon = 0$, the potential does not present curvature, as shown by the straight line b. When $\epsilon < 0$ (curve a), the curve of the potential is flatter near obstacles (high potential) and steeper close to the goals (low potential). Thus, this can be seen as a low preference region, because it is harder for the robot to enter this region than to leave. On the other hand, a high preference region is defined when $\epsilon > 0$ (curve c). In this case, it is easier for the robot to enter this region than to leave, because the slope in the potential is increased near obstacles and decreased close to the goals.

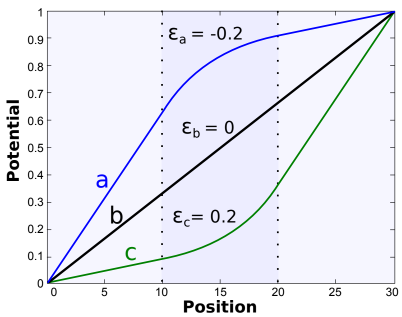


Fig. 1. Changes in the curvature of the potential field by varying the preference (ϵ) of the central region. Curve b is the standard potential calculated without preferences. Curve a presents the potential calculated for a negative preference value, while curve c is the result for a positive preference value.

By using only two fixed boundary conditions in the obstacles and unexplored cells, the BVP exploration is always guided to the largest and closest unexplored regions. This approach works well when there is no uncertainty in the robot localization. However, an effective integrated exploration strategy balances the navigation to unexplored regions with re-visiting tasks in order to improve the localization of the robot and the quality of the resulting map. Preferences can be applied on BVP exploration to favor the passage over specific regions. This is good, for instance, in sparse environments, because it is possible to keep the robot close to the walls and minimize localization errors. Yet, the robot always moves towards the regions with smaller potential – even with preferences. As a result, re-visiting actions are unlikely to happen.

IV. Time Driven BVP Integrated Exploration

Our integrated exploration approach, called Time-driven BVP (TDBVP), adds another boundary condition to the BVP exploration. In addition to the high potentials in obstacles (p=1), and the low potentials in unexplored areas (p=0), we set a variable potential in the center of the areas visited by the robot, generating rails where we want the robot to move. These three different boundary conditions are used to compute the potentials of the remaining cells, according to Eq. 1.

We compute the Voronoi skeleton of the known free-space (thinning) to extract the center cells [15]. Then, to define the potential of those cells, we compute a 1-D harmonic function, along the Voronoi skeleton, based on the time of the last visit made by the robot

$$\nabla^2 \, p(s) = 0 \tag{2}$$

where $p(s)$ is the potential of the center cells visited at step s. The idea is that cells not visited for a long time have lower potential (i.e. are more attractive) than recently visited cells. For that reason, the boundary conditions of Eq. 2 are the potentials of the most recently visited cells ($p(s_{newest}) = 1$) and the potentials of the oldest cells ($p(s_{oldest}) = 0$). Since this is a 1D function, its solution is a straight line between the oldest and the newest cells. Thus, we can compute the potential of the center cells analytically through

$$p(s) = \frac{s - s_{oldest}}{s_{newest} - s_{oldest}} \tag{3}$$

The potentials of the center cells not visited yet, such as recently visualized cells, are set to a fixed negative value ($p = -0.5$). Therefore, when the robot has to choose between an unexplored area and a visited area, it will prefer the unexplored area (unless we insert distortions in the potential, as we will discuss in next session). When the robot has to choose between two visited areas, it will prefer the one visited the longest time ago. After the robot returns to the oldest area, it will become the newest while another one will be the oldest. Hence, in our approach the goal, i.e. the oldest cell, changes over time and never ceases to exist.

We can infer that the environment will always be entirely explored, even though the navigation decision is performed locally by the robot according to the potentials of its surroundings. Firstly, because the potential in the visited area is still higher than the potentials of unexplored areas. And secondly, because the only way that this would not happen, would be if the robot could get stuck in some section of the map, which is not possible. That is, after spending some time inside a region, the potentials of unvisited regions will fall, attracting the robot toward them. The longer it takes to visit a cell the stronger its attractive force. However, we must point out that in sparse environments the thinning algorithm will not cover all free regions seen by the sensors. Thus, in this cases we recommend the additional use of partial thinning relaxations [16], to ensure that the robot will stay close to walls, delaying the entrance on *featureless* regions.

Fig. 2 shows an example of our integrated exploration method in a simulated environment. The robot is shown in red, its trajectory in blue, and the obstacles in brown. Observe that the zone just behind the robot is the lightest area of the map, along with the surroundings of the walls (in black), because their potentials are practically equal to 1. The darkest area of the map is the top right zone between the two inner walls, which was visited only in the beginning of the experiment – the elder cells claiming for a visit.

An important advantage of our strategy in relation to the BVP exploration described in Section III, is that our potential field can always be computed using only a small window around the robot. In our approach, the potential of the cells surrounding the robot is primarily affected by the nearby center cells and obstacles. Thus, there is no need to compute the potentials in the whole map, making this computation independent of the size of the environment. In contrast, the potentials in the traditional BVP exploration are affected by

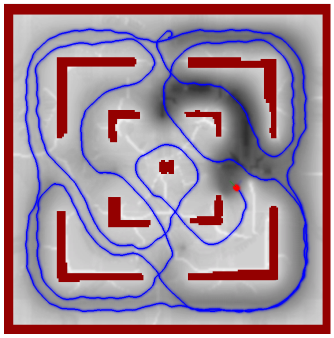


Fig. 2. Example of the potential field based on the time of visit. The dark areas are the regions visited the longest time ago.

cells that can be very far in the environment. For example, when the robot is almost finishing the exploration and only a distant small portion of the map remains unexplored, the potential can take a very long time to converge and start attracting the robot.

V. Dynamic Activation of Preferences

As discussed in Section III, the potential field inside a region can be distorted by inserting a preference parameter in the potential computation. This distortion can be seen as a form of control over the robot navigation in situations of indecision. For instance, bifurcations zones with openings of similar potentials cause unpredictability in the robot movement.

In the integrated exploration problem, the robot has to return to regions already visited, aiming to improve its localization. This is the idea behind the active loop closure behavior, as studied by Stachniss et al. [2]. Our approach deals with loop closures by setting dynamic preferences in the front of the robot. We divide the front of the robot (half semi-circular region in front of the robot) in two (left side and right side), and apply different preferences on each side. If we put a positive preference on the left and a negative preference on the right, the potential field beneath the region will be distorted to the left. The contrary will happen if we invert the preferences.

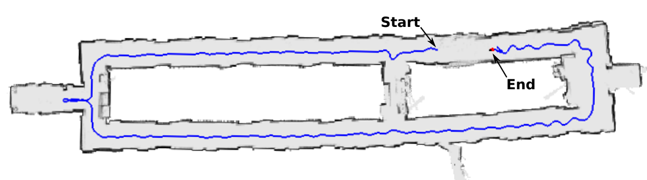
If we maintain the preference fixed for turning to a side during a loop closure, whenever the robot reaches a bifurcation with balanced potentials, it will choose to turn to this side. However, if the potential outside the loop is significantly lower than the potential inside the loop, the robot will resume the exploration outside the loop. As a result, there is an emergent balance in our approach between the loop closure behavior and the exploratory behavior. This balance is controlled implicitly by the values of preferences and boundary conditions.

That said, using always the same preferences configuration is not the better strategy, because depending on the choice of the preferences, the robot might even end up avoiding loop closures. So, we adapt the preferences using a simple algorithm based on the brief history of turns performed by the robot. When the robot sensors detect a large opening to the left or to the right (i.e. a sudden increase in the range of the corresponding laser readings), the method tries to set the preferences to the side of the observed opening. Yet, to avoid changing them too quickly, the preference side is only changed when two subsequent decisions are the same.

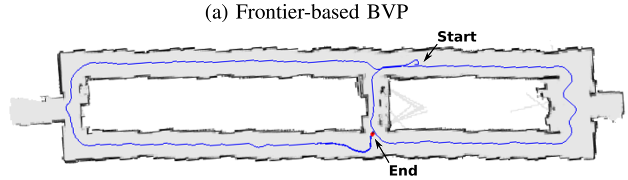
VI. Experiments

We evaluate our method comparing it with the frontier-based BVP exploration in real and simulated environments. Both strategies were combined with a Rao-Blackwellized Particle Filter SLAM algorithm, with laser range-finder and occupancy grids [17].

The experiments in the real environment were performed in a building floor of $72m \times 13m$ containing two loops (with $90m$ and $60m$ of length). Fig. 3 presents the maps obtained by both methods using 300 particles in SLAM. We can see in Fig. 3(a) that the frontier-based BVP exploration did not close the two loops separately, spoiling SLAM quality. This happened because the small passageway at the center of the map is too narrow to attract the robot when there are other near unexplored frontiers. We also observe in the ending of the frontier-based exploration that the robot trajectory was not stable. This is a result of the increase in the BVP's convergence time associated to the map growth. The slow update of the potential field generates abrupt changes in the potential gradient that makes the robot oscillate. We could stop the robot and wait for the full convergence of BVP to avoid the trajectory oscillation, unfortunately, this would render the method useless for real-time applications. Fig. 3(b) shows the resulting map of the time-driven approach. We see that the robot separately closed both loops, due to its potential distortion strategy, and also presented a smooth trajectory, due to the local window computation of the potential field.



(a) Frontier-based BVP

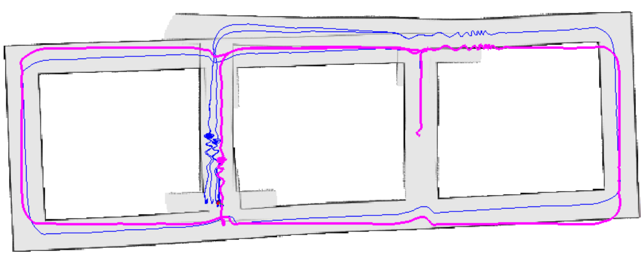


(b) Time-Driven BVP

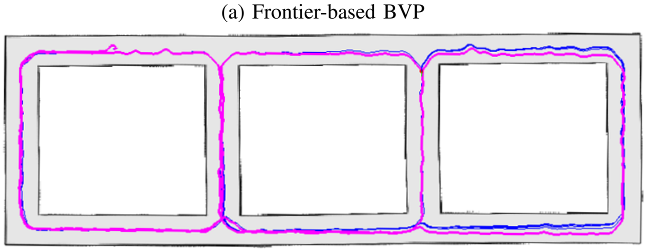
Fig. 3. Experiments in a real environment. The robot trajectory is shown in blue.

The simulated experiments were performed in two different scenarios, adding uncertainty to the sensors measurements. The first scenario is composed of three adjacent loops (with $72m$ of length each), while the second is composed of nested loops (the larger has $88m$ of length). For each scenario, we chose 10 different starting positions to perform the test for each method. In all simulated experiments, we used 200 SLAM particles.

Fig. 4 presents the best visual results obtained in the adjacent loops scenario. The trajectories of the remaining particles at the end of the process are shown in blue, while the correct trajectory of the robot is shown in magenta. Fig. 4(a) depicts the best map generated by the frontier-based BVP exploration. The robot does not try to revisit known places, in fact, it spends too much time in the external long corridors guided to unexplored areas. As a result, its localization only gets worse with time. Later, when the robot finally returns to visited areas, there is not enough diversity to recover the quality of the map. Another issue is that the robot trajectory oscillates, like in the real environment experiment. As the goal changes abruptly to a distant region, the potential can take too long to converge. Fig. 4(b) shows the best map generated by our approach. As expected, the robot tends to close loop by loop, which is good to improve its localization. The estimated trajectory in the SLAM still deviates slightly from the exact one, but the consistency of the map is maintained.



(a) Frontier-based BVP

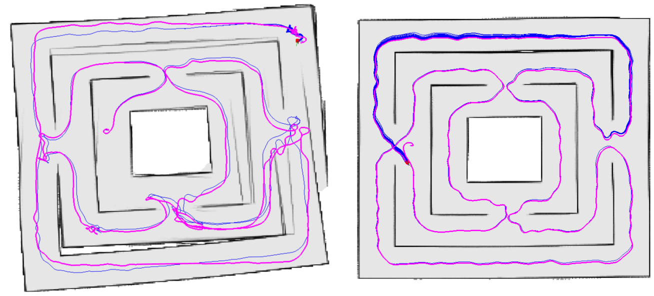


(b) Time-Driven BVP

Fig. 4. Experiments in the adjacent loops scenario. The trajectories of the particles are shown in blue, while the correct trajectory of the robot is shown in magenta.

Fig. 5 shows the best results obtained in the nested loops scenario. As shown in Fig. 5(a), the frontier-based strategy does not proved to be suitable for the situation. The problem is that one badly consolidated loop can directly damage the closure of subsequent ones. In Fig. 5(b), we show the map obtained with our method, and, once again, the result is visually better.

Besides the visual analysis, we also compare the methods by the error in the robot position. At each iteration of the process, the particle filter error ($PFErr$) is computed by the mean distance between each particle position and the real robot position (which is known in a simulated experiment). The final mean error $\overline{PFErr}$ is the mean of $PFErr$ over all iterations.



(a) Frontier-based BVP (b) Time-Driven BVP

Fig. 5. Experiments in the nested loops scenario. The trajectories of the particles are shown in blue, while the correct trajectory of the robot is shown in magenta.

Table I shows the results for the adjacent loops scenario. The number of exploration steps was just slightly larger using our method (1.24%). Thus, apparently, our loop closure behavior does not slow down the exploration process too much in this environment. Regarding the mean error measures, our approach presented a gain of 83.06%. Applying a statistical sign test [18] to the results of 10 runs, we prove that the mean error measures obtained by our method in this environment are significantly larger than the ones obtained by the frontier based BVP (p-value=0.001).

	Frontier-based BVP		Time-driven BVP		diff
	μ	σ	μ	σ	%
Steps	1947	319.80	1971.1	186.67	+1.24
$\overline{PFErr}$	2.3489	1.1733	0.3979	0.1335	-83.06

TABLE I

RESULTS OF THE EXPERIMENTS IN THE ADJACENT LOOPS SCENARIO.

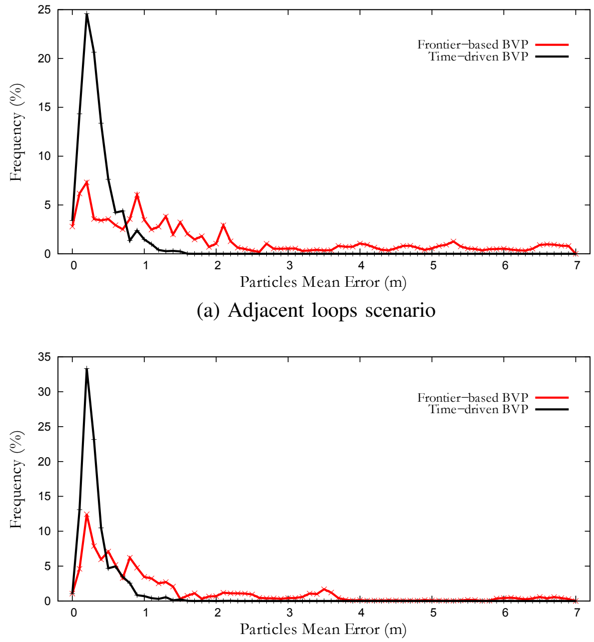
Table II shows the results for the nested loops scenario. This time, the number of exploration steps was smaller with our approach than with BVP (-1.41%), nonetheless, the difference was not significant. The mean errors are also smaller using the proposed method (56.98% smaller). After running the sign test, we observe that our approach performed significantly better in this environment than the frontier-based BVP exploration (p-value=0.001).

	Frontier-based BVP		Time-driven BVP		diff
	μ	σ	μ	σ	%
Steps	1627.4	254.21	1604.4	290.76	-1.41
$\overline{PFErr}$	1.4732	1.2783	0.6338	0.1312	-56.98

TABLE II

RESULTS OF THE EXPERIMENTS IN THE NESTED LOOPS SCENARIO.

Finally, we present the histograms of the mean position error in the adjacent loops and in the nested loops scenarios, Fig. 6(a) and Fig. 6(b) respectively. We detect that in our approach, most of the time – 97% in the first scenario and 98% in the second – the error was smaller than $1m$, while in the frontier-based BVP approach these percentages are only 45% in the adjacent loops and 62% in the nested loops.



(a) Adjacent loops scenario

(b) Nested loops scenario

Fig. 6. Histograms of the mean error for the simulated experiments.

VII. Conclusions

In this paper we propose a strategy that blends harmonic fields and a modified Voronoi Skeleton of the known area. These techniques are used to guide the robot to unexplored regions or to the "oldest" explored area. Our key contributions are: (i) a potential field that never ceases to exist, even if the whole map is known; (ii) a loop-closing behavior that emerges naturally from the TDBVP equations; (iii) a less expensive approach than traditional BVP-Exploration, as the potential field is computed only inside a local window, instead of the whole map.

Experiments show that TDBVP presents statistically significant improvements in terms of localization errors in simulated environments. We can also see in the results that in simulated and real environments TDBVP presents better map quality than the traditional "greedy" BVP-Exploration. Moreover, our strategy does not demand substantial increase in the exploration time. This is interesting, because the traditional BVP tends to use the smallest path to the goal and also to avoid revisits.

Our idea can be easily integrated to topological SLAM strategies since most of them create a path as the robot moves (be it a skeleton or a given sequence of positions). Even though the exploration strategy is fairly simple and effective, we believe that it is possible to gain more control over the harmonic potential field. We continue to explore new ways to improve the integrated exploration strategy (e.g., by incorporating utility and cost metrics.

We are planning to extend the algorithm to 3D environments where approaches using BVP are still not real-time. Finally. our algorithm is well suited for cluttered environments. When we consider sparse environments there are still some issues that need to be addressed. First, we need to test the algorithm using partial relaxations. Second, the robot may be traveling in a rail that suddenly disappears. In this case, the robot will move to the next nearby rail. However, it might not be in the local window. If that happens, the robot can try to move towards the next unexplored region inside the window. In the event that none of this conditions are satisfied, we need to drive the robot towards the Voronoi diagram, relying on its property of accessibility [19].

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