Mathics

A free, open-source alternative to Mathematica

The Mathics Team

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Part I.

Manual

1. Introduction

Mathics—to be pronounced like "Mathematics" without the "emat"—is a general-purpose computer algebra system (CAS). It is meant to be a free, open-source alternative to *Mathematica*®. It is free both as in "free beer" and as in "freedom". There are various online mirrors running *Mathics* but it is also possible to run *Mathics* locally. A list of mirrors can be found at the *Mathics* homepage, `https://mathics.org`.

The programming language of *Mathics* is meant to resemble *Wolfram*'s famous *Mathematica*® as much as possible. However, *Mathics* is in no way affiliated or supported by *Wolfram*. *Mathics* will probably never have the power to compete with *Mathematica*® in industrial applications; yet, it might be an interesting alternative for educational purposes.

For implementation details see `https://github.com/mathics/Mathics/wiki`.

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Why yet another CAS?

Mathematica® is great, but it has one big disadvantage: It is not free. On the one hand, people might not be able or willing to pay hundreds of dollars for it; on the other hand, they would still not be able to see what's going on "inside" the program to understand their computations better. That's what free software is for!

Mathics aims at combining the best of both worlds: the beauty of *Mathematica*® backed by a free, extensible Python core which includes a rich set of Python tools numeric computation, `https://numpy.org/numpy`, and symbolic mathematics, `https://sympy.org`.

Of course, there are drawbacks to the *Mathematica*® language, despite all its beauty. It does not really provide object orientation and especially encapsulation, which might be crucial for big software projects. Nevertheless, *Wolfram* still managed to create their amazing *Wolfram | Alpha* entirely with *Mathematica*®, so it can't be too bad!

However, it is not even the intention of *Mathics* to be used in large-scale projects and calculations—at least not as the main framework—but rather as a tool for quick explorations and in educating people who might later switch to *Mathematica*®.

What does it offer?

Some of the most important features of *Mathics* are

- a powerful functional programming language,
- a system driven by pattern matching and rules application,
- rationals, complex numbers, and arbitrary-precision arithmetic,
- lots of list and structure manipulation routines,
- an interactive graphical user interface right in the Web browser using MathML (apart from a command line interface),
- creation of graphics (e.g. plots) and display in the browser using SVG for 2D graphics and WebGL for 3D graphics,
- export of results to LaTeX (using Asymptote for graphics),
- a very easy way of defining new functions in Python,
- an integrated documentation and testing system.

What is missing?

There are lots of ways in which *Mathics* could still be improved.

Most notably, performance is still slow, so any serious usage in cutting-edge industry or research will fail, unfortunately. Although Cython can be used to speed up parts of *Mathics*, more is needed to speed up pattern matching. Replacing recursion with iteration may help here.

Apart from performance issues, new features such as more functions in various mathematical fields like calculus, number theory, or graph theory are still to be added.

In the future we intend to make better use the the graphics available in the excellent packages:

- sympy plotting,`https://docs.sympy.org/latest/modules/plotting.html`
- mathplotlib pyplot, `https://matplotlib.org/api/pyplot_api.html`, and
- networkx, `https://networkx.github.io/`

Who is behind it?

Mathics was created by Jan Pöschk in 2011. From 2013 to about 2017 it had been maintained mostly by Angus Griffith and Ben Jones. Since then, a number of others have been people involved in *Mathics*; the list can be found in the `AUTHORS.txt` file, `https://github.com/mathics/Mathics/blob/master/AUTHORS.txt`.

If you have any ideas on how to improve *Mathics* or even want to help out yourself, please contact us!

Welcome to *Mathics*, have fun!

2. Installation and Running

Mathics runs natively on a computer that has Python or PyPy 3.6 or later installed. Since *Mathics* relies on *sympy* which in turn relies on *numpy*, you will need at least those installed.

Since installation may change, see `https://github.com/mathics/Mathics/wiki/Installing-and-Running` for the most recent instructions for installing from PyPI, source, or from *docker*.

3. Language Tutorials

The following sections are introductions to the basic principles of the language of *Mathics*. A few examples and functions are presented. Only their most common usages are listed; for a full description of a Symbols possible arguments, options, etc., see its entry in the Reference of Built-in Symbols.
However if you google for “Mathematica Tutorials” you will find easily dozens of other tutorials which are applicable. Be warned though that *Mathics* does not yet offer the full range and features and capabilities of *Mathematica*®.

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Basic calculations

Mathics can be used to calculate basic stuff:

```
>> 1 + 2
   3
```

To submit a command to *Mathics*, press `Shift+Return` in the Web interface or `Return` in the console interface. The result will be printed in a new line below your query.
Mathics understands all basic arithmetic operators and applies the usual operator precedence. Use parentheses when needed:

```
>> 1 - 2 * (3 + 5)/ 4
   -3
```

The multiplication can be omitted:

```
>> 1 - 2 (3 + 5)/ 4
   -3

>> 2 4
   8
```

Powers can be entered using `^`:

```
>> 3 ^ 4
   81
```

Integer divisions yield rational numbers:

```
>> 6 / 4
```

$\frac{3}{2}$

To convert the result to a floating point number, apply the function `N`:

```
>> N[6 / 4]
   1.5
```

As you can see, functions are applied using square braces `[` and `]`, in contrast to the common notation of `(` and `)`. At first hand, this might seem strange, but this distinction between function application and precedence change is necessary to allow some general syntax structures, as you will see later.
Mathics provides many common mathematical functions and constants, e.g.:

```
>> Log[E]
   1

>> Sin[Pi]
   0

>> Cos[0.5]
   0.877583
```

When entering floating point numbers in your query, *Mathics* will perform a numerical evaluation and present a numerical result, pretty much like if you had applied `N`.

Of course, *Mathics* has complex numbers:

```
>> Sqrt[-4]
```
$2I$

```
>> I ^ 2
```
-1

```
>> (3 + 2 I)^ 4
```
$-119 + 120I$

```
>> (3 + 2 I)^ (2.5 - I)
```
$43.663 + 8.28556I$

```
>> Tan[I + 0.5]
```
$0.195577 + 0.842966I$

`Abs` calculates absolute values:

```
>> Abs[-3]
```
3

```
>> Abs[3 + 4 I]
```
5

Mathics can operate with pretty huge numbers:

```
>> 100!
```
93 326 215 443 944 152 681 699~
~238 856 266 700 490 715 968~
~264 381 621 468 592 963 895~
~217 599 993 229 915 608 941~
~463 976 156 518 286 253 697 920~
~827 223 758 251 185 210 916 864~
~000 000 000 000 000 000 000 000

(`!` denotes the factorial function.) The precision of numerical evaluation can be set:

```
>> N[Pi, 100]
```
3.141592653589793238462643~
~383279502884197169399375~
~105820974944592307816406~
~286208998628034825342117068

Division by zero is forbidden:

```
>> 1 / 0
```
Infinite expression 1/0 encountered.

ComplexInfinity

Other expressions involving `Infinity` are evaluated:

```
>> Infinity + 2 Infinity
```
∞

In contrast to combinatorial belief, `0^0` is undefined:

```
>> 0 ^ 0
```
Indeterminate expression 0^0 *encountered.*

Indeterminate

The result of the previous query to *Mathics* can be accessed by `%`:

```
>> 3 + 4
```
7

```
>> % ^ 2
```
49

Symbols and Assignments

Symbols need not be declared in *Mathics*, they can just be entered and remain variable:

```
>> x
```
x

Basic simplifications are performed:

```
>> x + 2 x
```
$3x$

Symbols can have any name that consists of characters and digits:

```
>> iAm1Symbol ^ 2
```
$\mathrm{iAm1Symbol}^2$

You can assign values to symbols:

```
>> a = 2
```
2

```
>> a ^ 3
```
8

```
>> a = 4
```
4

```
>> a ^ 3
```
64

Assigning a value returns that value. If you want to suppress the output of any result, add a `;` to the end of your query:

```
>> a = 4;
```

Values can be copied from one variable to another:

```
>> b = a;
```

Now changing `a` does not affect `b`:

```
>> a = 3;
```

```
>> b
```
4

Such a dependency can be achieved by using "delayed assignment" with the `:=` operator (which does not return anything, as the right side is not even evaluated):

```
>> b := a ^ 2
```

```
>> b
```
9

```
>> a = 5;
```

```
>> b
```
25

Comparisons and Boolean Logic

Values can be compared for equality using the operator `==`:

```
>> 3 == 3
```
True

```
>> 3 == 4
```
False

The special symbols `True` and `False` are used to denote truth values. Naturally, there are inequality comparisons as well:

```
>> 3 > 4
```
False

Inequalities can be chained:

```
>> 3 < 4 >= 2 != 1
```
True

Truth values can be negated using `!` (logical *not*) and combined using `&&` (logical *and*) and `||` (logical *or*):

```
>> !True
```
False

```
>> !False
```
True

```
>> 3 < 4 && 6 > 5
```
True

`&&` has higher precedence than `||`, i.e. it binds stronger:

```
>> True && True || False && False
```
True

```
>> True && (True || False)&& False
```
False

Strings

Strings can be entered with `"` as delimiters:

```
>> "Hello world!"
```
Hello world!

As you can see, quotation marks are not printed in the output by default. This can be changed by using `InputForm`:

```
>> InputForm["Hello world!"]
```
"Hello world!"

Strings can be joined using `<>`:

```
>> "Hello" <> " " <> "world!"
```
Hello world!

Numbers cannot be joined to strings:

```
>> "Debian" <> 6
```
Stringexpected.

Debian<>6

They have to be converted to strings using `ToString` first:

```
>> "Debian" <> ToString[6]
```
Debian6

Lists

Lists can be entered in *Mathics* with curly braces `{` and `}`:

```
>> mylist = {a, b, c, d}
```
$\{a, b, c, d\}$

There are various functions for constructing lists:

```
>> Range[5]
```
$\{1, 2, 3, 4, 5\}$

```
>> Array[f, 4]
```
$\{f[1], f[2], f[3], f[4]\}$

```
>> ConstantArray[x, 4]
```
$\{x, x, x, x\}$

```
>> Table[n ^ 2, {n, 2, 5}]
```
$\{4, 9, 16, 25\}$

The number of elements of a list can be determined with `Length`:

```
>> Length[mylist]
```
4

Elements can be extracted using double square braces:

```
>> mylist[[3]]
```
c

Negative indices count from the end:

```
>> mylist[[-3]]
```
b

Lists can be nested:

```
>> mymatrix = {{1, 2}, {3, 4}, {5,
   6}};
```

There are alternate forms to display lists:

```
>> TableForm[mymatrix]
```

1	2
3	4
5	6

```
>> MatrixForm[mymatrix]
```

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

There are various ways of extracting elements from a list:

```
>> mymatrix[[2, 1]]
```
3

```
>> mymatrix[[;;, 2]]
```
$\{2,4,6\}$

```
>> Take[mylist, 3]
```
$\{a,b,c\}$

```
>> Take[mylist, -2]
```
$\{c,d\}$

```
>> Drop[mylist, 2]
```
$\{c,d\}$

```
>> First[mymatrix]
```
$\{1,2\}$

```
>> Last[mylist]
```
d

```
>> Most[mylist]
```
$\{a,b,c\}$

```
>> Rest[mylist]
```
$\{b,c,d\}$

Lists can be used to assign values to multiple variables at once:

```
>> {a, b} = {1, 2};
```

```
>> a
```
1

```
>> b
```
2

Many operations, like addition and multiplication, "thread" over lists, i.e. lists are combined element-wise:

```
>> {1, 2, 3} + {4, 5, 6}
```
$\{5,7,9\}$

```
>> {1, 2, 3} * {4, 5, 6}
```
$\{4,10,18\}$

It is an error to combine lists with unequal lengths:

```
>> {1, 2} + {4, 5, 6}
```
$Objectsofunequallengthcannotbecombined.$

$\{1,2\} + \{4,5,6\}$

The Structure of Things

Every expression in *Mathics* is built upon the same principle: it consists of a *head* and an arbitrary number of *children*, unless it is an *atom*, i.e. it can not be subdivided any further. To put it another way: everything is a function call. This can be best seen when displaying expressions in their "full form":

```
>> FullForm[a + b + c]
```
Plus $[a,b,c]$

Nested calculations are nested function calls:

```
>> FullForm[a + b * (c + d)]
```
Plus $[a,$ Times $[b,$ Plus $[c,d]]]$

Even lists are function calls of the function `List`:

```
>> FullForm[{1, 2, 3}]
```
List $[1,2,3]$

The head of an expression can be determined with `Head`:

```
>> Head[a + b + c]
```
Plus

The children of an expression can be accessed like list elements:

```
>> (a + b + c)[[2]]
```
b

The head is the 0th element:

```
>> (a + b + c)[[0]]
```
Plus

The head of an expression can be exchanged using the function `Apply`:

```
>> Apply[g, f[x, y]]
```
$g[x, y]$

```
>> Apply[Plus, a * b * c]
```
$a + b + c$

`Apply` can be written using the operator `@@`:

```
>> Times @@ {1, 2, 3, 4}
```
24

(This exchanges the head `List` of `{1, 2, 3, 4}` with `Times`, and then the expression `Times[1, 2, 3, 4]` is evaluated, yielding 24.) `Apply` can also be applied on a certain *level* of an expression:

```
>> Apply[f, {{1, 2}, {3, 4}}, {1}]
```
$\{f[1,2], f[3,4]\}$

Or even on a range of levels:

```
>> Apply[f, {{1, 2}, {3, 4}}, {0,
   2}]
```
$f[f[1,2], f[3,4]]$

`Apply` is similar to `Map` (`/@`):

```
>> Map[f, {1, 2, 3, 4}]
```
$\{f[1], f[2], f[3], f[4]\}$

```
>> f /@ {{1, 2}, {3, 4}}
```
$\{f[\{1,2\}], f[\{3,4\}]\}$

The atoms of *Mathics* are numbers, symbols, and strings. `AtomQ` tests whether an expression is an atom:

```
>> AtomQ[5]
```
True

```
>> AtomQ[a + b]
```
False

The full form of rational and complex numbers looks like they were compound expressions:

```
>> FullForm[3 / 5]
```
Rational[3, 5]

```
>> FullForm[3 + 4 I]
```
Complex[3, 4]

However, they are still atoms, thus unaffected by applying functions, for instance:

```
>> f @@ Complex[3, 4]
```
$3 + 4I$

Nevertheless, every atom has a head:

```
>> Head /@ {1, 1/2, 2.0, I, "a
   string", x}
```
{Integer, Rational, Real, Complex, String, Symbol}

The operator `===` tests whether two expressions are the same on a structural level:

```
>> 3 === 3
```
True

```
>> 3 == 3.0
```
True

But

```
>> 3 === 3.0
```
False

because `3` (an `Integer`) and `3.0` (a `Real`) are structurally different.

Functions and Patterns

Functions can be defined in the following way:

```
>> f[x_] := x ^ 2
```

This tells *Mathics* to replace every occurrence of `f` with one (arbitrary) parameter `x` with `x ^ 2`.

```
>> f[3]
```
9

```
>> f[a]
```
a^2

The definition of `f` does not specify anything for two parameters, so any such call will stay unevaluated:

```
>> f[1, 2]
```
$f[1,2]$

In fact, *functions* in *Mathics* are just one aspect of *patterns*: `f[x_]` is a pattern that *matches* expressions like `f[3]` and `f[a]`. The following patterns are available:

`_` or `Blank[]`
: matches one expression.

`Pattern[`x`, `p`]`
: matches the pattern p and stores the value in x.

x`_` or `Pattern[`x`, Blank[]]`
: matches one expression and stores it in x.

`__` or `BlankSequence[]`
: matches a sequence of one or more expressions.

`___` or `BlankNullSequence[]`
: matches a sequence of zero or more expressions.

`_`h or `Blank[`h`]`
: matches one expression with head h.

x`_`h or `Pattern[`x`, Blank[`h`]]`
: matches one expression with head h and stores it in x.

p `|` q or `Alternatives[`p`, `q`]`
: matches either pattern p or q.

p `?` t or `PatternTest[`p`, `t`]`
: matches p if the test t`[`p`]` yields `True`.

p `/;` c or `Condition[`p`, `c`]`
: matches p if condition c holds.

`Verbatim[`p`]`
: matches an expression that equals p, without regarding patterns inside p.

As before, patterns can be used to define functions:

```
>> g[s___] := Plus[s] ^ 2

>> g[1, 2, 3]
   36
```

`MatchQ[`e`, `p`]` tests whether e matches p:

```
>> MatchQ[a + b, x_ + y_]
   True

>> MatchQ[6, _Integer]
   True
```

`ReplaceAll` (`/.`) replaces all occurrences of a pattern in an expression using a `Rule` given by `->`:

```
>> {2, "a", 3, 2.5, "b", c} /.
   x_Integer -> x ^ 2
```

$\{4, \mathrm{a}, 9, 2.5, \mathrm{b}, c\}$

You can also specify a list of rules:

```
>> {2, "a", 3, 2.5, "b", c} /. {
   x_Integer -> x ^ 2.0, y_String
   -> 10}
```

$\{4., 10, 9., 2.5, 10, c\}$

`ReplaceRepeated` (`//.`) applies a set of rules repeatedly, until the expression doesn't change anymore:

```
>> {2, "a", 3, 2.5, "b", c} //. {
   x_Integer -> x ^ 2.0, y_String
   -> 10}
```

$\{4., 100., 9., 2.5, 100., c\}$

There is a "delayed" version of `Rule` which can be specified by `:>` (similar to the relation of `:=` to `=`):

```
>> a :> 1 + 2
```

$a{:>}1+2$

```
>> a -> 1 + 2
```

$a- > 3$

This is useful when the right side of a rule should not be evaluated immediately (before matching):

```
>> {1, 2} /. x_Integer -> N[x]
```

$\{1, 2\}$

Here, `N` is applied to `x` before the actual matching, simply yielding `x`. With a delayed rule this can be avoided:

```
>> {1, 2} /. x_Integer :> N[x]
```

$\{1., 2.\}$

While `ReplaceAll` and `ReplaceRepeated` simply take the first possible match into account, `ReplaceList` returns a list of all possible matches. This can be used to get all subsequences of a list, for instance:

```
>> ReplaceList[{a, b, c}, {___, x__
   , ___} -> {x}]
```

$\{\{a\}, \{a, b\}, \{a, b, c\}, \{b\}, \{b, c\}, \{c\}\}$

`ReplaceAll` would just return the first expression:

```
>> ReplaceAll[{a, b, c}, {___, x__,
    ___} -> {x}]
```

$\{a\}$

In addition to defining functions as rules for certain patterns, there are *pure* functions that can be defined using the `&` postfix operator, where everything before it is treated as the function body and `#` can be used as argument placeholder:

```
>> h = # ^ 2 &;

>> h[3]
   9
```

Multiple arguments can simply be indexed:

```
>> sum = #1 + #2 &;
```

```
>> sum[4, 6]
```

10

It is also possible to name arguments using `Function`:

```
>> prod = Function[{x, y}, x * y];
```

```
>> prod[4, 6]
```

24

Pure functions are very handy when functions are used only locally, e.g., when combined with operators like `Map`:

```
>> # ^ 2 & /@ Range[5]
```

$\{1, 4, 9, 16, 25\}$

Sort according to the second part of a list:

```
>> Sort[{{x, 10}, {y, 2}, {z, 5}},
   #1[[2]] < #2[[2]] &]
```

$\{\{y, 2\}, \{z, 5\}, \{x, 10\}\}$

Functions can be applied using prefix or postfix notation, in addition to using `[]`:

```
>> h @ 3
```

9

```
>> 3 // h
```

9

Control Statements

Like most programming languages, *Mathics* has common control statements for conditions, loops, etc.:

`If[`*cond*`, `*pos*`, `*neg*`]`
: returns *pos* if *cond* evaluates to `True`, and *neg* if it evaluates to `False`.

`Which[`*cond1*`, `*expr1*`, `*cond2*`, `*expr2*`, ...]`
: yields *expr1* if *cond1* evaluates to `True`, *expr2* if *cond2* evaluates to `True`, etc.

`Do[`*expr*`, {`*i*`, `*max*`}]`
: evaluates *expr* *max* times, substituting *i* in *expr* with values from 1 to *max*.

`For[`*start*`, `*test*`, `*incr*`, `*body*`]`
: evaluates *start*, and then iteratively *body* and *incr* as long as *test* evaluates to `True`.

`While[`*test*`, `*body*`]`
: evaluates *body* as long as *test* evaluates to `True`.

`Nest[`*f*`, `*expr*`, `*n*`]`
: returns an expression with *f* applied *n* times to *expr*.

`NestWhile[`*f*`, `*expr*`, `*test*`]`
: applies a function *f* repeatedly on an expression *expr*, until applying *test* on the result no longer yields `True`.

`FixedPoint[`*f*`, `*expr*`]`
: starting with *expr*, repeatedly applies *f* until the result no longer changes.

```
>> If[2 < 3, a, b]
```

a

```
>> x = 3; Which[x < 2, a, x > 4, b,
    x < 5, c]
```

c

Compound statements can be entered with `;`. The result of a compound expression is its last part or `Null` if it ends with a `;`.

```
>> 1; 2; 3
```

3

```
>> 1; 2; 3;
```

Inside `For`, `While`, and `Do` loops, `Break[]` exits the loop and `Continue[]` continues to the next iteration.

```
>> For[i = 1, i <= 5, i++, If[i ==
   4, Break[]]; Print[i]]
```

1
2
3

Scoping

By default, all symbols are "global" in *Mathics*, i.e. they can be read and written in any part of your program. However, sometimes "local" variables are needed in order not to disturb the global namespace. *Mathics* provides two ways to support this:

- *lexical scoping* by `Module`, and
- *dynamic scoping* by `Block`.

`Module[{`*vars*`}, `*expr*`]`
: localizes variables by giving them a temporary name of the form `name$number`, where number is the current value of `$ModuleNumber`. Each time a module is evaluated, `$ModuleNumber` is incremented.

`Block[{`*vars*`}, `*expr*`]`
: temporarily stores the definitions of certain variables, evaluates *expr* with reset values and restores the original definitions afterwards.

Both scoping constructs shield inner variables from affecting outer ones:

```
>> t = 3;

>> Module[{t}, t = 2]
   2

>> Block[{t}, t = 2]
   2

>> t
   3
```

`Module` creates new variables:

```
>> y = x ^ 3;

>> Module[{x = 2}, x * y]
```
$2x^3$

`Block` does not:

```
>> Block[{x = 2}, x * y]
   16
```

Thus, `Block` can be used to temporarily assign a value to a variable:

```
>> expr = x ^ 2 + x;

>> Block[{x = 3}, expr]
   12

>> x
```
x

`Block` can also be used to temporarily change the value of system parameters:

```
>> Block[{$RecursionLimit = 30}, x
   = 2 x]
```
Recursiondepthof30exceeded.

$Aborted

```
>> f[x_] := f[x + 1]; Block[{
   $IterationLimit = 30}, f[1]]
```
Iterationlimitof30exceeded.

$Aborted

It is common to use scoping constructs for function definitions with local variables:

```
>> fac[n_] := Module[{k, p}, p = 1;
    For[k = 1, k <= n, ++k, p *= k
   ]; p]

>> fac[10]
   3 628 800

>> 10!
   3 628 800
```

Formatting Output

The way results are formatted for output in *Mathics* is rather sophisticated, as compatibility to the way *Mathematica*® does things is one of the design goals. It can be summed up in the following procedure:

1. The result of the query is calculated.
2. The result is stored in `Out` (which `%` is a shortcut for).
3. Any `Format` rules for the desired output form are applied to the result. In the console version of *Mathics*, the result is formatted as `OutputForm`; `MathMLForm` for the `StandardForm` is used in the interactive Web version; and `TeXForm` for the `StandardForm` is used to generate the LaTeX version of this documentation.
4. `MakeBoxes` is applied to the formatted result, again given either `OutputForm`, `MathMLForm`, or `TeXForm` depending on the execution context of *Mathics*. This yields a new expression consisting of "box constructs".
5. The boxes are turned into an ordinary string and displayed in the console, sent to the browser, or written to the documentation LaTeX file.

As a consequence, there are various ways to implement your own formatting strategy for custom objects.
You can specify how a symbol shall be formatted by assigning values to `Format`:

```
>> Format[x] = "y";
```

```
>> x
```
y

This will apply to `MathMLForm`, `OutputForm`, `StandardForm`, `TeXForm`, and `TraditionalForm`.

```
>> x // InputForm
```
x

You can specify a specific form in the assignment to `Format`:

```
>> Format[x, TeXForm] = "z";
```

```
>> x // TeXForm
```
\text{z}

Special formats might not be very relevant for individual symbols, but rather for custom functions (objects):

```
>> Format[r[args___]] = "<an r
   object>";
```

```
>> r[1, 2, 3]
```
<an r object>

You can use several helper functions to format expressions:

`Infix[`*expr*`, `*op*`]`
: formats the arguments of *expr* with infix operator *op*.

`Prefix[`*expr*`, `*op*`]`
: formats the argument of *expr* with prefix operator *op*.

`Postfix[`*expr*`, `*op*`]`
: formats the argument of *expr* with postfix operator *op*.

`StringForm[`*form*`, `*arg1*`, `*arg2*`, ...]`
: formats arguments using a format string.

```
>> Format[r[args___]] = Infix[{args
   }, "~"];
```

```
>> r[1, 2, 3]
```
$1 \sim 2 \sim 3$

```
>> StringForm["`1` and `2`", n, m]
```
n and m

There are several methods to display expressions in 2-D:

`Row[{...}]`
: displays expressions in a row.

`Grid[{{...}}]`
: displays a matrix in two-dimensional form.

`Subscript[`*expr*`, `*i1*`, `*i2*`, ...]`
: displays *expr* with subscript indices *i1*, *i2*, ...

`Superscript[`*expr*`, `*exp*`]`
: displays *expr* with superscript (exponent) *exp*.

```
>> Grid[{{a, b}, {c, d}}]
```
$$\begin{matrix} a & b \\ c & d \end{matrix}$$

```
>> Subscript[a, 1, 2] // TeXForm
```
a_{1,2}

If you want even more low-level control of how expressions are displayed, you can override `MakeBoxes`:

```
>> MakeBoxes[b, StandardForm] = "c
   ";
```

```
>> b
```
c

This will even apply to `TeXForm`, because `TeXForm` implies `StandardForm`:

```
>> b // TeXForm
```
c

Except some other form is applied first:

```
>> b // OutputForm // TeXForm
```
b

`MakeBoxes` for another form:

```
>> MakeBoxes[b, TeXForm] = "d";
```

```
>> b // TeXForm
```
d

You can cause a much bigger mess by overriding `MakeBoxes` than by sticking to `Format`, e.g. generate invalid XML:

```
>> MakeBoxes[c, MathMLForm] = "<not
    closed";
```

```
>> c // MathMLForm
```
<not closed

However, this will not affect formatting of expressions involving `c`:

```
>> c + 1 // MathMLForm
```

<math display="block"><mstyle mathvariant="sans-serif"> <mrow><mn>1</mn> <mo>+</mo> <mi>c</mi> </mrow></mstyle></math>

That's because `MathMLForm` will, when not overridden for a special case, call `StandardForm` first. `Format` will produce escaped output:

```
>> Format[d, MathMLForm] = "<not
   closed";
```

```
>> d // MathMLForm
```

<math display="block"> <mtext><not closed</mtext> </math>

```
>> d + 1 // MathMLForm
```

<math display="block"><mstyle mathvariant="sans-serif"><mrow> <mn>1</mn> <mo>+</mo> <mtext><not closed</mtext> </mrow></mstyle></math>

For instance, you can override `MakeBoxes` to format lists in a different way:

```
>> MakeBoxes[{items___},
   StandardForm] := RowBox[{"[",
   Sequence @@ Riffle[MakeBoxes /@
   {items}, " "], "]"}]
```

```
>> {1, 2, 3}
```

[123]

However, this will not be accepted as input to *Mathics* anymore:

```
>> [1 2 3]
```

```
>> Clear[MakeBoxes]
```

By the way, `MakeBoxes` is the only built-in symbol that is not protected by default:

```
>> Attributes[MakeBoxes]
```

{HoldAllComplete}

`MakeBoxes` must return a valid box construct:

```
>> MakeBoxes[squared[args___],
   StandardForm] := squared[args] ^
    2
```

```
>> squared[1, 2]
```

Power[squared[1,2], 2]isnotavalidboxstructure.

```
>> squared[1, 2] // TeXForm
```

Power[squared[1,2], 2]isnotavalidboxstructure.

= The desired effect can be achieved in the following way:

```
>> MakeBoxes[squared[args___],
   StandardForm] := SuperscriptBox[
   RowBox[{MakeBoxes[squared], "[",
    RowBox[Riffle[MakeBoxes[#]& /@
   {args}, ","]], "]"}], 2]
```

```
>> squared[1, 2]
```

squared $[1,2]^2$

You can view the box structure of a formatted expression using `ToBoxes`:

```
>> ToBoxes[m + n]
```

RowBox [{m, +, n}]

The list elements in this `RowBox` are strings, though string delimiters are not shown in the default output form:

```
>> InputForm[%]
```

RowBox [{"m", "+", "n"}]

Graphics Introduction Examples

Two-dimensional graphics can be created using the function `Graphics` and a list of graphics primitives. For three-dimensional graphics see the following section. The following primitives are available:

`Circle[{`*x*`, `*y*`}, `*r*`]`
: draws a circle.

`Disk[{`*x*`, `*y*`}, `*r*`]`
: draws a filled disk.

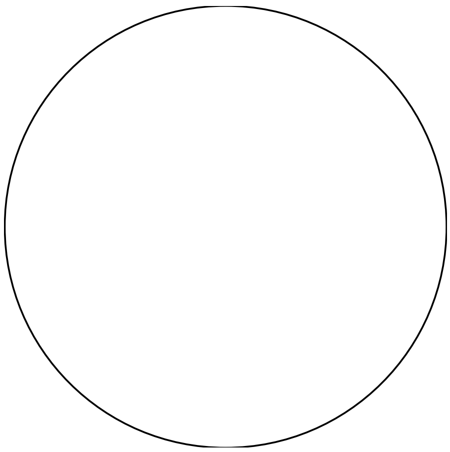
`Rectangle[{`*x1*`, `*y1*`}, {`*x2*`, `*y2*`}]`
: draws a filled rectangle.

`Polygon[{{`*x1*`, `*y1*`}, {`*x2*`, `*y2*`}, ...}]`
: draws a filled polygon.

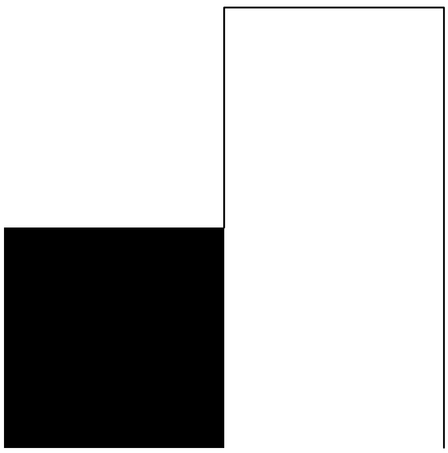
`Line[{{`*x1*`, `*y1*`}, {`*x2*`, `*y2*`}, ...}]`
: draws a line.

`Text[`*text*`, {`*x*`, `*y*`}]`
: draws text in a graphics.

```
>> Graphics[{Circle[{0, 0}, 1]}]
```



```
>> Graphics[{Line[{{0, 0}, {0, 1},
   {1, 1}, {1, -1}}], Rectangle[{0,
    0}, {-1, -1}]}]
```



Colors can be added in the list of graphics primitives to change the drawing color. The following ways to specify colors are supported:

`RGBColor[`*r*`, `*g*`, `*b*`]`
: specifies a color using red, green, and blue.

`CMYKColor[`*c*`, `*m*`, `*y*`, `*k*`]`
: specifies a color using cyan, magenta, yellow, and black.

`Hue[`*h*`, `*s*`, `*b*`]`
: specifies a color using hue, saturation, and brightness.

`GrayLevel[`*l*`]`
: specifies a color using a gray level.

All components range from 0 to 1. Each color function can be supplied with an additional argument specifying the desired opacity ("alpha") of the color. There are many predefined colors, such as `Black`, `White`, `Red`, `Green`, `Blue`, etc.

```
>> Graphics[{Red, Disk[]}]
```

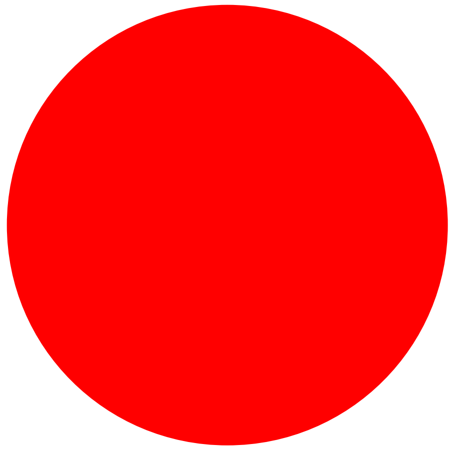
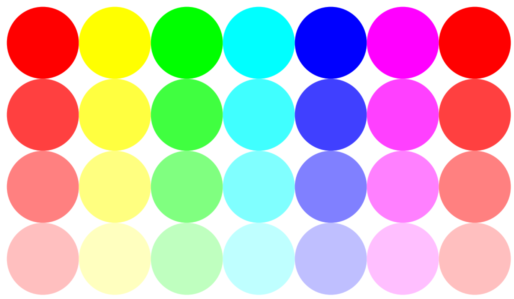


Table of hues:

```
>> Graphics[Table[{Hue[h, s], Disk
   [{12h, 8s}]}, {h, 0, 1, 1/6}, {s
   , 0, 1, 1/4}]]
```



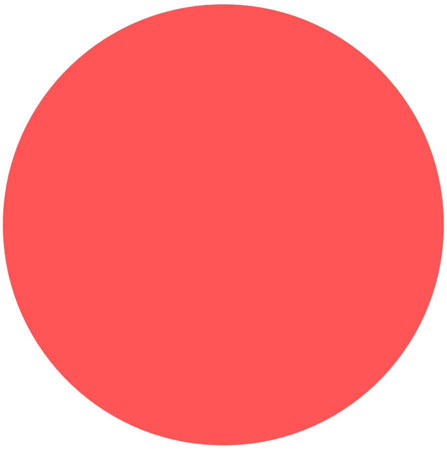
Colors can be mixed and altered using the following functions:

`Blend[{`*color1*`, `*color2*`}, `*ratio*`]`
: mixes *color1* and *color2* with *ratio*, where a ratio of 0 returns *color1* and a ratio of 1 returns *color2*.

`Lighter[`*color*`]`
: makes *color* lighter (mixes it with `White`).

`Darker[`*color*`]`
: makes *color* darker (mixes it with `Black`).

```
>> Graphics[{Lighter[Red], Disk[]}]
```



`Graphics` produces a `GraphicsBox`:

```
>> Head[ToBoxes[Graphics[{Circle
   []}]]]
```

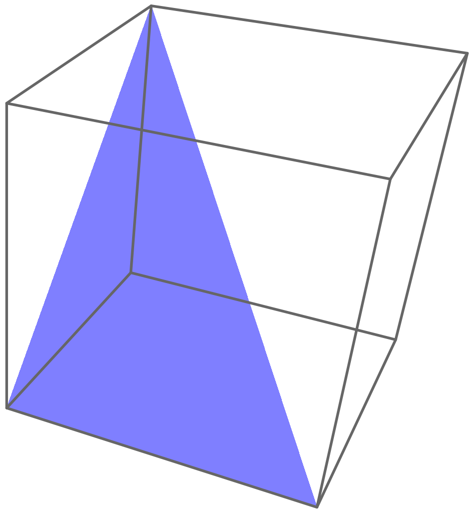
GraphicsBox

3D Graphics

Three-dimensional graphics are created using the function `Graphics3D` and a list of 3D primitives. The following primitives are supported so far:

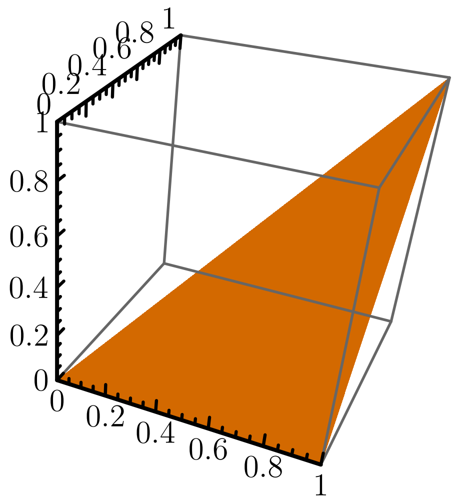
`Polygon[{{`*x1*`,` *y1*`,` *z1*`}, {`*x2*`,` *y2*`,` *z3*`}, ...}]`
    draws a filled polygon.
`Line[{{`*x1*`,` *y1*`,` *z1*`}, {`*x2*`,` *y2*`,` *z3*`}, ...}]`
    draws a line.
`Point[{`*x1*`,` *y1*`,` *z1*`}]`
    draws a point.

```
>> Graphics3D[Polygon[{{0,0,0},
   {0,1,1}, {1,0,0}}]]
```



Colors can also be added to three-dimensional primitives.

```
>> Graphics3D[{Orange, Polygon
   [{{0,0,0}, {1,1,1}, {1,0,0}}]},
   Axes->True]
```



`Graphics3D` produces a `Graphics3DBox`:

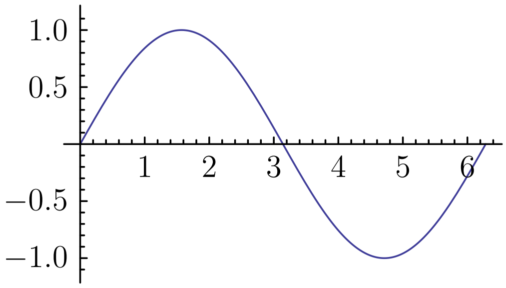
```
>> Head[ToBoxes[Graphics3D[{Polygon
   []}]]]
```

Graphics3DBox

Plotting Introduction Examples

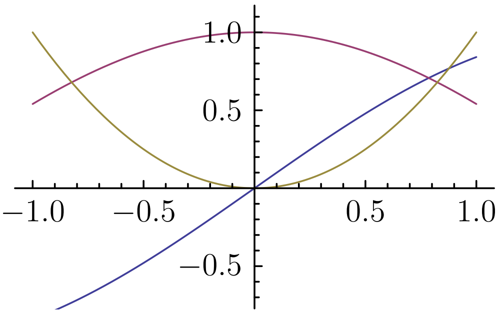
Mathics can plot functions:

```
>> Plot[Sin[x], {x, 0, 2 Pi}]
```



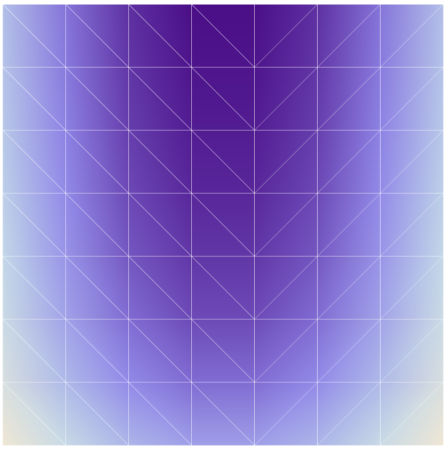
You can also plot multiple functions at once:

```
>> Plot[{Sin[x], Cos[x], x ^ 2}, {x
   , -1, 1}]
```



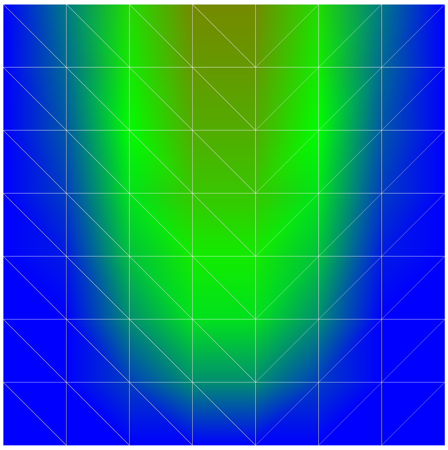
Two-dimensional functions can be plotted using `DensityPlot`:

```
>> DensityPlot[x ^ 2 + 1 / y, {x,
   -1, 1}, {y, 1, 4}]
```



You can use a custom coloring function:

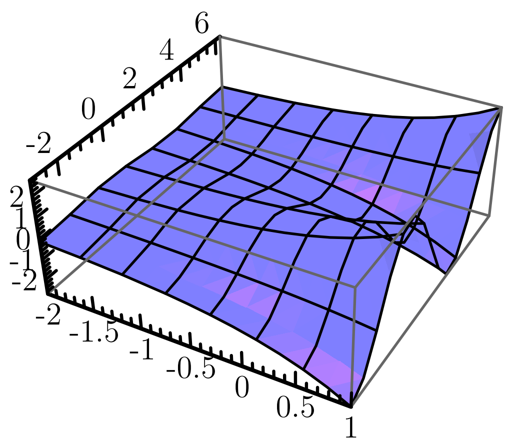
```
>> DensityPlot[x ^ 2 + 1 / y, {x,
   -1, 1}, {y, 1, 4}, ColorFunction
    -> (Blend[{Red, Green, Blue},
   #]&)]
```



One problem with `DensityPlot` is that it's still very slow, basically due to function evaluation being pretty slow in general—and `DensityPlot` has to evaluate a lot of functions.

Three-dimensional plots are supported as well:

```
>> Plot3D[Exp[x] Cos[y], {x, -2,
   1}, {y, -Pi, 2 Pi}]
```



4. Examples

Contents

Curve sketching

Let's sketch the function

```
>> f[x_] := 4 x / (x ^ 2 + 3 x + 5)
```

The derivatives are

```
>> {f'[x], f''[x], f'''[x]} //
   Together
```

$$\left\{\frac{-4\left(-5+x^2\right)}{\left(5+3x+x^2\right)^2}, \frac{8\left(-15-15x+x^3\right)}{\left(5+3x+x^2\right)^3}, \frac{-24\left(-20-60x-30x^2+x^4\right)}{\left(5+3x+x^2\right)^4}\right\}$$

To get the extreme values of `f`, compute the zeroes of the first derivatives:

```
>> extremes = Solve[f'[x] == 0, x]
```

$$\left\{\left\{x->-\sqrt{5}\right\},\left\{x->\sqrt{5}\right\}\right\}$$

And test the second derivative:

```
>> f''[x] /. extremes // N
```

$$\{1.65086, -0.064079\}$$

Thus, there is a local maximum at `x = Sqrt[5]` and a local minimum at `x = -Sqrt[5]`. Compute the inflection points numerically, chopping imaginary parts close to 0:

```
>> inflections = Solve[f''[x] == 0,
    x] // N // Chop
```

$$\{\{x->-1.0852\},\{x->-3.21463\},\{x->4.29983\}\}$$

Insert into the third derivative:

```
>> f'''[x] /. inflections
```

$$\{-3.67683, 0.694905, 0.00671894\}$$

Being different from 0, all three points are actual inflection points. `f` is not defined where its denominator is 0:

```
>> Solve[Denominator[f[x]] == 0, x]
```

$$\left\{\left\{x->-\frac{3}{2}-\frac{I}{2}\sqrt{11}\right\},\left\{x->-\frac{3}{2}+\frac{I}{2}\sqrt{11}\right\}\right\}$$

These are non-real numbers, consequently `f` is defined on all real numbers. The behaviour of `f` at the boundaries of its definition:

```
>> Limit[f[x], x -> Infinity]
```

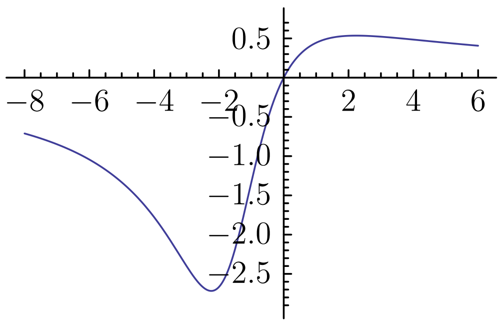
$$0$$

```
>> Limit[f[x], x -> -Infinity]
```

$$0$$

Finally, let's plot `f`:

```
>> Plot[f[x], {x, -8, 6}]
```



Linear algebra

Let's consider the matrix

```
>> A = {{1, 1, 0}, {1, 0, 1}, {0,
   1, 1}};
```

```
>> MatrixForm[A]
```

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

We can compute its eigenvalues and eigenvectors:

```
>> Eigenvalues[A]
```

$\{2, -1, 1\}$

```
>> Eigenvectors[A]
```

$\{\{1,1,1\}, \{1, -2, 1\}, \{-1, 0, 1\}\}$

This yields the diagonalization of `A`:

```
>> T = Transpose[Eigenvectors[A]];
   MatrixForm[T]
```

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & -2 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

```
>> Inverse[T] . A . T // MatrixForm
```

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
>> % == DiagonalMatrix[Eigenvalues[
   A]]
```

True

We can solve linear systems:

```
>> LinearSolve[A, {1, 2, 3}]
```

$\{0, 1, 2\}$

```
>> A . %
```

$\{1, 2, 3\}$

In this case, the solution is unique:

```
>> NullSpace[A]
```

$\{\}$

Let's consider a singular matrix:

```
>> B = {{1, 2, 3}, {4, 5, 6}, {7,
   8, 9}};
```

```
>> MatrixRank[B]
```

2

```
>> s = LinearSolve[B, {1, 2, 3}]
```

$$\left\{-\frac{1}{3}, \frac{2}{3}, 0\right\}$$

```
>> NullSpace[B]
```

$\{\{1, -2, 1\}\}$

```
>> B . (RandomInteger[100] * %[[1]]
    + s)
```

$\{1, 2, 3\}$

Dice

Let's play with dice in this example. A `Dice` object shall represent the outcome of a series of rolling a dice with six faces, e.g.:

```
>> Dice[1, 6, 4, 4]
```

Dice $[1, 6, 4, 4]$

Like in most games, the ordering of the individual throws does not matter. We can express this by making `Dice` `Orderless`:

```
>> SetAttributes[Dice, Orderless]
```

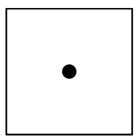
```
>> Dice[1, 6, 4, 4]
```

Dice $[1, 4, 4, 6]$

A dice object shall be displayed as a rectangle with the given number of points in it, positioned like on a traditional dice:

```
>> Format[Dice[n_Integer?(1 <= # <=
    6 &)]] := Block[{p = 0.2, r =
   0.05}, Graphics[{EdgeForm[Black
   ], White, Rectangle[], Black,
   EdgeForm[], If[OddQ[n], Disk
   [{0.5, 0.5}, r]], If[MemberQ[{2,
    3, 4, 5, 6}, n], Disk[{p, p}, r
   ]], If[MemberQ[{2, 3, 4, 5, 6},
   n], Disk[{1 - p, 1 - p}, r]], If
   [MemberQ[{4, 5, 6}, n], Disk[{p,
    1 - p}, r]], If[MemberQ[{4, 5,
   6}, n], Disk[{1 - p, p}, r]], If
   [n === 6, {Disk[{p, 0.5}, r],
   Disk[{1 - p, 0.5}, r]}]},
   ImageSize -> Tiny]]
```

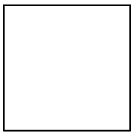
```
>> Dice[1]
```



The empty series of dice shall be displayed as an empty dice:

```
>> Format[Dice[]] := Graphics[{
   EdgeForm[Black], White,
   Rectangle[]}, ImageSize -> Tiny]
```

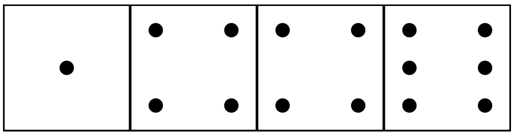
```
>> Dice[]
```



Any non-empty series of dice shall be displayed as a row of individual dice:

```
>> Format[Dice[d___Integer?(1 <= #
   <= 6 &)]] := Row[Dice /@ {d}]
```

```
>> Dice[1, 6, 4, 4]
```



Note that *Mathics* will automatically sort the given format rules according to their "generality", so the rule for the empty dice does not get overridden by the rule for a series of dice. We can still see the original form by using `InputForm`:

```
>> Dice[1, 6, 4, 4] // InputForm
```

Dice [1, 4, 4, 6]

We want to combine `Dice` objects using the + operator:

```
>> Dice[a___] + Dice[b___] ^:= Dice
   [Sequence @@ {a, b}]
```

The `^:=` (`UpSetDelayed`) tells *Mathics* to associate this rule with `Dice` instead of `Plus`, which is protected—we would have to unprotect it first:

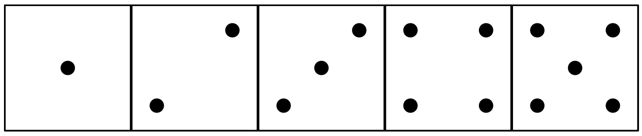
```
>> Dice[a___] + Dice[b___] := Dice[
   Sequence @@ {a, b}]
```

TagPlusinDice[a___]
+ Dice[b___]isProtected.

$Failed

We can now combine dice:

```
>> Dice[1, 5] + Dice[3, 2] + Dice
   [4]
```



Let's write a function that returns the sum of the rolled dice:

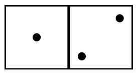
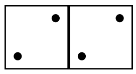
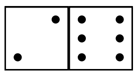
```
>> DiceSum[Dice[d___]] := Plus @@ {
   d}
```

```
>> DiceSum @ Dice[1, 2, 5]
```

8

And now let's put some dice into a table:

```
>> Table[{Dice[Sequence @@ d],
   DiceSum @ Dice[Sequence @@ d]},
   {d, {{1, 2}, {2, 2}, {2, 6}}}]
   // TableForm
```

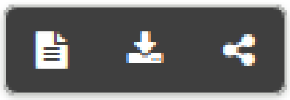
	3
	4
	8

It is not very sophisticated from a mathematical point of view, but it's beautiful.

5. Django-based Web Interface

In the future we plan on providing an interface to Jupyter as a separate package. However currently as part *Mathics*, we distribute a browser-based interface using Django 3.1. Since a Jupyter-based interface seems preferable to the home-grown interface described here, it is doubtful whether there will be future improvements to the this interface.

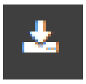
When you enter Mathics in the top after the Mathics logo and the word “Mathics” you’ll see a *menubar*.
It looks like this:



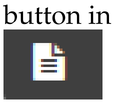
Contents

Saving, Loading, and Deleting Worksheets

<subsection title=“Saving Worksheets”>
Worksheets exist in the browser window only and are not stored on the server, by default. To save all your queries and results, use the *Save* button which is the middle graphic of the menu bar. It looks like this:



Depending on browser, desktop, and OS-settings, the "Ctrl+S" key combination may do the same thing.
<subsection title=“Loading and Deleting Worksheets”>
Saved worksheets can be loaded or deleted using the *File Open* button which is the left-most button in the menu bar. It looks like this:



Depending on browser, desktop, and OS-settings, the "Ctrl+O" key combination may do the same thing.
A popup menu should appear with the list of saved worksheets with an option to either load or delete the worksheet.

Persistence of Mathics Definitions in a Session

When you use the Django-based Web interface of *Mathics*, a browser session is created. Cookies have to be enabled to allow this. Your session holds a key which is used to access your definitions that are stored in a database on the server. As long as you don’t clear the cookies in your browser, your definitions will remain even when you close and re-open the browser.
This implies that you should not store sensitive, private information in *Mathics* variables when using the online Web interface. In addition to their values being stored in a database on the server, your queries might be saved for debugging purposes. However, the fact that they are transmitted over plain HTTP should make you aware that you should not transmit any sensitive information. When you want to do calculations with that kind of stuff, simply install *Mathics* locally!
If you are using a public terminal, to erase all your definitions and close the browser window. When you use *Mathics* in a browser, use the command `Quit[]` or its alias, `Exit[]`.
Normally, when you reload the current page in a browser using the default url, e.g

`http:localhost:8000`, all of the previous input and output disappears, even though definitions as described above do not, unless `Quit[]` or `Exit[]` is entered as described above.

However if you want a URL that will that records the input entered the *Generate Input Hash* button does this. The button looks like this:

For example, assuming you have a *Mathics* server running at port 8000 on `localhost`, and you enter the url `http://localhost:8000/#cXVlcmllcz14`, you should see a single line of input containing `x` entered.

Of course, what the value of this is when evaluated depends on whether `x` has been previously defined.

Keyboard Commands

There are some keyboard commands you can use in the Django-based Web interface of *Mathics*.

`Shift+Return`
: This evaluates the current cell (the most important one, for sure). On the right-hand side you may also see an "=" button which can be clicked to do the same thing.

`Ctrl+D`
: This moves the cursor over to the documentation pane on the right-hand side. From here you can preform a search for a pre-defined *Mathics* function, or symbol. Clicking on the "?" symbol on the right-hand side does the same thing.

`Ctrl+C`
: This moves the cursor back to document code pane area where you type *Mathics* expressions

`Ctrl+S`
: Save worksheet

`Ctrl+O`
: Open worksheet

Keyboard commands behavior depends the browser used, the operating system, desktop settings, and customization. We hook into the desktop “Open the current document” and “Save the current document” functions that many desktops provide. For example see: `https://help.ubuntu.com/community/KeyboardShortcuts#Finding_keyboard_shortcuts`

Often, these shortcut keyboard command are only recognized when a textfield has focus; otherwise,the browser might do some browser-specific actions, like setting a bookmark etc.

Part II.

Reference of Built-in Symbols

I. Evaluation

Contents

Evaluate

`Evaluate[`*expr*`]`
: forces evaluation of *expr*, even if it occurs inside a held argument or a `Hold` form.

Create a function *f* with a held argument:

```
>> SetAttributes[f, HoldAll]
```

```
>> f[1 + 2]
```
$f[1+2]$

`Evaluate` forces evaluation of the argument, even though *f* has the `HoldAll` attribute:

```
>> f[Evaluate[1 + 2]]
```
$f[3]$

```
>> Hold[Evaluate[1 + 2]]
```
Hold [3]

```
>> HoldComplete[Evaluate[1 + 2]]
```
HoldComplete [Evaluate [1 + 2]]

```
>> Evaluate[Sequence[1, 2]]
```
Sequence [1, 2]

$HistoryLength

`$HistoryLength`
: specifies the maximum number of `In` and `Out` entries.

```
>> $HistoryLength
```
100

```
>> $HistoryLength = 1;
```

```
>> 42
```
42

```
>> %
```
42

```
>> %%
```
%3

```
>> $HistoryLength = 0;
```

```
>> 42
```
42

```
>> %
```
%7

Hold

`Hold[`*expr*`]`
: prevents *expr* from being evaluated.

```
>> Attributes[Hold]
```
{HoldAll, Protected}

HoldComplete

`HoldComplete[`*expr*`]`
: prevents *expr* from being evaluated, and also prevents `Sequence` objects from being spliced into argument lists.

>> `Attributes[HoldComplete]`

{HoldAllComplete, Protected}

HoldForm

`HoldForm[`*expr*`]`
: is equivalent to `Hold[`*expr*`]`, but prints as *expr*.

>> `HoldForm[1 + 2 + 3]`

$1+2+3$

`HoldForm` has attribute `HoldAll`:

>> `Attributes[HoldForm]`

{HoldAll, Protected}

In

`In[`*k*`]`
: gives the *k*th line of input.

>> `x = 1`

1

>> `x = x + 1`

2

>> `Do[In[2], {3}]`

>> `x`

5

>> `In[-1]`

5

>> `Definition[In]`

$$\begin{aligned} \text{Attributes}[\text{In}] &= \{\text{Protected}\} \\ \text{In}[6] &= \text{Definition}[\text{In}] \\ \text{In}[5] &= \text{In}[-1] \\ \text{In}[4] &= x \\ \text{In}[3] &= \text{Do}\left[\text{In}[2], \{3\}\right] \\ \text{In}[2] &= x = x+1 \\ \text{In}[1] &= x = 1 \end{aligned}$$

$IterationLimit

`$IterationLimit`
: specifies the maximum number of times a reevaluation of an expression may happen.

Calculations terminated by `$IterationLimit` return `$Aborted`:
> $IterationLimit = 1000 # FIX Later # #> ClearAll[f]; # #> f[x_, 0] := x; f[x_, n_] := Module[{y = x + 1}, f[y, n - 1]]; # #> Block[{$IterationLimit = 20}, f[0, 100]] # = 100 # #> ClearAll[f];

$Line

`$Line`
: holds the current input line number.

>> `$Line`

1

>> `$Line`

2

>> `$Line = 12;`

>> `2 * 5`

10

>> `Out[13]`

10

>> `$Line = -1;`

Non − negativeintegerexpected.

Out

`Out[`*k*`]`
`%`*k*
: gives the result of the *k*th input line.

`%`, `%%`, etc.
: gives the result of the previous input line, of the line before the previous input line, etc.

>> `42`

42

>> `%`

42

```
>> 43;
```

```
>> %
```
43

```
>> 44
```
44

```
>> %1
```
42

```
>> %%
```
44

```
>> Hold[Out[-1]]
```
Hold [%]

```
>> Hold[%4]
```
Hold [%4]

```
>> Out[0]
```
Out [0]

Quit

`Quit[]`
: Terminates the Mathics session.

`Quit[n]`
: Terminates the mathics session with exit code *n*.

`Exit[]`
: Terminates the Mathics session.

`Exit[n]`
: Terminates the mathics session with exit code *n*.

$RecursionLimit

`$RecursionLimit`
: specifies the maximum allowable recursion depth after which a calculation is terminated.

Calculations terminated by `$RecursionLimit` return `$Aborted`:

```
>> a = a + a
```
Recursiondepthof200exceeded.

$Aborted

```
>> $RecursionLimit
```
200

```
>> $RecursionLimit = x;
```
Cannotset$RecursionLimittox; valuemustbeanintegerbetween

```
>> $RecursionLimit = 512
```
512

```
>> a = a + a
```
Recursiondepthof512exceeded.

$Aborted

ReleaseHold

`ReleaseHold[expr]`
: removes any `Hold`, `HoldForm`, `HoldPattern` or `HoldComplete` head from *expr*.

```
>> x = 3;
```

```
>> Hold[x]
```
Hold [x]

```
>> ReleaseHold[Hold[x]]
```
3

```
>> ReleaseHold[y]
```
y

Sequence

`Sequence[x1, x2, ...]`
: represents a sequence of arguments to a function.

`Sequence` is automatically spliced in, except when a function has attribute `SequenceHold` (like assignment functions).

```
>> f[x, Sequence[a, b], y]
```
$f\left[x,a,b,y\right]$

```
>> Attributes[Set]
```
{HoldFirst, Protected, SequenceHold}

```
>> a = Sequence[b, c];
```

```
>> a
```
Sequence [b, c]

Apply `Sequence` to a list to splice in arguments:

```
>> list = {1, 2, 3};
```

```
>> f[Sequence @@ list]
```

$f[1,2,3]$

Inside `Hold` or a function with a held argument, `Sequence` is spliced in at the first level of the argument:

```
>> Hold[a, Sequence[b, c], d]
```

$\text{Hold}[a,b,c,d]$

If `Sequence` appears at a deeper level, it is left unevaluated:

```
>> Hold[{a, Sequence[b, c], d}]
```

$\text{Hold}\left[\{a,\text{Sequence}[b,c],d\}\right]$

Unevaluated

`Unevaluated[`*expr*`]`
: temporarily leaves *expr* in an unevaluated form when it appears as a function argument.

`Unevaluated` is automatically removed when function arguments are evaluated:

```
>> Sqrt[Unevaluated[x]]
```

$\sqrt{x}$

```
>> Length[Unevaluated[1+2+3+4]]
```

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`Unevaluated` has attribute `HoldAllComplete`:

```
>> Attributes[Unevaluated]
```

$\{\text{HoldAllComplete},\text{Protected}\}$

`Unevaluated` is maintained for arguments to non-executed functions:

```
>> f[Unevaluated[x]]
```

$f[\text{Unevaluated}[x]]$

Likewise, its kept in flattened arguments and sequences:

```
>> Attributes[f] = {Flat};
```

```
>> f[a, Unevaluated[f[b, c]]]
```

$f[a,\text{Unevaluated}[b],\text{Unevaluated}[c]]$

```
>> g[a, Sequence[Unevaluated[b],
   Unevaluated[c]]]
```

$g[a,\text{Unevaluated}[b],\text{Unevaluated}[c]]$

However, unevaluated sequences are kept:

```
>> g[Unevaluated[Sequence[a, b, c
   ]]]
```

$g\left[\text{Unevaluated}\left[\text{Sequence}[a,b,c]\right]\right]$

II. Control Statements

Contents

Abort

`Abort[]`
: aborts an evaluation completely and returns `$Aborted`.

```
>> Print["a"]; Abort[]; Print["b"]
```

a

$Aborted

Break

`Break[]`
: exits a `For`, `While`, or `Do` loop.

```
>> n = 0;
```

```
>> While[True, If[n>10, Break[]]; n
   =n+1]
```

```
>> n
```

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Catch

`Catch[‘expr‘]`
: returns the argument of the first Throw generated in the evaluation of expr.

`Catch[‘expr‘, ‘form‘]`
: returns value from the first Throw[‘value‘,‘tag‘] for which form matches ‘tag‘.

`Catch[‘expr‘,‘form‘, ‘f‘]`
: returns the argument of the first ‘Throw‘ generated in the evaluation of ‘expr‘.

Exit to the enclosing Catch as soon as Throw is evaluated: « Catch[r; s; Throw[t]; u; v] = t
Define a function that can “throw an exception”: « f[x_] := If[x > 12, Throw[overflow], x!] = ... The result of Catch is just what is thrown by Throw: « Catch[f[1] + f[15]] = overflow « Catch[f[1]+f[4]] = 24

CompoundExpression (;)

`CompoundExpression[`*e1*`, `*e2*`, ...]`
e1`; `*e2*`; ...`
: evaluates its arguments in turn, returning the last result.

```
>> a; b; c; d
```

d

If the last argument is omitted, `Null` is taken:

```
>> a;
```

Continue

`Continue[]`
: continues with the next iteration in a `For`, `While`, or `Do` loop.

```
>> For[i=1, i<=8, i=i+1, If[Mod[i
,2] == 0, Continue[]]; Print[i]]
```

1
3
5
7

Do

`Do[`*expr*`, {`*max*`}]`
: evaluates *expr* *max* times.

`Do[`*expr*`, {`*i*`, `*max*`}]`
: evaluates *expr* *max* times, substituting *i* in *expr* with values from 1 to *max*.

`Do[`*expr*`, {`*i*`, `*min*`, `*max*`}]`
: starts with *i* = *max*.

`Do[`*expr*`, {`*i*`, `*min*`, `*max*`, `*step*`}]`
: uses a step size of *step*.

`Do[`*expr*`, {`*i*`, {`*i1*`, `*i2*`, ...}}]`
: uses values *i1*, *i2*, ... for *i*.

`Do[`*expr*`, {`*i*`, `*imin*`, `*imax*`}, {`*j*`, `*jmin*`, `*jmax*`}, ...]`
: evaluates *expr* for each *j* from *jmin* to *jmax*, for each *i* from *imin* to *imax*, etc.

```
>> Do[Print[i], {i, 2, 4}]
```

2
3
4

```
>> Do[Print[{i, j}], {i,1,2}, {j
,3,5}]
```

$\{1,3\}$
$\{1,4\}$
$\{1,5\}$
$\{2,3\}$
$\{2,4\}$
$\{2,5\}$

You can use `Break[]` and `Continue[]` inside `Do`:

```
>> Do[If[i > 10, Break[], If[Mod[i,
 2] == 0, Continue[]]; Print[i
]], {i, 5, 20}]
```

5
7
9

FixedPoint

`FixedPoint[`*f*`, `*expr*`]`
: starting with *expr*, iteratively applies *f* until the result no longer changes.

`FixedPoint[`*f*`, `*expr*`, `*n*`]`
: performs at most *n* iterations.

```
>> FixedPoint[Cos, 1.0]
```

0.739085

```
>> FixedPoint[#+1 &, 1, 20]
```

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FixedPointList

`FixedPointList[`*f*`, `*expr*`]`
: starting with *expr*, iteratively applies *f* until the result no longer changes, and returns a list of all intermediate results.

`FixedPointList[`*f*`, `*expr*`, `*n*`]`
: performs at most *n* iterations.

```
>> FixedPointList[Cos, 1.0, 4]
```

{1., 0.540302, 0.857~
 ~553, 0.65429, 0.79348}

Observe the convergence of Newton's method for approximating square roots:

```
>> newton[n_] := FixedPointList
[.5(# + n/#)&, 1.];
```

```
>> newton[9]
```

{1., 5., 3.4, 3.02353, 3.00009, 3., 3., 3.}

Plot the "hailstone" sequence of a number:

```
>> collatz[1] := 1;
```

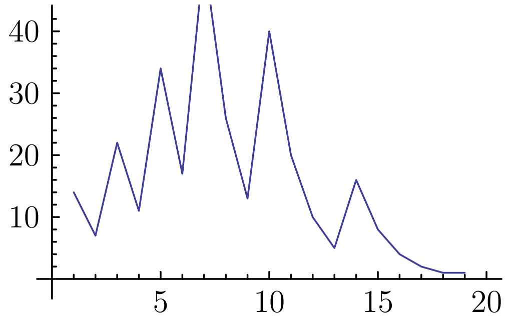
```
>> collatz[x_ ? EvenQ] := x / 2;
```

```
>> collatz[x_] := 3 x + 1;
```

```
>> list = FixedPointList[collatz, 14]
```

{14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 1}

```
>> ListLinePlot[list]
```



For

`For[start, test, incr, body]`
: evaluates *start*, and then iteratively *body* and *incr* as long as *test* evaluates to `True`.

`For[start, test, incr]`
: evaluates only *incr* and no *body*.

`For[start, test]`
: runs the loop without any body.

Compute the factorial of 10 using `For`:

```
>> n := 1
```

```
>> For[i=1, i<=10, i=i+1, n = n * i]
```

```
>> n
```

3 628 800

```
>> n == 10!
```

True

If

`If[cond, pos, neg]`
: returns *pos* if *cond* evaluates to `True`, and *neg* if it evaluates to `False`.

`If[cond, pos, neg, other]`
: returns *other* if *cond* evaluates to neither `True` nor `False`.

`If[cond, pos]`
: returns `Null` if *cond* evaluates to `False`.

```
>> If[1<2, a, b]
```

a

If the second branch is not specified, `Null` is taken:

```
>> If[1<2, a]
```

a

```
>> If[False, a] //FullForm
```

Null

You might use comments (inside `(*` and `*)`) to make the branches of `If` more readable:

```
>> If[a, (*then*)b, (*else*)c];
```

Interrupt

`Interrupt[]`
: Interrupt an evaluation and returns `$Aborted`.

```
>> Print["a"]; Interrupt[]; Print["b"]
```

a

$Aborted

Nest

`Nest[f, expr, n]`
: starting with *expr*, iteratively applies *f* *n* times and returns the final result.

```
>> Nest[f, x, 3]
```

$f[f[f[x]]]$

```
>> Nest[(1+#)^ 2 &, x, 2]
```

$\left(1 + (1 + x)^2\right)^2$

NestList

`NestList[f, expr, n]`
: starting with *expr*, iteratively applies *f* *n* times and returns a list of all intermediate results.

```
>> NestList[f, x, 3]
```

$\{x, f[x], f[f[x]], f[f[f[x]]]\}$

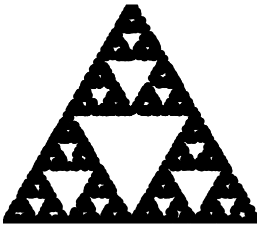
```
>> NestList[2 # &, 1, 8]
```
$\{1,2,4,8,16,32,64,128,256\}$

Chaos game rendition of the Sierpinski triangle:

```
>> vertices = {{0,0}, {1,0}, {.5,
   .5 Sqrt[3]}};
```

```
>> points = NestList[.5(vertices[[
   RandomInteger[{1,3}] ]] + #)&,
   {0.,0.}, 2000];
```

```
>> Graphics[Point[points],
   ImageSize->Small]
```



NestWhile

`NestWhile[`*f*`, `*expr*`, `*test*`]`
applies a function *f* repeatedly on an expression *expr*, until applying *test* on the result no longer yields `True`.

`NestWhile[`*f*`, `*expr*`, `*test*`, `*m*`]`
supplies the last *m* results to *test* (default value: 1).

`NestWhile[`*f*`, `*expr*`, `*test*`, All]`
supplies all results gained so far to *test*.

Divide by 2 until the result is no longer an integer:

```
>> NestWhile[#/2&, 10000, IntegerQ]
```
$\frac{625}{2}$

Return

`Return[`*expr*`]`
aborts a function call and returns *expr*.

```
>> f[x_] := (If[x < 0, Return[0]];
   x)
```

```
>> f[-1]
```
0

```
>> Do[If[i > 3, Return[]]; Print[i
   ], {i, 10}]
```
1
2
3

`Return` only exits from the innermost control flow construct.

```
>> g[x_] := (Do[If[x < 0, Return
   [0]], {i, {2, 1, 0, -1}}]; x)
```

```
>> g[-1]
```
-1

Switch

`Switch[`*expr*`, `*pattern1*`, `*value1*`, `*pattern2*`, `*value2*`, ...]`
yields the first *value* for which *expr* matches the corresponding *pattern*.

```
>> Switch[2, 1, x, 2, y, 3, z]
```
y

```
>> Switch[5, 1, x, 2, y]
```
Switch $[5,1,x,2,y]$

```
>> Switch[5, 1, x, 2, y, _, z]
```
z

```
>> Switch[2, 1]
```
Switchcalledwith2arguments.Switchmustbecalledwithanoddn

Switch $[2,1]$

Throw

`Throw['value']`
stops evaluation and returns 'value' as the value of the nearest enclosing Catch.

`Catch['value', 'tag']`
is caught only by 'Catch[expr,form]', where tag matches form.

Using Throw can affect the structure of what is returned by a function:
« NestList[#^2 + 1 &, 1, 7] = ... « Catch[NestList[If[# > 1000, Throw[#], #^2 + 1] &, 1, 7]] = 458330 « Throw[1] = Null

Which

`Which[`*cond1*`, `*expr1*`, `*cond2*`, `*expr2*`, ...]`
: yields *expr1* if *cond1* evaluates to `True`, *expr2* if *cond2* evaluates to `True`, etc.

```
>> n = 5;
```

```
>> Which[n == 3, x, n == 5, y]
```
y

```
>> f[x_] := Which[x < 0, -x, x ==
   0, 0, x > 0, x]
```

```
>> f[-3]
```
3

If no test yields `True`, `Which` returns `Null`:

```
>> Which[False, a]
```

If a test does not evaluate to `True` or `False`, evaluation stops and a `Which` expression containing the remaining cases is returned:

```
>> Which[False, a, x, b, True, c]
```
Which $[x, b, \text{True}, c]$

`Which` must be called with an even number of arguments:

```
>> Which[a, b, c]
```
Whichcalledwith3arguments.

Which $[a, b, c]$

While

`While[`*test*`, `*body*`]`
: evaluates *body* as long as *test* evaluates to `True`.

`While[`*test*`]`
: runs the loop without any body.

Compute the GCD of two numbers:

```
>> {a, b} = {27, 6};
```

```
>> While[b != 0, {a, b} = {b, Mod[a
   , b]}];
```

```
>> a
```
3

III. Date and Time

Contents

AbsoluteTime

`AbsoluteTime[]`
: gives the local time in seconds since epoch January 1, 1900, in your time zone.

`AbsoluteTime[{`*y*`, `*m*`, `*d*`, `*h*`, `*m*`, `*s*`}]`
: gives the absolute time specification corresponding to a date list.

`AbsoluteTime["`*string*`"]`
: gives the absolute time specification for a given date string.

`AbsoluteTime[{"`*string*`",{`*e1*`, `*e2*`, ...}}]`
: takgs the date string to contain the elements "*ei*".

```
>> AbsoluteTime[]
```
3.82644×10^9

```
>> AbsoluteTime[{2000}]
```
3 155 673 600

```
>> AbsoluteTime[{"01/02/03", {"Day
   ", "Month", "YearShort"}}]
```
3 253 046 400

```
>> AbsoluteTime["6 June 1991"]
```
2 885 155 200

```
>> AbsoluteTime[{"6-6-91", {"Day",
   "Month", "YearShort"}}]
```
2 885 155 200

AbsoluteTiming

`AbsoluteTiming[`*expr*`]`
: evaluates *expr*, returning a list of the absolute number of seconds in real time that have elapsed, together with the result obtained.

```
>> AbsoluteTiming[50!]
```
{0.00024724, 30 414 093 201 713 378~
~043 612 608 166 064 768 844 377~
~641 568 960 512 000 000 000 000}

```
>> Attributes[AbsoluteTiming]
```
{HoldAll, Protected}

DateDifference

`DateDifference[`*date1*`, `*date2*`]`
: returns the difference between *date1* and *date2* in days.

`DateDifference[`*date1*`, `*date2*`, `*unit*`]`
: returns the difference in the specified *unit*.

`DateDifference[`*date1*`, `*date2*`, {`*unit1*`, `*unit2*`, ...}]`
: represents the difference as a list of integer multiples of each *unit*, with any remainder expressed in the smallest unit.

```
>> DateDifference[{2042, 1, 4},
   {2057, 1, 1}]
```
5 476

```
>> DateDifference[{1936, 8, 14},
   {2000, 12, 1}, "Year"]
```
{64.3425, Year}

```
>> DateDifference[{2010, 6, 1},
   {2015, 1, 1}, "Hour"]
```
{40 200, Hour}

```
>> DateDifference[{2003, 8, 11},
   {2003, 10, 19}, {"Week", "Day"}]
```
{{9, Week} , {6, Day}}

DateList

`DateList[]`
: returns the current local time in the form {*year*, *month*, *day*, *hour*, *minute*, *second*}.

`DateList[`*time*`]`
: returns a formatted date for the number of seconds *time* since epoch Jan 1 1900.

`DateList[{`*y*`, `*m*`, `*d*`, `*h*`, `*m*`, `*s*`}]`
: converts an incomplete date list to the standard representation.

`DateString[`*string*`]`
: returns the formatted date list of a date string specification.

`DateString[`*string*`, {`*e1*`, `*e2*`, ...}]`
: returns the formatted date list of a *string* obtained from elements *ei*.

```
>> DateList[0]
```
{1 900, 1, 1, 0, 0, 0.}

```
>> DateList[3155673600]
```
{2 000, 1, 1, 0, 0, 0.}

```
>> DateList[{2003, 5, 0.5, 0.1,
   0.767}]
```
{2 003, 4, 30, 12, 6, 46.02}

```
>> DateList[{2012, 1, 300., 10}]
```
{2 012, 10, 26, 10, 0, 0.}

```
>> DateList["31/10/1991"]
```
{1 991, 10, 31, 0, 0, 0.}

```
>> DateList["1/10/1991"]
```
*Theinterpretationof*1/10/
1 991*isambiguous.*

{1 991, 1, 10, 0, 0, 0.}

```
>> DateList[{"31/10/91", {"Day", "
   Month", "YearShort"}}]
```
{1 991, 10, 31, 0, 0, 0.}

```
>> DateList[{"31 10/91", {"Day", "
   ", "Month", "/", "YearShort"}}]
```
{1 991, 10, 31, 0, 0, 0.}

If not specified, the current year assumed

```
>> DateList[{"5/18", {"Month", "Day
   "}}]
```
{2 021, 5, 18, 0, 0, 0.}

DatePlus

`DatePlus[`*date*`, `*n*`]`
: finds the date *n* days after *date*.

`DatePlus[`*date*`, {`*n*`, "`*unit*`"}]`
: finds the date *n* units after *date*.

`DatePlus[`*date*`, {{`*n1*`, "`*unit1*`"}, {`*n2*`, "`*unit2*`"}, ...}]`
: finds the date which is *n_i* specified units after *date*.

`DatePlus[`*n*`]`
: finds the date *n* days after the current date.

`DatePlus[`*offset*`]`
: finds the date which is offset from the current date.

Add 73 days to Feb 5, 2010:

```
>> DatePlus[{2010, 2, 5}, 73]
```
{2 010, 4, 19}

Add 8 weeks and 1 day to March 16, 1999:

```
>> DatePlus[{2010, 2, 5}, {{8, "
   Week"}, {1, "Day"}}]
```
{2 010, 4, 3}

DateString

`DateString[]`
: returns the current local time and date as a string.

`DateString[`*elem*`]`
: returns the time formatted according to *elems*.

`DateString[{`*e1*`, `*e2*`, ...}]`
: concatinates the time formatted according to elements *ei*.

`DateString[`*time*`]`
: returns the date string of an AbsoluteTime.

`DateString[{`*y*`, `*m*`, `*d*`, `*h*`, `*m*`, `*s*`}]`
: returns the date string of a date list specification.

`DateString[`*string*`]`
: returns the formatted date string of a date string specification.

`DateString[`*spec*`, `*elems*`]`
: formats the time in turns of *elems*. Both *spec* and *elems* can take any of the above formats.

The current date and time:

```
>> DateString[];
```

```
>> DateString[{1991, 10, 31, 0, 0},
    {"Day", " ", "MonthName", " ",
   "Year"}]
```

31 October 1 991

```
>> DateString[{2007, 4, 15, 0}]
```

Sun 15 Apr 2 007 00:00:00

```
>> DateString[{1979, 3, 14}, {"
   DayName", " ", "Month", "-", "
   YearShort"}]
```

Wednesday 03-79

Non-integer values are accepted too:

```
>> DateString[{1991, 6, 6.5}]
```

Thu 6 Jun 1 991 12:00:00

$DateStringFormat

`$DateStringFormat`
: gives the format used for dates generated by `DateString`.

```
>> $DateStringFormat
```

$\{$DateTimeShort$\}$

EasterSunday

`EasterSunday[`*year*`]`
: returns the date of the Gregorian Easter Sunday as {year, month, day}.

```
>> EasterSunday[2000]
```

$\{2\,000, 4, 23\}$

```
>> EasterSunday[2030]
```

$\{2\,030, 4, 21\}$

Pause

`Pause[n]`
: pauses for *n* seconds.

```
>> Pause[0.5]
```

SessionTime

`SessionTime[]`
: returns the total time in seconds since this session started.

```
>> SessionTime[]
```

92.8816

$SystemTimeZone

`$SystemTimeZone`
: gives the current time zone for the computer system on which Mathics is being run.

```
>> $SystemTimeZone
```

$-5.$

TimeConstrained

`TimeConstrained[`*expr*`, `*t*`]`
: evaluates *expr*, stopping after *t* seconds.

`TimeConstrained[`*expr*`, `*t*`, `*failexpr*`]`
: returns *failexpr* if the time constraint is not met.

```
>> TimeConstrained[Integrate[Sin[x
   ]^1000000,x],1]
```

$Aborted

```
>> TimeConstrained[Integrate[Sin[x
   ]^1000000,x], 1, Integrate[Cos[x
   ],x]]
```

$\text{Sin}[x]$

```
>> s=TimeConstrained[Integrate[Sin[
   x] ^ 3, x], a]
```

$Number of seconds a is not a positive machine - sized number or Infinity.$

$$\text{TimeConstrained}\left[\int \text{Sin}[x]^3\, dx, a\right]$$

```
>> a=1; s
```

$$-\text{Cos}[x] + \frac{\text{Cos}[x]^3}{3}$$

Possible issues: for certain time-consuming functions (like simplify) which are based on sympy or other libraries, it is possible that the evaluation continues after the timeout. However, at the end of the evaluation, the function will return $
Aborted and the results will not affect the state of the mathics kernel.

TimeRemaining

`TimeRemaining[]`
: Gives the number of seconds remaining until the earliest enclosing `TimeConstrained` will request the current computation to stop.

`TimeConstrained[`*expr*`, `*t*`, `*failexpr*`]`
: returns *failexpr* if the time constraint is not met.

If TimeConstrained is called out of a TimeConstrained expression, returns 'Infinity'

```
>> TimeRemaining[]
```

∞

```
>> TimeConstrained[1+2; Print[
   TimeRemaining[]], 0.9]
```

0.899331

TimeUsed

`TimeUsed[]`
: returns the total CPU time used for this session, in seconds.

```
>> TimeUsed[]
```

321.542

$TimeZone

`$TimeZone`
: gives the current time zone to assume for dates and times.

```
>> $TimeZone
```

$-5.$

Timing

`Timing[`*expr*`]`
: measures the processor time taken to evaluate *expr*. It returns a list containing the measured time in seconds and the result of the evaluation.

```
>> Timing[50!]
```

{0.00024226, 30 414 093 201 713 378~
~043 612 608 166 064 768 844 377~
~641 568 960 512 000 000 000 000}

```
>> Attributes[Timing]
```

{HoldAll, Protected}

IV. Graphics (3D)

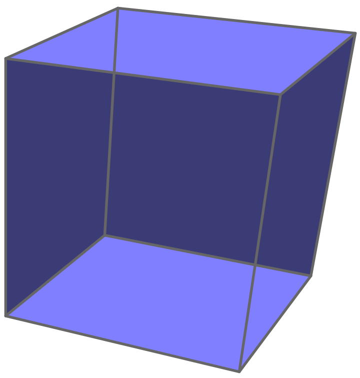
Contents

Cuboid

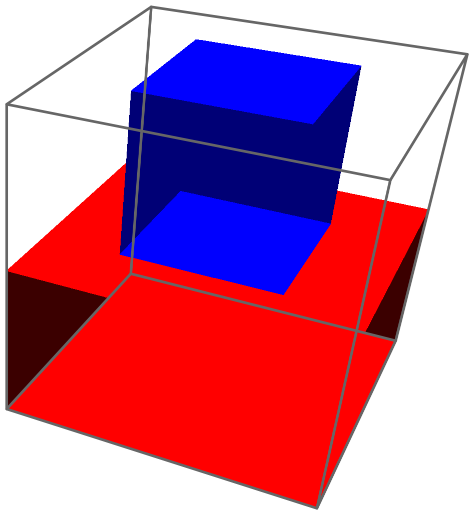
`Cuboid[{`*xmin*`, `*ymin*`, `*zmin*`}]`
: is a unit cube.

`Cuboid[{`*xmin*`, `*ymin*`, `*zmin*`}, {`*xmax*`, `*ymax*`, `*zmax*`}]`
: represents a cuboid extending from {*xmin*, *ymin*, *zmin*} to {*xmax*, *ymax*, *zmax*}.

```
>> Graphics3D[Cuboid[{0, 0, 1}]]
```



```
>> Graphics3D[{Red, Cuboid[{0, 0, 0}, {1, 1, 0.5}], Blue, Cuboid[{0.25, 0.25, 0.5}, {0.75, 0.75, 1}]}]
```

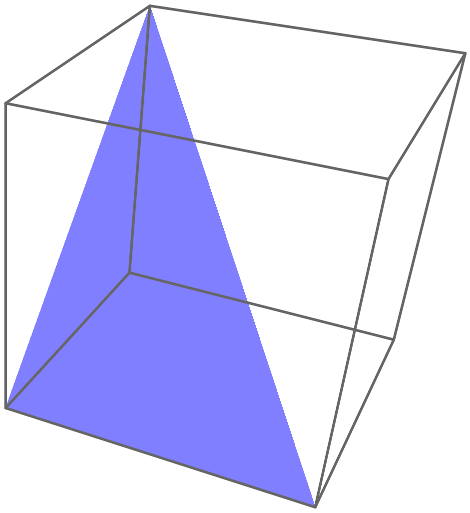


Graphics3D

`Graphics3D[`*primitives*`, `*options*`]`
: represents a three-dimensional graphic.

See also the Section "Plotting" for a list of Plot options.

>> `Graphics3D[Polygon[{{0,0,0}, {0,1,1}, {1,0,0}}]]`



In `TeXForm`, `Graphics3D` creates Asymptote figures:

>> `Graphics3D[Sphere[]] // TeXForm`

\begin{asy}
import three;
import solids;
size(6.6667cm, 6.6667cm);
currentprojection=perspective(2.6,-4.8,4.0);
currentlight=light(rgb(0.5,0.5,1),
specular=red, (2,0,2), (2,2,2), (0,2,2));
draw(surface(sphere((0, 0, 0), 1)),
rgb(1,1,1));
draw(((-1,-1,-1)–(1,-1,-1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((-1,1,-1)–(1,1,-1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((-1,-1,1)–(1,-1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((-1,1,1)–(1,1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((-1,-1,-1)–(-1,1,-1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((1,-1,-1)–(1,1,-1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((-1,-1,1)–(-1,1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((1,-1,1)–(1,1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((-1,-1,-1)–(-1,-1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((1,-1,-1)–(1,-1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((-1,1,-1)–(-1,1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((1,1,-1)–(1,1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
\end{asy}

Graphics3DBox

Line3DBox

Point3DBox

Polygon3DBox

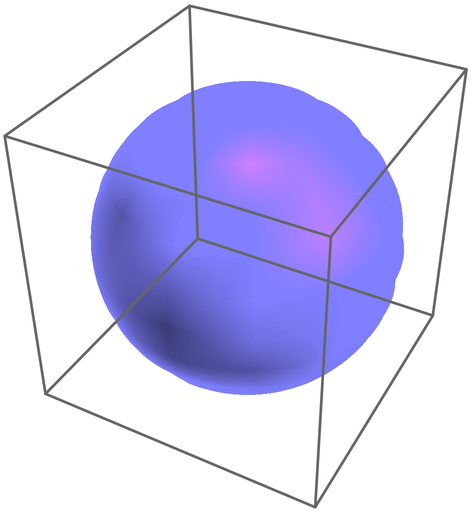
Sphere

`Sphere[{`x`, `y`, `z`}]`
: is a sphere of radius 1 centered at the point $\{x, y, z\}$.

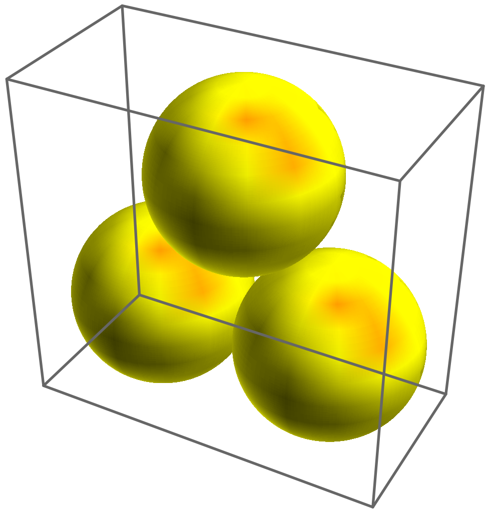
`Sphere[{`x`, `y`, `z`}, `r`]`
: is a sphere of radius r centered at the point $\{x, y, z\}$.

`Sphere[{{`$x1$`, `$y1$`, `$z1$`}, {`$x2$`, `$y2$`, `$z2$`}, ... }, `r`]`
: is a collection spheres of radius r centered at the points $\{x1, y2, z2\}$, $\{x2, y2, z2\}$, ...

```
>> Graphics3D[Sphere[{0, 0, 0}, 1]]
```



```
>> Graphics3D[{Yellow, Sphere[{{-1, 0, 0}, {1, 0, 0}, {0, 0, Sqrt[3.]}}, 1]}]
```



Sphere3DBox

V. Input and Output

Contents

BaseForm

`BaseForm[`*expr*`, `*n*`]`
prints numbers in *expr* in base *n*.

```
>> BaseForm[33, 2]
```
$100\,001_2$

```
>> BaseForm[234, 16]
```
ea_{16}

```
>> BaseForm[12.3, 2]
```
$1\,100.010011001100110011_2$

```
>> BaseForm[-42, 16]
```
$-2a_{16}$

```
>> BaseForm[x, 2]
```
x

```
>> BaseForm[12, 3] // FullForm
```
BaseForm [12, 3]

Bases must be between 2 and 36:

```
>> BaseForm[12, -3]
```
Positivemachine −sizedintegerexpectedatposition2inBaseForm[12, − 3].

MakeBoxes[*BaseForm*[12, − 3], *StandardForm*]*isnotavalidboxstructure.*

```
>> BaseForm[12, 100]
```
*Requestedbase*100*mustbebetween*2*and*36.

MakeBoxes[*BaseForm*[12, 100], *StandardForm*]*isnotavalidboxstructure.*

BoxData

`BoxData[...]`
is a low-level representation of the contents of a typesetting cell.

ButtonBox

`ButtonBox[boxes]`
: is a low-level box construct that represents a button in a notebook expression.

Center

`Center`
: is used with the `ColumnAlignments` option to `Grid` or `TableForm` to specify a centered column.

Check

`Check[expr, failexpr]`
: evaluates *expr*, and returns the result, unless messages were generated, in which case it evaluates and *failexpr* will be returned.

`Check[expr, failexpr, {s1::t1,s2::t2,...}]`
: checks only for the specified messages.

Return err when a message is generated:

```
>> Check[1/0, err]
```

Infiniteexpression1/0encountered.

err

Check only for specific messages:

```
>> Check[Sin[0^0], err, Sin::argx]
```

Indeterminateexpression0^0encountered.

Indeterminate

```
>> Check[1/0, err, Power::infy]
```

Infiniteexpression1/0encountered.

err

Format

`Format[expr]`
: holds values specifying how *expr* should be printed.

Assign values to `Format` to control how particular expressions should be formatted when printed to the user.

```
>> Format[f[x___]] := Infix[{x}, "~"]
```

```
>> f[1, 2, 3]
```

$1 \sim 2 \sim 3$

```
>> f[1]
```

1

Raw objects cannot be formatted:

```
>> Format[3] = "three";
```

Cannotassigntorawobject3.

Format types must be symbols:

```
>> Format[r, a + b] = "r";
```

Formattypea + bisnotasymbol.

Formats must be attached to the head of an expression:

```
>> f /: Format[g[f]] = "my f";
```

Tagfnotfoundortoodeepforanassignedrule.

FullForm

`FullForm[expr]`
: displays the underlying form of *expr*.

```
>> FullForm[a + b * c]
```

Plus [a, Times [b, c]]

```
>> FullForm[2/3]
```

Rational [2, 3]

```
>> FullForm["A string"]
```

"A string"

General

`General`
: is a symbol to which all general-purpose messages are assigned.

```
>> General::argr
```

'1' called with 1 argument;
  '2' arguments are expected.

```
>> Message[Rule::argr, Rule, 2]
```

Rulecalledwith1argument; 2argumentsareexpected.

Grid

`Grid[{{`*a1*`, `*a2*`, ...}, {`*b1*`, `*b2*`, ...}, ...}]`
: formats several expressions inside a `GridBox`.

```
>> Grid[{{a, b}, {c, d}}]
```

a	*b*
c	*d*

GridBox

`GridBox[{{...}, {...}}]`
: is a box construct that represents a sequence of boxes arranged in a grid.

» MathMLForm[TableForm[{{a,b},{c,d}}]] # =

...

Infix

`Infix[`*expr*`, `*oper*`, `*prec*`, `*assoc*`]`
: displays *expr* with the infix operator *oper*, with precedence *prec* and associativity *assoc*.

`Infix` can be used with `Format` to display certain forms with user-defined infix notation:

```
>> Format[g[x_, y_]] := Infix[{x, y
   }, "#", 350, Left]
```

```
>> g[a, g[b, c]]
```

a# (*b*#*c*)

```
>> g[g[a, b], c]
```

a#*b*#*c*

```
>> g[a + b, c]
```

$(a+b)$ #*c*

```
>> g[a * b, c]
```

ab#*c*

```
>> g[a, b] + c
```

$c+a$#*b*

```
>> g[a, b] * c
```

c (*a*#*b*)

```
>> Infix[{a, b, c}, {"+", "-"}]
```

$a+b-c$

InputForm

`InputForm[`*expr*`]`
: displays *expr* in an unambiguous form suitable for input.

```
>> InputForm[a + b * c]
```

$a+b*c$

```
>> InputForm["A string"]
```

"A string"

```
>> InputForm[f'[x]]
```

Derivative [1] $[f]$ $[x]$

```
>> InputForm[Derivative[1, 0][f][x
   ]]
```

Derivative [1, 0] $[f]$ $[x]$

InterpretationBox

`InterpretationBox[{...}]`
: is a low-level box construct that displays as boxes but is interpreted on input as expr.

Left

`Left`
: is used with operator formatting constructs to specify a left-associative operator.

MakeBoxes

`MakeBoxes[`*expr*`]`
: is a low-level formatting primitive that converts *expr* to box form, without evaluating it.

`\( ... \)`
: directly inputs box objects.

String representation of boxes

```
>> \(x \^ 2\)
```
SuperscriptBox [x, 2]

```
>> \(x \_ 2\)
```
SubscriptBox [x, 2]

```
>> \( a \+ b \% c\)
```
UnderoverscriptBox [a, b, c]

```
>> \( a \& b \% c\)
```
UnderoverscriptBox [a, c, b]

```
>> \(x \& y \)
```
OverscriptBox [x, y]

```
>> \(x \+ y \)
```
UnderscriptBox [x, y]

MathMLForm

`MathMLForm[`*expr*`]`
: displays *expr* as a MathML expression.

```
>> MathMLForm[HoldForm[Sqrt[a^3]]]
```
<math display="block"><mstyle mathvariant="sans-serif"> <msqrt><msup><mi>a</mi> <mn>3</mn></msup> </msqrt></mstyle></math>

```
>> MathMLForm[\[Mu]]
```
<math display="block"><mstyle mathvariant="sans-serif"> <mi></mi></mstyle></math>

This can causes the TeX to fail # » MathMLForm[Graphics[Text[“”]]] # = ...
= ...

MatrixForm

`MatrixForm[`*m*`]`
: displays a matrix *m*, hiding the underlying list structure.

```
>> Array[a,{4,3}]//MatrixForm
```
$$\begin{pmatrix} a[1,1] & a[1,2] & a[1,3] \\ a[2,1] & a[2,2] & a[2,3] \\ a[3,1] & a[3,2] & a[3,3] \\ a[4,1] & a[4,2] & a[4,3] \end{pmatrix}$$

Message

`Message[`*symbol*`::`*msg*`, `*expr1*`, `*expr2*`, ...]`
: displays the specified message, replacing placeholders in the message text with the corresponding expressions.

```
>> a::b = "Hello world!"
```
Hello world!

```
>> Message[a::b]
```
Helloworld!

```
>> a::c := "Hello `1`, Mr 00`2`!"
```

```
>> Message[a::c, "you", 3 + 4]
```
Helloyou, *Mr*007!

MessageName (::)

`MessageName[`*symbol*`, `*tag*`]`
symbol`::`*tag*
: identifies a message.

`MessageName` is the head of message IDs of the form `symbol::tag`.

```
>> FullForm[a::b]
```
MessageName [*a*, "b"]

The second parameter `tag` is interpreted as a string.

```
>> FullForm[a::"b"]
```
MessageName [*a*, "b"]

NonAssociative

`NonAssociative`
: is used with operator formatting constructs to specify a non-associative operator.

NumberForm

`NumberForm[`*expr*`, `*n*`]`
: prints a real number *expr* with *n*-digits of precision.

`NumberForm[`*expr*`, {`*n*`, `*f*`}]`
: prints with *n*-digits and *f* digits to the right of the decimal point.

```
>> NumberForm[N[Pi], 10]
```
3.141592654

```
>> NumberForm[N[Pi], {10, 5}]
```
3.14159

Off

`Off[`*symbol*`::`*tag*`]`
: turns a message off so it is no longer printed.

```
>> Off[Power::infy]
```

```
>> 1 / 0
```
ComplexInfinity

```
>> Off[Power::indet, Syntax::com]
```

```
>> {0 ^ 0,}
```
{Indeterminate, Null}

On

`On[`*symbol*`::`*tag*`]`
: turns a message on for printing.

```
>> Off[Power::infy]
```

```
>> 1 / 0
```
ComplexInfinity

```
>> On[Power::infy]
```

```
>> 1 / 0
```
Infiniteexpression1/0encountered.

ComplexInfinity

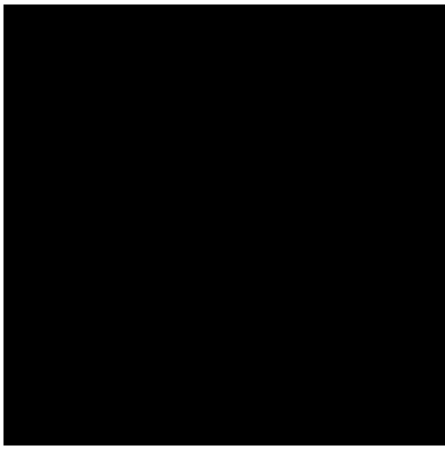
OutputForm

`OutputForm[`*expr*`]`
: displays *expr* in a plain-text form.

```
>> OutputForm[f'[x]]
```
$f'[x]$

```
>> OutputForm[Derivative[1, 0][f][x
   ]]
```
Derivative $[1,0]$ $[f]$ $[x]$

```
>> OutputForm["A string"]
```
A string

```
>> OutputForm[Graphics[Rectangle
   []]]
```


Postfix (//)

x `//` *f*
: is equivalent to $f[x]$.

```
>> b // a
```
$a[b]$

```
>> c // b // a
```
$a[b[c]]$

The postfix operator `//` is parsed to an expression before evaluation:

```
>> Hold[x // a // b // c // d // e
   // f]
```
Hold $[f[e[d[c[b[a[x]]]]]]]$

Precedence

`Precedence[op]`
: returns the precedence of the built-in operator *op*.

```
>> Precedence[Plus]
```
310.

```
>> Precedence[Plus] < Precedence[Times]
```
True

Unknown symbols have precedence 670:

```
>> Precedence[f]
```
670.

Other expressions have precedence 1000:

```
>> Precedence[a + b]
```
1 000.

Prefix (@)

f @ *x*
: is equivalent to *f* [*x*] .

```
>> a @ b
```
$a[b]$

```
>> a @ b @ c
```
$a[b[c]]$

```
>> Format[p[x_]] := Prefix[{x}, "*"]
```

```
>> p[3]
```
$*3$

```
>> Format[q[x_]] := Prefix[{x}, "~", 350]
```

```
>> q[a+b]
```
$\sim (a+b)$

```
>> q[a*b]
```
$\sim ab$

```
>> q[a]+b
```
$b+ \sim a$

The prefix operator @ is parsed to an expression before evaluation:

```
>> Hold[a @ b @ c @ d @ e @ f @ x]
```
$\text{Hold}[a[b[c[d[e[f[x]]]]]]]$

Print

`Print[expr, ...]`
: prints each *expr* in string form.

```
>> Print["Hello world!"]
```
Helloworld!

```
>> Print["The answer is ", 7 * 6, "."]
```
*Theansweris*42.

PythonForm

`PythonForm[expr]`
: returns an approximate equivalent of *expr* in Python, when that is possible. We assume that Python has sympy imported. No explicit import will be include in the result.

```
>> PythonForm[Infinity]
```
math.inf

```
>> PythonForm[Pi]
```
sympy.pi

```
>> E // PythonForm
```
sympy.E

```
>> {1, 2, 3} // PythonForm
```
[1, 2, 3]

Quiet

`Quiet[`*expr*`, {`*s1*`::`*t1*`, ...}]`
: evaluates *expr*, without messages `{`*s1*`::`*t1*`, ...}` being displayed.

`Quiet[`*expr*`, All]`
: evaluates *expr*, without any messages being displayed.

`Quiet[`*expr*`, None]`
: evaluates *expr*, without all messages being displayed.

`Quiet[`*expr*`, `*off*`, `*on*`]`
: evaluates *expr*, with messages *off* being suppressed, but messages *on* being displayed.

```
>> a::b = "Hello";
```

```
>> Quiet[x+x, {a::b}]
```
$2x$

```
>> Quiet[Message[a::b]; x+x, {a::b
}]
```
$2x$

```
>> Message[a::b]; y=Quiet[Message[a
::b]; x+x, {a::b}]; Message[a::b
]; y
```
Hello
Hello
$2x$

```
>> Quiet[expr, All, All]
```
Arguments2and3ofQuiet[expr, All, All]shouldnotbothbeAll.

Quiet [expr, All, All]

```
>> Quiet[x + x, {a::b}, {a::b}]
```
InQuiet[x + x, {a :: b}, {a :: b}]themessagename(s){a :: b}appearinboththeli... *switchon.*

Quiet $[x + x, \{a::b\}, \{a::b\}]$

Right

`Right`
: is used with operator formatting constructs to specify a right-associative operator.

Row

`Row[{`*expr*`, ...}]`
: formats several expressions inside a `RowBox`.

RowBox

`RowBox[{...}]`
: is a box construct that represents a sequence of boxes arranged in a horizontal row.

StandardForm

`StandardForm[`*expr*`]`
: displays *expr* in the default form.

```
>> StandardForm[a + b * c]
```
$a + bc$

```
>> StandardForm["A string"]
```
A string

`StandardForm` is used by default:

```
>> "A string"
```
A string

```
>> f'[x]
```
$f'[x]$

StringForm

`StringForm[`*str*`, `*expr1*`, `*expr2*`, ...]`
: displays the string *str*, replacing placeholders in *str* with the corresponding expressions.

```
>> StringForm["`1` bla `2` blub ``
bla `2`", a, b, c]
```
a bla *b* blub *c* bla *b*

Style

StyleBox

`StyleBox[boxes, options]`
: is a low-level representation of boxes to be shown with the specified option settings.

`StyleBox[boxes, style]`
: uses the option setting for the specified style in the current notebook.

Subscript

`Subscript[a, i]`
: displays as *a_i*.

```
>> Subscript[x,1,2,3] // TeXForm
```

x_{1,2,3}

SubscriptBox

Subsuperscript

`Subsuperscript[a, b, c]`
: displays as a_b^c.

```
>> Subsuperscript[a, b, c] //
   TeXForm
```

a_b^c

SubsuperscriptBox

Superscript

`Superscript[x, y]`
: displays as *x^y*.

```
>> Superscript[x,3] // TeXForm
```

x^3

SuperscriptBox

SympyForm

`SympyForm[expr]`
: returns an Sympy *expr* in Python. Sympy is used internally to implement a number of Mathics functions, like Simplify.

```
>> SympyForm[Pi^2]
```

pi**2

```
>> E^2 + 3E // SympyForm
```

exp(2) + 3*E

Syntax

`Syntax`
: is a symbol to which all syntax messages are assigned.

```
>> 1 +
```

```
>> Sin[1)
```

```
>> ^ 2
```

```
>> 1.5``
```

TableForm

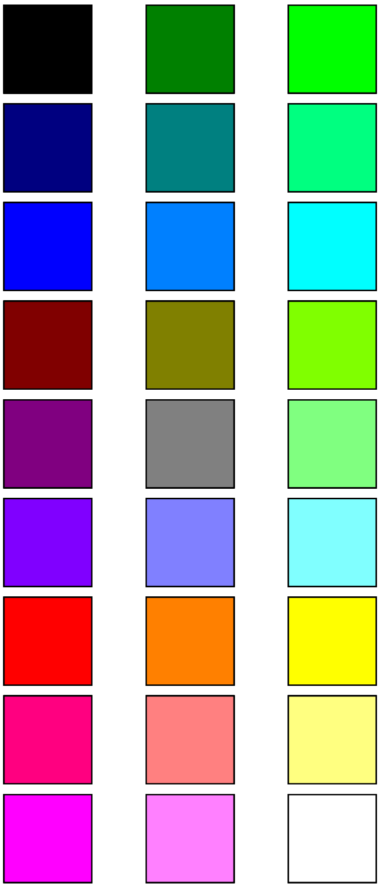
`TableForm[expr]`
: displays *expr* as a table.

```
>> TableForm[Array[a, {3,2}],
   TableDepth->1]
```

$\{a[1,1], a[1,2]\}$
$\{a[2,1], a[2,2]\}$
$\{a[3,1], a[3,2]\}$

A table of Graphics:

```
>> Table[Style[Graphics[{EdgeForm[{
   Black}], RGBColor[r,g,b],
   Rectangle[]}],
   ImageSizeMultipliers->{0.2, 1}],
    {r,0,1,1/2}, {g,0,1,1/2}, {b
   ,0,1,1/2}] // TableForm
```



TagBox

`TagBox[boxes, tag]`
: is a low-level box construct that displays as boxes but is interpreted on input as expr

TeXForm

`TeXForm[`*expr*`]`
: displays *expr* using TeX math mode commands.

```
>> TeXForm[HoldForm[Sqrt[a^3]]]
```
\sqrt{a^3}

TemplateBox

`TemplateBox[{`*box_1*`,` *box_2*`,...}, tag]`
: is a low-level box structure that parameterizes the display and evaluation of the boxes *box_i* .

TextData

`TextData[...]`
: is a low-level representation of the contents of a textual cell.

ToBoxes

`ToBoxes[`*expr*`]`
: evaluates *expr* and converts the result to box form.

Unlike `MakeBoxes`, `ToBoxes` evaluates its argument:

```
>> ToBoxes[a + a]
```
RowBox [{2, , a}]

```
>> ToBoxes[a + b]
```
RowBox [{a, +, b}]

```
>> ToBoxes[a ^ b] // FullForm
```
SuperscriptBox["a", "b"]

TooltipBox

`TooltipBox[{...}]`
: undocumented...

$UseSansSerif

`$UseSansSerif`
: controls whether the Web interfaces use a Sans-Serif font.

When set True, the output in MathMLForm uses SansSerif fonts instead of the standard fonts.

```
>> $UseSansSerif
   True

>> $UseSansSerif = False
   False
```

VI. Plotting

Contents

Axis

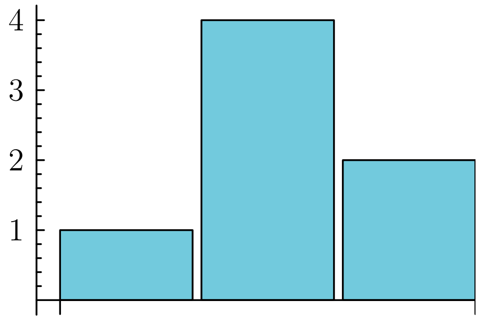
`Axis`
is a possible value for the `Filling` option.

```
>> ListLinePlot[Table[Sin[x], {x,
   -5, 5, 0.5}], Filling->Axis]
```

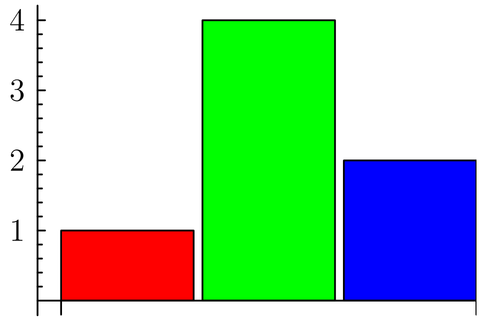
BarChart

`BarChart[{`*b1*`, `*b2*` ...}]`
makes a bar chart with lengths *b1*, *b2*, ...

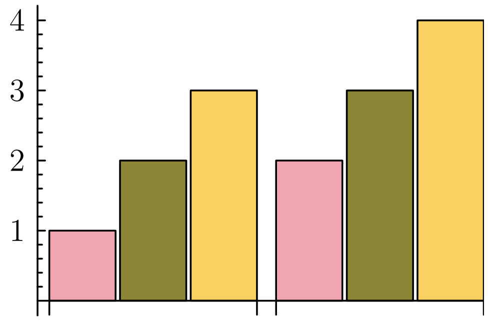
```
>> BarChart[{1, 4, 2}]
```



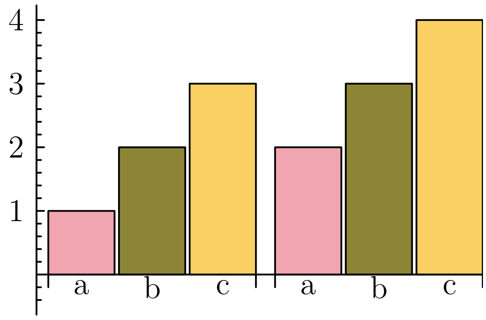

```
>> BarChart[{1, 4, 2}, ChartStyle
   -> {Red, Green, Blue}]
```



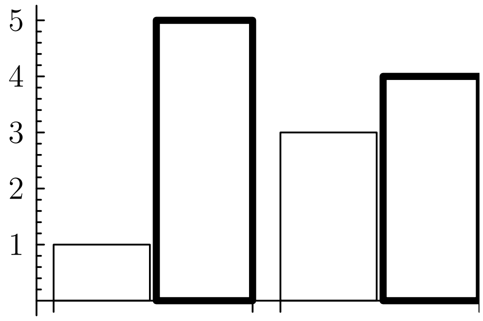

```
>> BarChart[{{1, 2, 3}, {2, 3, 4}}]
```



```
>> BarChart[{{1, 2, 3}, {2, 3, 4}},
    ChartLabels -> {"a", "b", "c"}]
```




```
>> BarChart[{{1, 5}, {3, 4}},
   ChartStyle -> {{EdgeForm[Thin],
   White}, {EdgeForm[Thick], White
   }}]
```



Bottom

`Bottom`
  is a possible value for the `Filling` option.

```
>> ListLinePlot[Table[Sin[x], {x,
   -5, 5, 0.5}], Filling->Bottom]
```

ColorData

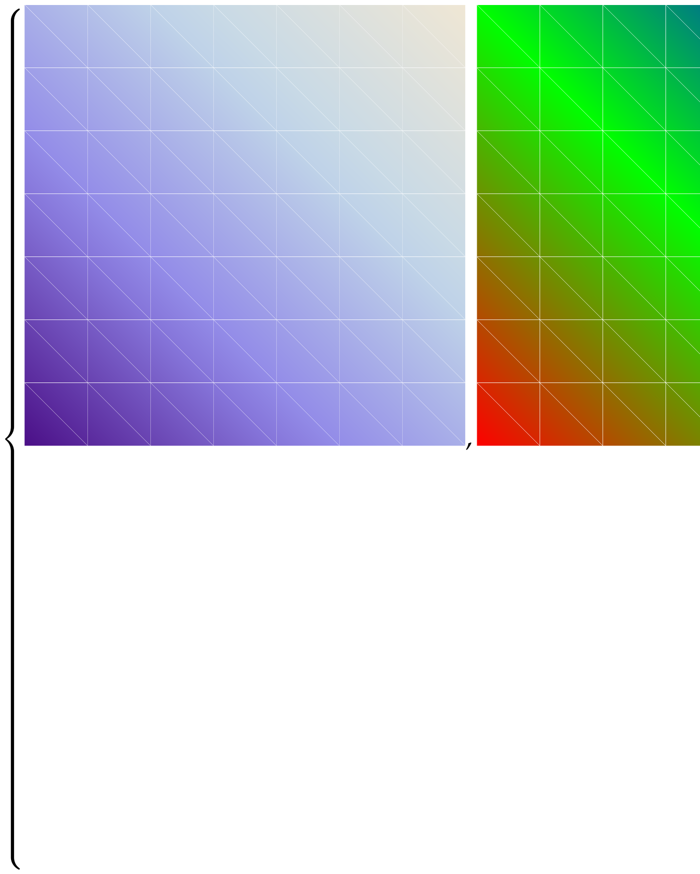
`ColorData["`*name*`"]`
  returns a color function with the given *name*.

Define a user-defined color function:

```
>> Unprotect[ColorData]; ColorData
   ["test"] := ColorDataFunction["
   test", "Gradients", {0, 1},
   Blend[{Red, Green, Blue}, #1]
   &]; Protect[ColorData]
```

Compare it to the default color function, `LakeColors`:

```
>> {DensityPlot[x + y, {x, -1, 1},
   {y, -1, 1}], DensityPlot[x + y,
   {x, -1, 1}, {y, -1, 1},
   ColorFunction->"test"]}
```

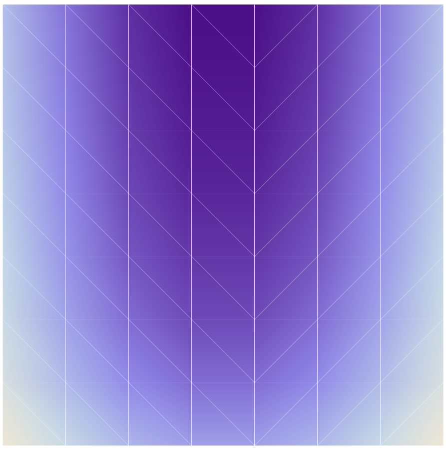


ColorDataFunction

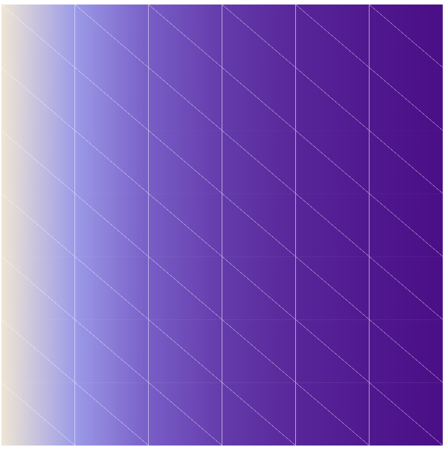
DensityPlot

`DensityPlot[`*f*`, {`*x*`, `*xmin*`, `*xmax*`}, {`*y*`, `*ymin*`, `*ymax*`}]`
  plots a density plot of *f* with *x* ranging from *xmin* to *xmax* and *y* ranging from *ymin* to *ymax*.

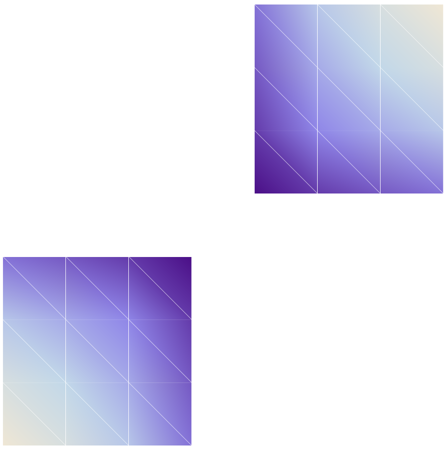
```
>> DensityPlot[x ^ 2 + 1 / y, {x,
   -1, 1}, {y, 1, 4}]
```



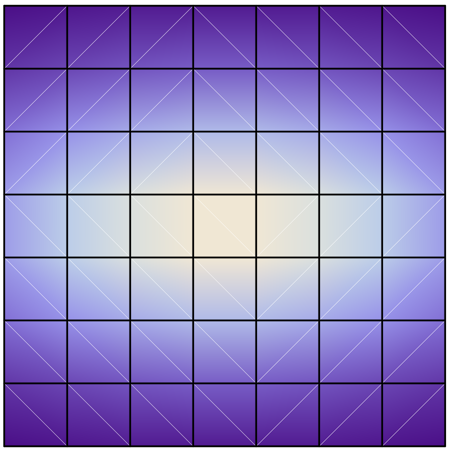
```
>> DensityPlot[1 / x, {x, 0, 1}, {y
   , 0, 1}]
```



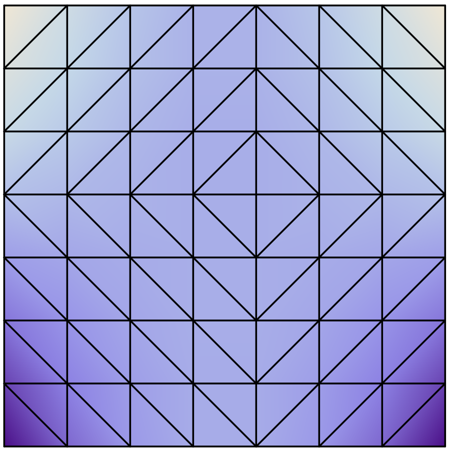
```
>> DensityPlot[Sqrt[x * y], {x, -1,
    1}, {y, -1, 1}]
```



```
>> DensityPlot[1/(x^2 + y^2 + 1), {
   x, -1, 1}, {y, -2,2}, Mesh->Full
   ]
```



```
>> DensityPlot[x^2 y, {x, -1, 1}, {
   y, -1, 1}, Mesh->All]
```



Filling

`Filling Top |Bottom|Axis`
: is a an option to Plot to specify what filling to add under point, curves, and surfaces

```
>> ListLinePlot[Table[Sin[x], {x,
   -5, 5, 0.5}], Filling->Axis]
```

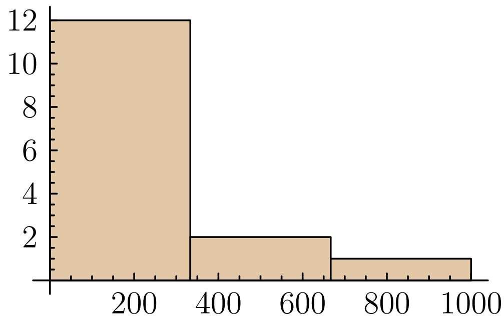
Full

`Full`
: is a possible value for the `Mesh` and `PlotRange` options.

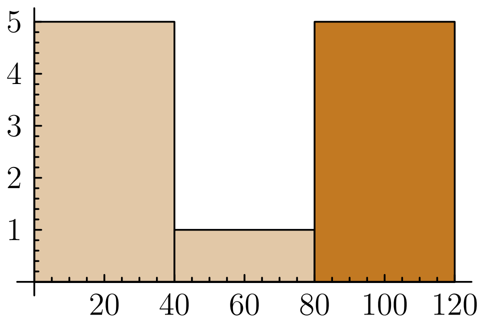
Histogram

`Histogram[{`*x1*`, `*x2*` ...}]`
: plots a histogram using the values *x1*, *x2*, ...

```
>> Histogram[{3, 8, 10, 100, 1000,
   500, 300, 200, 10, 20, 200, 100,
    200, 300, 500}]
```




```
>> Histogram[{{1, 2, 10, 5, 50,
   20}, {90, 100, 101, 120, 80}}]
```



ImageSize

`ImageSize`
: is an option that specifies the overall size of an image to display.

Specifications for both width and height can be any of the following:

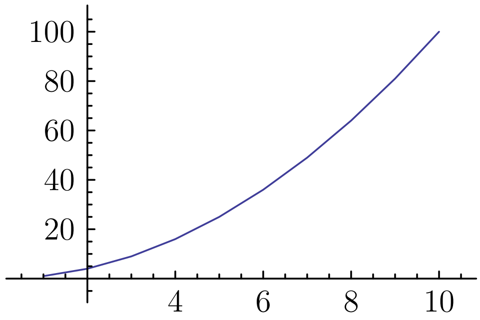
Automatic
: determined by location or other dimension (default)

Tiny, Small, Medium, Large
: pre defined absolute sizes

Joined

`Joined` *boolean*
: is an option for `Plot` that gives whether to join points to make lines.

```
>> ListPlot[Table[n ^ 2, {n, 10}],
   Joined->True]
```



ListLinePlot

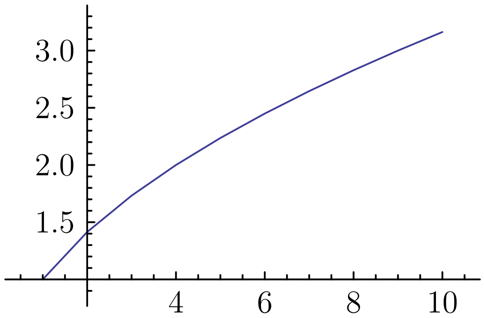
`ListLinePlot[{`*y_1*`, `*y_2*`, ...}]`
: plots a line through a list of y-values, assuming integer x-values 1, 2, 3, ...

`ListLinePlot[{{`*x_1*`, `*y_1*`}, {`*x_2*`, `*y_2*`}, ...}]`
: plots a line through a list of x, y pairs.

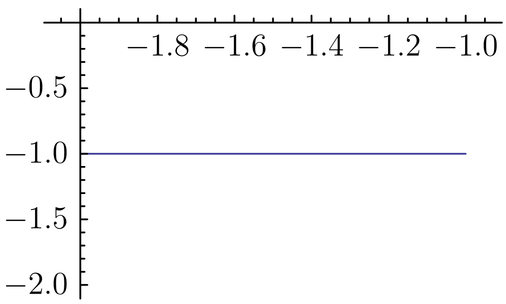
`ListLinePlot[{`*list_1*`, `*list_2*`, ...}]`
: plots several lines.

ListPlot accepts a superset of the Graphics options.

```
>> ListLinePlot[Table[{n, n ^ 0.5},
   {n, 10}]]
```




```
>> ListLinePlot[{{-2, -1}, {-1,
   -1}}]
```



ListPlot

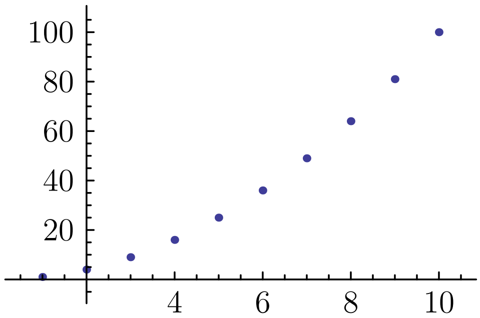
`ListPlot[{`*y_1*`, `*y_2*`, ...}]`
: plots a list of y-values, assuming integer x-values 1, 2, 3, ...

`ListPlot[{{`*x_1*`, `*y_1*`}, {`*x_2*`, `*y_2*`}, ...}]`
: plots a list of x, y pairs.

`ListPlot[{`*list_1*`, `*list_2*`, ...}]`
: plots several lists of points.

ListPlot accepts a superset of the Graphics options.

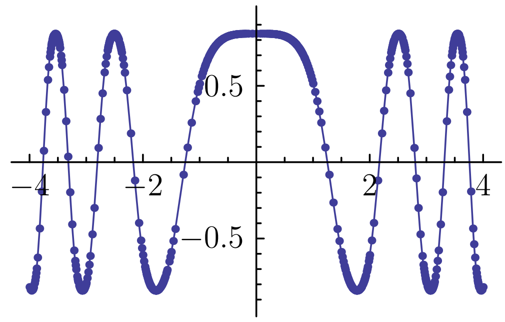
```
>> ListPlot[Table[n ^ 2, {n, 10}]]
```



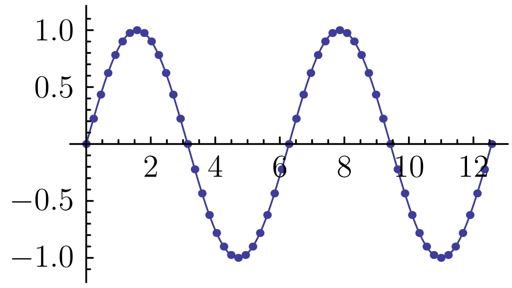
Mesh

`Mesh`
: is an option for `Plot` that specifies the mesh to be drawn. The default is `Mesh->None`.

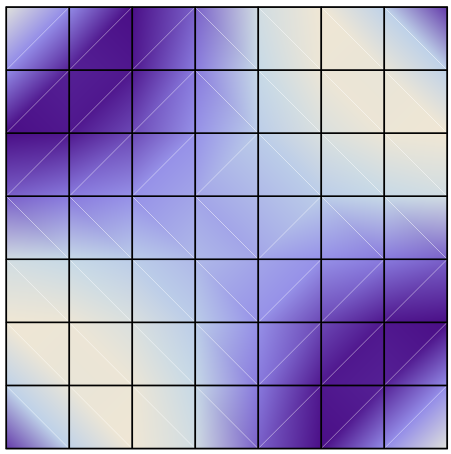
```
>> Plot[Sin[Cos[x^2]],{x,-4,4},Mesh
   ->All]
```



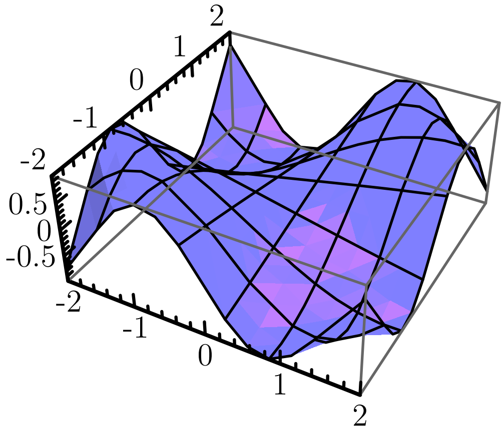

```
>> Plot[Sin[x], {x,0,4 Pi}, Mesh->
   Full]
```




```
>> DensityPlot[Sin[x y], {x, -2,
   2}, {y, -2, 2}, Mesh->Full]
```



```
>> Plot3D[Sin[x y], {x, -2, 2}, {y,
   -2, 2}, Mesh->Full]
```



ParametricPlot

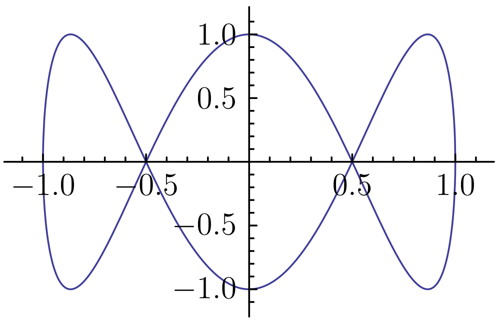
`ParametricPlot[{`*f_x*`, `*f_y*`}, {`*u*`, `*umin*`, `*umax*`}]`
plots a parametric function *f* with the parameter *u* ranging from *umin* to *umax*.

`ParametricPlot[{{`*f_x*`, `*f_y*`}, {`*g_x*`, `*g_y*`}, ...}, {`*u*`, `*umin*`, `*umax*`}]`
plots several parametric functions *f*, *g*, ...

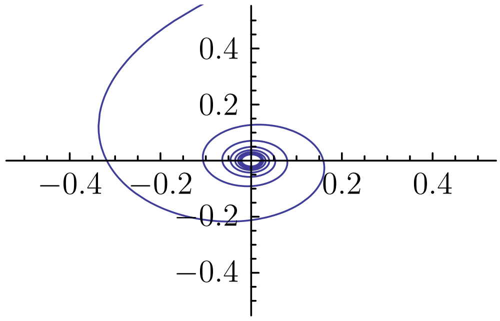
`ParametricPlot[{`*f_x*`, `*f_y*`}, {`*u*`, `*umin*`, `*umax*`}, {`*v*`, `*vmin*`, `*vmax*`}]`
plots a parametric area.

`ParametricPlot[{{`*f_x*`, `*f_y*`}, {`*g_x*`, `*g_y*`}, ...}, {`*u*`, `*umin*`, `*umax*`}, {`*v*`, `*vmin*`, `*vmax*`}]`
plots several parametric areas.

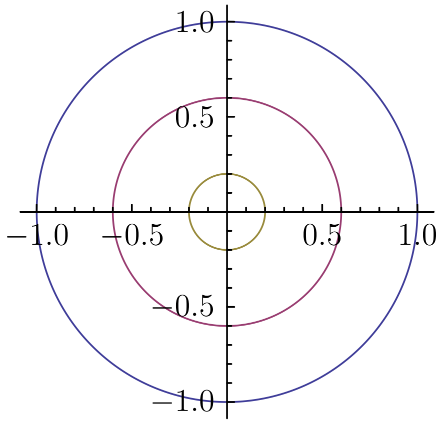
```
>> ParametricPlot[{Sin[u], Cos[3 u
   ]}, {u, 0, 2 Pi}]
```




```
>> ParametricPlot[{Cos[u] / u, Sin[
   u] / u}, {u, 0, 50}, PlotRange
   ->0.5]
```



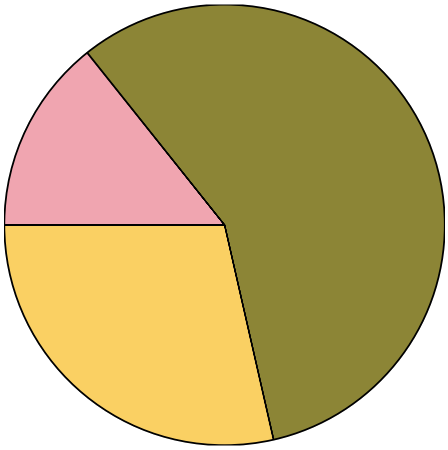

```
>> ParametricPlot[{{Sin[u], Cos[u
   ]},{0.6 Sin[u], 0.6 Cos[u]},
   {0.2 Sin[u], 0.2 Cos[u]}}, {u,
   0, 2 Pi}, PlotRange->1,
   AspectRatio->1]
```



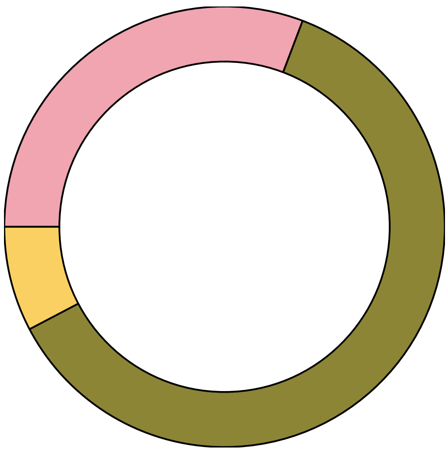
PieChart

`PieChart[{`*p1*`, `*p2* `...}]`
draws a pie chart.

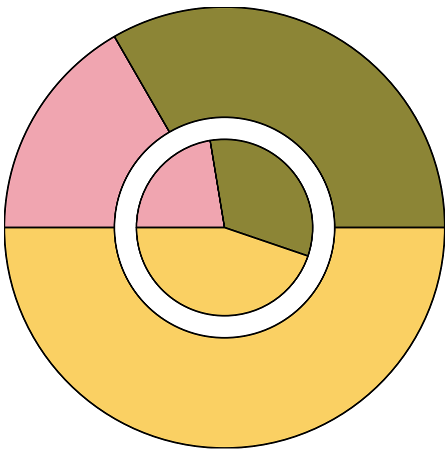
```
>>  PieChart[{1, 4, 2}]
```



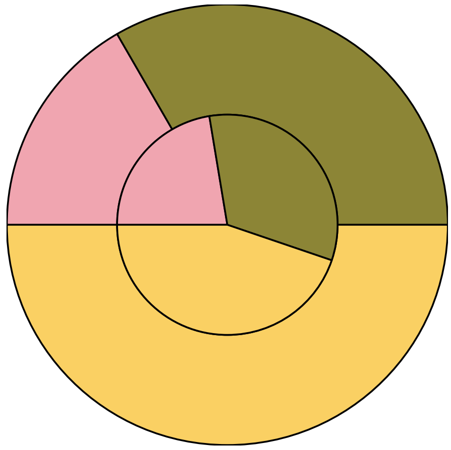
```
>>  PieChart[{8, 16, 2},
    SectorOrigin -> {Automatic,
    1.5}]
```



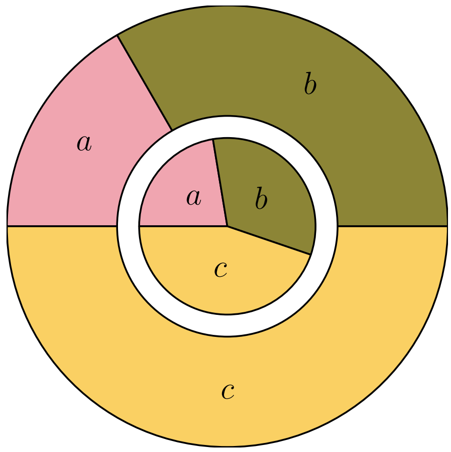
```
>>  PieChart[{{10, 20, 30}, {15, 22,
     30}}]
```



```
>>  PieChart[{{10, 20, 30}, {15, 22,
     30}}, SectorSpacing -> None]
```

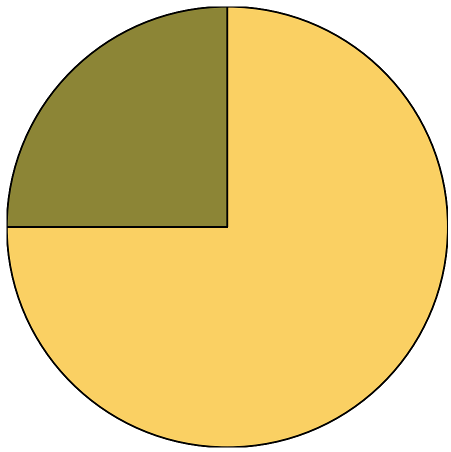


```
>>  PieChart[{{10, 20, 30}, {15, 22,
     30}}, ChartLabels -> {a, b, c}]
```



Negative values are clipped to 0.

```
>>  PieChart[{1, -1, 3}]
```

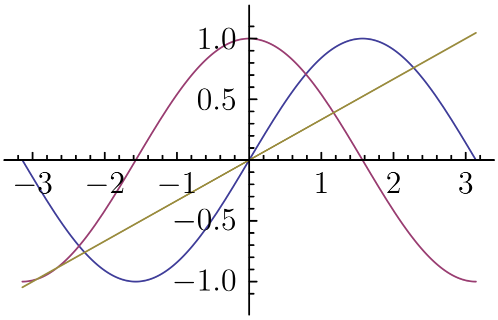


Plot

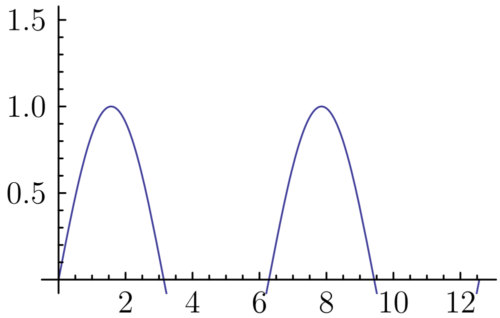
`Plot[`*f*`, {`*x*`, `*xmin*`, `*xmax*`}]`
: plots *f* with *x* ranging from *xmin* to *xmax*.

`Plot[{`*f1*`, `*f2*`, ...}, {`*x*`, `*xmin*`, `*xmax*`}]`
: plots several functions *f1*, *f2*, ...

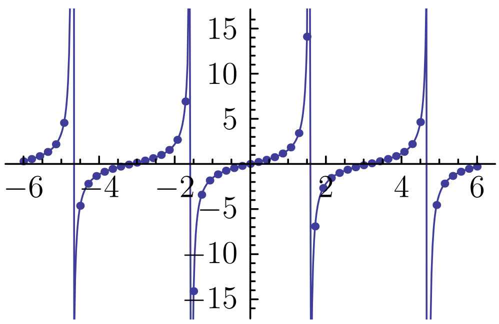
```
>> Plot[{Sin[x], Cos[x], x / 3}, {x
   , -Pi, Pi}]
```



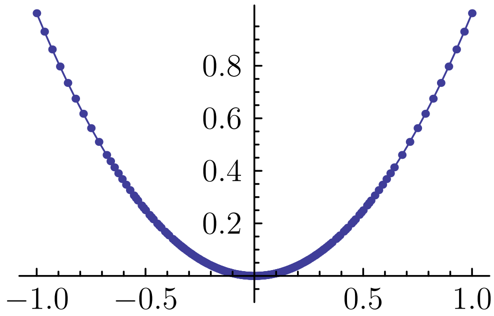
```
>> Plot[Sin[x], {x, 0, 4 Pi},
   PlotRange->{{0, 4 Pi}, {0,
   1.5}}]
```



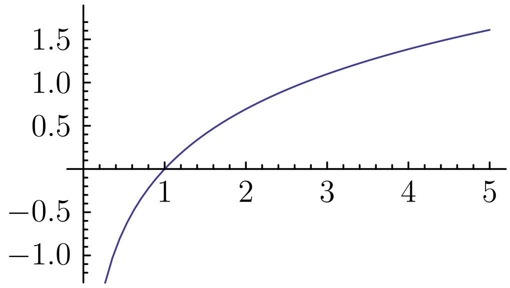
```
>> Plot[Tan[x], {x, -6, 6}, Mesh->
   Full]
```



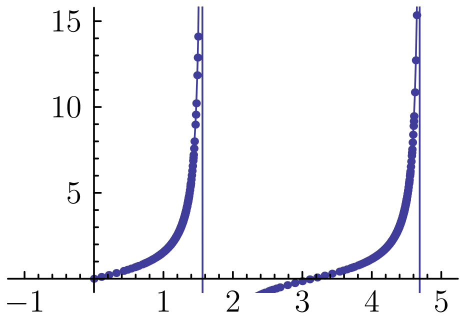
```
>> Plot[x^2, {x, -1, 1},
   MaxRecursion->5, Mesh->All]
```



```
>> Plot[Log[x], {x, 0, 5},
   MaxRecursion->0]
```

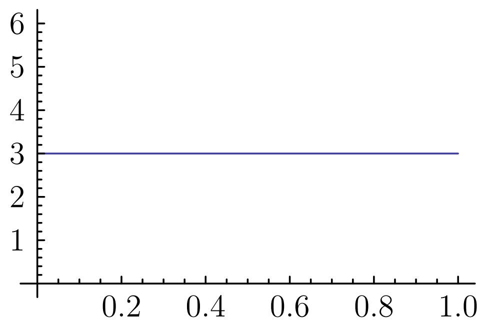


```
>> Plot[Tan[x], {x, 0, 6}, Mesh->
   All, PlotRange->{{-1, 5}, {0,
   15}}, MaxRecursion->10]
```



A constant function:

```
>> Plot[3, {x, 0, 1}]
```

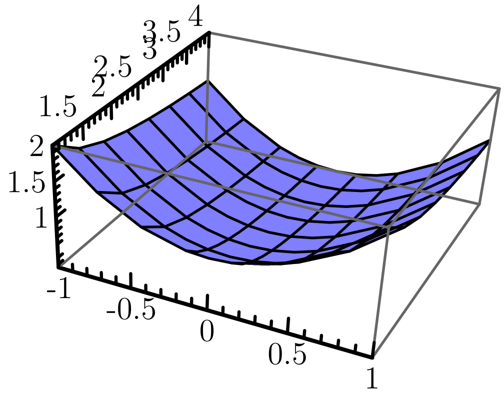


Plot3D

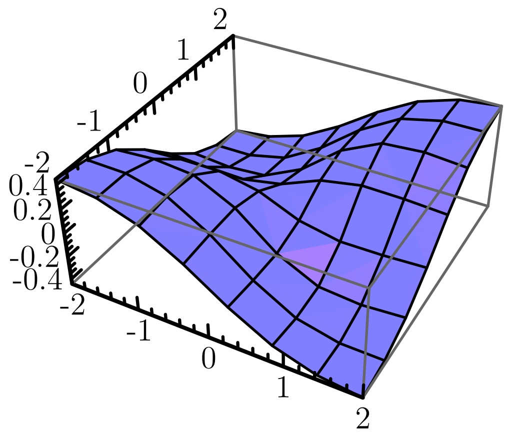
`Plot3D[`*f*`, {`*x*`, `*xmin*`, `*xmax*`}, {`*y*`, `*ymin*`, `*ymax*`}]`

creates a three-dimensional plot of *f* with *x* ranging from *xmin* to *xmax* and *y* ranging from *ymin* to *ymax*.

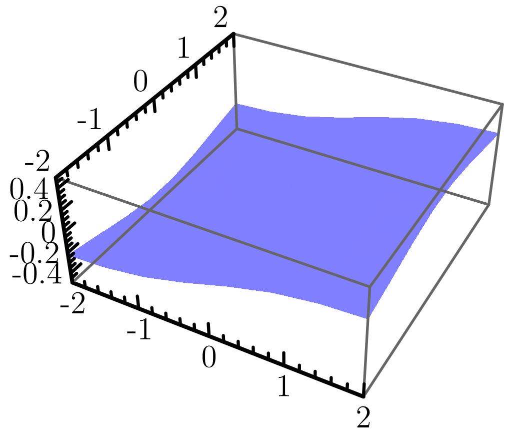
```
>> Plot3D[x ^ 2 + 1 / y, {x, -1,
1}, {y, 1, 4}]
```



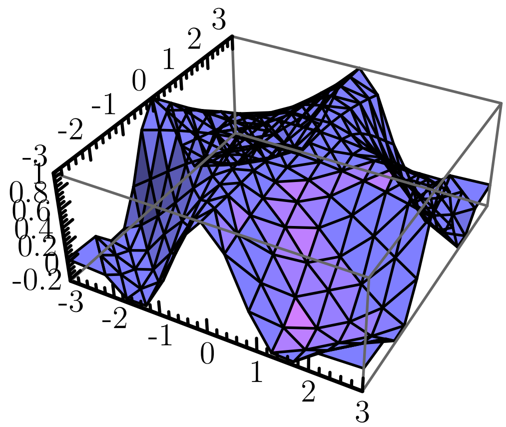

```
>> Plot3D[x y / (x ^ 2 + y ^ 2 + 1)
, {x, -2, 2}, {y, -2, 2}]
```



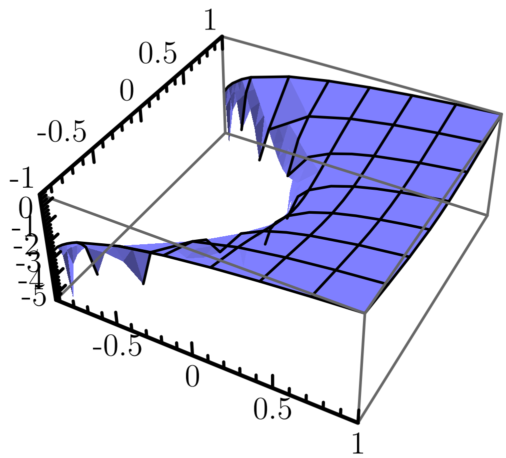

```
>> Plot3D[x / (x ^ 2 + y ^ 2 + 1),
{x, -2, 2}, {y, -2, 2}, Mesh->
None]
```




```
>> Plot3D[Sin[x y] /(x y), {x, -3,
3}, {y, -3, 3}, Mesh->All]
```



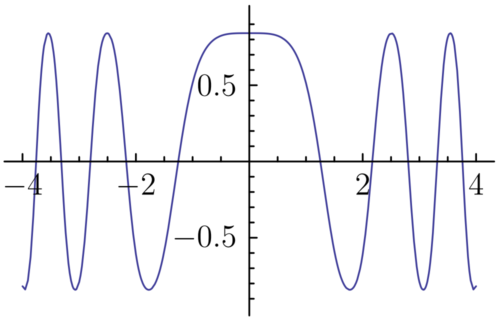

```
>> Plot3D[Log[x + y^2], {x, -1, 1},
 {y, -1, 1}]
```



PlotPoints

`PlotPoints` *n*
A number specifies how many initial sample points to use.

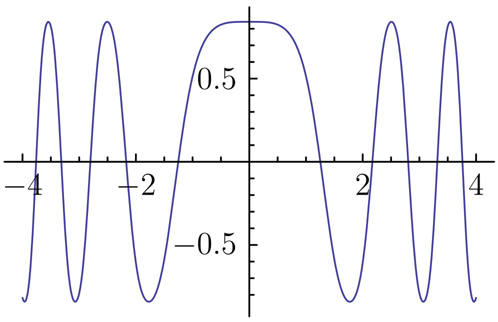
```
>> Plot[Sin[Cos[x^2]],{x,-4,4},
PlotPoints->22]
```



PlotRange

`PlotRange` *n* | `All` | `Automatic`
is an option for `Plot` that gives the range of coordinates to include in a plot.

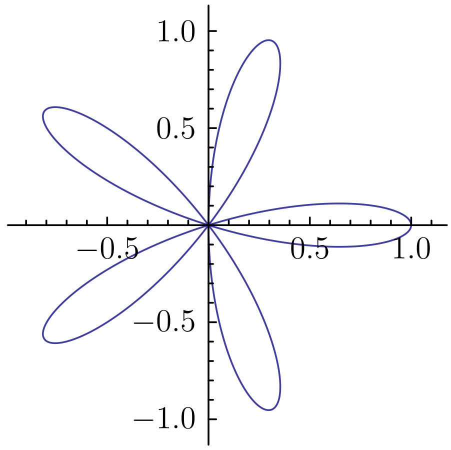
```
>> Plot[Sin[Cos[x^2]],{x,-4,4},
PlotRange -> All]
```



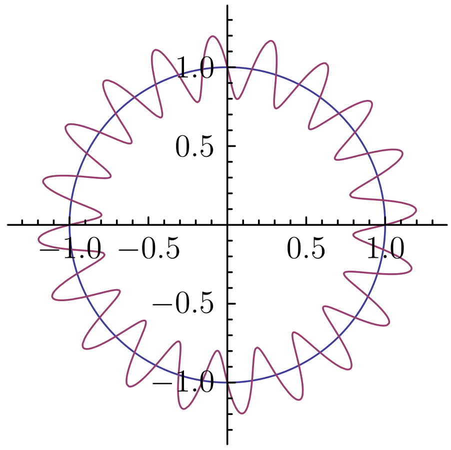
PolarPlot

`PolarPlot[`*r*`, {`*t*`,` *tmin*`,` *tmax*`}]`
creates a polar plot of *r* with angle *t* ranging from *tmin* to *tmax*.

```
>> PolarPlot[Cos[5t], {t, 0, Pi}]
```



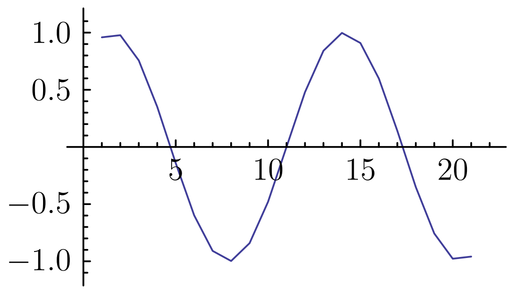

```
>> PolarPlot[{1, 1 + Sin[20 t] /
5}, {t, 0, 2 Pi}]
```



Top

`Top`
is a possible value for the `Filling` option.

```
>> ListLinePlot[Table[Sin[x], {x,
-5, 5, 0.5}], Filling->Axis|Top|
Bottom]
```



VII. Logic

Contents

AllTrue

`AllTrue[{`*expr1*`, `*expr2*`, ...}, `*test*`]`
: returns True if all applications of *test* to *expr1*, *expr2*, ... evaluate to True.

`AllTrue[`*list*`, `*test*`, `*level*`]`
: returns True if all applications of *test* to items of *list* at *level* evaluate to True.

`AllTrue[`*test*`]`
: gives an operator that may be applied to expressions.

```
>> AllTrue[{2, 4, 6}, EvenQ]
   True
```

```
>> AllTrue[{2, 4, 7}, EvenQ]
   False
```

And (&&)

`And[`*expr1*`, `*expr2*`, ...]`
expr1 `&&` *expr2* `&& ...`
: evaluates each expression in turn, returning `False` as soon as an expression evaluates to `False`. If all expressions evaluate to `True`, `And` returns `True`.

```
>> True && True && False
   False
```

If an expression does not evaluate to `True` or `False`, `And` returns a result in symbolic form:

```
>> a && b && True && c
```
a&&b&&c

AnyTrue

`AnyTrue[{`*expr1*`, `*expr2*`, ...}, `*test*`]`
: returns True if any application of *test* to *expr1*, *expr2*, ... evaluates to True.

`AnyTrue[`*list*`, `*test*`, `*level*`]`
: returns True if any application of *test* to items of *list* at *level* evaluates to True.

`AnyTrue[`*test*`]`
: gives an operator that may be applied to expressions.

```
>> AnyTrue[{1, 3, 5}, EvenQ]
   False
```

```
>> AnyTrue[{1, 4, 5}, EvenQ]
   True
```

Equivalent (===)

`Equivalent[`*expr1*`, `*expr2*`, ...]`
expr1 `===` *expr2* `===...`
: is equivalent to (*expr1* && *expr2* && ...) `||` (!*expr1* && !*expr2* && ...)

```
>> Equivalent[True, True, False]
   False
```

If all expressions do not evaluate to `True` or `False`, `Equivalent` returns a result in symbolic form:

```
>> Equivalent[a, b, c]
```
abc

Otherwise, `Equivalent` returns a result in DNF

```
>> Equivalent[a, b, True, c]
```

a&&*b*&&*c*

False

`False`
: represents the Boolean false value.

Implies (=>)

`Implies[`*expr1*`, `*expr2*`]`
expr1 `=>` *expr2*
: evaluates each expression in turn, returning `True` as soon as the first expression evaluates to `False`. If the first expression evaluates to `True`, `Implies` returns the second expression.

```
>> Implies[False, a]
```

True

```
>> Implies[True, a]
```

a

If an expression does not evaluate to `True` or `False`, `Implies` returns a result in symbolic form:

```
>> Implies[a, Implies[b, Implies[
   True, c]]]
```

abc

NoneTrue

`NoneTrue[{`*expr1*`, `*expr2*`, ...}, `*test*`]`
: returns True if no application of *test* to *expr1*, *expr2*, ... evaluates to True.

`NoneTrue[`*list*`, `*test*`, `*level*`]`
: returns True if no application of *test* to items of *list* at *level* evaluates to True.

`NoneTrue[`*test*`]`
: gives an operator that may be applied to expressions.

```
>> NoneTrue[{1, 3, 5}, EvenQ]
```

True

```
>> NoneTrue[{1, 4, 5}, EvenQ]
```

False

Not (!)

`Not[`*expr*`]`
`!`*expr*
: negates the logical expression *expr*.

```
>> !True
```

False

```
>> !False
```

True

```
>> !b
```

!*b*

Or (||)

`Or[`*expr1*`, `*expr2*`, ...]`
expr1 `||` *expr2* `|| ...`
: evaluates each expression in turn, returning `True` as soon as an expression evaluates to `True`. If all expressions evaluate to `False`, `Or` returns `False`.

```
>> False || True
```

True

If an expression does not evaluate to `True` or `False`, `Or` returns a result in symbolic form:

```
>> a || False || b
```

a||*b*

True

`True`
: represents the Boolean true value.

Xor (`xor`)

`Xor[`*expr1*`, `*expr2*`, ...]`
expr1 `xor` *expr2* `xor...`

evaluates each expression in turn, returning `True` as soon as not all expressions evaluate to the same value. If all expressions evaluate to the same value, `Xor` returns `False`.

```
>> Xor[False, True]
```
True

```
>> Xor[True, True]
```
False

If an expression does not evaluate to `True` or `False`, `Xor` returns a result in symbolic form:

```
>> Xor[a, False, b]
```
ab

VIII. Scoping

Contents

Begin

`Begin[`*context*`]`
: temporarily sets the current context to *context*.

```
>> Begin["test`"]
```
test`

```
>> {$Context, $ContextPath}
```
{test`, {Global`, System`}}

```
>> Context[newsymbol]
```
test`

```
>> End[]
```
test`

```
>> End[]
```
Nopreviouscontextdefined.

Global`

BeginPackage

`BeginPackage[`*context*`]`
: starts the package given by *context*.

The *context* argument must be a valid context name. `BeginPackage` changes the values of `$Context` and `$ContextPath`, setting the current context to *context*.

```
>> BeginPackage["test`"]
```
test`

Block

`Block[{`*x*`, `*y*`, ...}, `*expr*`]`
: temporarily removes the definitions of the given variables, evaluates *expr*, and restores the original definitions afterwards.

`Block[{`*x*=*x0*`, `*y*=*y0*`, ...}, `*expr*`]`
: assigns temporary values to the variables during the evaluation of *expr*.

```
>> n = 10
```
10

```
>> Block[{n = 5}, n ^ 2]
```
25

```
>> n
```
10

Values assigned to block variables are evaluated at the beginning of the block. Keep in mind that the result of `Block` is evaluated again, so a returned block variable will get its original value.

```
>> Block[{x = n+2, n}, {x, n}]
```
{12, 10}

If the variable specification is not of the described form, an error message is raised:

```
>> Block[{x + y}, x]
```
Localvariablespecificationcontainsx+y, whichisnotasymboloranassignmenttoasymbol.

x

Variable names may not appear more than once:

```
>> Block[{x, x}, x]
```
Duplicatelocalvariablexfoundinlocalvariablespecification.

x

Context

`Context[`*symbol*`]`
: yields the name of the context where *symbol* is defined in.

`Context[]`
: returns the value of `$Context`.

```
>> Context[a]
```
Global‘

```
>> Context[b‘c]
```
b‘

```
>> InputForm[Context[]]
```
"Global‘"

$ContextPath

`$ContextPath`
: is the search path for contexts.

```
>> $ContextPath // InputForm
```
{"Global‘", "System‘"}

System‘Private‘$ContextPathStack

`System‘Private‘$ContextPathStack`
: is an internal variable tracking the values of `$ContextPath` saved by `Begin` and `BeginPackage`.

System‘Private‘$ContextStack

`System‘Private‘$ContextStack`
: is an internal variable tracking the values of `$Context` saved by `Begin` and `BeginPackage`.

$Context

`$Context`
: is the current context.

```
>> $Context
```
Global‘

Contexts

`Contexts[]`
: yields a list of all contexts.

```
>> x = 5;
```

```
>> Contexts[] // InputForm
```
{"CombinatoricaOld‘", "Global‘", "ImportExport‘", "Internal‘", "Settings‘", "System‘", "System‘Convert‘B64Dump‘", "System‘Convert‘CommonDump‘", "System‘Convert‘Image‘", "System‘Convert‘JSONDump‘", "System‘Convert‘TableDump‘", "System‘Convert‘TextDump‘", "System‘ConvertersDump‘", "System‘Private‘", "XML‘", "XML‘Parser‘"}

End

`End[]`
: ends a context started by `Begin`.

EndPackage

`EndPackage[]`
marks the end of a package, undoing a previous `BeginPackage`.

After `EndPackage`, the values of `$Context` and `$ContextPath` at the time of the `BeginPackage` call are restored, with the new package's context prepended to $ContextPath.

Module

`Module[{vars}, expr]`
localizes variables by giving them a temporary name of the form `name$number`, where number is the current value of `$ModuleNumber`. Each time a module is evaluated, `$ModuleNumber` is incremented.

```
>> x = 10;

>> Module[{x=x}, x=x+1; x]
   11

>> x
   10

>> t === Module[{t}, t]
   False
```

Initial values are evaluated immediately:

```
>> Module[{t=x}, x = x + 1; t]
   10

>> x
   11
```

Variables inside other scoping constructs are not affected by the renaming of `Module`:

```
>> Module[{a}, Block[{a}, a]]
   a

>> Module[{a}, Block[{}, a]]
   a$5
```

$ModuleNumber

`$ModuleNumber`
is the current "serial number" to be used for local module variables.

```
>> Unprotect[$ModuleNumber]

>> $ModuleNumber = 20;

>> Module[{x}, x]
   x$20

>> $ModuleNumber = x;
   Cannotset$ModuleNumbertox;valuemustbeapositiveinteger.
```

Unique

`Unique[]`
generates a new symbol and gives a name of the form `$number`.
`Unique[x]`
generates a new symbol and gives a name of the form `x$number`.
`Unique[{x, y, ...}]`
generates a list of new symbols.
`Unique[‘‘xxx’]’`
generates a new symbol and gives a name of the form `xxxnumber`.

Create a unique symbol with no particular name:

```
>> Unique[]
   $9

>> Unique[sym]
   sym$1
```

Create a unique symbol whose name begins with x:

```
>> Unique["x"]
   x10
```

Each use of Unique[symbol] increments $ModuleNumber:

```
>> {$ModuleNumber, Unique[x],
   $ModuleNumber}
   {2, x$2, 3}
```

Unique[symbol] creates symbols in the same way Module does:

```
>> {Module[{x}, x], Unique[x]}
   {x$3, x$4}
```

Unique with more arguments

```
>> Unique[{x, "s"}, Flat ^ Listable
   ^ Orderless]
```

Flat^{L}istable^{O}rderlessisnotaknownattribute.

$\text{Unique}\left[\{x, \text{s}\}, \text{Flat}^{\text{Listable}^{\text{Orderless}}}\right]$

Unique call without symbol argument

```
>> Unique[x + y]
```

x + yisnotasymboloravalidsymbolname.

$\text{Unique}[x + y]$

With

`With[{x=x0, y=y0, ...}, expr]`
: specifies that all occurrences of the symbols *x*, *y*, ... in *expr* should be replaced by *x0*, *y0*, ...

```
>> n = 10
```

10

Evaluate an expression with x locally set to 5:
`With` works even without evaluation:

```
>> With[{x = a}, (1 + x^2)&]
```

$1 + a^2\&$

Use `With` to insert values into held expressions

```
>> With[{x=y}, Hold[x]]
```

$\text{Hold}[y]$

```
>> Table[With[{i=j}, Hold[i]],{j
   ,1,4}]
```

$\{\text{Hold}[1], \text{Hold}[2], \text{Hold}[3], \text{Hold}[4]\}$

```
>> x=5; With[{x=x}, Hold[x]]
```

$\text{Hold}[5]$

```
>> {Block[{x = 3}, Hold[x]], With[{
   x = 3}, Hold[x]]}
```

$\{\text{Hold}[x], \text{Hold}[3]\}$

```
>> x=.; ReleaseHold /@ %
```

$\{x, 3\}$

```
>> With[{e = y}, Function[{x,y}, e*
   x*y]]
```

Function [{x\$, y\$} , *y*x\$y\$]

IX. Random number generation

Random numbers are generated using the Mersenne Twister.

Contents

Random

Legacy function. Superseded by RandomReal, RandomInteger and RandomComplex.

RandomChoice

`RandomChoice[`*items*`]`
: randomly picks one item from *items*.

`RandomChoice[`*items*`, `*n*`]`
: randomly picks *n* items from *items*. Each pick in the *n* picks happens from the given set of *items*, so each item can be picked any number of times.

`RandomChoice[`*items*`, {`*n1*`, `*n2*`, ...}]`
: randomly picks items from *items* and arranges the picked items in the nested list structure described by {*n1*, *n2*, ...}.

`RandomChoice[`*weights*` -> `*items*`, `*n*`]`
: randomly picks *n* items from *items* and uses the corresponding numeric values in *weights* to determine how probable it is for each item in *items* to get picked (in the long run, items with higher weights will get picked more often than ones with lower weight).

`RandomChoice[`*weights*` -> `*items*`]`
: randomly picks one items from *items* using weights *weights*.

`RandomChoice[`*weights*` -> `*items*`, {`*n1*`, `*n2*`, ...}]`
: randomly picks a structured list of items from *items* using weights *weights*.

```
>> SeedRandom[42]
```

```
>> RandomChoice[{a, b, c}]
```
$\{c\}$

```
>> SeedRandom[42]
```

```
>> RandomChoice[{a, b, c}, 20]
```
$\{c,a,c,c,a,a,c,b,c,c,$
$c,c,a,c,b,a,b,b,b,b\}$

```
>> SeedRandom[42]
```

```
>> RandomChoice[{"a", {1, 2}, x, {}}, 10]
```
$\{x, \{\}, \mathrm{a}, x, x, \{\}, \mathrm{a}, \mathrm{a}, x, \{1,2\}\}$

```
>> SeedRandom[42]
```

```
>> RandomChoice[{a, b, c}, {5, 2}]
```
$\{\{c,a\}, \{c,c\}, \{a,a\}, \{c,b\}, \{c,c\}\}$

```
>> SeedRandom[42]
```

```
>> RandomChoice[{1, 100, 5} -> {a, b, c}, 20]
```
$\{b,b,b,b,b,b,b,b,b,b,$
$b,c,b,b,b,b,b,b,b,b\}$

RandomComplex

`RandomComplex[{`*z_min*`, `*z_max*`}]`
: yields a pseudorandom complex number in the rectangle with complex corners *z_min* and *z_max*.

`RandomComplex[`*z_max*`]`
: yields a pseudorandom complex number in the rectangle with corners at the origin and at *z_max*.

`RandomComplex[]`
: yields a pseudorandom complex number with real and imaginary parts from 0 to 1.

`RandomComplex[`*range*`, `*n*`]`
: gives a list of *n* pseudorandom complex numbers.

`RandomComplex[`*range*`, {`*n1*`, `*n2*`, ...}]`
: gives a nested list of pseudorandom complex numbers.

```
>> RandomComplex[]
```
$0.909199 + 0.871613I$

```
>> RandomComplex[{1+I, 5+5I}]
```
$4.55479 + 4.88347I$

```
>> RandomComplex[1+I, 5]
```
$\{0.407429 + 0.964898I, 0.318\tilde{}\ \tilde{}244 + 0.82312I, 0.348569 + 0.760967I, 0.769704 + 0.587\tilde{}\ \tilde{}049I, 0.729443 + 0.519018I\}$

```
>> RandomComplex[{1+I, 2+2I}, {2, 2}]
```
$\{\{1.51303 + 1.90058I, 1.808 + 1.00296I\}, \{1.59168 + 1.379\tilde{}\ \tilde{}83I, 1.74807 + 1.24582I\}\}$

RandomInteger

`RandomInteger[{`*min*`, `*max*`}]`
: yields a pseudorandom integer in the range from *min* to *max* inclusive.

`RandomInteger[`*max*`]`
: yields a pseudorandom integer in the range from 0 to *max* inclusive.

`RandomInteger[]`
: gives 0 or 1.

`RandomInteger[`*range*`, `*n*`]`
: gives a list of *n* pseudorandom integers.

`RandomInteger[`*range*`, {`*n1*`, `*n2*`, ...}]`
: gives a nested list of pseudorandom integers.

```
>> RandomInteger[{1, 5}]
```
4

```
>> RandomInteger[100, {2, 3}] // TableForm
```

13	16	31
22	60	22

Calling `RandomInteger` changes `$RandomState`:

```
>> previousState = $RandomState;
```

```
>> RandomInteger[]
```
0

```
>> $RandomState != previousState
```
True

RandomReal

`RandomReal[{`*min*`, `*max*`}]`
: yields a pseudorandom real number in the range from *min* to *max*.

`RandomReal[`*max*`]`
: yields a pseudorandom real number in the range from 0 to *max*.

`RandomReal[]`
: yields a pseudorandom real number in the range from 0 to 1.

`RandomReal[`*range*`, `*n*`]`
: gives a list of *n* pseudorandom real numbers.

`RandomReal[`*range*`, {`*n1*`, `*n2*`, ...}]`
: gives a nested list of pseudorandom real numbers.

>> `RandomReal[]`
0.545943

>> `RandomReal[{1, 5}]`
1.0481

RandomSample

`RandomSample[`*items*`]`
: randomly picks one item from *items*.

`RandomSample[`*items*`, `*n*`]`
: randomly picks *n* items from *items*. Each pick in the *n* picks happens after the previous items picked have been removed from *items*, so each item can be picked at most once.

`RandomSample[`*items*`, {`*n1*`, `*n2*`, ...}]`
: randomly picks items from *items* and arranges the picked items in the nested list structure described by {*n1*, *n2*, ...}. Each item gets picked at most once.

`RandomSample[`*weights*` -> `*items*`, `*n*`]`
: randomly picks *n* items from *items* and uses the corresponding numeric values in *weights* to determine how probable it is for each item in *items* to get picked (in the long run, items with higher weights will get picked more often than ones with lower weight). Each item gets picked at most once.

`RandomSample[`*weights*` -> `*items*`]`
: randomly picks one items from *items* using weights *weights*. Each item gets picked at most once.

`RandomSample[`*weights*` -> `*items*`, {`*n1*`, `*n2*`, ...}]`
: randomly picks a structured list of items from *items* using weights *weights*. Each item gets picked at most once.

>> `SeedRandom[42]`

>> `RandomSample[{a, b, c}]`
$\{a\}$

>> `SeedRandom[42]`

>> `RandomSample[{a, b, c, d, e, f, g, h}, 7]`
$\{b, f, a, h, c, e, d\}$

>> `SeedRandom[42]`

>> `RandomSample[{"a", {1, 2}, x, {}}, 3]`
{{1,2},{},a}

>> `SeedRandom[42]`

>> `RandomSample[Range[100], {2, 3}]`
$\{\{84, 54, 71\}, \{46, 45, 40\}\}$

>> `SeedRandom[42]`

>> `RandomSample[Range[100] -> Range[100], 5]`
$\{62, 98, 86, 78, 40\}$

$RandomState

`$RandomState`
: is a long number representing the internal state of the pseudorandom number generator.

>> `Mod[$RandomState, 10^100]`
5 882 368 839 901 494 183 273 475~
~747 155 858 448 014 086 417 618~
~023 479 609 412 276 978 828 651~
~158 804 942 079 827 675 173 687 342

>> `IntegerLength[$RandomState]`
6 466

So far, it is not possible to assign values to `$RandomState`.

>> `$RandomState = 42`
Itisnotpossibletochangetherandomstate.
42

Not even to its own value:

>> `$RandomState = $RandomState;`
Itisnotpossibletochangetherandomstate.

SeedRandom

`SeedRandom[`*n*`]`
: resets the pseudorandom generator with seed *n*.

`SeedRandom[]`
: uses the current date and time as the seed.

`SeedRandom` can be used to get reproducible random numbers:

```
>> SeedRandom[42]
```

```
>> RandomInteger[100]
```

51

```
>> RandomInteger[100]
```

92

```
>> SeedRandom[42]
```

```
>> RandomInteger[100]
```

51

```
>> RandomInteger[100]
```

92

String seeds are supported as well:

```
>> SeedRandom["Mathics"]
```

```
>> RandomInteger[100]
```

27

Calling `SeedRandom` without arguments will seed the random number generator to a random state:

```
>> SeedRandom[]
```

```
>> RandomInteger[100]
```

35

X. Combinatorial Functions

Contents

Binomial

`Binomial[`*n*`, `*k*`]`
: gives the binomial coefficient *n* choose *k*.

```
>> Binomial[5, 3]
10
```

`Binomial` supports inexact numbers:

```
>> Binomial[10.5,3.2]
165.286
```

Some special cases:

```
>> Binomial[10, -2]
0
```

```
>> Binomial[-10.5, -3.5]
0.
```

DiceDissimilarity

`DiceDissimilarity[`*u*`, `*v*`]`
: returns the Dice dissimilarity between the two boolean 1-D lists *u* and *v*, which is defined as (c_tf + c_ft) / (2 * c_tt + c_ft + c_tf), where n is len(*u*) and c_ij is the number of occurrences of *u*[k]=i and *v*[k]=j for k<n.

```
>> DiceDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
```

$\frac{1}{2}$

Fibonacci

`Fibonacci[`*n*`]`
: computes the *n*th Fibonacci number.

```
>> Fibonacci[0]
0
```

```
>> Fibonacci[1]
1
```

```
>> Fibonacci[10]
55
```

```
>> Fibonacci[200]
280 571 172 992 510 140 037 ˜
˜611 932 413 038 677 189 525
```

JaccardDissimilarity

`JaccardDissimilarity[`*u*`, `*v*`]`
: returns the Jaccard-Needham dissimilarity between the two boolean 1-D lists *u* and *v*, which is defined as (c_tf + c_ft) / (c_tt + c_ft + c_tf), where n is len(*u*) and c_ij is the number of occurrences of *u*[k]=i and *v*[k]=j for k<n.

```
>> JaccardDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
```

$\frac{2}{3}$

MatchingDissimilarity

`MatchingDissimilarity[u, v]`
: returns the Matching dissimilarity between the two boolean 1-D lists *u* and *v*, which is defined as (c_tf + c_ft) / n, where n is len(*u*) and c_ij is the number of occurrences of *u*[k]=i and *v*[k]=j for k<n.

```
>> MatchingDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
```

$\frac{4}{7}$

Multinomial

`Multinomial[n1, n2, ...]`
: gives the multinomial coefficient `(n1+n2+...)!/(n1!n2!...)`.

```
>> Multinomial[2, 3, 4, 5]
```

2 522 520

```
>> Multinomial[]
```

1

Multinomial is expressed in terms of `Binomial`:

```
>> Multinomial[a, b, c]
```

Binomial $[a, a]$ Binomial $[a + b, b]$ Binomial $[a + b + c, c]$

`Multinomial[n-k, k]` is equivalent to `Binomial[n, k]`.

```
>> Multinomial[2, 3]
```

10

RogersTanimotoDissimilarity

`RogersTanimotoDissimilarity[u, v]`
: returns the Rogers-Tanimoto dissimilarity between the two boolean 1-D lists *u* and *v*, which is defined as R / (c_tt + c_ff + R) where n is len(*u*), c_ij is the number of occurrences of *u*[k]=i and *v*[k]=j for k<n, and R = 2 * (c_tf + c_ft).

```
>> RogersTanimotoDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
```

$\frac{8}{11}$

RussellRaoDissimilarity

`RussellRaoDissimilarity[u, v]`
: returns the Russell-Rao dissimilarity between the two boolean 1-D lists *u* and *v*, which is defined as (n - c_tt) / c_tt where n is len(*u*) and c_ij is the number of occurrences of *u*[k]=i and *v*[k]=j for k<n.

```
>> RussellRaoDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
```

$\frac{5}{7}$

SokalSneathDissimilarity

`SokalSneathDissimilarity[u, v]`
: returns the Sokal-Sneath dissimilarity between the two boolean 1-D lists *u* and *v*, which is defined as R / (c_tt + R) where n is len(*u*), c_ij is the number of occurrences of *u*[k]=i and *v*[k]=j for k<n, and R = 2 * (c_tf + c_ft).

```
>> SokalSneathDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
```

$\frac{4}{5}$

StirlingS1

`StirlingS1[n, m]`
: gives the Stirling number of the first kind $ _n^m$.

Integer mathematical function, suitable for both symbolic and numerical manipulation. gives the number of permutations of n elements that contain exactly m cycles.

```
>> StirlingS1[50, 1]
```

$-608\,281\,864\,034\,267\,560\,872$~
~$252\,163\,321\,295\,376\,887\,552$~
~$831\,379\,210\,240\,000\,000\,000$

StirlingS2

`StirlingS2[`*n*`, `*m*`]`
: gives the Stirling number of the second kind _n^m.

returns the number of ways of partitioning a set of n elements into m non empty subsets.

```
>> Table[StirlingS2[10, m], {m, 10}]
```

$\{1, 511, 9\,330, 34\,105, 42\,525, 22\,827, 5\,880, 750, 45, 1\}$

Subsets

`Subsets[`*list*`]`
: finds a list of all possible subsets of *list*.

`Subsets[`*list*`, `*n*`]`
: finds a list of all possible subsets containing at most n elements.

`Subsets[`*list*`, {`*n*`}]`
: finds a list of all possible subsets containing exactly n elements.

`Subsets[`*list*`, {`*min*`, `*max*`}]`
: finds a list of all possible subsets containing between *min* and *max* elements.

`Subsets[`*list*`, `*spec*`, `*n*`]`
: finds a list of the first n possible subsets.

`Subsets[`*list*`, `*spec*`, {`*n*`}]`
: finds the nth possible subset.

All possible subsets (power set):

```
>> Subsets[{a, b, c}]
```

$\{\{\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

All possible subsets containing up to 2 elements:

```
>> Subsets[{a, b, c, d}, 2]
```

$\{\{\}, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$

Subsets containing exactly 2 elements:

```
>> Subsets[{a, b, c, d}, {2}]
```

$\{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$

The first 5 subsets containing 3 elements:

```
>> Subsets[{a, b, c, d, e}, {3}, 5]
```

$\{\{a, b, c\}, \{a, b, d\}, \{a, b, e\}, \{a, c, d\}, \{a, c, e\}\}$

All subsets with even length:

```
>> Subsets[{a, b, c, d, e}, {0, 5, 2}]
```

$\{\{\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, e\}, \{b, c\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}, \{a, b, c, d\}, \{a, b, c, e\}, \{a, b, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}\}$

The 25th subset:

```
>> Subsets[Range[5], All, {25}]
```

$\{\{2, 4, 5\}\}$

The odd-numbered subsets of {a,b,c,d} in reverse order:

```
>> Subsets[{a, b, c, d}, All, {15, 1, -2}]
```

$\{\{b, c, d\}, \{a, b, d\}, \{c, d\}, \{b, c\}, \{a, c\}, \{d\}, \{b\}, \{\}\}$

YuleDissimilarity

`YuleDissimilarity[`*u*`, `*v*`]`
: returns the Yule dissimilarity between the two boolean 1-D lists *u* and *v*, which is defined as R / (c_tt * c_ff + R / 2) where n is len(*u*), c_ij is the number of occurrences of *u*[k]=i and *v*[k]=j for k<n, and R = 2 * c_tf * c_ft.

```
>> YuleDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
```

$\frac{6}{5}$

XI. Global System Information

Contents

$Aborted

`$Aborted`
: is returned by a calculation that has been aborted.

$ByteOrdering

`$ByteOrdering`
: returns the native ordering of bytes in binary data on your computer system.

>> **`$ByteOrdering`**

-1

$CommandLine

`$CommandLine`
: is a list of strings passed on the command line to launch the Mathics session.

>> **`$CommandLine`**

{mathics/test.py, -ot, -k}

Environment

`Environment[`*var*`]`
: gives the value of an operating system environment variable.

>> **`Environment["HOME"]`**

/home/rocky

$Failed

`$Failed`
: is returned by some functions in the event of an error.

GetEnvironment

`GetEnvironment["`*var*`"]`
: gives the setting corresponding to the variable “var” in the operating system environment.

>> **`GetEnvironment["HOME"]`**

HOME− > /home/rocky

$Machine

`$Machine`
: returns a string describing the type of computer system on which the Mathics is being run.

```
>> $Machine
```
linux

$MachineName

`$MachineName`
: is a string that gives the assigned name of the computer on which Mathics is being run, if such a name is defined.

```
>> $MachineName
```
muffin

MathicsVersion

`MathicsVersion`
: this string is the version of Mathics we are running.

```
>> MathicsVersion
```
2.1.0

Names

`Names["`*pattern*`"]`
: returns the list of names matching *pattern*.

```
>> Names["List"]
```
{List}

The wildcard `*` matches any character:

```
>> Names["List*"]
```
{List, ListLinePlot, ListPlot, ListQ, Listable}

The wildcard `@` matches only lowercase characters:

```
>> Names["List@"]
```
{Listable}

```
>> x = 5;
```

```
>> Names["Global`*"]
```
{x}

The number of built-in symbols:

```
>> Length[Names["System`*"]]
```
1 088

$Packages

`$Packages`
: returns a list of the contexts corresponding to all packages which have been loaded into Mathics.

```
>> $Packages
```
{ImportExport`, XML`, Internal`, System`, Global`}

$ParentProcessID

`$ParentProcesID`
: gives the ID assigned to the process which invokes the *Mathics* by the operating system under which it is run.

```
>> $ParentProcessID
```
3 751 183

$ProcessID

`$ProcessID`
: gives the ID assigned to the *Mathics* process by the operating system under which it is run.

```
>> $ProcessID
```
3 751 185

$ProcessorType

`$ProcessorType`
: gives a string giving the architecture of the processor on which the *Mathics* is being run.

>> `$ProcessorType`

x86_64

Run

`Run[`*command*`]`
: runs command as an external operating system command, returning the exit code obtained.

>> `Run["date"]`

0

$ScriptCommandLine

`$ScriptCommandLine`
: is a list of string arguments when running the kernel is script mode.

>> `$ScriptCommandLine`

{}

$SystemID

`$SystemID`
: is a short string that identifies the type of computer system on which the *Mathics* is being run.

>> `$SystemID`

linux

$SystemWordLength

`$SystemWordLength`
: gives the effective number of bits in raw machine words on the computer system where *Mathics* is running.

>> `$SystemWordLength`

64

$UserName

$UserName
: returns a string describing the type of computer system on which *Mathics* is being run.

>> `$UserName`

rocky

$Version

`$Version`
: returns a string with the current Mathics version and the versions of relevant libraries.

>> `$Version`

Mathics 2.1.0 on CPython 3.6.12 (default, Oct 24 2 020, 10:34:18) using SymPy 1.7.1, mpmath 1.1.0, numpy 1.19.2, cython 0.29.21

$VersionNumber

`$VersionNumber`
: is a real number which gives the current Wolfram Language version that *Mathics* tries to be compatible with.

>> `$VersionNumber`

6.

XII. The Main Loop

Contents

$Post

$Post
is a global variable whose value, if set, is applied to every output expression.

$Pre

$Pre
is a global variable whose value, if set, is applied to every input expression.

Set *Timing* as the $Pre function, stores the enlapsed time in a variable, stores just the result in Out[$Line] and print a formated version showing the enlapsed time

```
>> $Pre := (Print["[Processing
   input...]"];#1)&

>> $Post := (Print["[Storing result
   ...]"]; #1)&
```

[Processinginput...]

[Storingresult...]

```
>> $PrePrint := (Print["The result
   is:"]; {TimeUsed[], #1})&
```

[Processinginput...]

[Storingresult...]

```
>> 2 + 2
```

[Processinginput...]

[Storingresult...]

Theresultis :

$\{403.586, 4\}$

```
>> $Pre = .; $Post = .; $PrePrint =
   .; $EnlapsedTime = .;
```

[Processinginput...]

```
>> 2 + 2
```

4

$PrePrint

$PrePrint
is a global variable whose value, if set, is applied to every output expression before it is printed.

$PreRead

$PreRead
is a global variable whose value, if set, is applied to the text or box form of every input expression before it is fed to the parser.
(Not implemented yet)

$SyntaxHandler

$SyntaxHandler
is a global variable whose value, if set, is applied to any input string that is found to contain a syntax error.
(Not implemented yet)

XIII. Quantities

Contents

KnownUnitQ

`KnownUnitQ[`*unit*`]`
: returns True if *unit* is a canonical unit, and False otherwise.

```
>> KnownUnitQ["Feet"]
True
```

```
>> KnownUnitQ["Foo"]
False
```

Quantity

`Quantity[`*magnitude*`, `*unit*`]`
: represents a quantity with size *magnitude* and unit specified by *unit*.

`Quantity[`*unit*`]`
: assumes the magnitude of the specified *unit* to be 1.

```
>> Quantity["Kilogram"]
1kilogram
```

```
>> Quantity[10, "Meters"]
10meter
```

```
>> Quantity[{10,20}, "Meters"]
{10meter, 20meter}
```

QuantityMagnitude

`QuantityMagnitude[`*quantity*`]`
: gives the amount of the specified *quantity*.

`QuantityMagnitude[`*quantity*`, `*unit*`]`
: gives the value corresponding to *quantity* when converted to *unit*.

```
>> QuantityMagnitude[Quantity["
   Kilogram"]]
1
```

```
>> QuantityMagnitude[Quantity[10, "
   Meters"]]
10
```

```
>> QuantityMagnitude[Quantity
   [{10,20}, "Meters"]]
{10, 20}
```

QuantityQ

`QuantityQ[`*expr*`]`
: return True if *expr* is a valid Association object, and False otherwise.

```
>> QuantityQ[Quantity[3, "Meters"]]
True
```

```
>> QuantityQ[Quantity[3, "Maters"]]
Unabletointerpretunitspecification Maters.
False
```

QuantityUnit

`QuantityUnit[`*quantity*`]`
: returns the unit associated with the specified *quantity*.

```
>> QuantityUnit[Quantity["Kilogram
   "]]
```

kilogram

```
>> QuantityUnit[Quantity[10, "
   Meters"]]
```

meter

```
>> QuantityUnit[Quantity[{10,20}, "
   Meters"]]
```

{meter, meter}

UnitConvert

`UnitConvert[`*quantity*`, `*targetunit*`]`
: converts the specified *quantity* to the specified *targetunit*.

`UnitConvert[quantity]`
: converts the specified *quantity* to its "SIBase" units.

Convert from miles to kilometers:

```
>> UnitConvert[Quantity[5.2, "miles
   "], "kilometers"]
```

8.36859kilometer

Convert a Quantity object to the appropriate SI base units:

```
>> UnitConvert[Quantity[3.8, "
   Pounds"]]
```

1.72365kilogram

XIV. Solving Recurrence Equations

Contents

RSolve

`RSolve[`*eqn*`, `*a*`[`*n*`], `*n*`]`
solves a recurrence equation for the function *a*[*n*].

Solve a difference equation:

```
>> RSolve[a[n] == a[n+1], a[n], n]
```

$\{\{a[n]->C[0]\}\}$

No boundary conditions gives two general paramaters:

```
>> RSolve[{a[n + 2] == a[n]}, a, n]
```

$\{\{a->(\text{Function}[\{n\}, C[0]+C[1]-1^{\wedge}n])\}\}$

Include one boundary condition:

```
>> RSolve[{a[n + 2] == a[n], a[0]
   == 1}, a, n]
```

$\{\{a->(\text{Function}[\{n\}, 1-C[1]+C[1]-1^{\wedge}n])\}\}$

Geta "pure function" solution for a with two boundary conditions:

```
>> RSolve[{a[n + 2] == a[n], a[0]
   == 1, a[1] == 4}, a, n]
```

$$\left\{\left\{a->\left(\text{Function}\left[\{n\}, \frac{5}{2}-\frac{3-1^{\wedge}n}{2}\right]\right)\right\}\right\}$$

XV. Patterns and Rules

Some examples:

```
>> a + b + c /. a + b -> t
```

$c + t$

```
>> a + 2 + b + c + x * y /.
   n_Integer + s__Symbol + rest_ ->
    {n, s, rest}
```

$\{2, a, b + c + xy\}$

```
>> f[a, b, c, d] /. f[first_,
   rest___] -> {first, {rest}}
```

$\{a, \{b, c, d\}\}$

Tests and Conditions:

```
>> f[4] /. f[x_?(# > 0&)] -> x ^ 2
```

16

```
>> f[4] /. f[x_] /; x > 0 -> x ^ 2
```

16

Leaves in the beginning of a pattern rather match fewer leaves:

```
>> f[a, b, c, d] /. f[start__,
   end__] -> {{start}, {end}}
```

$\{\{a\}, \{b, c, d\}\}$

Optional arguments using `Optional`:

```
>> f[a] /. f[x_, y_:3] -> {x, y}
```

$\{a, 3\}$

Options using `OptionsPattern` and `OptionValue`:

```
>> f[y, a->3] /. f[x_,
   OptionsPattern[{a->2, b->5}]] ->
    {x, OptionValue[a], OptionValue
   [b]}
```

$\{y, 3, 5\}$

The attributes `Flat`, `Orderless`, and `OneIdentity` affect pattern matching.

Contents

Alternatives (|)

`Alternatives[`*p1*`, `*p2*`, ..., `*p_i*`]`
p1 `|` *p2* `| ... |` *p_i*
    is a pattern that matches any of the patterns '*p1*, *p2*, ..., *p_i*'.

```
>> a+b+c+d/.(a|b)->t
```

$c + d + 2t$

Alternatives can also be used for string expressions

```
>> StringReplace["0123 3210", "1" |
    "2" -> "X"]
```

0XX3 3XX0

Blank

`Blank[]`
`_`
: represents any single expression in a pattern.

`Blank[`*h*`]`
`_`*h*
: represents any expression with head *h*.

```
>> MatchQ[a + b, _]
   True
```

Patterns of the form _*h* can be used to test the types of objects:

```
>> MatchQ[42, _Integer]
   True
```

```
>> MatchQ[1.0, _Integer]
   False
```

```
>> {42, 1.0, x} /. {_Integer -> "
   integer", _Real -> "real"} //
   InputForm
```

{"integer", "real", x}

`Blank` only matches a single expression:

```
>> MatchQ[f[1, 2], f[_]]
   False
```

BlankNullSequence

`BlankNullSequence[]`
`___`
: represents any sequence of expression leaves in a pattern, including an empty sequence.

`BlankNullSequence` is like `BlankSequence`, except it can match an empty sequence:

```
>> MatchQ[f[], f[___]]
   True
```

BlankSequence

`BlankSequence[]`
`__`
: represents any non-empty sequence of expression leaves in a pattern.

`BlankSequence[`*h*`]`
`__`*h*
: represents any sequence of leaves, all of which have head *h*.

Use a `BlankSequence` pattern to stand for a non-empty sequence of arguments:

```
>> MatchQ[f[1, 2, 3], f[__]]
   True
```

```
>> MatchQ[f[], f[__]]
   False
```

__*h* will match only if all leaves have head *h*:

```
>> MatchQ[f[1, 2, 3], f[__Integer]]
   True
```

```
>> MatchQ[f[1, 2.0, 3], f[__Integer
   ]]
   False
```

The value captured by a named `BlankSequence` pattern is a `Sequence` object:

```
>> f[1, 2, 3] /. f[x__] -> x
```

Sequence $[1, 2, 3]$

Condition (/;)

`Condition[`*pattern*`, `*expr*`]`
pattern `/;` *expr*
: places an additional constraint on *pattern* that only allows it to match if *expr* evaluates to `True`.

The controlling expression of a `Condition` can use variables from the pattern:

```
>> f[3] /. f[x_] /; x>0 -> t
```

t

```
>> f[-3] /. f[x_] /; x>0 -> t
```

$f[-3]$

`Condition` can be used in an assignment:

```
>> f[x_] := p[x] /; x>0
```

```
>> f[3]
```
$p[3]$

```
>> f[-3]
```
$f[-3]$

Except

`Except[`*c*`]`
: represents a pattern object that matches any expression except those matching *c*.

`Except[`*c*`, `*p*`]`
: represents a pattern object that matches *p* but not *c*.

```
>> Cases[{x, a, b, x, c}, Except[x
   ]]
```
$\{a, b, c\}$

```
>> Cases[{a, 0, b, 1, c, 2, 3},
   Except[1, _Integer]]
```
$\{0, 2, 3\}$

Except can also be used for string expressions:

```
>> StringReplace["Hello world!",
   Except[LetterCharacter] -> ""]
```
Helloworld

HoldPattern

`HoldPattern[`*expr*`]`
: is equivalent to *expr* for pattern matching, but maintains it in an unevaluated form.

```
>> HoldPattern[x + x]
```
HoldPattern $[x + x]$

```
>> x /. HoldPattern[x] -> t
```
t

`HoldPattern` has attribute `HoldAll`:

```
>> Attributes[HoldPattern]
```
{HoldAll, Protected}

Longest

```
>> StringCases["aabaaab", Longest["
   a" ~~__ ~~"b"]]
```
{aabaaab}

```
>> StringCases["aabaaab", Longest[
   RegularExpression["a+b"]]]
```
{aab, aaab}

MatchQ

`MatchQ[`*expr*`, `*form*`]`
: tests whether *expr* matches *form*.

```
>> MatchQ[123, _Integer]
```
True

```
>> MatchQ[123, _Real]
```
False

```
>> MatchQ[_Integer][123]
```
True

Optional (:)

`Optional[`*patt*`, `*default*`]`
patt : *default*
: is a pattern which matches *patt*, which if omitted should be replaced by *default*.

```
>> f[x_, y_:1] := {x, y}
```

```
>> f[1, 2]
```
$\{1, 2\}$

```
>> f[a]
```
$\{a, 1\}$

Note that *symb* : *patt* represents a `Pattern` object. However, there is no disambiguity, since *symb* has to be a symbol in this case.

```
>> x:_ // FullForm
```
Pattern $[x$, Blank []]

```
>> _:d // FullForm
```
Optional [Blank [] , d]

```
>> x:_+y_:d // FullForm
```
Pattern $[x, \text{Plus}[\text{Blank}[],$
$\text{Optional}[\text{Pattern}[y, \text{Blank}[]], d]]]$

`s_.` is equivalent to `Optional[s_]` and represents an optional parameter which, if omitted, gets its value from `Default`.

```
>> FullForm[s_.]
```
Optional [Pattern [s, Blank []]]

```
>> Default[h, k_] := k
```

```
>> h[a] /. h[x_, y_.] -> {x, y}
```
$\{a, 2\}$

OptionsPattern

`OptionsPattern[`*f*`]`
: is a pattern that stands for a sequence of options given to a function, with default values taken from `Options[`*f*`]`. The options can be of the form *opt*`->`*value* or *opt*`:>`*value*, and might be in arbitrarily nested lists.

`OptionsPattern[{`*opt1*`->`*value1*`, ...}]`
: takes explicit default values from the given list. The list may also contain symbols *f*, for which `Options[`*f*`]` is taken into account; it may be arbitrarily nested. `OptionsPattern[{}]` does not use any default values.

The option values can be accessed using `OptionValue`.

```
>> f[x_, OptionsPattern[{n->2}]] :=
    x ^ OptionValue[n]
```

```
>> f[x]
```
x^2

```
>> f[x, n->3]
```
x^3

Delayed rules as options:

```
>> e = f[x, n:>a]
```
x^a

```
>> a = 5;
```

```
>> e
```
x^5

Options might be given in nested lists:

```
>> f[x, {{{n->4}}}]
```
x^4

PatternTest (?)

`PatternTest[`*pattern*`, `*test*`]`
pattern `?` *test*
: constrains *pattern* to match *expr* only if the evaluation of *test*`[`*expr*`]` yields `True`.

```
>> MatchQ[3, _Integer?(#>0&)]
```
True

```
>> MatchQ[-3, _Integer?(#>0&)]
```
False

Pattern

`Pattern[`*symb*`, `*patt*`]`
symb `:` *patt*
: assigns the name *symb* to the pattern *patt*.

symb_head
: is equivalent to *symb* `:` *_head* (accordingly with __ and ___).

symb `:` *patt* `:` *default*
: is a pattern with name *symb* and default value *default*, equivalent to `Optional[`*patt* `:` *symb*`, `*default*`]`.

```
>> FullForm[a_b]
```
Pattern [a, Blank [b]]

```
>> FullForm[a:_:b]
```
Optional [Pattern [a, Blank []] , b]

`Pattern` has attribute `HoldFirst`, so it does not evaluate its name:

```
>> x = 2
```
2

```
>> x_
```
x_

Nested `Pattern` assign multiple names to the same pattern. Still, the last parameter is the default value.

```
>> f[y] /. f[a:b,_:d] -> {a, b}
```
$f[y]$

This is equivalent to:

```
>> f[a] /. f[a:_:b] -> {a, b}
```

$\{a, b\}$

FullForm:

```
>> FullForm[a:b:c:d:e]
```

Optional [Pattern [a, b] , Optional [Pattern [c, d] , e]]

```
>> f[] /. f[a:_:b] -> {a, b}
```

$\{b, b\}$

Repeated (..)

Repeated[*pattern*]
: matches one or more occurrences of *pattern*.

```
>> a_Integer.. // FullForm
```

Repeated [Pattern [a, Blank [Integer]]]

```
>> 0..1//FullForm
```

Repeated [0]

```
>> {{}, {a}, {a, b}, {a, a, a}, {a, a, a, a}} /. {Repeated[x : a | b, 3]} -> x
```

$\{\{\}, a, \{a, b\}, a, \{a, a, a, a\}\}$

```
>> f[x, 0, 0, 0] /. f[x, s:0..] -> s
```

Sequence [0, 0, 0]

RepeatedNull (...)

RepeatedNull[*pattern*]
: matches zero or more occurrences of *pattern*.

```
>> a___Integer...//FullForm
```

RepeatedNull [Pattern [a, BlankNullSequence [Integer]]]

```
>> f[x] /. f[x, 0...] -> t
```

t

Replace

Replace[*expr*, *x* -> *y*]
: yields the result of replacing *expr* with *y* if it matches the pattern *x*.

Replace[*expr*, *x* -> *y*, *levelspec*]
: replaces only subexpressions at levels specified through *levelspec*.

Replace[*expr*, {*x* -> *y*, ...}]
: performs replacement with multiple rules, yielding a single result expression.

Replace[*expr*, {{*a* -> *b*, ...}, {*c* -> *d*, ...}, ...}]
: returns a list containing the result of performing each set of replacements.

```
>> Replace[x, {x -> 2}]
```

2

By default, only the top level is searched for matches

```
>> Replace[1 + x, {x -> 2}]
```

$1 + x$

```
>> Replace[x, {{x -> 1}, {x -> 2}}]
```

$\{1, 2\}$

Replace stops after the first replacement

```
>> Replace[x, {x -> {}, _List -> y}]
```

$\{\}$

Replace replaces the deepest levels first

```
>> Replace[x[1], {x[1] -> y, 1 -> 2}, All]
```

x [2]

By default, heads are not replaced

```
>> Replace[x[x[y]], x -> z, All]
```

x [x [y]]

Heads can be replaced using the Heads option

```
>> Replace[x[x[y]], x -> z, All, Heads -> True]
```

z [z [y]]

Note that heads are handled at the level of leaves

```
>> Replace[x[x[y]], x -> z, {1}, Heads -> True]
```

z [x [y]]

You can use Replace as an operator

```
>> Replace[{x_ -> x + 1}][10]
```
11

ReplaceAll (/.)

`ReplaceAll[`*expr*`, ` *x* ` -> ` *y*`]`
expr `/.` *x* `->` *y*
: yields the result of replacing all subexpressions of *expr* matching the pattern *x* with *y*.

expr `/. {`*x* `->` *y*`, ...}`
: performs replacement with multiple rules, yielding a single result expression.

expr `/. {{`*a* `->` *b*`, ...}, {`*c* `->` *d*`, ...}, ...}`
: returns a list containing the result of performing each set of replacements.

```
>> a+b+c /. c->d
```
$a + b + d$

```
>> g[a+b+c,a]/.g[x_+y_,x_]->{x,y}
```
$\{a, b + c\}$

If *rules* is a list of lists, a list of all possible respective replacements is returned:

```
>> {a, b} /. {{a->x, b->y}, {a->u, b->v}}
```
$\{\{x, y\}, \{u, v\}\}$

The list can be arbitrarily nested:

```
>> {a, b} /. {{{a->x, b->y}, {a->w, b->z}}, {a->u, b->v}}
```
$\{\{\{x, y\}, \{w, z\}\}, \{u, v\}\}$

```
>> {a, b} /. {{{a->x, b->y}, a->w, b->z}, {a->u, b->v}}
```
Elementsof$\{\{a->x, b->y\}, a->w, b->z\}$areamixtureoflistsandnonlists.

$\{a, b\} /. \{\{a->x, b->y\}, a->w, b->z\}, \{u, v\}\}$

ReplaceAll also can be used as an operator:

```
>> ReplaceAll[{a -> 1}][{a, b}]
```
$\{1, b\}$

ReplaceAll replaces the shallowest levels first:

```
>> ReplaceAll[x[1], {x[1] -> y, 1 -> 2}]
```
y

ReplaceList

`ReplaceList[`*expr*`, ` *rules*`]`
: returns a list of all possible results of applying *rules* to *expr*.

Get all subsequences of a list:

```
>> ReplaceList[{a, b, c}, {___, x__, ___} -> {x}]
```
$\{\{a\}, \{a, b\}, \{a, b, c\}, \{b\}, \{b, c\}, \{c\}\}$

You can specify the maximum number of items:

```
>> ReplaceList[{a, b, c}, {___, x__, ___} -> {x}, 3]
```
$\{\{a\}, \{a, b\}, \{a, b, c\}\}$

```
>> ReplaceList[{a, b, c}, {___, x__, ___} -> {x}, 0]
```
$\{\}$

If no rule matches, an empty list is returned:

```
>> ReplaceList[a, b->x]
```
$\{\}$

Like in `ReplaceAll`, *rules* can be a nested list:

```
>> ReplaceList[{a, b, c}, {{{___, x__, ___} -> {x}}, {{a, b, c} -> t}}, 2]
```
$\{\{\{a\}, \{a, b\}\}, \{t\}\}$

```
>> ReplaceList[expr, {}, -1]
```
Non$-$negativeintegerorInfinityexpectedatposition3.

ReplaceList [expr, {}, −1]

Possible matches for a sum:

```
>> ReplaceList[a + b + c, x_ + y_ -> {x, y}]
```
$\{\{a, b + c\}, \{b, a + c\}, \{c, a + b\}, \{a + b, c\}, \{a + c, b\}, \{b + c, a\}\}$

ReplaceRepeated (//.)

`ReplaceRepeated[`*expr*`, ` *x* ` -> ` *y*`]`
expr `//.` *x* `->` *y*
: repeatedly applies the rule *x* `->` *y* to *expr* until the result no longer changes.

```
>> a+b+c //. c->d
```
$a + b + d$

Simplification of logarithms:
```
>> logrules = {Log[x_ * y_] :> Log[
   x] + Log[y], Log[x_ ^ y_] :> y *
    Log[x]};
```
```
>> Log[a * (b * c)^ d ^ e * f] //.
   logrules
```
$\mathrm{Log}\left[a\right]+\mathrm{Log}\left[f\right]+\left(\mathrm{Log}\left[b\right]+\mathrm{Log}\left[c\right]\right)d^{e}$

`ReplaceAll` just performs a single replacement:
```
>> Log[a * (b * c)^ d ^ e * f] /.
   logrules
```
$\mathrm{Log}\left[a\right]+\mathrm{Log}\left[f\left(bc\right)^{d^{e}}\right]$

RuleDelayed (:>)

`RuleDelayed[`*x*`, `*y*`]`
x `:>` *y*
: represents a rule replacing *x* with *y*, with *y* held unevaluated.

```
>> Attributes[RuleDelayed]
```
{HoldRest, Protected, SequenceHold}

Rule (->)

`Rule[`*x*`, `*y*`]`
x `->` *y*
: represents a rule replacing *x* with *y*.

```
>> a+b+c /. c->d
```
$a + b + d$

```
>> {x,x^2,y} /. x->3
```
$\{3, 9, y\}$

Shortest

Verbatim

`Verbatim[`*expr*`]`
: prevents pattern constructs in *expr* from taking effect, allowing them to match themselves.

Create a pattern matching `Blank`:
```
>> _ /. Verbatim[_]->t
```
t

```
>> x /. Verbatim[_]->t
```
x

Without `Verbatim`, `Blank` has its normal effect:
```
>> x /. _->t
```
t

XVI. Mathematical Functions

Basic arithmetic functions, including complex number arithmetic.

Contents

Abs

`Abs[`x`]`
returns the absolute value of x.

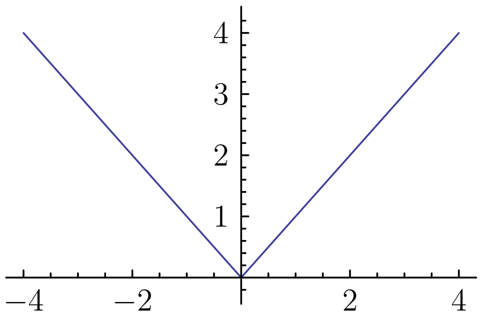
>> `Abs[-3]`
3

`Abs` returns the magnitude of complex numbers:

>> `Abs[3 + I]`
$\sqrt{10}$

>> `Abs[3.0 + I]`
3.16228

>> `Plot[Abs[x], {x, -4, 4}]`



Boole

`Boole[expr]`
returns 1 if expr is True and 0 if expr is False.

>> `Boole[2 == 2]`
1

>> `Boole[7 < 5]`
0

>> `Boole[a == 7]`
Boole $[a{=}{=}7]$

Complex

`Complex`
is the head of complex numbers.
`Complex[`a`, `b`]`
constructs the complex number a `+ I` b.

>> `Head[2 + 3*I]`
Complex

```
>> Complex[1, 2/3]
```
$1+\frac{2I}{3}$

```
>> Abs[Complex[3, 4]]
```
5

Conjugate

`Conjugate[z]`
: returns the complex conjugate of the complex number z.

```
>> Conjugate[3 + 4 I]
```
$3-4I$

```
>> Conjugate[3]
```
3

```
>> Conjugate[a + b * I]
```
$\text{Conjugate}[a]-I\text{Conjugate}[b]$

```
>> Conjugate[{{1, 2 + I 4, a + I b
}, {I}}]
```
$\{\{1,2-4I,\text{Conjugate}[a]-I\text{Conjugate}[b]\},\{-I\}\}$

```
>> Conjugate[1.5 + 2.5 I]
```
$1.5-2.5I$

CubeRoot

`CubeRoot[n]`
: finds the real-valued cube root of the given n.

```
>> CubeRoot[16]
```
$22^{\frac{1}{3}}$

DirectedInfinity

`DirectedInfinity[z]`
: represents an infinite multiple of the complex number z.

`DirectedInfinity[]`
: is the same as `ComplexInfinity`.

```
>> DirectedInfinity[1]
```
∞

```
>> DirectedInfinity[]
```
ComplexInfinity

```
>> DirectedInfinity[1 + I]
```
$\left(\frac{1}{2}+\frac{I}{2}\right)\sqrt{2}\infty$

```
>> 1 / DirectedInfinity[1 + I]
```
0

```
>> DirectedInfinity[1] +
DirectedInfinity[-1]
```
Indeterminateexpression − Infinity + Infinityencountered.

Indeterminate

```
>> DirectedInfinity[0]
```
Indeterminateexpression0Infinityencountered.

Indeterminate

Divide (/)

`Divide[a, b]`
`a / b`
: represents the division of a by b.

```
>> 30 / 5
```
6

```
>> 1 / 8
```
$\frac{1}{8}$

```
>> Pi / 4
```
$\frac{\text{Pi}}{4}$

Use `N` or a decimal point to force numeric evaluation:

```
>> Pi / 4.0
```
0.785398

```
>> 1 / 8
```
$\frac{1}{8}$

```
>> N[%]
```
0.125

Nested divisions:

```
>> a / b / c
```
$\frac{a}{bc}$

```
>> a / (b / c)
```
$\frac{ac}{b}$

```
>> a / b / (c / (d / e))
```
$\frac{ad}{bce}$

```
>> a / (b ^ 2 * c ^ 3 / e)
```
$\frac{ae}{b^2c^3}$

ExactNumberQ

`ExactNumberQ[`*expr*`]`
: returns `True` if *expr* is an exact number, and `False` otherwise.

```
>> ExactNumberQ[10]
```
True

```
>> ExactNumberQ[4.0]
```
False

```
>> ExactNumberQ[n]
```
False

`ExactNumberQ` can be applied to complex numbers:

```
>> ExactNumberQ[1 + I]
```
True

```
>> ExactNumberQ[1 + 1. I]
```
False

Factorial (!)

`Factorial[`*n*`]`
n`!`
: computes the factorial of *n*.

```
>> 20!
```
2 432 902 008 176 640 000

`Factorial` handles numeric (real and complex) values using the gamma function:

```
>> 10.5!
```
1.18994×10^7

```
>> (-3.0+1.5*I)!
```
$0.0427943 - 0.00461565I$

However, the value at poles is `ComplexInfinity`:

```
>> (-1.)!
```
ComplexInfinity

`Factorial` has the same operator (`!`) as `Not`, but with higher precedence:

```
>> !a! //FullForm
```
Not [Factorial [*a*]]

Gamma

`Gamma[`*z*`]`
: is the gamma function on the complex number *z*.

`Gamma[`*z*`, `*x*`]`
: is the upper incomplete gamma function.

`Gamma[`*z*`, `*x0*`, `*x1*`]`
: is equivalent to `Gamma[`*z*`, `*x0*`] - Gamma[`*z*`, `*x1*`]`.

`Gamma[`*z*`]` is equivalent to `(`*z*` - 1)!`:

```
>> Simplify[Gamma[z] - (z - 1)!]
```
0

Exact arguments:

```
>> Gamma[8]
```
5 040

```
>> Gamma[1/2]
```
$\sqrt{\text{Pi}}$

```
>> Gamma[1, x]
```
E^{-x}

```
>> Gamma[0, x]
```
ExpIntegralE [1, *x*]

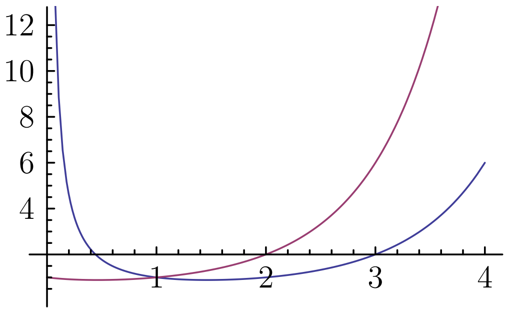
Numeric arguments:

```
>> Gamma[123.78]
```
4.21078×10^{204}

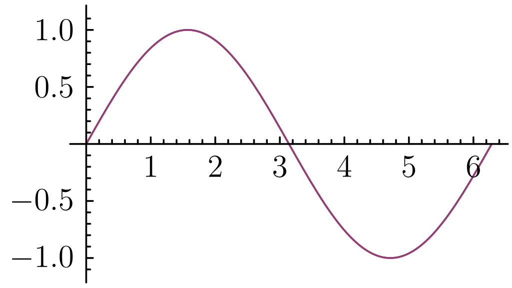
```
>> Gamma[1. + I]
```
$0.498016 - 0.15495I$

Both `Gamma` and `Factorial` functions are continuous:

```
>> Plot[{Gamma[x], x!}, {x, 0, 4}]
```




```
>> Plot[{Sin[a], Im[E^(I a)]}, {a, 0, 2 Pi}]
```



HarmonicNumber

`HarmonicNumber[n]`
returns the *n*th harmonic number.

```
>> Table[HarmonicNumber[n], {n, 8}]
```

$$\left\{1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, \frac{137}{60}, \frac{49}{20}, \frac{363}{140}, \frac{761}{280}\right\}$$

```
>> HarmonicNumber[3.8]
```

2.03806

I

`I`
represents the imaginary number `Sqrt[-1]`.

```
>> I^2
```

-1

```
>> (3+I)*(3-I)
```

10

Im

`Im[z]`
returns the imaginary component of the complex number *z*.

```
>> Im[3+4I]
```

4

InexactNumberQ

`InexactNumberQ[`*expr*`]`
returns `True` if *expr* is not an exact number, and `False` otherwise.

```
>> InexactNumberQ[a]
```

False

```
>> InexactNumberQ[3.0]
```

True

```
>> InexactNumberQ[2/3]
```

False

`InexactNumberQ` can be applied to complex numbers:

```
>> InexactNumberQ[4.0+I]
```

True

IntegerQ

`IntegerQ[`*expr*`]`
returns `True` if *expr* is an integer, and `False` otherwise.

```
>> IntegerQ[3]
```

True

```
>> IntegerQ[Pi]
```

False

Integer

`Integer`
is the head of integers.

```
>> Head[5]
```
Integer

MachineNumberQ

`MachineNumberQ[expr]`
: returns `True` if *expr* is a machine-precision real or complex number.

= True

```
>> MachineNumberQ
   [3.14159265358979324]
```
False

```
>> MachineNumberQ[1.5 + 2.3 I]
```
True

```
>> MachineNumberQ
   [2.71828182845904524 +
   3.14159265358979324 I]
```
False

Minus (-)

`Minus[expr]`
: is the negation of *expr*.

```
>> -a //FullForm
```
Times [− 1, *a*]

`Minus` automatically distributes:

```
>> -(x - 2/3)
```
$\frac{2}{3} - x$

`Minus` threads over lists:

```
>> -Range[10]
```
$\{-1, -2, -3, -4, -5, -6, -7, -8, -9, -10\}$

NumberQ

`NumberQ[expr]`
: returns `True` if *expr* is an explicit number, and `False` otherwise.

```
>> NumberQ[3+I]
```
True

```
>> NumberQ[5!]
```
True

```
>> NumberQ[Pi]
```
False

Piecewise

`Piecewise[{{expr1, cond1}, ...}]`
: represents a piecewise function.

`Piecewise[{{expr1, cond1}, ...}, expr]`
: represents a piecewise function with default `expr`.

Heaviside function

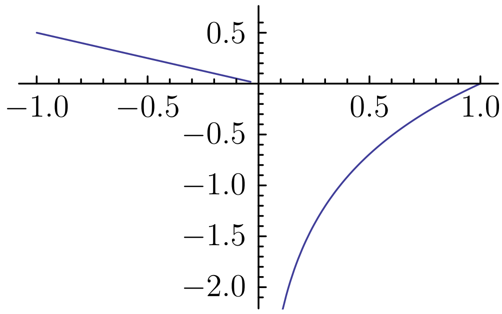
```
>> Piecewise[{{0, x <= 0}}, 1]
```
Piecewise $[\{\{0, x{<=}0\}\}, 1]$

```
>> Integrate[Piecewise[{{1, x <=
   0}, {-1, x > 0}}], x]
```
Piecewise $[\{\{x, x{<=}0\}, \{-x, \text{True}\}\}]$

```
>> Integrate[Piecewise[{{1, x <=
   0}, {-1, x > 0}}], {x, -1, 2}]
```
-1

Piecewise defaults to 0 if no other case is matching.

```
>> Piecewise[{{1, False}}]
```
0

```
>> Plot[Piecewise[{{Log[x], x > 0},
    {x*-0.5, x < 0}}], {x, -1, 1}]
```



```
>> Piecewise[{{0 ^ 0, False}}, -1]
```
-1

Plus (+)

`Plus[a, b, ...]`
$a + b + ...$
represents the sum of the terms a, b, ...

```
>> 1 + 2
```
3

`Plus` performs basic simplification of terms:

```
>> a + b + a
```
$2a + b$

```
>> a + a + 3 * a
```
$5a$

```
>> a + b + 4.5 + a + b + a + 2 + 1.5 b
```
$6.5 + 3a + 3.5b$

Apply `Plus` on a list to sum up its elements:

```
>> Plus @@ {2, 4, 6}
```
12

The sum of the first 1000 integers:

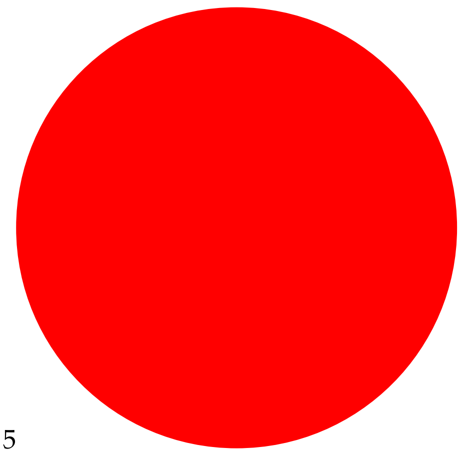
```
>> Plus @@ Range[1000]
```
500 500

`Plus` has default value 0:

```
>> DefaultValues[Plus]
```
{HoldPattern [Default [Plus]] :>0}

```
>> a /. n_. + x_ :> {n, x}
```
$\{0, a\}$

The sum of 2 red circles and 3 red circles is...

```
>> 2 Graphics[{Red,Disk[]}] + 3 Graphics[{Red,Disk[]}]
```


5

Pochhammer

`Pochhammer[a, n]`
is the Pochhammer symbol (a)_n.

```
>> Pochhammer[4, 8]
```
6 652 800

PossibleZeroQ

`PossibleZeroQ[expr]`
returns `True` if basic symbolic and numerical methods suggest that expr has value zero, and `False` otherwise.

Test whether a numeric expression is zero:

```
>> PossibleZeroQ[E^(I Pi/4)- (-1)^(1/4)]
```
True

The determination is approximate.
Test whether a symbolic expression is likely to be identically zero:

```
>> PossibleZeroQ[(x + 1)(x - 1)- x^2 + 1]
```
True

```
>> PossibleZeroQ[(E + Pi)^2 - E^2 - Pi^2 - 2 E Pi]
```
True

Show that a numeric expression is nonzero:

```
>> PossibleZeroQ[E^Pi - Pi^E]
```
False

```
>> PossibleZeroQ[1/x + 1/y - (x + y)/(x y)]
```
True

Decide that a numeric expression is zero, based on approximate computations:

```
>> PossibleZeroQ[2^(2 I)- 2^(-2 I)- 2 I Sin[Log[4]]]
```
True

```
>> PossibleZeroQ[Sqrt[x^2] - x]
```
False

Power (^)

`Power[`*a*`, `*b*`]`
a `^` *b*
: represents *a* raised to the power of *b*.

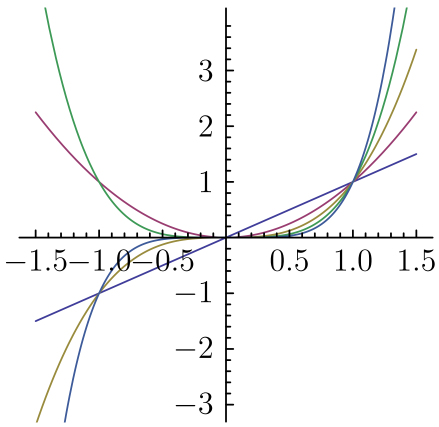
```
>> 4 ^ (1/2)
```
2

```
>> 4 ^ (1/3)
```
$2^{\frac{2}{3}}$

```
>> 3^123
```
48 519 278 097 689 642 681 ~
~155 855 396 759 336 072 ~
~749 841 943 521 979 872 827

```
>> (y ^ 2)^ (1/2)
```
$\sqrt{y^2}$

```
>> (y ^ 2)^ 3
```
y^6

```
>> Plot[Evaluate[Table[x^y, {y, 1,
5}]], {x, -1.5, 1.5},
AspectRatio -> 1]
```


Use a decimal point to force numeric evaluation:

```
>> 4.0 ^ (1/3)
```
1.5874

`Power` has default value 1 for its second argument:

```
>> DefaultValues[Power]
```
{HoldPattern [Default [Power, 2]] :>1}

```
>> a /. x_ ^ n_. :> {x, n}
```
$\{a, 1\}$

`Power` can be used with complex numbers:

```
>> (1.5 + 1.0 I)^ 3.5
```
$-3.68294 + 6.95139I$

```
>> (1.5 + 1.0 I)^ (3.5 + 1.5 I)
```
$-3.19182 + 0.645659I$

Product

`Product[`*expr*`, {`*i*`, `*imin*`, `*imax*`}]`
: evaluates the discrete product of *expr* with *i* ranging from *imin* to *imax*.

`Product[`*expr*`, {`*i*`, `*imax*`}]`
: same as `Product[`*expr*`, {`*i*`, 1, `*imax*`}]`.

`Product[`*expr*`, {`*i*`, `*imin*`, `*imax*`, `*di*`}]`
: *i* ranges from *imin* to *imax* in steps of *di*.

`Product[`*expr*`, {`*i*`, `*imin*`, `*imax*`}, {`*j*`, `*jmin*`, `*jmax*`}, ...]`
: evaluates *expr* as a multiple product, with {*i*, ...}, {*j*, ...}, ... being in outermost-to-innermost order.

```
>> Product[k, {k, 1, 10}]
```
3 628 800

```
>> 10!
```
3 628 800

```
>> Product[x^k, {k, 2, 20, 2}]
```
x^{110}

```
>> Product[2 ^ i, {i, 1, n}]
```
$2^{\frac{n}{2}+\frac{n^2}{2}}$

```
>> Product[f[i], {i, 1, 7}]
```
$f[1]f[2]f[3]f[4]f[5]f[6]f[7]$

Symbolic products involving the factorial are evaluated:

```
>> Product[k, {k, 3, n}]
```
$\frac{n!}{2}$

Evaluate the *n*th primorial:

```
>> primorial[0] = 1;
```

```
>> primorial[n_Integer] := Product[
Prime[k], {k, 1, n}];
```

```
>> primorial[12]
```
7 420 738 134 810

Rational

`Rational`
is the head of rational numbers.
`Rational[`*a*`, `*b*`]`
constructs the rational number *a* / *b*.

>> `Head[1/2]`
Rational

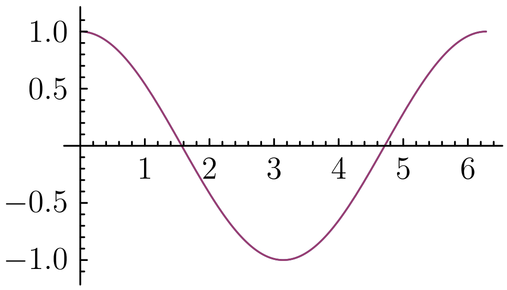
>> `Rational[1, 2]`
$\frac{1}{2}$

Re

`Re[`*z*`]`
returns the real component of the complex number *z*.

>> `Re[3+4I]`
3

>> `Plot[{Cos[a], Re[E^(I a)]}, {a, 0, 2 Pi}]`



RealNumberQ

`RealNumberQ[`*expr*`]`
returns `True` if *expr* is an explicit number with no imaginary component.

>> `RealNumberQ[10]`
True

>> `RealNumberQ[4.0]`
True

>> `RealNumberQ[1+I]`
False

>> `RealNumberQ[0 * I]`
True

>> `RealNumberQ[0.0 * I]`
True

Real

`Real`
is the head of real (inexact) numbers.

>> `x = 3. ^ -20;`

>> `InputForm[x]`
$2.8679719907924413*^{\wedge} - 10$

>> `Head[x]`
Real

Sign

`Sign[`*x*`]`
return -1, 0, or 1 depending on whether *x* is negative, zero, or positive.

>> `Sign[19]`
1

>> `Sign[-6]`
-1

>> `Sign[0]`
0

>> `Sign[{-5, -10, 15, 20, 0}]`
$\{-1, -1, 1, 1, 0\}$

>> `Sign[3 - 4*I]`
$\frac{3}{5} - \frac{4I}{5}$

Sqrt

`Sqrt[`*expr*`]`
returns the square root of *expr*.

>> `Sqrt[4]`
2

>> `Sqrt[5]`

$\sqrt{5}$

>> `Sqrt[5] // N`

2.23607

>> `Sqrt[a]^2`

a

Complex numbers:

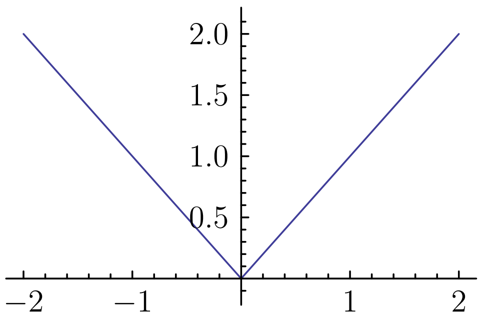
>> `Sqrt[-4]`

$2I$

>> `I == Sqrt[-1]`

True

>> `Plot[Sqrt[a^2], {a, -2, 2}]`



Subtract (-)

`Subtract[`*a*`, `*b*`]`
a - *b*
represents the subtraction of *b* from *a*.

>> `5 - 3`

2

>> `a - b // FullForm`

Plus$[a, \text{Times}[-1, b]]$

>> `a - b - c`

$a - b - c$

>> `a - (b - c)`

$a - b + c$

Sum

`Sum[`*expr*`, {`*i*`, `*imin*`, `*imax*`}]`
evaluates the discrete sum of *expr* with *i* ranging from *imin* to *imax*.
`Sum[`*expr*`, {`*i*`, `*imax*`}]`
same as `Sum[`*expr*`, {`*i*`, 1, `*imax*`}]`.
`Sum[`*expr*`, {`*i*`, `*imin*`, `*imax*`, `*di*`}]`
i ranges from *imin* to *imax* in steps of *di*.
`Sum[`*expr*`, {`*i*`, `*imin*`, `*imax*`}, {`*j*`, `*jmin*`, `*jmax*`}, ...]`
evaluates *expr* as a multiple sum, with {*i*, ...}, {*j*, ...}, ... being in outermost-to-innermost order.

>> `Sum[k, {k, 1, 10}]`

55

Double sum:

>> `Sum[i * j, {i, 1, 10}, {j, 1, 10}]`

3 025

Symbolic sums are evaluated:

>> `Sum[k, {k, 1, n}]`

$\frac{n(1+n)}{2}$

>> `Sum[k, {k, n, 2 n}]`

$\frac{3n(1+n)}{2}$

>> `Sum[k, {k, I, I + 1}]`

$1 + 2I$

>> `Sum[1 / k ^ 2, {k, 1, n}]`

HarmonicNumber$[n, 2]$

>> `Sum[f[i], {i, 1, 7}]`

$f[1] + f[2] + f[3] + f[4] + f[5] + f[6] + f[7]$

Verify algebraic identities:

>> `Sum[x ^ 2, {x, 1, y}] - y * (y + 1)* (2 * y + 1)/ 6`

0

Infinite sums:

>> `Sum[1 / 2 ^ i, {i, 1, Infinity}]`

1

>> `Sum[1 / k ^ 2, {k, 1, Infinity}]`

$\frac{\text{Pi}^2}{6}$

Times (*)

`Times[`*a*`, `*b*`, ...]`
a `*` *b* `* ...`
a *b* `...`
    represents the product of the terms *a*, *b*, ...

```
>> 10 * 2
```
20

```
>> 10 2
```
20

```
>> a * a
```
a^2

```
>> x ^ 10 * x ^ -2
```
x^8

```
>> {1, 2, 3} * 4
```
$\{4, 8, 12\}$

```
>> Times @@ {1, 2, 3, 4}
```
24

```
>> IntegerLength[Times@@Range
[5000]]
```
16 326

`Times` has default value 1:

```
>> DefaultValues[Times]
```
{HoldPattern [Default [Times]] :>1}

```
>> a /. n_. * x_ :> {n, x}
```
$\{1, a\}$

XVII. Functional Programming

Contents

Composition

`Composition[`*f*`, `*g*`]`
: returns the composition of two functions f and g.

```
>> Composition[f, g][x]
```
$f[g[x]]$

```
>> Composition[f, g, h][x, y, z]
```
$f[g[h[x,y,z]]]$

```
>> Composition[]
```
Identity

```
>> Composition[][x]
```
x

```
>> Attributes[Composition]
```
{Flat, OneIdentity, Protected}

```
>> Composition[f, Composition[g, h]]
```
Composition $[f, g, h]$

Function (&)

`Function[`*body*`]`
body `&`
: represents a pure function with parameters `#1`, `#2`, etc.

`Function[{`*x1*`, `*x2*`, ...}, `*body*`]`
: represents a pure function with parameters *x1*, *x2*, etc.

`Function[{`*x1*`, `*x2*`, ...}, `*body*`, `*attr*`]`
: assume that the function has the attributes *attr*.

```
>> f := # ^ 2 &
```

```
>> f[3]
```
9

```
>> #^3& /@ {1, 2, 3}
```
{1, 8, 27}

```
>> #1+#2&[4, 5]
```
9

You can use `Function` with named parameters:

```
>> Function[{x, y}, x * y][2, 3]
```
6

Parameters are renamed, when necessary, to avoid confusion:

```
>> Function[{x}, Function[{y}, f[x, y]]][y]
```
Function $[\{y\$\}, f[y, y\$]]$

```
>> Function[{y}, f[x, y]] /. x->y
```
Function $[\{y\}, f[y, y]]$

```
>> Function[y, Function[x, y^x]][x
   ][y]
```
x^y

```
>> Function[x, Function[y, x^y]][x
   ][y]
```
x^y

Slots in inner functions are not affected by outer function application:

```
>> g[#] & [h[#]] & [5]
```
$g[h[5]]$

Identity

`Identity[`*x*`]`
: is the identity function, which returns *x* unchanged.

```
>> Identity[x]
```
x

```
>> Identity[x, y]
```
Identity $[x, y]$

Slot

`#`*n*
: represents the *n*th argument to a pure function.

`#`
: is short-hand for `#1`.

`#0`
: represents the pure function itself.

```
>> #
```
#1

Unused arguments are simply ignored:

```
>> {#1, #2, #3}&[1, 2, 3, 4, 5]
```
$\{1, 2, 3\}$

Recursive pure functions can be written using `#0`:

```
>> If[#1<=1, 1, #1 #0[#1-1]]& [10]
```
3 628 800

SlotSequence

`##`
: is the sequence of arguments supplied to a pure function.

`##`*n*
: starts with the *n*th argument.

```
>> Plus[##]& [1, 2, 3]
```
6

```
>> Plus[##2]& [1, 2, 3]
```
5

```
>> FullForm[##]
```
SlotSequence[1]

XVIII. Compilation

Contents

Compile

`Compile[{x1, x2, ...}, expr]`
: Compiles *expr* assuming each *xi* is a *Real* number.

`Compile[{{x1, t1} {x2, t1} ...}, expr]`
: Compiles assuming each *xi* matches type *ti*.

Compilation is performed using llvmlite , or Python's builtin "compile" function.

```
>> cf = Compile[{x, y}, x + 2 y]
```

CompiledFunction $\left[\{x, y\}, x + 2y, -\mathit{CompiledCode}-\right]$

```
>> cf[2.5, 4.3]
```

11.1

```
>> cf = Compile[{{x, _Real}}, Sin[x]]
```

CompiledFunction $\left[\{x\}, \mathrm{Sin}[x], -\mathit{CompiledCode}-\right]$

```
>> cf[1.4]
```

0.98545

Compile supports basic flow control:

```
>> cf = Compile[{{x, _Real}, {y, _Integer}}, If[x == 0.0 && y <= 0, 0.0, Sin[x ^ y] + 1 / Min[x, 0.5]] + 0.5]
```

CompiledFunction $\left[\{x, y\}, 0.5 + \mathrm{If}\left[x{=}{=}0.\&\&y{<}{=}0, 0., \mathrm{Sin}\left[x^y\right] + \frac{1}{\mathrm{Min}[x, 0.5]}\right], -\mathit{CompiledCode}-\right]$

```
>> cf[3.5, 2]
```

2.18888

Loops and variable assignments are supported usinv Python builtin "compile" function:

```
>> Compile[{{a, _Integer}, {b, _Integer}}, While[b != 0, {a, b} = {b, Mod[a, b]}]; a] (* GCD of a, b *)
```

CompiledFunction $\left[\{a, b\}, a, -\mathit{PythonizedCode}-\right]$

CompiledCodeBox

Used internally by *CompileCode[]*.

CompiledFunction

,

`CompiledFunction[args...]`
: represents compiled code for evaluating a compiled function.

```
>> sqr = Compile[{x}, x x]
```

CompiledFunction$\left[\{x\},\ x^2,\ -CompiledCode-\right]$

```
>> Head[sqr]
```

CompiledFunction

```
>> sqr[2]
```

4.

XIX. Options and Default Arguments

Contents

Default

`Default[`*f*`]`
gives the default value for an omitted paramter of *f*.

`Default[`*f*`, `*k*`]`
gives the default value for a parameter on the *k*th position.

`Default[`*f*`, `*k*`, `*n*`]`
gives the default value for the *k*th parameter out of *n*.

Assign values to `Default` to specify default values.

```
>> Default[f] = 1
```
1

```
>> f[x_.] := x ^ 2
```

```
>> f[]
```
1

Default values are stored in `DefaultValues`:

```
>> DefaultValues[f]
```
$\{\text{HoldPattern}\ [\text{Default}\ [f]] :> 1\}$

You can use patterns for *k* and *n*:

```
>> Default[h, k_, n_] := {k, n}
```

Note that the position of a parameter is relative to the pattern, not the matching expression:

```
>> h[] /. h[___, ___, x_., y_., ___
   ] -> {x, y}
```
$\{\{3,5\}, \{4,5\}\}$

FilterRules

`FilterRules[`*rules*`, `*pattern*`]`
gives those *rules* that have a left side that matches *pattern*.

`FilterRules[`*rules*`, {`*pattern1*`, `*pattern2*`, ...}]`
gives those *rules* that have a left side that match at least one of *pattern1*, *pattern2*, ...

```
>> FilterRules[{x -> 100, y ->
   1000}, x]
```
$\{x- > 100\}$

```
>> FilterRules[{x -> 100, y ->
   1000, z -> 10000}, {a, b, x, z}]
```
$\{x- > 100, z- > 10\,000\}$

NotOptionQ

`NotOptionQ[`*expr*`]`
returns `True` if *expr* does not have the form of a valid option specification.

```
>> NotOptionQ[x]
```
True

```
>> NotOptionQ[2]
```
True

```
>> NotOptionQ["abc"]
```
True

```
>> NotOptionQ[a -> True]
```
False

OptionQ

`OptionQ[`*expr*`]`
: returns `True` if *expr* has the form of a valid option specification.

Examples of option specifications:

```
>> OptionQ[a -> True]
```
True

```
>> OptionQ[a :> True]
```
True

```
>> OptionQ[{a -> True}]
```
True

```
>> OptionQ[{a :> True}]
```
True

Options lists are flattened when are applyied, so

```
>> OptionQ[{a -> True, {b->1, "c
   "->2}}]
```
True

```
>> OptionQ[{a -> True, {b->1, c}}]
```
False

```
>> OptionQ[{a -> True, F[b->1,c
   ->2]}]
```
False

`OptionQ` returns `False` if its argument is not a valid option specification:

```
>> OptionQ[x]
```
False

OptionValue

`OptionValue[`*name*`]`
: gives the value of the option *name* as specified in a call to a function with `OptionsPattern`.

`OptionValue[`*f*`, `*name*`]`
: recover the value of the option *name* associated to the symbol *f*.

`OptionValue[`*f*`, `*optvals*`, `*name*`]`
: recover the value of the option *name* associated to the symbol *f*, extracting the values from *optvals* if available.

`OptionValue[$\ldots$, `*list*`]`
: recover the value of the options in *list* .

```
>> f[a->3] /. f[OptionsPattern[{}]]
   -> {OptionValue[a]}
```
$\{3\}$

Unavailable options generate a message:

```
>> f[a->3] /. f[OptionsPattern[{}]]
   -> {OptionValue[b]}
```
Optionnamebnotfound.

$\{b\}$

The argument of `OptionValue` must be a symbol:

```
>> f[a->3] /. f[OptionsPattern[{}]]
   -> {OptionValue[a+b]}
```
Argumenta
+ batposition1isexpectedtobeasymbol.

$\{\text{OptionValue}[a+b]\}$

However, it can be evaluated dynamically:

```
>> f[a->5] /. f[OptionsPattern[{}]]
   -> {OptionValue[Symbol["a"]]}
```
$\{5\}$

Options

`Options[`*f*`]`
: gives a list of optional arguments to *f* and their default values.

You can assign values to `Options` to specify options.

```
>> Options[f] = {n -> 2}
```
$\{n- > 2\}$

```
>> Options[f]
```
$\{n{:>}2\}$

```
>> f[x_, OptionsPattern[f]] := x ^
   OptionValue[n]
```

```
>> f[x]
```
x^2

```
>> f[x, n -> 3]
```
x^3

Delayed option rules are evaluated just when the corresponding `OptionValue` is called:

```
>> f[a :> Print["value"]] /. f[
   OptionsPattern[{}]] :> (
   OptionValue[a]; Print["between
   "]; OptionValue[a]);
```

value
between
value

In contrast to that, normal option rules are evaluated immediately:

```
>> f[a -> Print["value"]] /. f[
   OptionsPattern[{}]] :> (
   OptionValue[a]; Print["between
   "]; OptionValue[a]);
```

value
between

Options must be rules or delayed rules:

```
>> Options[f] = {a}
```

$\{a\}isnotavalidlistofoptionrules.$

$\{a\}$

A single rule need not be given inside a list:

```
>> Options[f] = a -> b
```

$a- > b$

```
>> Options[f]
```

$\{a{:>}b\}$

Options can only be assigned to symbols:

```
>> Options[a + b] = {a -> b}
```

$Argumenta$
$+ batposition1isexpectedtobeasymbol.$

$\{a- > b\}$

XX. Comparison

Contents

BooleanQ

`BooleanQ[`*expr*`]`
: returns `True` if *expr* is either `True` or `False`.

```
>> BooleanQ[True]
True

>> BooleanQ[False]
True

>> BooleanQ[a]
False

>> BooleanQ[1 < 2]
True
```

Equal (==)

`Equal[`*x*`, `*y*`]`
x `==` *y*
: yields `True` if *x* and *y* are known to be equal, or `False` if *x* and *y* are known to be unequal.

lhs `==` *rhs*
: represents the equation *lhs* = *rhs*.

```
>> a==a
True

>> a==b
a==b

>> 1==1.
True
```

Strings are allowed:

```
>> Equal["11", "11"]
True

>> Equal["121", "11"]
False
```

Comparision to mismatched types is False:

```
>> Equal[11, "11"]
False
```

Lists are compared based on their elements:

```
>> {{1}, {2}} == {{1}, {2}}
True

>> {1, 2} == {1, 2, 3}
False
```

Real values are considered equal if they only differ in their last digits:

```
>> 0.739085133215160642 ==
   0.739085133215160641
True

>> 0.73908513321516064200000000 ==
   0.73908513321516064100000000
False
```

Comparisons are done using the lower precision:

```
>> N[E, 100] == N[E, 150]
True
```

Symbolic constants are compared numerically:

>> `E > 1`

True

>> `Pi == 3.14`

False

Greater (>)

`Greater[`*x*`, `*y*`]`
x `>` *y*
: yields `True` if x is known to be greater than y.

lhs `>` *rhs*
: represents the inequality $lhs > rhs$.

>> `a > b > c //FullForm`

Greater $[a,b,c]$

>> `Greater[3, 2, 1]`

True

GreaterEqual (>=)

`GreaterEqual[`*x*`, `*y*`]`
x `>=` *y*
: yields `True` if x is known to be greater than or equal to y.

lhs `>=` *rhs*
: represents the inequality *lhs rhs*.

Inequality

`Inequality`
: is the head of expressions involving different inequality operators (at least temporarily). Thus, it is possible to write chains of inequalities.

>> `a < b <= c`

$a < b \&\& b <= c$

>> `Inequality[a, Greater, b, LessEqual, c]`

$a > b \&\& b <= c$

>> `1 < 2 <= 3`

True

>> `1 < 2 > 0`

True

>> `1 < 2 < -1`

False

Less (<)

`Less[`*x*`, `*y*`]`
x `<` *y*
: yields `True` if x is known to be less than y.

lhs `<` *rhs*
: represents the inequality $lhs < rhs$.

LessEqual (<=)

`LessEqual[`*x*`, `*y*`]`
x `<=` *y*
: yields `True` if x is known to be less than or equal to y.

lhs `<=` *rhs*
: represents the inequality *lhs rhs*.

Max

`Max[`*e_1*`, `*e_2*`, ..., `*e_i*`]`
: returns the expression with the greatest value among the *e_i*.

Maximum of a series of values:

>> `Max[4, -8, 1]`

4

>> `Max[E - Pi, Pi, E + Pi, 2 E]`

$E + \mathrm{Pi}$

`Max` flattens lists in its arguments:

>> `Max[{1,2},3,{-3,3.5,-Infinity},{{1/2}}]`

3.5

`Max` with symbolic arguments remains in symbolic form:

>> `Max[x, y]`

Max $[x,y]$

```
>> Max[5, x, -3, y, 40]
```
$\text{Max}\left[40, x, y\right]$

With no arguments, `Max` gives `-Infinity`:
```
>> Max[]
```
$-\infty$

Min

`Min[e_1, e_2, ..., e_i]`
: returns the expression with the lowest value among the *e_i*.

Minimum of a series of values:
```
>> Min[4, -8, 1]
```
-8

```
>> Min[E - Pi, Pi, E + Pi, 2 E]
```
$E - \text{Pi}$

`Min` flattens lists in its arguments:
```
>> Min[{1,2},3,{-3,3.5,-Infinity
   },{{1/2}}]
```
$-\infty$

`Min` with symbolic arguments remains in symbolic form:
```
>> Min[x, y]
```
$\text{Min}\left[x, y\right]$

```
>> Min[5, x, -3, y, 40]
```
$\text{Min}\left[-3, x, y\right]$

With no arguments, `Min` gives `Infinity`:
```
>> Min[]
```
∞

Negative

`Negative[x]`
: returns `True` if *x* is a negative real number.

```
>> Negative[0]
```
False

```
>> Negative[-3]
```
True

```
>> Negative[10/7]
```
False

```
>> Negative[1+2I]
```
False

```
>> Negative[a + b]
```
$\text{Negative}\left[a + b\right]$

NonNegative

`NonNegative[x]`
: returns `True` if *x* is a positive real number or zero.

```
>> {Positive[0], NonNegative[0]}
```
$\{\text{False}, \text{True}\}$

NonPositive

`NonPositive[x]`
: returns `True` if *x* is a negative real number or zero.

```
>> {Negative[0], NonPositive[0]}
```
$\{\text{False}, \text{True}\}$

Positive

`Positive[x]`
: returns `True` if *x* is a positive real number.

```
>> Positive[1]
```
True

`Positive` returns `False` if *x* is zero or a complex number:
```
>> Positive[0]
```
False

```
>> Positive[1 + 2 I]
```
False

SameQ (===)

`SameQ[`*x*`, `*y*`]`
x `===` *y*
: returns `True` if *x* and *y* are structurally identical.

Any object is the same as itself:

```
>> a===a
   True
```

Unlike `Equal`, `SameQ` only yields `True` if *x* and *y* have the same type:

```
>> {1==1., 1===1.}
   {True, False}
```

SympyComparison

TrueQ

`TrueQ[`*expr*`]`
: returns `True` if and only if *expr* is `True`.

```
>> TrueQ[True]
   True

>> TrueQ[False]
   False

>> TrueQ[a]
   False
```

Unequal (!=)

`Unequal[`*x*`, `*y*`]`
x `!=` *y*
: yields `False` if *x* and *y* are known to be equal, or `True` if *x* and *y* are known to be unequal.

lhs `==` *rhs*
: represents the inequality *lhs rhs*.

```
>> 1 != 1.
   False
```

Strings are allowed: Unequal["11", "11"] = False
Equal["121", "11"] = True
Comparision to mismatched types is True: Equal[11, "11"] = True

Lists are compared based on their elements:

```
>> {1} != {2}
   True

>> {1, 2} != {1, 2}
   False

>> {a} != {a}
   False

>> "a" != "b"
   True

>> "a" != "a"
   False
```

UnsameQ (=!=)

`UnsameQ[`*x*`, `*y*`]`
x `=!=` *y*
: returns `True` if *x* and *y* are not structurally identical.

```
>> a=!=a
   False

>> 1=!=1.
   True
```

ValueQ

`ValueQ[`*expr*`]`
: returns `True` if and only if *expr* is defined.

```
>> ValueQ[x]
   False

>> x = 1;

>> ValueQ[x]
   True
```

XXI. Linear algebra

Contents

BrayCurtisDistance

`BrayCurtisDistance[`u`, `v`]`
: returns the Bray Curtis distance between u and v.

```
>> BrayCurtisDistance[-7, 5]
```
6

```
>> BrayCurtisDistance[{-1, -1}, {10, 10}]
```
$\frac{11}{9}$

CanberraDistance

`CanberraDistance[`u`, `v`]`
: returns the canberra distance between u and v, which is a weighted version of the Manhattan distance.

```
>> CanberraDistance[-7, 5]
```
1

```
>> CanberraDistance[{-1, -1}, {1, 1}]
```
2

ChessboardDistance

`ChessboardDistance[`u`, `v`]`
: returns the chessboard distance (also known as Chebyshev distance) between u and v, which is the number of moves a king on a chessboard needs to get from square u to square v.

```
>> ChessboardDistance[-7, 5]
```
12

```
>> ChessboardDistance[{-1, -1}, {1, 1}]
```
2

CosineDistance

`CosineDistance[`u`, `v`]`
: returns the cosine distance between u and v.

```
>> N[CosineDistance[{7, 9}, {71, 89}]]
```
0.0000759646

```
>> CosineDistance[{a, b}, {c, d}]
```

$$1 + \frac{-ac - bd}{\sqrt{\mathrm{Abs}[a]^2 + \mathrm{Abs}[b]^2}\sqrt{\mathrm{Abs}[c]^2 + \mathrm{Abs}[d]^2}}$$

Cross

`Cross[`*a*`, `*b*`]`
: computes the vector cross product of *a* and *b*.

```
>> Cross[{x1, y1, z1}, {x2, y2, z2}]
```

$\{y1z2 - y2z1, -x1z2 + x2z1, x1y2 - x2y1\}$

```
>> Cross[{x, y}]
```

$\{-y, x\}$

```
>> Cross[{1, 2}, {3, 4, 5}]
```

Theargumentsareexpectedtobevectorsofequallength, andthenumberofargumentsisexpectedtobe1lessthantheirlength.

Cross [{1,2} , {3,4,5}]

DesignMatrix

`DesignMatrix[`*m*`, `*f*`, `*x*`]`
: returns the design matrix.

```
>> DesignMatrix[{{2, 1}, {3, 4}, {5, 3}, {7, 6}}, x, x]
```

$\{\{1,2\}, \{1,3\}, \{1,5\}, \{1,7\}\}$

```
>> DesignMatrix[{{2, 1}, {3, 4}, {5, 3}, {7, 6}}, f[x], x]
```

$\{\{1, f[2]\}, \{1, f[3]\}, \{1, f[5]\}, \{1, f[7]\}\}$

Det

`Det[`*m*`]`
: computes the determinant of the matrix *m*.

```
>> Det[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}]
```

-2

Symbolic determinant:

```
>> Det[{{a, b, c}, {d, e, f}, {g, h, i}}]
```

$aei - afh - bdi + bfg + cdh - ceg$

Eigensystem

`Eigensystem[`*m*`]`
: returns the list `{Eigenvalues[`*m*`], Eigenvectors[`*m*`]}`.

```
>> Eigensystem[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}]
```

$\{\{2, -1, 1\}, \{\{1,1,1\}, \{1, -2, 1\}, \{-1,0,1\}\}\}$

Eigenvalues

`Eigenvalues[`*m*`]`
: computes the eigenvalues of the matrix *m*. By default Sympy's routine is used. Sometimes this is slow and less good than the corresponding mpmath routine. Use option Method->"mpmath" if you want to use mpmath's routine instead.

Numeric eigenvalues are sorted in order of decreasing absolute value:

```
>> Eigenvalues[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}]
```

$\{2, -1, 1\}$

Symbolic eigenvalues:

```
>> Eigenvalues[{{Cos[theta],Sin[theta],0},{-Sin[theta],Cos[theta],0},{0,0,1}}] // Sort
```

$$\Big\{1, \mathrm{Cos}[\mathrm{theta}] + \sqrt{(-1 + \mathrm{Cos}[\mathrm{theta}])(1 + \mathrm{Cos}[\mathrm{theta}])}, \mathrm{Cos}[\mathrm{theta}] - \sqrt{(-1 + \mathrm{Cos}[\mathrm{theta}])(1 + \mathrm{Cos}[\mathrm{theta}])}\Big\}$$

>> **`Eigenvalues[{{7, 1}, {-4, 3}}]`**

$\{5, 5\}$

>> **`Eigenvalues[{{7, 1}, {-4, 3}}]`**

$\{5, 5\}$

Eigenvectors

`Eigenvectors[`*m*`]`
: computes the eigenvectors of the matrix *m*.

>> **`Eigenvectors[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}]`**

$\{\{1, 1, 1\}, \{1, -2, 1\}, \{-1, 0, 1\}\}$

>> **`Eigenvectors[{{1, 0, 0}, {0, 1, 0}, {0, 0, 0}}]`**

$\{\{0, 1, 0\}, \{1, 0, 0\}, \{0, 0, 1\}\}$

>> **`Eigenvectors[{{2, 0, 0}, {0, -1, 0}, {0, 0, 0}}]`**

$\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}\}$

>> **`Eigenvectors[{{0.1, 0.2}, {0.8, 0.5}}]`**

{{−0.35551821648126701667̃6˜
˜297559501705929896062805897˜
˜153500209120909839738411406˜
˜528939551208168268203735351˜
˜562500000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜0000000000000, − 1.150481˜
˜115772866118834236549972506˜
˜478611688789589714534394707˜
˜980136107750013252370990812˜
˜778472900390625000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜0000000000000000000000000000
}, {−0.628960169645094045731˜
˜745684302104224901929314653˜
˜543850901708147770746704097˜
˜177826042752712965011596679˜
˜687500000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜0000000000000, 0.777437524˜
˜821136041447958386087174831˜
˜147822934682708885214954348˜
˜721189125726027668861206620˜
˜931625366210937500000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜000000000000000000000000000˜
˜00000000000000000000000000}}

EuclideanDistance

`EuclideanDistance[`u`, `v`]`
returns the euclidean distance between u and v.

```
>> EuclideanDistance[-7, 5]
```
12

```
>> EuclideanDistance[{-1, -1}, {1, 1}]
```
$2\sqrt{2}$

```
>> EuclideanDistance[{a, b}, {c, d}]
```
$\sqrt{\text{Abs}[a-c]^2+\text{Abs}[b-d]^2}$

FittedModel

Inverse

`Inverse[`m`]`
computes the inverse of the matrix m.

```
>> Inverse[{{1, 2, 0}, {2, 3, 0}, {3, 4, 1}}]
```
$\{\{-3,2,0\},\{2,-1,0\},\{1,-2,1\}\}$

```
>> Inverse[{{1, 0}, {0, 0}}]
```
Thematrix{{1,0},{0,0}}issingular.

Inverse [{{1,0},{0,0}}]

```
>> Inverse[{{1, 0, 0}, {0, Sqrt[3]/2, 1/2}, {0,-1 / 2, Sqrt[3]/2}}]
```
$\left\{\{1,0,0\},\left\{0,\frac{\sqrt{3}}{2},-\frac{1}{2}\right\},\left\{0,\frac{1}{2},\frac{\sqrt{3}}{2}\right\}\right\}$

LeastSquares

`LeastSquares[`m`, `b`]`
computes the least squares solution to $m\ x = b$, finding an x that solves for b optimally.

```
>> LeastSquares[{{1, 2}, {2, 3}, {5, 6}}, {1, 5, 3}]
```
$\left\{-\frac{28}{13},\frac{31}{13}\right\}$

```
>> Simplify[LeastSquares[{{1, 2}, {2, 3}, {5, 6}}, {1, x, 3}]]
```
$\left\{\frac{12}{13}-\frac{8x}{13},-\frac{4}{13}+\frac{7x}{13}\right\}$

```
>> LeastSquares[{{1, 1, 1}, {1, 1, 2}}, {1, 3}]
```
Solvingforunderdeterminedsystemnotimplemented.

LeastSquares [{{1,1,1},{1,1,2}},{1,3}]

LinearModelFit

`LinearModelFit[`m`, `f`, `x`]`
returns the design matrix.

```
>> m = LinearModelFit[{{2, 1}, {3, 4}, {5, 3}, {7, 6}}, x, x];
```

```
>> m["BasisFunctions"]
```
$\{1,x\}$

```
>> m["BestFit"]
```
$0.186441+0.779661x$

```
>> m["BestFitParameters"]
```
$\{0.186441,0.779661\}$

```
>> m["DesignMatrix"]
```
$\{\{1,2\},\{1,3\},\{1,5\},\{1,7\}\}$

```
>> m["Function"]
```
0.186441 + 0.779661#1&

```
>> m["Response"]
```
$\{1,4,3,6\}$

```
>> m["FitResiduals"]
```
$\{-0.745763, 1.47458, -1.08475, 0.355932\}$

```
>> m = LinearModelFit[{{2, 2, 1}, {3, 2, 4}, {5, 6, 3}, {7, 9, 6}}, {Sin[x], Cos[y]}, {x, y}];
```

```
>> m["BasisFunctions"]
```
$\{1, \mathrm{Sin}[x], \mathrm{Cos}[y]\}$

```
>> m["Function"]
```
$3.33077 - 5.65221\mathrm{Cos}[\#2] - 5.01042\mathrm{Sin}[\#1]\,\&$

```
>> m = LinearModelFit[{{{1, 4}, {1, 5}, {1, 7}}, {1, 2, 3}}];
```

```
>> m["BasisFunctions"]
```
$\{\#1, \#2\}$

```
>> m["FitResiduals"]
```
$\{-0.142857, 0.214286, -0.0714286\}$

LinearSolve

`LinearSolve[`*matrix*`,` *right*`]`
: solves the linear equation system *matrix* `.` *x* `=` *right* and returns one corresponding solution *x*.

```
>> LinearSolve[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}, {1, 2, 3}]
```
$\{0, 1, 2\}$

Test the solution:
```
>> {{1, 1, 0}, {1, 0, 1}, {0, 1, 1}} . {0, 1, 2}
```
$\{1, 2, 3\}$

If there are several solutions, one arbitrary solution is returned:
```
>> LinearSolve[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}, {1, 1, 1}]
```
$\{-1, 1, 0\}$

Infeasible systems are reported:
```
>> LinearSolve[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}, {1, -2, 3}]
```
Linearequationencounteredthathasnosolution.

$\mathrm{LinearSolve}[\{\{1,2,3\}, \{4,5,6\}, \{7,8,9\}\}, \{1, -2, 3\}]$

ManhattanDistance

`ManhattanDistance[`*u*`,` *v*`]`
: returns the Manhattan distance between *u* and *v*, which is the number of horizontal or vertical moves in the gridlike Manhattan city layout to get from *u* to *v*.

```
>> ManhattanDistance[-7, 5]
```
12

```
>> ManhattanDistance[{-1, -1}, {1, 1}]
```
4

MatrixExp

`MatrixExp[`*m*`]`
: computes the exponential of the matrix *m*.

```
>> MatrixExp[{{0, 2}, {0, 1}}]
```
$\{\{1, -2 + 2E\}, \{0, E\}\}$

```
>> MatrixExp[{{1.5, 0.5}, {0.5, 2.0}}]
```
$\{\{5.16266, 3.02952\}, \{3.02952, 8.19218\}\}$

MatrixPower

`MatrixPower[`*m*`,` *n*`]`
: computes the *n*th power of a matrix *m*.

```
>> MatrixPower[{{1, 2}, {1, 1}}, 10]
```
$\{\{3363, 4756\}, \{2378, 3363\}\}$

```
>> MatrixPower[{{1, 2}, {2, 5}}, -3]
```
$\{\{169, -70\}, \{-70, 29\}\}$

MatrixRank

`MatrixRank[`*matrix*`]`
: returns the rank of *matrix*.

```
>> MatrixRank[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}]
```
2

```
>> MatrixRank[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}]
```
3

```
>> MatrixRank[{{a, b}, {3 a, 3 b}}]
```
1

Norm

`Norm[`*m*`, `*l*`]`
: computes the l-norm of matrix m (currently only works for vectors!).

`Norm[`*m*`]`
: computes the 2-norm of matrix m (currently only works for vectors!).

```
>> Norm[{1, 2, 3, 4}, 2]
```
$\sqrt{30}$

```
>> Norm[{10, 100, 200}, 1]
```
310

```
>> Norm[{a, b, c}]
```
$\sqrt{\text{Abs}[a]^2 + \text{Abs}[b]^2 + \text{Abs}[c]^2}$

```
>> Norm[{-100, 2, 3, 4}, Infinity]
```
100

```
>> Norm[1 + I]
```
$\sqrt{2}$

Normalize

`Normalize[`*v*`]`
: calculates the normalized vector *v*.

`Normalize[`*z*`]`
: calculates the normalized complex number *z*.

```
>> Normalize[{1, 1, 1, 1}]
```
$\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}$

```
>> Normalize[1 + I]
```
$\left(\frac{1}{2} + \frac{I}{2}\right)\sqrt{2}$

NullSpace

`NullSpace[`*matrix*`]`
: returns a list of vectors that span the nullspace of *matrix*.

```
>> NullSpace[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}]
```
$\{\{1, -2, 1\}\}$

```
>> A = {{1, 1, 0}, {1, 0, 1}, {0, 1, 1}};
```

```
>> NullSpace[A]
```
$\{\}$

```
>> MatrixRank[A]
```
3

PseudoInverse

`PseudoInverse[`*m*`]`
: computes the Moore-Penrose pseudoinverse of the matrix *m*. If *m* is invertible, the pseudoinverse equals the inverse.

```
>> PseudoInverse[{{1, 2}, {2, 3}, {3, 4}}]
```
$\left\{\left\{-\frac{11}{6}, -\frac{1}{3}, \frac{7}{6}\right\}, \left\{\frac{4}{3}, \frac{1}{3}, -\frac{2}{3}\right\}\right\}$

```
>> PseudoInverse[{{1, 2, 0}, {2, 3, 0}, {3, 4, 1}}]
```

$\{\{-3,2,0\},\{2,-1,0\},\{1,-2,1\}\}$

```
>> PseudoInverse[{{1.0, 2.5}, {2.5, 1.0}}]
```

$\{\{-0.190476,0.47619\},\{0.47619,-0.190476\}\}$

QRDecomposition

`QRDecomposition[`*m*`]`
: computes the QR decomposition of the matrix *m*.

```
>> QRDecomposition[{{1, 2}, {3, 4}, {5, 6}}]
```

$$\left\{\left\{\left\{\frac{\sqrt{35}}{35},\frac{3\sqrt{35}}{35},\frac{\sqrt{35}}{7}\right\},\left\{\frac{13\sqrt{210}}{210},\frac{2\sqrt{210}}{105},-\frac{\sqrt{210}}{42}\right\}\right\},\left\{\left\{\sqrt{35},\frac{44\sqrt{35}}{35}\right\},\left\{0,\frac{2\sqrt{210}}{35}\right\}\right\}\right\}$$

RowReduce

`RowReduce[`*matrix*`]`
: returns the reduced row-echelon form of *matrix*.

```
>> RowReduce[{{1, 0, a}, {1, 1, b}}]
```

$\{\{1,0,a\},\{0,1,-a+b\}\}$

```
>> RowReduce[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

SingularValueDecomposition

`SingularValueDecomposition[`*m*`]`
: calculates the singular value decomposition for the matrix *m*.

`SingularValueDecomposition` returns u, s, w such that m=u s v, uu=1, vv=1, and s is diagonal.

```
>> SingularValueDecomposition[{{1.5, 2.0}, {2.5, 3.0}}]
```

$\{\{\{0.538954,0.842335\},\{0.842335,-0.538954\}\},\{\{4.63555,0.\},\{0.,0.107862\}\},\{\{0.628678,0.777666\},\{-0.777666,0.628678\}\}\}$

SquaredEuclideanDistance

`SquaredEuclideanDistance[`*u*`, `*v*`]`
: returns squared the euclidean distance between *u* and *v*.

```
>> SquaredEuclideanDistance[-7, 5]
```

144

```
>> SquaredEuclideanDistance[{-1, -1}, {1, 1}]
```

8

Tr

`Tr[`*m*`]`
: computes the trace of the matrix *m*.

```
>> Tr[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}]
```

15

Symbolic trace:

```
>> Tr[{{a, b, c}, {d, e, f}, {g, h, i}}]
```

$a+e+i$

VectorAngle

`VectorAngle[`*u*`, `*v*`]`
gives the angles between vectors *u* and *v*

\>> **`VectorAngle[{1, 0}, {0, 1}]`**

$\frac{\text{Pi}}{2}$

\>> **`VectorAngle[{1, 2}, {3, 1}]`**

$\frac{\text{Pi}}{4}$

\>> **`VectorAngle[{1, 1, 0}, {1, 0, 1}]`**

$\frac{\text{Pi}}{3}$

XXII. Attributes

There are several builtin-attributes which have a predefined meaning in *Mathics*. However, you can set any symbol as an attribute, in contrast to *Mathematica*®.

Contents

Attributes

`Attributes`[*symbol*]
returns the attributes of *symbol*.
`Attributes`[*symbol*] = {*attr1*, *attr2*}
sets the attributes of *symbol*, replacing any existing attributes.

```
>> Attributes[Plus]
```
{Flat, Listable, NumericFunction, OneIdentity, Orderless, Protected}

`Attributes` always considers the head of an expression:

```
>> Attributes[a + b + c]
```
{Flat, Listable, NumericFunction, OneIdentity, Orderless, Protected}

You can assign values to `Attributes` to set attributes:

```
>> Attributes[f] = {Flat, Orderless
   }
```
{Flat, Orderless}

```
>> f[b, f[a, c]]
```
$f[a, b, c]$

Attributes must be symbols:

```
>> Attributes[f] := {a + b}
```
Argumenta
+ batposition1isexpectedtobeasymbol.
$Failed

Use `Symbol` to convert strings to symbols:

```
>> Attributes[f] = Symbol["Listable
   "]
```
Listable

```
>> Attributes[f]
```
{Listable}

ClearAttributes

`ClearAttributes`[*symbol*, *attrib*]
removes *attrib* from *symbol*'s attributes.

```
>> SetAttributes[f, Flat]
```

```
>> Attributes[f]
```
{Flat}

```
>> ClearAttributes[f, Flat]
```

```
>> Attributes[f]
```
{}

Attributes that are not even set are simply ignored:

```
>> ClearAttributes[{f}, {Flat}]
```

```
>> Attributes[f]
```
{}

Constant

`Constant`
is an attribute that indicates that a symbol is a constant.

Mathematical constants like `E` have attribute `Constant`:

```
>> Attributes[E]
```
{Constant, Protected, ReadProtected}

Constant symbols cannot be used as variables in `Solve` and related functions:

```
>> Solve[x + E == 0, E]
```
Eisnotavalidvariable.
Solve $[E + x{=}{=}0, E]$

Flat

`Flat`
is an attribute that specifies that nested occurrences of a function should be automatically flattened.

A symbol with the `Flat` attribute represents an associative mathematical operation:

```
>> SetAttributes[f, Flat]
```

```
>> f[a, f[b, c]]
```
$f[a, b, c]$

`Flat` is taken into account in pattern matching:

```
>> f[a, b, c] /. f[a, b] -> d
```
$f[d, c]$

HoldAll

`HoldAll`
is an attribute specifying that all arguments of a function should be left unevaluated.

```
>> Attributes[Function]
```
{HoldAll, Protected}

HoldAllComplete

`HoldAllComplete`
is an attribute that includes the effects of `HoldAll` and `SequenceHold`, and also protects the function from being affected by the upvalues of any arguments.

`HoldAllComplete` even prevents upvalues from being used, and includes `SequenceHold`.

```
>> SetAttributes[f, HoldAllComplete
   ]
```

```
>> f[a] ^= 3;
```

```
>> f[a]
```
$f[a]$

```
>> f[Sequence[a, b]]
```
$f[\text{Sequence}[a, b]]$

HoldFirst

`HoldFirst`
is an attribute specifying that the first argument of a function should be left unevaluated.

```
>> Attributes[Set]
```
{HoldFirst, Protected, SequenceHold}

HoldRest

`HoldRest`
is an attribute specifying that all but the first argument of a function should be left unevaluated.

```
>> Attributes[If]
```
{HoldRest, Protected}

Listable

`Listable`
: is an attribute specifying that a function should be automatically applied to each element of a list.

```
>> SetAttributes[f, Listable]
```

```
>> f[{1, 2, 3}, {4, 5, 6}]
```
$\{f[1,4], f[2,5], f[3,6]\}$

```
>> f[{1, 2, 3}, 4]
```
$\{f[1,4], f[2,4], f[3,4]\}$

```
>> {{1, 2}, {3, 4}} + {5, 6}
```
$\{\{6,7\}, \{9,10\}\}$

Locked

`Locked`
: is an attribute that prevents attributes on a symbol from being modified.

The attributes of `Locked` symbols cannot be modified:

```
>> Attributes[lock] = {Flat, Locked
   };
```

```
>> SetAttributes[lock, {}]
```
Symbollockislocked.

```
>> ClearAttributes[lock, Flat]
```
Symbollockislocked.

```
>> Attributes[lock] = {}
```
Symbollockislocked.

{}

```
>> Attributes[lock]
```
{Flat, Locked}

However, their values might be modified (as long as they are not `Protected` too):

```
>> lock = 3
```
3

NHoldAll

`NHoldAll`
: is an attribute that protects all arguments of a function from numeric evaluation.

```
>> N[f[2, 3]]
```
$f[2.,3.]$

```
>> SetAttributes[f, NHoldAll]
```

```
>> N[f[2, 3]]
```
$f[2,3]$

NHoldFirst

`NHoldFirst`
: is an attribute that protects the first argument of a function from numeric evaluation.

NHoldRest

`NHoldRest`
: is an attribute that protects all but the first argument of a function from numeric evaluation.

OneIdentity

`OneIdentity`
: is an attribute specifying that $f[x]$ should be treated as equivalent to x in pattern matching.

`OneIdentity` affects pattern matching:

```
>> SetAttributes[f, OneIdentity]
```

```
>> a /. f[args___] -> {args}
```
$\{a\}$

It does not affect evaluation:

```
>> f[a]
```
$f[a]$

Orderless

`Orderless`
: is an attribute that can be assigned to a symbol *f* to indicate that the elements *ei* in expressions of the form *f*[*e1*, *e2*, ...] should automatically be sorted into canonical order. This property is accounted for in pattern matching.

The leaves of an `Orderless` function are automatically sorted:

```
>> SetAttributes[f, Orderless]

>> f[c, a, b, a + b, 3, 1.0]
```
$f[1., 3, a, b, c, a + b]$

A symbol with the `Orderless` attribute represents a commutative mathematical operation.

```
>> f[a, b] == f[b, a]
```
True

`Orderless` affects pattern matching:

```
>> SetAttributes[f, Flat]

>> f[a, b, c] /. f[a, c] -> d
```
$f[b, d]$

Protect

`Protect`[*s1*, *s2*, ...]
: sets the attribute `Protected` for the symbols *si*.

`Protect`[*str1*, *str2*, ...]
: protects all symbols whose names textually match *stri*.

```
>> A = {1, 2, 3};

>> Protect[A]

>> A[[2]] = 4;
```
SymbolAisProtected.

```
>> A
```
$\{1, 2, 3\}$

Protected

`Protected`
: is an attribute that prevents values on a symbol from being modified.

Values of `Protected` symbols cannot be modified:

```
>> Attributes[p] = {Protected};

>> p = 2;
```
SymbolpisProtected.

```
>> f[p] ^= 3;
```
Tagpinf[p]isProtected.

```
>> Format[p] = "text";
```
SymbolpisProtected.

However, attributes might still be set:

```
>> SetAttributes[p, Flat]

>> Attributes[p]
```
{Flat, Protected}

Thus, you can easily remove the attribute `Protected`:

```
>> Attributes[p] = {};

>> p = 2
```
2

You can also use `Protect` or `Unprotect`, resp.

```
>> Protect[p]

>> Attributes[p]
```
{Protected}

```
>> Unprotect[p]
```

If a symbol is `Protected` and `Locked`, it can never be changed again:

```
>> SetAttributes[p, {Protected,
   Locked}]

>> p = 2
```
SymbolpisProtected.

2

```
>> Unprotect[p]
```
Symbolpislocked.

ReadProtected

`ReadProtected`
: is an attribute that prevents values on a symbol from being read.

Values associated with `ReadProtected` symbols cannot be seen in `Definition`:

```
>> ClearAll[p]
```

```
>> p = 3;
```

```
>> Definition[p]
```

$$p = 3$$

```
>> SetAttributes[p, ReadProtected]
```

```
>> Definition[p]
```

$$\text{Attributes}[p] = \{\text{ReadProtected}\}$$

SequenceHold

`SequenceHold`
: is an attribute that prevents `Sequence` objects from being spliced into a function's arguments.

Normally, `Sequence` will be spliced into a function:

```
>> f[Sequence[a, b]]
```

$f[a, b]$

It does not for `SequenceHold` functions:

```
>> SetAttributes[f, SequenceHold]
```

```
>> f[Sequence[a, b]]
```

$f[\text{Sequence}[a, b]]$

E.g., `Set` has attribute `SequenceHold` to allow assignment of sequences to variables:

```
>> s = Sequence[a, b];
```

```
>> s
```

$\text{Sequence}[a, b]$

```
>> Plus[s]
```

$a + b$

SetAttributes

`SetAttributes[`*symbol*, *attrib*`]`
: adds *attrib* to the list of *symbol*'s attributes.

```
>> SetAttributes[f, Flat]
```

```
>> Attributes[f]
```

$\{\text{Flat}\}$

Multiple attributes can be set at the same time using lists:

```
>> SetAttributes[{f, g}, {Flat, Orderless}]
```

```
>> Attributes[g]
```

$\{\text{Flat}, \text{Orderless}\}$

Unprotect

`Unprotect[`*s1*, *s2*, ...`]`
: removes the attribute `Protected` for the symbols *si*.

`Unprotect[`*str*`]`
: unprotects symbols whose names textually match *str*.

XXIII. Assignment

Contents

AddTo (+=)

`AddTo[`*x*`,` *dx*`]`
x `+=` *dx*
is equivalent to *x* `=` *x* `+` *dx*.

```
>> a = 10;
```

```
>> a += 2
```
12

```
>> a
```
12

Clear

`Clear[`*symb1*`,` *symb2*`, ...]`
clears all values of the given symbols. The arguments can also be given as strings containing symbol names.

```
>> x = 2;
```

```
>> Clear[x]
```

```
>> x
```
x

```
>> x = 2;
```

```
>> y = 3;
```

```
>> Clear["Global`*"]
```

```
>> x
```
x

```
>> y
```
y

`ClearAll` may not be called for `Protected` symbols.

```
>> Clear[Sin]
```
SymbolSinisProtected.

The values and rules associated with built-in symbols will not get lost when applying `Clear` (after unprotecting them):

```
>> Unprotect[Sin]
```

```
>> Clear[Sin]
```

```
>> Sin[Pi]
```
0

`Clear` does not remove attributes, messages, options, and default values associated with the symbols. Use `ClearAll` to do so.

```
>> Attributes[r] = {Flat, Orderless
   };
```

```
>> Clear["r"]
```

```
>> Attributes[r]
```
{Flat, Orderless}

ClearAll

`ClearAll[`*symb1*`, `*symb2*`, ...]`
: clears all values, attributes, messages and options associated with the given symbols. The arguments can also be given as strings containing symbol names.

```
>> x = 2;

>> ClearAll[x]

>> x
```
x

```
>> Attributes[r] = {Flat, Orderless
   };

>> ClearAll[r]

>> Attributes[r]
```
{}

`ClearAll` may not be called for `Protected` or `Locked` symbols.

```
>> Attributes[lock] = {Locked};

>> ClearAll[lock]
```
Symbollockislocked.

Decrement (--)

`Decrement[`*x*`]`
x`--`
: decrements *x* by 1, returning the original value of *x*.

```
>> a = 5;

>> a--
```
5

```
>> a
```
4

DefaultValues

`DefaultValues[`*symbol*`]`
: gives the list of default values associated with *symbol*.

```
>> Default[f, 1] = 4
```
4

```
>> DefaultValues[f]
```
$\{\text{HoldPattern}\left[\text{Default}\left[f,1\right]\right]:>4\}$

You can assign values to `DefaultValues`:

```
>> DefaultValues[g] = {Default[g]
   -> 3};

>> Default[g, 1]
```
3

```
>> g[x_.] := {x}

>> g[a]
```
$\{a\}$

```
>> g[]
```
{3}

Definition

`Definition[`*symbol*`]`
: prints as the user-defined values and rules associated with *symbol*.

`Definition` does not print information for `ReadProtected` symbols. `Definition` uses `InputForm` to format values.

```
>> a = 2;

>> Definition[a]
```
$$a = 2$$

```
>> f[x_] := x ^ 2

>> g[f] ^:= 2

>> Definition[f]
```
$$f\left[\text{x\_}\right] = x^2$$
$$g\left[f\right]\ \text{^}{=}2$$

Definition of a rather evolved (though meaningless) symbol:

```
>> Attributes[r] := {Orderless}
```

```
>> Format[r[args___]] := Infix[{
   args}, "~"]
```

```
>> N[r] := 3.5
```

```
>> Default[r, 1] := 2
```

```
>> r::msg := "My message"
```

```
>> Options[r] := {Opt -> 3}
```

```
>> r[arg_., OptionsPattern[r]] := {
   arg, OptionValue[Opt]}
```

Some usage:

```
>> r[z, x, y]
```

$x \sim y \sim z$

```
>> N[r]
```

3.5

```
>> r[]
```

$\{2, 3\}$

```
>> r[5, Opt->7]
```

$\{5, 7\}$

Its definition:

```
>> Definition[r]
```

$$\begin{aligned}
&\text{Attributes}[r] = \{\text{Orderless}\} \\
&\text{arg_.} \sim \text{OptionsPattern}[r] \\
&\quad = \{\text{arg}, \text{OptionValue}[\text{Opt}]\} \\
&N[r, \text{MachinePrecision}] = 3.5 \\
&\text{Format}[\text{args___}, \text{MathMLForm}] \\
&= \text{Infix}[\{\text{args}\}, "\sim"] \\
&\text{Format}[\text{args___}, \\
&\text{OutputForm}] = \text{Infix}[\{\text{args}\}, "\sim"] \\
&\text{Format}[\text{args___}, \text{StandardForm}] \\
&= \text{Infix}[\{\text{args}\}, "\sim"] \\
&\text{Format}[\text{args___}, \\
&\text{TeXForm}] = \text{Infix}[\{\text{args}\}, "\sim"] \\
&\text{Format}[\text{args___}, \text{TraditionalForm}] \\
&= \text{Infix}[\{\text{args}\}, "\sim"] \\
&\text{Default}[r, 1] = 2 \\
&\text{Options}[r] = \{\text{Opt}- > 3\}
\end{aligned}$$

For `ReadProtected` symbols, `Definition` just prints attributes, default values and options:

```
>> SetAttributes[r, ReadProtected]
```

```
>> Definition[r]
```

$$\begin{aligned}
\text{Attributes}[r] &= \{\text{Orderless}, \\
&\quad \text{ReadProtected}\} \\
\text{Default}[r, 1] &= 2 \\
\text{Options}[r] &= \{\text{Opt}- > 3\}
\end{aligned}$$

This is the same for built-in symbols:

```
>> Definition[Plus]
```

$$\begin{aligned}
\text{Attributes}[\text{Plus}] = \{&\text{Flat}, \text{Listable}, \\
&\text{NumericFunction}, \\
&\text{OneIdentity}, \\
&\text{Orderless}, \text{Protected}\} \\
\text{Default}[\text{Plus}] = 0&
\end{aligned}$$

```
>> Definition[Level]
```

$$\begin{aligned}
\text{Attributes}[\text{Level}] &= \{\text{Protected}\} \\
\text{Options}[\text{Level}] &= \{\text{Heads}- > \text{False}\}
\end{aligned}$$

`ReadProtected` can be removed, unless the symbol is locked:

```
>> ClearAttributes[r, ReadProtected
   ]
```

`Clear` clears values:

```
>> Clear[r]
```

```
>> Definition[r]
```

$$\begin{aligned}
\text{Attributes}[r] &= \{\text{Orderless}\} \\
\text{Default}[r, 1] &= 2 \\
\text{Options}[r] &= \{\text{Opt}- > 3\}
\end{aligned}$$

`ClearAll` clears everything:

```
>> ClearAll[r]
```

```
>> Definition[r]
```

Null

If a symbol is not defined at all, `Null` is printed:

```
>> Definition[x]
```

Null

DivideBy (/=)

`DivideBy[`*x*`, `*dx*`]`
x `/=` *dx*
    is equivalent to *x* `=` *x* `/` *dx*.

```
>> a = 10;
```

```
>> a /= 2
```

5

```
>> a
   5
```

DownValues

`DownValues[`*symbol*`]`
: gives the list of downvalues associated with *symbol*.

`DownValues` uses `HoldPattern` and `RuleDelayed` to protect the downvalues from being evaluated. Moreover, it has attribute `HoldAll` to get the specified symbol instead of its value.

```
>> f[x_] := x ^ 2

>> DownValues[f]
```
$\{\text{HoldPattern}\left[f\left[\text{x\_}\right]\right] :> x^2\}$

Mathics will sort the rules you assign to a symbol according to their specificity. If it cannot decide which rule is more special, the newer one will get higher precedence.

```
>> f[x_Integer] := 2

>> f[x_Real] := 3

>> DownValues[f]
```
$\{\text{HoldPattern}\left[f\left[\text{x\_Real}\right]\right] :> 3, \text{HoldPattern}\left[f\left[\text{x\_Integer}\right]\right] :> 2, \text{HoldPattern}\left[f\left[\text{x\_}\right]\right] :> x^2\}$

```
>> f[3]
   2

>> f[3.]
   3

>> f[a]
```
a^2

The default order of patterns can be computed using `Sort` with `PatternsOrderedQ`:

```
>> Sort[{x_, x_Integer},
   PatternsOrderedQ]
```
{x_Integer, x_}

By assigning values to `DownValues`, you can override the default ordering:

```
>> DownValues[g] := {g[x_] :> x ^
   2, g[x_Integer] :> x}

>> g[2]
   4
```

Fibonacci numbers:

```
>> DownValues[fib] := {fib[0] -> 0,
    fib[1] -> 1, fib[n_] :> fib[n -
    1] + fib[n - 2]}

>> fib[5]
   5
```

Increment (++)

`Increment[`*x*`]`
x`++`
: increments *x* by 1, returning the original value of *x*.

```
>> a = 2;

>> a++
   2

>> a
   3
```

Grouping of `Increment`, `PreIncrement` and `Plus`:

```
>> ++++a+++++2//Hold//FullForm
```
Hold [Plus [PreIncrement [PreIncrement [Increment [Increment [*a*]]]] , 2]]

Information (??)

`Information[`*symbol*`]`
: Prints information about a *symbol*

`Information` does not print information for `ReadProtected` symbols. `Information` uses `InputForm` to format values.

LoadModule

`LoadModule[`*module*`]`
: 'Load Mathics definitions from the python module *module*

```
>> LoadModule["nomodule"]
```
Pythonmodulenomoduledoesnotexist.

$Failed

```
>> LoadModule["sys"]
```
Pythonmodulesysisnotapymathicsmodule.

$Failed

Messages

`Messages[symbol]`
: gives the list of messages associated with *symbol*.

```
>> a::b = "foo"
```
foo

```
>> Messages[a]
```
{HoldPattern [*a*::b] :>foo}

```
>> Messages[a] = {a::c :> "bar"};
```

```
>> a::c // InputForm
```
"bar"

```
>> Message[a::c]
```
bar

NValues

`NValues[symbol]`
: gives the list of numerical values associated with *symbol*.

```
>> NValues[a]
```
{}

```
>> N[a] = 3;
```

```
>> NValues[a]
```
{HoldPattern [*N* [*a*, MachinePrecision]] :>3}

You can assign values to `NValues`:

```
>> NValues[b] := {N[b, MachinePrecision] :> 2}
```

```
>> N[b]
```
2.

Be sure to use `SetDelayed`, otherwise the left-hand side of the transformation rule will be evaluated immediately, causing the head of `N` to get lost. Furthermore, you have to include the precision in the rules; `MachinePrecision` will not be inserted automatically:

```
>> NValues[c] := {N[c] :> 3}
```

```
>> N[c]
```
c

Mathics will gracefully assign any list of rules to `NValues`; however, inappropriate rules will never be used:

```
>> NValues[d] = {foo -> bar};
```

```
>> NValues[d]
```
{HoldPattern [foo] :>bar}

```
>> N[d]
```
d

OwnValues

`OwnValues[symbol]`
: gives the list of ownvalues associated with *symbol*.

```
>> x = 3;
```

```
>> x = 2;
```

```
>> OwnValues[x]
```
{HoldPattern [*x*] :>2}

```
>> x := y
```

```
>> OwnValues[x]
```
{HoldPattern [*x*] :>*y*}

```
>> y = 5;
```

```
>> OwnValues[x]
```
{HoldPattern [*x*] :>*y*}

```
>> Hold[x] /. OwnValues[x]
```
Hold [*y*]

```
>> Hold[x] /. OwnValues[x] // ReleaseHold
```
5

PreDecrement (--)

`PreDecrement[`*x*`]`
`--`*x*
: decrements *x* by 1, returning the new value of *x*.

`--`*a* is equivalent to *a* `=` *a* `- 1`:

```
>> a = 2;
>> --a
1
>> a
1
```

PreIncrement (++)

`PreIncrement[`*x*`]`
`++`*x*
: increments *x* by 1, returning the new value of *x*.

`++`*a* is equivalent to *a* `=` *a* `+ 1`:

```
>> a = 2;
>> ++a
3
>> a
3
```

Set (=)

`Set[`*expr*`, ` *value*`]`
expr = *value*
: evaluates *value* and assigns it to *expr*.

{*s1*, *s2*, *s3*} = {*v1*, *v2*, *v3*}
: sets multiple symbols (*s1*, *s2*, ...) to the corresponding values (*v1*, *v2*, ...).

`Set` can be used to give a symbol a value:

```
>> a = 3
3
>> a
3
```

An assignment like this creates an ownvalue:

```
>> OwnValues[a]
{HoldPattern[a]:>3}
```

You can set multiple values at once using lists:

```
>> {a, b, c} = {10, 2, 3}
{10,2,3}
>> {a, b, {c, {d}}} = {1, 2, {{c1,
   c2}, {a}}}
{1,2, {{c1,c2} , {10}}}
>> d
10
```

`Set` evaluates its right-hand side immediately and assigns it to the left-hand side:

```
>> a
1
>> x = a
1
>> a = 2
2
>> x
1
```

`Set` always returns the right-hand side, which you can again use in an assignment:

```
>> a = b = c = 2;
>> a == b == c == 2
True
```

`Set` supports assignments to parts:

```
>> A = {{1, 2}, {3, 4}};
>> A[[1, 2]] = 5
5
>> A
{{1,5} , {3,4}}
>> A[[;;, 2]] = {6, 7}
{6,7}
>> A
{{1,6} , {3,7}}
```

Set a submatrix:

```
>> B = {{1, 2, 3}, {4, 5, 6}, {7,
   8, 9}};
>> B[[1;;2, 2;;-1]] = {{t, u}, {y,
   z}};
>> B
{{1,t,u} , {4,y,z} , {7,8,9}}
```

SetDelayed (:=)

`SetDelayed[`*expr*`, `*value*`]`
expr := *value*
: assigns *value* to *expr*, without evaluating *value*.

`SetDelayed` is like `Set`, except it has attribute `HoldAll`, thus it does not evaluate the right-hand side immediately, but evaluates it when needed.

```
>> Attributes[SetDelayed]
```
{HoldAll, Protected, SequenceHold}

```
>> a = 1
```
1

```
>> x := a
```

```
>> x
```
1

Changing the value of a affects x:

```
>> a = 2
```
2

```
>> x
```
2

`Condition` (`/;`) can be used with `SetDelayed` to make an assignment that only holds if a condition is satisfied:

```
>> f[x_] := p[x] /; x>0
```

```
>> f[3]
```
$p[3]$

```
>> f[-3]
```
$f[-3]$

It also works if the condition is set in the LHS:

```
>> F[x_, y_] /; x < y /; x>0 := x /
    y;
```

```
>> F[x_, y_] := y / x;
```

```
>> F[2, 3]
```
$\frac{2}{3}$

```
>> F[3, 2]
```
$\frac{2}{3}$

```
>> F[-3, 2]
```
$-\frac{2}{3}$

SubValues

`SubValues[`*symbol*`]`
: gives the list of subvalues associated with *symbol*.

```
>> f[1][x_] := x
```

```
>> f[2][x_] := x ^ 2
```

```
>> SubValues[f]
```
$\left\{\text{HoldPattern}\left[f[2][x\_]\right] :> x^2, \text{HoldPattern}\left[f[1][x\_]\right] :> x\right\}$

```
>> Definition[f]
```
$f[2][x\_] = x^2$
$f[1][x\_] = x$

SubtractFrom (-=)

`SubtractFrom[`*x*`, `*dx*`]`
x `-=` *dx*
: is equivalent to *x* `=` *x* `-` *dx*.

```
>> a = 10;
```

```
>> a -= 2
```
8

```
>> a
```
8

TagSet

`TagSet[`*f*`, `*expr*`, `*value*`]`
f `/:` *expr* `=` *value*
: assigns *value* to *expr*, associating the corresponding rule with the symbol *f*.

Create an upvalue without using `UpSet`:

```
>> x /: f[x] = 2
```
2

```
>> f[x]
```
2

```
>> DownValues[f]
```
{}

```
>> UpValues[x]
```
{HoldPattern [f [x]] :>2}

The symbol f must appear as the ultimate head of *lhs* or as the head of a leaf in *lhs*:

```
>> x /: f[g[x]] = 3;
```
Tagxnotfoundortoodeepforanassignedrule.

```
>> g /: f[g[x]] = 3;
```

```
>> f[g[x]]
```
3

TagSetDelayed

`TagSetDelayed[`*f*`, `*expr*`, `*value*`]`
f `/:` *expr* `:=` *value*
    is the delayed version of `TagSet`.

TimesBy (`*=`)

`TimesBy[`*x*`, `*dx*`]`
x `*=` *dx*
    is equivalent to *x* `=` *x* `*` *dx*.

```
>> a = 10;
```

```
>> a *= 2
```
20

```
>> a
```
20

Unset (`=.`)

`Unset[`*x*`]`
x`=.`
    removes any value belonging to *x*.

```
>> a = 2
```
2

```
>> a =.
```

```
>> a
```
a

Unsetting an already unset or never defined variable will not change anything:

```
>> a =.
```

```
>> b =.
```

`Unset` can unset particular function values. It will print a message if no corresponding rule is found.

```
>> f[x_] =.
```
Assignmentonfforf[x_]notfound.

$Failed

```
>> f[x_] := x ^ 2
```

```
>> f[3]
```
9

```
>> f[x_] =.
```

```
>> f[3]
```
f [3]

You can also unset `OwnValues`, `DownValues`, `SubValues`, and `UpValues` directly. This is equivalent to setting them to `{}`.

```
>> f[x_] = x; f[0] = 1;
```

```
>> DownValues[f] =.
```

```
>> f[2]
```
f [2]

`Unset` threads over lists:

```
>> a = b = 3;
```

```
>> {a, {b}} =.
```
{Null, {Null}}

UpSet (`^=`)

$f[x]$ ^= *expression*
    evaluates *expression* and assigns it to the value of $f[x]$, associating the value with x.

`UpSet` creates an upvalue:

```
>> a[b] ^= 3;
```

```
>> DownValues[a]
```
{}

```
>> UpValues[b]
```
{HoldPattern[a[b]]:>3}

```
>> a ^= 3
```
Nonatomicexpressionexpected.

3

You can use `UpSet` to specify special values like format values. However, these values will not be saved in `UpValues`:

```
>> Format[r] ^= "custom";
```

```
>> r
```
custom

```
>> UpValues[r]
```
{}

UpSetDelayed (^:=)

`UpSetDelayed[`*expression*`,` *value*`]`
expression `^:=` *value*
: assigns *expression* to the value of $f[x]$ (without evaluating *expression*), associating the value with x.

```
>> a[b] ^:= x
```

```
>> x = 2;
```

```
>> a[b]
```
2

```
>> UpValues[b]
```
{HoldPattern[a[b]]:>x}

UpValues

`UpValues[`*symbol*`]`
: gives the list of upvalues associated with *symbol*.

```
>> a + b ^= 2
```
2

```
>> UpValues[a]
```
{HoldPattern[$a + b$]:>2}

```
>> UpValues[b]
```
{HoldPattern[$a + b$]:>2}

You can assign values to `UpValues`:

```
>> UpValues[pi] := {Sin[pi] :> 0}
```

```
>> Sin[pi]
```
0

XXIV. Tensors

Contents

ArrayDepth

`ArrayDepth[`*a*`]`
: returns the depth of the non-ragged array *a*, defined as `Length[Dimensions[`*a*`]]`.

```
>> ArrayDepth[{{a,b},{c,d}}]
```
2

```
>> ArrayDepth[x]
```
0

ArrayQ

`ArrayQ[`*expr*`]`
: tests whether *expr* is a full array.

`ArrayQ[`*expr*`, `*pattern*`]`
: also tests whether the array depth of *expr* matches *pattern*.

`ArrayQ[`*expr*`, `*pattern*`, `*test*`]`
: furthermore tests whether *test* yields `True` for all elements of *expr*. `ArrayQ[`*expr*`]` is equivalent to `ArrayQ[`*expr*`, _, True&]`.

```
>> ArrayQ[a]
```
False

```
>> ArrayQ[{a}]
```
True

```
>> ArrayQ[{{{a}},{{b,c}}}]
```
False

```
>> ArrayQ[{{a, b}, {c, d}}, 2, SymbolQ]
```
True

DiagonalMatrix

`DiagonalMatrix[`*list*`]`
: gives a matrix with the values in *list* on its diagonal and zeroes elsewhere.

```
>> DiagonalMatrix[{1, 2, 3}]
```
$\{\{1,0,0\},\{0,2,0\},\{0,0,3\}\}$

```
>> MatrixForm[%]
```
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Dimensions

`Dimensions[`*expr*`]`
: returns a list of the dimensions of the expression *expr*.

A vector of length 3:

```
>> Dimensions[{a, b, c}]
```
$\{3\}$

A 3x2 matrix:

```
>> Dimensions[{{a, b}, {c, d}, {e, f}}]
```
$\{3,2\}$

Ragged arrays are not taken into account:

```
>> Dimensions[{{a, b}, {b, c}, {c, d, e}}]
```

$\{3\}$

The expression can have any head:

```
>> Dimensions[f[f[a, b, c]]]
```

$\{1,3\}$

Dot (.)

`Dot[`*x*`, `*y*`]`
x `.` *y*
: computes the vector dot product or matrix product *x* . *y*.

Scalar product of vectors:

```
>> {a, b, c} . {x, y, z}
```

$ax + by + cz$

Product of matrices and vectors:

```
>> {{a, b}, {c, d}} . {x, y}
```

$\{ax + by, cx + dy\}$

Matrix product:

```
>> {{a, b}, {c, d}} . {{r, s}, {t, u}}
```

$\{\{ar + bt, as + bu\}, \{cr + dt, cs + du\}\}$

```
>> a . b
```

$a.b$

IdentityMatrix

`IdentityMatrix[`*n*`]`
: gives the identity matrix with *n* rows and columns.

```
>> IdentityMatrix[3]
```

$\{\{1,0,0\}, \{0,1,0\}, \{0,0,1\}\}$

Inner

`Inner[`*f*`, `*x*`, `*y*`, `*g*`]`
: computes a generalised inner product of *x* and *y*, using a multiplication function *f* and an addition function *g*.

```
>> Inner[f, {a, b}, {x, y}, g]
```

$g[f[a,x], f[b,y]]$

Inner can be used to compute a dot product:

```
>> Inner[Times, {a, b}, {c, d}, Plus] == {a, b} . {c, d}
```

True

The inner product of two boolean matrices:

```
>> Inner[And, {{False, False}, {False, True}}, {{True, False}, {True, True}}, Or]
```

{{False, False}, {True, True}}

Inner works with tensors of any depth:

```
>> Inner[f, {{{a, b}}, {{c, d}}}, {{1}, {2}}, g]
```

$\{\{\{g[f[a,1], f[b,2]]\}\}, \{\{g[f[c,1], f[d,2]]\}\}\}$

MatrixQ

`MatrixQ[`*m*`]`
: returns `True` if *m* is a list of equal-length lists.

`MatrixQ[`*m*`, `*f*`]`
: only returns `True` if *f*[*x*] returns `True` for each element *x* of the matrix *m*.

```
>> MatrixQ[{{1, 3}, {4.0, 3/2}}, NumberQ]
```

True

Outer

`Outer[`*f*`, `*x*`, `*y*`]`
: computes a generalised outer product of *x* and *y*, using the function *f* in place of multiplication.

```
>> Outer[f, {a, b}, {1, 2, 3}]
```
{{f[a,1], f[a,2], f[a,3]}, {f[b,1], f[b,2], f[b,3]}}

Outer product of two matrices:
```
>> Outer[Times, {{a, b}, {c, d}}, {{1, 2}, {3, 4}}]
```
{{{{a, 2a}, {3a, 4a}}, {{b, 2b}, {3b, 4b}}}, {{{c, 2c}, {3c, 4c}}, {{d, 2d}, {3d, 4d}}}}

`Outer` of multiple lists:
```
>> Outer[f, {a, b}, {x, y, z}, {1, 2}]
```
{{{f[a,x,1], f[a,x,2]}, {f[a,y,1], f[a,y,2]}, {f[a,z,1], f[a,z,2]}}, {{f[b,x,1], f[b,x,2]}, {f[b,y,1], f[b,y,2]}, {f[b,z,1], f[b,z,2]}}}

Arrays can be ragged:
```
>> Outer[Times, {{1, 2}}, {{a, b}, {c, d, e}}]
```
{{{{a, b}, {c, d, e}}, {{2a, 2b}, {2c, 2d, 2e}}}}

Word combinations:
```
>> Outer[StringJoin, {"", "re", "un"}, {"cover", "draw", "wind"}, {"", "ing", "s"}] // InputForm
```
{{{"cover", "covering", "covers"}, {"draw", "drawing", "draws"}, {"wind", "winding", "winds"}}, {{"recover", "recovering", "recovers"}, {"redraw", "redrawing", "redraws"}, {"rewind", "rewinding", "rewinds"}}, {{"uncover", "uncovering", "uncovers"}, {"undraw", "undrawing", "undraws"}, {"unwind", "unwinding", "unwinds"}}}

Compositions of trigonometric functions:
```
>> trigs = Outer[Composition, {Sin, Cos, Tan}, {ArcSin, ArcCos, ArcTan}]
```
{{Composition[Sin, ArcSin], Composition[Sin, ArcCos], Composition[Sin, ArcTan]}, {Composition[Cos, ArcSin], Composition[Cos, ArcCos], Composition[Cos, ArcTan]}, {Composition[Tan, ArcSin], Composition[Tan, ArcCos], Composition[Tan, ArcTan]}}

Evaluate at 0:
```
>> Map[#[0] &, trigs, {2}]
```
{{0, 1, 0}, {1, 0, 1}, {0, ComplexInfinity, 0}}

Transpose

`Tranpose[m]`
: transposes rows and columns in the matrix *m*.

```
>> Transpose[{{1, 2, 3}, {4, 5, 6}}]
```
{{1, 4}, {2, 5}, {3, 6}}

```
>> MatrixForm[%]
```
$$\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

VectorQ

`VectorQ[v]`
: returns `True` if *v* is a list of elements which are not themselves lists.

`VectorQ[v, f]`
: returns `True` if *v* is a vector and *f*[*x*] returns `True` for each element *x* of *v*.

```
>> VectorQ[{a, b, c}]
```
True

XXV. Structure

Contents

Apply (@@)

`Apply[`*f*`, ` *expr*`]`
f `@@` *expr*
: replaces the head of *expr* with *f*.

`Apply[`*f*`, ` *expr*`, ` *levelspec*`]`
: applies *f* on the parts specified by *levelspec*.

```
>> f @@ {1, 2, 3}
```
$f[1,2,3]$

```
>> Plus @@ {1, 2, 3}
```
6

The head of *expr* need not be `List`:
```
>> f @@ (a + b + c)
```
$f[a,b,c]$

Apply on level 1:
```
>> Apply[f, {a + b, g[c, d, e * f], 3}, {1}]
```
$\{f[a,b], f[c,d,ef], 3\}$

The default level is 0:
```
>> Apply[f, {a, b, c}, {0}]
```
$f[a,b,c]$

Range of levels, including negative level (counting from bottom):
```
>> Apply[f, {{{{{a}}}}}, {2, -3}]
```
$\{\{f[f[\{a\}]]\}\}$

Convert all operations to lists:
```
>> Apply[List, a + b * c ^ e * f[g], {0, Infinity}]
```
$\{a, \{b, \{g\}, \{c,e\}\}\}$

ApplyLevel (@@@)

`ApplyLevel[`*f*`, ` *expr*`]`
f `@@@` *expr*
: is equivalent to `Apply[`*f*`, ` *expr*`, {1}]`.

```
>> f @@@ {{a, b}, {c, d}}
```
$\{f[a,b], f[c,d]\}$

AtomQ

`AtomQ[`*x*`]`
: is true if *x* is an atom (an object such as a number or string, which cannot be divided into subexpressions using `Part`).

```
>> AtomQ[x]
```
True

```
>> AtomQ[1.2]
True
```

```
>> AtomQ[2 + I]
True
```

```
>> AtomQ[2 / 3]
True
```

```
>> AtomQ[x + y]
False
```

CombinatoricaOld‘BinarySearch

`CombinatoricaOld‘BinarySearch[`*l*`, `*k*`]`
: searches the list *l*, which has to be sorted, for key *k* and returns its index in *l*. If *k* does not exist in *l*, `BinarySearch` returns (a + b) / 2, where a and b are the indices between which *k* would have to be inserted in order to maintain the sorting order in *l*. Please note that *k* and the elements in *l* need to be comparable under a strict total order (see https://en.wikipedia.org/wiki/Total_order).

`CombinatoricaOld‘BinarySearch[`*l*`, `*k*`, `*f*`]`
: the index of $k in the elements of *l* if *f* is applied to the latter prior to comparison. Note that *f* needs to yield a sorted sequence if applied to the elements of $l.

```
>> CombinatoricaOld‘BinarySearch
   [{3, 4, 10, 100, 123}, 100]
4
```

```
>> CombinatoricaOld‘BinarySearch
   [{2, 3, 9}, 7] // N
2.5
```

```
>> CombinatoricaOld‘BinarySearch
   [{2, 7, 9, 10}, 3] // N
1.5
```

```
>> CombinatoricaOld‘BinarySearch
   [{-10, 5, 8, 10}, -100] // N
0.5
```

```
>> CombinatoricaOld‘BinarySearch
   [{-10, 5, 8, 10}, 20] // N
4.5
```

```
>> CombinatoricaOld‘BinarySearch[{{
   a, 1}, {b, 7}}, 7, #[[2]]&]
2
```

ByteCount

`ByteCount[`*expr*`]`
: gives the internal memory space used by *expr*, in bytes.

The results may heavily depend on the Python implementation in use.

Depth

`Depth[`*expr*`]`
: gives the depth of *expr*.

The depth of an expression is defined as one plus the maximum number of `Part` indices required to reach any part of *expr*, except for heads.

```
>> Depth[x]
1
```

```
>> Depth[x + y]
2
```

```
>> Depth[{{{{x}}}}]
5
```

Complex numbers are atomic, and hence have depth 1:

```
>> Depth[1 + 2 I]
1
```

`Depth` ignores heads:

```
>> Depth[f[a, b][c]]
2
```

Flatten

`Flatten[`*expr*`]`
: flattens out nested lists in *expr*.

`Flatten[`*expr*`, `*n*`]`
: stops flattening at level *n*.

`Flatten[`*expr*`, `*n*`, `*h*`]`
: flattens expressions with head *h* instead of `List`.

```
>> Flatten[{{a, b}, {c, {d}, e}, {f
   , {g, h}}}]
```
$\{a, b, c, d, e, f, g, h\}$

```
>> Flatten[{{a, b}, {c, {e}, e}, {f
   , {g, h}}}, 1]
```
$\{a, b, c, \{e\}, e, f, \{g, h\}\}$

```
>> Flatten[f[a, f[b, f[c, d]], e],
   Infinity, f]
```
$f[a, b, c, d, e]$

```
>> Flatten[{{a, b}, {c, d}}, {{2},
   {1}}]
```
$\{\{a, c\}, \{b, d\}\}$

```
>> Flatten[{{a, b}, {c, d}}, {{1,
   2}}]
```
$\{a, b, c, d\}$

Flatten also works in irregularly shaped arrays

```
>> Flatten[{{1, 2, 3}, {4}, {6, 7},
    {8, 9, 10}}, {{2}, {1}}]
```
$\{\{1, 4, 6, 8\}, \{2, 7, 9\}, \{3, 10\}\}$

FreeQ

`FreeQ[`*expr*`, ` *x*`]`
: returns `True` if *expr* does not contain the expression *x*.

```
>> FreeQ[y, x]
```
True

```
>> FreeQ[a+b+c, a+b]
```
False

```
>> FreeQ[{1, 2, a^(a+b)}, Plus]
```
False

```
>> FreeQ[a+b, x_+y_+z_]
```
True

```
>> FreeQ[a+b+c, x_+y_+z_]
```
False

```
>> FreeQ[x_+y_+z_][a+b]
```
True

Head

`Head[`*expr*`]`
: returns the head of the expression or atom *expr*.

```
>> Head[a * b]
```
Times

```
>> Head[6]
```
Integer

```
>> Head[x]
```
Symbol

Map (/@)

`Map[`*f*`, ` *expr*`]` or *f* `/@` *expr*
: applies *f* to each part on the first level of *expr*.

`Map[`*f*`, ` *expr*`, ` *levelspec*`]`
: applies *f* to each level specified by *levelspec* of *expr*.

```
>> f /@ {1, 2, 3}
```
$\{f[1], f[2], f[3]\}$

```
>> #^2& /@ {1, 2, 3, 4}
```
$\{1, 4, 9, 16\}$

Map *f* on the second level:

```
>> Map[f, {{a, b}, {c, d, e}}, {2}]
```
$\{\{f[a], f[b]\}, \{f[c], f[d], f[e]\}\}$

Include heads:

```
>> Map[f, a + b + c, Heads->True]
```
$f[\text{Plus}][f[a], f[b], f[c]]$

MapAt

`MapAt[`*f*`, `*expr*`, `*n*`]`
: applies *f* to the element at position *n* in *expr*. If *n* is negative, the position is counted from the end.

`MapAt[f, `*expr*`, {`*i*`, `*j*` ...}]`
: applies *f* to the part of *expr* at position {*i*, *j*, ...}.

`MapAt[`*f*`,`*pos*`]`
: represents an operator form of MapAt that can be applied to an expression.

Map *f* onto the part at position 2:

```
>> MapAt[f, {a, b, c, d}, 2]
```

$\{a, f[b], c, d\}$

Map *f* onto multiple parts:

```
>> MapAt[f, {a, b, c, d}, {{1}, {4}}]
```

$\{f[a], b, c, f[d]\}$

Map *f* onto the at the end:

```
>> MapAt[f, {a, b, c, d}, -1]
```

$\{a, b, c, f[d]\}$

Map *f* onto an association:

```
>> MapAt[f, <|"a" -> 1, "b" -> 2, "c" -> 3, "d" -> 4, "e" -> 5|>, 3]
```

$\{a->1, b->2, c->f[3], d->4, e->5\}$

Use negative position in an association:

```
>> MapAt[f, <|"a" -> 1, "b" -> 2, "c" -> 3, "d" -> 4|>, -3]
```

$\{a->1, b->f[2], c->3, d->4\}$

Use the operator form of MapAt:

```
>> MapAt[f, 1][{a, b, c, d}]
```

$\{f[a], b, c, d\}$

MapIndexed

`MapIndexed[`*f*`, `*expr*`]`
: applies *f* to each part on the first level of *expr*, including the part positions in the call to *f*.

`MapIndexed[`*f*`, `*expr*`, `*levelspec*`]`
: applies *f* to each level specified by *levelspec* of *expr*.

```
>> MapIndexed[f, {a, b, c}]
```

$\{f[a, \{1\}], f[b, \{2\}], f[c, \{3\}]\}$

Include heads (index 0):

```
>> MapIndexed[f, {a, b, c}, Heads->True]
```

$f[\text{List}, \{0\}][f[a, \{1\}], f[b, \{2\}], f[c, \{3\}]]$

Map on levels 0 through 1 (outer expression gets index `{}`):

```
>> MapIndexed[f, a + b + c * d, {0, 1}]
```

$f[f[a, \{1\}] + f[b, \{2\}] + f[cd, \{3\}], \{\}]$

Get the positions of atoms in an expression (convert operations to `List` first to disable `Listable` functions):

```
>> expr = a + b * f[g] * c ^ e;
```

```
>> listified = Apply[List, expr, {0, Infinity}];
```

```
>> MapIndexed[#2 &, listified, {-1}]
```

$\{\{1\}, \{\{2,1\}, \{\{2,2,1\}\}, \{\{2,3,1\}, \{2,3,2\}\}\}\}$

Replace the heads with their positions, too:

```
>> MapIndexed[#2 &, listified, {-1}, Heads -> True]
```

$\{0\}[\{1\}, \{2,0\}[\{2,1\}, \{2,2,0\}[\{2,2,1\}], \{2,3,0\}[\{2,3,1\}, \{2,3,2\}]]]$

The positions are given in the same format as used by `Extract`. Thus, mapping `Extract` on the indices given by `MapIndexed` re-constructs the original expression:

```
>> MapIndexed[Extract[expr, #2] &,
   listified, {-1}, Heads -> True]
```

$a + bf[g]c^e$

MapThread

'MapThread[*f*, {{*a1*, *a2*, ...}, {*b1*, *b2*, ...}, ...}]
: returns {*f*[*a1*, *b1*, ...], *f*[*a2*, *b2*, ...], ...}.

`MapThread[`*f*`, {`*expr1*`, `*expr2*`, ...}, `*n*`]`
: applies *f* at level *n*.

```
>> MapThread[f, {{a, b, c}, {1, 2,
   3}}]
```

$\{f[a,1], f[b,2], f[c,3]\}$

```
>> MapThread[f, {{{a, b}, {c, d}},
   {{e, f}, {g, h}}}, 2]
```

$\{\{f[a,e], f[b,f]\}, \{f[c,g], f[d,h]\}\}$

Null

`Null`
: is the implicit result of expressions that do not yield a result.

```
>> FullForm[a:=b]
```

Null

It is not displayed in StandardForm,

```
>> a:=b
```

in contrast to the empty string:

```
>> ""
```

Operate

`Operate[`*p*`, `*expr*`]`
: applies *p* to the head of *expr*.

`Operate[`*p*`, `*expr*`, `*n*`]`
: applies *p* to the *n*th head of *expr*.

```
>> Operate[p, f[a, b]]
```

$p[f][a,b]$

The default value of *n* is 1:

```
>> Operate[p, f[a, b], 1]
```

$p[f][a,b]$

With *n*=0, `Operate` acts like `Apply`:

```
>> Operate[p, f[a][b][c], 0]
```

$p[f[a][b][c]]$

Order

`Order[`*x*`, `*y*`]`
: returns a number indicating the canonical ordering of *x* and *y*. 1 indicates that *x* is before *y*, -1 that *y* is before *x*. 0 indicates that there is no specific ordering. Uses the same order as `Sort`.

```
>> Order[7, 11]
```

1

```
>> Order[100, 10]
```

-1

```
>> Order[x, z]
```

1

```
>> Order[x, x]
```

0

OrderedQ

`OrderedQ[`*a*`, `*b*`]`
: is `True` if *a* sorts before *b* according to canonical ordering.

```
>> OrderedQ[a, b]
```

True

```
>> OrderedQ[b, a]
```

False

PatternsOrderedQ

`PatternsOrderedQ[`*patt1*`, `*patt2*`]`
: returns `True` if pattern *patt1* would be applied before *patt2* according to canonical pattern ordering.

```
>> PatternsOrderedQ[x__, x_]
```
False

```
>> PatternsOrderedQ[x_, x__]
```
True

```
>> PatternsOrderedQ[b, a]
```
True

Scan

`Scan[f, expr]`
: applies f to each element of *expr* and returns `Null`.

'Scan[*f*, *expr*, *levelspec*]
: applies f to each level specified by *levelspec* of *expr*.

```
>> Scan[Print, {1, 2, 3}]
```
1
2
3

Sort

`Sort[list]`
: sorts *list* (or the leaves of any other expression) according to canonical ordering.

`Sort[list, p]`
: sorts using p to determine the order of two elements.

```
>> Sort[{4, 1.0, a, 3+I}]
```
$\{1., 3 + I, 4, a\}$

Sort uses `OrderedQ` to determine ordering by default. You can sort patterns according to their precedence using `PatternsOrderedQ`:

```
>> Sort[{items___, item_,
   OptionsPattern[], item_symbol,
   item_?test}, PatternsOrderedQ]
```
{item_symbol, item_? test, item_,
items___, OptionsPattern []}

When sorting patterns, values of atoms do not matter:

```
>> Sort[{a, b/;t}, PatternsOrderedQ
   ]
```
$\{b/;t, a\}$

```
>> Sort[{2+c_, 1+b__},
   PatternsOrderedQ]
```
{2 + c_, 1 + b__}

```
>> Sort[{x_ + n_*y_, x_ + y_},
   PatternsOrderedQ]
```
{x_ + n_y_, x_ + y_}

SortBy

`SortBy[list, f]`
: sorts *list* (or the leaves of any other expression) according to canonical ordering of the keys that are extracted from the *list*'s elements using $f. Chunks of leaves that appear the same under $f are sorted according to their natural order (without applying $f).

`SortBy[f]`
: creates an operator function that, when applied, sorts by $f.

```
>> SortBy[{{5, 1}, {10, -1}}, Last]
```
$\{\{10, -1\}, \{5, 1\}\}$

```
>> SortBy[Total][{{5, 1}, {10,
   -9}}]
```
$\{\{10, -9\}, \{5, 1\}\}$

SymbolName

`SymbolName[s]`
: returns the name of the symbol s (without any leading context name).

```
>> SymbolName[x] // InputForm
```
"x"

SymbolQ

`SymbolQ[x]`
: is `True` if x is a symbol, or `False` otherwise.

```
>> SymbolQ[a]
```
True

```
>> SymbolQ[1]
```
False

```
>> SymbolQ[a + b]
```
False

Symbol

`Symbol`
: is the head of symbols.

```
>> Head[x]
```
Symbol

You can use `Symbol` to create symbols from strings:

```
>> Symbol["x"] + Symbol["x"]
```
$2x$

Thread

`Thread[`f`[`$args$`]]`
: threads f over any lists that appear in *args*.

`Thread[`f`[`$args$`], `h`]`
: threads over any parts with head h.

```
>> Thread[f[{a, b, c}]]
```
$\{f[a], f[b], f[c]\}$

```
>> Thread[f[{a, b, c}, t]]
```
$\{f[a,t], f[b,t], f[c,t]\}$

```
>> Thread[f[a + b + c], Plus]
```
$f[a] + f[b] + f[c]$

Functions with attribute `Listable` are automatically threaded over lists:

```
>> {a, b, c} + {d, e, f} + g
```
$\{a+d+g, b+e+g, c+f+g\}$

Through

`Through[`p`[`f`][`x`]]`
: gives $p[f[x]]$.

```
>> Through[f[g][x]]
```
$f[g[x]]$

```
>> Through[p[f, g][x]]
```
$p[f[x], g[x]]$

XXVI. Exponential, Trigonometric and Hyperbolic Functions

Mathics basically supports all important trigonometric and hyperbolic functions.
Numerical values and derivatives can be computed; however, most special exact values and simplification rules are not implemented yet.

Contents

AnglePath

`AnglePath[{`*phi1*`, `*phi2*`, ...}]`
: returns the points formed by a turtle starting at {0, 0} and angled at 0 degrees going through the turns given by angles *phi1*, *phi2*, ... and using distance 1 for each step.

`AnglePath[{{`*r1*`, `*phi1*`}, {`*r2*`, `*phi2*`}, ...}]`
: instead of using 1 as distance, use *r1*, *r2*, ... as distances for the respective steps.

`AngleVector[`*phi0*`, {`*phi1*`, `*phi2*`, ...}]`
: returns the points on a path formed by a turtle starting with direction *phi0* instead of 0.

`AngleVector[{`*x*`, `*y*`}, {`*phi1*`, `*phi2*`, ...}]`
: returns the points on a path formed by a turtle starting at {$x, $y} instead of {0, 0}.

`AngleVector[{{`*x*`, `*y*`}, `*phi0*`}, {`*phi1*`, `*phi2*`, ...}]`
: specifies initial position {*x*, *y*} and initial direction *phi0*.

`AngleVector[{{`*x*`, `*y*`}, {`*dx*`, `*dy*`}}, {`*phi1*`, `*phi2*`, ...}]`
: specifies initial position {*x*, *y*} and a slope {*dx*, *dy*} that is understood to be the initial direction of the turtle.

```
>> AnglePath[{90 Degree, 90 Degree,
   90 Degree, 90 Degree}]
```

{{0,0}, {0,1}, {−1, 1}, {−1,0}, {0,0}}

```
>> AnglePath[{{1, 1}, 90 Degree},
   {{1, 90 Degree}, {2, 90 Degree},
   {1, 90 Degree}, {2, 90 Degree
   }}]
```

{{1,1}, {0,1}, {0, − 1}, {1, − 1}, {1,1}}

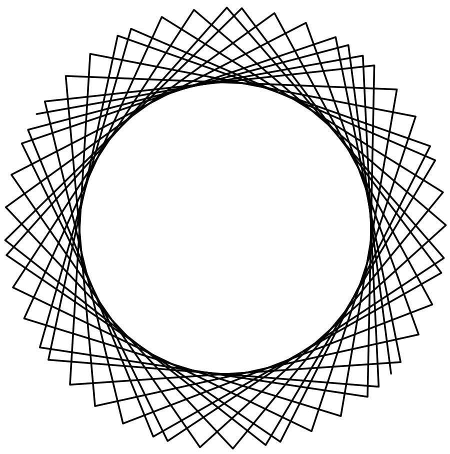
```
>> AnglePath[{a, b}]
```

{{0,0}, {Cos [a] , Sin [a]}, {Cos [a] + Cos [$a + b$] , Sin [a] + Sin [$a + b$]}}

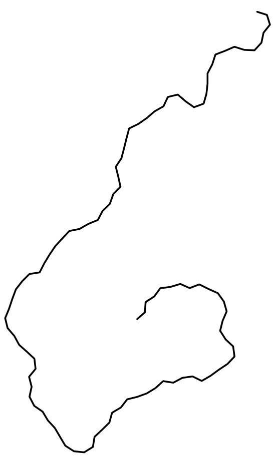
```
>> Precision[Part[AnglePath[{N[1/3,
   100], N[2/3, 100]}], 2, 1]]
```

100.

```
>> Graphics[Line[AnglePath[Table
   [1.7, {50}]]]]
```



```
>> Graphics[Line[AnglePath[
   RandomReal[{-1, 1}, {100}]]]]
```



AngleVector

`AngleVector[`*phi*`]`
: returns the point at angle *phi* on the unit circle.

`AngleVector[{`*r*`,` *phi*`}]`
: returns the point at angle *phi* on a circle of radius *r*.

`AngleVector[{`*x*`,` *y*`},` *phi*`]`
: returns the point at angle *phi* on a circle of radius 1 centered at {*x*, *y*}.

`AngleVector[{`*x*`,` *y*`}, {`*r*`,` *phi*`}]`
: returns point at angle *phi* on a circle of radius *r* centered at {*x*, *y*}.

```
>> AngleVector[90 Degree]
```

{0,1}

```
>> AngleVector[{1, 10}, a]
```

{1 + Cos [a] , 10 + Sin [a]}

ArcCos

`ArcCos[`*z*`]`
: returns the inverse cosine of *z*.

```
>> ArcCos[1]
```

0

```
>> ArcCos[0]
```

$\frac{\text{Pi}}{2}$

```
>> Integrate[ArcCos[x], {x, -1, 1}]
```

Pi

ArcCosh

`ArcCosh[`*z*`]`
: returns the inverse hyperbolic cosine of *z*.

```
>> ArcCosh[0]
```

$\frac{I}{2}\text{Pi}$

```
>> ArcCosh[0.]
```

0. + 1.5708I

>> `ArcCosh[0.00000000000000000000000000000000000000]`

1.570796326794896619~
~231321691639751442I

ArcCot

`ArcCot[z]`
: returns the inverse cotangent of z.

>> `ArcCot[0]`

$\frac{\text{Pi}}{2}$

>> `ArcCot[1]`

$\frac{\text{Pi}}{4}$

ArcCoth

`ArcCoth[z]`
: returns the inverse hyperbolic cotangent of z.

>> `ArcCoth[0]`

$\frac{I}{2}\text{Pi}$

>> `ArcCoth[1]`

∞

>> `ArcCoth[0.0]`

$0. + 1.5708I$

>> `ArcCoth[0.5]`

$0.549306 - 1.5708I$

ArcCsc

`ArcCsc[z]`
: returns the inverse cosecant of z.

>> `ArcCsc[1]`

$\frac{\text{Pi}}{2}$

>> `ArcCsc[-1]`

$-\frac{\text{Pi}}{2}$

ArcCsch

`ArcCsch[z]`
: returns the inverse hyperbolic cosecant of z.

>> `ArcCsch[0]`

ComplexInfinity

>> `ArcCsch[1.0]`

0.881374

ArcSec

`ArcSec[z]`
: returns the inverse secant of z.

>> `ArcSec[1]`

0

>> `ArcSec[-1]`

Pi

ArcSech

`ArcSech[z]`
: returns the inverse hyperbolic secant of z.

>> `ArcSech[0]`

∞

>> `ArcSech[1]`

0

>> `ArcSech[0.5]`

1.31696

ArcSin

`ArcSin[z]`
: returns the inverse sine of z.

>> `ArcSin[0]`

0

```
>> ArcSin[1]
```
$\frac{\text{Pi}}{2}$

ArcSinh

`ArcSinh[z]`
: returns the inverse hyperbolic sine of z.

```
>> ArcSinh[0]
```
0

```
>> ArcSinh[0.]
```
0.

```
>> ArcSinh[1.0]
```
0.881374

ArcTan

`ArcTan[z]`
: returns the inverse tangent of z.

```
>> ArcTan[1]
```
$\frac{\text{Pi}}{4}$

```
>> ArcTan[1.0]
```
0.785398

```
>> ArcTan[-1.0]
```
-0.785398

```
>> ArcTan[1, 1]
```
$\frac{\text{Pi}}{4}$

ArcTanh

`ArcTanh[z]`
: returns the inverse hyperbolic tangent of z.

```
>> ArcTanh[0]
```
0

```
>> ArcTanh[1]
```
∞

```
>> ArcTanh[0]
```
0

```
>> ArcTanh[.5 + 2 I]
```
$0.0964156 + 1.12656I$

```
>> ArcTanh[2 + I]
```
$\text{ArcTanh}[2 + I]$

Cos

`Cos[z]`
: returns the cosine of z.

```
>> Cos[3 Pi]
```
-1

Cosh

`Cosh[z]`
: returns the hyperbolic cosine of z.

```
>> Cosh[0]
```
1

Cot

`Cot[z]`
: returns the cotangent of z.

```
>> Cot[0]
```
ComplexInfinity

```
>> Cot[1.]
```
0.642093

Coth

`Coth[z]`
: returns the hyperbolic cotangent of z.

```
>> Coth[0]
```
ComplexInfinity

Csc

`Csc[z]`
returns the cosecant of z.

```
>> Csc[0]
```
ComplexInfinity

```
>> Csc[1] (* Csc[1] in Mathematica *)
```
$$\frac{1}{\text{Sin}[1]}$$

```
>> Csc[1.]
```
1.1884

Csch

`Csch[z]`
returns the hyperbolic cosecant of z.

```
>> Csch[0]
```
ComplexInfinity

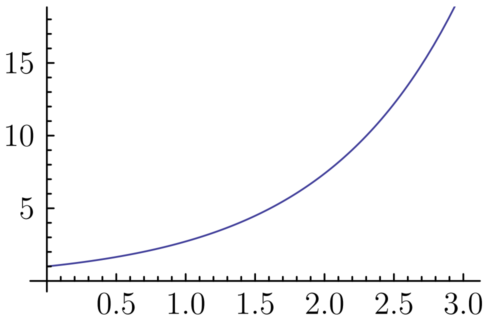
Exp

`Exp[z]`
returns the exponential function of z.

```
>> Exp[1]
```
E

```
>> Exp[10.0]
```
22 026.5

```
>> Exp[x] //FullForm
```
Power $[E, x]$

```
>> Plot[Exp[x], {x, 0, 3}]
```


Haversine

`Haversine[z]`
returns the haversine function of z.

```
>> Haversine[1.5]
```
0.464631

```
>> Haversine[0.5 + 2I]
```
$-1.15082 + 0.869405I$

InverseHaversine

`InverseHaversine[z]`
returns the inverse haversine function of z.

```
>> InverseHaversine[0.5]
```
1.5708

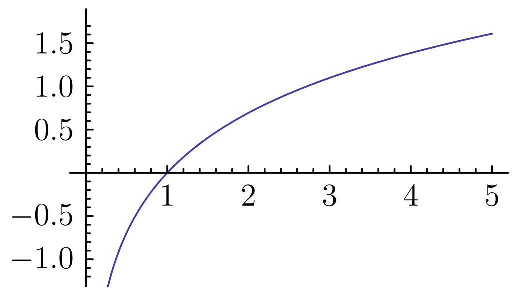
```
>> InverseHaversine[1 + 2.5 I]
```
$1.76459 + 2.33097I$

Log

`Log[z]`
returns the natural logarithm of z.

```
>> Log[{0, 1, E, E * E, E ^ 3, E ^ x}]
```
$\{-\infty, 0, 1, 2, 3, \text{Log}[E^x]\}$

```
>> Log[0.]
```
Indeterminate

```
>> Plot[Log[x], {x, 0, 5}]
```


Log10

`Log10[`z`]`
: returns the base-10 logarithm of z.

```
>> Log10[1000]
```
3

```
>> Log10[{2., 5.}]
```
$\{0.30103, 0.69897\}$

```
>> Log10[E ^ 3]
```
$$\frac{3}{\text{Log}[10]}$$

Log2

`Log2[`z`]`
: returns the base-2 logarithm of z.

```
>> Log2[4 ^ 8]
```
16

```
>> Log2[5.6]
```
2.48543

```
>> Log2[E ^ 2]
```
$$\frac{2}{\text{Log}[2]}$$

LogisticSigmoid

`LogisticSigmoid[`z`]`
: returns the logistic sigmoid of z.

```
>> LogisticSigmoid[0.5]
```
0.622459

```
>> LogisticSigmoid[0.5 + 2.3 I]
```
$1.06475 + 0.808177I$

```
>> LogisticSigmoid[{-0.2, 0.1,
   0.3}]
```
$\{0.450166, 0.524979, 0.574443\}$

Sec

`Sec[`z`]`
: returns the secant of z.

```
>> Sec[0]
```
1

```
>> Sec[1] (* Sec[1] in Mathematica
   *)
```
$$\frac{1}{\text{Cos}[1]}$$

```
>> Sec[1.]
```
1.85082

Sech

`Sech[`z`]`
: returns the hyperbolic secant of z.

```
>> Sech[0]
```
1

Sin

`Sin[`z`]`
: returns the sine of z.

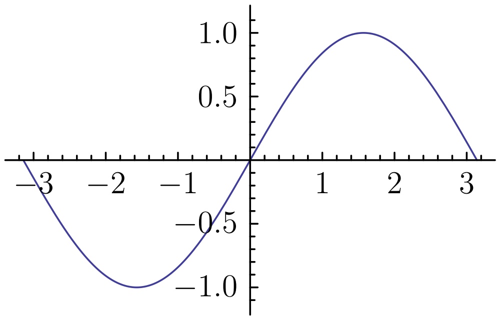
```
>> Sin[0]
```
0

```
>> Sin[0.5]
```
0.479426

```
>> Sin[3 Pi]
```
0

```
>> Sin[1.0 + I]
```
$1.29846 + 0.634964I$

>> `Plot[Sin[x], {x, -Pi, Pi}]`



Sinh

`Sinh[`*z*`]`
returns the hyperbolic sine of *z*.

>> `Sinh[0]`
0

Tan

`Tan[`*z*`]`
returns the tangent of *z*.

>> `Tan[0]`
0

>> `Tan[Pi / 2]`
ComplexInfinity

Tanh

`Tanh[`*z*`]`
returns the hyperbolic tangent of *z*.

>> `Tanh[0]`
0

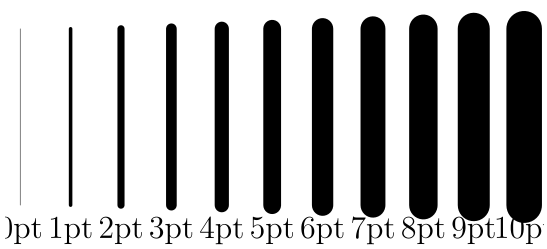
XXVII. Drawing Graphics

Contents

AbsoluteThickness

`AbsoluteThickness[`*p*`]`
: sets the line thickness for subsequent graphics primitives to *p* points.

```
>> Graphics[Table[{
   AbsoluteThickness[t], Line[{{20
   t, 10}, {20 t, 80}}], Text[
   ToString[t]<>"pt", {20 t, 0}]},
   {t, 0, 10}]]
```



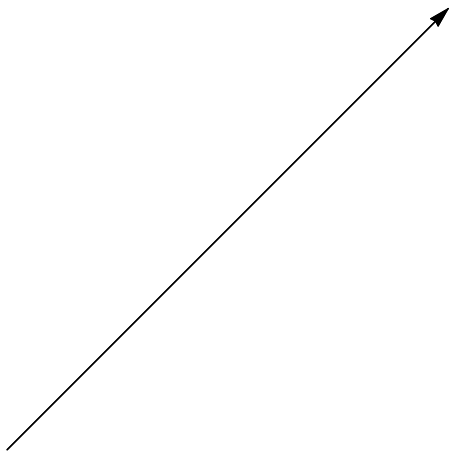
Arrow

`Arrow[{`*p1*`, `*p2*`}]`
: represents a line from *p1* to *p2* that ends with an arrow at *p2*.

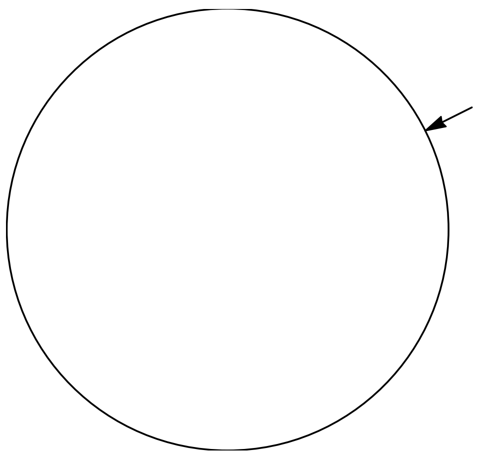
`Arrow[{`*p1*`, `*p2*`}, `*s*`]`
: represents a line with arrow that keeps a distance of *s* from *p1* and *p2*.

`Arrow[{`*point_1*`, `*point_2*`}, {`*s1*`, `*s2*`}]`
: represents a line with arrow that keeps a distance of *s1* from *p1* and a distance of *s2* from *p2*.

```
>> Graphics[Arrow[{{0,0}, {1,1}}]]
```

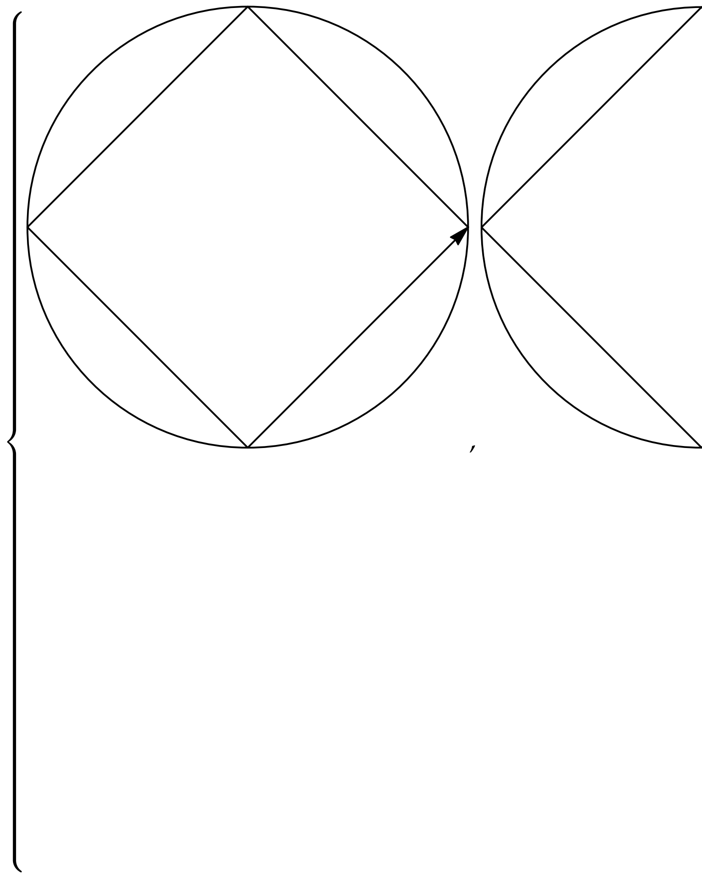


```
>> Graphics[{Circle[], Arrow[{{2,
   1}, {0, 0}}, 1]}]
```



Keeping distances may happen across multiple segments:

```
>> Table[Graphics[{Circle[], Arrow[
   Table[{Cos[phi],Sin[phi]},{phi
   ,0,2*Pi,Pi/2}],{d, d}]}],{d
   ,0,2,0.5}]
```



ArrowBox

Arrowheads

`Arrowheads[`*s*`]`
: specifies that Arrow[] draws one arrow of size *s* (relative to width of image, defaults to 0.04).

`Arrowheads[{`*spec1*`, `*spec2*`, ..., `*specn*`}]`
: specifies that Arrow[] draws n arrows as defined by *spec1*, *spec2*, ... *specn*.

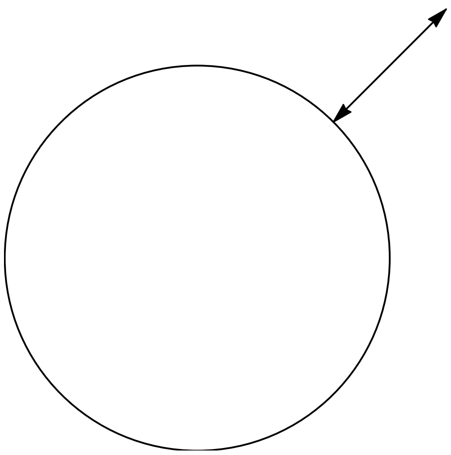
`Arrowheads[{{`*s*`}}]`
: specifies that one arrow of size *s* should be drawn.

`Arrowheads[{{`*s*`, `*pos*`}}]`
: specifies that one arrow of size *s* should be drawn at position *pos* (for the arrow to be on the line, *pos* has to be between 0, i.e. the start for the line, and 1, i.e. the end of the line).

`Arrowheads[{{`*s*`, `*pos*`, `*g*`}}]`
: specifies that one arrow of size *s* should be drawn at position *pos* using Graphics *g*.

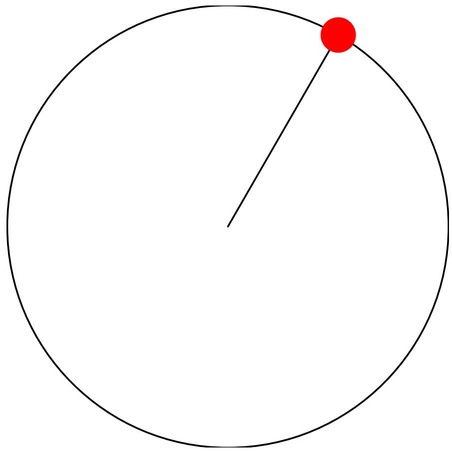
Arrows on both ends can be achieved using negative sizes:

```
>> Graphics[{Circle[],Arrowheads
   [{-0.04, 0.04}], Arrow[{{0, 0},
   {2, 2}}, {1,1}]}]
```

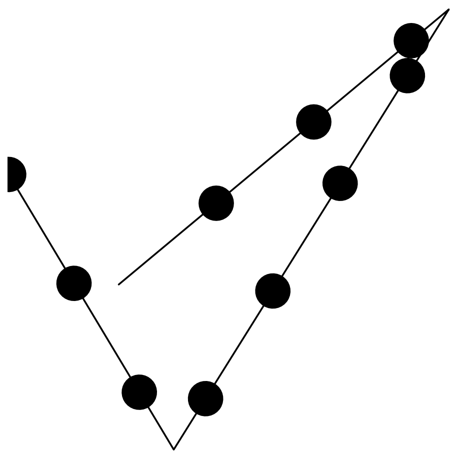


You may also specify our own arrow shapes:

```
>> Graphics[{Circle[], Arrowheads
   [{{0.04, 1, Graphics[{Red, Disk
   []}]}}], Arrow[{{0, 0}, {Cos[Pi
   /3],Sin[Pi/3]}}]}]
```



```
>> Graphics[{Arrowheads[Table
   [{0.04, i/10, Graphics[Disk
   []]},{i,1,10}]], Arrow[{{0, 0},
   {6, 5}, {1, -3}, {-2, 2}}]}]
```



Automatic

`Automatic`
: is used to specify an automatically computed option value.

`Automatic` is the default for `PlotRange`, `ImageSize`, and other graphical options:

```
>> Cases[Options[Plot], HoldPattern
   [_ :> Automatic]]
```

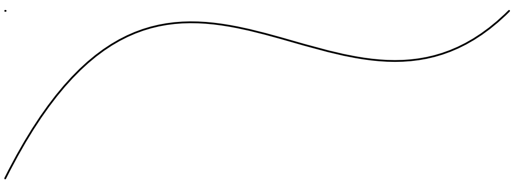
{Background:>Automatic,
Exclusions:>Automatic,
ImageSize:>Automatic,
MaxRecursion:>Automatic,
PlotRange:>Automatic,
PlotRangePadding:>Automatic}

BernsteinBasis

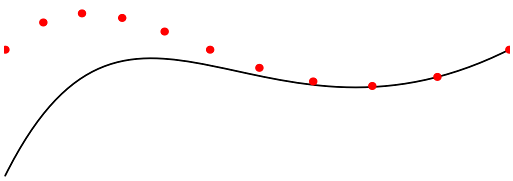
BezierCurve

`BezierCurve[{`*p1*`, `*p2*` ...}]`
: represents a bezier curve with *p1*, *p2* as control points.

```
>> Graphics[BezierCurve[{{0, 0},{1,
    1},{2, -1},{3, 0}}]]
```



```
>> Module[{p={{0, 0},{1, 1},{2,
   -1},{4, 0}}}, Graphics[{
   BezierCurve[p], Red, Point[Table
   [BezierFunction[p][x], {x, 0, 1,
    0.1}]]}]]
```



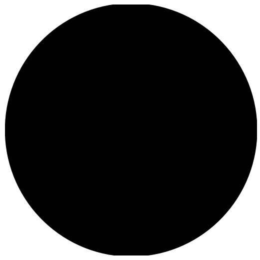
BezierCurveBox

BezierFunction

Black

`Black`
: represents the color black in graphics.

```
>> Graphics[{EdgeForm[Black], Black
   , Disk[]}, ImageSize->Small]
```



```
>> Black // ToBoxes
```

StyleBox [GraphicsBox [
{EdgeForm [GrayLevel [0]] ,
GrayLevel [0] , RectangleBox [{0,
0}] } , $OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

```
>> Black
```

Blend

`Blend[{`*c1*`, `*c2*`}]`
: represents the color between *c1* and *c2*.

`Blend[{`*c1*`, `*c2*`}, `*x*`]`
: represents the color formed by blending *c1* and *c2* with factors 1 - *x* and *x* respectively.

`Blend[{`*c1*`, `*c2*`, ..., `*cn*`}, `*x*`]`
: blends between the colors *c1* to *cn* according to the factor *x*.

```
>> Blend[{Red, Blue}]
```

```
>> Blend[{Red, Blue}, 0.3]
```

```
>> Blend[{Red, Blue, Green}, 0.75]
```

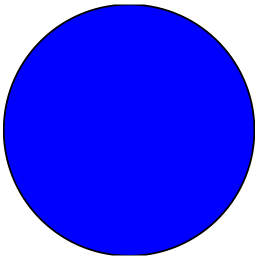
```
>> Graphics[Table[{Blend[{Red,
Green, Blue}, x], Rectangle[{10
x, 0}]}, {x, 0, 1, 1/10}]]
```

```
>> Graphics[Table[{Blend[{RGBColor
[1, 0.5, 0, 0.5], RGBColor[0, 0,
 1, 0.5]}, x], Disk[{5x, 0}]}, {
x, 0, 1, 1/10}]]
```

Blue

`Blue`
: represents the color blue in graphics.

```
>> Graphics[{EdgeForm[Black], Blue,
 Disk[]}, ImageSize->Small]
```



```
>> Blue // ToBoxes
```

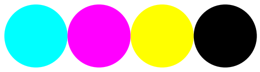
StyleBox [GraphicsBox [
{EdgeForm [GrayLevel [
0]] , RGBColor [0, 0, 1] ,
RectangleBox [{0, 0}] } ,
\$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

```
>> Blue
```

CMYKColor

`CMYKColor[c, m, y, k]`
: represents a color with the specified cyan, magenta, yellow and black components.

```
>> Graphics[MapIndexed[{CMYKColor
@@ #1, Disk[2*#2 ~Join~{0}]} &,
IdentityMatrix[4]], ImageSize->
Small]
```



Circle

`Circle[{cx, cy}, r]`
: draws a circle with center (cx, cy) and radius r.

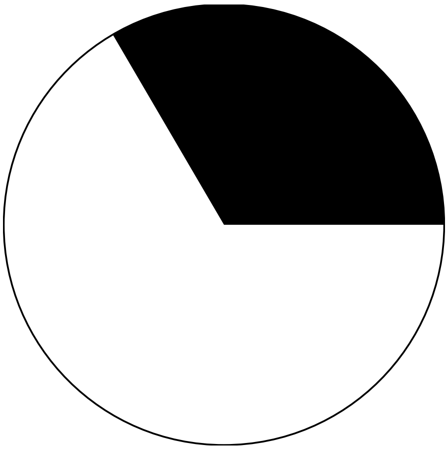
`Circle[{cx, cy}, {rx, ry}]`
: draws an ellipse.

`Circle[{cx, cy}]`
: chooses radius 1.

`Circle[]`
: chooses center (0, 0) and radius 1.

```
>> Graphics[{Red, Circle[{0, 0},
{2, 1}]}]
```

```
>> Graphics[{Circle[], Disk[{0, 0},
 {1, 1}, {0, 2.1}]}]
```



CircleBox

ColorDistance

`ColorDistance[`*c1*`, `*c2*`]`
: returns a measure of color distance between the colors *c1* and *c2*.

`ColorDistance[`*list*`, `*c2*`]`
: returns a list of color distances between the colors in *list* and *c2*.

The option DistanceFunction specifies the method used to measure the color distance. Available options are:
CIE76: euclidean distance in the LABColor space CIE94: euclidean distance in the LCHColor space CIE2000 or CIEDE2000: CIE94 distance with corrections CMC: Colour Measurement Committee metric (1984) DeltaL: difference in the L component of LCHColor DeltaC: difference in the C component of LCHColor DeltaH: difference in the H component of LCHColor
It is also possible to specify a custom distance

```
>> ColorDistance[Magenta, Green]
```

2.2507

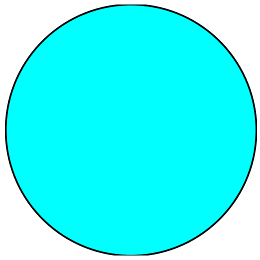
```
>> ColorDistance[{Red, Blue}, {
Green, Yellow}, DistanceFunction
 -> {"CMC", "Perceptibility"}]
```

{1.0495, 1.27455}

Cyan

`Cyan`
: represents the color cyan in graphics.

```
>> Graphics[{EdgeForm[Black], Cyan,
 Disk[]}, ImageSize->Small]
```



```
>> Cyan // ToBoxes
```

StyleBox [GraphicsBox [
{EdgeForm [GrayLevel [
0]] , RGBColor [0, 1, 1] ,
RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

```
>> Cyan
```

Darker

`Darker[`*c*`, `*f*`]`
: is equivalent to `Blend[{`*c*`, Black}, `*f*`]`.

`Darker[`*c*`]`
: is equivalent to `Darker[`*c*`, 1/3]`.

```
>> Graphics[Table[{Darker[Yellow, x
], Disk[{12x, 0}]}, {x, 0, 1,
1/6}]]
```

Directive

Disk

`Disk[{cx, cy}, r]`
: fills a circle with center (*cx*, *cy*) and radius *r*.

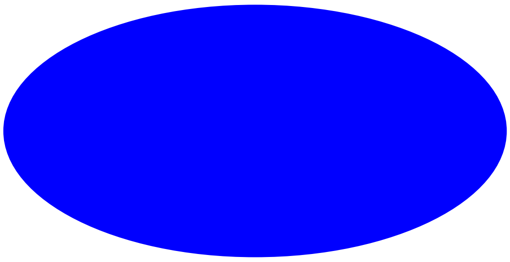
`Disk[{cx, cy}, {rx, ry}]`
: fills an ellipse.

`Disk[{cx, cy}]`
: chooses radius 1.

`Disk[]`
: chooses center (0, 0) and radius 1.

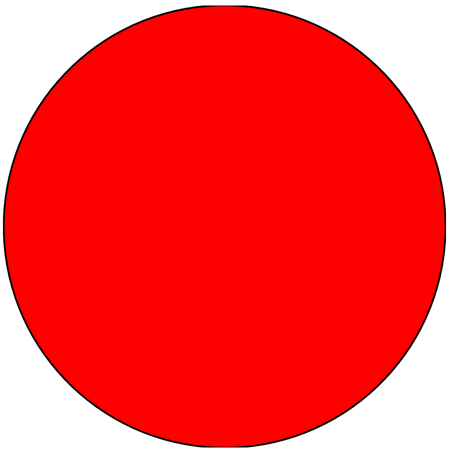
`Disk[{x, y}, ..., {t1, t2}]`
: is a sector from angle *t1* to *t2*.

```
>> Graphics[{Blue, Disk[{0, 0}, {2, 1}]}]
```



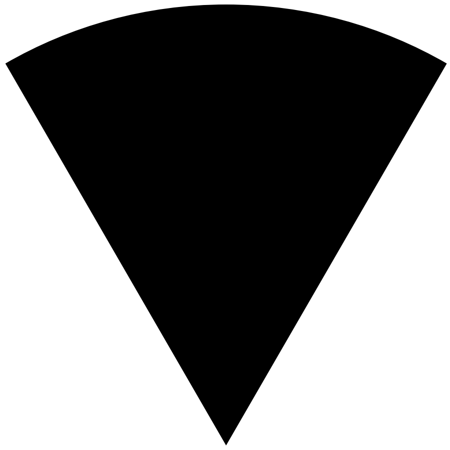
The outer border can be drawn using `EdgeForm`:

```
>> Graphics[{EdgeForm[Black], Red, Disk[]}]
```

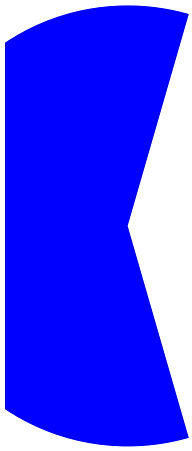


Disk can also draw sectors of circles and ellipses

```
>> Graphics[Disk[{0, 0}, 1, {Pi / 3, 2 Pi / 3}]]
```



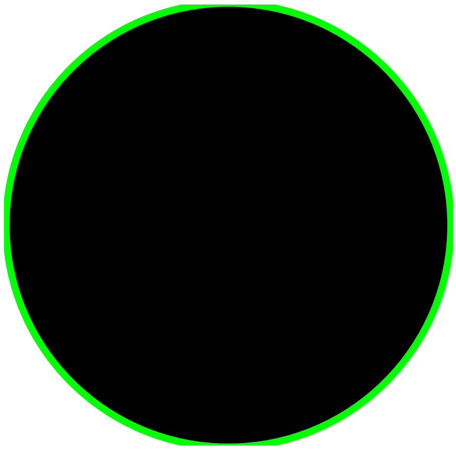
```
>> Graphics[{Blue, Disk[{0, 0}, {1, 2}, {Pi / 3, 5 Pi / 3}]}]
```



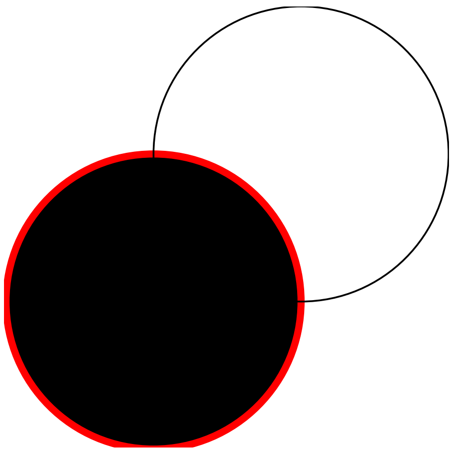
DiskBox

EdgeForm

```
>> Graphics[{EdgeForm[{Thick, Green
   }], Disk[]}]
```



```
>> Graphics[{Style[Disk[],EdgeForm
   [{Thick,Red}]], Circle[{1,1}]}]
```

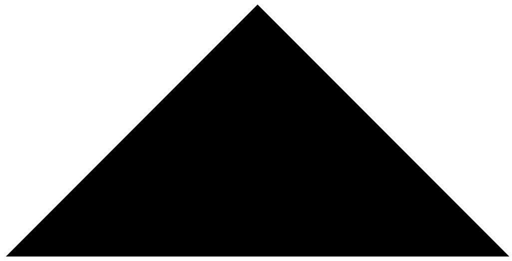


FaceForm

FilledCurve

`FilledCurve[{`*segment1*`, `*segment2*` ...}]`
represents a filled curve.

```
>> Graphics[FilledCurve[{Line[{{0,
   0}, {1, 1}, {2, 0}}]}]]
```



```
>> Graphics[FilledCurve[{
   BezierCurve[{{0, 0}, {1, 1}, {2,
    0}}], Line[{{3, 0}, {0, 2}}]}]]
```

FilledCurveBox

FontColor

`FontColor`
is an option for Style to set the font color.

Graphics

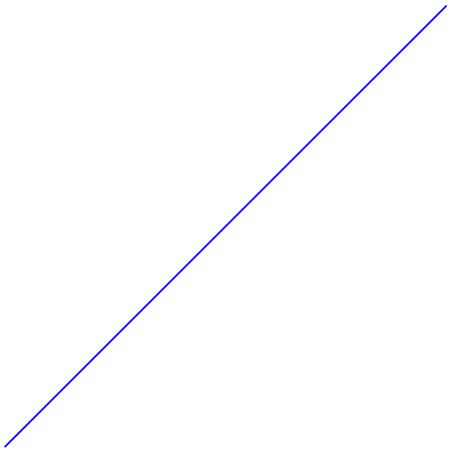
`Graphics[`*primitives*`, `*options*`]`
represents a graphic.

Options include:

- Axes
- TicksStyle
- AxesStyle
- LabelStyle
- AspectRatio
- PlotRange
- PlotRangePadding
- ImageSize
- Background

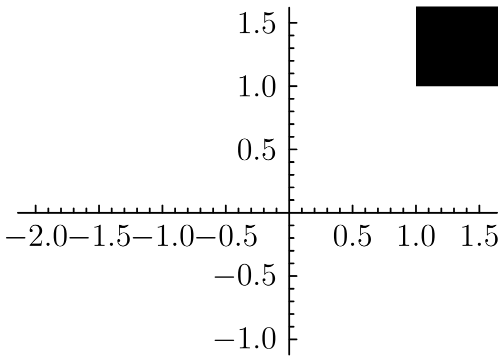
•

>> `Graphics[{Blue, Line[{{0,0}, {1,1}}]}]`



`Graphics` supports `PlotRange`:

>> `Graphics[{Rectangle[{1, 1}]}, Axes -> True, PlotRange -> {{-2, 1.5}, {-1, 1.5}}]`



>> `Graphics[{Rectangle[],Red,Disk[{1,0}]},PlotRange->{{0,1},{0,1}}]`

`Graphics` produces `GraphicsBox` boxes:

>> `Graphics[Rectangle[]] // ToBoxes // Head`

GraphicsBox

In `TeXForm`, `Graphics` produces Asymptote figures:

>> `Graphics[Circle[]] // TeXForm`

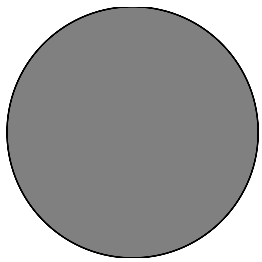
\begin{asy}
usepackage("amsmath");
size(5.8556cm, 5.8333cm);
draw(ellipse((175,175),175,175), rgb(0, 0, 0)+linewidth(0.66667));
clip(box((-0.33333,0.33333), (350.33,349.67)));
\end{asy}

GraphicsBox

Gray

`Gray`
represents the color gray in graphics.

>> `Graphics[{EdgeForm[Black], Gray, Disk[]}, ImageSize->Small]`



```
>> Gray // ToBoxes
```

StyleBox [GraphicsBox [
{EdgeForm [GrayLevel [0]] ,
GrayLevel [0.5] , RectangleBox [{0,
0}] } , $OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1,1}]

```
>> Gray
```

GrayLevel

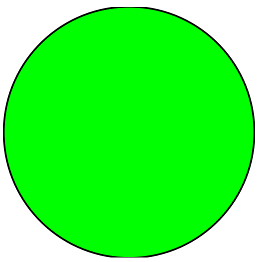
`GrayLevel[`*g*`]`
: represents a shade of gray specified by *g*, ranging from 0 (black) to 1 (white).

`GrayLevel[`*g*`, `*a*`]`
: represents a shade of gray specified by *g* with opacity *a*.

Green

`Green`
: represents the color green in graphics.

```
>> Graphics[{EdgeForm[Black], Green
   , Disk[]}, ImageSize->Small]
```



```
>> Green // ToBoxes
```

StyleBox [GraphicsBox [
{EdgeForm [GrayLevel [
0]] , RGBColor [0, 1, 0] ,
RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1,1}]

```
>> Green
```

Hue

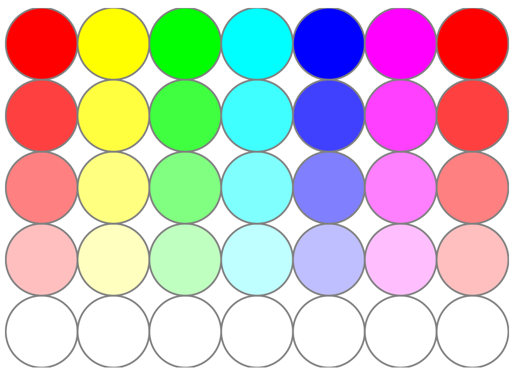
`Hue[`*h*`, `*s*`, `*l*`, `*a*`]`
: represents the color with hue *h*, saturation *s*, lightness *l* and opacity *a*.

`Hue[`*h*`, `*s*`, `*l*`]`
: is equivalent to `Hue[`*h*`, `*s*`, `*l*`, 1]`.

`Hue[`*h*`, `*s*`]`
: is equivalent to `Hue[`*h*`, `*s*`, 1, 1]`.

`Hue[`*h*`]`
: is equivalent to `Hue[`*h*`, 1, 1, 1]`.

```
>> Graphics[Table[{EdgeForm[Gray],
   Hue[h, s], Disk[{12h, 8s}]}, {h,
    0, 1, 1/6}, {s, 0, 1, 1/4}]]
```



```
>> Graphics[Table[{EdgeForm[{
   GrayLevel[0, 0.5]}], Hue[(-11+q
   +10r)/72, 1, 1, 0.6], Disk[(8-r)
   {Cos[2Pi q/12], Sin[2Pi q/12]},
   (8-r)/3]}, {r, 6}, {q, 12}]]
```

Inset

InsetBox

LABColor

`LABColor[`*l*`, `*a*`, `*b*`]`
: represents a color with the specified lightness, red/green and yellow/blue components in the CIE 1976 L*a*b* (CIELAB) color space.

LCHColor

`LCHColor[`*l*`, `*c*`, `*h*`]`
: represents a color with the specified lightness, chroma and hue components in the CIELCh CIELab cube color space.

LUVColor

`LCHColor[`*l*`, `*u*`, `*v*`]`
: represents a color with the specified components in the CIE 1976 L*u*v* (CIELUV) color space.

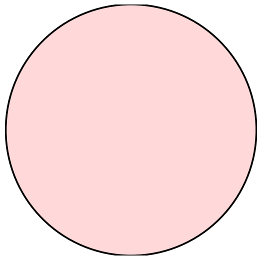
Large

`ImageSize -> Large`
: produces a large image.

LightRed

`LightRed`
: represents the color light red in graphics.

```
>> Graphics[{EdgeForm[Black],
   LightRed, Disk[]}, ImageSize->
   Small]
```



```
>> LightRed // ToBoxes
```

StyleBox [GraphicsBox [{EdgeForm [GrayLevel [0]] , RGBColor [1., 0.85, 0.85] , RectangleBox [{0, 0}] } , $OptionSyntax− > Ignore, AspectRatio− > Automatic, Axes− > False, AxesStyle− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle− > {}] , ImageSizeMultipliers− > {1, 1}]

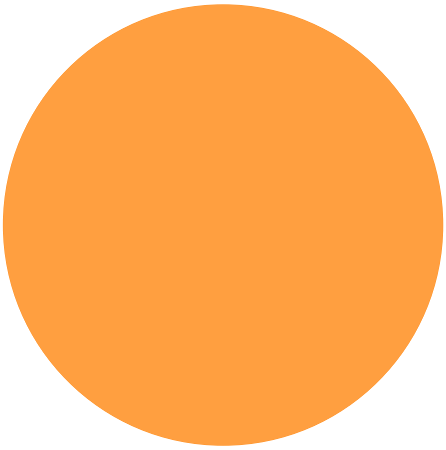
Lighter

`Lighter[`*c*`, `*f*`]`
: is equivalent to `Blend[{`*c*`, White}, `*f*`]`.

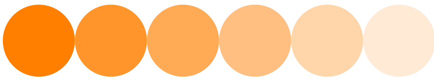
`Lighter[`*c*`]`
: is equivalent to `Lighter[`*c*`, 1/3]`.

```
>> Lighter[Orange, 1/4]
```

```
>> Graphics[{Lighter[Orange, 1/4],
   Disk[]}]
```



```
>> Graphics[Table[{Lighter[Orange,
   x], Disk[{12x, 0}]}, {x, 0, 1,
   1/6}]]
```

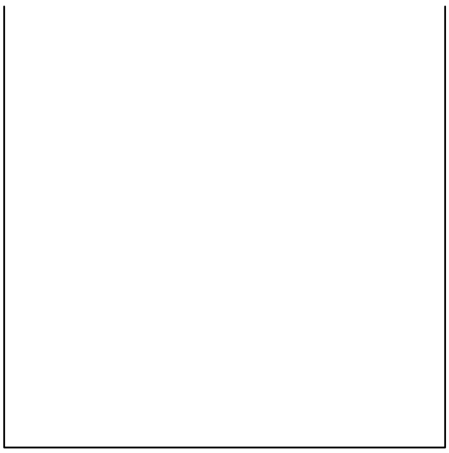


Line

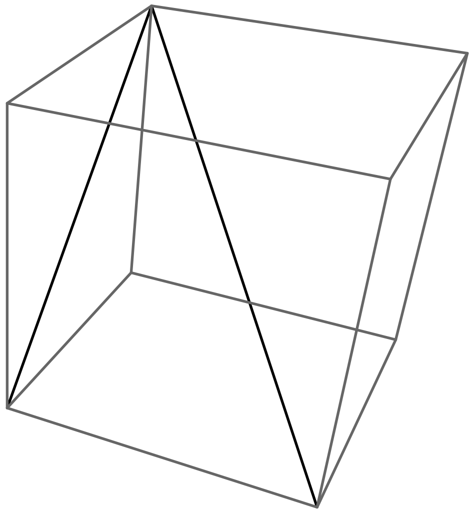
`Line[{`*point_1*`, `*point_2*` ...}]`
    represents the line primitive.

`Line[{{`*p_11*`, `*p_12*`, ...}, {`*p_21*`, `*p_22*`, ...}, ...}]`
    represents a number of line primitives.

```
>> Graphics[Line
   [{{0,1},{0,0},{1,0},{1,1}}]]
```



```
>> Graphics3D[Line
   [{{0,0,0},{0,1,1},{1,0,0}}]]
```

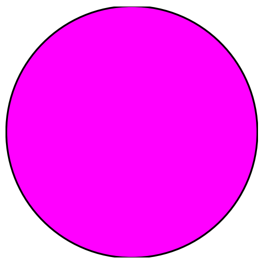


LineBox

Magenta

`Magenta`
    represents the color magenta in graphics.

```
>> Graphics[{EdgeForm[Black],
   Magenta, Disk[]}, ImageSize->
   Small]
```



>> `Magenta // ToBoxes`

StyleBox [GraphicsBox [
{EdgeForm [GrayLevel [
0]] , RGBColor [1, 0, 1] ,
RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

>> `Magenta`

Medium

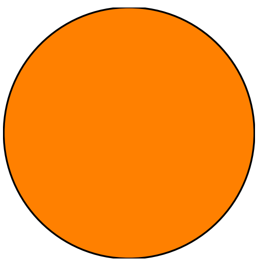
`ImageSize -> Medium`
produces a medium-sized image.

Offset

Orange

`Orange`
represents the color orange in graphics.

>> `Graphics[{EdgeForm[Black], Orange, Disk[]}, ImageSize->Small]`



>> `Orange // ToBoxes`

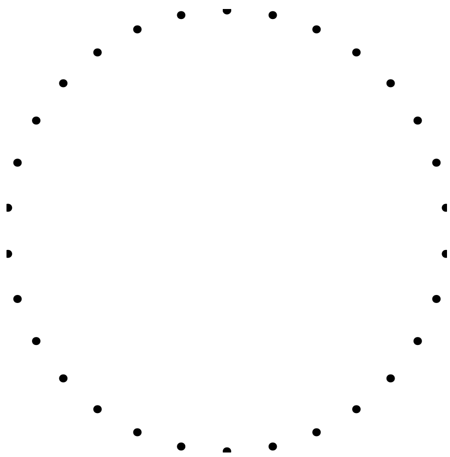
StyleBox [GraphicsBox [
{EdgeForm [GrayLevel [
0]] , RGBColor [1, 0.5, 0] ,
RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

Point

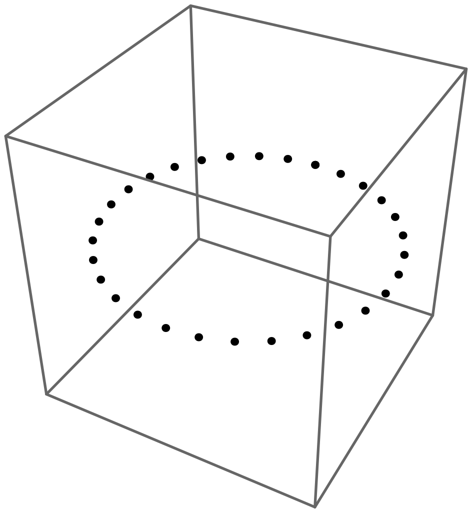
`Point[{`*point_1*`, `*point_2*` ...}]`
represents the point primitive.
`Point[{{`*p_11*`, `*p_12*`, ...}, {`*p_21*`, `*p_22*`, ...}, ...}]`
represents a number of point primitives.

>> `Graphics[Point[{0,0}]]`

>> `Graphics[Point[Table[{Sin[t], Cos[t]}, {t, 0, 2. Pi, Pi / 15.}]]]`



```
>> Graphics3D[Point[Table[{Sin[t],
   Cos[t], 0}, {t, 0, 2. Pi, Pi /
   15.}]]]
```



PointBox

PointSize

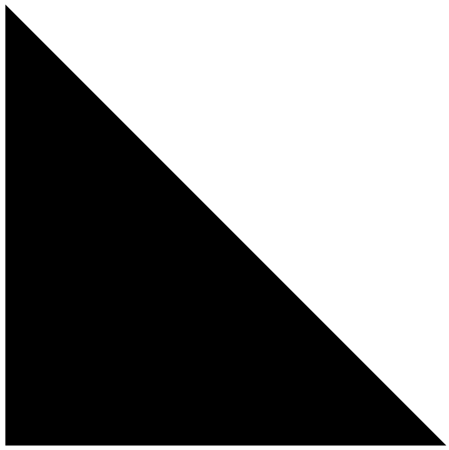
`PointSize[`*t*`]`
: sets the diameter of points to *t*, which is relative to the overall width.

Polygon

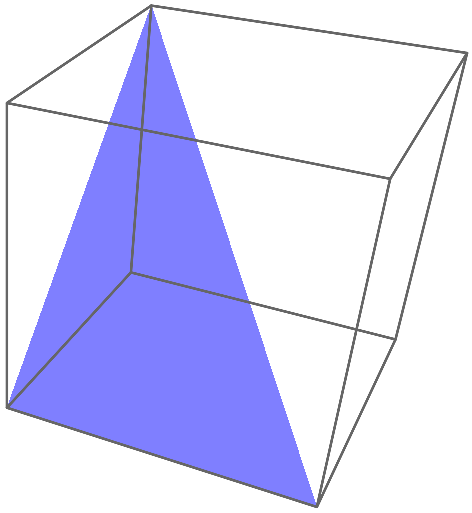
`Polygon[{`*point_1*`, `*point_2*` ...}]`
: represents the filled polygon primitive.

`Polygon[{{`*p_11*`, `*p_12*`, ...}, {`*p_21*`, `*p_22*`, ...}, ...}]`
: represents a number of filled polygon primitives.

```
>> Graphics[Polygon
   [{{1,0},{0,0},{0,1}}]]
```



```
>> Graphics3D[Polygon
   [{{0,0,0},{0,1,1},{1,0,0}}]]
```

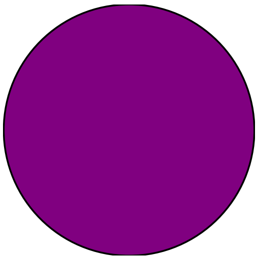


PolygonBox

Purple

`Purple`
: represents the color purple in graphics.

```
>> Graphics[{EdgeForm[Black],
   Purple, Disk[]}, ImageSize->
   Small]
```



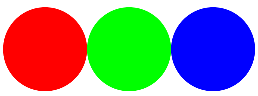
```
>> Purple // ToBoxes
```

StyleBox [GraphicsBox [
{EdgeForm [GrayLevel [
0]] , RGBColor [0.5, 0, 0.5] ,
RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

RGBColor

`RGBColor[`*r*`, `*g*`, `*b*`]`
: represents a color with the specified red, green and blue components.

```
>> Graphics[MapIndexed[{RGBColor @@
    #1, Disk[2*#2 ~Join~{0}]} &,
   IdentityMatrix[3]], ImageSize->
   Small]
```



```
>> RGBColor[0, 1, 0]
```

```
>> RGBColor[0, 1, 0] // ToBoxes
```

StyleBox [GraphicsBox [
{EdgeForm [GrayLevel [
0]] , RGBColor [0, 1, 0] ,
RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

Rectangle

`Rectangle[{`*xmin*`, `*ymin*`}]`
: represents a unit square with bottom-left corner at {*xmin*, *ymin*}.

'Rectangle[{*xmin*, *ymin*}, {*xmax*, *ymax*}]
: is a rectange extending from {*xmin*, *ymin*} to {*xmax*, *ymax*}.

```
>> Graphics[Rectangle[]]
```

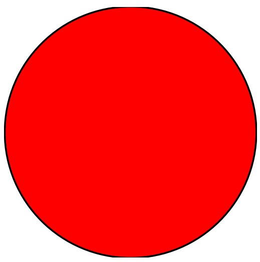
>> `Graphics[{Blue, Rectangle[{0.5, 0}], Orange, Rectangle[{0, 0.5}]}]`

RectangleBox

Red

`Red`
    represents the color red in graphics.

>> `Graphics[{EdgeForm[Black], Red, Disk[]}, ImageSize->Small]`



>> `Red // ToBoxes`

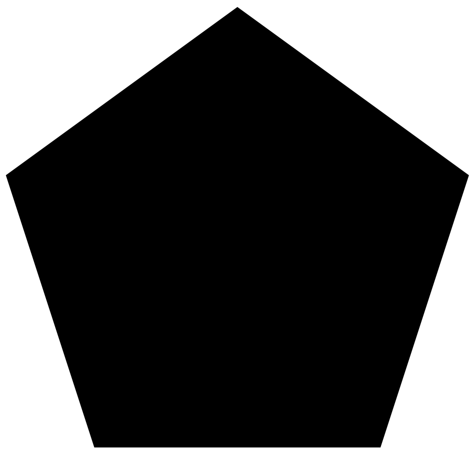
StyleBox [GraphicsBox [{EdgeForm [GrayLevel [0]] , RGBColor [1, 0, 0] , RectangleBox [{0, 0}] } , $OptionSyntax− > Ignore, AspectRatio− > Automatic, Axes− > False, AxesStyle− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle− > {}] , ImageSizeMultipliers− > {1, 1}]

>> `Red`

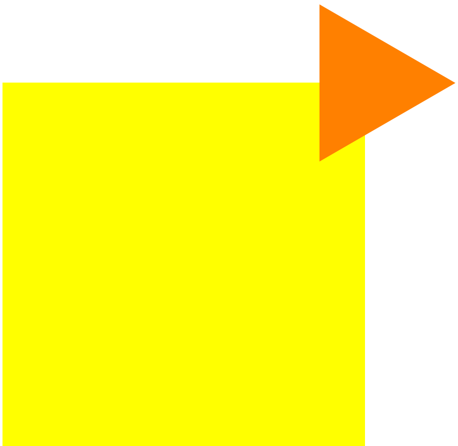
RegularPolygon

`RegularPolygon[n]`
    gives the regular polygon with *n* edges.
`RegularPolygon[r, n]`
    gives the regular polygon with *n* edges and radius *r*.
`RegularPolygon[{r, phi}, n]`
    gives the regular polygon with radius *r* with one vertex drawn at angle *phi*.
`RegularPolygon[{$x, $y}, r, n]`
    gives the regular polygon centered at the position {$x, $y}.

>> `Graphics[RegularPolygon[5]]`



>> `Graphics[{Yellow, Rectangle[], Orange, RegularPolygon[{1, 1}, {0.25, 0}, 3]}]`



RegularPolygonBox

Show

`Show[`*graphics*`, `*options*`]`
: shows graphics with the specified options added.

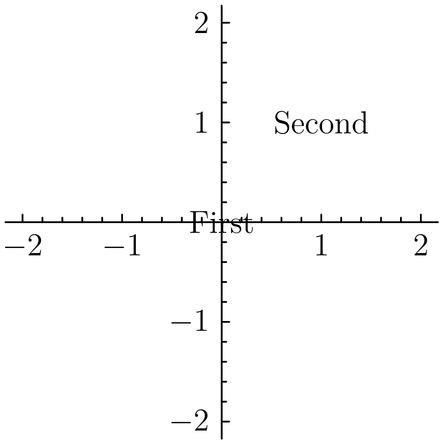
Small

`ImageSize -> Small`
: produces a small image.

Text

`Text["`*text*`", {`*x*`, `*y*`}]`
: draws *text* centered on position `{`*x*`, `*y*`}`.

>> `Graphics[{Text["First", {0, 0}], Text["Second", {1, 1}]}, Axes->True, PlotRange->{{-2, 2}, {-2, 2}}]`



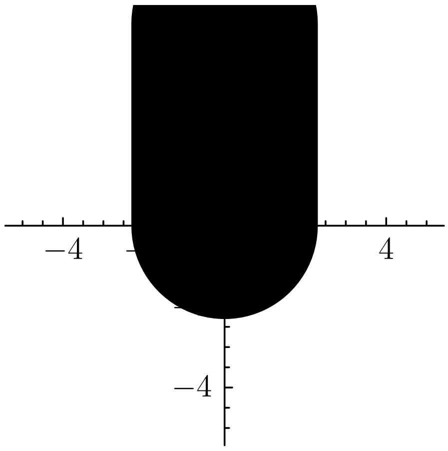
Thick

`Thick`
: sets the line width for subsequent graphics primitives to 2pt.

Thickness

`Thickness[`*t*`]`
: sets the line thickness for subsequent graphics primitives to *t* times the size of the plot area.

>> `Graphics[{Thickness[0.2], Line[{{0, 0}, {0, 5}}]}, Axes->True, PlotRange->{{-5, 5}, {-5, 5}}]`



Thin

`Thin`
sets the line width for subsequent graphics primitives to 0.5pt.

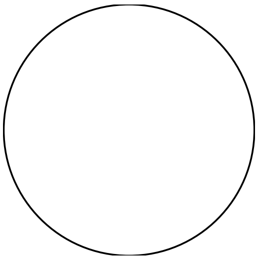
Tiny

`ImageSize -> Tiny`
produces a tiny image.

White

`White`
represents the color white in graphics.

```
>> Graphics[{EdgeForm[Black], White
   , Disk[]}, ImageSize->Small]
```



```
>> White // ToBoxes
```

StyleBox [GraphicsBox [{EdgeForm [GrayLevel [0]] , GrayLevel [1] , RectangleBox [{0, 0}] } , $OptionSyntax− > Ignore, AspectRatio− > Automatic, Axes− > False, AxesStyle− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle− > {}] , ImageSizeMultipliers− > {1, 1}]

```
>> White
```

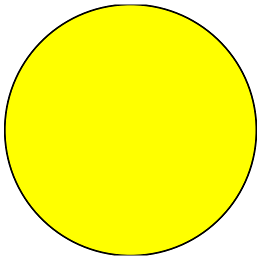
XYZColor

`XYZColor[x, y, z]`
represents a color with the specified components in the CIE 1931 XYZ color space.

Yellow

`Yellow`
represents the color yellow in graphics.

```
>> Graphics[{EdgeForm[Black],
   Yellow, Disk[]}, ImageSize->
   Small]
```



```
>> Yellow // ToBoxes
```

StyleBox [GraphicsBox [{EdgeForm [GrayLevel [0]] , RGBColor [1, 1, 0] , RectangleBox [{0, 0}] } , $OptionSyntax− > Ignore, AspectRatio− > Automatic, Axes− > False, AxesStyle− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle− > {}] , ImageSizeMultipliers− > {1, 1}]

```
>> Yellow
```

XXVIII. Manipulate

Contents

Manipulate

`Manipulate[`*expr1*`, {`*u*`, `*u_min*`, `*u_max*`}]`
: interactively compute and display an expression with different values of *u*.

`Manipulate[`*expr1*`, {`*u*`, `*u_min*`, `*u_max*`, `*du*`}]`
: allows *u* to vary between *u_min* and *u_max* in steps of *du*.

`Manipulate[`*expr1*`, {{`*u*`, `*u_init*`}, `*u_min*`, `*u_max*`, ...}]`
: starts with initial value of *u_init*.

`Manipulate[`*expr1*`, {{`*u*`, `*u_init*`, `*u_lbl*`}, ...}]`
: labels the *u* controll by *u_lbl*.

`Manipulate[`*expr1*`, {`*u*`, {`*u_1*`, `*u_2*`, ...}}]`
: sets *u* to take discrete values *u_1, u_2, ...* .

`Manipulate[`*expr1*`, {`*u*`, ...}, {`*v*`, ...}, ...]`
: control each of *u, v, ...* .

System`Private`ManipulateParameter

XXIX. Number theoretic functions

Contents

ContinuedFraction

`ContinuedFraction[`*x*`, `*n*`]`
: generate the first *n* terms in the continued fraction reprentation of *x*.

`ContinuedFraction[`*x*`]`
: the complete continued fraction representation for a rational or quadradic irrational number.

>> `ContinuedFraction[Pi, 10]`

$\{3, 7, 15, 1, 292, 1, 1, 1, 2, 1\}$

>> `ContinuedFraction[(1 + 2 Sqrt[3])/5]`

$\{0, 1, \{8, 3, 34, 3\}\}$

>> `ContinuedFraction[Sqrt[70]]`

$\{8, \{2, 1, 2, 1, 2, 16\}\}$

CoprimeQ

`CoprimeQ[`*x*`, `*y*`]`
: tests whether *x* and *y* are coprime by computing their greatest common divisor.

>> `CoprimeQ[7, 9]`

True

>> `CoprimeQ[-4, 9]`

True

>> `CoprimeQ[12, 15]`

False

CoprimeQ also works for complex numbers

>> `CoprimeQ[1+2I, 1-I]`

True

>> `CoprimeQ[4+2I, 6+3I]`

True

>> `CoprimeQ[2, 3, 5]`

True

>> `CoprimeQ[2, 4, 5]`

False

Divisors

`Divisors[`*n*`]`
: returns a list of the integers that divide *n*.

>> `Divisors[96]`

$\{1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96\}$

>> `Divisors[704]`

$\{1, 2, 4, 8, 11, 16, 22, 32, 44, 64, 88, 176, 352, 704\}$

```
>> Divisors[{87, 106, 202, 305}]
```
$\{\{1,3,29,87\}, \{1,2,53,106\}, \{1,2,101,202\}, \{1,5,61,305\}\}$

EvenQ

`EvenQ[x]`
: returns `True` if x is even, and `False` otherwise.

```
>> EvenQ[4]
```
True

```
>> EvenQ[-3]
```
False

```
>> EvenQ[n]
```
False

FactorInteger

`FactorInteger[n]`
: returns the factorization of n as a list of factors and exponents.

```
>> factors = FactorInteger[2010]
```
$\{\{2,1\}, \{3,1\}, \{5,1\}, \{67,1\}\}$

To get back the original number:

```
>> Times @@ Power @@@ factors
```
2 010

`FactorInteger` factors rationals using negative exponents:

```
>> FactorInteger[2010 / 2011]
```
$\{\{2,1\}, \{3,1\}, \{5,1\}, \{67,1\}, \{2011, -1\}\}$

FractionalPart

`FractionalPart[n]`
: finds the fractional part of n.

```
>> FractionalPart[4.1]
```
0.1

```
>> FractionalPart[-5.25]
```
-0.25

FromContinuedFraction

`FromContinuedFraction[list]`
: reconstructs a number from the list of its continued fraction terms.

```
>> FromContinuedFraction[{3, 7, 15,
    1, 292, 1, 1, 1, 2, 1}]
```
$\frac{1146408}{364913}$

```
>> FromContinuedFraction[Range[5]]
```
$\frac{225}{157}$

GCD

`GCD[n1, n2, ...]`
: computes the greatest common divisor of the given integers.

```
>> GCD[20, 30]
```
10

```
>> GCD[10, y]
```
$\mathrm{GCD}[10, y]$

`GCD` is `Listable`:

```
>> GCD[4, {10, 11, 12, 13, 14}]
```
$\{2,1,4,1,2\}$

`GCD` does not work for rational numbers and Gaussian integers yet.

IntegerExponent

`IntegerExponent[n, b]`
: gives the highest exponent of b that divides n.

```
>> IntegerExponent[16, 2]
```
4

```
>> IntegerExponent[-510000]
```
4

```
>> IntegerExponent[10, b]
```
$\mathrm{IntegerExponent}[10, b]$

LCM

`LCM[`$n1$`, `$n2$`, ...]`
: computes the least common multiple of the given integers.

```
>> LCM[15, 20]
```
60

```
>> LCM[20, 30, 40, 50]
```
600

MantissaExponent

`MantissaExponent[`n`]`
: finds a list containing the mantissa and exponent of a given number n.

`MantissaExponent[`n`, `b`]`
: finds the base b mantissa and exponent of n.

```
>> MantissaExponent[2.5*10^20]
```
$\{0.25, 21\}$

```
>> MantissaExponent[125.24]
```
$\{0.12524, 3\}$

```
>> MantissaExponent[125., 2]
```
$\{0.976563, 7\}$

```
>> MantissaExponent[10, b]
```
MantissaExponent $[10, b]$

Mod

`Mod[`x`, `m`]`
: returns x modulo m.

```
>> Mod[14, 6]
```
2

```
>> Mod[-3, 4]
```
1

```
>> Mod[-3, -4]
```
-3

```
>> Mod[5, 0]
```
Theargument0shouldbenonzero.

Mod $[5, 0]$

NextPrime

`NextPrime[`n`]`
: gives the next prime after n.

`NextPrime[`n`,`k`]`
: gives the kth prime after n.

```
>> NextPrime[10000]
```
10 007

```
>> NextPrime[100, -5]
```
73

```
>> NextPrime[10, -5]
```
-2

```
>> NextPrime[100, 5]
```
113

```
>> NextPrime[5.5, 100]
```
563

```
>> NextPrime[5, 10.5]
```
NextPrime $[5, 10.5]$

OddQ

`OddQ[`x`]`
: returns `True` if x is odd, and `False` otherwise.

```
>> OddQ[-3]
```
True

```
>> OddQ[0]
```
False

PartitionsP

`PartitionsP[`n`]`
: return the number $p(n)$ of unrestricted partitions of the integer n.

```
>> Table[PartitionsP[k], {k, -2, 12}]
```
$\{0,0,1,1,2,3,5,7,11,15,22,30,42,56,77\}$

PowerMod

`PowerMod[x, y, m]`
computes x^y modulo m.

```
>> PowerMod[2, 10000000, 3]
```
1

```
>> PowerMod[3, -2, 10]
```
9

```
>> PowerMod[0, -1, 2]
```
0isnotinvertiblemodulo2.

PowerMod $[0, -1, 2]$

```
>> PowerMod[5, 2, 0]
```
Theargument0shouldbenonzero.

PowerMod $[5, 2, 0]$

`PowerMod` does not support rational coefficients (roots) yet.

Prime

`Prime[n]`
returns the nth prime number.

```
>> Prime[1]
```
2

```
>> Prime[167]
```
991

PrimePi

`PrimePi[x]`
gives the number of primes less than or equal to x.

```
>> PrimePi[100]
```
25

```
>> PrimePi[-1]
```
0

```
>> PrimePi[3.5]
```
2

```
>> PrimePi[E]
```
1

PrimePowerQ

`PrimePowerQ[n]`
returns `True` if n is a power of a prime number.

```
>> PrimePowerQ[9]
```
True

```
>> PrimePowerQ[52142]
```
False

```
>> PrimePowerQ[-8]
```
True

```
>> PrimePowerQ[371293]
```
True

PrimeQ

`PrimeQ[n]`
returns `True` if n is a prime number.

For very large numbers, `PrimeQ` uses probabilistic prime testing, so it might be wrong sometimes (a number might be composite even though `PrimeQ` says it is prime). The algorithm might be changed in the future.

```
>> PrimeQ[2]
```
True

```
>> PrimeQ[-3]
```
True

```
>> PrimeQ[137]
```
True

```
>> PrimeQ[2 ^ 127 - 1]
```
True

All prime numbers between 1 and 100:

```
>> Select[Range[100], PrimeQ]
```
$\{2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97\}$

`PrimeQ` has attribute `Listable`:

>> `PrimeQ[Range[20]]`

{False, True, True, False, True, False, True, False, False, False, True, False, True, False, False, False, True, False, True, False}

Quotient

`Quotient[m, n]`
: computes the integer quotient of *m* and *n*.

>> `Quotient[23, 7]`

3

QuotientRemainder

`QuotientRemainder[m, n]`
: computes a list of the quotient and remainder from division of *m* by *n*.

>> `QuotientRemainder[23, 7]`

{3, 2}

RandomPrime

`RandomPrime[{`*imin*`, $imax}]`
: gives a random prime between *imin* and *imax*.

`RandomPrime[`*imax*`]`
: gives a random prime between 2 and *imax*.

`RandomPrime[`*range*`, `*n*`]`
: gives a list of *n* random primes in *range*.

>> `RandomPrime[{14, 17}]`

17

>> `RandomPrime[{14, 16}, 1]`

Therearenoprimesinthespecifiedinterval.

RandomPrime[{14, 16}, 1]

>> `RandomPrime[{8,12}, 3]`

{11, 11, 11}

>> `RandomPrime[{10,30}, {2,5}]`

{{29, 29, 29, 29, 29}, {29, 29, 29, 29, 29}}

XXX. XML

Contents

XML‘PlaintextImport

```
>> StringReplace[StringTake[Import
   ["ExampleData/InventionNo1.xml",
    "Plaintext"],31],
   FromCharacterCode[10]->"/"]
```

MuseScore 1.2/2 012-09-12/5.7/40

XML‘TagsImport

```
>> Take[Import["ExampleData/
   InventionNo1.xml", "Tags"], 10]
```

{accidental, alter, arpeggiate, articulations, attributes, backup, bar-style, barline, beam, beat-type}

XMLElement

XML‘Parser‘XMLGet

XML‘Parser‘XMLGetString

```
>> Head[XML‘Parser‘XMLGetString["<a
   ></a>"]]
```

XMLObject [Document]

XMLObject

XML‘XMLObjectImport

```
>> Part[Import["ExampleData/
   InventionNo1.xml", "XMLObject"],
    2, 3, 1]
```

XMLElement [identification, {}, {XMLElement [encoding, {}, {XMLElement [software, {}, {MuseScore 1.2}], XMLElement [encoding-date, {}, {2 012-09-12}]}]}]

```
>> Part[Import["ExampleData/
   Namespaces.xml"], 2]
```

XMLElement [book, {{http://www.w3.org/2 000/xmlns/, xmlns} − > urn:loc.gov:books}, {XMLElement [title, {}, {Cheaper by the Dozen}], XMLElement [{urn:ISBN:0-395-36 341-6, number}, {}, {1 568 491 379}], XMLElement [notes, {}, {XMLElement [p, {{http://www.w3.org/2 000/xmlns/, xmlns} − > http://www.w3.org/1 999/xhtml}, {This is a, XMLElement [i, {}, {funny, book!}]}]}]}]

XXXI. Differential Equations

Contents

C

C[n]
: represents the nth constant in a solution to a differential equation.

DSolve

DSolve[eq, y[x], x]
: solves a differential equation for the function y[x].

```
>> DSolve[y''[x] == 0, y[x], x]
```

$\{\{y[x]->xC[2]+C[1]\}\}$

```
>> DSolve[y''[x] == y[x], y[x], x]
```

$\left\{\left\{y[x]->C[1]E^{-x}+C[2]E^{x}\right\}\right\}$

```
>> DSolve[y''[x] == y[x], y, x]
```

$\left\{\left\{y->\left(\text{Function}\left[\{x\},\ C[1]E^{-x}+C[2]E^{x}\right]\right)\right\}\right\}$

DSolve can also solve basic PDE

```
>> DSolve[D[f[x, y], x] / f[x, y] +
    3 D[f[x, y], y] / f[x, y] == 2,
    f, {x, y}]
```

$\left\{\left\{f->\left(\text{Function}\left[\{x,y\},\ E^{\frac{x}{5}+\frac{3y}{5}}C[1]\left[3x-y\right]\right]\right)\right\}\right\}$

```
>> DSolve[D[f[x, y], x] x + D[f[x,
y], y] y == 2, f[x, y], {x, y}]
```

$\left\{\left\{f\left[x,y\right]->2\text{Log}[x]+C[1]\left[\frac{y}{x}\right]\right\}\right\}$

```
>> DSolve[D[y[x, t], t] + 2 D[y[x,
t], x] == 0, y[x, t], {x, t}]
```

$\{\{y[x,t]->C[1][-2t+x]\}\}$

XXXII. Strings and Characters

Contents

$CharacterEncoding

`CharacterEncoding`
specifies the default character encoding to use if no other encoding is specified.

$CharacterEncodings

CharacterRange

`CharacterRange["`*a*`‘ ‘,"`*b*`"]`
returns a list of the Unicode characters from *a* to *b* inclusive.

```
>> CharacterRange["a", "e"]
```
{a, b, c, d, e}

```
>> CharacterRange["b", "a"]
```
{}

Characters

`Characters["`*string*`"]`
returns a list of the characters in *string*.

```
>> Characters["abc"]
```
{a, b, c}

DamerauLevenshteinDistance

`DamerauLevenshteinDistance[`*a*`, `*b*`]`
: returns the Damerau-Levenshtein distance of *a* and *b*, which is defined as the minimum number of transpositions, insertions, deletions and substitutions needed to transform one into the other. In contrast to EditDistance, DamerauLevenshteinDistance counts transposition of adjacent items (e.g. "ab" into "ba") as one operation of change.

```
>> DamerauLevenshteinDistance["
   kitten", "kitchen"]
```
2

```
>> DamerauLevenshteinDistance["abc
   ", "ac"]
```
1

```
>> DamerauLevenshteinDistance["abc
   ", "acb"]
```
1

```
>> DamerauLevenshteinDistance["azbc
   ", "abxyc"]
```
3

The IgnoreCase option makes DamerauLevenshteinDistance ignore the case of letters:

```
>> DamerauLevenshteinDistance["time
   ", "Thyme"]
```
3

```
>> DamerauLevenshteinDistance["time
   ", "Thyme", IgnoreCase -> True]
```
2

DamerauLevenshteinDistance also works on lists:

```
>> DamerauLevenshteinDistance[{1, E
   , 2, Pi}, {1, E, Pi, 2}]
```
1

DigitCharacter

`DigitCharacter`
: represents the digits 0-9.

```
>> StringMatchQ["1", DigitCharacter
   ]
```
True

```
>> StringMatchQ["a", DigitCharacter
   ]
```
False

```
>> StringMatchQ["12",
   DigitCharacter]
```
False

```
>> StringMatchQ["123245",
   DigitCharacter..]
```
True

DigitQ

`DigitQ[`*string*`]` yields `True` if all the characters in the *string* are digits, and yields `False` otherwise.

```
>> DigitQ["9"]
```
True

```
>> DigitQ["a"]
```
False

```
>> DigitQ
   ["01001101011000010111010001101000011010010110001
```
True

```
>> DigitQ["-123456789"]
```
False

EditDistance

`EditDistance[`*a*`, `*b*`]`
: returns the Levenshtein distance of *a* and *b*, which is defined as the minimum number of insertions, deletions and substitutions on the constituents of *a* and *b* needed to transform one into the other.

```
>> EditDistance["kitten", "kitchen
   "]
```
2

```
>> EditDistance["abc", "ac"]
```
1

```
>> EditDistance["abc", "acb"]
```

2

```
>> EditDistance["azbc", "abxyc"]
```

3

The IgnoreCase option makes EditDistance ignore the case of letters:

```
>> EditDistance["time", "Thyme"]
```

3

```
>> EditDistance["time", "Thyme",
   IgnoreCase -> True]
```

2

EditDistance also works on lists:

```
>> EditDistance[{1, E, 2, Pi}, {1,
   E, Pi, 2}]
```

2

EndOfLine

`EndOfString`
represents the end of a line in a string.

```
>> StringReplace["aba\nbba\na\nab",
    "a" ~~EndOfLine -> "c"]
```

abc
bbc
c
ab

```
>> StringSplit["abc\ndef\nhij",
   EndOfLine]
```

{abc,
def,
hij}

EndOfString

`EndOfString`
represents the end of a string.

Test whether strings end with "e":

```
>> StringMatchQ[#, __ ~~"e" ~~
   EndOfString] &/@ {"apple", "
   banana", "artichoke"}
```

$\{$True, False, True$\}$

```
>> StringReplace["aab\nabb", "b" ~~
   EndOfString -> "c"]
```

aab
abc

FromCharacterCode

`FromCharacterCode[`*n*`]`
returns the character corresponding to Unicode codepoint *n*.

`FromCharacterCode[{`*n1*`, `*n2*`, ...}]`
returns a string with characters corresponding to *n_i*.

`FromCharacterCode[{{`*n11*`, `*n12*`, ...}, {`*n21*`, `*n22*`, ...}, ...}]`
returns a list of strings.

```
>> FromCharacterCode[100]
```

d

```
>> FromCharacterCode[228, "ISO8859
   -1"]
```

ä

```
>> FromCharacterCode[{100, 101,
   102}]
```

def

```
>> ToCharacterCode[%]
```

$\{100, 101, 102\}$

```
>> FromCharacterCode[{{97, 98, 99},
    {100, 101, 102}}]
```

$\{$abc, def$\}$

```
>> ToCharacterCode["abc 123"] //
   FromCharacterCode
```

abc 123

HammingDistance

`HammingDistance[`*u*`, `*v*`]`
returns the Hamming distance between *u* and *v*, i.e. the number of different elements. *u* and *v* may be lists or strings.

```
>> HammingDistance[{1, 0, 1, 0},
   {1, 0, 0, 1}]
```

2

```
>> HammingDistance["time", "dime"]
```
1

```
>> HammingDistance["TIME", "dime",
   IgnoreCase -> True]
```
1

HexidecimalCharacter

`HexidecimalCharacter`
: represents the characters 0-9, a-f and A-F.

```
>> StringMatchQ[#,
   HexidecimalCharacter] & /@ {"a",
   "1", "A", "x", "H", " ", "."}
```
{True, True, True, False, False, False, False}

LetterCharacter

`LetterCharacter`
: represents letters.

```
>> StringMatchQ[#, LetterCharacter]
   & /@ {"a", "1", "A", " ", "."}
```
{True, False, True, False, False}

LetterCharacter also matches unicode characters.

```
>> StringMatchQ["\[Lambda]",
   LetterCharacter]
```
True

LetterQ

`LetterQ[`*string*`]` yields `True` if all the characters in the *string* are letters, and yields `False` otherwise.

```
>> LetterQ["m"]
```
True

```
>> LetterQ["9"]
```
False

```
>> LetterQ["Mathics"]
```
True

```
>> LetterQ["Welcome to Mathics"]
```
False

LowerCaseQ

`LowerCaseQ[`*s*`]`
: returns True if *s* consists wholly of lower case characters.

```
>> LowerCaseQ["abc"]
```
True

An empty string returns True.

```
>> LowerCaseQ[""]
```
True

NumberString

`NumberString`
: represents the characters in a number.

```
>> StringMatchQ["1234",
   NumberString]
```
True

```
>> StringMatchQ["1234.5",
   NumberString]
```
True

```
>> StringMatchQ["1.2‘20",
   NumberString]
```
False

RegularExpression

`RegularExpression[‘‘regex’]’`
: represents the regex specified by the string $“regex”$.

```
>> StringSplit["1.23, 4.56 7.89",
   RegularExpression["(\\s|,)+"]]
```
{1.23, 4.56, 7.89}

RemoveDiacritics

`RemoveDiacritics[s]`
: returns a version of *s* with all diacritics removed.

```
>> RemoveDiacritics["en prononçant pêcher et pécher"]
```
en prononcant pecher et pecher

```
>> RemoveDiacritics["piñata"]
```
pinata

StartOfLine

`StartOfString`
: represents the start of a line in a string.

```
>> StringReplace["aba\nbba\na\nab", StartOfLine ~~"a" -> "c"]
```
cba
 bba
 c
 cb

```
>> StringSplit["abc\ndef\nhij", StartOfLine]
```
{abc
 , def
 , hij}

StartOfString

`StartOfString`
: represents the start of a string.

Test whether strings start with "a":

```
>> StringMatchQ[#, StartOfString ~~ "a" ~~__] &/@ {"apple", "banana", "artichoke"}
```
{True, False, True}

```
>> StringReplace["aba\nabb", StartOfString ~~"a" -> "c"]
```
cba
 abb

StringCases

`StringCases["string", pattern]`
: gives all occurences of *pattern* in *string*.

`StringReplace["string", pattern -> form]`
: gives all instances of *form* that stem from occurences of *pattern* in *string*.

`StringCases["string", {pattern1, pattern2, ...}]`
: gives all occurences of *pattern1*, *pattern2*, ...

`StringReplace["string", pattern, n]`
: gives only the first *n* occurences.

`StringReplace[{"string1‘‘,"string2", ...}, pattern]`
: gives occurences in *string1*, *string2*, ...

```
>> StringCases["axbaxxb", "a" ~~x_ ~~"b"]
```
{axb}

```
>> StringCases["axbaxxb", "a" ~~x__ ~~"b"]
```
{axbaxxb}

```
>> StringCases["axbaxxb", Shortest["a" ~~x__ ~~"b"]]
```
{axb, axxb}

```
>> StringCases["-abc- def -uvw- xyz", Shortest["-" ~~x__ ~~"-"] -> x]
```
{abc, uvw}

```
>> StringCases["-öhi- -abc- -.-", "-" ~~x : WordCharacter .. ~~"-" -> x]
```
{öhi, abc}

```
>> StringCases["abc-abc xyz-uvw", Shortest[x : WordCharacter .. ~~ "-" ~~x_] -> x]
```
{abc}

```
>> StringCases["abba", {"a" -> 10, "b" -> 20}, 2]
```
{10, 20}

```
>> StringCases["a#ä_123", WordCharacter]
```
{a, ä, 1, 2, 3}

```
>> StringCases["a#ä_123",
   LetterCharacter]
```

{a, ä}

StringContainsQ

`StringContainsQ["`*string*`", `*patt*`]`
: returns True if any part of *string* matches *patt*, and returns False otherwise.

`StringContainsQ[{‘‘s1’, "s2", ...}, patt]’`
: returns the list of results for each element of string list.

`StringContainsQ[patt]`
: represents an operator form of StringContainsQ that can be applied to an expression.

```
>> StringContainsQ["mathics", "m" ~
   ~__ ~~"s"]
```

True

```
>> StringContainsQ["mathics", "a" ~
   ~__ ~~"m"]
```

False

```
>> StringContainsQ["Mathics", "MA"
   , IgnoreCase -> True]
```

True

```
>> StringContainsQ[{"g", "a", "laxy
   ", "universe", "sun"}, "u"]
```

{False, False, False, True, True}

```
>> StringContainsQ["e" ~~___ ~~"u"]
    /@ {"The Sun", "Mercury", "
   Venus", "Earth", "Mars", "
   Jupiter", "Saturn", "Uranus", "
   Neptune"}
```

{True, True, True, False, False, False, False, False, True}

StringDrop

`StringDrop["`*string*`", `*n*`]`
: gives *string* with the first *n* characters dropped.

`StringDrop["`*string*`", -`*n*`]`
: gives *string* with the last *n* characters dropped.

`StringDrop["`*string*`", {`*n*`}]`
: gives *string* with the *n*th character dropped.

`StringDrop["`*string*`", {`*m*`, `*n*`}]`
: gives *string* with the characters *m* through *n* dropped.

```
>> StringDrop["abcde", 2]
```

cde

```
>> StringDrop["abcde", -2]
```

abc

```
>> StringDrop["abcde", {2}]
```

acde

```
>> StringDrop["abcde", {2,3}]
```

ade

```
>> StringDrop["abcd",{3,2}]
```

abcd

```
>> StringDrop["abcd",0]
```

abcd

StringExpression (~~)

`StringExpression[s_1, s_2, ...]`
: represents a sequence of strings and symbolic string objects *s_i*.

```
>> "a" ~~"b" // FullForm
```

"ab"

StringFreeQ

`StringFreeQ["`*string*`", `*patt*`]`
: returns True if no substring in *string* matches the string expression *patt*, and returns False otherwise.

`StringFreeQ[{‘‘s1’, "s2", ...}, patt]’`
: returns the list of results for each element of string list.

`StringFreeQ[‘‘string’, {p1, p2, ...}]’`
: returns True if no substring matches any of the *pi*.

`StringFreeQ[patt]`
: represents an operator form of StringFreeQ that can be applied to an expression.

```
>> StringFreeQ["mathics", "m" ~~__
   ~~"s"]
```
False

```
>> StringFreeQ["mathics", "a" ~~__
   ~~"m"]
```
True

```
>> StringFreeQ["Mathics", "MA" ,
   IgnoreCase -> True]
```
False

```
>> StringFreeQ[{"g", "a", "laxy", "
   universe", "sun"}, "u"]
```
{True, True, True, False, False}

```
>> StringFreeQ["e" ~~___ ~~"u"] /@
   {"The Sun", "Mercury", "Venus",
   "Earth", "Mars", "Jupiter", "
   Saturn", "Uranus", "Neptune"}
```
{False, False, False, True,
True, True, True, True, False}

```
>> StringFreeQ[{"A", "Galaxy", "Far
   ", "Far", "Away"}, {"F" ~~__ ~~"
   r", "aw" ~~___}, IgnoreCase ->
   True]
```
{True, True, False, False, False}

StringInsert

`StringInsert["`*string*`‘‘,"`*snew*`", `*n*`]`
: yields a string with *snew* inserted starting at position *n* in *string*.

`StringInsert["`*string*`‘‘,"`*snew*`", -`*n*`]`
: inserts a at position *n* from the end of "*string*".

`StringInsert["`*string*`‘‘,"`*snew*`", {`*n_1*`, `*n_2*`, ...}]`
: inserts a copy of *snew* at each position *n_i* in *string*; the *n_i* are taken before any insertion is done.

`StringInsert[{`*s_1*`, `*s_2*`, ...}, "`*snew*`", `*n*`]`
: gives the list of resutls for each of the *s_i*.

```
>> StringInsert["noting", "h", 4]
```
nothing

```
>> StringInsert["note", "d", -1]
```
noted

```
>> StringInsert["here", "t", -5]
```
there

```
>> StringInsert["adac", "he", {1,
   5}]
```
headache

```
>> StringInsert[{"something", "
   sometimes"}, " ", 5]
```
{some thing, some times}

```
>> StringInsert["1234567890123456",
   ".", Range[-16, -4, 3]]
```
1.234.567.890.123.456

StringJoin (<>)

`StringJoin["`*s1*`‘‘,"`*s2*`", ...]`
: returns the concatenation of the strings *s1*, *s2*, .

```
>> StringJoin["a", "b", "c"]
```
abc

```
>> "a" <> "b" <> "c" // InputForm
```
"abc"

`StringJoin` flattens lists out:

```
>> StringJoin[{"a", "b"}] // InputForm
```

"ab"

```
>> Print[StringJoin[{"Hello", " ", {"world"}}, "!"]]
```

Helloworld!

StringLength

`StringLength["string"]`
gives the length of *string*.

```
>> StringLength["abc"]
```

3

`StringLength` is listable:

```
>> StringLength[{"a", "bc"}]
```

{1, 2}

```
>> StringLength[x]
```

Stringexpected.

StringLength[*x*]

StringMatchQ

```
>> StringMatchQ["abc", "abc"]
```

True

```
>> StringMatchQ["abc", "abd"]
```

False

```
>> StringMatchQ["15a94xcZ6", (DigitCharacter | LetterCharacter)..]
```

True

Use StringMatchQ as an operator

```
>> StringMatchQ[LetterCharacter]["a"]
```

True

StringPosition

`StringPosition["string", patt]`
gives a list of starting and ending positions where *patt* matches "*string*".

`StringPosition["string", patt, n]`
returns the first *n* matches only.

`StringPosition["string", {patt1, patt2, ...}, n]`
matches multiple patterns.

`StringPosition[{s1, s2, ...}, patt]`
returns a list of matches for multiple strings.

```
>> StringPosition["123ABCxyABCzzzABCABC", "ABC"]
```

{{4, 6}, {9, 11}, {15, 17}, {18, 20}}

```
>> StringPosition["123ABCxyABCzzzABCABC", "ABC", 2]
```

{{4, 6}, {9, 11}}

`StringPosition` can be useful for searching through text.

```
>> data = Import["ExampleData/EinsteinSzilLetter.txt"];
```

```
>> StringPosition[data, "uranium"]
```

{{299, 305}, {870, 876}, {1 538, 1~~544}, {1 671, 1 677}, {2 300, 2 306}, {2 784, 2 790}, {3 093, 3 099}}

StringQ

`StringQ[expr]`
returns `True` if *expr* is a `String`, or `False` otherwise.

```
>> StringQ["abc"]
```

True

```
>> StringQ[1.5]
```

False

```
>> Select[{"12", 1, 3, 5, "yz", x, y}, StringQ]
```

{12, yz}

StringRepeat

`StringRepeat["string", n]`
: gives *string* repeated *n* times.

`StringRepeat["string", n, max]`
: gives *string* repeated *n* times, but not more than *max* characters.

```
>> StringRepeat["abc", 3]
```
abcabcabc

```
>> StringRepeat["abc", 10, 7]
```
abcabca

StringReplace

`StringReplace["string‘‘,"a"->"b"]`
: replaces each occurrence of *old* with *new* in *string*.

`StringReplace["string", {"s1"->"sp1‘‘, "s2"->"sp2"}]`
: performs multiple replacements of each *si* by the corresponding *spi* in *string*.

`StringReplace["string", srules, n]`
: only performs the first *n* replacements.

`StringReplace[{"string1‘‘,"string2", ...}, srules]`
: performs the replacements specified by *srules* on a list of strings.

StringReplace replaces all occurrences of one substring with another:

```
>> StringReplace["xyxyxyyyxxxyyxy",
    "xy" -> "A"]
```
AAAyyxxAyA

Multiple replacements can be supplied:

```
>> StringReplace["xyzwxyzwxxyzxyzw
   ", {"xyz" -> "A", "w" -> "BCD"}]
```
ABCDABCDxAABCD

Only replace the first 2 occurences:

```
>> StringReplace["xyxyxyyyxxxyyxy",
    "xy" -> "A", 2]
```
AAxyyyxxxyyxy

Also works for multiple rules:

```
>> StringReplace["abba", {"a" -> "A
   ", "b" -> "B"}, 2]
```
ABba

StringReplace acts on lists of strings too:

```
>> StringReplace[{"xyxyxxy", "
   yxyxyxxxyyxy"}, "xy" -> "A"]
```
{AAxA, yAAxxAyA}

StringReplace also can be used as an operator:

```
>> StringReplace["y" -> "ies"]["
   city"]
```
cities

StringRiffle

`StringRiffle[{s1, s2, s3, ...}]`
: returns a new string by concatenating all the *si*, with spaces inserted between them.

`StringRiffle[list, sep]`
: inserts the separator *sep* between all elements in *list*.

`StringRiffle[list, {‘‘left’, "sep", "right"}]’`
: use *left* and *right* as delimiters after concatenation.

```
>> StringRiffle[{"a", "b", "c", "d
   ", "e"}]
```
a b c d e

```
>> StringRiffle[{"a", "b", "c", "d
   ", "e"}, ", "]
```
a, b, c, d, e

```
>> StringRiffle[{"a", "b", "c", "d
   ", "e"}, {"(", " ", ")"}]
```
(a b c d e)

StringSplit

`StringSplit["s"]`
: splits the string *s* at whitespace, discarding the whitespace and returning a list of strings.

`StringSplit["s``,"d"]`
: splits *s* at the delimiter *d*.

`StringSplit[s, {"d1``,"d2", ...}]`
: splits *s* using multiple delimiters.

`StringSplit[{s_1, $s_2, ...}, {"d1``, "d2", ...}]`
: returns a list with the result of applying the function to each element.

```
>> StringSplit["abc,123", ","]
```
{abc, 123}

```
>> StringSplit["abc 123"]
```
{abc, 123}

```
>> StringSplit["abc,123.456", {",", "."}]
```
{abc, 123, 456}

```
>> StringSplit["a b c", RegularExpression[" +"]]
```
{a, b, c}

```
>> StringSplit[{"a b", "c d"}, RegularExpression[" +"]]
```
{{a, b} , {c, d}}

StringTake

`StringTake["string", n]`
: gives the first *n* characters in *string*.

`StringTake["string", -n]`
: gives the last *n* characters in *string*.

`StringTake["string", {n}]`
: gives the *n*th character in *string*.

`StringTake["string", {m, n}]`
: gives characters *m* through *n* in *string*.

`StringTake["string", {m, n, s}]`
: gives characters *m* through *n* in steps of *s*.

```
>> StringTake["abcde", 2]
```
ab

```
>> StringTake["abcde", 0]
```

```
>> StringTake["abcde", -2]
```
de

```
>> StringTake["abcde", {2}]
```
b

```
>> StringTake["abcd", {2,3}]
```
bc

```
>> StringTake["abcdefgh", {1, 5, 2}]
```
ace

StringTake also supports standard sequence specifications

```
>> StringTake["abcdef", All]
```
abcdef

StringTrim

`StringTrim[s]`
: returns a version of *s* with whitespace removed from start and end.

```
>> StringJoin["a", StringTrim[" \tb \n "], "c"]
```
abc

```
>> StringTrim["ababaxababyaabab", RegularExpression["(ab)+"]]
```
axababya

String

`String`
: is the head of strings.

```
>> Head["abc"]
```
String

```
>> "abc"
```
abc

Use `InputForm` to display quotes around strings:

```
>> InputForm["abc"]
```
"abc"

`FullForm` also displays quotes:

```
>> FullForm["abc" + 2]
```
Plus [2, "abc"]

$SystemCharacterEncoding

$SystemCharacterEncoding

ToCharacterCode

`ToCharacterCode["`*string*`"]`
converts the string to a list of character codes (Unicode codepoints).

`ToCharacterCode[{"`*string1*`‘‘,"`*string2*`", ...}]`
converts a list of strings to character codes.

```
>> ToCharacterCode["abc"]
```
{97, 98, 99}

```
>> FromCharacterCode[%]
```
abc

```
>> ToCharacterCode["\[Alpha]\[Beta
]\[Gamma]"]
```
{945, 946, 947}

```
>> ToCharacterCode["ä", "UTF8"]
```
{195, 164}

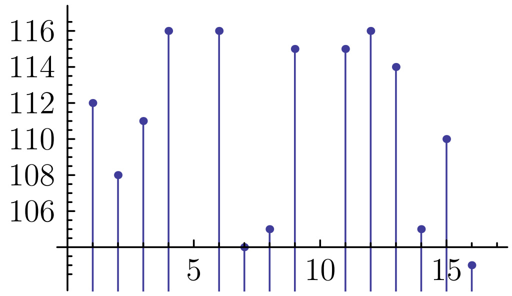
```
>> ToCharacterCode["ä", "ISO8859
-1"]
```
{228}

```
>> ToCharacterCode[{"ab", "c"}]
```
{{97, 98}, {99}}

```
>> ToCharacterCode[{"ab", x}]
```
Stringorlistofstringsexpectedatposition1inToCharacterCode[{ab, x}].

ToCharacterCode [{ab, x}]

```
>> ListPlot[ToCharacterCode["plot
this string"], Filling -> Axis]
```



ToExpression

`ToExpression[`*input*`]`
intepretes a given string as Mathics input.

`ToExpression[`*input*`, `*form*`]`
reads the given input in the specified *form*.

`ToExpression[`*input*`, `*form*`, `*h*`]`
applies the head *h* to the expression before evaluating it.

```
>> ToExpression["1 + 2"]
```
3

```
>> ToExpression["{2, 3, 1}",
InputForm, Max]
```
3

ToLowerCase

`ToLowerCase[`*s*`]`
returns *s* in all lower case.

```
>> ToLowerCase["New York"]
```
new york

ToString

`ToString[`*expr*`]`
returns a string representation of *expr*.

```
>> ToString[2]
```
2

```
>> ToString[2] // InputForm
```
"2"

```
>> ToString[a+b]
```
a + b

```
>> "U" <> 2
```
Stringexpected.

U<>2

```
>> "U" <> ToString[2]
```
U2

ToUpperCase

`ToUpperCase[s]`
: returns *s* in all upper case.

```
>> ToUpperCase["New York"]
```
NEW YORK

Transliterate

`Transliterate[s]`
: transliterates a text in some script into an ASCII string.

The following examples were taken from # https://en.wikipedia.org/wiki/Iliad, # https://en.wikipedia.org/wiki/Russian_language, and # https://en.wikipedia.org/wiki/Hiragana

UpperCaseQ

`UpperCaseQ[s]`
: returns True if *s* consists wholly of upper case characters.

```
>> UpperCaseQ["ABC"]
```
True

An empty string returns True.

```
>> UpperCaseQ[""]
```
True

Whitespace

`Whitespace`
: represents a sequence of whitespace characters.

```
>> StringMatchQ["\r \n", Whitespace
   ]
```
True

```
>> StringSplit["a \n b \r\n c d",
   Whitespace]
```
$\{a, b, c, d\}$

```
>> StringReplace[" this has leading
    and trailing whitespace \n ", (
   StartOfString ~~Whitespace)| (
   Whitespace ~~EndOfString)-> ""]
   <> " removed" // FullForm
```
"this has leading and trailing whitespace removed"

WhitespaceCharacter

`WhitespaceCharacter`
: represents a single whitespace character.

```
>> StringMatchQ["\n",
   WhitespaceCharacter]
```
True

```
>> StringSplit["a\nb\r\nc\rd",
   WhitespaceCharacter]
```
$\{a, b, c, d\}$

For sequences of whitespace characters use `Whitespace`:

```
>> StringMatchQ[" \n",
   WhitespaceCharacter]
```
False

```
>> StringMatchQ[" \n", Whitespace]
```
True

WordBoundary

`WordBoundary`
: represents the boundary between words.

```
>> StringReplace["apple banana
   orange artichoke", "e" ~~
   WordBoundary -> "E"]
```
applE banana orangE artichokE

WordCharacter

`WordCharacter`
: represents a single letter or digit character.

```
>> StringMatchQ[#, WordCharacter]
   &/@ {"1", "a", "A", ",", " "}
```

{True, True, True, False, False}

Test whether a string is alphanumeric:

```
>> StringMatchQ["abc123DEF",
   WordCharacter..]
```

True

```
>> StringMatchQ["$b;123",
   WordCharacter..]
```

False

XXXIII. Optimization

Contents

Maximize

`Maximize[`f`, `x`]`
compute the maximum of f respect x that change between a and b

```
>> Maximize[-2 x^2 - 3 x + 5, x]
```

$$\left\{\left\{\frac{49}{8},\left\{x->-\frac{3}{4}\right\}\right\}\right\}$$

#» Maximize[1 - (x y - 3)^2, {x, y}] = {{1, {x -> 3, y -> 1}}}
#» Maximize[{x - 2 y, x^2 + y^2 <= 1}, {x, y}] = {{Sqrt[5], {x -> Sqrt[5] / 5, y -> -2 Sqrt[5] / 5}}}

Minimize

`Minimize[`f`, `x`]`
compute the minimum of f respect x that change between a and b

```
>> Minimize[2 x^2 - 3 x + 5, x]
```

$$\left\{\left\{\frac{31}{8},\left\{x->\frac{3}{4}\right\}\right\}\right\}$$

#» Minimize[(x y - 3)^2 + 1, {x, y}] = {{1, {x -> 3, y -> 1}}}
#» Minimize[{x - 2 y, x^2 + y^2 <= 1}, {x, y}] = {{-Sqrt[5], {x -> -Sqrt[5] / 5, y -> 2 Sqrt[5] / 5}}}

XXXIV. Image[] and image related functions.

Note that you (currently) need scikit-image installed in order for this module to work.

Contents

Binarize

`Binarize[`*image*`]`
: gives a binarized version of *image*, in which each pixel is either 0 or 1.

`Binarize[`*image*`, `*t*`]`
: map values $x > t$ to 1, and values $x <= t$ to 0.

`Binarize[`*image*`, {`*t1*`, `*t2*`}]`
: map $t1 < x < t2$ to 1, and all other values to 0.

```
>> img = Import["ExampleData/lena.tif"];
```

```
>> Binarize[img]
```
−Image−

```
>> Binarize[img, 0.7]
```
−Image−

```
>> Binarize[img, {0.2, 0.6}]
```
−Image−

BinaryImageQ

`BinaryImageQ[$image]`
: returns True if the pixels of $image are binary bit values, and False otherwise.

```
>> img = Import["ExampleData/lena.tif"];
```

```
>> BinaryImageQ[img]
```
False

```
>> BinaryImageQ[Binarize[img]]
```
True

Blur

`Blur[`*image*`]`
  gives a blurred version of *image*.
`Blur[`*image*`, `*r*`]`
  blurs *image* with a kernel of size *r*.

```
>> lena = Import["ExampleData/lena.tif"];
```

```
>> Blur[lena]
```
-Image-

```
>> Blur[lena, 5]
```
-Image-

BoxMatrix

`BoxMatrix[$s]`
  Gives a box shaped kernel of size 2 *s* + 1.

```
>> BoxMatrix[3]
```
{{1,1,1,1,1,1,1}, {1,1,1,1,1,1,1}, {1,1,1,1,1,1,1}, {1,1,1,1,1,1,1}, {1,1,1,1,1,1,1}, {1,1,1,1,1,1,1}, {1,1,1,1,1,1,1}}

Closing

`Closing[`*image*`, `*ker*`]`
  Gives the morphological closing of *image* with respect to structuring element *ker*.

```
>> ein = Import["ExampleData/Einstein.jpg"];
```

```
>> Closing[ein, 2.5]
```
-Image-

ColorCombine

`ColorCombine[`*channels*`, `*colorspace*`]`
  Gives an image with *colorspace* and the respective components described by the given channels.

```
>> ColorCombine[{{{1, 0}, {0, 0.75}}, {{0, 1}, {0, 0.25}}, {{0, 0}, {1, 0.5}}}, "RGB"]
```
-Image-

ColorConvert

`ColorConvert[`*c*`, `*colspace*`]`
  returns the representation of *c* in the color space *colspace*. *c* may be a color or an image.

Valid values for *colspace* are:
CMYK: convert to CMYKColor Grayscale: convert to GrayLevel HSB: convert to Hue LAB: concert to LABColor LCH: convert to LCHColor LUV: convert to LUVColor RGB: convert to RGBColor XYZ: convert to XYZColor

ColorNegate

`ColorNegate[`*image*`]`
  Gives a version of *image* with all colors negated.

ColorQuantize

`ColorQuantize[`*image*`, `*n*`]`
  gives a version of *image* using only *n* colors.

```
>> img = Import["ExampleData/lena.tif"];
```

```
>> ColorQuantize[img, 6]
```
-Image-

ColorSeparate

`ColorSeparate[`*image*`]`
  Gives each channel of *image* as a separate grayscale image.

Colorize

`Colorize[`*values*`]`
: returns an image where each number in the rectangular matrix *values* is a pixel and each occurence of the same number is displayed in the same unique color, which is different from the colors of all non-identical numbers.

`Colorize[`*image*`]`
: gives a colorized version of *image*.

```
>> Colorize[{{1.3, 2.1, 1.5}, {1.3,
    1.3, 2.1}, {1.3, 2.1, 1.5}}]
```

−Image−

```
>> Colorize[{{1, 2}, {2, 2}, {2,
   3}}, ColorFunction -> (Blend[{
   White, Blue}, #]&)]
```

−Image−

DiamondMatrix

`DiamondMatrix[$s]`
: Gives a diamond shaped kernel of size 2 $s + 1$.

```
>> DiamondMatrix[3]
```

{{0,0,0,1,0,0,0}, {0,0,1,1,1,0,0}, {0,1,1,1,1,1,0}, {1,1,1,1,1,1,1}, {0,1,1,1,1,1,0}, {0,0,1,1,1,0,0}, {0,0,0,1,0,0,0}}

Dilation

`Dilation[`*image*`, `*ker*`]`
: Gives the morphological dilation of *image* with respect to structuring element *ker*.

```
>> ein = Import["ExampleData/
   Einstein.jpg"];
```

```
>> Dilation[ein, 2.5]
```

−Image−

DiskMatrix

`DiskMatrix[$s]`
: Gives a disk shaped kernel of size 2 $s + 1$.

```
>> DiskMatrix[3]
```

{{0,0,1,1,1,0,0}, {0,1,1,1,1,1,0}, {1,1,1,1,1,1,1}, {1,1,1,1,1,1,1}, {1,1,1,1,1,1,1}, {0,1,1,1,1,1,0}, {0,0,1,1,1,0,0}}

DominantColors

`DominantColors[`*image*`]`
: gives a list of colors which are dominant in the given image.

`DominantColors[`*image*`, `*n*`]`
: returns at most *n* colors.

`DominantColors[`*image*`, `*n*`, `*prop*`]`
: returns the given property *prop*, which may be “Color” (return RGB colors), “LABColor” (return LAB colors), “Count” (return the number of pixels a dominant color covers), “Coverage” (return the fraction of the image a dominant color covers), or “CoverageImage” (return a black and white image indicating with white the parts that are covered by a dominant color).

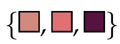
The option “ColorCoverage” specifies the minimum amount of coverage needed to include a dominant color in the result.
The option “MinColorDistance” specifies the distance (in LAB color space) up to which colors are merged and thus regarded as belonging to the same dominant color.

```
>> img = Import["ExampleData/lena.
   tif"]
```

−Image−

```
>> DominantColors[img]
```

```
>> DominantColors[img, 3]
```



```
>> DominantColors[img, 3, "Coverage
   "]
```

$$\left\{\frac{28\,579}{131\,072}, \frac{751}{4\,096}, \frac{23\,841}{131\,072}\right\}$$

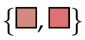
```
>> DominantColors[img, 3, "
   CoverageImage"]
```

{*−Image−*, *− Image−*, *− Image−*}

```
>> DominantColors[img, 3, "Count"]
```

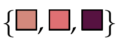
{57 158, 48 064, 47 682}

```
>> DominantColors[img, 2, "LABColor
   "]
```



```
>> DominantColors[img,
   MinColorDistance -> 0.5]
```

```
>> DominantColors[img,
   ColorCoverage -> 0.15]
```



EdgeDetect

`EdgeDetect[`*image*`]`
: returns an image showing the edges in *image*.

```
>> lena = Import["ExampleData/lena.
   tif"];
```

```
>> EdgeDetect[lena]
```

−Image−

```
>> EdgeDetect[lena, 5]
```

−Image−

```
>> EdgeDetect[lena, 4, 0.5]
```

−Image−

Erosion

`Erosion[`*image*`,` *ker*`]`
: Gives the morphological erosion of *image* with respect to structuring element *ker*.

```
>> ein = Import["ExampleData/
   Einstein.jpg"];
```

```
>> Erosion[ein, 2.5]
```

−Image−

GaussianFilter

`GaussianFilter[`*image*`,` *r*`]`
: blurs *image* using a Gaussian blur filter of radius *r*.

```
>> lena = Import["ExampleData/lena.
   tif"];
```

```
>> GaussianFilter[lena, 2.5]
```

−Image−

ImageAdd

`ImageAdd[`*image*`,` *expr_1*`,` *expr_2*`, ...]`
: adds all *expr_i* to *image* where each *expr_i* must be an image or a real number.

```
>> i = Image[{{0, 0.5, 0.2, 0.1,
   0.9}, {1.0, 0.1, 0.3, 0.8,
   0.6}}];
```

```
>> ImageAdd[i, 0.5]
```

−Image−

```
>> ImageAdd[i, i]
```

−Image−

```
>> ein = Import["ExampleData/
   Einstein.jpg"];
```

```
>> noise = RandomImage[{-0.1, 0.1},
    ImageDimensions[ein]];
```

```
>> ImageAdd[noise, ein]
```

−Image−

```
>> lena = Import["ExampleData/lena.
   tif"];
```

```
>> noise = RandomImage[{-0.2, 0.2},
    ImageDimensions[lena],
   ColorSpace -> "RGB"];
```

```
>> ImageAdd[noise, lena]
```

−Image−

ImageAdjust

`ImageAdjust[`*image*`]`
: adjusts the levels in *image*.

`ImageAdjust[`*image*`, `*c*`]`
: adjusts the contrast in *image* by *c*.

`ImageAdjust[`*image*`, {`*c*`, `*b*`}]`
: adjusts the contrast *c*, and brightness *b* in *image*.

`ImageAdjust[`*image*`, {`*c*`, `*b*`, `*g*`}]`
: adjusts the contrast *c*, brightness *b*, and gamma *g* in *image*.

```
>> lena = Import["ExampleData/lena.
   tif"];
```

```
>> ImageAdjust[lena]
```

−Image−

ImageAspectRatio

`ImageAspectRatio[`*image*`]`
: gives the aspect ratio of *image*.

```
>> img = Import["ExampleData/lena.
   tif"];
```

```
>> ImageAspectRatio[img]
```

1

```
>> ImageAspectRatio[Image[{{0, 1},
   {1, 0}, {1, 1}}]]
```

$\frac{3}{2}$

Image

ImageBox

ImageChannels

`ImageChannels[`*image*`]`
: gives the number of channels in *image*.

```
>> ImageChannels[Image[{{0, 1}, {1,
    0}}]]
```

1

```
>> img = Import["ExampleData/lena.
   tif"];
```

```
>> ImageChannels[img]
```

3

ImageColorSpace

`ImageColorSpace[`*image*`]`
: gives *image*'s color space, e.g. "RGB" or "CMYK".

```
>> img = Import["ExampleData/lena.
   tif"];
```

```
>> ImageColorSpace[img]
```

RGB

ImageConvolve

`ImageConvolve[`*image*`, `*kernel*`]`
: Computes the convolution of *image* using *kernel*.

```
>> img = Import["ExampleData/lena.
   tif"];
```

```
>> ImageConvolve[img, DiamondMatrix
   [5] / 61]
```

−Image−

```
>> ImageConvolve[img, DiskMatrix[5]
    / 97]
```

−Image−

```
>> ImageConvolve[img, BoxMatrix[5]
   / 121]
```

−Image−

ImageData

`ImageData[`*image*`]`
: gives a list of all color values of *image* as a matrix.

`ImageData[`*image*`, `*stype*`]`
: gives a list of color values in type *stype*.

```
>> img = Image[{{0.2, 0.4}, {0.9, 0.6}, {0.5, 0.8}}];
```

```
>> ImageData[img]
```
$\{\{0.2, 0.4\}, \{0.9, 0.6\}, \{0.5, 0.8\}\}$

```
>> ImageData[img, "Byte"]
```
$\{\{51, 102\}, \{229, 153\}, \{127, 204\}\}$

```
>> ImageData[Image[{{0, 1}, {1, 0}, {1, 1}}], "Bit"]
```
$\{\{0, 1\}, \{1, 0\}, \{1, 1\}\}$

ImageDimensions

`ImageDimensions[`*image*`]`
: Returns the dimensions of *image* in pixels.

```
>> lena = Import["ExampleData/lena.tif"];
```

```
>> ImageDimensions[lena]
```
$\{512, 512\}$

```
>> ImageDimensions[RandomImage[1, {50, 70}]]
```
$\{50, 70\}$

ImageExport

ImageImport

```
>> Import["ExampleData/Einstein.jpg"]
```
−Image−

```
>> Import["ExampleData/MadTeaParty.gif"]
```
−Image−

```
>> Import["ExampleData/moon.tif"]
```
−Image−

ImageMultiply

`ImageMultiply[`*image*`, `*expr_1*`, `*expr_2*`, ...]`
: multiplies all *expr_i* with *image* where each *expr_i* must be an image or a real number.

```
>> i = Image[{{0, 0.5, 0.2, 0.1, 0.9}, {1.0, 0.1, 0.3, 0.8, 0.6}}];
```

```
>> ImageMultiply[i, 0.2]
```
−Image−

```
>> ImageMultiply[i, i]
```
−Image−

```
>> ein = Import["ExampleData/Einstein.jpg"];
```

```
>> noise = RandomImage[{0.7, 1.3}, ImageDimensions[ein]];
```

```
>> ImageMultiply[noise, ein]
```
−Image−

ImagePartition

`ImagePartition[`*image*`, `*s*`]`
: Partitions an image into an array of *s* x *s* pixel subimages.

`ImagePartition[`*image*`, {`*w*`, `*h*`}]`
: Partitions an image into an array of *w* x *h* pixel subimages.

```
>> lena = Import["ExampleData/lena.tif"];
```

```
>> ImageDimensions[lena]
```
$\{512, 512\}$

```
>> ImagePartition[lena, 256]
```
{{*−Image−*, *−Image−*}, {*−Image−*, *−Image−*}}

```
>> ImagePartition[lena, {512, 128}]
```

{{−*Image*−}, {−*Image*−}, {−*Image*−}, {−*Image*−}}

ImageQ

`ImageQ[Image[$pixels]]`
returns True if $pixels has dimensions from which an Image can be constructed, and False otherwise.

```
>> ImageQ[Image[{{0, 1}, {1, 0}}]]
```

True

```
>> ImageQ[Image[{{{0, 0, 0}, {0, 1, 0}}, {{0, 1, 0}, {0, 1, 1}}}]]
```

True

```
>> ImageQ[Image[{{{0, 0, 0}, {0, 1}}, {{0, 1, 0}, {0, 1, 1}}}]]
```

False

```
>> ImageQ[Image[{1, 0, 1}]]
```

False

```
>> ImageQ["abc"]
```

False

ImageReflect

`ImageReflect[`*image*`]`
Flips *image* top to bottom.

`ImageReflect[`*image*`, `*side*`]`
Flips *image* so that *side* is interchanged with its opposite.

`ImageReflect[`*image*`, `*side_1* `->` *side_2*`]`
Flips *image* so that *side_1* is interchanged with *side_2*.

```
>> ein = Import["ExampleData/Einstein.jpg"];
```

```
>> ImageReflect[ein]
```

−*Image*−

```
>> ImageReflect[ein, Left]
```

−*Image*−

```
>> ImageReflect[ein, Left -> Top]
```

−*Image*−

ImageResize

`ImageResize[`*image*`, `*width*`]`
`ImageResize[`*image*`, {`*width*`, `*height*`}]`

```
>> ein = Import["ExampleData/Einstein.jpg"];
```

```
>> ImageDimensions[ein]
```

$\{615, 768\}$

```
>> ImageResize[ein, {400, 600}]
```

−*Image*−

```
>> ImageResize[ein, 256]
```

−*Image*−

```
>> ImageDimensions[%]
```

$\{256, 320\}$

The default sampling method is Bicubic

```
>> ImageResize[ein, 256, Resampling -> "Bicubic"]
```

−*Image*−

```
>> ImageResize[ein, 256, Resampling -> "Nearest"]
```

−*Image*−

```
>> ImageResize[ein, 256, Resampling -> "Gaussian"]
```

−*Image*−

ImageRotate

`ImageRotate[`*image*`]`
Rotates *image* 90 degrees counterclockwise.

`ImageRotate[`*image*`, `*theta*`]`
Rotates *image* by a given angle *theta*

```
>> ein = Import["ExampleData/Einstein.jpg"];
```

```
>> ImageRotate[ein]
```

−*Image*−

```
>> ImageRotate[ein, 45 Degree]
```

−*Image*−

```
>> ImageRotate[ein, Pi / 2]
```
—Image—

ImageSubtract

`ImageSubtract[`*image*`, `*expr_1*`, `*expr_2*`, ...]`
: subtracts all *expr_i* from *image* where each *expr_i* must be an image or a real number.

```
>> i = Image[{{0, 0.5, 0.2, 0.1, 0.9}, {1.0, 0.1, 0.3, 0.8, 0.6}}];
```

```
>> ImageSubtract[i, 0.2]
```
—Image—

```
>> ImageSubtract[i, i]
```
—Image—

ImageTake

`ImageTake[`*image*`, `*n*`]`
: gives the first *n* rows of *image*.

`ImageTake[`*image*`, -`*n*`]`
: gives the last *n* rows of *image*.

`ImageTake[`*image*`, {`*r1*`, `*r2*`}]`
: gives rows *r1, ..., r2* of *image*.

`ImageTake[`*image*`, {`*r1*`, `*r2*`}, {`*c1*`, `*c2*`}]`
: gives a cropped version of *image*.

ImageType

`ImageType[`*image*`]`
: gives the interval storage type of *image*, e.g. "Real", "Bit32", or "Bit".

```
>> img = Import["ExampleData/lena.tif"];
```

```
>> ImageType[img]
```
Byte

```
>> ImageType[Image[{{0, 1}, {1, 0}}]]
```
Real

```
>> ImageType[Binarize[img]]
```
Bit

MaxFilter

`MaxFilter[`*image*`, `*r*`]`
: gives *image* with a maximum filter of radius *r* applied on it. This always picks the largest value in the filter's area.

```
>> lena = Import["ExampleData/lena.tif"];
```

```
>> MaxFilter[lena, 5]
```
—Image—

MedianFilter

`MedianFilter[`*image*`, `*r*`]`
: gives *image* with a median filter of radius *r* applied on it. This always picks the median value in the filter's area.

```
>> lena = Import["ExampleData/lena.tif"];
```

```
>> MedianFilter[lena, 5]
```
—Image—

MinFilter

`MinFilter[`*image*`, `*r*`]`
: gives *image* with a minimum filter of radius *r* applied on it. This always picks the smallest value in the filter's area.

```
>> lena = Import["ExampleData/lena.tif"];
```

```
>> MinFilter[lena, 5]
```
—Image—

MorphologicalComponents

Opening

`Opening[`*image*`, `*ker*`]`
: Gives the morphological opening of *image* with respect to structuring element *ker*.

```
>> ein = Import["ExampleData/Einstein.jpg"];
```

```
>> Opening[ein, 2.5]
```
−Image−

PillowImageFilter

PixelValue

`PixelValue[`*image*`, {`*x*`, `*y*`}]`
: gives the value of the pixel at position {*x*, *y*} in *image*.

```
>> lena = Import["ExampleData/lena.tif"];
```

```
>> PixelValue[lena, {1, 1}]
```
$\{0.321569, 0.0862745, 0.223529\}$

PixelValuePositions

`PixelValuePositions[`*image*`, `*val*`]`
: gives the positions of all pixels in *image* that have value *val*.

```
>> PixelValuePositions[Image[{{0, 1}, {1, 0}, {1, 1}}], 1]
```
$\{\{1,1\}, \{1,2\}, \{2,1\}, \{2,3\}\}$

```
>> PixelValuePositions[Image[{{0.2, 0.4}, {0.9, 0.6}, {0.3, 0.8}}], 0.5, 0.15]
```
$\{\{2,2\}, \{2,3\}\}$

```
>> img = Import["ExampleData/lena.tif"];
```

```
>> PixelValuePositions[img, 3 / 255, 0.5 / 255]
```
$\{\{180,192,2\}, \{181,192,2\}, \{181,193,2\}, \{188,204,2\}, \{265,314,2\}, \{364,77,2\}, \{365,72,2\}, \{365,73,2\}, \{365,77,2\}, \{366,70,2\}, \{367,65,2\}\}$

```
>> PixelValue[img, {180, 192}]
```
$\{0.25098, 0.0117647, 0.215686\}$

RandomImage

`RandomImage[`*max*`]`
: creates an image of random pixels with values 0 to *max*.

`RandomImage[{`*min*`, `*max*`}]`
: creates an image of random pixels with values *min* to *max*.

`RandomImage[..., `*size*`]`
: creates an image of the given *size*.

```
>> RandomImage[1, {100, 100}]
```
−Image−

Sharpen

`Sharpen[`*image*`]`
: gives a sharpened version of *image*.

`Sharpen[`*image*`, `*r*`]`
: sharpens *image* with a kernel of size *r*.

```
>> lena = Import["ExampleData/lena.tif"];
```

```
>> Sharpen[lena]
```
−Image−

```
>> Sharpen[lena, 5]
```
−Image−

TextRecognize

`TextRecognize[{`*image*`}]`
: Recognizes text in *image* and returns it as string.

Threshold

`Threshold[`*image*`]`
: gives a value suitable for binarizing *image*.

The option "Method" may be "Cluster" (use Otsu's threshold), "Median", or "Mean".

```
>> img = Import["ExampleData/lena.
   tif"];
```

```
>> Threshold[img]
```

0.456739

```
>> Binarize[img, %]
```

−Image−

```
>> Threshold[img, Method -> "Mean"]
```

0.486458

```
>> Threshold[img, Method -> "Median
   "]
```

0.504726

WordCloud

`WordCloud[{`*word1*`, `*word2*`, ...}]`
: Gives a word cloud with the given list of words.

`WordCloud[{`*weight1*` -> `*word1*`, `*weight2*` -> `*word2*`, ...}]`
: Gives a word cloud with the words weighted using the given weights.

`WordCloud[{`*weight1*`, `*weight2*`, ...} -> {`*word1*`, `*word2*`, ...}]`
: Also gives a word cloud with the words weighted using the given weights.

`WordCloud[{{`*word1*`, `*weight1*`}, {`*word2*`, `*weight2*`}, ...}]`
: Gives a word cloud with the words weighted using the given weights.

```
>> WordCloud[StringSplit[Import["
   ExampleData/EinsteinSzilLetter.
   txt"]]]
```

−Image−

```
>> WordCloud[Range[50] -> ToString
   /@ Range[50]]
```

−Image−

XXXV. Special Functions

Contents

AiryAi

`AiryAi[`x`]`
returns the Airy function Ai(x).

Exact values:

```
>> AiryAi[0]
```

$$\frac{3^{\frac{1}{3}}}{3\text{Gamma}\left[\frac{2}{3}\right]}$$

`AiryAi` can be evaluated numerically:

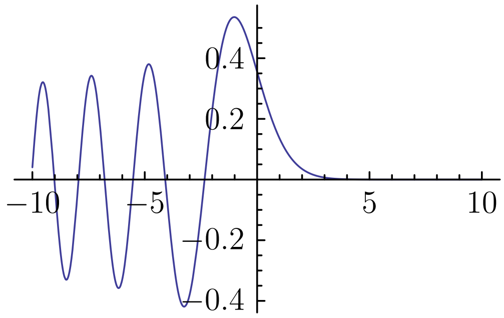
```
>> AiryAi[0.5]
```

0.231694

```
>> AiryAi[0.5 + I]
```

$0.157118 - 0.24104I$

```
>> Plot[AiryAi[x], {x, -10, 10}]
```



AiryAiPrime

`AiryAiPrime[`x`]`
returns the derivative of the Airy function `AiryAi[`x`]`.

Exact values:

```
>> AiryAiPrime[0]
```

$$-\frac{3^{\frac{2}{3}}}{3\text{Gamma}\left[\frac{1}{3}\right]}$$

Numeric evaluation:

```
>> AiryAiPrime[0.5]
```
-0.224911

AiryAiZero

`AiryAiZero[`k`]`
: returns the kth zero of the Airy function $\mathrm{Ai}(z)$.

```
>> N[AiryAiZero[1]]
```
-2.33811

AiryBi

`AiryBi[`x`]`
: returns the Airy function of the second kind $\mathrm{Bi}(x)$.

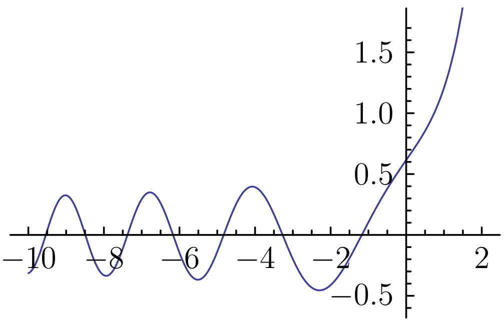
Exact values:

```
>> AiryBi[0]
```
$$\frac{3^{\frac{5}{6}}}{3\mathrm{Gamma}\left[\frac{2}{3}\right]}$$

Numeric evaluation:

```
>> AiryBi[0.5]
```
0.854277

```
>> AiryBi[0.5 + I]
```
$0.688145 + 0.370815I$

```
>> Plot[AiryBi[x], {x, -10, 2}]
```


AiryBiPrime

`AiryBiPrime[`x`]`
: returns the derivative of the Airy function of the second kind `AiryBi[`x`]`.

Exact values:

```
>> AiryBiPrime[0]
```
$$\frac{3^{\frac{1}{6}}}{\mathrm{Gamma}\left[\frac{1}{3}\right]}$$

Numeric evaluation:

```
>> AiryBiPrime[0.5]
```
0.544573

AiryBiZero

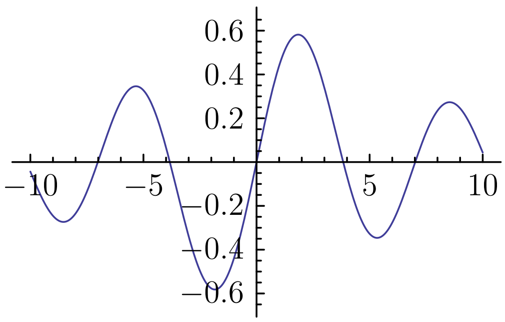
`AiryBiZero[`k`]`
: returns the kth zero of the Airy function $\mathrm{Bi}(z)$.

```
>> N[AiryBiZero[1]]
```
-1.17371

AngerJ

`AngerJ[`n`, `z`]`
: returns the Anger function J_n(z).

```
>> AngerJ[1.5, 3.5]
```
0.294479

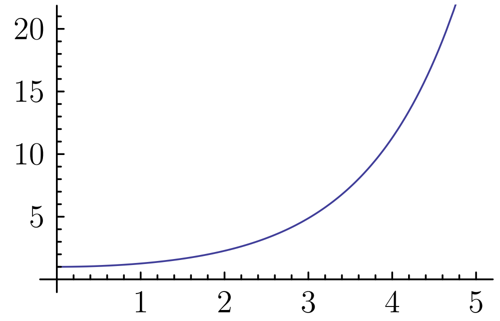
```
>> Plot[AngerJ[1, x], {x, -10, 10}]
```


BesselI

`BesselI[n, z]`
: returns the modified Bessel function of the first kind I_*n*(*z*).

```
>> BesselI[1.5, 4]
8.17263
```

```
>> Plot[BesselI[0, x], {x, 0, 5}]
```

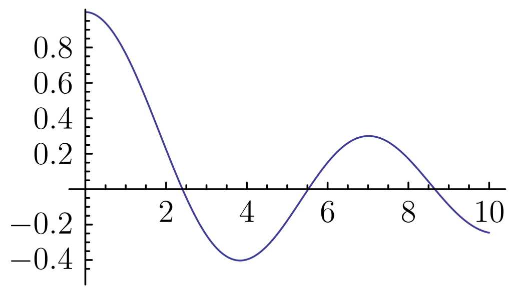


BesselJ

`BesselJ[n, z]`
: returns the Bessel function of the first kind J_*n*(*z*).

```
>> BesselJ[0, 5.2]
-0.11029
```

```
>> Plot[BesselJ[0, x], {x, 0, 10}]
```



BesselJZero

`BesselJZero[n, k]`
: returns the *k*th zero of the Bessel function of the first kind J_*n*(*z*).

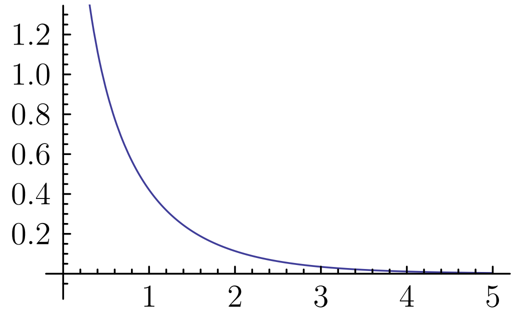
```
>> N[BesselJZero[0, 1]]
2.40483
```

BesselK

`BesselK[n, z]`
: returns the modified Bessel function of the second kind K_*n*(*z*).

```
>> BesselK[1.5, 4]
0.014347
```

```
>> Plot[BesselK[0, x], {x, 0, 5}]
```

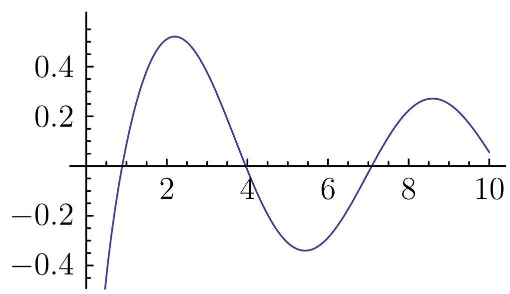


BesselY

`BesselY[n, z]`
: returns the Bessel function of the second kind Y_*n*(*z*).

```
>> BesselY[1.5, 4]
0.367112
```

```
>> Plot[BesselY[0, x], {x, 0, 10}]
```



BesselYZero

`BesselYZero[n, k]`
: returns the *k*th zero of the Bessel function of the second kind Y_*n*(*z*).

```
>> N[BesselYZero[0, 1]]
0.893577
```

ChebyshevT

`ChebyshevT[`*n*`, `*x*`]`
returns the Chebyshev polynomial of the first kind T_*n*(*x*).

>> `ChebyshevT[8, x]`
$1 - 32x^2 + 160x^4 - 256x^6 + 128x^8$

>> `ChebyshevT[1 - I, 0.5]`
$0.800143 + 1.08198I$

ChebyshevU

`ChebyshevU[`*n*`, `*x*`]`
returns the Chebyshev polynomial of the second kind U_*n*(*x*).

>> `ChebyshevU[8, x]`
$1 - 40x^2 + 240x^4 - 448x^6 + 256x^8$

>> `ChebyshevU[1 - I, 0.5]`
$1.60029 + 0.721322I$

Erf

`Erf[`*z*`]`
returns the error function of *z*.
`Erf[`*z0*`, `*z1*`]`
returns the result of `Erf[`*z1*`] - Erf[`*z0*`]`.

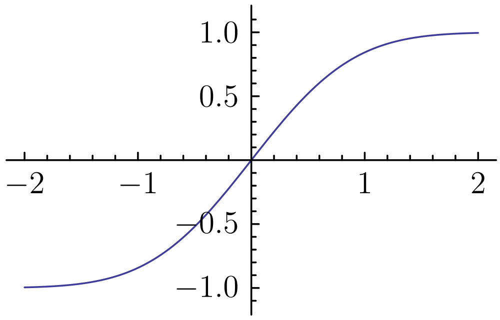
`Erf[`*x*`]` is an odd function:
>> `Erf[-x]`
−Erf[*x*]

>> `Erf[1.0]`
0.842701

>> `Erf[0]`
0

>> `{Erf[0, x], Erf[x, 0]}`
{Erf[*x*], − Erf[*x*]}

>> `Plot[Erf[x], {x, -2, 2}]`



Erfc

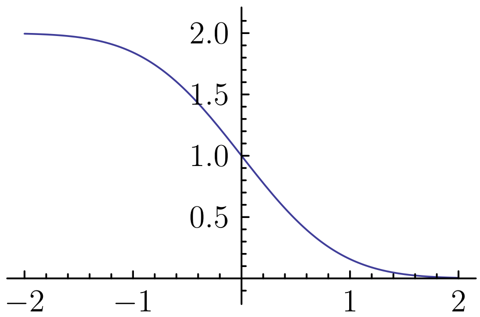
`Erfc[`*z*`]`
returns the complementary error function of *z*.

>> `Erfc[-x] / 2`
$\frac{2 - \mathrm{Erfc}[x]}{2}$

>> `Erfc[1.0]`
0.157299

>> `Erfc[0]`
1

>> `Plot[Erfc[x], {x, -2, 2}]`



ExpIntegralE

`ExpIntegralE[`*n*`, `*z*`]`
returns the exponential integral function $E_n(z)$.

>> `ExpIntegralE[2.0, 2.0]`
0.0375343

ExpIntegralEi

`ExpIntegralEi[z]`
: returns the exponential integral function $Ei(z)$.

```
>> ExpIntegralEi[2.0]
```
4.95423

FresnelC

`FresnelC[z]`
: is the Fresnel C integral $C(z)$.

```
>> FresnelC[{0, Infinity}]
```
$\left\{0, \frac{1}{2}\right\}$

```
>> Integrate[Cos[x^2 Pi/2], {x, 0, z}]
```
$$\frac{\text{FresnelC}\,[z]\,\text{Gamma}\left[\frac{1}{4}\right]}{4\text{Gamma}\left[\frac{5}{4}\right]}$$

FresnelS

`FresnelS[z]`
: is the Fresnel S integral $S(z)$.

```
>> FresnelS[{0, Infinity}]
```
$\left\{0, \frac{1}{2}\right\}$

```
>> Integrate[Sin[x^2 Pi/2], {x, 0, z}]
```
$$\frac{3\text{FresnelS}\,[z]\,\text{Gamma}\left[\frac{3}{4}\right]}{4\text{Gamma}\left[\frac{7}{4}\right]}$$

GegenbauerC

`GegenbauerC[n, m, x]`
: returns the Gegenbauer polynomial C_n^(m)(x).

```
>> GegenbauerC[6, 1, x]
```
$-1 + 24x^2 - 80x^4 + 64x^6$

```
>> GegenbauerC[4 - I, 1 + 2 I, 0.7]
```
$-3.2621 - 24.9739I$

HankelH1

`HankelH1[n, z]`
: returns the Hankel function of the first kind H_n^1 (z).

```
>> HankelH1[1.5, 4]
```
$0.185286 + 0.367112I$

HankelH2

`HankelH2[n, z]`
: returns the Hankel function of the second kind H_n^2 (z).

```
>> HankelH2[1.5, 4]
```
$0.185286 - 0.367112I$

HermiteH

`HermiteH[n, x]`
: returns the Hermite polynomial H_n(x).

```
>> HermiteH[8, x]
```
$1\,680 - 13\,440x^2 + 13\,440x^4 - 3\,584x^6 + 256x^8$

```
>> HermiteH[3, 1 + I]
```
$-28 + 4I$

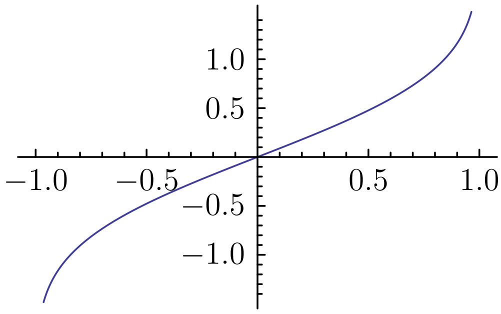
```
>> HermiteH[4.2, 2]
```
77.5291

InverseErf

`InverseErf[z]`
: returns the inverse error function of z.

```
>> InverseErf /@ {-1, 0, 1}
```
$\{-\infty, 0, \infty\}$

>> `Plot[InverseErf[x], {x, -1, 1}]`



`InverseErf[z]` only returns numeric values for `-1 <= z <= 1`:

>> `InverseErf /@ {0.9, 1.0, 1.1}`

{1.16309, ∞, InverseErf[1.1]}

InverseErfc

`InverseErfc[z]`
returns the inverse complementary error function of z.

>> `InverseErfc /@ {0, 1, 2}`

$\{\infty, 0, -\infty\}$

JacobiP

`JacobiP[n, a, b, x]`
returns the Jacobi polynomial P_n^(a,b)(x).

>> `JacobiP[1, a, b, z]`

$$\frac{a}{2} - \frac{b}{2} + z\left(1 + \frac{a}{2} + \frac{b}{2}\right)$$

>> `JacobiP[3.5 + I, 3, 2, 4 - I]`

1410.02 + 5797.3I

KelvinBei

`KelvinBei[z]`
returns the Kelvin function bei(z).
`KelvinBei[n, z]`
returns the Kelvin function bei_n(z).

>> `KelvinBei[0.5]`

0.0624932

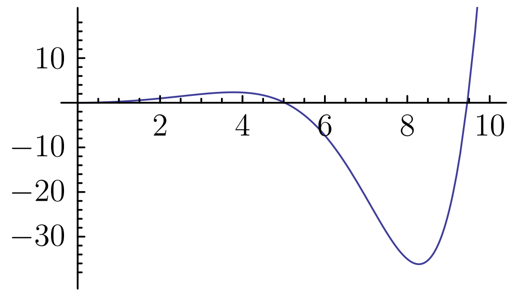
>> `KelvinBei[1.5 + I]`

0.326323 + 0.755606I

>> `KelvinBei[0.5, 0.25]`

0.370153

>> `Plot[KelvinBei[x], {x, 0, 10}]`



KelvinBer

`KelvinBer[z]`
returns the Kelvin function ber(z).
`KelvinBer[n, z]`
returns the Kelvin function ber_n(z).

>> `KelvinBer[0.5]`

0.999023

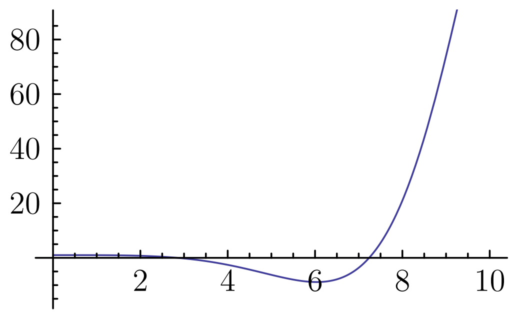
>> `KelvinBer[1.5 + I]`

1.1162 − 0.117944I

>> `KelvinBer[0.5, 0.25]`

0.148824

>> `Plot[KelvinBer[x], {x, 0, 10}]`



KelvinKei

`KelvinKei[z]`
returns the Kelvin function kei(z).
`KelvinKei[n, z]`
returns the Kelvin function kei_n(z).

>> `KelvinKei[0.5]`

-0.671582

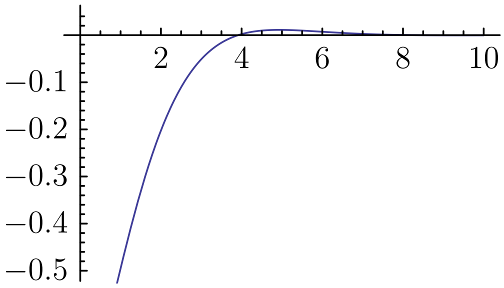
>> `KelvinKei[1.5 + I]`

$-0.248994 + 0.303326I$

>> `KelvinKei[0.5, 0.25]`

-2.0517

>> `Plot[KelvinKei[x], {x, 0, 10}]`



KelvinKer

`KelvinKer[`z`]`
: returns the Kelvin function ker(z).

`KelvinKer[`n`, `z`]`
: returns the Kelvin function ker_n(z).

>> `KelvinKer[0.5]`

0.855906

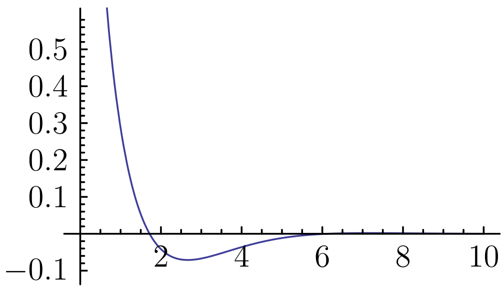
>> `KelvinKer[1.5 + I]`

$-0.167162 - 0.184404I$

>> `KelvinKer[0.5, 0.25]`

0.450023

>> `Plot[KelvinKer[x], {x, 0, 10}]`



LaguerreL

`LaguerreL[`n`, `x`]`
: returns the Laguerre polynomial L_n(x).

`LaguerreL[`n`, `a`, `x`]`
: returns the generalised Laguerre polynomial L^a_n(x).

>> `LaguerreL[8, x]`

$$1 - 8x + 14x^2 - \frac{28x^3}{3} + \frac{35x^4}{12} - \frac{7x^5}{15} + \frac{7x^6}{180} - \frac{x^7}{630} + \frac{x^8}{40320}$$

>> `LaguerreL[3/2, 1.7]`

-0.947134

>> `LaguerreL[5, 2, x]`

$$21 - 35x + \frac{35x^2}{2} - \frac{7x^3}{2} + \frac{7x^4}{24} - \frac{x^5}{120}$$

LegendreP

`LegendreP[`n`, `x`]`
: returns the Legendre polynomial P_n(x).

`LegendreP[`n`, `m`, `x`]`
: returns the associated Legendre polynomial P^m_n(x).

>> `LegendreP[4, x]`

$$\frac{3}{8} - \frac{15x^2}{4} + \frac{35x^4}{8}$$

>> `LegendreP[5/2, 1.5]`

4.17762

>> `LegendreP[1.75, 1.4, 0.53]`

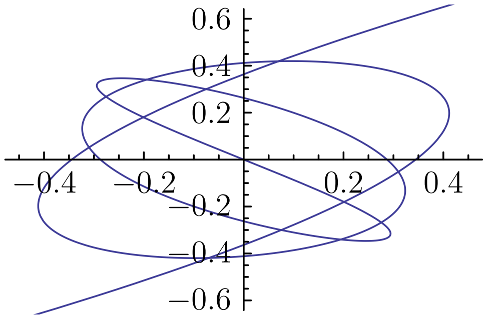
-1.32619

>> `LegendreP[1.6, 3.1, 1.5]`

$-0.303998 - 1.91937I$

`LegendreP` can be used to draw generalized Lissajous figures:

```
>> ParametricPlot[ {LegendreP[7, x
   ], LegendreP[5, x]}, {x, -1, 1}]
```



LegendreQ

`LegendreQ[n, x]`
: returns the Legendre function of the second kind Q_n(x).

`LegendreQ[n, m, x]`
: returns the associated Legendre function of the second Q^m_n(x).

```
>> LegendreQ[5/2, 1.5]
   0.036211 − 6.56219I

>> LegendreQ[1.75, 1.4, 0.53]
   2.05499

>> LegendreQ[1.6, 3.1, 1.5]
   −1.71931 − 7.70273I
```

LerchPhi

`LerchPhi[z,s,a]`
: gives the Lerch transcendent (z,s,a).

```
>> LerchPhi[2, 3, -1.5]
   19.3893 − 2.1346I

>> LerchPhi[1, 2, 1/4]
   17.1973
```

ProductLog

`ProductLog[z]`
: returns the value of the Lambert W function at z.

The defining equation:

```
>> z == ProductLog[z] * E ^
   ProductLog[z]
   True
```

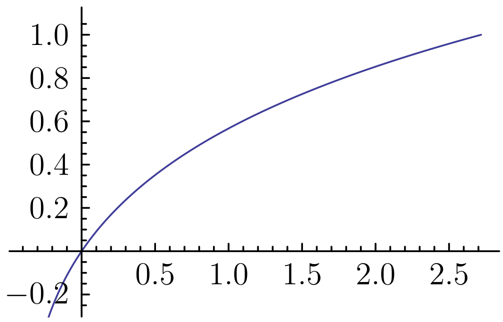
Some special values:

```
>> ProductLog[0]
   0

>> ProductLog[E]
   1
```

The graph of `ProductLog`:

```
>> Plot[ProductLog[x], {x, -1/E, E
   }]
```



SphericalHarmonicY

`SphericalHarmonicY[l, m, theta, phi]`
: returns the spherical harmonic function Y_l^m(theta, phi).

```
>> SphericalHarmonicY[3/4, 0.5, Pi
   /5, Pi/3]
```

0.254247 + 0.14679I

```
>> SphericalHarmonicY[3, 1, theta,
   phi]
```

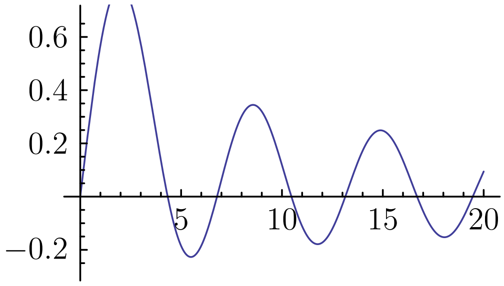
$$\frac{\sqrt{21}\left(1-5\mathrm{Cos}\,[\mathrm{theta}]^2\right)E^{I\mathrm{phi}}\mathrm{Sin}\,[\mathrm{theta}]}{8\sqrt{\mathrm{Pi}}}$$

StruveH

`StruveH[n, z]`
: returns the Struve function H_n(z).

```
>> StruveH[1.5, 3.5]
   1.13192
```

>> `Plot[StruveH[0, x], {x, 0, 20}]`

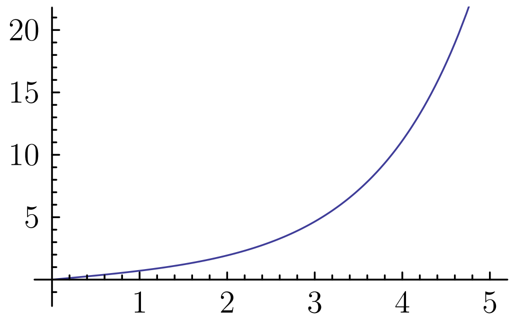


StruveL

`StruveL[n, z]`
returns the modified Struve function L_n(z).

>> `StruveL[1.5, 3.5]`
4.41126

>> `Plot[StruveL[0, x], {x, 0, 5}]`

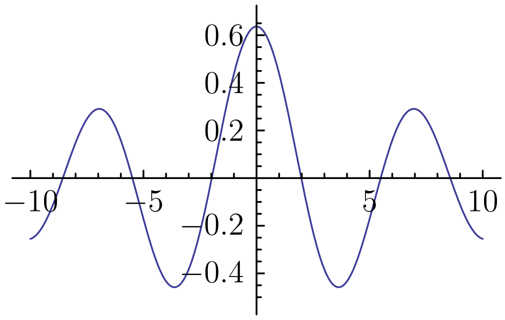


WeberE

`WeberE[n, z]`
returns the Weber function E_n(z).

>> `WeberE[1.5, 3.5]`
−0.397256

>> `Plot[WeberE[1, x], {x, -10, 10}]`



Zeta

`Zeta[z]`
returns the Riemann zeta function of z.

>> `Zeta[2]`
$\frac{\mathrm{Pi}^2}{6}$

>> `Zeta[-2.5 + I]`
$0.0235936 + 0.00140781I$

XXXVI. Physical and Chemical data

Contents

ElementData

`ElementData["`*name*`‘ ‘,"`*property*`"]`
: gives the value of the *property* for the chemical specified by *name*.

`ElementData[`*n*`, "`*property*`"]`
: gives the value of the *property* for the *n*th chemical element.

```
>> ElementData[74]
```

Tungsten

```
>> ElementData["He", "
   AbsoluteBoilingPoint"]
```

4.22

```
>> ElementData["Carbon", "
   IonizationEnergies"]
```

{1 086.5, 2 352.6, 4 620.5
, 6 222.7, 37 831, 47 277.}

```
>> ElementData[16, "
   ElectronConfigurationString"]
```

[Ne] 3s2 3p4

```
>> ElementData[73, "
   ElectronConfiguration"]
```

{{2} , {2, 6} , {2, 6, 10} , {2,
6, 10, 14} , {2, 6, 3} , {2}}

The number of known elements:

```
>> Length[ElementData[All]]
```

118

Some properties are not appropriate for certain elements:

```
>> ElementData["He", "
   ElectroNegativity"]
```

Missing [NotApplicable]

Some data is missing:

```
>> ElementData["Tc", "SpecificHeat
   "]
```

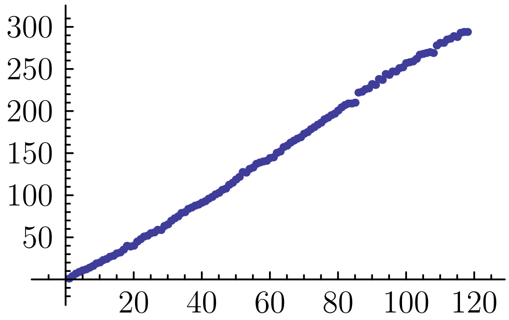
Missing [NotAvailable]

All the known properties:

```
>> ElementData["Properties"]
```

{Abbreviation,
AbsoluteBoilingPoint,
AbsoluteMeltingPoint,
AtomicNumber, AtomicRadius,
AtomicWeight, Block, BoilingPoint,
BrinellHardness, BulkModulus,
CovalentRadius, CrustAbundance,
Density, DiscoveryYear,
ElectroNegativity, ElectronAffinity,
ElectronConfiguration,
ElectronConfigurationString,
ElectronShellConfiguration,
FusionHeat, Group,
IonizationEnergies, LiquidDensity,
MeltingPoint, MohsHardness,
Name, Period, PoissonRatio,
Series, ShearModulus,
SpecificHeat, StandardName,
ThermalConductivity,
VanDerWaalsRadius,
VaporizationHeat,
VickersHardness, YoungModulus}

>> ListPlot[Table[ElementData[z, "
AtomicWeight"], {z, 118}]]



XXXVII. Calculus

Contents

Complexes

`Complexes`
is the set of complex numbers.

D

`D[`*f*`, `*x*`]`
gives the partial derivative of f with respect to x.
`D[`*f*`, `*x*`, `*y*`, ...]`
differentiates successively with respect to x, y, etc.
`D[`*f*`, {`*x*`, `*n*`}]`
gives the multiple derivative of order n.
`D[`*f*`, {{`*x1*`, `*x2*`, ...}}]`
gives the vector derivative of f with respect to *x1*, *x2*, etc.

First-order derivative of a polynomial:

```
>> D[x^3 + x^2, x]
```
$2x + 3x^2$

Second-order derivative:

```
>> D[x^3 + x^2, {x, 2}]
```
$2 + 6x$

Trigonometric derivatives:

```
>> D[Sin[Cos[x]], x]
```
$-\mathrm{Cos}\left[\mathrm{Cos}\left[x\right]\right]\mathrm{Sin}\left[x\right]$

```
>> D[Sin[x], {x, 2}]
```
$-\mathrm{Sin}\left[x\right]$

```
>> D[Cos[t], {t, 2}]
```
$-\mathrm{Cos}\left[t\right]$

Unknown variables are treated as constant:

```
>> D[y, x]
```
0

```
>> D[x, x]
```
1

```
>> D[x + y, x]
```
1

Derivatives of unknown functions are represented using `Derivative`:

```
>> D[f[x], x]
```
$f'\left[x\right]$

```
>> D[f[x, x], x]
```
$f^{(0,1)}\left[x,x\right] + f^{(1,0)}\left[x,x\right]$

```
>> D[f[x, x], x] // InputForm
```
$\mathrm{Derivative}\left[0,1\right]\left[f\right]\left[x,x\right] + \mathrm{Derivative}\left[1,0\right]\left[f\right]\left[x,x\right]$

Chain rule:

```
>> D[f[2x+1, 2y, x+y], x]
```
$2f^{(1,0,0)}\left[1+2x,2y,x+y\right] + f^{(0,0,1)}\left[1+2x,2y,x+y\right]$

```
>> D[f[x^2, x, 2y], {x,2}, y] // Expand
```

$8xf^{(1,1,1)}\left[x^2,x,2y\right]+8x^2f^{(2,0,1)}\left[x^2,x,2y\right]+2f^{(0,2,1)}\left[x^2,x,2y\right]+4f^{(1,0,1)}\left[x^2,x,2y\right]$

Compute the gradient vector of a function:

```
>> D[x ^ 3 * Cos[y], {{x, y}}]
```

$\left\{3x^2\mathrm{Cos}\left[y\right],-x^3\mathrm{Sin}\left[y\right]\right\}$

Hesse matrix:

```
>> D[Sin[x] * Cos[y], {{x,y}, 2}]
```

$\{\{-\mathrm{Cos}\left[y\right]\mathrm{Sin}\left[x\right],-\mathrm{Cos}\left[x\right]\mathrm{Sin}\left[y\right]\},\{-\mathrm{Cos}\left[x\right]\mathrm{Sin}\left[y\right],-\mathrm{Cos}\left[y\right]\mathrm{Sin}\left[x\right]\}\}$

Derivative (')

`Derivative[`*n*`][`*f*`]`
: represents the *n*th derivative of the function *f*.

`Derivative[`*n1*`, `*n2*`, ...][`*f*`]`
: represents a multivariate derivative.

```
>> Derivative[1][Sin]
```

Cos [#1] &

```
>> Derivative[3][Sin]
```

−Cos [#1] &

```
>> Derivative[2][# ^ 3&]
```

6#1&

`Derivative` can be entered using `'`:

```
>> Sin'[x]
```

$\mathrm{Cos}\left[x\right]$

```
>> (# ^ 4&)''
```

$12\#1^2\&$

```
>> f'[x] // InputForm
```

$\mathrm{Derivative}\left[1\right]\left[f\right]\left[x\right]$

```
>> Derivative[1][#2 Sin[#1]+Cos[#2]&]
```

Cos [#1] #2&

```
>> Derivative[1,2][#2^3 Sin[#1]+Cos[#2]&]
```

6Cos [#1] #2&

Deriving with respect to an unknown parameter yields 0:

```
>> Derivative[1,2,1][#2^3 Sin[#1]+Cos[#2]&]
```

0&

The 0th derivative of any expression is the expression itself:

```
>> Derivative[0,0,0][a+b+c]
```

$a+b+c$

You can calculate the derivative of custom functions:

```
>> f[x_] := x ^ 2
```

```
>> f'[x]
```

$2x$

Unknown derivatives:

```
>> Derivative[2, 1][h]
```

$h^{(2,1)}$

```
>> Derivative[2, 0, 1, 0][h[g]]
```

$h\left[g\right]^{(2,0,1,0)}$

DiscreteLimit

`DiscreteLimit[`*f*`, `*k*`->Infinity]`
: gives the limit of the sequence *f* as *k* tends to infinity.

```
>> DiscreteLimit[n/(n + 1), n -> Infinity]
```

1

```
>> DiscreteLimit[f[n], n -> Infinity]
```

$f\left[\infty\right]$

FindRoot

`FindRoot[`*f*`, {`*x*`, `*x0*`}]`
searches for a numerical root of *f*, starting from *x=x0*.

`FindRoot[`*lhs* `==` *rhs*`, {`*x*`, `*x0*`}]`
tries to solve the equation *lhs* `==` *rhs*.

`FindRoot` uses Newton's method, so the function of interest should have a first derivative.

```
>> FindRoot[Cos[x], {x, 1}]
```
$\{x->1.5708\}$

```
>> FindRoot[Sin[x] + Exp[x],{x, 0}]
```
$\{x->-0.588533\}$

```
>> FindRoot[Sin[x] + Exp[x] == Pi,{
   x, 0}]
```
$\{x->0.866815\}$

`FindRoot` has attribute `HoldAll` and effectively uses `Block` to localize *x*. However, in the result *x* will eventually still be replaced by its value.

```
>> x = 3;

>> FindRoot[Tan[x] + Sin[x] == Pi,
   {x, 1}]
```
$\{3->1.14911\}$

```
>> Clear[x]
```

`FindRoot` stops after 100 iterations:

```
>> FindRoot[x^2 + x + 1, {x, 1}]
```
Themaximumnumberofiterationswasexceeded.Theresultmightbeinaccurate.

$\{x->-1.\}$

Find complex roots:

```
>> FindRoot[x ^ 2 + x + 1, {x, -I}]
```
$\{x->-0.5-0.866025I\}$

The function has to return numerical values:

```
>> FindRoot[f[x] == 0, {x, 0}]
```
Thefunctionvalueisnotanumberatx = 0..

$\text{FindRoot}\left[f[x]-0,\{x,0\}\right]$

The derivative must not be 0:

```
>> FindRoot[Sin[x] == x, {x, 0}]
```
Encounteredasingularderivativeatthepointx = 0..

$\text{FindRoot}\left[\text{Sin}[x]-x,\{x,0\}\right]$

Integers

`Integers`
is the set of integer numbers.

Limit a solution to integer numbers:

```
>> Solve[-4 - 4 x + x^4 + x^5 == 0,
    x, Integers]
```
$\{\{x->-1\}\}$

```
>> Solve[x^4 == 4, x, Integers]
```
$\{\}$

Integrate

`Integrate[`*f*`, `*x*`]`
integrates *f* with respect to *x*. The result does not contain the additive integration constant.

`Integrate[`*f*`, {`*x*`, `*a*`, `*b*`}]`
computes the definite integral of *f* with respect to *x* from *a* to *b*.

Integrate a polynomial:

```
>> Integrate[6 x ^ 2 + 3 x ^ 2 - 4
   x + 10, x]
```
$10x-2x^2+3x^3$

Integrate trigonometric functions:

```
>> Integrate[Sin[x] ^ 5, x]
```
$-\text{Cos}[x]-\frac{\text{Cos}[x]^5}{5}+\frac{2\text{Cos}[x]^3}{3}$

Definite integrals:

```
>> Integrate[x ^ 2 + x, {x, 1, 3}]
```
$\frac{38}{3}$

```
>> Integrate[Sin[x], {x, 0, Pi/2}]
```
1

Some other integrals:

```
>> Integrate[1 / (1 - 4 x + x^2), x
   ]
```
$-\frac{\sqrt{3}\text{Log}\left[-2+\sqrt{3}+x\right]}{6}+\frac{\sqrt{3}\text{Log}\left[-2-\sqrt{3}+x\right]}{6}$

```
>> Integrate[4 Sin[x] Cos[x], x]
```
$2\mathrm{Sin}[x]^2$

Integration in TeX:
```
>> Integrate[f[x], {x, a, b}] // TeXForm
```
\int_a^b f\left[x\right] \, dx

Sometimes there is a loss of precision during integration. You can check the precision of your result with the following sequence of commands.
```
>> Integrate[Abs[Sin[phi]], {phi, 0, 2Pi}] // N
```
4.
```
>> % // Precision
```
MachinePrecision
```
>> Integrate[ArcSin[x / 3], x]
```
$x\mathrm{ArcSin}\left[\frac{x}{3}\right] + \sqrt{9 - x^2}$
```
>> Integrate[f'[x], {x, a, b}]
```
$f[b] - f[a]$

Limit

`Limit[`*expr*`, `*x*`->`*x0*`]`
: gives the limit of *expr* as *x* approaches *x0*.

`Limit[`*expr*`, `*x*`->`*x0*`, Direction->1]`
: approaches *x0* from smaller values.

`Limit[`*expr*`, `*x*`->`*x0*`, Direction->-1]`
: approaches *x0* from larger values.

```
>> Limit[x, x->2]
```
2
```
>> Limit[Sin[x] / x, x->0]
```
1
```
>> Limit[1/x, x->0, Direction->-1]
```
∞
```
>> Limit[1/x, x->0, Direction->1]
```
$-\infty$

Reals

`Reals`
: is the set of real numbers.

Limit a solution to real numbers:
```
>> Solve[x^3 == 1, x, Reals]
```
$\{\{x- > 1\}\}$

Root

`Root[`*f*`, `*i*`]`
: represents the i-th complex root of the polynomial *f*

```
>> Root[#1 ^ 2 - 1&, 1]
```
-1
```
>> Root[#1 ^ 2 - 1&, 2]
```
1

Roots that can't be represented by radicals:
```
>> Root[#1 ^ 5 + 2 #1 + 1&, 2]
```
$\mathrm{Root}\left[\#1^5 + 2\#1 + 1\&, 2\right]$

Solve

`Solve[`*equation*`, `*vars*`]`
: attempts to solve *equation* for the variables *vars*.

`Solve[`*equation*`, `*vars*`, `*domain*`]`
: restricts variables to *domain*, which can be `Complexes` or `Reals` or `Integers`.

```
>> Solve[x ^ 2 - 3 x == 4, x]
```
$\{\{x- > -1\}, \{x- > 4\}\}$
```
>> Solve[4 y - 8 == 0, y]
```
$\{\{y- > 2\}\}$

Apply the solution:
```
>> sol = Solve[2 x^2 - 10 x - 12 == 0, x]
```
$\{\{x- > -1\}, \{x- > 6\}\}$
```
>> x /. sol
```
$\{-1, 6\}$

Contradiction:
```
>> Solve[x + 1 == x, x]
```
$\{\}$

Tautology:

```
>> Solve[x ^ 2 == x ^ 2, x]
```
$\{\{\}\}$

Rational equations:

```
>> Solve[x / (x ^ 2 + 1)== 1, x]
```
$$\left\{\left\{x->\frac{1}{2}-\frac{I}{2}\sqrt{3}\right\},\left\{x->\frac{1}{2}+\frac{I}{2}\sqrt{3}\right\}\right\}$$

```
>> Solve[(x^2 + 3 x + 2)/(4 x - 2)
   == 0, x]
```
$\{\{x->-2\},\{x->-1\}\}$

Transcendental equations:

```
>> Solve[Cos[x] == 0, x]
```
$$\left\{\left\{x->\frac{\text{Pi}}{2}\right\},\left\{x->\frac{3\text{Pi}}{2}\right\}\right\}$$

Solve can only solve equations with respect to symbols or functions:

```
>> Solve[f[x + y] == 3, f[x + y]]
```
$\{\{f[x+y]->3\}\}$

```
>> Solve[a + b == 2, a + b]
```
a + bisnotavalidvariable.

Solve $[a+b{=}{=}2,a+b]$

This happens when solving with respect to an assigned symbol:

```
>> x = 3;
```

```
>> Solve[x == 2, x]
```
3isnotavalidvariable.

Solve [False, 3]

```
>> Clear[x]
```

```
>> Solve[a < b, a]
```
a < bisnotawell − formedequation.

Solve $[a<b,a]$

Solve a system of equations:

```
>> eqs = {3 x ^ 2 - 3 y == 0, 3 y ^
    2 - 3 x == 0};
```

```
>> sol = Solve[eqs, {x, y}] //
   Simplify
```
$$\left\{\{x->0,y->0\},\{x->1,y->1\},\left\{x->-\frac{1}{2}+\frac{I}{2}\sqrt{3},y->-\frac{1}{2}-\frac{I}{2}\sqrt{3}\right\},\left\{x->\frac{\left(1-I\sqrt{3}\right)^2}{4},y->-\frac{1}{2}+\frac{I}{2}\sqrt{3}\right\}\right\}$$

```
>> eqs /. sol // Simplify
```
{{True, True}, {True, True}, {False, False}, {True, True}}

An underdetermined system:

```
>> Solve[x^2 == 1 && z^2 == -1, {x,
    y, z}]
```
Equationsmaynotgivesolutionsforall"solve"variables.

$\{\{x->-1,z->-I\},\{x->-1,z->I\},\{x->1,z->-I\},\{x->1,z->I\}\}$

Domain specification:

```
>> Solve[x^2 == -1, x, Reals]
```
$\{\}$

```
>> Solve[x^2 == 1, x, Reals]
```
$\{\{x->-1\},\{x->1\}\}$

```
>> Solve[x^2 == -1, x, Complexes]
```
$\{\{x->-I\},\{x->I\}\}$

```
>> Solve[4 - 4 * x^2 - x^4 + x^6 ==
    0, x, Integers]
```
$\{\{x->-1\},\{x->1\}\}$

XXXVIII. Integer Functions

Contents

BitLength

`BitLength[`*x*`]`
: gives the number of bits needed to represent the integer x. x's sign is ignored.

```
>> BitLength[1023]
10

>> BitLength[100]
7

>> BitLength[-5]
3

>> BitLength[0]
0
```

Ceiling

`Ceiling[`*x*`]`
: gives the first integer greater than x.

```
>> Ceiling[1.2]
2

>> Ceiling[3/2]
2
```

For complex x, take the ceiling of real an imaginary parts.

```
>> Ceiling[1.3 + 0.7 I]
2 + I
```

DigitCount

`DigitCount[`*n*`, `*b*`, `*d*`]`
: returns the number of times digit d occurs in the base b representation of n.

`DigitCount[`*n*`, `*b*`]`
: returns a list indicating the number of times each digit occurs in the base b representation of n.

`DigitCount[`*n*`, `*b*`]`
: returns a list indicating the number of times each digit occurs in the decimal representation of n.

```
>> DigitCount[1022]
{1, 2, 0, 0, 0, 0, 0, 0, 0, 1}

>> DigitCount[Floor[Pi * 10^100]]
{8, 12, 12, 10, 8, 9, 8, 12, 14, 8}

>> DigitCount[1022, 2]
{9, 1}

>> DigitCount[1022, 2, 1]
9
```

Floor

`Floor[`*x*`]`
: gives the smallest integer less than or equal to x.

`Floor[`*x*`, `*a*`]`
: gives the smallest multiple of a less than or equal to x.

```
>> Floor[10.4]
```
10

```
>> Floor[10/3]
```
3

```
>> Floor[10]
```
10

```
>> Floor[21, 2]
```
20

```
>> Floor[2.6, 0.5]
```
2.5

```
>> Floor[-10.4]
```
-11

For complex x, take the floor of real an imaginary parts.

```
>> Floor[1.5 + 2.7 I]
```
$1 + 2I$

For negative a, the smallest multiple of a greater than or equal to x is returned.

```
>> Floor[10.4, -1]
```
11

```
>> Floor[-10.4, -1]
```
-10

FromDigits

`FromDigits[`*l*`]`
: returns the integer corresponding to the decimal representation given by l. l can be a list of digits or a string.

`FromDigits[`*l*`, `*b*`]`
: returns the integer corresponding to the base b representation given by l. l can be a list of digits or a string.

```
>> FromDigits["123"]
```
123

```
>> FromDigits[{1, 2, 3}]
```
123

```
>> FromDigits[{1, 0, 1}, 1000]
```
1 000 001

FromDigits can handle symbolic input:

```
>> FromDigits[{a, b, c}, 5]
```
$c + 5 (5a + b)$

Note that FromDigits does not automatically detect if you are providing a non-decimal representation:

```
>> FromDigits["a0"]
```
100

```
>> FromDigits["a0", 16]
```
160

FromDigits on empty lists or strings returns 0:

```
>> FromDigits[{}]
```
0

```
>> FromDigits[""]
```
0

IntegerDigits

`IntegerDigits[`*n*`]`
: returns the decimal representation of integer x as list of digits. x's sign is ignored.

`IntegerDigits[`*n*`, `*b*`]`
: returns the base b representation of integer x as list of digits. x's sign is ignored.

`IntegerDigits[`*n*`, `*b*`, `*length*`]`
: returns a list of length *length*. If the number is too short, the list gets padded with 0 on the left. If the number is too long, the *length* least significant digits are returned.

```
>> IntegerDigits[12345]
```
$\{1, 2, 3, 4, 5\}$

```
>> IntegerDigits[-500]
```
$\{5, 0, 0\}$

```
>> IntegerDigits[12345, 10, 8]
```
$\{0, 0, 0, 1, 2, 3, 4, 5\}$

```
>> IntegerDigits[12345, 10, 3]
```
$\{3, 4, 5\}$

```
>> IntegerDigits[11, 2]
```
$\{1, 0, 1, 1\}$

```
>> IntegerDigits[123, 8]
```
$\{1, 7, 3\}$

```
>> IntegerDigits[98765, 20]
```
$\{12, 6, 18, 5\}$

IntegerLength

`IntegerLength[`x`]`
: gives the number of digits in the base-10 representation of x.

`IntegerLength[`x`, `b`]`
: gives the number of base-b digits in x.

```
>> IntegerLength[123456]
```
6

```
>> IntegerLength[10^10000]
```
10 001

```
>> IntegerLength[-10^1000]
```
1 001

`IntegerLength` with base 2:

```
>> IntegerLength[8, 2]
```
4

Check that `IntegerLength` is correct for the first 100 powers of 10:

```
>> IntegerLength /@ (10 ^ Range
   [100])== Range[2, 101]
```
True

The base must be greater than 1:

```
>> IntegerLength[3, -2]
```
Base − 2isnotanintegergreaterthan1.

IntegerLength [3, − 2]

`0` is a special case:

```
>> IntegerLength[0]
```
0

IntegerReverse

`IntegerReverse[`n`]`
: returns the integer that has the reverse decimal representation of x without sign.

`IntegerReverse[`n`, `b`]`
: returns the integer that has the reverse base b represenation of x without sign.

```
>> IntegerReverse[1234]
```
4 321

```
>> IntegerReverse[1022, 2]
```
511

```
>> IntegerReverse[-123]
```
321

IntegerString

`IntegerString[`n`]`
: returns the decimal representation of integer x as string. x's sign is ignored.

`IntegerString[`n`, `b`]`
: returns the base b representation of integer x as string. x's sign is ignored.

`IntegerString[`n`, `b`, `*length*`]`
: returns a string of length *length*. If the number is too short, the string gets padded with 0 on the left. If the number is too long, the *length* least significant digits are returned.

For bases > 10, alphabetic characters a, b, ... are used to represent digits 11, 12, ... Note that base must be an integer in the range from 2 to 36.

```
>> IntegerString[12345]
```
12 345

```
>> IntegerString[-500]
```
500

```
>> IntegerString[12345, 10, 8]
```
00 012 345

```
>> IntegerString[12345, 10, 3]
```
345

```
>> IntegerString[11, 2]
```
1 011

```
>> IntegerString[123, 8]
```
173

```
>> IntegerString[32767, 16]
```
7fff

```
>> IntegerString[98765, 20]
```
c6i5

XXXIX. List Functions

Contents

Accumulate

`Accumulate[`*list*`]`
: accumulates the values of *list*, returning a new list.

```
>> Accumulate[{1, 2, 3}]
```
$\{1,3,6\}$

All

`All`
: is a possible value for `Span` and `Quiet`.

Append

`Append[`*expr*`, `*elem*`]`
: returns *expr* with *elem* appended.

```
>> Append[{1, 2, 3}, 4]
```
{1,2,3,4}

`Append` works on expressions with heads other than `List`:

```
>> Append[f[a, b], c]
```
$f[a,b,c]$

Unlike `Join`, `Append` does not flatten lists in *item*:

```
>> Append[{a, b}, {c, d}]
```
$\{a,b,\{c,d\}\}$

AppendTo

`AppendTo[`*s*`, `*item*`]`
: append *item* to value of *s* and sets *s* to the result.

```
>> s = {};
```

```
>> AppendTo[s, 1]
```
{1}

```
>> s
```
{1}

`Append` works on expressions with heads other than `List`:

```
>> y = f[];
```

```
>> AppendTo[y, x]
```
$f[x]$

```
>> y
```
$f[x]$

Array

`Array[`*f*`, `*n*`]`
: returns the *n*-element list `{`*f*`[1], ..., `*f*`[`*n*`]}`.

`Array[`*f*`, `*n*`, `*a*`]`
: returns the *n*-element list `{`*f*`[`*a*`], ..., `*f*`[`*a*` + `*n*`]}`.

`Array[`*f*`, {`*n*`, `*m*`}, {`*a*`, `*b*`}]`
: returns an *n*-by-*m* matrix created by applying *f* to indices ranging from `(`*a*`, `*b*`)` to `(`*a*` + `*n*`, `*b*` + `*m*`)`.

`Array[`*f*`, `*dims*`, `*origins*`, `*h*`]`
: returns an expression with the specified dimensions and index origins, with head *h* (instead of `List`).

```
>> Array[f, 4]
```
$\{f[1],f[2],f[3],f[4]\}$

```
>> Array[f, {2, 3}]
```
$\{\{f[1,1],f[1,2],f[1,3]\},\{f[2,1],f[2,2],f[2,3]\}\}$

```
>> Array[f, {2, 3}, 3]
```
$\{\{f[3,3],f[3,4],f[3,5]\},\{f[4,3],f[4,4],f[4,5]\}\}$

```
>> Array[f, {2, 3}, {4, 6}]
```
$\{\{f[4,6],f[4,7],f[4,8]\},\{f[5,6],f[5,7],f[5,8]\}\}$

```
>> Array[f, {2, 3}, 1, Plus]
```
$f[1,1]+f[1,2]+f[1,3]+f[2,1]+f[2,2]+f[2,3]$

Association

`Association[`*key1*` -> `*val1*`, `*key2*` -> `*val2*`, ...]`

`<|`*key1*` -> `*val1*`, `*key2*` -> `*val2*`, ...|>`
: represents an association between keys and values.

`Association` is the head of associations:

```
>> Head[<|a -> x, b -> y, c -> z|>]
```
Association

```
>> <|a -> x, b -> y|>
```
$<|a->x,b->y|>$

`>> Association[{a -> x, b -> y}]`

$<|a- > x, b- > y|>$

Associations can be nested:

`>> <|a -> x, b -> y, <|a -> z, d -> t|>|>`

$<|a- > z, b- > y, d- > t|>$

AssociationQ

`AssociationQ[`*expr*`]`
: return True if *expr* is a valid Association object, and False otherwise.

`>> AssociationQ[<|a -> 1, b :> 2|>]`

True

`>> AssociationQ[<|a, b|>]`

False

ByteArray

`ByteArray[{`*b_1*`, `*b_2*`, ...}]`
: Represents a sequence of Bytes *b_1*, *b_2*, ...

`ByteArray[‘‘string’]’`
: Constructs a byte array where bytes comes from decode a b64 encoded String

`>> A=ByteArray[{1, 25, 3}]`

ByteArray ["ARkD"]

`>> A[[2]]`

25

`>> Normal[A]`

$\{1, 25, 3\}$

`>> ToString[A]`

ByteArray["ARkD"]

`>> ByteArray["ARkD"]`

ByteArray ["ARkD"]

`>> B=ByteArray["asy"]`

ThefirstargumentinBytearray[asy]shouldbeaB64encondedstringoravectorofintegers.

$Failed

Cases

`Cases[`*list*`, `*pattern*`]`
: returns the elements of *list* that match *pattern*.

`Cases[`*list*`, `*pattern*`, `*ls*`]`
: returns the elements matching at levelspec *ls*.

`>> Cases[{a, 1, 2.5, "string"}, _Integer|_Real]`

$\{1, 2.5\}$

`>> Cases[_Complex][{1, 2I, 3, 4-I, 5}]`

$\{2I, 4 - I\}$

Catenate

`Catenate[{`*l1*`, `*l2*`, ...}]`
: concatenates the lists *l1*, *l2*, ...

`>> Catenate[{{1, 2, 3}, {4, 5}}]`

$\{1, 2, 3, 4, 5\}$

CentralMoment

`CentralMoment[`*list*`, `*r*`]`
: gives the the *r*th central moment (i.e. the *r*th moment about the mean) of *list*.

`>> CentralMoment[{1.1, 1.2, 1.4, 2.1, 2.4}, 4]`

0.100845

ClusteringComponents

`ClusteringComponents[`*list*`]`
: forms clusters from *list* and returns a list of cluster indices, in which each element shows the index of the cluster in which the corresponding element in *list* ended up.

`ClusteringComponents[`*list*`, `*k*`]`
: forms *k* clusters from *list* and returns a list of cluster indices, in which each element shows the index of the cluster in which the corresponding element in *list* ended up.

For more detailed documentation regarding options and behavior, see FindClusters[].

```
>> ClusteringComponents[{1, 2, 3,
   1, 2, 10, 100}]
```
$\{1,1,1,1,1,1,2\}$

```
>> ClusteringComponents[{10, 100,
   20}, Method -> "KMeans"]
```
$\{1,0,1\}$

Complement

`Complement[`*all*`, `*e1*`, `*e2*`, ...]`
: returns an expression containing the elements in the set *all* that are not in any of *e1*, *e2*, etc.

`Complement[`*all*`, `*e1*`, `*e2*`, ..., SameTest->`*test*`]`
: applies *test* to the elements in *all* and each of the *ei* to determine equality.

The sets *all*, *e1*, etc can have any head, which must all match. The returned expression has the same head as the input expressions. The expression will be sorted and each element will only occur once.

```
>> Complement[{a, b, c}, {a, c}]
```
$\{b\}$

```
>> Complement[{a, b, c}, {a, c}, {b
   }]
```
$\{\}$

```
>> Complement[f[z, y, x, w], f[x],
   f[x, z]]
```
$f[w,y]$

```
>> Complement[{c, b, a}]
```
$\{a,b,c\}$

ConstantArray

`ConstantArray[`*expr*`, `*n*`]`
: returns a list of *n* copies of *expr*.

```
>> ConstantArray[a, 3]
```
$\{a,a,a\}$

```
>> ConstantArray[a, {2, 3}]
```
$\{\{a,a,a\},\{a,a,a\}\}$

ContainsOnly

`ContainsOnly[`*list1*`, `*list2*`]`
: yields True if *list1* contains only elements that appear in *list2*.

```
>> ContainsOnly[{b, a, a}, {a, b, c
   }]
```
True

The first list contains elements not present in the second list:

```
>> ContainsOnly[{b, a, d}, {a, b, c
   }]
```
False

```
>> ContainsOnly[{}, {a, b, c}]
```
True

Use Equal as the comparison function to have numerical tolerance:

```
>> ContainsOnly[{a, 1.0}, {1, a, b
   }, {SameTest -> Equal}]
```
True

Correlation

`Correlation[`*a*`, `*b*`]`
: computes Pearson's correlation of two equal-sized vectors *a* and *b*.

An example from Wikipedia:

```
>> Correlation[{10, 8, 13, 9, 11,
   14, 6, 4, 12, 7, 5}, {8.04,
   6.95, 7.58, 8.81, 8.33, 9.96,
   7.24, 4.26, 10.84, 4.82, 5.68}]
```

0.816421

Count

`Count[`*list*`, `*pattern*`]`
: returns the number of times *pattern* appears in *list*.

`Count[`*list*`, `*pattern*`, `*ls*`]`
: counts the elements matching at levelspec *ls*.

```
>> Count[{3, 7, 10, 7, 5, 3, 7,
   10}, 3]
```

2

```
>> Count[{{a, a}, {a, a, a}, a}, a,
    {2}]
```

5

Covariance

`Covariance[`*a*`, `*b*`]`
: computes the covariance between the equal-sized vectors *a* and *b*.

```
>> Covariance[{0.2, 0.3, 0.1},
   {0.3, 0.3, -0.2}]
```

0.025

Delete

`Delete[`*expr*`, `*i*`]`
: deletes the element at position *i* in *expr*. The position is counted from the end if *i* is negative.

`Delete[`*expr*`, {`*m*`, `*n*`, ...}]`
: deletes the element at position {*m*, *n*, ...}.

`Delete[`*expr*`, {{`*m1*`, `*n1*`, ...}, {`*m2*`, `*n2*`, ...}, ...}]`
: deletes the elements at several positions.

Delete the element at position 3:

```
>> Delete[{a, b, c, d}, 3]
```

$\{a,b,d\}$

Delete at position 2 from the end:

```
>> Delete[{a, b, c, d}, -2]
```

$\{a,b,d\}$

Delete at positions 1 and 3:

```
>> Delete[{a, b, c, d}, {{1}, {3}}]
```

$\{b,d\}$

Delete in a 2D array:

```
>> Delete[{{a, b}, {c, d}}, {2, 1}]
```

$\{\{a,b\},\{d\}\}$

Deleting the head of a whole expression gives a Sequence object:

```
>> Delete[{a, b, c}, 0]
```

Sequence $[a,b,c]$

Delete in an expression with any head:

```
>> Delete[f[a, b, c, d], 3]
```

$f[a,b,d]$

Delete a head to splice in its arguments:

```
>> Delete[f[a, b, u + v, c], {3,
   0}]
```

$f[a,b,u,v,c]$

```
>> Delete[{a, b, c}, 0]
```

Sequence $[a,b,c]$

Delete without the position:

```
>> Delete[{a, b, c, d}]
```

Deletecalledwith1argument;2argumentsareexpected.

Delete $[\{a,b,c,d\}]$

Delete with many arguments:

```
>> Delete[{a, b, c, d}, 1, 2]
```
Deletecalledwith3arguments; 2argumentsareexpected.

Delete [$\{a,b,c,d\}$,1,2]

Delete the element out of range:

```
>> Delete[{a, b, c, d}, 5]
```
Part{5}of{a,b,c,d}doesnotexist.

Delete [$\{a,b,c,d\}$,5]

Delete the position not integer:

```
>> Delete[{a, b, c, d}, {1, n}]
```
Positionspecificationnin{a, b,c,d}isnotamachine − sizedintegeroralistofmachine − sizedintegers.

Delete [$\{a,b,c,d\}$, $\{1,n\}$]

DeleteCases

`DeleteCases[`*list*`, `*pattern*`]`
: returns the elements of *list* that do not match *pattern*.

`DeleteCases[`*list*`, `*pattern*`, `*levelspec*`]`
: removes all parts of $list on levels specified by *levelspec* that match pattern (not fully implemented).

`DeleteCases[`*list*`, `*pattern*`, `*levelspec*`, `*n*`]`
: removes the first *n* parts of *list* that match *pattern*.

```
>> DeleteCases[{a, 1, 2.5, "string
   "}, _Integer|_Real]
```
$\{a,\text{string}\}$

```
>> DeleteCases[{a, b, 1, c, 2, 3},
   _Symbol]
```
$\{1,2,3\}$

DeleteDuplicates

`DeleteDuplicates[`*list*`]`
: deletes duplicates from *list*.

`DeleteDuplicates[`*list*`, `*test*`]`
: deletes elements from *list* based on whether the function *test* yields `True` on pairs of elements. DeleteDuplicates does not change the order of the remaining elements.

```
>> DeleteDuplicates[{1, 7, 8, 4, 3,
    4, 1, 9, 9, 2, 1}]
```
$\{1,7,8,4,3,9,2\}$

```
>> DeleteDuplicates[{3,2,1,2,3,4},
   Less]
```
$\{3,2,1\}$

DisjointQ

`DisjointQ[`*a*`, `*b*`]`
: gives True if $a and $b are disjoint, or False if $a and $b have any common elements.

Drop

`Drop[`*expr*`, `*n*`]`
: returns *expr* with the first *n* leaves removed.

```
>> Drop[{a, b, c, d}, 3]
```
$\{d\}$

```
>> Drop[{a, b, c, d}, -2]
```
$\{a,b\}$

```
>> Drop[{a, b, c, d, e}, {2, -2}]
```
$\{a,e\}$

Drop a submatrix:

```
>> A = Table[i*10 + j, {i, 4}, {j,
   4}]
```
$\{\{11,12,13,14\},\{21,22,23,24\},\{31,32,33,34\},\{41,42,43,44\}\}$

```
>> Drop[A, {2, 3}, {2, 3}]
```
$\{\{11,14\},\{41,44\}\}$

Extract

`Extract[`*expr*`, ` *list*`]`
: extracts parts of *expr* specified by *list*.

`Extract[`*expr*`, {`*list1*`, ` *list2*`, ...}]`
: extracts a list of parts.

`Extract[`*expr*`, ` *i*`, ` *j*`, ...]` is equivalent to `Part[`*expr*`, {`*i*`, ` *j*`, ...}]`.

```
>> Extract[a + b + c, {2}]
```
b

```
>> Extract[{{a, b}, {c, d}}, {{1}, {2, 2}}]
```
$\{\{a,b\},d\}$

Failure

Failure[*tag*, *assoc*]
: represents a failure of a type indicated by *tag*, with details given by the association *assoc*.

FindClusters

`FindClusters[`*list*`]`
: returns a list of clusters formed from the elements of *list*. The number of cluster is determined automatically.

`FindClusters[`*list*`, ` *k*`]`
: returns a list of *k* clusters formed from the elements of *list*.

```
>> FindClusters[{1, 2, 20, 10, 11, 40, 19, 42}]
```
$\{\{1,2,20,10,11,19\},\{40,42\}\}$

```
>> FindClusters[{25, 100, 17, 20}]
```
$\{\{25,17,20\},\{100\}\}$

```
>> FindClusters[{3, 6, 1, 100, 20, 5, 25, 17, -10, 2}]
```
$\{\{3,6,1,5,-10,2\},\{100\},\{20,25,17\}\}$

```
>> FindClusters[{1, 2, 10, 11, 20, 21}]
```
$\{\{1,2\},\{10,11\},\{20,21\}\}$

```
>> FindClusters[{1, 2, 10, 11, 20, 21}, 2]
```
$\{\{1,2,10,11\},\{20,21\}\}$

```
>> FindClusters[{1 -> a, 2 -> b, 10 -> c}]
```
$\{\{a,b\},\{c\}\}$

```
>> FindClusters[{1, 2, 5} -> {a, b, c}]
```
$\{\{a,b\},\{c\}\}$

```
>> FindClusters[{1, 2, 3, 1, 2, 10, 100}, Method -> "Agglomerate"]
```
$\{\{1,2,3,1,2,10\},\{100\}\}$

```
>> FindClusters[{1, 2, 3, 10, 17, 18}, Method -> "Agglomerate"]
```
$\{\{1,2,3\},\{10\},\{17,18\}\}$

```
>> FindClusters[{{1}, {5, 6}, {7}, {2, 4}}, DistanceFunction -> (Abs[Length[#1] - Length[#2]]&)]
```
$\{\{\{1\},\{7\}\},\{\{5,6\},\{2,4\}\}\}$

```
>> FindClusters[{"meep", "heap", "deep", "weep", "sheep", "leap", "keep"}, 3]
```
{{meep, deep, weep, keep}, {heap, leap}, {sheep}}

FindClusters' automatic distance function detection supports scalars, numeric tensors, boolean vectors and strings.

The Method option must be either "Agglomerate" or "Optimize". If not specified, it defaults to "Optimize". Note that the Agglomerate and Optimize methods usually produce different clusterings.

The runtime of the Agglomerate method is quadratic in the number of clustered points n, builds the clustering from the bottom up, and is exact (no element of randomness). The Optimize method's runtime is linear in n, Optimize builds the clustering from top down, and uses random sampling.

First

`First[expr]`
: returns the first element in *expr*.

`First[expr]` is equivalent to `expr[[1]]`.

```
>> First[{a, b, c}]
```
a

```
>> First[a + b + c]
```
a

```
>> First[x]
```
Nonatomicexpressionexpected.

First [*x*]

FirstCase

FirstCase[{*e1*, *e2*, ...}, *pattern*]
: gives the first *ei* to match *pattern*, or $Missing["NotFound"]$ if none matching pattern is found.

FirstCase[{*e1*,*e2*, ...}, *pattern* -> *rhs*]
: gives the value of *rhs* corresponding to the first *ei* to match pattern.

FirstCase[*expr*, *pattern*, *default*]
: gives *default* if no element matching *pattern* is found.

FirstCase[*expr*, *pattern*, *default*, *levelspec*]
: finds only objects that appear on levels specified by *levelspec*.

FirstCase[*pattern*]
: represents an operator form of FirstCase that can be applied to an expression.

FirstPosition

`FirstPosition[expr, pattern]`
: gives the position of the first element in *expr* that matches *pattern*, or Missing["NotFound"] if no such element is found.

`FirstPosition[expr, pattern, default]`
: gives default if no element matching *pattern* is found.

`FirstPosition[expr, pattern, default, levelspec]`
: finds only objects that appear on levels specified by *levelspec*.

```
>> FirstPosition[{a, b, a, a, b, c,
    b}, b]
```
{2}

```
>> FirstPosition[{{a, a, b}, {b, a,
    a}, {a, b, a}}, b]
```
{1,3}

```
>> FirstPosition[{x, y, z}, b]
```
Missing [NotFound]

Find the first position at which x^2 to appears:

```
>> FirstPosition[{1 + x^2, 5, x^4,
   a + (1 + x^2)^2}, x^2]
```
{1,2}

Fold

`Fold[f, x, list]`
: returns the result of iteratively applying the binary operator *f* to each element of *list*, starting with *x*.

`Fold[f, list]`
: is equivalent to `Fold[f, First[list], Rest[list]]`.

```
>> Fold[Plus, 5, {1, 1, 1}]
```
8

```
>> Fold[f, 5, {1, 2, 3}]
```
$f[f[f[5,1],2],3]$

FoldList

`FoldList[f, x, list]`
: returns a list starting with *x*, where each element is the result of applying the binary operator *f* to the previous result and the next element of *list*.

`FoldList[f, list]`
: is equivalent to `FoldList[f, First[list], Rest[list]]`.

```
>> FoldList[f, x, {1, 2, 3}]
```
$\{x, f[x,1], f[f[x,1],2], f[f[f[x,1],2],3]\}$

>> `FoldList[Times, {1, 2, 3}]`

$\{1,2,6\}$

Gather

`Gather[`*list*`, `*test*`]`
: gathers leaves of *list* into sub lists of items that are the same according to *test*.

`Gather[`*list*`]`
: gathers leaves of *list* into sub lists of items that are the same.

The order of the items inside the sub lists is the same as in the original list.

>> `Gather[{1, 7, 3, 7, 2, 3, 9}]`

$\{\{1\},\{7,7\},\{3,3\},\{2\},\{9\}\}$

>> `Gather[{1/3, 2/6, 1/9}]`

$\left\{\left\{\frac{1}{3},\frac{1}{3}\right\},\left\{\frac{1}{9}\right\}\right\}$

GatherBy

`GatherBy[`*list*`, `*f*`]`
: gathers leaves of *list* into sub lists of items whose image under $f identical.

`GatherBy[`*list*`, {`*f*`, `*g*`, ...}]`
: gathers leaves of *list* into sub lists of items whose image under $f identical. Then, gathers these sub lists again into sub sub lists, that are identical under $g.

>> `GatherBy[{{1, 3}, {2, 2}, {1, 1}}, Total]`

$\{\{\{1,3\},\{2,2\}\},\{\{1,1\}\}\}$

>> `GatherBy[{"xy", "abc", "ab"}, StringLength]`

$\{\{\mathrm{xy},\mathrm{ab}\},\{\mathrm{abc}\}\}$

>> `GatherBy[{{2, 0}, {1, 5}, {1, 0}}, Last]`

$\{\{\{2,0\},\{1,0\}\},\{\{1,5\}\}\}$

>> `GatherBy[{{1, 2}, {2, 1}, {3, 5}, {5, 1}, {2, 2, 2}}, {Total, Length}]`

$\{\{\{\{1,2\},\{2,1\}\}\},\{\{\{3,5\}\}\},\{\{\{5,1\}\},\{\{2,2,2\}\}\}\}$

Insert

`Insert[`*list*`, `*elem*`, `*n*`]`
: inserts *elem* at position *n* in *list*. When *n* is negative, the position is counted from the end.

>> `Insert[{a,b,c,d,e}, x, 3]`

$\{a,b,x,c,d,e\}$

>> `Insert[{a,b,c,d,e}, x, -2]`

$\{a,b,c,d,x,e\}$

IntersectingQ

`IntersectingQ[`*a*`, `*b*`]`
: gives True if there are any common elements in $a and $b, or False if $a and $b are disjoint.

Intersection

`Intersection[`*a*`, `*b*`, ...]`
: gives the intersection of the sets. The resulting list will be sorted and each element will only occur once.

>> `Intersection[{1000, 100, 10, 1}, {1, 5, 10, 15}]`

$\{1,10\}$

>> `Intersection[{{a, b}, {x, y}}, {{x, x}, {x, y}, {x, z}}]`

$\{\{x,y\}\}$

>> `Intersection[{c, b, a}]`

$\{a,b,c\}$

```
>> Intersection[{1, 2, 3}, {2, 3, 4}, SameTest->Less]
```
$\{3\}$

Join

`Join[l1, l2]`
: concatenates the lists *l1* and *l2*.

`Join` concatenates lists:
```
>> Join[{a, b}, {c, d, e}]
```
$\{a, b, c, d, e\}$

```
>> Join[{{a, b}, {c, d}}, {{1, 2}, {3, 4}}]
```
$\{\{a, b\}, \{c, d\}, \{1, 2\}, \{3, 4\}\}$

The concatenated expressions may have any head:
```
>> Join[a + b, c + d, e + f]
```
$a + b + c + d + e + f$

However, it must be the same for all expressions:
```
>> Join[a + b, c * d]
```
HeadsPlusandTimesareexpectedtobethesame.

$\text{Join}[a + b, cd]$

Key

Key[*key*]
: represents a key used to access a value in an association.

Key[*key*][*assoc*]

Keys

`Keys[<|key1 -> val1, key2 -> val2, ...|>]`
: return a list of the keys *keyi* in an association.

`Keys[{key1 -> val1, key2 -> val2, ...}]`
: return a list of the *keyi* in a list of rules.

```
>> Keys[<|a -> x, b -> y|>]
```
$\{a, b\}$

```
>> Keys[{a -> x, b -> y}]
```
$\{a, b\}$

Keys automatically threads over lists:
```
>> Keys[{<|a -> x, b -> y|>, {w -> z, {}}}]
```
$\{\{a, b\}, \{w, \{\}\}\}$

Keys are listed in the order of their appearance:
```
>> Keys[{c -> z, b -> y, a -> x}]
```
$\{c, b, a\}$

Kurtosis

`Kurtosis[list]`
: gives the Pearson measure of kurtosis for *list* (a measure of existing outliers).

```
>> Kurtosis[{1.1, 1.2, 1.4, 2.1, 2.4}]
```
1.42098

Last

`Last[expr]`
: returns the last element in *expr*.

`Last[expr]` is equivalent to *expr*`[[-1]]`.
```
>> Last[{a, b, c}]
```
c

```
>> Last[x]
```
Nonatomicexpressionexpected.

$\text{Last}[x]$

LeafCount

`LeafCount[expr]`
: returns the total number of indivisible subexpressions in *expr*.

```
>> LeafCount[1 + x + y^a]
```
6

```
>> LeafCount[f[x, y]]
```
3

>> `LeafCount[{1 / 3, 1 + I}]`

7

>> `LeafCount[Sqrt[2]]`

5

>> `LeafCount[100!]`

1

Length

`Length[`*expr*`]`
: returns the number of leaves in *expr*.

Length of a list:

>> `Length[{1, 2, 3}]`

3

`Length` operates on the `FullForm` of expressions:

>> `Length[Exp[x]]`

2

>> `FullForm[Exp[x]]`

Power $[E, x]$

The length of atoms is 0:

>> `Length[a]`

0

Note that rational and complex numbers are atoms, although their `FullForm` might suggest the opposite:

>> `Length[1/3]`

0

>> `FullForm[1/3]`

Rational $[1, 3]$

Level

`Level[`*expr*`, `*levelspec*`]`
: gives a list of all subexpressions of *expr* at the level(s) specified by *levelspec*.

Level uses standard level specifications:

n
: levels 1 through *n*

`Infinity`
: all levels from level 1

`{`*n*`}`
: level *n* only

`{`*m*`, `*n*`}`
: levels *m* through *n*

Level 0 corresponds to the whole expression.
A negative level $-n$ consists of parts with depth n.
Level -1 is the set of atoms in an expression:

>> `Level[a + b ^ 3 * f[2 x ^ 2], {-1}]`

$\{a, b, 3, 2, x, 2\}$

>> `Level[{{{{a}}}}, 3]`

$\{\{a\}, \{\{a\}\}, \{\{\{a\}\}\}\}$

>> `Level[{{{{a}}}}, -4]`

$\{\{\{\{a\}\}\}\}$

>> `Level[{{{{a}}}}, -5]`

$\{\}$

>> `Level[h0[h1[h2[h3[a]]]], {0, -1}]`

$\{a, \mathrm{h3}[a], \mathrm{h2}[\mathrm{h3}[a]], \mathrm{h1}[\mathrm{h2}[\mathrm{h3}[a]]], \mathrm{h0}[\mathrm{h1}[\mathrm{h2}[\mathrm{h3}[a]]]]\}$

Use the option `Heads -> True` to include heads:

>> `Level[{{{{a}}}}, 3, Heads -> True]`

$\{\mathrm{List}, \mathrm{List}, \mathrm{List}, \{a\}, \{\{a\}\}, \{\{\{a\}\}\}\}$

>> `Level[x^2 + y^3, 3, Heads -> True]`

$\left\{\mathrm{Plus}, \mathrm{Power}, x, 2, x^2, \mathrm{Power}, y, 3, y^3\right\}$

>> `Level[a ^ 2 + 2 * b, {-1}, Heads -> True]`

$\{\mathrm{Plus}, \mathrm{Power}, a, 2, \mathrm{Times}, 2, b\}$

>> `Level[f[g[h]][x], {-1}, Heads -> True]`

$\{f, g, h, x\}$

>> `Level[f[g[h]][x], {-2, -1}, Heads -> True]`

$\{f, g, h, g[h], x, f[g[h]][x]\}$

LevelQ

`LevelQ[`*expr*`]`
: tests whether *expr* is a valid level specification.

```
>> LevelQ[2]
```
True

```
>> LevelQ[{2, 4}]
```
True

```
>> LevelQ[Infinity]
```
True

```
>> LevelQ[a + b]
```
False

List

`List[`*e1*`, `*e2*`, ..., `*ei*`]`
`{`*e1*`, `*e2*`, ..., `*ei*`}`
: represents a list containing the elements *e1*...*ei*.

`List` is the head of lists:

```
>> Head[{1, 2, 3}]
```
List

Lists can be nested:

```
>> {{a, b, {c, d}}}
```
$\{\{a, b, \{c, d\}\}\}$

ListQ

`ListQ[`*expr*`]`
: tests whether *expr* is a `List`.

```
>> ListQ[{1, 2, 3}]
```
True

```
>> ListQ[{{1, 2}, {3, 4}}]
```
True

```
>> ListQ[x]
```
False

Lookup

Lookup[*assoc*, *key*]
: looks up the value associated with *key* in the association *assoc*, or Missing[*KeyAbsent*].

Mean

`Mean[`*list*`]`
: returns the statistical mean of *list*.

```
>> Mean[{26, 64, 36}]
```
42

```
>> Mean[{1, 1, 2, 3, 5, 8}]
```
$\frac{10}{3}$

```
>> Mean[{a, b}]
```
$\frac{a + b}{2}$

Median

`Median[`*list*`]`
: returns the median of *list*.

```
>> Median[{26, 64, 36}]
```
36

For lists with an even number of elements, Median returns the mean of the two middle values:

```
>> Median[{-11, 38, 501, 1183}]
```
$\frac{539}{2}$

Passing a matrix returns the medians of the respective columns:

```
>> Median[{{100, 1, 10, 50}, {-1,
   1, -2, 2}}]
```
$\left\{\frac{99}{2}, 1, 4, 26\right\}$

MemberQ

`MemberQ[`*list*`, `*pattern*`]`
: returns `True` if *pattern* matches any element of *list*, or `False` otherwise.

```
>> MemberQ[{a, b, c}, b]
```
True

```
>> MemberQ[{a, b, c}, d]
```
False

```
>> MemberQ[{"a", b, f[x]}, _?NumericQ]
```
False

```
>> MemberQ[_List][{{}}]
```
True

Most

`Most[`*expr*`]`
: returns *expr* with the last element removed.

`Most[`*expr*`]` is equivalent to *expr*`[[;;-2]]`.

```
>> Most[{a, b, c}]
```
$\{a, b\}$

```
>> Most[a + b + c]
```
$a + b$

```
>> Most[x]
```
Nonatomicexpressionexpected.

Most[x]

Nearest

`Nearest[`*list*`, `*x*`]`
: returns the one item in *list* that is nearest to *x*.

`Nearest[`*list*`, `*x*`, `*n*`]`
: returns the *n* nearest items.

`Nearest[`*list*`, `*x*`, {`*n*`, `*r*`}]`
: returns up to *n* nearest items that are not farther from *x* than *r*.

`Nearest[{`*p1*` -> `*q1*`, `*p2*` -> `*q2*`, ...}, `*x*`]`
: returns *q1*, *q2*, ... but measures the distances using *p1*, *p2*, ...

`Nearest[{`*p1*`, `*p2*`, ...} -> {`*q1*`, `*q2*`, ...}, `*x*`]`
: returns *q1*, *q2*, ... but measures the distances using *p1*, *p2*, ...

```
>> Nearest[{5, 2.5, 10, 11, 15, 8.5, 14}, 12]
```
$\{11\}$

Return all items within a distance of 5:

```
>> Nearest[{5, 2.5, 10, 11, 15, 8.5, 14}, 12, {All, 5}]
```
$\{11, 10, 14\}$

```
>> Nearest[{Blue -> "blue", White -> "white", Red -> "red", Green -> "green"}, {Orange, Gray}]
```
{{red}, {white}}

```
>> Nearest[{{0, 1}, {1, 2}, {2, 3}} -> {a, b, c}, {1.1, 2}]
```
$\{b\}$

None

`None`
: is a possible value for `Span` and `Quiet`.

Normal

`Normal[expr_]`
: Brings especial expressions to a normal expression from different especial forms.

NotListQ

`NotListQ[`*expr*`]`
: returns true if *expr* is not a list.

PadLeft

`PadLeft[`*list*`, `*n*`]`
: pads *list* to length *n* by adding 0 on the left.

`PadLeft[`*list*`, `*n*`, `*x*`]`
: pads *list* to length *n* by adding *x* on the left.

`PadLeft[`*list*`, {`*n1*`, $n2, ...}, `*x*`]`
: pads *list* to lengths *n1*, *n2* at levels 1, 2, ... respectively by adding *x* on the left.

`PadLeft[`*list*`, `*n*`, `*x*`, `*m*`]`
: pads *list* to length *n* by adding *x* on the left and adding a margin of *m* on the right.

`PadLeft[`*list*`, `*n*`, `*x*`, {`*m1*`, `*m2*`, ...}]`
: pads *list* to length *n* by adding *x* on the left and adding margins of *m1*, *m2*, ... on levels 1, 2, ... on the right.

`PadLeft[`*list*`]`
: turns the ragged list *list* into a regular list by adding 0 on the left.

>> **PadLeft[{1, 2, 3}, 5]**

$\{0,0,1,2,3\}$

>> **PadLeft[x[a, b, c], 5]**

$x[0,0,a,b,c]$

>> **PadLeft[{1, 2, 3}, 2]**

$\{2,3\}$

>> **PadLeft[{{}, {1, 2}, {1, 2, 3}}]**

$\{\{0,0,0\},\{0,1,2\},\{1,2,3\}\}$

>> **PadLeft[{1, 2, 3}, 10, {a, b, c}, 2]**

$\{b,c,a,b,c,1,2,3,a,b\}$

>> **PadLeft[{{1, 2, 3}}, {5, 2}, x, 1]**

$\{\{x,x\},\{x,x\},\{x,x\},\{3,x\},\{x,x\}\}$

PadRight

`PadRight[`*list*`, `*n*`]`
: pads *list* to length *n* by adding 0 on the right.

`PadRight[`*list*`, `*n*`, `*x*`]`
: pads *list* to length *n* by adding *x* on the right.

`PadRight[`*list*`, {`*n1*`, $n2, ...}, `*x*`]`
: pads *list* to lengths *n1*, *n2* at levels 1, 2, ... respectively by adding *x* on the right.

`PadRight[`*list*`, `*n*`, `*x*`, `*m*`]`
: pads *list* to length *n* by adding *x* on the left and adding a margin of *m* on the left.

`PadRight[`*list*`, `*n*`, `*x*`, {`*m1*`, `*m2*`, ...}]`
: pads *list* to length *n* by adding *x* on the right and adding margins of *m1*, *m2*, ... on levels 1, 2, ... on the left.

`PadRight[`*list*`]`
: turns the ragged list *list* into a regular list by adding 0 on the right.

>> **PadRight[{1, 2, 3}, 5]**

$\{1,2,3,0,0\}$

>> **PadRight[x[a, b, c], 5]**

$x[a,b,c,0,0]$

>> **PadRight[{1, 2, 3}, 2]**

$\{1,2\}$

>> **PadRight[{{}, {1, 2}, {1, 2, 3}}]**

$\{\{0,0,0\},\{1,2,0\},\{1,2,3\}\}$

>> **PadRight[{1, 2, 3}, 10, {a, b, c}, 2]**

$\{b,c,1,2,3,a,b,c,a,b\}$

>> **PadRight[{{1, 2, 3}}, {5, 2}, x, 1]**

$\{\{x,x\},\{x,1\},\{x,x\},\{x,x\},\{x,x\}\}$

Part

`Part[`*expr*`, `*i*`]`
: returns part *i* of *expr*.

Extract an element from a list:

```
>> A = {a, b, c, d};
```

```
>> A[[3]]
```
c

Negative indices count from the end:

```
>> {a, b, c}[[-2]]
```
b

`Part` can be applied on any expression, not necessarily lists.

```
>> (a + b + c)[[2]]
```
b

expr`[[0]]` gives the head of *expr*:

```
>> (a + b + c)[[0]]
```
Plus

Parts of nested lists:

```
>> M = {{a, b}, {c, d}};
```

```
>> M[[1, 2]]
```
b

You can use `Span` to specify a range of parts:

```
>> {1, 2, 3, 4}[[2;;4]]
```
$\{2,3,4\}$

```
>> {1, 2, 3, 4}[[2;;-1]]
```
$\{2,3,4\}$

A list of parts extracts elements at certain indices:

```
>> {a, b, c, d}[[{1, 3, 3}]]
```
$\{a,c,c\}$

Get a certain column of a matrix:

```
>> B = {{a, b, c}, {d, e, f}, {g, h
   , i}};
```

```
>> B[[;;, 2]]
```
$\{b,e,h\}$

Extract a submatrix of 1st and 3rd row and the two last columns:

```
>> B = {{1, 2, 3}, {4, 5, 6}, {7,
   8, 9}};
```

```
>> B[[{1, 3}, -2;;-1]]
```
$\{\{2,3\},\{8,9\}\}$

Further examples:

```
>> (a+b+c+d)[[-1;;-2]]
```
0

```
>> x[[2]]
```
Partspecificationislongerthandepthofobject.

$x[[2]]$

Assignments to parts are possible:

```
>> B[[;;, 2]] = {10, 11, 12}
```
$\{10,11,12\}$

```
>> B
```
$\{\{1,10,3\},\{4,11,6\},\{7,12,9\}\}$

```
>> B[[;;, 3]] = 13
```
13

```
>> B
```
$\{\{1,10,13\},\{4,11,13\},\{7,12,13\}\}$

```
>> B[[1;;-2]] = t;
```

```
>> B
```
$\{t,t,\{7,12,13\}\}$

```
>> F = Table[i*j*k, {i, 1, 3}, {j,
   1, 3}, {k, 1, 3}];
```

```
>> F[[;; All, 2 ;; 3, 2]] = t;
```

```
>> F
```
$\{\{\{1,2,3\},\{2,t,6\},\{3,t,9\}\},$
$\{\{2,4,6\},\{4,t,12\},\{6,t,18\}\},$
$\{\{3,6,9\},\{6,t,18\},\{9,t,27\}\}\}$

```
>> F[[;; All, 1 ;; 2, 3 ;; 3]] = k;
```

```
>> F
```
$\{\{\{1,2,k\},\{2,t,k\},\{3,t,9\}\},$
$\{\{2,4,k\},\{4,t,k\},\{6,t,18\}\},$
$\{\{3,6,k\},\{6,t,k\},\{9,t,27\}\}\}$

Of course, part specifications have precedence over most arithmetic operations:

```
>> A[[1]] + B[[2]] + C[[3]] // Hold
    // FullForm
```
Hold [Plus [Part [A, 1], Part [B, 2], Part [C, 3]]]

Partition

`Partition[`*list*`, `*n*`]`
: partitions *list* into sublists of length *n*.

`Parition[`*list*`, `*n*`, `*d*`]`
: partitions *list* into sublists of length *n* which overlap *d* indicies.

```
>> Partition[{a, b, c, d, e, f}, 2]
```
$\{\{a,b\},\{c,d\},\{e,f\}\}$

```
>> Partition[{a, b, c, d, e, f}, 3, 1]
```
$\{\{a,b,c\},\{b,c,d\},\{c,d,e\},\{d,e,f\}\}$

Permutations

`Permutations[`*list*`]`
: gives all possible orderings of the items in *list*.

`Permutations[`*list*`, `*n*`]`
: gives permutations up to length *n*.

`Permutations[`*list*`, {`*n*`}]`
: gives permutations of length *n*.

```
>> Permutations[{y, 1, x}]
```
$\{\{y,1,x\},\{y,x,1\},\{1,y,x\},\{1,x,y\},\{x,y,1\},\{x,1,y\}\}$

Elements are differentiated by their position in *list*, not their value.

```
>> Permutations[{a, b, b}]
```
$\{\{a,b,b\},\{a,b,b\},\{b,a,b\},\{b,b,a\},\{b,a,b\},\{b,b,a\}\}$

```
>> Permutations[{1, 2, 3}, 2]
```
$\{\{\},\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,1\},\{2,3\},\{3,1\},\{3,2\}\}$

```
>> Permutations[{1, 2, 3}, {2}]
```
$\{\{1,2\},\{1,3\},\{2,1\},\{2,3\},\{3,1\},\{3,2\}\}$

Pick

`Pick[`*list*`, `*sel*`]`
: returns those items in *list* that are True in *sel*.

`Pick[`*list*`, `*sel*`, `*patt*`]`
: returns those items in *list* that match *patt* in *sel*.

```
>> Pick[{a, b, c}, {False, True, False}]
```
$\{b\}$

```
>> Pick[f[g[1, 2], h[3, 4]], {{True, False}, {False, True}}]
```
$f\left[g[1],h[4]\right]$

```
>> Pick[{a, b, c, d, e}, {1, 2, 3.5, 4, 5.5}, _Integer]
```
$\{a,b,d\}$

Position

`Position[`*expr*`, `*patt*`]`
: returns the list of positions for which *expr* matches *patt*.

`Position[`*expr*`, `*patt*`, `*ls*`]`
: returns the positions on levels specified by levelspec *ls*.

```
>> Position[{1, 2, 2, 1, 2, 3, 2}, 2]
```
$\{\{2\},\{3\},\{5\},\{7\}\}$

Find positions upto 3 levels deep

```
>> Position[{1 + Sin[x], x, (Tan[x] - y)^2}, x, 3]
```
$\{\{1,2,1\},\{2\}\}$

Find all powers of x

```
>> Position[{1 + x^2, x y ^ 2, 4 y, x ^ z}, x^_]
```
$\{\{1,2\},\{4\}\}$

Use Position as an operator

```
>> Position[_Integer][{1.5, 2, 2.5}]
```
$\{\{2\}\}$

Prepend

`Prepend[`*expr*`, `*item*`]`
: returns *expr* with *item* prepended to its leaves.

`Prepend` is similar to `Append`, but adds *item* to the beginning of *expr*:

```
>> Prepend[{2, 3, 4}, 1]
```
$\{1,2,3,4\}$

`Prepend` works on expressions with heads other than `List`:

```
>> Prepend[f[b, c], a]
```
$f[a,b,c]$

Unlike `Join`, `Prepend` does not flatten lists in *item*:

```
>> Prepend[{c, d}, {a, b}]
```
$\{\{a,b\},c,d\}$

PrependTo

`PrependTo[`*s*`, `*item*`]`
: prepends *item* to value of *s* and sets *s* to the result.

Assign s to a list

```
>> s = {1, 2, 4, 9}
```
$\{1,2,4,9\}$

Add a new value at the beginning of the list:

```
>> PrependTo[s, 0]
```
$\{0,1,2,4,9\}$

The value assigned to s has changed:

```
>> s
```
$\{0,1,2,4,9\}$

`PrependTo` works with a head other than `List`:

```
>> y = f[a, b, c];
```

```
>> PrependTo[y, x]
```
$f[x,a,b,c]$

```
>> y
```
$f[x,a,b,c]$

Quantile

`Quantile[`*list*`, `*q*`]`
: returns the *q*th quantile of *list*.

```
>> Quantile[Range[11], 1/3]
```
4

```
>> Quantile[Range[16], 1/4]
```
5

Quartiles

`Quartiles[`*list*`]`
: returns the 1/4, 1/2, and 3/4 quantiles of *list*.

```
>> Quartiles[Range[25]]
```
$$\left\{\frac{27}{4},13,\frac{77}{4}\right\}$$

Range

`Range[`*n*`]`
: returns a list of integers from 1 to *n*.

`Range[`*a*`, `*b*`]`
: returns a list of integers from *a* to *b*.

```
>> Range[5]
```
$\{1,2,3,4,5\}$

```
>> Range[-3, 2]
```
$\{-3,-2,-1,0,1,2\}$

```
>> Range[0, 2, 1/3]
```
$$\left\{0,\frac{1}{3},\frac{2}{3},1,\frac{4}{3},\frac{5}{3},2\right\}$$

RankedMax

`RankedMax[`*list*`, `*n*`]`
: returns the *n*th largest element of *list* (with $n = 1$ yielding the largest element, $n = 2$ yielding the second largest element, and so on).

```
>> RankedMax[{482, 17, 181, -12},
   2]
```

181

RankedMin

`RankedMin[`*list*`, `*n*`]`
: returns the *n*th smallest element of *list* (with $n = 1$ yielding the smallest element, $n = 2$ yielding the second smallest element, and so on).

```
>> RankedMin[{482, 17, 181, -12},
   2]
```

17

Reap

`Reap[`*expr*`]`
: gives the result of evaluating *expr*, together with all values sown during this evaluation. Values sown with different tags are given in different lists.

`Reap[`*expr*`, `*pattern*`]`
: only yields values sown with a tag matching *pattern*. `Reap[`*expr*`]` is equivalent to `Reap[`*expr*`, _]`.

`Reap[`*expr*`, {`*pattern1*`, `*pattern2*`, ...}]`
: uses multiple patterns.

`Reap[`*expr*`, `*pattern*`, `*f*`]`
: applies *f* on each tag and the corresponding values sown in the form *f*`[tag, {e1, e2, ...}]`.

```
>> Reap[Sow[3]; Sow[1]]
```

$\{1, \{\{3, 1\}\}\}$

```
>> Reap[Sow[2, {x, x, x}]; Sow[3, x
   ]; Sow[4, y]; Sow[4, 1], {
   _Symbol, _Integer, x}, f]
```

$\{4, \{\{f[x, \{2, 2, 2, 3\}], f[y, \{4\}]\}, \{f[1, \{4\}]\}, \{f[x, \{2, 2, 2, 3\}]\}\}\}$

Find the unique elements of a list, keeping their order:

```
>> Reap[Sow[Null, {a, a, b, d, c, a
   }], _, # &][[2]]
```

$\{a, b, d, c\}$

Sown values are reaped by the innermost matching `Reap`:

```
>> Reap[Reap[Sow[a, x]; Sow[b, 1],
   _Symbol, Print["Inner: ",
   #1]&];, _, f]
```

Inner : x

$\{\text{Null}, \{f[1, \{b\}]\}\}$

When no value is sown, an empty list is returned:

```
>> Reap[x]
```

$\{x, \{\}\}$

ReplacePart

`ReplacePart[`*expr*`, `*i*` -> `*new*`]`
: replaces part *i* in *expr* with *new*.

`ReplacePart[`*expr*`, {{`*i*`, `*j*`} -> `*e1*`, {`*k*`, `*l*`} -> `*e2*`}]`
: replaces parts *i* and *j* with *e1*, and parts *k* and *l* with *e2*.

```
>> ReplacePart[{a, b, c}, 1 -> t]
```

$\{t, b, c\}$

```
>> ReplacePart[{{a, b}, {c, d}},
   {2, 1} -> t]
```

$\{\{a, b\}, \{t, d\}\}$

```
>> ReplacePart[{{a, b}, {c, d}},
   {{2, 1} -> t, {1, 1} -> t}]
```

$\{\{t, b\}, \{t, d\}\}$

```
>> ReplacePart[{a, b, c}, {{1},
   {2}} -> t]
```

$\{t, t, c\}$

Delayed rules are evaluated once for each replacement:

```
>> n = 1;
```

```
>> ReplacePart[{a, b, c, d}, {{1},
   {3}} :> n++]
```

$\{1, b, 2, d\}$

Non-existing parts are simply ignored:

```
>> ReplacePart[{a, b, c}, 4 -> t]
```
$\{a, b, c\}$

You can replace heads by replacing part 0:

```
>> ReplacePart[{a, b, c}, 0 ->
   Times]
```
abc

(This is equivalent to `Apply`.)
Negative part numbers count from the end:

```
>> ReplacePart[{a, b, c}, -1 -> t]
```
$\{a, b, t\}$

Rest

`Rest[`*expr*`]`
: returns *expr* with the first element removed.

`Rest[`*expr*`]` is equivalent to *expr*`[[2;;]]`.

```
>> Rest[{a, b, c}]
```
$\{b, c\}$

```
>> Rest[a + b + c]
```
$b + c$

```
>> Rest[x]
```
Nonatomicexpressionexpected.

Rest[x]

Reverse

`Reverse[`*expr*`]`
: reverses the order of *expr*'s items (on the top level)

`Reverse[`*expr*`, `*n*`]`
: reverses the order of items in *expr* on level *n*

`Reverse[`*expr*`, {`*n1*`, `*n2*`, ...}]`
: reverses the order of items in *expr* on levels *n1*, *n2*, ...

```
>> Reverse[{1, 2, 3}]
```
$\{3, 2, 1\}$

```
>> Reverse[x[a, b, c]]
```
$x[c, b, a]$

```
>> Reverse[{{1, 2}, {3, 4}}, 1]
```
$\{\{3,4\}, \{1,2\}\}$

```
>> Reverse[{{1, 2}, {3, 4}}, 2]
```
$\{\{2,1\}, \{4,3\}\}$

```
>> Reverse[{{1, 2}, {3, 4}}, {1,
   2}]
```
$\{\{4,3\}, \{2,1\}\}$

Riffle

`Riffle[`*list*`, `*x*`]`
: inserts a copy of *x* between each element of *list*.

`Riffle[{`*a1*`, `*a2*`, ...}, {`*b1*`, `*b2*`, ...}]`
: interleaves the elements of both lists, returning `{`*a1*`, `*b1*`, `*a2*`, `*b2*`, ...}`.

```
>> Riffle[{a, b, c}, x]
```
$\{a, x, b, x, c\}$

```
>> Riffle[{a, b, c}, {x, y, z}]
```
$\{a, x, b, y, c, z\}$

```
>> Riffle[{a, b, c, d, e, f}, {x, y
   , z}]
```
$\{a, x, b, y, c, z, d, x, e, y, f\}$

RotateLeft

`RotateLeft[`*expr*`]`
: rotates the items of *expr*' by one item to the left.

`RotateLeft[`*expr*`, `*n*`]`
: rotates the items of *expr*' by *n* items to the left.

`RotateLeft[`*expr*`, {`*n1*`, `*n2*`, ...}]`
: rotates the items of *expr*' by *n1* items to the left at the first level, by *n2* items to the left at the second level, and so on.

```
>> RotateLeft[{1, 2, 3}]
```
$\{2, 3, 1\}$

```
>> RotateLeft[Range[10], 3]
```
$\{4, 5, 6, 7, 8, 9, 10, 1, 2, 3\}$

```
>> RotateLeft[x[a, b, c], 2]
```
$x[c,a,b]$

```
>> RotateLeft[{{a, b, c}, {d, e, f
   }, {g, h, i}}, {1, 2}]
```
$\{\{f,d,e\},\{i,g,h\},\{c,a,b\}\}$

RotateRight

`RotateRight[`*expr*`]`
: rotates the items of *expr'* by one item to the right.

`RotateRight[`*expr*`, `*n*`]`
: rotates the items of *expr'* by *n* items to the right.

`RotateRight[`*expr*`, {`*n1*`, `*n2*`, ...}]`
: rotates the items of *expr'* by *n1* items to the right at the first level, by *n2* items to the right at the second level, and so on.

```
>> RotateRight[{1, 2, 3}]
```
$\{3,1,2\}$

```
>> RotateRight[Range[10], 3]
```
$\{8,9,10,1,2,3,4,5,6,7\}$

```
>> RotateRight[x[a, b, c], 2]
```
$x[b,c,a]$

```
>> RotateRight[{{a, b, c}, {d, e, f
   }, {g, h, i}}, {1, 2}]
```
$\{\{h,i,g\},\{b,c,a\},\{e,f,d\}\}$

Select

`Select[{`*e1*`, `*e2*`, ...}, `*f*`]`
: returns a list of the elements *ei* for which *f*[*ei*] returns `True`.

Find numbers greater than zero:
```
>> Select[{-3, 0, 1, 3, a}, #>0&]
```
$\{1,3\}$

`Select` works on an expression with any head:
```
>> Select[f[a, 2, 3], NumberQ]
```
$f[2,3]$

```
>> Select[a, True]
```
Nonatomicexpressionexpected.

Select $[a,$ True$]$

Skewness

`Skewness[`*list*`]`
: gives Pearson's moment coefficient of skewness for *list* (a measure for estimating the symmetry of a distribution).

```
>> Skewness[{1.1, 1.2, 1.4, 2.1,
   2.4}]
```
0.407041

Sow

`Sow[`*e*`]`
: sends the value *e* to the innermost `Reap`.

`Sow[`*e*`, `*tag*`]`
: sows *e* using *tag*. `Sow[`*e*`]` is equivalent to `Sow[`*e*`, Null]`.

`Sow[`*e*`, {`*tag1*`, `*tag2*`, ...}]`
: uses multiple tags.

Span (;;)

`Span`
: is the head of span ranges like `1;;3`.

```
>> ;; // FullForm
```
Span[1, All]

```
>> 1;;4;;2 // FullForm
```
Span[1, 4, 2]

```
>> 2;;-2 // FullForm
```
Span[2, −2]

```
>> ;;3 // FullForm
```
Span[1, 3]

Split

`Split[`*list*`]`
: splits *list* into collections of consecutive identical elements.

`Split[`*list*`, `*test*`]`
: splits *list* based on whether the function *test* yields `True` on consecutive elements.

```
>> Split[{x, x, x, y, x, y, y, z}]
```
$\{\{x,x,x\},\{y\},\{x\},\{y,y\},\{z\}\}$

Split into increasing or decreasing runs of elements

```
>> Split[{1, 5, 6, 3, 6, 1, 6, 3, 4, 5, 4}, Less]
```
$\{\{1,5,6\},\{3,6\},\{1,6\},\{3,4,5\},\{4\}\}$

```
>> Split[{1, 5, 6, 3, 6, 1, 6, 3, 4, 5, 4}, Greater]
```
$\{\{1\},\{5\},\{6,3\},\{6,1\},\{6,3\},\{4\},\{5,4\}\}$

Split based on first element

```
>> Split[{x -> a, x -> y, 2 -> a, z -> c, z -> a}, First[#1] === First[#2] &]
```
$\{\{x->a,x->y\},\{2->a\},\{z->c,z->a\}\}$

SplitBy

`SplitBy[`*list*`, `*f*`]`
: splits *list* into collections of consecutive elements that give the same result when *f* is applied.

```
>> SplitBy[Range[1, 3, 1/3], Round]
```
$\left\{\left\{1,\frac{4}{3}\right\},\left\{\frac{5}{3},2,\frac{7}{3}\right\},\left\{\frac{8}{3},3\right\}\right\}$

```
>> SplitBy[{1, 2, 1, 1.2}, {Round, Identity}]
```
$\{\{\{1\}\},\{\{2\}\},\{\{1\},\{1.2\}\}\}$

StandardDeviation

`StandardDeviation[`*list*`]`
: computes the standard deviation of $list. *list* may consist of numerical values or symbols. Numerical values may be real or complex.

StandardDeviation[{{*a1*, *a2*, ...}, {*b1*, *b2*, ...}, ...}] will yield {StandardDeviation[{*a1*, *b1*, ...}, StandardDeviation[{*a2*, *b2*, ...}], ...}.

```
>> StandardDeviation[{1, 2, 3}]
```
1

```
>> StandardDeviation[{7, -5, 101, 100}]
```
$\frac{\sqrt{13297}}{2}$

```
>> StandardDeviation[{a, a}]
```
0

```
>> StandardDeviation[{{1, 10}, {-1, 20}}]
```
$\left\{\sqrt{2},5\sqrt{2}\right\}$

SubsetQ

`SubsetQ[`*list1*`, `*list2*`]`
: returns True if *list2* is a subset of *list1*, and False otherwise.

```
>> SubsetQ[{1, 2, 3}, {3, 1}]
```
True

The empty list is a subset of every list:

```
>> SubsetQ[{}, {}]
```
True

```
>> SubsetQ[{1, 2, 3}, {}]
```
True

Every list is a subset of itself:

```
>> SubsetQ[{1, 2, 3}, {1, 2, 3}]
```
True

Table

`Table[`*expr*`, {`*i*`, `*n*`}]`
: evaluates *expr* with *i* ranging from 1 to *n*, returning a list of the results.

`Table[`*expr*`, {`*i*`, `*start*`, `*stop*`, `*step*`}]`
: evaluates *expr* with *i* ranging from *start* to *stop*, incrementing by *step*.

`Table[`*expr*`, {`*i*`, {`*e1*`, `*e2*`, ..., `*ei*`}}]`
: evaluates *expr* with *i* taking on the values *e1*, *e2*, ..., *ei*.

```
>> Table[x, {4}]
```
$\{x, x, x, x\}$

```
>> n = 0;
```

```
>> Table[n = n + 1, {5}]
```
$\{1, 2, 3, 4, 5\}$

```
>> Table[i, {i, 4}]
```
$\{1, 2, 3, 4\}$

```
>> Table[i, {i, 2, 5}]
```
$\{2, 3, 4, 5\}$

```
>> Table[i, {i, 2, 6, 2}]
```
$\{2, 4, 6\}$

```
>> Table[i, {i, Pi, 2 Pi, Pi / 2}]
```
$\left\{\mathrm{Pi}, \frac{3\mathrm{Pi}}{2}, 2\mathrm{Pi}\right\}$

```
>> Table[x^2, {x, {a, b, c}}]
```
$\left\{a^2, b^2, c^2\right\}$

`Table` supports multi-dimensional tables:
```
>> Table[{i, j}, {i, {a, b}}, {j,
   1, 2}]
```
$\{\{\{a, 1\}, \{a, 2\}\}, \{\{b, 1\}, \{b, 2\}\}\}$

Take

`Take[`*expr*`, `*n*`]`
: returns *expr* with all but the first *n* leaves removed.

```
>> Take[{a, b, c, d}, 3]
```
$\{a, b, c\}$

```
>> Take[{a, b, c, d}, -2]
```
$\{c, d\}$

```
>> Take[{a, b, c, d, e}, {2, -2}]
```
$\{b, c, d\}$

Take a submatrix:
```
>> A = {{a, b, c}, {d, e, f}};
```

```
>> Take[A, 2, 2]
```
$\{\{a, b\}, \{d, e\}\}$

Take a single column:
```
>> Take[A, All, {2}]
```
$\{\{b\}, \{e\}\}$

TakeLargest

`TakeLargest[`*list*`, `*f*`, `*n*`]`
: returns the a sorted list of the *n* largest items in *list*.

```
>> TakeLargest[{100, -1, 50, 10},
   2]
```
$\{100, 50\}$

None, Null, Indeterminate and expressions with head Missing are ignored by default:
```
>> TakeLargest[{-8, 150, Missing[
   abc]}, 2]
```
$\{150, -8\}$

You may specify which items are ignored using the option ExcludedForms:
```
>> TakeLargest[{-8, 150, Missing[
   abc]}, 2, ExcludedForms -> {}]
```
$\{\mathrm{Missing}[\mathrm{abc}], 150\}$

TakeLargestBy

`TakeLargestBy[`*list*`, `*f*`, `*n*`]`
: returns the a sorted list of the *n* largest items in *list* using *f* to retrieve the items' keys to compare them.

For details on how to use the ExcludedForms option, see TakeLargest[].

>> `TakeLargestBy[{{1, -1}, {10, 100}, {23, 7, 8}, {5, 1}}, Total, 2]`

$\{\{10, 100\}, \{23, 7, 8\}\}$

>> `TakeLargestBy[{"abc", "ab", "x"}, StringLength, 1]`

{abc}

TakeSmallest

`TakeSmallest[`*list*`, `*f*`, `*n*`]`
: returns the a sorted list of the *n* smallest items in *list*.

For details on how to use the ExcludedForms option, see TakeLargest[].

>> `TakeSmallest[{100, -1, 50, 10}, 2]`

$\{-1, 10\}$

TakeSmallestBy

`TakeSmallestBy[`*list*`, `*f*`, `*n*`]`
: returns the a sorted list of the *n* smallest items in *list* using *f* to retrieve the items' keys to compare them.

For details on how to use the ExcludedForms option, see TakeLargest[].

>> `TakeSmallestBy[{{1, -1}, {10, 100}, {23, 7, 8}, {5, 1}}, Total, 2]`

$\{\{1, -1\}, \{5, 1\}\}$

>> `TakeSmallestBy[{"abc", "ab", "x"}, StringLength, 1]`

{x}

Tally

`Tally[`*list*`]`
: counts and returns the number of occurences of objects and returns the result as a list of pairs {object, count}.

`Tally[`*list*`, `*test*`]`
: counts the number of occurences of objects and uses $test to determine if two objects should be counted in the same bin.

>> `Tally[{a, b, c, b, a}]`

$\{\{a, 2\}, \{b, 2\}, \{c, 1\}\}$

Tally always returns items in the order as they first appear in *list*:

>> `Tally[{b, b, a, a, a, d, d, d, d, c}]`

$\{\{b, 2\}, \{a, 3\}, \{d, 4\}, \{c, 1\}\}$

Total

`Total[`*list*`]`
: adds all values in *list*.

`Total[`*list*`, `*n*`]`
: adds all values up to level *n*.

`Total[`*list*`, {`*n*`}]`
: totals only the values at level {*n*}.

`Total[`*list*`, {`*n_1*`, `*n_2*`}]`
: totals at levels {*n_1*, *n_2*}.

>> `Total[{1, 2, 3}]`

6

>> `Total[{{1, 2, 3}, {4, 5, 6}, {7, 8 ,9}}]`

$\{12, 15, 18\}$

Total over rows and columns

>> `Total[{{1, 2, 3}, {4, 5, 6}, {7, 8 ,9}}, 2]`

45

Total over rows instead of columns

>> `Total[{{1, 2, 3}, {4, 5, 6}, {7, 8 ,9}}, {2}]`

$\{6, 15, 24\}$

Tuples

`Tuples[`*list*`, `*n*`]`
: returns a list of all *n*-tuples of elements in *list*.

`Tuples[{`*list1*`, `*list2*`, ...}]`
: returns a list of tuples with elements from the given lists.

```
>> Tuples[{a, b, c}, 2]
```
$\{\{a,a\},\{a,b\},\{a,c\},\{b,a\},\{b,b\},\{b,c\},\{c,a\},\{c,b\},\{c,c\}\}$

```
>> Tuples[{}, 2]
```
$\{\}$

```
>> Tuples[{a, b, c}, 0]
```
$\{\{\}\}$

```
>> Tuples[{{a, b}, {1, 2, 3}}]
```
$\{\{a,1\},\{a,2\},\{a,3\},\{b,1\},\{b,2\},\{b,3\}\}$

The head of *list* need not be `List`:

```
>> Tuples[f[a, b, c], 2]
```
$\{f[a,a],f[a,b],f[a,c],f[b,a],f[b,b],f[b,c],f[c,a],f[c,b],f[c,c]\}$

However, when specifying multiple expressions, `List` is always used:

```
>> Tuples[{f[a, b], g[c, d]}]
```
$\{\{a,c\},\{a,d\},\{b,c\},\{b,d\}\}$

Union

`Union[`*a*`, `*b*`, ...]`
: gives the union of the given set or sets. The resulting list will be sorted and each element will only occur once.

```
>> Union[{5, 1, 3, 7, 1, 8, 3}]
```
$\{1,3,5,7,8\}$

```
>> Union[{a, b, c}, {c, d, e}]
```
$\{a,b,c,d,e\}$

```
>> Union[{c, b, a}]
```
$\{a,b,c\}$

```
>> Union[{{a, 1}, {b, 2}}, {{c, 1}, {d, 3}}, SameTest->(SameQ[Last[#1],Last[#2]]&)]
```
$\{\{b,2\},\{c,1\},\{d,3\}\}$

```
>> Union[{1, 2, 3}, {2, 3, 4}, SameTest->Less]
```
$\{1,2,2,3,4\}$

UnitVector

`UnitVector[`*n*`, `*k*`]`
: returns the *n*-dimensional unit vector with a 1 in position *k*.

`UnitVector[`*k*`]`
: is equivalent to `UnitVector[2, `*k*`]`.

```
>> UnitVector[2]
```
$\{0,1\}$

```
>> UnitVector[4, 3]
```
$\{0,0,1,0\}$

Values

`Values[<|`*key1*` -> `*val1*`, `*key2*` -> `*val2*`, ...|>]`
: return a list of the values *vali* in an association.

`Values[{`*key1*` -> `*val1*`, `*key2*` -> `*val2*`, ...}]`
: return a list of the *vali* in a list of rules.

```
>> Values[<|a -> x, b -> y|>]
```
$\{x,y\}$

```
>> Values[{a -> x, b -> y}]
```
$\{x,y\}$

Values automatically threads over lists:

```
>> Values[{<|a -> x, b -> y|>, {c -> z, {}}}]
```
$\{\{x,y\},\{z,\{\}\}\}$

Values are listed in the order of their appearance:

```
>> Values[{c -> z, b -> y, a -> x}]
```
$\{z,y,x\}$

Variance

`Variance[`*list*`]`
: computes the variance of $list. *list* may consist of numerical values or symbols. Numerical values may be real or complex.

Variance[{{*a1*, *a2*, ...}, {*b1*, *b2*, ...}, ...}] will yield {Variance[{*a1*, *b1*, ...}, Variance[{*a2*, *b2*, ...}], ...}.

>> **`Variance[{1, 2, 3}]`**

1

>> **`Variance[{7, -5, 101, 3}]`**

$$\frac{7475}{3}$$

>> **`Variance[{1 + 2I, 3 - 10I}]`**

74

>> **`Variance[{a, a}]`**

0

>> **`Variance[{{1, 3, 5}, {4, 10, 100}}]`**

$$\left\{\frac{9}{2},\frac{49}{2},\frac{9025}{2}\right\}$$

XL. Mathematical Constants

Numeric, Arithmetic, or Symbolic constants like Pi, E, or Infinity.

Contents

Catalan

`Catalan`
is Catalan's constant with numerical value 0.915966.

>> `Catalan // N`
0.915965594177219

>> `N[Catalan, 20]`
0.91596559417721901505

ComplexInfinity

`ComplexInfinity`
represents an infinite complex quantity of undetermined direction.

>> `1 / ComplexInfinity`
0

>> `ComplexInfinity * Infinity`
ComplexInfinity

>> `FullForm[ComplexInfinity]`
DirectedInfinity []

Degree

`Degree`
is the number of radians in one degree. It has a numerical value of π / 180.

>> `Cos[60 Degree]`
$\frac{1}{2}$

Degree has the value of Pi / 180

>> `Degree == Pi / 180`
True

E

`E`
is the constant with numerical value 2.71828.

>> `N[E]`
2.71828

>> `N[E, 50]`
2.7182818284590452353602874713526624977572470937000

EulerGamma

`EulerGamma`
is Euler's constant with numerial value 0.577216.

>> `EulerGamma // N`
0.577216

>> `N[EulerGamma, 40]`
0.5772156649015328606̃
~065120900824024310422

Glaisher

`Glaisher`
is Glaisher's constant, with numerical value 1.28243.

>> `N[Glaisher]`
1.28242712910062

>> `N[Glaisher, 50]`
1.282427129100622636875342~
~568869791727767688927325

1.282427129100621954194139107130467891693115234375

GoldenRatio

`GoldenRatio`
is the golden ratio, = (1+Sqrt[5])/2.

>> `GoldenRatio // N`
1.61803398874989

>> `N[GoldenRatio, 40]`
1.618033988749894848~
~204586834365638117720

Indeterminate

`Indeterminate`
represents an indeterminate result.

>> `0^0`
Indeterminateexpression0^0encountered.
Indeterminate

>> `Tan[Indeterminate]`
Indeterminate

Infinity

`Infinity`
represents an infinite real quantity.

>> `1 / Infinity`
0

>> `Infinity + 100`
∞

Use `Infinity` in sum and limit calculations:

>> `Sum[1/x^2, {x, 1, Infinity}]`
$\frac{\mathrm{Pi}^2}{6}$

Khinchin

`Khinchin`
is Khinchin's constant, with numerical value 2.68545.

>> `N[Khinchin]`
2.68545200106531

>> `N[Khinchin, 50]`
2.685452001065306445309714~
~8354817956938203822939945

= 2.68545200106530757011569221504032611846923828125

MPMathConstant

Representation of a constant in mpmath, e.g. Pi, E, I, etc.

NumpyConstant

Representation of a constant in numpy, e.g. Pi, E, etc.

Pi

`Pi`
is the constant π.

```
>> N[Pi]
```
3.14159

Force using the value given from numpy to compute Pi.

```
>> N[Pi, Method->"numpy"]
```
3.14159

Force using the value given from sympy to compute Pi to 3 places, two places after the decimal point.
Note that sympy is the default method.

```
>> N[Pi, 3, Method->"sympy"]
```
3.14

```
>> N[Pi, 50]
```
3.14159265358979323846264338327950288419716939937510

```
>> Attributes[Pi]
```
{Constant, Protected, ReadProtected}

SympyConstant

Representation of a constant in Sympy, e.g. Pi, E, I, Catalan, etc.

XLI. Algebraic Manipulation

Contents

Apart

`Apart[`*expr*`]`
: writes *expr* as a sum of individual fractions.

`Apart[`*expr*`, `*var*`]`
: treats *var* as the main variable.

```
>> Apart[1 / (x^2 + 5x + 6)]
```

$$\frac{1}{2+x} - \frac{1}{3+x}$$

When several variables are involved, the results can be different depending on the main variable:

```
>> Apart[1 / (x^2 - y^2), x]
```

$$-\frac{1}{2y\left(x+y\right)} + \frac{1}{2y\left(x-y\right)}$$

```
>> Apart[1 / (x^2 - y^2), y]
```

$$\frac{1}{2x\left(x+y\right)} + \frac{1}{2x\left(x-y\right)}$$

`Apart` is `Listable`:

```
>> Apart[{1 / (x^2 + 5x + 6)}]
```

$$\left\{\frac{1}{2+x} - \frac{1}{3+x}\right\}$$

But it does not touch other expressions:

```
>> Sin[1 / (x ^ 2 - y ^ 2)] //
   Apart
```

$$\text{Sin}\left[\frac{1}{x^2-y^2}\right]$$

Cancel

`Cancel[`*expr*`]`
: cancels out common factors in numerators and denominators.

```
>> Cancel[x / x ^ 2]
```

$$\frac{1}{x}$$

`Cancel` threads over sums:

```
>> Cancel[x / x ^ 2 + y / y ^ 2]
```

$$\frac{1}{x} + \frac{1}{y}$$

```
>> Cancel[f[x] / x + x * f[x] / x ^
   2]
```

$$\frac{2f\left[x\right]}{x}$$

Coefficient

`Coefficient[expr, form]`
: returns the coefficient of *form* in the polynomial *expr*.

`Coefficient[expr, form, n]`
: return the coefficient of *form*^*n* in *expr*.

```
>> Coefficient[(x + y)^4, (x^2)* (y
   ^2)]
```

6

```
>> Coefficient[a x^2 + b y^3 + c x + d y + 5, x]
```
c

```
>> Coefficient[(x + 3 y)^5, x]
```
$405y^4$

```
>> Coefficient[(x + 3 y)^5, x * y^4]
```
405

```
>> Coefficient[(x + 2)/(y - 3)+ (x + 3)/(y - 2), x]
```
$$\frac{1}{-3+y}+\frac{1}{-2+y}$$

```
>> Coefficient[x*Cos[x + 3] + 6*y, x]
```
$\mathrm{Cos}[3+x]$

```
>> Coefficient[(x + 1)^3, x, 2]
```
3

```
>> Coefficient[a x^2 + b y^3 + c x + d y + 5, y, 3]
```
b

```
>> Coefficient[(x + 2)^3 + (x + 3)^2, x, 0]
```
17

```
>> Coefficient[(x + 2)^3 + (x + 3)^2, y, 0]
```
$(2+x)^3+(3+x)^2$

```
>> Coefficient[a x^2 + b y^3 + c x + d y + 5, x, 0]
```
$5+by^3+dy$

CoefficientList

`CoefficientList[poly, var]`
: returns a list of coefficients of powers of *var* in *poly*, starting with power 0.

`CoefficientList[poly, {var1, var2, ...}]`
: returns an array of coefficients of the *vari*.

```
>> CoefficientList[(x + 3)^5, x]
```
$\{243,405,270,90,15,1\}$

```
>> CoefficientList[(x + y)^4, x]
```
$\left\{y^4,4y^3,6y^2,4y,1\right\}$

```
>> CoefficientList[a x^2 + b y^3 + c x + d y + 5, x]
```
$\left\{5+by^3+dy,c,a\right\}$

```
>> CoefficientList[(x + 2)/(y - 3)+ x/(y - 2), x]
```
$$\left\{\frac{2}{-3+y},\frac{1}{-3+y}+\frac{1}{-2+y}\right\}$$

```
>> CoefficientList[(x + y)^3, z]
```
$\left\{(x+y)^3\right\}$

```
>> CoefficientList[a x^2 + b y^3 + c x + d y + 5, {x, y}]
```
$\{\{5,d,0,b\},\{c,0,0,0\},\{a,0,0,0\}\}$

```
>> CoefficientList[(x - 2 y + 3 z)^3, {x, y, z}]
```
$\{\{\{0,0,0,27\},\{0,0,-54,0\},\{0,36,0,0\},\{-8,0,0,0\}\},\{\{0,0,27,0\},\{0,-36,0,0\},\{12,0,0,0\},\{0,0,0,0\}\},\{\{0,9,0,0\},\{-6,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{1,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\}\}$

Denominator

`Denominator[expr]`
: gives the denominator in *expr*.

```
>> Denominator[a / b]
```
b

```
>> Denominator[2 / 3]
```
3

```
>> Denominator[a + b]
```
1

Expand

`Expand[`*expr*`]`
: expands out positive integer powers and products of sums in *expr*, as well as trigonometric identities.

```
>> Expand[(x + y)^ 3]
```
$$x^3 + 3x^2y + 3xy^2 + y^3$$

```
>> Expand[(a + b)(a + c + d)]
```
$$a^2 + ab + ac + ad + bc + bd$$

```
>> Expand[(a + b)(a + c + d)(e + f)
   + e a a]
```
$$2a^2e + a^2f + abe + abf + ace + acf + ade + adf + bce + bcf + bde + bdf$$

```
>> Expand[(a + b)^ 2 * (c + d)]
```
$$a^2c + a^2d + 2abc + 2abd + b^2c + b^2d$$

```
>> Expand[(x + y)^ 2 + x y]
```
$$x^2 + 3xy + y^2$$

```
>> Expand[((a + b)(c + d))^ 2 + b
   (1 + a)]
```
$$a^2c^2 + 2a^2cd + a^2d^2 + b + ab + 2abc^2 + 4abcd + 2abd^2 + b^2c^2 + 2b^2cd + b^2d^2$$

Expand expands items in lists and rules:
```
>> Expand[{4 (x + y), 2 (x + y)-> 4
    (x + y)}]
```
$$\{4x + 4y, 2x + 2y- > 4x + 4y\}$$

Expand expands trigonometric identities
```
>> Expand[Sin[x + y], Trig -> True]
```
$$\mathrm{Cos}\,[x]\,\mathrm{Sin}\,[y] + \mathrm{Cos}\,[y]\,\mathrm{Sin}\,[x]$$

Expand does not change any other expression.
```
>> Expand[Sin[x (1 + y)]]
```
$$\mathrm{Sin}\,[x\,(1 + y)]$$

Expand also works in Galois fields
```
>> Expand[(1 + a)^12, Modulus -> 3]
```
$$1 + a^3 + a^9 + a^{12}$$

```
>> Expand[(1 + a)^12, Modulus -> 4]
```
$$1 + 2a^2 + 3a^4 + 3a^8 + 2a^{10} + a^{12}$$

ExpandAll

`ExpandAll[`*expr*`]`
: expands out negative integer powers and products of sums in *expr*.

```
>> ExpandAll[(a + b)^ 2 / (c + d)
   ^2]
```
$$\frac{a^2}{c^2 + 2cd + d^2} + \frac{2ab}{c^2 + 2cd + d^2} + \frac{b^2}{c^2 + 2cd + d^2}$$

`ExpandAll` descends into sub expressions
```
>> ExpandAll[(a + Sin[x (1 + y)])
   ^2]
```
$$2a\mathrm{Sin}\,[x + xy] + a^2 + \mathrm{Sin}\,[x + xy]^2$$

`ExpandAll` also expands heads
```
>> ExpandAll[((1 + x)(1 + y))[x]]
```
$$(1 + x + y + xy)\,[x]$$

`ExpandAll` can also work in finite fields
```
>> ExpandAll[(1 + a)^ 6 / (x + y)
   ^3, Modulus -> 3]
```
$$\frac{1 + 2a^3 + a^6}{x^3 + y^3}$$

ExpandDenominator

`ExpandDenominator[`*expr*`]`
: expands out negative integer powers and products of sums in *expr*.

```
>> ExpandDenominator[(a + b)^ 2 /
   ((c + d)^2 (e + f))]
```
$$\frac{(a + b)^2}{c^2e + c^2f + 2cde + 2cdf + d^2e + d^2f}$$

Exponent

`Exponent[expr, form]`
: returns the maximum power with which *form* appears in the expanded form of *expr*.

`Exponent[expr, form, h]`
: applies *h* to the set of exponents with which *form* appears in *expr*.

```
>> Exponent[5 x^2 - 3 x + 7, x]
```
2

```
>> Exponent[(x^3 + 1)^2 + 1, x]
```
6

```
>> Exponent[x^(n + 1)+ Sqrt[x] + 1, x]
```
$\text{Max}\left[\frac{1}{2}, 1+n\right]$

```
>> Exponent[x / y, y]
```
-1

```
>> Exponent[(x^2 + 1)^3 - 1, x, Min]
```
2

```
>> Exponent[1 - 2 x^2 + a x^3, x, List]
```
$\{0,2,3\}$

```
>> Exponent[0, x]
```
$-\infty$

```
>> Exponent[1, x]
```
0

Factor

`Factor[`*expr*`]`
: factors the polynomial expression *expr*.

```
>> Factor[x ^ 2 + 2 x + 1]
```
$(1+x)^2$

```
>> Factor[1 / (x^2+2x+1)+ 1 / (x^4+2x^2+1)]
```
$$\frac{2+2x+3x^2+x^4}{(1+x)^2\left(1+x^2\right)^2}$$

FactorTermsList

`FactorTermsList[poly]`
: returns a list of 2 elements. The first element is the numerical factor in *poly*. The second one is the remaining of the polynomial with numerical factor removed

`FactorTermsList[poly, {x1, x2, ...}]`
: returns a list of factors in *poly*. The first element is the numerical factor in *poly*. The next ones are factors that are independent of variables lists which are created by removing each variable *xi* from right to left. The last one is the remaining of polynomial after dividing *poly* to all previous factors

```
>> FactorTermsList[2 x^2 - 2]
```
$\left\{2, -1+x^2\right\}$

```
>> FactorTermsList[x^2 - 2 x + 1]
```
$\left\{1, 1-2x+x^2\right\}$

```
>> f = 3 (-1 + 2 x)(-1 + y)(1 - a)
```
$3(-1+2x)\left(-1+y\right)(1-a)$

```
>> FactorTermsList[f]
```
$\{-3, -1+a-2ax-ay$
$+2x+y-2xy+2axy\}$

```
>> FactorTermsList[f, x]
```
$\{-3, 1-a-y+ay, -1+2x\}$

```
>> FactorTermsList[f, {x, y}]
```
$\{-3, -1+a, -1+y, -1+2x\}$

MinimalPolynomial

`MinimalPolynomial[s, x]`
: gives the minimal polynomial in x for which the algebraic number s is a root.

```
>> MinimalPolynomial[7, x]
```
$-7+x$

```
>> MinimalPolynomial[Sqrt[2] + Sqrt[3], x]
```
$1-10x^2+x^4$

```
>> MinimalPolynomial[Sqrt[1 + Sqrt
   [3]], x]
```
$-2 - 2x^2 + x^4$

```
>> MinimalPolynomial[Sqrt[I + Sqrt
   [6]], x]
```
$49 - 10x^4 + x^8$

Missing

Numerator

`Numerator[`*expr*`]`
: gives the numerator in *expr*.

```
>> Numerator[a / b]
```
a

```
>> Numerator[2 / 3]
```
2

```
>> Numerator[a + b]
```
$a + b$

PolynomialQ

`PolynomialQ[expr, var]`
: returns True if *expr* is a polynomial in *var*, and returns False otherwise.

`PolynomialQ[expr, {var1, ...}]`
: tests whether *expr* is a polynomial in the *vari*.

```
>> PolynomialQ[x^3 - 2 x/y + 3xz, x
   ]
```
True

```
>> PolynomialQ[x^3 - 2 x/y + 3xz, y
   ]
```
False

```
>> PolynomialQ[f[a] + f[a]^2, f[a]]
```
True

```
>> PolynomialQ[x^2 + axy^2 - bSin[c
   ], {x, y}]
```
True

```
>> PolynomialQ[x^2 + axy^2 - bSin[c
   ], {a, b, c}]
```
False

PowerExpand

`PowerExpand[`*expr*`]`
: expands out powers of the form `(x^y)^z` and `(x*y)^z` in *expr*.

```
>> PowerExpand[(a ^ b)^ c]
```
a^{bc}

```
>> PowerExpand[(a * b)^ c]
```
$a^c b^c$

`PowerExpand` is not correct without certain assumptions:

```
>> PowerExpand[(x ^ 2)^ (1/2)]
```
x

Simplify

`Simplify[`*expr*`]`
: simplifies *expr*.

```
>> Simplify[2*Sin[x]^2 + 2*Cos[x
   ]^2]
```
2

```
>> Simplify[x]
```
x

```
>> Simplify[f[x]]
```
$f[x]$

Together

`Together[`*expr*`]`
: writes sums of fractions in *expr* together.

```
>> Together[a / c + b / c]
```
$\frac{a + b}{c}$

`Together` operates on lists:

```
>> Together[{x / (y+1)+ x / (y+1)
   ^2}]
```

$$\left\{\frac{x\left(2+y\right)}{\left(1+y\right)^{2}}\right\}$$

But it does not touch other functions:

```
>> Together[f[a / c + b / c]]
```

$$f\left[\frac{a}{c}+\frac{b}{c}\right]$$

UpTo

Variables

`Variables[`*expr*`]`
: gives a list of the variables that appear in the polynomial *expr*.

```
>> Variables[a x^2 + b x + c]
```

$\{a, b, c, x\}$

```
>> Variables[{a + b x, c y^2 + x
   /2}]
```

$\{a, b, c, x, y\}$

```
>> Variables[x + Sin[y]]
```

$\{x, \mathrm{Sin}\left[y\right]\}$

XLII. Numeric Evaluation

Support for numeric evaluation with arbitrary precision is just a proof-of-concept. Precision is not "guarded" through the evaluation process. Only integer precision is supported. However, things like `N[Pi, 100]` should work as expected.

Contents

Chop

`Chop[`*expr*`]`
: replaces floating point numbers close to 0 by 0.

`Chop[`*expr*`, `*delta*`]`
: uses a tolerance of *delta*. The default tolerance is `10^-10`.

>> `Chop[10.0 ^ -16]`

0

>> `Chop[10.0 ^ -9]`

$1. \times 10^{-9}$

>> `Chop[10 ^ -11 I]`

$\frac{I}{100\,000\,000\,000}$

>> `Chop[0. + 10 ^ -11 I]`

0

Hash

`Hash[`*expr*`]`
: returns an integer hash for the given *expr*.

`Hash[`*expr*`, `*type*`]`
: returns an integer hash of the specified *type* for the given *expr*.
The types supported are "MD5", "Adler32", "CRC32", "SHA", "SHA224", "SHA256", "SHA384", and "SHA512".

`Hash[`*expr*`, `*type*`, `*format*`]`
: Returns the hash in the specified format.

> Hash["The Adventures of Huckleberry Finn"]
= 213425047836523694663619736686226550816
> Hash["The Adventures of Huckleberry Finn", "SHA256"] = 95092649594590384288057183408609254918934351
> Hash[1/3] = 56073172797010645108327809727054836008
> Hash[{a, b, {c, {d, e, f}}}] = 135682164776235407777080772547528
> Hash[SomeHead[3.1415]] = 58042316473471877315442015469706

>> `Hash[{a, b, c}, "xyzstr"]`

Hash $[\{a, b, c\}$, xyzstr, Integer$]$

IntegerDigits

`IntegerDigits[`*n*`]`
: returns a list of the base-10 digits in the integer n.

`IntegerDigits[`*n*`, `*base*`]`
: returns a list of the base-*base* digits in n.

`IntegerDigits[`*n*`, `*base*`, `*length*`]`
: returns a list of length *length*, truncating or padding with zeroes on the left as necessary.

```
>> IntegerDigits[76543]
```
$\{7,6,5,4,3\}$

The sign of n is discarded:

```
>> IntegerDigits[-76543]
```
$\{7,6,5,4,3\}$

```
>> IntegerDigits[15, 16]
```
$\{15\}$

```
>> IntegerDigits[1234, 16]
```
$\{4,13,2\}$

```
>> IntegerDigits[1234, 10, 5]
```
$\{0,1,2,3,4\}$

$MachineEpsilon

`$MachineEpsilon`
: is the distance between `1.0` and the next nearest representable machine-precision number.

```
>> $MachineEpsilon
```
2.22045×10^{-16}

```
>> x = 1.0 + {0.4, 0.5, 0.6}
   $MachineEpsilon;
```

```
>> x - 1
```
$\left\{0.,0.,2.22045 \times 10^{-16}\right\}$

MachinePrecision

`MachinePrecision`
: represents the precision of machine precision numbers.

```
>> N[MachinePrecision]
```
15.9546

```
>> N[MachinePrecision, 30]
```
15.9545897701910033463281614204

$MachinePrecision

`$MachinePrecision`
: is the number of decimal digits of precision for machine-precision numbers.

```
>> $MachinePrecision
```
15.9546

$MaxPrecision

`$MaxPrecision`
: represents the maximum number of digits of precision permitted in abitrary-precision numbers.

```
>> $MaxPrecision
```
∞

```
>> $MaxPrecision = 10;
```

```
>> N[Pi, 11]
```
Requestedprecision11islargerthan$MaxPrecision.Usingcurre

= Infinityspecifiesthatanyprecisionshouldbeallowed.

3.141592654

$MinPrecision

`$MinPrecision`
: represents the minimum number of digits of precision permitted in abitrary-precision numbers.

```
>> $MinPrecision
```
0

```
>> $MinPrecision = 10;
```

```
>> N[Pi, 9]
```

Requestedprecision9issmallerthan$MinPrecision.UsingCurrent$MinPrecisionof10.instead.

3.141592654

N

`N[`*expr*`, `*prec*`]`
: evaluates *expr* numerically with a precision of *prec* digits.

```
>> N[Pi, 50]
```

3.14159265358979323846264338327950288419716939937510

```
>> N[1/7]
```

0.142857

```
>> N[1/7, 5]
```

0.14286

You can manually assign numerical values to symbols. When you do not specify a precision, `MachinePrecision` is taken.

```
>> N[a] = 10.9
```

10.9

```
>> a
```

a

`N` automatically threads over expressions, except when a symbol has attributes `NHoldAll`, `NHoldFirst`, or `NHoldRest`.

```
>> N[a + b]
```

10.9 + *b*

```
>> N[a, 20]
```

a

```
>> N[a, 20] = 11;
```

```
>> N[a + b, 20]
```

11.000000000000000000 + *b*

```
>> N[f[a, b]]
```

f [10.9, *b*]

```
>> SetAttributes[f, NHoldAll]
```

```
>> N[f[a, b]]
```

f [*a*, *b*]

The precision can be a pattern:

```
>> N[c, p_?(#>10&)] := p
```

```
>> N[c, 3]
```

c

```
>> N[c, 11]
```

11.000000000

You can also use `UpSet` or `TagSet` to specify values for `N`:

```
>> N[d] ^= 5;
```

However, the value will not be stored in `UpValues`, but in `NValues` (as for `Set`):

```
>> UpValues[d]
```

{}

```
>> NValues[d]
```

{HoldPattern [*N* [*d*, MachinePrecision]] :>5}

```
>> e /: N[e] = 6;
```

```
>> N[e]
```

6.

Values for `N[`*expr*`]` must be associated with the head of *expr*:

```
>> f /: N[e[f]] = 7;
```

Tagfnotfoundortoodeepforanassignedrule.

You can use `Condition`:

```
>> N[g[x_, y_], p_] := x + y * Pi
   /; x + y > 3
```

```
>> SetAttributes[g, NHoldRest]
```

```
>> N[g[1, 1]]
```

g [1., 1]

```
>> N[g[2, 2]] // InputForm
```

8.283185307179586

The precision of the result is no higher than the precision of the input

```
>> N[Exp[0.1], 100]
```

1.10517

```
>> % // Precision
```

MachinePrecision

```
>> N[Exp[1/10], 100]
```
1.105170918075647624811707˜
˜826490246668224547194737˜
˜518718792863289440967966˜
˜747654302989143318970748654

```
>> % // Precision
```
100.

```
>> N[Exp[1.0`20], 100]
```
2.7182818284590452354

```
>> % // Precision
```
20.

NIntegrate

`NIntegrate[`*expr*`, `*interval*`]`
: returns a numeric approximation to the definite integral of *expr* with limits *interval* and with a precision of *prec* digits.

`NIntegrate[`*expr*`, `*interval1*`, `*interval2*`, ...]`
: returns a numeric approximation to the multiple integral of *expr* with limits *interval1*, *interval2* and with a precision of *prec* digits.

```
>> NIntegrate[Exp[-x],{x,0,Infinity
   },Tolerance->1*^-6]
```
1.

```
>> NIntegrate[Exp[x],{x,-Infinity,
   0},Tolerance->1*^-6]
```
1.

```
>> NIntegrate[Exp[-x^2/2.],{x,-
   Infinity, Infinity},Tolerance
   ->1*^-6]
```
2.50663

```
>> Table[1./NIntegrate[x^k,{x,0,1},
   Tolerance->1*^-6], {k,0,6}]
```
Thespecifiedmethodfailedtoreturnanumber.Falling

$\{1., 2., 3., 4., 5., 6., 7.\}$

```
>> NIntegrate[1 / z, {z, -1 - I, 1
   - I, 1 + I, -1 + I, -1 - I},
   Tolerance->1.*^-4]
```
Integrationoveracomplexdomainisnotimplementedyet

$$\text{NIntegrate}\left[\frac{1}{z}, \{z, -1-I, 1-I, 1+I, -1+I, -1-I\}, \text{Tolerance}->0.0001\right]$$

Integrate singularities with weak divergences:

```
>> Table[ NIntegrate[x^(1./k-1.), {
   x,0,1.}, Tolerance->1*^-6], {k
   ,1,7.} ]
```
$\{1., 2., 3., 4., 5., 6., 7.\}$

Mutiple Integrals :

```
>> NIntegrate[x * y,{x, 0, 1}, {y,
   0, 1}]
```
0.25

NumericQ

`NumericQ[`*expr*`]`
: tests whether *expr* represents a numeric quantity.

```
>> NumericQ[2]
```
True

```
>> NumericQ[Sqrt[Pi]]
```
True

```
>> NumberQ[Sqrt[Pi]]
```
False

Precision

`Precision[`*expr*`]`
: examines the number of significant digits of *expr*.

This is rather a proof-of-concept than a full implementation. Precision of compound expression is not supported yet.

```
>> Precision[1]
```
∞

```
>> Precision[1/2]
```
∞

```
>> Precision[0.5]
```
MachinePrecision

Rationalize

`Rationalize[x]`
: converts a real number x to a nearby rational number.

`Rationalize[x, dx]`
: finds the rational number within dx of x with the smallest denominator.

```
>> Rationalize[2.2]
```
$\frac{11}{5}$

Not all numbers can be well approximated.

```
>> Rationalize[N[Pi]]
```
3.14159

Find the exact rational representation of `N[Pi]`

```
>> Rationalize[N[Pi], 0]
```
$\frac{245\,850\,922}{78\,256\,779}$

RealDigits

`RealDigits[n]`
: returns the decimal representation of the real number n as list of digits, together with the number of digits that are to the left of the decimal point.

`RealDigits[n, b]`
: returns a list of base_b representation of the real number n.

`RealDigits[n, b, len]`
: returns a list of len digits.

`RealDigits[n, b, len, p]`
: return len digits starting with the coefficient of b^p

Return the list of digits and exponent:

```
>> RealDigits[123.55555]
```
$\{\{1,2,3,5,5,5,5,5,$
$0,0,0,0,0,0,0,0\},3\}$

```
>> RealDigits[0.000012355555]
```
$\{\{1,2,3,5,5,5,5,5,0,$
$0,0,0,0,0,0,0\},-4\}$

```
>> RealDigits[-123.55555]
```
$\{\{1,2,3,5,5,5,5,5,$
$0,0,0,0,0,0,0,0\},3\}$

Return 25 digits of in base 10:

```
>> RealDigits[Pi, 10, 25]
```
$\{\{3,1,4,1,5,9,2,6,5,3,5,8,9,$
$7,9,3,2,3,8,4,6,2,6,4,3\},1\}$

Return an explicit recurring decimal form:

```
>> RealDigits[19 / 7]
```
$\{\{2, \{7,1,4,2,8,5\}\},1\}$

20 digits starting with the coefficient of 10^-5:

```
>> RealDigits[Pi, 10, 20, -5]
```
$\{\{9,2,6,5,3,5,8,9,7,9,3,$
$2,3,8,4,6,2,6,4,3\},-4\}$

The 10000th digit of is an 8:

```
>> RealDigits[Pi, 10, 1, -10000]
```
$\{\{8\},-9\,999\}$

RealDigits gives Indeterminate if more digits than the precision are requested:

```
>> RealDigits[123.45, 10, 18]
```
$\{\{1,2,3,4,5,0,0,0,0,0,0,$
$0,0,0,0,0,\text{Indeterminate},$
$\text{Indeterminate}\},3\}$

Internal‘RealValuedNumberQ

Internal‘RealValuedNumericQ

Round

`Round[expr]`
: rounds $expr$ to the nearest integer.

`Round[expr, k]`
: rounds $expr$ to the closest multiple of k.

```
>> Round[10.6]
```
11

```
>> Round[0.06, 0.1]
```
0.1

>> `Round[0.04, 0.1]`

0.

Constants can be rounded too

>> `Round[Pi, .5]`

3.

>> `Round[Pi^2]`

10

Round to exact value

>> `Round[2.6, 1/3]`

$\frac{8}{3}$

>> `Round[10, Pi]`

3Pi

Round complex numbers

>> `Round[6/(2 + 3 I)]`

$1 - I$

>> `Round[1 + 2 I, 2 I]`

$2I$

Round Negative numbers too

>> `Round[-1.4]`

-1

Expressions other than numbers remain unevaluated:

>> `Round[x]`

Round $[x]$

>> `Round[1.5, k]`

Round $[1.5, k]$

XLIII. File Operations

Contents

AbsoluteFileName

`AbsoluteFileName["`*name*`"]`
returns the absolute version of the given filename.

\>> `AbsoluteFileName["ExampleData/sunflowers.jpg"]`

/src/external-vcs/github/mathics/Mathics/matl

BinaryRead

`BinaryRead[`*stream*`]`
reads one byte from the stream as an integer from 0 to 255.

`BinaryRead[`*stream*`, `*type*`]`
reads one object of specified type from the stream.

`BinaryRead[`*stream*`, {`*type1*`, `*type2*`, ...}]`
reads a sequence of objects of specified types.

```
>> strm = OpenWrite[BinaryFormat -> True]
```
OutputStream [/tmp/tmpgpbutnnt, 184]

```
>> BinaryWrite[strm, {97, 98, 99}]
```
OutputStream [/tmp/tmpgpbutnnt, 184]

```
>> Close[strm]
```
/tmp/tmpgpbutnnt

```
>> strm = OpenRead[%, BinaryFormat -> True]
```
InputStream [/tmp/tmpgpbutnnt, 185]

```
>> BinaryRead[strm, {"Character8", "Character8", "Character8"}]
```
{a, b, c}

```
>> Close[strm];
```

BinaryWrite

`BinaryWrite[`*channel*`, `*b*`]`
: writes a single byte given as an integer from 0 to 255.

`BinaryWrite[`*channel*`, {b1, b2, ...}]`
: writes a sequence of byte.

`BinaryWrite[`*channel*`, ‘‘string’]’`
: writes the raw characters in a string.

`BinaryWrite[`*channel*`, `*x*`, `*type*`]`
: writes *x* as the specified type.

`BinaryWrite[`*channel*`, {`*x1*`, `*x2*`, ...}, `*type*`]`
: writes a sequence of objects as the specified type.

`BinaryWrite[`*channel*`, {`*x1*`, `*x2*`, ...}, {`*type1*`, `*type2*`, ...}]`
: writes a sequence of objects using a sequence of specified types.

```
>> strm = OpenWrite[BinaryFormat -> True]
```
OutputStream [/tmp/tmpsz1vavp4, 310]

```
>> BinaryWrite[strm, {39, 4, 122}]
```
OutputStream [/tmp/tmpsz1vavp4, 310]

```
>> Close[strm]
```
/tmp/tmpsz1vavp4

```
>> strm = OpenRead[%, BinaryFormat -> True]
```
InputStream [/tmp/tmpsz1vavp4, 311]

```
>> BinaryRead[strm]
```
39

```
>> BinaryRead[strm, "Byte"]
```
4

```
>> BinaryRead[strm, "Character8"]
```
z

```
>> Close[strm];
```

Write a String

```
>> strm = OpenWrite[BinaryFormat -> True]
```
OutputStream [/tmp/tmp5uqo8fc3, 312]

```
>> BinaryWrite[strm, "abc123"]
```
OutputStream [/tmp/tmp5uqo8fc3, 312]

```
>> Close[%]
```
/tmp/tmp5uqo8fc3

Read as Bytes

```
>> strm = OpenRead[%, BinaryFormat -> True]
```
InputStream [/tmp/tmp5uqo8fc3, 313]

```
>> BinaryRead[strm, {"Character8", "Character8", "Character8", "Character8", "Character8", "Character8", "Character8"}]
```
{a, b, c, 1, 2, 3, EndOfFile}

```
>> Close[strm]
```
/tmp/tmp5uqo8fc3

Read as Characters

```
>> strm = OpenRead[%, BinaryFormat
-> True]
```
InputStream [/tmp/tmp5uqo8fc3, 314]

```
>> BinaryRead[strm, {"Byte", "Byte
", "Byte", "Byte", "Byte", "Byte
", "Byte"}]
```
{97, 98, 99, 49, 50, 51, EndOfFile}

```
>> Close[strm]
```
/tmp/tmp5uqo8fc3

Write Type
```
>> strm = OpenWrite[BinaryFormat ->
 True]
```
OutputStream [/tmp/tmpxzu4q67m, 315]

```
>> BinaryWrite[strm, 97, "Byte"]
```
OutputStream [/tmp/tmpxzu4q67m, 315]

```
>> BinaryWrite[strm, {97, 98, 99},
{"Byte", "Byte", "Byte"}]
```
OutputStream [/tmp/tmpxzu4q67m, 315]

```
>> Close[%]
```
/tmp/tmpxzu4q67m

Byte

`Byte`
  is a data type for `Read`.

Character

`Character`
  is a data type for `Read`.

Close

`Close[`*stream*`]`
  closes an input or output stream.

```
>> Close[StringToStream["123abc"]]
```
String

```
>> Close[OpenWrite[]]
```
/tmp/tmpf_1jpm7c

Compress

`Compress[`*expr*`]`
  gives a compressed string representation of *expr*.

```
>> Compress[N[Pi, 10]]
```
eJwz1jM0MTS1NDIzNQEADRsCNw==

CopyDirectory

`CopyDirectory["`*dir1*`‘‘,"`*dir2*`"]`
  copies directory *dir1* to *dir2*.

CopyFile

`CopyFile["`*file1*`‘‘,"`*file2*`"]`
  copies *file1* to *file2*.

```
>> CopyFile["ExampleData/sunflowers
.jpg", "MathicsSunflowers.jpg"]
```
MathicsSunflowers.jpg

```
>> DeleteFile["MathicsSunflowers.
jpg"]
```

CreateDirectory

`CreateDirectory["`*dir*`"]`
  creates a directory called *dir*.
`CreateDirectory[]`
  creates a temporary directory.

```
>> dir = CreateDirectory[]
```
/tmp/mq90htb2v

CreateFile

`CreateFile[‘‘filename’]’`
: Creates a file named “filename” temporary file, but do not open it.

`CreateFile[]`
: Creates a temporary file, but do not open it.

CreateTemporary

`CreateTemporary[]`
: Creates a temporary file, but do not open it.

DeleteDirectory

`DeleteDirectory["`*dir*`"]`
: deletes a directory called *dir*.

```
>> dir = CreateDirectory[]
```
/tmp/mivzw_j7l

```
>> DeleteDirectory[dir]
```

```
>> DirectoryQ[dir]
```
False

DeleteFile

`Delete["`*file*`"]`
: deletes *file*.

`Delete[{"`*file1*`‘‘,"`*file2*`", ...}]`
: deletes a list of files.

```
>> CopyFile["ExampleData/sunflowers
   .jpg", "MathicsSunflowers.jpg"];
```

```
>> DeleteFile["MathicsSunflowers.
   jpg"]
```

```
>> CopyFile["ExampleData/sunflowers
   .jpg", "MathicsSunflowers1.jpg
   "];
```

```
>> CopyFile["ExampleData/sunflowers
   .jpg", "MathicsSunflowers2.jpg
   "];
```

```
>> DeleteFile[{"MathicsSunflowers1.
   jpg", "MathicsSunflowers2.jpg"}]
```

Directory

`Directory[]`
: returns the current working directory.

```
>> Directory[]
```
/src/external-vcs/github/mathics/Mathics

DirectoryName

`DirectoryName["`*name*`"]`
: extracts the directory name from a filename.

```
>> DirectoryName["a/b/c"]
```
a/b

```
>> DirectoryName["a/b/c", 2]
```
a

DirectoryQ

`DirectoryQ["`*name*`"]`
: returns `True` if the directory called *name* exists and `False` otherwise.

```
>> DirectoryQ["ExampleData/"]
```
True

```
>> DirectoryQ["ExampleData/
   MythicalSubdir/"]
```
False

DirectoryStack

`DirectoryStack[]`
: returns the directory stack.

>> `DirectoryStack[]`

{/src/external-vcs/github/mathics/Mathics}

EndOfFile

`EndOfFile`
: is returned by `Read` when the end of an input stream is reached.

ExpandFileName

`ExpandFileName["`*name*`"]`
: expands *name* to an absolute filename for your system.

>> `ExpandFileName["ExampleData/sunflowers.jpg"]`

/src/external-vcs/github/mathics/Mathics/ExampleData/sunflowers.jpg

Expression

`Expression`
: is a data type for `Read`.

File

FileBaseName

`FileBaseName["`*file*`"]`
: gives the base name for the specified file name.

>> `FileBaseName["file.txt"]`

file

>> `FileBaseName["file.tar.gz"]`

file.tar

FileByteCount

`FileByteCount[`*file*`]`
: returns the number of bytes in *file*.

>> `FileByteCount["ExampleData/sunflowers.jpg"]`

142 286

FileDate

`FileDate[`*file*`, `*types*`]`
: returns the time and date at which the file was last modified.

>> `FileDate["ExampleData/sunflowers.jpg"]`

{2 120, 9, 7, 7, 16, 33.2822}

>> `FileDate["ExampleData/sunflowers.jpg", "Access"]`

{2 121, 4, 4, 8, 50, 39.9091}

>> `FileDate["ExampleData/sunflowers.jpg", "Creation"]`

Missing [NotApplicable]

>> `FileDate["ExampleData/sunflowers.jpg", "Change"]`

{2 120, 9, 7, 7, 16, 33.2822}

>> `FileDate["ExampleData/sunflowers.jpg", "Modification"]`

{2 120, 9, 7, 7, 16, 33.2822}

>> `FileDate["ExampleData/sunflowers.jpg", "Rules"]`

{Access− > {2 121, 4, 4, 8, 50, 39.9091} , Creation− > Missing [NotApplicable] , Change− > { 2 120, 9, 7, 7, 16, 33.282~ ~2} , Modification− > { 2 120, 9, 7, 7, 16, 33.2822}}

FileExistsQ

`FileExistsQ["`*file*`"]`
: returns `True` if *file* exists and `False` otherwise.

>> `FileExistsQ["ExampleData/sunflowers.jpg"]`

True

```
>> FileExistsQ["ExampleData/
sunflowers.png"]
```

False

FileExtension

`FileExtension["`*file*`"]`
gives the extension for the specified file name.

```
>> FileExtension["file.txt"]
```

txt

```
>> FileExtension["file.tar.gz"]
```

gz

FileHash

`FileHash[`*file*`]`
returns an integer hash for the given *file*.
`FileHash[`*file*`, `*type*`]`
returns an integer hash of the specified *type* for the given *file*.
The types supported are “MD5”, “Adler32”, “CRC32”, “SHA”, “SHA224”, “SHA256”, “SHA384”, and “SHA512”.
`FileHash[`*file*`, `*type*`, `*format*`]`
gives a hash code in the specified format.

```
>> FileHash["ExampleData/sunflowers
.jpg"]
```

109 937 059 621 979 839 ~
~952 736 809 235 486 742 106

```
>> FileHash["ExampleData/sunflowers
.jpg", "MD5"]
```

109 937 059 621 979 839 ~
~952 736 809 235 486 742 106

```
>> FileHash["ExampleData/sunflowers
.jpg", "Adler32"]
```

1 607 049 478

```
>> FileHash["ExampleData/sunflowers
.jpg", "SHA256"]
```

111 619 807 552 579 450 300 684 600 ~
~241 129 773 909 359 865 098 672 ~
~286 468 229 443 390 003 894 913 065

FileInformation

`FileInformation["`*file*`"]`
returns information about *file*.

This function is totally undocumented in MMA!

```
>> FileInformation["ExampleData/
sunflowers.jpg"]
```

{File
−> /src/external-vcs/github/mathics/Mathics/ExampleD
FileType− > File, ByteCount− >
142 286, Date− > 6.96413×10^{9}}

FileNameDepth

`FileNameDepth["`*name*`"]`
gives the number of path parts in the given filename.

```
>> FileNameDepth["a/b/c"]
```

3

```
>> FileNameDepth["a/b/c/"]
```

3

FileNameJoin

`FileNameJoin[{"`*dir_1*`‘‘,"`*dir_2*`", ...}]`
joins the *dir_i* together into one path.
`FileNameJoin[..., OperatingSystem->‘‘os’]’`
yields a file name in the format for the specified operating system. Possible choices are “Windows”, “MacOSX”, and “Unix”.

```
>> FileNameJoin[{"dir1", "dir2", "
dir3"}]
```

dir1/dir2/dir3

```
>> FileNameJoin[{"dir1", "dir2", "
dir3"}, OperatingSystem -> "Unix
"]
```

dir1/dir2/dir3

```
>> FileNameJoin[{"dir1", "dir2", "
   dir3"}, OperatingSystem -> "
   Windows"]
```

dir1\dir2\dir3

FileNameSplit

`FileNameSplit["`*filenams*`"]`
: splits a *filename* into a list of parts.

```
>> FileNameSplit["example/path/file
   .txt"]
```

{example, path, file.txt}

FileNameTake

`FileNameTake["`*file*`"]`
: returns the last path element in the file name *name*.

`FileNameTake["`*file*`", `*n*`]`
: returns the first *n* path elements in the file name *name*.

`FileNameTake["`*file*`", $-n$]`
: returns the last *n* path elements in the file name *name*.

FileNames

`FileNames[]`
: Returns a list with the filenames in the current working folder.

`FileNames[`*form*`]`
: Returns a list with the filenames in the current working folder that matches with *form*.

`FileNames[{`*form_1*`, `*form_2*`, ...}]`
: Returns a list with the filenames in the current working folder that matches with one of *form_1*, *form_2*, ...

`FileNames[{`*form_1*`, `*form_2*`, ...},{`*dir_1*`, `*dir_2*`, ...}]`
: Looks into the directories *dir_1*, *dir_2*, ...

`FileNames[{`*form_1*`, `*form_2*`, ...},{`*dir_1*`, `*dir_2*`, ...}]`
: Looks into the directories *dir_1*, *dir_2*, ...

`FileNames[{`*forms*`, `*dirs*`, `*n*`]`
: Look for files up to the level *n*.

```
>> SetDirectory[
   $InstallationDirectory <> "/
   autoload"];
```

```
>> FileNames["*.m", "formats"]//
   Length
```

0

```
>> FileNames["*.m", "formats", 3]//
   Length
```

12

```
>> FileNames["*.m", "formats",
   Infinity]//Length
```

12

FilePrint

`FilePrint[`*file*`]`
: prints the raw contents of *file*.

FileType

`FileType["`*file*`"]`
: gives the type of a file, a string. This is typically `File`, `Directory` or `None`.

```
>> FileType["ExampleData/sunflowers
   .jpg"]
```

File

```
>> FileType["ExampleData"]
```

Directory

```
>> FileType["ExampleData/
   nonexistant"]
```

None

Find

`Find[`*stream*`, `*text*`]`
: find the first line in *stream* that contains *text*.

```
>> str = OpenRead["ExampleData/
   EinsteinSzilLetter.txt"];
```

>> `Find[str, "uranium"]`

in manuscript, leads me
to expect that the element
uranium may be turned into

>> `Find[str, "uranium"]`

become possible to set up
a nuclear chain reaction in
a large mass of uranium,

>> `Close[str]`

ExampleData/EinsteinSzilLetter.txt

>> `str = OpenRead["ExampleData/EinsteinSzilLetter.txt"];`

>> `Find[str, {"energy", "power"} ]`

a new and important source
of energy in the immediate
future. Certain aspects

>> `Find[str, {"energy", "power"} ]`

by which vast amounts of
power and large quantities
of new radium-like

>> `Close[str]`

ExampleData/EinsteinSzilLetter.txt

FindFile

`FindFile[`*name*`]`
: searches `$Path` for the given filename.

>> `FindFile["ExampleData/sunflowers.jpg"]`

/src/external-vcs/github/mathics/Mathics/mathics/data/ExampleData/sunflowers.jpg

>> `FindFile["VectorAnalysis`"]`

/src/external-vcs/github/mathics/Mathics/mathics/packages/VectorAnalysis/Kernel/init.m

>> `FindFile["VectorAnalysis`VectorAnalysis`"]`

/src/external-vcs/github/mathics/Mathics/mathics/packages/VectorAnalysis/VectorAnalysis.m

FindList

`FindList[`*file*`, `*text*`]`
: returns a list of all lines in *file* that contain *text*.

`FindList[`*file*`, {`*text1*`, `*text2*`, ...}]`
: returns a list of all lines in *file* that contain any of the specified string.

`FindList[{`*file1*`, `*file2*`, ...}, ...]`
: returns a list of all lines in any of the *filei* that contain the specified strings.

>> `str = FindList["ExampleData/EinsteinSzilLetter.txt", "uranium"];`

>> `FindList["ExampleData/EinsteinSzilLetter.txt", "uranium", 1]`

{in manuscript, leads me
to expect that the element
uranium may be turned into}

Get (<<)

`<<`*name*
: reads a file and evaluates each expression, returning only the last one.

>> `filename = $TemporaryDirectory <> "/example_file";`

>> `Put[x + y, filename]`

>> `Get[filename]`

"x"cannotbefollowedby"
text{+}y"(line1of"/tmp/example_file").

>> `filename = $TemporaryDirectory <> "/example_file";`

>> `Put[x + y, 2x^2 + 4z!, Cos[x] + I Sin[x], filename]`

>> `Get[filename]`

"x"cannotbefollowedby"
text{+}y"(line1of"/tmp/example_file").

>> `DeleteFile[filename]`

$HomeDirectory

`$HomeDirectory`
: returns the users HOME directory.

```
>> $HomeDirectory
```
/home/rocky

$InitialDirectory

`$InitialDirectory`
: returns the directory from which *Mathics* was started.

```
>> $InitialDirectory
```
/src/external-vcs/github/mathics/Mathics

$Input

`$Input`
: is the name of the stream from which input is currently being read.

```
>> $Input
```

$InputFileName

`$InputFileName`
: is the name of the file from which input is currently being read.

While in interactive mode, `$InputFileName` is "".

```
>> $InputFileName
```
/tmp/example_file

InputStream

`InputStream[`*name*`, `*n*`]`
: represents an input stream.

```
>> str = StringToStream["Mathics is cool!"]
```
InputStream [String, 375]

```
>> Close[str]
```
String

$InstallationDirectory

`$InstallationDirectory`
: returns the directory in which *Mathics* was installed.

```
>> $InstallationDirectory
```
/src/external-vcs/github/mathics/Mathics/mathics

Needs

`Needs["context`"]`
: loads the specified context if not already in `$Packages`.

```
>> Needs["VectorAnalysis`"]
```

Number

`Number`
: is a data type for `Read`.

OpenAppend

`OpenAppend[``file']'`
: opens a file and returns an OutputStream to which writes are appended.

```
>> OpenAppend[]
```
OutputStream [
/tmp/tmphi1fwdet, 385]

OpenRead

`OpenRead[‘‘file’]’`
: opens a file and returns an InputStream.

```
>> OpenRead["ExampleData/EinsteinSzilLetter.txt"]
```

InputStream [ExampleData/EinsteinSzilLetter.txt, 391]

```
>> OpenRead["https://raw.githubusercontent.com/mathics/Mathics/master/README.rst"]
```

InputStream [https://raw.githubusercontent.com/mathics/Mathics/master/README.rst, 392]

```
>> Close[%];
```

OpenWrite

`OpenWrite[‘‘file’]’`
: opens a file and returns an OutputStream.

```
>> OpenWrite[]
```

OutputStream [/tmp/tmpa4sbgv4e, 396]

$OperatingSystem

`$OperatingSystem`
: gives the type of operating system running Mathics.

```
>> $OperatingSystem
```

Unix

OutputStream

`OutputStream[name, n]`
: represents an output stream.

```
>> OpenWrite[]
```

OutputStream [/tmp/tmpypoe6anr, 399]

```
>> Close[%]
```

/tmp/tmpypoe6anr

ParentDirectory

`ParentDirectory[]`
: returns the parent of the current working directory.

`ParentDirectory["dir"]`
: returns the parent *dir*.

```
>> ParentDirectory[]
```

/src/external-vcs/github/mathics/Mathics/mathics

$Path

`$Path`
: returns the list of directories to search when looking for a file.

```
>> $Path
```

{., /home/rocky, /src/external-vcs/github/mathics/Mathics/mathics/data, /src/external-vcs/github/mathics/Mathics/mathics/packa

$PathnameSeparator

`$PathnameSeparator`
: returns a string for the seperator in paths.

```
>> $PathnameSeparator
```

/

Put (>>)

expr >> *filename*
: write *expr* to a file.

`Put[`*expr1*`,` *expr2*`, ...,` *filename*`]`
: write a sequence of expressions to a file.

```
>> Put[40!, fortyfactorial]
```
fortyfactorialisnotstring, InputStream[], orOutputStream[]

815 915 283 247 897 734 345 ˜
˜611 269 596 115 894 272 ˜
˜000 000 000»fortyfactorial

```
>> filename = $TemporaryDirectory
<> "/fortyfactorial";

>> Put[40!, filename]

>> FilePrint[filename]
```
815 915 283 247 897 734 345 611 ˜
˜269 596 115 894 272 000 000 000

```
>> Get[filename]
```
815 915 283 247 897 734 345 611 ˜
˜269 596 115 894 272 000 000 000

```
>> DeleteFile[filename]

>> filename = $TemporaryDirectory
<> "/fiftyfactorial";

>> Put[10!, 20!, 30!, filename]

>> FilePrint[filename]
```
3 628 800
2 432 902 008 176 640 000
265 252 859 812 191 ˜
˜058 636 308 480 000 000

```
>> DeleteFile[filename]
```

=

```
>> filename = $TemporaryDirectory
<> "/example_file";

>> Put[x + y, 2x^2 + 4z!, Cos[x] +
I Sin[x], filename]

>> FilePrint[filename]
```
x
$text\{+\}y$
$2 * x\{\}^{\{}$
$wedge\}2$
$text\{+\}4 * z!$

$text\{Cos\}$
$left[x$
$right]$
$text\{+\}I *$
$text\{Sin\}$
$left[x$
$right]$

```
>> DeleteFile[filename]
```

PutAppend (>>>)

expr >>> *filename*
 append *expr* to a file.
PutAppend[*expr1*, *expr2*, ..., $‘‘ filename’$]’
 write a sequence of expressions to a file.

```
>> Put[50!, "factorials"]

>> FilePrint["factorials"]
```
30 414 093 201 713 378 043 612 ˜
˜608 166 064 768 844 377 641 ˜
˜568 960 512 000 000 000 000

```
>> PutAppend[10!, 20!, 30!, "
factorials"]

>> FilePrint["factorials"]
```
30 414 093 201 713 378 043 612 ˜
˜608 166 064 768 844 377 641 ˜
˜568 960 512 000 000 000 000
3 628 800
2 432 902 008 176 640 000
265 252 859 812 191 ˜
˜058 636 308 480 000 000

```
>> 60! >>> "factorials"
```

```
>> FilePrint["factorials"]
```
30 414 093 201 713 378 043 612 ~
  ~608 166 064 768 844 377 641 ~
  ~568 960 512 000 000 000 000
3 628 800
2 432 902 008 176 640 000
265 252 859 812 191 ~
  ~058 636 308 480 000 000
8 320 987 112 741 390 144 ~
  ~276 341 183 223 364 380 754 ~
  ~172 606 361 245 952 449 277 ~
  ~696 409 600 000 000 000 000

```
>> "string" >>> factorials
```

```
>> FilePrint["factorials"]
```
30 414 093 201 713 378 043 612 ~
  ~608 166 064 768 844 377 641 ~
  ~568 960 512 000 000 000 000
3 628 800
2 432 902 008 176 640 000
265 252 859 812 191 ~
  ~058 636 308 480 000 000
8 320 987 112 741 390 144 ~
  ~276 341 183 223 364 380 754 ~
  ~172 606 361 245 952 449 277 ~
  ~696 409 600 000 000 000 000
"string"

Read

`Read[stream]`
: reads the input stream and returns one expression.

`Read[stream, type]`
: reads the input stream and returns an object of the given type.

```
>> str = StringToStream["abc123"];
```

```
>> Read[str, String]
```
abc123

```
>> str = StringToStream["abc 123"];
```

```
>> Read[str, Word]
```
abc

```
>> Read[str, Word]
```
123

```
>> str = StringToStream["123, 4"];
```

```
>> Read[str, Number]
```
123

```
>> Read[str, Number]
```
4

```
>> str = StringToStream["123 abc"];
```

```
>> Read[str, {Number, Word}]
```
{123, abc}

Multiple lines:

```
>> str = StringToStream["\"Tengo
una\nvaca lechera.\""]; Read[str
]
```
Tengo una
  vaca lechera.

ReadList

`ReadList["file"]`
: Reads all the expressions until the end of file.

`ReadList["file", type]`
: Reads objects of a specified type until the end of file.

`ReadList["file", {type1, type2, ...}]`
: Reads a sequence of specified types until the end of file.

```
>> ReadList[StringToStream["a 1 b
2"], {Word, Number}]
```
{{a, 1}, {b, 2}}

```
>> str = StringToStream["\"abc123
\""];
```

```
>> ReadList[str]
```
{abc123}

```
>> InputForm[%]
```
{"abc123"}

Record

`Record`
: is a data type for `Read`.

RenameDirectory

`RenameDirectory["`*dir1*`‘ ‘,"`*dir2*`"]`
    renames directory *dir1* to *dir2*.

RenameFile

`RenameFile["`*file1*`‘ ‘,"`*file2*`"]`
    renames *file1* to *file2*.

```
>> CopyFile["ExampleData/sunflowers
   .jpg", "MathicsSunflowers.jpg"]
```
MathicsSunflowers.jpg

```
>> RenameFile["MathicsSunflowers.
   jpg", "MathicsSunnyFlowers.jpg"]
```
MathicsSunnyFlowers.jpg

```
>> DeleteFile["MathicsSunnyFlowers.
   jpg"]
```

ResetDirectory

`ResetDirectory[]`
    pops a directory from the directory stack and returns it.

```
>> ResetDirectory[]
```
/src/external-vcs/github/mathics/Mathics/matl

$RootDirectory

`$RootDirectory`
    returns the system root directory.

```
>> $RootDirectory
```
/

SetDirectory

`SetDirectory[`*dir*`]`
    sets the current working directory to *dir*.

```
>> SetDirectory[]
```
/home/rocky

SetFileDate

`SetFileDate["`*file*`"]`
    set the file access and modification dates of *file* to the current date.

`SetFileDate["`*file*`", `*date*`]`
    set the file access and modification dates of *file* to the specified date list.

`SetFileDate["`*file*`", `*date*`, "`*type*`"]`
    set the file date of *file* to the specified date list. The "*type*“ can be one of ”*Access*“, ”*Creation*“, ”*Modification*", or `All`.

Create a temporary file (for example purposes)

```
>> tmpfilename =
   $TemporaryDirectory <> "/tmp0";
```

```
>> Close[OpenWrite[tmpfilename]];
```

```
>> SetFileDate[tmpfilename, {2002,
   1, 1, 0, 0, 0.}, "Access"];
```

```
>> FileDate[tmpfilename, "Access"]
```
$\{2002, 1, 1, 0, 0, 0.\}$

SetStreamPosition

`SetStreamPosition[`*stream*`, `*n*`]`
    sets the current position in a stream.

```
>> str = StringToStream["Mathics is
    cool!"]
```
InputStream [String, 482]

```
>> SetStreamPosition[str, 8]
```
8

```
>> Read[str, Word]
```
is

```
>> SetStreamPosition[str, Infinity]
```
16

Skip

`Skip[`*stream*`, `*type*`]`
: skips ahead in an input steream by one object of the specified *type*.

`Skip[`*stream*`, `*type*`, `*n*`]`
: skips ahead in an input steream by *n* objects of the specified *type*.

```
>> str = StringToStream["a b c d"];
```

```
>> Read[str, Word]
```
a

```
>> Skip[str, Word]
```

```
>> Read[str, Word]
```
c

```
>> str = StringToStream["a b c d"];
```

```
>> Read[str, Word]
```
a

```
>> Skip[str, Word, 2]
```

```
>> Read[str, Word]
```
d

StreamPosition

`StreamPosition[`*stream*`]`
: returns the current position in a stream as an integer.

```
>> str = StringToStream["Mathics is cool!"]
```
InputStream [String, 491]

```
>> Read[str, Word]
```
Mathics

```
>> StreamPosition[str]
```
7

Streams

`Streams[]`
: returns a list of all open streams.

```
>> Streams[]
```
{InputStream [<stdin>, 0], OutputStream [<stdout>, 1], OutputStream [<stderr>, 2], OutputStream [/tmp/tmp6sce_u38, 384], OutputStream [/tmp/tmphi1fwdet, 385], InputStream [/src/external-vcs/github/mathics/Mathics/mathics/data/ 389], InputStream [/src/external-vcs/github/mathics/Mathics/mathics/data/ 391], OutputStream [/tmp/tmpcdofme7t, 395], OutputStream [/tmp/tmpa4sbgv4e, 396], InputStream [String, 449], InputStream [String, 459], InputStream [String, 460], InputStream [String, 461], InputStream [String, 462], InputStream [String, 463], InputStream [String, 464], InputStream [String, 465], InputStream [String, 466], InputStream [String, 467], InputStream [String, 468], InputStream [String, 469], InputStream [String, 470], InputStream [String, 472], InputStream [String, 473], InputStream [String, 474], InputStream [String, 475], InputStream [String, 476], InputStream [String, 480], InputStream [String, 481], InputStream [String, 482], InputStream [String, 485], InputStream [String, 486], InputStream [String, 487], InputStream [String, 488], InputStream [String, 489], InputStream [String, 490], InputStream [String, 491], OutputStream [/tmp/tmprn8h2x15, 492]}

StringToStream

`StringToStream[`*string*`]`
: converts a *string* to an open input stream.

```
>> strm = StringToStream["abc 123"]
```
InputStream [String, 495]

$TemporaryDirectory

`$TemporaryDirectory`
: returns the directory used for temporary files.

```
>> $TemporaryDirectory
```
/tmp

ToFileName

`ToFileName[{"`*dir_1*`‘‘,"`*dir_2*`", ...}]`
: joins the *dir_i* together into one path.

`ToFileName` has been superseded by `FileNameJoin`.

```
>> ToFileName[{"dir1", "dir2"}, "
   file"]
```
dir1/dir2/file

```
>> ToFileName["dir1", "file"]
```
dir1/file

```
>> ToFileName[{"dir1", "dir2", "
   dir3"}]
```
dir1/dir2/dir3

URLSave

`URLSave[‘‘url’]’`
: Save “url” in a temporary file.

`URLSave[‘‘url’,’ `*filename*`]`
: Save “url” in *filename*.

Uncompress

`Uncompress["`*string*`"]`
: recovers an expression from a string generated by `Compress`.

```
>> Compress["Mathics is cool"]
```
eJxT8k0sychMLlbILFZIzs/PUQIANFwF1w==

```
>> Uncompress[%]
```
Mathics is cool

```
>> a = x ^ 2 + y Sin[x] + 10 Log
   [15];
```

```
>> b = Compress[a];
```

```
>> Uncompress[b]
```
$x^2 + y\mathrm{Sin}[x] + 10\mathrm{Log}[15]$

Word

`Word`
: is a data type for `Read`.

Write

`Write[`*channel*`, `*expr1*`, `*expr2*`, ...]`
: writes the expressions to the output channel followed by a newline.

```
>> str = OpenWrite[]
```
OutputStream [
/tmp/tmppafnbov9, 500]

```
>> Write[str, 10 x + 15 y ^ 2]
```

```
>> Write[str, 3 Sin[z]]
```

```
>> Close[str]
```
/tmp/tmppafnbov9

```
>> str = OpenRead[%];
```

```
>> ReadList[str]
```
$\{10x + 15y^2, 3\mathrm{Sin}[z]\}$

WriteString

`WriteString[`*stream*`, $str1, `*str2*`, ... ]`
: writes the strings to the output stream.

```
>> str = OpenWrite[];
```

```
>> WriteString[str, "This is a test
    1"]
```

```
>> WriteString[str, "This is also a
    test 2"]
```

```
>> Close[str]
```

/tmp/tmp5_i9m7dp

```
>> FilePrint[%]
```

*Thisisatest*1*Thisisalsoatest*2

```
>> str = OpenWrite[];
```

```
>> WriteString[str, "This is a test
    1", "This is also a test 2"]
```

```
>> Close[str]
```

/tmp/tmpo4whswm_

```
>> FilePrint[%]
```

*Thisisatest*1*Thisisalsoatest*2

XLIV. Importing and Exporting

Contents

System'Convert'B64Dump'B64Decode

`System'Convert'B64Dump'B64Decode[`*string*`]`
: Decode *string* in Base64 coding to an expression.

>> `System'Convert'B64Dump'B64Decode["R!="]`

String"R! = "isnotavalidb64encodedstring.

$Failed

System'Convert'B64Dump'B64Encode

`System'Convert'B64Dump'B64Encode[`*expr*`]`
: Encodes *expr* in Base64 coding

>> `System'Convert'B64Dump'B64Encode["Hello world"]`

SGVsbG8gd29ybGQ=

>> `System'Convert'B64Dump'B64Decode[%]`

Hello world

>> `System'Convert'B64Dump'B64Encode[Integrate[f[x],{x,0,2}]]`

SW50ZWdyYXRlW2ZbeF0sIHt4LCAwLCAyfV0=

>> `System'Convert'B64Dump'B64Decode[%]`

Integrate[f[x], {x, 0, 2}]

System'Convert'CommonDump'RemoveLinearSyntax

`System'Convert'CommonDump'RemoveLinearSyntax[`*something*`]`
: Keine anung... Undocumented in wma

System'ConvertersDump'$extensionMappings

`$extensionMappings`
: Returns a list of associations between file extensions and file types.

System'ConvertersDump'$formatMappings

`$formatMappings`
: Returns a list of associations between file extensions and file types.

Export

`Export["`*file.ext*`", `*expr*`]`
: exports *expr* to a file, using the extension *ext* to determine the format.

`Export["`*file*`", `*expr*`, "`*format*`"]`
: exports *expr* to a file in the specified format.

`Export["`*file*`", `*exprs*`, `*elems*`]`
: exports *exprs* to a file as elements specified by *elems*.

$ExportFormats

`$ExportFormats`
: returns a list of file formats supported by Export.

```
>> $ExportFormats
```

{BMP, Base64, CSV, GIF, JPEG, JPEG2 000, PBM, PCX, PGM, PNG, PPM, SVG, TIFF, Text}

ExportString

`ExportString[`*expr*`, `*form*`]`
: exports *expr* to a string, in the format *form*.

`Export["`*file*`", `*exprs*`, `*elems*`]`
: exports *exprs* to a string as elements specified by *elems*.

```
>> ExportString
   [{{1,2,3,4},{3},{2},{4}}, "CSV"]
```

1,2,3,4
3,
2,
4,

```
>> ExportString[{1,2,3,4}, "CSV"]
```

1,
2,
3,
4,

```
>> ExportString[Integrate[f[x],{x
   ,0,2}], "SVG"]//Head
```

String

FileFormat

`FileFormat["`*name*`"]`
: attempts to determine what format `Import` should use to import specified file.

```
>> FileFormat["ExampleData/
   sunflowers.jpg"]
```

JPEG

```
>> FileFormat["ExampleData/
   EinsteinSzilLetter.txt"]
```

Text

```
>> FileFormat["ExampleData/lena.tif
   "]
```

TIFF

Import

`Import["`*file*`"]`
: imports data from a file.

`Import["`*file*`", `*elements*`]`
: imports the specified elements from a file.

`Import["http://`*url*`", ...]` and `Import["ftp://`*url*`", ...]`
: imports from a URL.

```
>> Import["ExampleData/ExampleData.
   txt", "Elements"]
```

{Data, Lines, Plaintext, String, Words}

```
>> Import["ExampleData/ExampleData.
   txt", "Lines"]
```

{Example File Format, Created by Angus, 0.629452 0.586355, 0.711009 0.687453, 0.246540 0.433973, 0.926871 0.887255, 0.825141 0.940900, 0.847035 0.127464, 0.054348 0.296494, 0.838545 0.247025, 0.838697 0.436220, 0.309496 0.833591}

```
>> Import["ExampleData/colors.json
"]
```

{colorsArray
−> {{colorName−> black,
rgbValue−> (0, 0, 0),
hexValue−> #000 000} ,
{colorName−> red,
rgbValue−> (255, 0, 0),
hexValue−> #FF0 000} ,
{colorName−> green,
rgbValue−> (0, 255, 0),
hexValue−> #00FF00} ,
{colorName−> blue,
rgbValue−> (0, 0, 255),
hexValue−> #0 000FF} ,
{colorName−> yellow,
rgbValue−> (255, 255, 0),
hexValue−> #FFFF00} ,
{colorName−> cyan,
rgbValue−> (0, 255, 255),
hexValue−> #00FFFF} ,
{colorName−> magenta,
rgbValue−> (255, 0, 255),
hexValue−> #FF00FF} ,
{colorName−> white,
rgbValue−> (255, 255, 255),
hexValue−> #FFFFFF}}}

$ImportFormats

`$ImportFormats`
returns a list of file formats supported by Import.

```
>> $ImportFormats
```

{BMP, Base64, CSV, GIF,
ICO, JPEG, JPEG2 000, JSON,
PBM, PCX, PGM, PNG, PPM,
Package, TGA, TIFF, Text, XML}

ImportString

`ImportString["`*data*`‘‘,"`*format*`"]`
imports data in the specified format from a string.

`ImportString["`*file*`", `*elements*`]`
imports the specified elements from a string.

`ImportString["`*data*`"]`
attempts to determine the format of the string from its content.

```
>> str = "Hello!\n This is a
testing text\n";
```

```
>> ImportString[str, "Elements"]
```

{Data, Lines, Plaintext, String, Words}

```
>> ImportString[str, "Lines"]
```

{Hello!, This is a testing text}

ImportExport‘RegisterExport

`RegisterExport["`*format*`", `*func*`]`
register *func* as the default function used when exporting from a file of type "*format*".

Simple text exporter

```
>> ExampleExporter1[filename_,
data_, opts___] := Module[{strm
= OpenWrite[filename], char =
data}, WriteString[strm, char];
Close[strm]]
```

```
>> ImportExport‘RegisterExport["
ExampleFormat1",
ExampleExporter1]
```

```
>> Export["sample.txt", "Encode
this string!", "ExampleFormat1
"];
```

```
>> FilePrint["sample.txt"]
```

Encodethisstring!

Very basic encrypted text exporter

```
>> ExampleExporter2[filename_,
   data_, opts___] := Module[{strm
   = OpenWrite[filename], char}, (*
    TODO: Check data *)char =
   FromCharacterCode[Mod[
   ToCharacterCode[data] - 84, 26]
   + 97]; WriteString[strm, char];
   Close[strm]]
```

```
>> ImportExport`RegisterExport["
   ExampleFormat2",
   ExampleExporter2]
```

```
>> Export["sample.txt", "
   encodethisstring", "
   ExampleFormat2"];
```

```
>> FilePrint["sample.txt"]
```

rapbqrguvffgevat

ImportExport'RegisterImport

`RegisterImport["`*format*`", `*defaultFunction*`]`
: register *defaultFunction* as the default function used when importing from a file of type "*format*".

`RegisterImport["`*format*`", {"`*elem1*`" :> `*conditionalFunction1*`, "`*elem2*`" :> `*conditionalFunction2*`, ..., `*defaultFunction*`}]`
: registers multiple elements (*elem1*, ...) and their corresponding converter functions (*conditionalFunction1*, ...) in addition to the *defaultFunction*.

`RegisterImport["`*format*`", {"`*conditionalFunctions*`, `*defaultFunction*`, "`*elem3*`" :> `*postFunction3*`, "`*elem4*`" :> `*postFunction4*`, ...}]`
: also registers additional elements (*elem3*, ...) whose converters (*postFunction3*, ...) act on output from the low-level funcions.

First, define the default function used to import the data.

```
>> ExampleFormat1Import[
   filename_String] := Module[{
   stream, head, data}, stream =
   OpenRead[filename]; head =
   ReadList[stream, String, 2];
   data = Partition[ReadList[stream
   , Number], 2]; Close[stream]; {"
   Header" -> head, "Data" -> data
   }]
```

`RegisterImport` is then used to register the above function to a new data format.

```
>> ImportExport`RegisterImport["
   ExampleFormat1",
   ExampleFormat1Import]
```

```
>> FilePrint["ExampleData/
   ExampleData.txt"]
```

ExampleFileFormat

CreatedbyAngus

0.6294520.586355

0.7110090.687453

0.2465400.433973

0.9268710.887255

0.8251410.940900

0.8470350.127464

0.0543480.296494

0.8385450.247025

0.8386970.436220

0.3094960.833591

```
>> Import["ExampleData/ExampleData.
   txt", {"ExampleFormat1", "
   Elements"}]
```

{Data, Header}

```
>> Import["ExampleData/ExampleData.
   txt", {"ExampleFormat1", "Header
   "}]
```

{Example File Format,
Created by Angus}

Conditional Importer:

```
>> ExampleFormat2DefaultImport[
   filename_String] := Module[{
   stream, head}, stream = OpenRead
   [filename]; head = ReadList[
   stream, String, 2]; Close[stream
   ]; {"Header" -> head}]
```

```
>> ExampleFormat2DataImport[
   filename_String] := Module[{
   stream, data}, stream = OpenRead
   [filename]; Skip[stream, String,
    2]; data = Partition[ReadList[
   stream, Number], 2]; Close[
   stream]; {"Data" -> data}]
```

```
>> ImportExport`RegisterImport["
   ExampleFormat2", {"Data" :>
   ExampleFormat2DataImport,
   ExampleFormat2DefaultImport}]
```

```
>> Import["ExampleData/ExampleData.
   txt", {"ExampleFormat2", "
   Elements"}]
```

{Data, Header}

```
>> Import["ExampleData/ExampleData.
   txt", {"ExampleFormat2", "Header
   "}]
```

{Example File Format,
Created by Angus}

```
>> Import["ExampleData/ExampleData.
   txt", {"ExampleFormat2", "Data
   "}] // Grid
```

0.629452	0.586355
0.711009	0.687453
0.24654	0.433973
0.926871	0.887255
0.825141	0.9409
0.847035	0.127464
0.054348	0.296494
0.838545	0.247025
0.838697	0.43622
0.309496	0.833591

URLFetch

`URLFetch[`*URL*`]`
Returns the content of *URL* as a string.

Part III.

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A. GNU General Public License

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pymimemagic

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