



A free, open-source alternative to Mathematica

The Mathics Team

October 3, 2020

Contents

I.	Manual	5
1.	Introduction	6
2.	Installation	8
3.	Language tutorials	10
4.	Examples	23
5.	Web interface	26
6.	Implementation	27
II.	Reference of built-in symbols	31
I.	Algebra	32
II.	Arithmetic functions	38
III.	Assignment	48
IV.	Attributes	58
V.	Calculus functions	63
VI.	Combinatorial	68
VII.	Compilation	71
VIII.	Comparison	72
IX.	Control statements	76
X.	Date and Time	81
XI.	Differential equation solver functions	85
XII.	Evaluation	86
XIII.	Exponential, trigonometric and hyperbolic functions	90
XIV.	Functional programming	97
XV.	Graphics	99
XVI.	Graphics (3D)	117
XVII.	Image[] and image related functions.	121

XVIII.	Input and Output	131
XIX.	Integer functions	140
XX.	lohooks	143
XXI.	Linear algebra	144
XXII.	List functions	151
XXIII.	Logic	174
XXIV.	Manipulate	177
XXV.	Quantities	178
XXVI.	Number theoretic functions	180
XXVII.	Numeric evaluation	185
XXVIII.	Options and default arguments	190
XXIX.	Patterns and rules	193
XXX.	Plotting	200
XXXI.	Physical and Chemical data	210
XXXII.	Random number generation	212
XXXIII.	Recurrence relation solvers	216
XXXIV.	Special functions	217
XXXV.	Scoping	226
XXXVI.	String functions	230
XXXVII.	Structure	243
XXXVIII.	System functions	250
XXXIX.	Tensor functions	253
XL.	XML	256
XLI.	Optimization	257
XLII.	File Operations	258
XLIII.	Importing and Exporting	272
III.	License	277
A.	GNU General Public License	278
B.	Included software and data	287

Part I.

Manual

1. Introduction

Mathics—to be pronounced like “Mathematics” without the “emat”—is a general-purpose computer algebra system (CAS). It is meant to be a free, open-source alternative to *Mathematica*®. It is free both as in “free beer” and as in “freedom”. There are various online mirrors running *Mathics* but it is also possible to run *Mathics* locally. A list of mirrors can be found at the *Mathics* homepage, <https://mathics.org>.

The programming language of *Mathics* is meant to resemble *Wolfram*’s famous *Mathematica*® as much as possible. However, *Mathics* is in no way affiliated or supported by *Wolfram*. *Mathics* will probably never have the power to compete with *Mathematica*® in industrial applications; yet, it might be an interesting alternative for educational purposes.

Contents

Why yet another CAS? . . .	6	What does it offer? . . .	6	Who is behind it? . . .	7
		What is missing? . . .	7		

Why yet another CAS?

Mathematica® is great, but it has one big disadvantage: It is not free. On the one hand, people might not be able or willing to pay hundreds of dollars for it; on the other hand, they would still not be able to see what’s going on “inside” the program to understand their computations better. That’s what free software is for!

Mathics aims at combining the best of both worlds: the beauty of *Mathematica*® backed by a free, extensible Python core.

Of course, there are drawbacks to the *Mathematica*® language, despite all its beauty. It does not really provide object orientation and especially encapsulation, which might be crucial for big software projects. Nevertheless, *Wolfram* still managed to create their amazing *Wolfram* | *Alpha* entirely with *Mathematica*®, so it can’t be too bad!

However, it is not even the intention of *Mathics* to be used in large-scale projects and calculations—at least not as the main framework—but rather as a tool for quick explorations and in educating people who might later switch to *Mathematica*®.

What does it offer?

Some of the most important features of *Mathics* are

- a powerful functional programming language,
- a system driven by pattern matching and rules application,
- rationals, complex numbers, and arbitrary-precision arithmetic,
- lots of list and structure manipulation routines,
- an interactive graphical user interface right in the Web browser using MathML (apart from a command line interface),
- creation of graphics (e.g. plots) and display in the browser using SVG for 2D graphics and WebGL for 3D graphics,
- export of results to L^AT_EX (using Asymptote for graphics),
- a very easy way of defining new functions in Python,
- an integrated documentation and testing system.

What is missing?

There are lots of ways in which *Mathics* could still be improved.

Most notably, performance is still very slow, so any serious usage in cutting-edge industry or research will fail, unfortunately. Speeding up pattern matching, maybe "out-sourcing" parts of it from Python to C, would certainly improve the whole *Mathics* experience.

Apart from performance issues, new features such as more functions in various mathematical fields like calculus, number theory, or graph theory are still to be added.

Who is behind it?

Mathics was created by Jan Pöschko. Since 2013 it has been maintained by Angus Griffith. A list of all people involved in *Mathics* can be found in the AUTHORS file.

If you have any ideas on how to improve *Mathics* or even want to help out yourself, please contact us!

Welcome to *Mathics*, have fun!

2. Installation

Contents

Browser requirements . . .	8	Installation prerequisites . . .	8	Setup	8
				Running <i>Mathics</i> . . .	9

Browser requirements

To use the online version of *Mathics* (anybody could run their own version), you need a decent version of a modern Web browser, such as Firefox, Chrome, or Safari. Opera is not supported “officially” as it obviously has some problems with mathematical text inside SVG graphics, but except from that everything should work pretty fine.

Installation prerequisites

To run *Mathics*, you need Python 2.7 or higher on your computer. Since version 0.9 *Mathics* also supports Python3. On most Linux distributions and on Mac OS X, Python is already included in the system by default. For Windows, you can get it from <http://www.python.org>. Anyway, the primary target platforms for *Mathics* are Linux (especially Debian and Ubuntu) and Mac OS X. If you are on Windows and want to help by providing an installer to make setup on Windows easier, feel very welcome!

Furthermore, SQLite support is needed. Debian/Ubuntu provides the package `libsqlite3-dev`. The packages `python-dev` and `python-setuptools` are needed as well. You can install all required packages by running

```
\# apt-get install python-dev
    libsqlite3-dev python-
    setuptools
```

(as super-user, i.e. either after having issued `su` or by preceding the command with `sudo`).

On Mac OS X

```
\# brew install sqlite3
```

If you are on Windows, please figure out yourself how to install SQLite.

Get the latest version of *Mathics* from <http://www.mathics.org>. You will need internet access for the installation of *Mathics*.

Setup

Simply run:

```
\# python setup.py install
```

In addition to installing *Mathics*, this will download the required Python packages `sympy`, `mpmath`, `django`, and `pysqlite` and install them in your Python site-packages directory (usually `/usr/lib/python3.x/site-packages` on Debian or `/Library/Frameworks/Python.framework/Versions/3.x/lib/python3.x/site-packages` on Mac OS X).

Two executable files will be created in a binary directory on your PATH (usually `/usr/bin` on Debian or `/Library/Frameworks/Python.framework/Versions/3.x/bin` on Mac OS X): `mathics` and `mathicsserver`.

Running *Mathics*

Run

```
$ mathics
```

to start the console version of *Mathics*.

Run

```
$ mathicsserver
```

to start the local Web server of *Mathics* which serves the web GUI interface. The first time this

command is run it will create the database file for saving your sessions. Issue

```
$ mathicsserver --help
```

to see a list of options.

You can set the used port by using the option `-p`, as in:

```
$ mathicsserver -p 8010
```

The default port for *Mathics* is 8000. Make sure you have the necessary privileges to start an ap-

plication that listens to this port. Otherwise, you will have to run *Mathics* as super-user.

By default, the Web server is only reachable from your local machine. To be able to access it from another computer, use the option `-e`. However, the server is only intended for local use, as it is a security risk to run it openly on a public Web server! This documentation does not cover how to setup *Mathics* for being used on a public server. Maybe you want to hire a *Mathics* developer to do that for you?!

3. Language tutorials

The following sections are introductions to the basic principles of the language of *Mathics*. A few examples and functions are presented. Only their most common usages are listed; for a full

description of their possible arguments, options, etc., see their entry in the Reference of built-in symbols.

Contents

Basic calculations . . .	11	Strings	12	Formatting output . . .	19
Symbols and assignments . . .	12	Lists	13	Graphics	21
Comparisons and Boolean logic . . .	12	The structure of things	14	3D Graphics	21
		Functions and patterns	16	Plotting	22
		Control statements . .	16		
		Scoping	17		

Basic calculations

Mathics can be used to calculate basic stuff:

```
>> 1 + 2
3
```

To submit a command to *Mathics*, press Shift+Return in the Web interface or Return in the console interface. The result will be printed in a new line below your query.

Mathics understands all basic arithmetic operators and applies the usual operator precedence. Use parentheses when needed:

```
>> 1 - 2 * (3 + 5) / 4
-3
```

The multiplication can be omitted:

```
>> 1 - 2 (3 + 5) / 4
-3
```

```
>> 2 4
8
```

Powers can be entered using \wedge :

```
>> 3 ^ 4
81
```

Integer divisions yield rational numbers:

```
>> 6 / 4
3
2
```

To convert the result to a floating point number, apply the function `N`:

```
>> N[6 / 4]
1.5
```

As you can see, functions are applied using square braces [and], in contrast to the common notation of (and). At first hand, this might seem strange, but this distinction between function application and precedence change is necessary to allow some general syntax structures, as you will see later.

Mathics provides many common mathematical functions and constants, e.g.:

```
>> Log[E]
1
```

```
>> Sin[Pi]
0
```

```
>> Cos[0.5]
0.877583
```

When entering floating point numbers in your query, *Mathics* will perform a numerical evaluation and present a numerical result, pretty much like if you had applied `N`.

Of course, *Mathics* has complex numbers:

```
>> Sqrt[-4]
2I
```

```
>> I ^ 2
-1

>> (3 + 2 I)^ 4
-119 + 120I

>> (3 + 2 I)^ (2.5 - I)
43.663 + 8.28556I

>> Tan[I + 0.5]
0.195577 + 0.842966I
```

Abs calculates absolute values:

```
>> Abs[-3]
3

>> Abs[3 + 4 I]
5
```

Mathics can operate with pretty huge numbers:

```
>> 100!
93 326 215 443 944 152 681 699 ~
~238 856 266 700 490 715 968 ~
~264 381 621 468 592 963 895 ~
~217 599 993 229 915 608 941 ~
~463 976 156 518 286 253 697 920 ~
~827 223 758 251 185 210 916 864 ~
~000 000 000 000 000 000 000 000
```

(! denotes the factorial function.) The precision of numerical evaluation can be set:

```
>> N[Pi, 100]
3.141592653589793238462643~
~383279502884197169399375~
~105820974944592307816406~
~286208998628034825342117068
```

Division by zero is forbidden:

```
>> 1 / 0
Indeterminate/0encountered.
ComplexInfinity
```

Other expressions involving Infinity are evaluated:

```
>> Infinity + 2 Infinity
∞
```

In contrast to combinatorial belief, 0^0 is undefined:

```
>> 0 ^ 0
Indeterminateexpression0^0encountered.
Indeterminate
```

The result of the previous query to Mathics can be accessed by %:

```
>> 3 + 4
7

>> % ^ 2
49
```

Symbols and assignments

Symbols need not be declared in Mathics, they can just be entered and remain variable:

```
>> x
x
```

Basic simplifications are performed:

```
>> x + 2 x
3x
```

Symbols can have any name that consists of characters and digits:

```
>> iAm1Symbol ^ 2
iAm1Symbol^2
```

You can assign values to symbols:

```
>> a = 2
2

>> a ^ 3
8

>> a = 4
4

>> a ^ 3
64
```

Assigning a value returns that value. If you want to suppress the output of any result, add a ; to the end of your query:

```
>> a = 4;
```

Values can be copied from one variable to another:

```
>> b = a;
```

Now changing a does not affect b:

```
>> a = 3;
```

```
>> b
4
```

Such a dependency can be achieved by using “delayed assignment” with the := operator (which does not return anything, as the right side is not even evaluated):

```
>> b := a ^ 2
```

```
>> b
9

>> a = 5;

>> b
25
```

Comparisons and Boolean logic

Values can be compared for equality using the operator `==`:

```
>> 3 == 3
True

>> 3 == 4
False
```

The special symbols `True` and `False` are used to denote truth values. Naturally, there are inequality comparisons as well:

```
>> 3 > 4
False
```

Inequalities can be chained:

```
>> 3 < 4 >= 2 != 1
True
```

Truth values can be negated using `!` (logical *not*) and combined using `&&` (logical *and*) and `||` (logical *or*):

```
>> !True
False

>> !False
True
```

```
>> 3 < 4 && 6 > 5
True
```

`&&` has higher precedence than `||`, i.e. it binds stronger:

```
>> True && True || False && False
True

>> True && (True || False) && False
False
```

Strings

Strings can be entered with `"` as delimiters:

```
>> "Hello world!"
Hello world!
```

As you can see, quotation marks are not printed

in the output by default. This can be changed by using `InputForm`:

```
>> InputForm["Hello world!"]
"Hello world!"
```

Strings can be joined using `<>`:

```
>> "Hello" <> " " <> "world!"
Hello world!
```

Numbers cannot be joined to strings:

```
>> "Debian" <> 6
Stringexpected.
Debian<>6
```

They have to be converted to strings using `ToString` first:

```
>> "Debian" <> ToString[6]
Debian6
```

Lists

Lists can be entered in *Mathics* with curly braces `{` and `}`:

```
>> mylist = {a, b, c, d}
{a,b,c,d}
```

There are various functions for constructing lists:

```
>> Range[5]
{1,2,3,4,5}

>> Array[f, 4]
{f[1],f[2],f[3],f[4]}

>> ConstantArray[x, 4]
{x,x,x,x}

>> Table[n ^ 2, {n, 2, 5}]
{4,9,16,25}
```

The number of elements of a list can be determined with `Length`:

```
>> Length[mylist]
4
```

Elements can be extracted using double square braces:

```
>> mylist[[3]]
c
```

Negative indices count from the end:

```
>> mylist[[-3]]
b
```

Lists can be nested:

```
>> mymatrix = {{1, 2}, {3, 4}, {5, 6}};
```

There are alternate forms to display lists:

```
>> TableForm[mymatrix]
```

```
1 2
3 4
5 6
```

```
>> MatrixForm[mymatrix]
```

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

There are various ways of extracting elements from a list:

```
>> mymatrix[[2, 1]]
```

```
3
```

```
>> mymatrix[;;, 2]
```

```
{2, 4, 6}
```

```
>> Take[mylist, 3]
```

```
{a, b, c}
```

```
>> Take[mylist, -2]
```

```
{c, d}
```

```
>> Drop[mylist, 2]
```

```
{c, d}
```

```
>> First[mymatrix]
```

```
{1, 2}
```

```
>> Last[mylist]
```

```
d
```

```
>> Most[mylist]
```

```
{a, b, c}
```

```
>> Rest[mylist]
```

```
{b, c, d}
```

Lists can be used to assign values to multiple variables at once:

```
>> {a, b} = {1, 2};
```

```
>> a
1
```

```
>> b
2
```

Many operations, like addition and multiplication, “thread” over lists, i.e. lists are combined element-wise:

```
>> {1, 2, 3} + {4, 5, 6}
{5, 7, 9}
```

```
>> {1, 2, 3} * {4, 5, 6}
{4, 10, 18}
```

It is an error to combine lists with unequal lengths:

```
>> {1, 2} + {4, 5, 6}
```

Objectsofunequallengthcannotbecombined.

```
{1, 2} + {4, 5, 6}
```

The structure of things

Every expression in *Mathics* is built upon the same principle: it consists of a *head* and an arbitrary number of *children*, unless it is an *atom*, i.e. it can not be subdivided any further. To put it another way: everything is a function call. This can be best seen when displaying expressions in their “full form”:

```
>> FullForm[a + b + c]
Plus[a, b, c]
```

Nested calculations are nested function calls:

```
>> FullForm[a + b * (c + d)]
Plus[a, Times[b, Plus[c, d]]]
```

Even lists are function calls of the function `List`:

```
>> FullForm[{1, 2, 3}]
List[1, 2, 3]
```

The head of an expression can be determined with `Head`:

```
>> Head[a + b + c]
Plus
```

The children of an expression can be accessed like list elements:

```
>> (a + b + c)[[2]]
b
```

The head is the 0th element:

```
>> (a + b + c)[[0]]
Plus
```

The head of an expression can be exchanged using the function `Apply`:

```
>> Apply[g, f[x, y]]
      g[x, y]

>> Apply[Plus, a * b * c]
      a + b + c
```

Apply can be written using the operator @@:

```
>> Times @@ {1, 2, 3, 4}
      24
```

(This exchanges the head List of {1, 2, 3, 4} with Times, and then the expression Times[1, 2, 3, 4] is evaluated, yielding 24.) Apply can also be applied on a certain *level* of an expression:

```
>> Apply[f, {{1, 2}, {3, 4}}, {1}]
      {f[1, 2], f[3, 4]}
```

Or even on a range of levels:

```
>> Apply[f, {{1, 2}, {3, 4}}, {0, 2}]
      f[f[1, 2], f[3, 4]]
```

Apply is similar to Map (/@):

```
>> Map[f, {1, 2, 3, 4}]
      {f[1], f[2], f[3], f[4]}

>> f /@ {{1, 2}, {3, 4}}
      {f[{1, 2}], f[{3, 4}]}
```

The atoms of *Mathics* are numbers, symbols, and strings. AtomQ tests whether an expression is an atom:

```
>> AtomQ[5]
      True

>> AtomQ[a + b]
      False
```

The full form of rational and complex numbers looks like they were compound expressions:

```
>> FullForm[3 / 5]
      Rational[3, 5]

>> FullForm[3 + 4 I]
      Complex[3, 4]
```

However, they are still atoms, thus unaffected by applying functions, for instance:

```
>> f @@ Complex[3, 4]
      3 + 4I
```

Nevertheless, every atom has a head:

```
>> Head /@ {1, 1/2, 2.0, I, "a string", x}
      {Integer, Rational, Real, Complex, String, Symbol}
```

The operator === tests whether two expressions are the same on a structural level:

```
>> 3 === 3
      True

>> 3 == 3.0
      True
```

But

```
>> 3 === 3.0
      False
```

because 3 (an Integer) and 3.0 (a Real) are structurally different.

Functions and patterns

Functions can be defined in the following way:

```
>> f[x_] := x ^ 2
```

This tells *Mathics* to replace every occurrence of f with one (arbitrary) parameter x with x^2 .

```
>> f[3]
      9

>> f[a]
      a2
```

The definition of f does not specify anything for two parameters, so any such call will stay unevaluated:

```
>> f[1, 2]
      f[1, 2]
```

In fact, *functions* in *Mathics* are just one aspect of *patterns*: f[x_] is a pattern that *matches* expressions like f[3] and f[a]. The following patterns are available:

```

_ or Blank[]
    matches one expression.
Pattern[x, p]
    matches the pattern p and stores the value
    in x.
x_ or Pattern[x, Blank[]]
    matches one expression and stores it in x.
__ or BlankSequence[]
    matches a sequence of one or more ex-
    pressions.
___ or BlankNullSequence[]
    matches a sequence of zero or more ex-
    pressions.
_h or Blank[h]
    matches one expression with head h.
x_h or Pattern[x, Blank[h]]
    matches one expression with head h and
    stores it in x.
p | q or Alternatives[p, q]
    matches either pattern p or q.
p ? t or PatternTest[p, t]
    matches p if the test t[p] yields True.
p /; c or Condition[p, c]
    matches p if condition c holds.
Verbatim[p]
    matches an expression that equals p,
    without regarding patterns inside p.

```

As before, patterns can be used to define func-
tions:

```

>> g[s___] := Plus[s] ^ 2

>> g[1, 2, 3]
36

```

MatchQ[*e*, *p*] tests whether *e* matches *p*:

```

>> MatchQ[a + b, x_ + y_]
True

>> MatchQ[6, _Integer]
True

```

ReplaceAll (/.) replaces all occurrences of a
pattern in an expression using a Rule given by
->:

```

>> {2, "a", 3, 2.5, "b", c} /.
    x_Integer -> x ^ 2
{4, a, 9, 2.5, b, c}

```

You can also specify a list of rules:

```

>> {2, "a", 3, 2.5, "b", c} /. {
    x_Integer -> x ^ 2.0, y_String
    -> 10}
{4., 10, 9., 2.5, 10, c}

```

ReplaceRepeated (//.) applies a set of rules re-
peatedly, until the expression doesn't change
anymore:

```

>> {2, "a", 3, 2.5, "b", c} //. {
    x_Integer -> x ^ 2.0, y_String
    -> 10}
{4., 100., 9., 2.5, 100., c}

```

There is a “delayed” version of Rule which can
be specified by :> (similar to the relation of := to
=):

```

>> a :> 1 + 2
a:>1 + 2

>> a -> 1 + 2
a- > 3

```

This is useful when the right side of a rule
should not be evaluated immediately (before
matching):

```

>> {1, 2} /. x_Integer -> N[x]
{1, 2}

```

Here, N is applied to *x* before the actual match-
ing, simply yielding *x*. With a delayed rule this
can be avoided:

```

>> {1, 2} /. x_Integer :> N[x]
{1., 2.}

```

While ReplaceAll and ReplaceRepeated sim-
ply take the first possible match into ac-
count, ReplaceList returns a list of all possi-
ble matches. This can be used to get all subse-
quences of a list, for instance:

```

>> ReplaceList[{a, b, c}, {___, x__,
    , ___} -> {x}]
{{a}, {a, b}, {a, b,
    c}, {b}, {b, c}, {c}}

```

ReplaceAll would just return the first expres-
sion:

```

>> ReplaceAll[{a, b, c}, {___, x__,
    ___} -> {x}]
{a}

```

In addition to defining functions as rules for cer-
tain patterns, there are *pure* functions that can be
defined using the & postfix operator, where ev-
erything before it is treated as the function body
and # can be used as argument placeholder:

```

>> h = # ^ 2 &;

>> h[3]
9

```

Multiple arguments can simply be indexed:

```
>> sum = #1 + #2 &;  
  
>> sum[4, 6]  
10
```

It is also possible to name arguments using Function:

```
>> prod = Function[{x, y}, x * y];  
  
>> prod[4, 6]  
24
```

Pure functions are very handy when functions are used only locally, e.g., when combined with operators like Map:

```
>> # ^ 2 & /@ Range[5]  
{1,4,9,16,25}
```

Sort according to the second part of a list:

```
>> Sort[{{x, 10}, {y, 2}, {z, 5}},  
#1[[2]] < #2[[2]] &]  
{{y,2}, {z,5}, {x,10}}
```

Functions can be applied using prefix or postfix notation, in addition to using []:

```
>> h @ 3  
9  
  
>> 3 // h  
9
```

Control statements

Like most programming languages, *Mathics* has common control statements for conditions, loops, etc.:

If[*cond*, *pos*, *neg*]
returns *pos* if *cond* evaluates to True, and *neg* if it evaluates to False.
Which[*cond1*, *expr1*, *cond2*, *expr2*, ...]
yields *expr1* if *cond1* evaluates to True, *expr2* if *cond2* evaluates to True, etc.
Do[*expr*, {*i*, *max*}]
evaluates *expr* *max* times, substituting *i* in *expr* with values from 1 to *max*.
For[*start*, *test*, *incr*, *body*]
evaluates *start*, and then iteratively *body* and *incr* as long as *test* evaluates to True.
While[*test*, *body*]
evaluates *body* as long as *test* evaluates to True.
Nest[*f*, *expr*, *n*]
returns an expression with *f* applied *n* times to *expr*.
NestWhile[*f*, *expr*, *test*]
applies a function *f* repeatedly on an expression *expr*, until applying *test* on the result no longer yields True.
FixedPoint[*f*, *expr*]
starting with *expr*, repeatedly applies *f* until the result no longer changes.

```
>> If[2 < 3, a, b]  
a  
  
>> x = 3; Which[x < 2, a, x > 4, b,  
x < 5, c]  
c
```

Compound statements can be entered with ;. The result of a compound expression is its last part or Null if it ends with a ;.

```
>> 1; 2; 3  
3  
  
>> 1; 2; 3;
```

Inside For, While, and Do loops, Break[] exits the loop and Continue[] continues to the next iteration.

```
>> For[i = 1, i <= 5, i++, If[i ==  
4, Break[]]; Print[i]]  
1  
2  
3
```


Scoping

By default, all symbols are “global” in *Mathics*, i.e. they can be read and written in any part of your program. However, sometimes “local” variables are needed in order not to disturb the global namespace. *Mathics* provides two ways to support this:

- *lexical scoping* by `Module`, and
- *dynamic scoping* by `Block`.

```
Module[{vars}, expr]
  localizes variables by giving them a temporary name of the form name$number, where number is the current value of $ModuleNumber. Each time a module is evaluated, $ModuleNumber is incremented.

Block[{vars}, expr]
  temporarily stores the definitions of certain variables, evaluates expr with reset values and restores the original definitions afterwards.
```

Both scoping constructs shield inner variables from affecting outer ones:

```
>> t = 3;

>> Module[{t}, t = 2]
2

>> Block[{t}, t = 2]
2

>> t
3
```

`Module` creates new variables:

```
>> y = x ^ 3;

>> Module[{x = 2}, x * y]
2x3
```

`Block` does not:

```
>> Block[{x = 2}, x * y]
16
```

Thus, `Block` can be used to temporarily assign a value to a variable:

```
>> expr = x ^ 2 + x;

>> Block[{x = 3}, expr]
12

>> x
x
```

`Block` can also be used to temporarily change the value of system parameters:

```
>> Block[{$RecursionLimit = 30}, x
= 2 x]
Recursiondepthof30exceeded.
$Aborted

>> f[x_] := f[x + 1]; Block[{
$IterationLimit = 30}, f[1]]
Iterationlimitof30exceeded.
$Aborted
```

It is common to use scoping constructs for function definitions with local variables:

```
>> fac[n_] := Module[{k, p}, p = 1;
  For[k = 1, k <= n, ++k, p *= k
]; p]

>> fac[10]
3 628 800

>> 10!
3 628 800
```

Formatting output

The way results are formatted for output in *Mathics* is rather sophisticated, as compatibility to the way *Mathematica*® does things is one of the design goals. It can be summed up in the following procedure:

1. The result of the query is calculated.
2. The result is stored in `Out` (which `%` is a shortcut for).
3. Any `Format` rules for the desired output form are applied to the result. In the console version of *Mathics*, the result is formatted as `OutputForm`; `MathMLForm` for the `StandardForm` is used in the interactive Web version; and `TeXForm` for the `StandardForm` is used to generate the \LaTeX version of this documentation.
4. `MakeBoxes` is applied to the formatted result, again given either `OutputForm`, `MathMLForm`, or `TeXForm` depending on the execution context of *Mathics*. This yields a new expression consisting of “box constructs”.
5. The boxes are turned into an ordinary string and displayed in the console, sent to the browser, or written to the documentation \LaTeX file.

As a consequence, there are various ways to implement your own formatting strategy for custom objects.

You can specify how a symbol shall be formatted by assigning values to `Format`:

```
>> Format[x] = "y";

>> x
y
```

This will apply to `MathMLForm`, `OutputForm`, `StandardForm`, `TeXForm`, and `TraditionalForm`.

```
>> x // InputForm
x
```

You can specify a specific form in the assignment to `Format`:

```
>> Format[x, TeXForm] = "z";

>> x // TeXForm
\text{z}
```

Special formats might not be very relevant for individual symbols, but rather for custom functions (objects):

```
>> Format[r[args___]] = "<an r
object>";

>> r[1, 2, 3]
<an r object>
```

You can use several helper functions to format expressions:

```
Infix[expr, op]
  formats the arguments of expr with infix
  operator op.
Prefix[expr, op]
  formats the argument of expr with prefix
  operator op.
Postfix[expr, op]
  formats the argument of expr with postfix
  operator op.
StringForm[form, arg1, arg2, ...]
  formats arguments using a format string.
```

```
>> Format[r[args___]] = Infix[{args
}, "~"];

>> r[1, 2, 3]
1 ~ 2 ~ 3

>> StringForm["'1' and '2'", n, m]
n and m
```

There are several methods to display expressions in 2-D:

```
Row[{...}]
  displays expressions in a row.
Grid[{{...}}]
  displays a matrix in two-dimensional
  form.
Subscript[expr, i1, i2, ...]
  displays expr with subscript indices i1, i2,
  ...
Superscript[expr, exp]
  displays expr with superscript (exponent)
  exp.
```

```
>> Grid[{{a, b}, {c, d}}]
  a  b
  c  d

>> Subscript[a, 1, 2] // TeXForm
a_{1,2}
```

If you want even more low-level control of how expressions are displayed, you can override `MakeBoxes`:

```
>> MakeBoxes[b, StandardForm] = "c
";

>> b
c
```

This will even apply to `TeXForm`, because `TeXForm` implies `StandardForm`:

```
>> b // TeXForm
c
```

Except some other form is applied first:

```
>> b // OutputForm // TeXForm
b
```

`MakeBoxes` for another form:

```
>> MakeBoxes[b, TeXForm] = "d";

>> b // TeXForm
d
```

You can cause a much bigger mess by overriding `MakeBoxes` than by sticking to `Format`, e.g. generate invalid XML:

```
>> MakeBoxes[c, MathMLForm] = "<not
closed";

>> c // MathMLForm
<not closed
```

However, this will not affect formatting of ex-

pressions involving c :

```
>> c + 1 // MathMLForm
<math display="block"><mrow>
  <mn>1</mn> <mo>+</mo>
  <mi>c</mi></mrow></math>
```

That's because MathMLForm will, when not overridden for a special case, call StandardForm first. Format will produce escaped output:

```
>> Format[d, MathMLForm] = "<not
  closed";

>> d // MathMLForm
<math display="block">
<mtext>&lt;not&nbsp;closed</mtext>
</math>

>> d + 1 // MathMLForm
<math display="block"><mrow>
<mn>1</mn> <mo>+</mo>
<mtext>&lt;not&nbsp;closed</mtext>
</mrow></math>
```

For instance, you can override MakeBoxes to format lists in a different way:

```
>> MakeBoxes[{items___},
  StandardForm] := RowBox[{"[",
  Sequence @@ Riffle[MakeBoxes /@
  {items}, " ", "]""]

>> {1, 2, 3}
[123]
```

However, this will not be accepted as input to *Mathics* anymore:

```
>> [1 2 3]

>> Clear[MakeBoxes]
```

By the way, MakeBoxes is the only built-in symbol that is not protected by default:

```
>> Attributes[MakeBoxes]
[HoldAllComplete]
```

MakeBoxes must return a valid box construct:

```
>> MakeBoxes[squared[args___],
  StandardForm] := squared[args] ^
  2

>> squared[1, 2]
Power[squared[1,2],
  2]isnotavalidboxstructure.
```

The desired effect can be achieved in the following way:

```
>> MakeBoxes[squared[args___],
  StandardForm] := SuperscriptBox[
  RowBox[{MakeBoxes[squared], "["},
  RowBox[Riffle[MakeBoxes[#] & /@
  {args}, " "], "]""], 2]

>> squared[1, 2]
squared[1,2]2
```

You can view the box structure of a formatted expression using ToBoxes:

```
>> ToBoxes[m + n]
RowBox[{m, +, n}]
```

The list elements in this RowBox are strings, though string delimiters are not shown in the default output form:

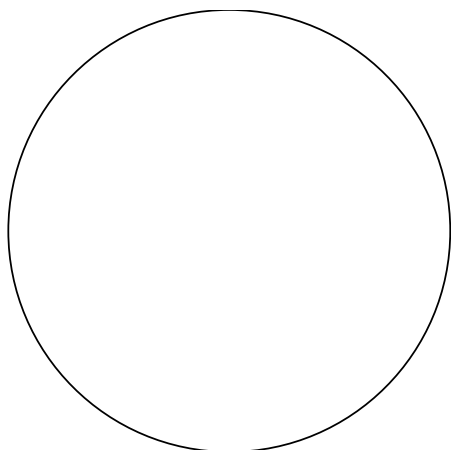
```
>> InputForm[%]
RowBox[{ "m", "+", "n" ]}
```

Graphics

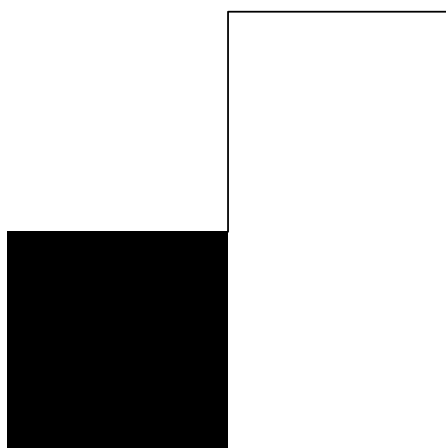
Two-dimensional graphics can be created using the function Graphics and a list of graphics primitives. For three-dimensional graphics see the following section. The following primitives are available:

```
Circle[{x, y}, r]
  draws a circle.
Disk[{x, y}, r]
  draws a filled disk.
Rectangle[{x1, y1}, {x2, y2}]
  draws a filled rectangle.
Polygon[{{x1, y1}, {x2, y2}, ...}]
  draws a filled polygon.
Line[{{x1, y1}, {x2, y2}, ...}]
  draws a line.
Text[text, {x, y}]
  draws text in a graphics.
```

```
>> Graphics[{Circle[{0, 0}, 1]}]
```



```
>> Graphics[{Line[{0, 0}, {0, 1}, {1, 1}, {1, -1}], Rectangle[{0, 0}, {-1, -1}]}]
```



Colors can be added in the list of graphics primitives to change the drawing color. The following ways to specify colors are supported:

`RGBColor[r, g, b]`
specifies a color using red, green, and blue.

`CMYKColor[c, m, y, k]`
specifies a color using cyan, magenta, yellow, and black.

`Hue[h, s, b]`
specifies a color using hue, saturation, and brightness.

`GrayLevel[l]`
specifies a color using a gray level.

All components range from 0 to 1. Each color

function can be supplied with an additional argument specifying the desired opacity ("alpha") of the color. There are many predefined colors,

such as Black, White, Red, Green, Blue, etc.

```
>> Graphics[{Red, Disk[]}]
```

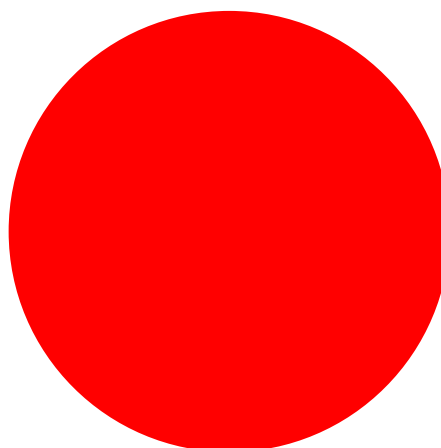
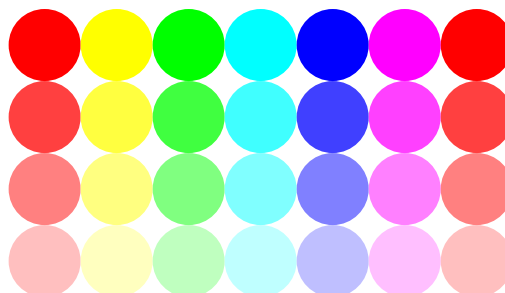


Table of hues:

```
>> Graphics[Table[{Hue[h, s], Disk[{12h, 8s]}], {h, 0, 1, 1/6}, {s, 0, 1, 1/4}]]
```



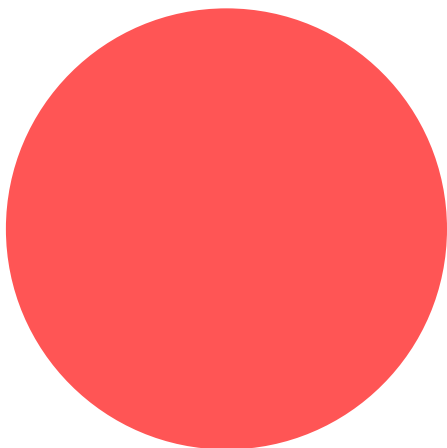
Colors can be mixed and altered using the following functions:

`Blend[{color1, color2}, ratio]`
mixes *color1* and *color2* with *ratio*, where a ratio of 0 returns *color1* and a ratio of 1 returns *color2*.

`Lighter[color]`
makes *color* lighter (mixes it with White).

`Darker[color]`
makes *color* darker (mixes it with Black).

```
>> Graphics[{Lighter[Red], Disk[]}]
```



Graphics produces a GraphicsBox:

```
>> Head[ToBoxes[Graphics[{Circle[]}]]]
```

GraphicsBox

3D Graphics

Three-dimensional graphics are created using the function Graphics3D and a list of 3D primitives. The following primitives are supported so far:

```
Polygon[{{x1, y1, z1}, {x2, y2, z3}, ...}]
```

draws a filled polygon.

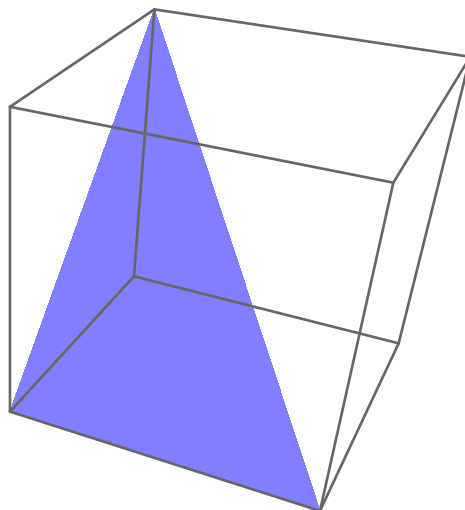
```
Line[{{x1, y1, z1}, {x2, y2, z3}, ...}]
```

draws a line.

```
Point[{x1, y1, z1}]
```

draws a point.

```
>> Graphics3D[Polygon[{{0,0,0}, {0,1,1}, {1,0,0}}]]
```



Colors can also be added to three-dimensional primitives.

```
>> Graphics3D[{0, Orange, Polygon[{{0,0,0}, {1,1,1}, {1,0,0}}]}], Axes->True]
```

```
Graphics3DBox[List[StyleBox[Graphics[List[EdgeForm[Graphics3DBox[Polygon[{{0,0,0}, {1,1,1}, {1,0,0}], Rule[ImageSize, 16]], Rule[ImageSizeMultipliers, List[1, 1]], Polygon3DBox[List[List[0, 0, 0], List[1, 1, 1], List[1, 0, 0]]], Rule[$OptionSyntax, Ignore], Rule[AspectRatio, Automatic], Rule[Axes, True], Rule[AxesStyle, List[]], Rule[Background, Automatic], Rule[BoxRatios, Automatic], Rule[ImageSize, Automatic], Rule[LabelStyle, List[]], Rule[Lighting, Automatic], Rule[PlotRange, Automatic], Rule[PlotRangePadding, Automatic], Rule[TicksStyle, List[]], Rule[ViewPoint, List[1.3, -2.4, 2.]]]isnotavalidboxstructure.
```

Graphics3D produces a Graphics3DBox:

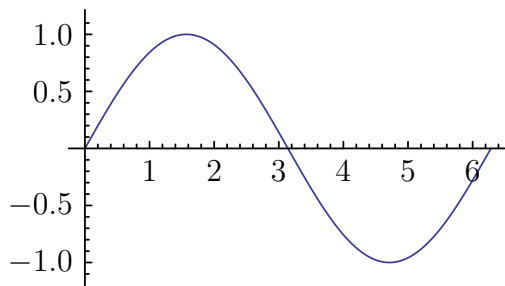
```
>> Head[ToBoxes[Graphics3D[{Polygon[]}]]]
```

Graphics3DBox

Plotting

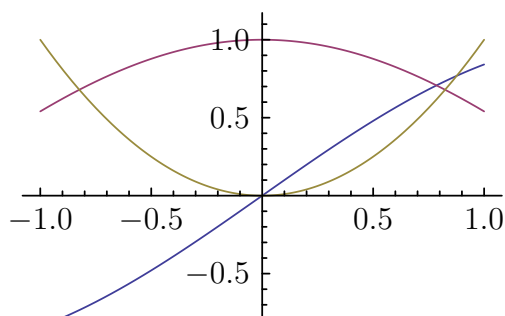
Mathics can plot functions:

```
>> Plot[Sin[x], {x, 0, 2 Pi}]
```



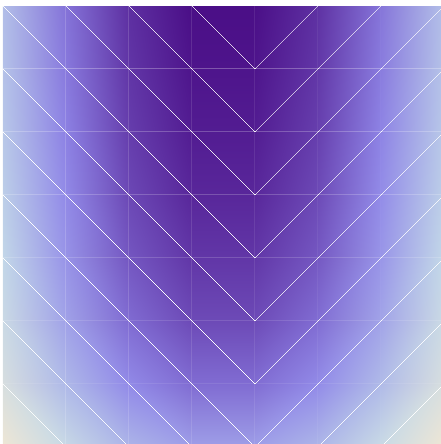
You can also plot multiple functions at once:

```
>> Plot[{Sin[x], Cos[x], x ^ 2}, {x, -1, 1}]
```



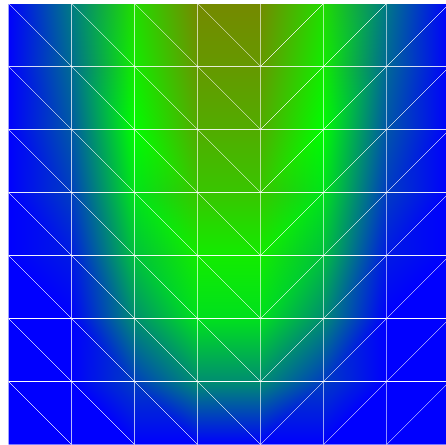
Two-dimensional functions can be plotted using `DensityPlot`:

```
>> DensityPlot[x ^ 2 + 1 / y, {x, -1, 1}, {y, 1, 4}]
```



You can use a custom coloring function:

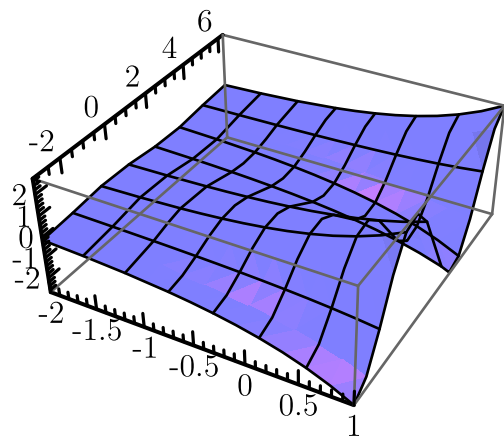
```
>> DensityPlot[x ^ 2 + 1 / y, {x, -1, 1}, {y, 1, 4}, ColorFunction -> (Blend[{Red, Green, Blue}, #]&)]
```



One problem with `DensityPlot` is that it's still very slow, basically due to function evaluation being pretty slow in general—and `DensityPlot` has to evaluate a lot of functions.

Three-dimensional plots are supported as well:

```
>> Plot3D[Exp[x] Cos[y], {x, -2, 1}, {y, -Pi, 2 Pi}]
```



4. Examples

Contents

Curve sketching	23	Linear algebra	24	Dice	25
-------------------------	----	--------------------------	----	----------------	----

Curve sketching

Let's sketch the function

```
>> f[x_] := 4 x / (x ^ 2 + 3 x + 5)
```

The derivatives are

```
>> {f'[x], f''[x], f'''[x]} //
```

Together

$$\left\{ \begin{array}{l} \frac{-4(-5+x^2)}{(5+3x+x^2)^2}, \\ \frac{8(-15-15x+x^3)}{(5+3x+x^2)^3}, \\ \frac{-24(-20-60x-30x^2+x^4)}{(5+3x+x^2)^4} \end{array} \right\}$$

To get the extreme values of f, compute the zeroes of the first derivatives:

```
>> extremes = Solve[f'[x] == 0, x]
```

$$\left\{ \left\{ x \rightarrow -\sqrt{5} \right\}, \left\{ x \rightarrow \sqrt{5} \right\} \right\}$$

And test the second derivative:

```
>> f''[x] /. extremes // N
```

$$\{1.65086, -0.064079\}$$

Thus, there is a local maximum at $x = \text{Sqrt}[5]$ and a local minimum at $x = -\text{Sqrt}[5]$. Compute the inflection points numerically, chopping imaginary parts close to 0:

```
>> inflections = Solve[f''[x] == 0, x] // N // Chop
```

$$\left\{ \left\{ x \rightarrow -1.0852 \right\}, \left\{ x \rightarrow -3.21463 \right\}, \left\{ x \rightarrow 4.29983 \right\} \right\}$$

Insert into the third derivative:

```
>> f'''[x] /. inflections
```

$$\{-3.67683, 0.694905, 0.00671894\}$$

Being different from 0, all three points are actual inflection points. f is not defined where its denominator is 0:

```
>> Solve[Denominator[f[x]] == 0, x]
```

$$\left\{ \left\{ x \rightarrow -\frac{3}{2} - \frac{I}{2}\sqrt{11} \right\}, \left\{ x \rightarrow -\frac{3}{2} + \frac{I}{2}\sqrt{11} \right\} \right\}$$

These are non-real numbers, consequently f is defined on all real numbers. The behaviour of f at the boundaries of its definition:

```
>> Limit[f[x], x -> Infinity]
```

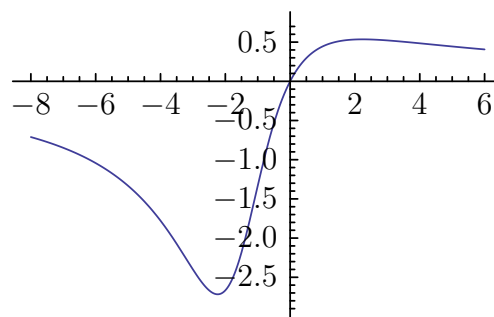
$$0$$

```
>> Limit[f[x], x -> -Infinity]
```

$$0$$

Finally, let's plot f:

```
>> Plot[f[x], {x, -8, 6}]
```



Linear algebra

Let's consider the matrix

```
>> A = {{1, 1, 0}, {1, 0, 1}, {0, 1, 1}};
```

```
>> MatrixForm[A]

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

```

We can compute its eigenvalues and eigenvectors:

```
>> Eigenvalues[A]
{2, -1, 1}

>> Eigenvectors[A]
{{1, 1, 1}, {1, -2, 1}, {-1, 0, 1}}
```

This yields the diagonalization of A:

```
>> T = Transpose[Eigenvectors[A]];
MatrixForm[T]

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & -2 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$


>> Inverse[T] . A . T // MatrixForm

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$


>> % == DiagonalMatrix[Eigenvalues[A]]
True
```

We can solve linear systems:

```
>> LinearSolve[A, {1, 2, 3}]
{0, 1, 2}

>> A . %
{1, 2, 3}
```

In this case, the solution is unique:

```
>> NullSpace[A]
{}
```

Let's consider a singular matrix:

```
>> B = {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}};

>> MatrixRank[B]
2

>> s = LinearSolve[B, {1, 2, 3}]
 $\left\{-\frac{1}{3}, \frac{2}{3}, 0\right\}$ 

>> NullSpace[B]
{{1, -2, 1}}
```

```
>> B . (RandomInteger[100] * %[[1]]
+ s)
{1, 2, 3}
```

Dice

Let's play with dice in this example. A Dice object shall represent the outcome of a series of rolling a dice with six faces, e.g.:

```
>> Dice[1, 6, 4, 4]
Dice[1, 6, 4, 4]
```

Like in most games, the ordering of the individual throws does not matter. We can express this by making Dice Orderless:

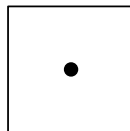
```
>> SetAttributes[Dice, Orderless]

>> Dice[1, 6, 4, 4]
Dice[1, 4, 4, 6]
```

A dice object shall be displayed as a rectangle with the given number of points in it, positioned like on a traditional dice:

```
>> Format[Dice[n_Integer?(1 <= # <=
6 &)] := Block[{p = 0.2, r =
0.05}, Graphics[{EdgeForm[Black],
White, Rectangle[], Black,
EdgeForm[], If[OddQ[n], Disk
[{0.5, 0.5}, r]], If[MemberQ[{2,
3, 4, 5, 6}, n], Disk[{p, p}, r]],
If[MemberQ[{2, 3, 4, 5, 6},
n], Disk[{1 - p, 1 - p}, r]], If
[MemberQ[{4, 5, 6}, n], Disk[{p,
1 - p}, r]], If[MemberQ[{4, 5,
6}, n], Disk[{1 - p, p}, r]], If
[n === 6, {Disk[{p, 0.5}, r],
Disk[{1 - p, 0.5}, r}]}],
ImageSize -> Tiny]]

>> Dice[1]
```

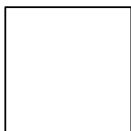


The empty series of dice shall be displayed as an empty dice:

```
>> Format[Dice[]] := Graphics[{
EdgeForm[Black], White,
Rectangle[]}, ImageSize -> Tiny]
```



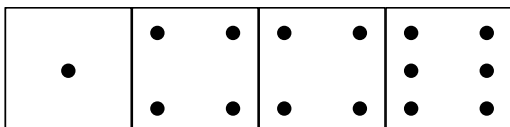
```
>> Dice[]
```



Any non-empty series of dice shall be displayed as a row of individual dice:

```
>> Format[Dice[d___Integer?(1 <= #
  <= 6 &)] := Row[Dice /@ {d}]
```

```
>> Dice[1, 6, 4, 4]
```



Note that *Mathics* will automatically sort the given format rules according to their “generality”, so the rule for the empty dice does not get overridden by the rule for a series of dice. We can still see the original form by using `InputForm`:

```
>> Dice[1, 6, 4, 4] // InputForm
Dice[1, 4, 4, 6]
```

We want to combine `Dice` objects using the `+` operator:

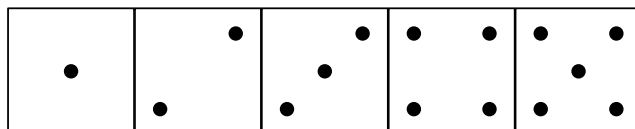
```
>> Dice[a___] + Dice[b___] ^:= Dice
  [Sequence @@ {a, b}]
```

The `^:=` (`UpSetDelayed`) tells *Mathics* to associate this rule with `Dice` instead of `Plus`, which is protected—we would have to unprotect it first:

```
>> Dice[a___] + Dice[b___] := Dice[
  Sequence @@ {a, b}]
TagPlusinDice[a___]
+ Dice[b___]isProtected.
$Failed
```

We can now combine dice:

```
>> Dice[1, 5] + Dice[3, 2] + Dice
  [4]
```



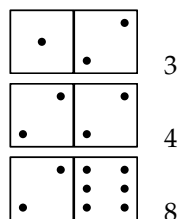
Let’s write a function that returns the sum of the rolled dice:

```
>> DiceSum[Dice[d___]] := Plus @@ {
  d}

>> DiceSum @ Dice[1, 2, 5]
8
```

And now let’s put some dice into a table:

```
>> Table[{Dice[Sequence @@ d],
  DiceSum @ Dice[Sequence @@ d]},
  {d, {{1, 2}, {2, 2}, {2, 6}}}]
// TableForm
```



It is not very sophisticated from a mathematical point of view, but it’s beautiful.

5. Web interface

Contents

Saving and loading worksheets	26	How definitions are stored	26	Keyboard commands	26
--	----	---	----	-----------------------------	----

Saving and loading worksheets

Worksheets exist in the browser window only and are not stored on the server, by default. To save all your queries and results, use the *Save* button in the menu bar. You have to login using your email address. If you don't have an account yet, leave the password field empty and a password will be sent to you. You will remain logged in until you press the *Logout* button in the upper right corner.

Saved worksheets can be loaded again using the *Load* button. Note that worksheet names are case-insensitive.

How definitions are stored

When you use the Web interface of *Mathics*, a browser session is created. Cookies have to be enabled to allow this. Your session holds a key which is used to access your definitions that are stored in a database on the server. As long as you don't clear the cookies in your browser, your definitions will remain even when you close and re-open the browser.

This implies that you should not store sensitive, private information in *Mathics* variables when using the online Web interface, of course. In addition to their values being stored in a database on the server, your queries might be saved for debugging purposes. However, the fact that

they are transmitted over plain HTTP should make you aware that you should not transmit any sensitive information. When you want to do calculations with that kind of stuff, simply install *Mathics* locally!

When you use *Mathics* on a public terminal, use the command `Quit[]` to erase all your definitions and close the browser window.

Keyboard commands

There are some keyboard commands you can use in the web interface of *Mathics*.

Shift+Return	Evaluate current cell (the most important one, for sure)
Ctrl+D	Focus documentation search
Ctrl+C	Back to document code
Ctrl+S	Save worksheet
Ctrl+O	Open worksheet

Unfortunately, keyboard commands do not work as expected in all browsers and under all operating systems. Often, they are only recognized when a textfield has focus; otherwise, the browser might do some browser-specific actions, like setting a bookmark etc.

6. Implementation

Contents

Developing	27	Documentation markup	28	Adding built-in symbols	30
Documentation and tests	27	Classes	28		

Developing

To start developing, check out the source directory. Run

```
$ python setup.py develop
```

This will temporarily overwrite the installed package in your Python library with a link to the current source directory. In addition, you might want to start the Django development server with

```
$ python manage.py runserver
```

It will restart automatically when you make changes to the source code.

Documentation and tests

One of the greatest features of *Mathics* is its integrated documentation and test system. Tests can be included right in the code as Python docstrings. All desired functionality should be covered by these tests to ensure that changes to the code don't break it. Execute

```
$ python test.py
```

to run all tests.

During a test run, the results of tests can be stored for the documentation, both in MathML and \LaTeX form, by executing

```
$ python test.py -o
```

The XML version of the documentation, which can be accessed in the Web interface, is updated immediately. To produce the \LaTeX documentation file, run:

```
$ python test.py -t
```

You can then create the PDF using \LaTeX . All required steps can be executed by

```
$ make latex
```

in the `doc/tex` directory, which uses `latexmk` to build the \LaTeX document. You just have to adjust the `Makefile` and `latexmkrc` to your environment. You need the *Asymptote* (version 2 at least) to generate the graphics in the documentation.

You can also run the tests for individual built-in symbols using

```
python test.py -s [name]
```

This will not re-create the corresponding documentation results, however. You have to run a complete test to do that.

Documentation markup

There is a lot of special markup syntax you can use in the documentation. It is kind of a mixture of XML, \LaTeX , Python doctest, and custom markup.

The following commands can be used to specify test cases.

```

>> query
    a test query.
: message
    a message in the result of the test query.
| print
    a printed line in the result of the test
    query.
= result
    the actual result of the test query.
. newline
    a newline in the test result.
$identifier$
    a variable identifier in Mathics code or in
    text.
#> query
    a test query that is not shown in the doc-
    umentation.
-Graphics-
    graphics in the test result.
...
    a part of the test result which is not
    checked in the test, e.g., for randomized
    or system-dependent output.

```

The following commands can be used to markup documentation text.

```

## comment
    a comment line that is not shown in the
    documentation.
<dl>list</dl>
    a definition list with <dt> and <dd> en-
    tries.
<dt>title
    the title of a description item.
<dd>description
    the description of a description item.
<ul>list</ul>
    an unordered list with <li> entries.
<ol>list</ol>
    an ordered list with <li> entries.
<li>item
    an item of an unordered or ordered list.
'code'
    inline Mathics code or other code.
<console>text</console>
    a console (shell/bash/Terminal) tran-
    script in its own paragraph.
<con>text</con>
    an inline console transcript.
<em>text</em>
    emphasized (italic) text.
<url>url</url>
    a URL.

    an image.
<ref label="label">
    a reference to an image.
\skip
    a vertical skip.
\LaTeX, \Mathematica, \Mathics
    special product and company names.
\'
    a single '.

```

To include images in the documentation, use the `img` tag, place an EPS file `src.eps` in `documentation/images` and run `images.sh` in the `doc` directory.

Classes

A UML diagram of the most important classes in *Mathics* can be seen in figure 6.1.

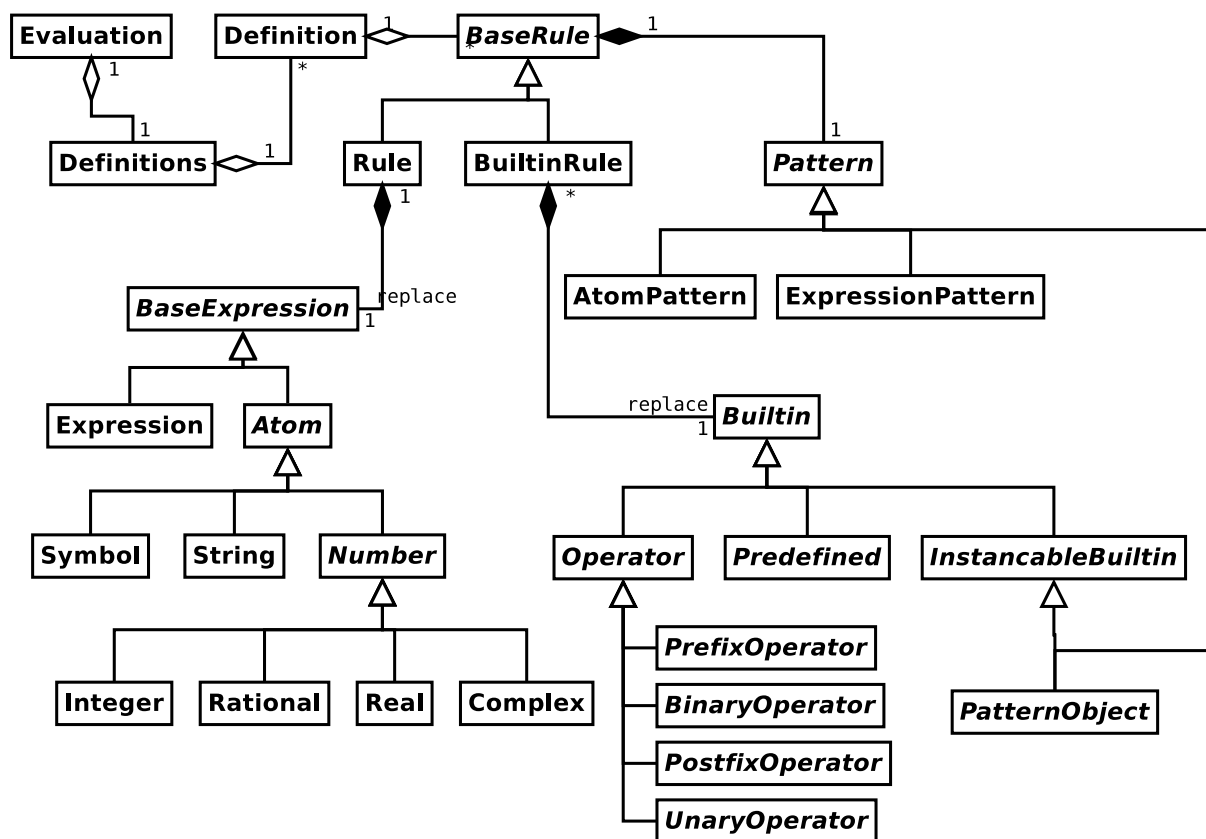


Figure 6.1.: UML class diagram

Adding built-in symbols

Adding new built-in symbols to *Mathics* is very easy. Either place a new module in the builtin directory and add it to the list of modules in builtin/___init___py or use an existing module. Create a new class derived from Builtin. If you want to add an operator, you should use one of the subclasses of Operator. Use SympyFunction for symbols that have a special meaning in SymPy.

To get an idea of how a built-in class can look like, consider the following implementation of If:

```
class If(Builtin):
    """
    <dl>
    <dt>'If[$cond$, $pos$, $neg$]'
    <dd>returns $pos$ if $cond$ evaluates to '
    True', and $neg$ if it evaluates to '
    False'.
    <dt>'If[$cond$, $pos$, $neg$, $other$]'
    <dd>returns $other$ if $cond$ evaluates to
    neither 'True' nor 'False'.
    <dt>'If[$cond$, $pos$]'
    <dd>returns 'Null' if $cond$ evaluates to '
    False'.
    </dl>
    >> If[1<2, a, b]
    = a
    If the second branch is not specified, 'Null'
    is taken:
    >> If[1<2, a]
    = a
    >> If[False, a] //FullForm
    = Null

    You might use comments (inside '(*' and '*)')
    to make the branches of 'If' more
    readable:
    >> If[a, (*then*) b, (*else*) c];
    """

    attributes = ['HoldRest']

    rules = {
```

```
        'If[condition_, t_]': 'If[condition, t,
            Null]',
    }

    def apply_3(self, condition, t, f, evaluation
    ):
        'If[condition_, t_, f_]

        if condition == Symbol('True'):
            return t.evaluate(evaluation)
        elif condition == Symbol('False'):
            return f.evaluate(evaluation)

    def apply_4(self, condition, t, f, u,
    evaluation):
        'If[condition_, t_, f_, u_]

        if condition == Symbol('True'):
            return t.evaluate(evaluation)
        elif condition == Symbol('False'):
            return f.evaluate(evaluation)
        else:
            return u.evaluate(evaluation)
```

The class starts with a Python *docstring* that specifies the documentation and tests for the symbol. A list (or tuple) attributes can be used to assign attributes to the symbol. Protected is assigned by default. A dictionary rules can be used to add custom rules that should be applied. Python functions starting with apply are converted to built-in rules. Their docstring is compiled to the corresponding *Mathics* pattern. Pattern variables used in the pattern are passed to the Python function by their same name, plus an additional evaluation object. This object is needed to evaluate further expressions, print messages in the Python code, etc. Unsurprisingly, the return value of the Python function is the expression which is replaced for the matched pattern. If the function does not return any value, the *Mathics* expression is left unchanged. Note that you have to return Symbol['Null'] explicitly if you want that.

Part II.

Reference of built-in symbols

I. Algebra

Contents

Apart	32	ExpandDenominator	34	PolynomialQ	36
Cancel	32	Exponent	35	PowerExpand	36
Coefficient	33	Factor	35	Simplify	36
CoefficientList	33	FactorTermsList	35	Together	37
Denominator	33	MinimalPolynomial	36	UpTo	37
Expand	34	Missing	36	Variables	37
ExpandAll	34	Numerator	36		

Apart

`Apart[expr]`
writes *expr* as a sum of individual fractions.
`Apart[expr, var]`
treats *var* as the main variable.

```
>> Apart[1 / (x^2 + 5x + 6)]
```

$$\frac{1}{2+x} - \frac{1}{3+x}$$

When several variables are involved, the results can be different depending on the main variable:

```
>> Apart[1 / (x^2 - y^2), x]
```

$$-\frac{1}{2y(x+y)} + \frac{1}{2y(x-y)}$$

```
>> Apart[1 / (x^2 - y^2), y]
```

$$\frac{1}{2x(x+y)} + \frac{1}{2x(x-y)}$$

Apart is Listable:

```
>> Apart[{1 / (x^2 + 5x + 6)}]
```

$$\left\{ \frac{1}{2+x} - \frac{1}{3+x} \right\}$$

But it does not touch other expressions:

```
>> Sin[1 / (x^2 - y^2)] //
```

Apart

$$\text{Sin} \left[\frac{1}{x^2 - y^2} \right]$$

Cancel

`Cancel[expr]`
cancels out common factors in numerators and denominators.

```
>> Cancel[x / x^2]
```

$$\frac{1}{x}$$

Cancel threads over sums:

```
>> Cancel[x / x^2 + y / y^2]
```

$$\frac{1}{x} + \frac{1}{y}$$

```
>> Cancel[f[x] / x + x * f[x] / x^2]
```

$$\frac{2f[x]}{x}$$

Coefficient

`Coefficient[expr, form]`
returns the coefficient of *form* in the polynomial *expr*.
`Coefficient[expr, form, n]`
return the coefficient of *form*ⁿ in *expr*.

```
>> Coefficient[(x + y)^4, (x^2)* (y^2)]
```

6


```

>> Coefficient[a x^2 + b y^3 + c x
+ d y + 5, x]
c

>> Coefficient[(x + 3 y)^5, x]
405 y^4

>> Coefficient[(x + 3 y)^5, x * y
^4]
405

>> Coefficient[(x + 2)/(y - 3) + (x
+ 3)/(y - 2), x]
1 / (-3 + y) + 1 / (-2 + y)

>> Coefficient[x * Cos[x + 3] + 6 * y,
x]
Cos[3 + x]

>> Coefficient[(x + 1)^3, x, 2]
3

>> Coefficient[a x^2 + b y^3 + c x
+ d y + 5, y, 3]
b

>> Coefficient[(x + 2)^3 + (x + 3)
^2, x, 0]
17

>> Coefficient[(x + 2)^3 + (x + 3)
^2, y, 0]
(2 + x)^3 + (3 + x)^2

>> Coefficient[a x^2 + b y^3 + c x
+ d y + 5, x, 0]
5 + b y^3 + d y

```

CoefficientList

`CoefficientList[poly, var]`
returns a list of coefficients of powers of *var* in *poly*, starting with power 0.

`CoefficientList[poly, {var1, var2, ...}]`
returns an array of coefficients of the *vari*.

```

>> CoefficientList[(x + 3)^5, x]
{243, 405, 270, 90, 15, 1}

```

```

>> CoefficientList[(x + y)^4, x]
{y^4, 4 y^3, 6 y^2, 4 y, 1}

>> CoefficientList[a x^2 + b y^3 +
c x + d y + 5, x]
{5 + b y^3 + d y, c, a}

>> CoefficientList[(x + 2)/(y - 3) +
x/(y - 2), x]
{2 / (-3 + y), 1 / (-3 + y) + 1 / (-2 + y)}

>> CoefficientList[(x + y)^3, z]
{(x + y)^3}

>> CoefficientList[a x^2 + b y^3 +
c x + d y + 5, {x, y}]
{{5, d, 0, b}, {c, 0, 0, 0}, {a, 0, 0, 0}}

>> CoefficientList[(x - 2 y + 3 z)
^3, {x, y, z}]
{{{0, 0, 0, 27}, {0, 0, -54, 0},
{0, 36, 0, 0}, {-8, 0, 0, 0}}, {{0,
0, 27, 0}, {0, -36, 0, 0}, {12,
0, 0, 0}, {0, 0, 0, 0}}, {{0, 9, 0,
0}, {-6, 0, 0, 0}, {0, 0, 0, 0},
{0, 0, 0, 0}}, {{1, 0, 0, 0}, {0, 0,
0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}

```

Denominator

`Denominator[expr]`
gives the denominator in *expr*.

```

>> Denominator[a / b]
b

>> Denominator[2 / 3]
3

>> Denominator[a + b]
1

```

Expand

Expand[expr]
expands out positive integer powers and products of sums in *expr*, as well as trigonometric identities.

```
>> Expand[(x + y)^3]
x^3 + 3x^2y + 3xy^2 + y^3

>> Expand[(a + b)(a + c + d)]
a^2 + ab + ac + ad + bc + bd

>> Expand[(a + b)(a + c + d)(e + f) + e a a]
2a^2e + a^2f + abe + abf + ace + acf
+ ade + adf + bce + bcf + bde + bdf

>> Expand[(a + b)^2 * (c + d)]
a^2c + a^2d + 2abc + 2abd + b^2c + b^2d

>> Expand[(x + y)^2 + x y]
x^2 + 3xy + y^2

>> Expand[((a + b)(c + d))^2 + b(1 + a)]
a^2c^2 + 2a^2cd + a^2d^2 + b + ab + 2abc^2
+ 4abcd + 2abd^2 + b^2c^2 + 2b^2cd + b^2d^2
```

Expand expands items in lists and rules:

```
>> Expand[{4 (x + y), 2 (x + y) -> 4 (x + y)}]
{4x + 4y, 2x + 2y -> 4x + 4y}
```

Expand expands trigonometric identities

```
>> Expand[Sin[x + y], Trig -> True]
Cos[x] Sin[y] + Cos[y] Sin[x]
```

Expand does not change any other expression.

```
>> Expand[Sin[x (1 + y)]]
Sin[x (1 + y)]
```

Expand also works in Galois fields

```
>> Expand[(1 + a)^12, Modulus -> 3]
1 + a^3 + a^9 + a^12

>> Expand[(1 + a)^12, Modulus -> 4]
1 + 2a^2 + 3a^4 + 3a^8 + 2a^10 + a^12
```

ExpandAll

ExpandAll[expr]
expands out negative integer powers and products of sums in *expr*.

```
>> ExpandAll[(a + b)^2 / (c + d)^2]
a^2 / (c^2 + 2cd + d^2) + 2ab / (c^2 + 2cd + d^2)
+ b^2 / (c^2 + 2cd + d^2)
```

ExpandAll descends into sub expressions

```
>> ExpandAll[(a + Sin[x (1 + y)])^2]
2a Sin[x + xy] + a^2 + Sin[x + xy]^2
```

ExpandAll also expands heads

```
>> ExpandAll[((1 + x)(1 + y))[x]]
(1 + x + y + xy)[x]
```

ExpandAll can also work in finite fields

```
>> ExpandAll[(1 + a)^6 / (x + y)^3, Modulus -> 3]
(1 + 2a^3 + a^6) / (x^3 + y^3)
```

ExpandDenominator

ExpandDenominator[expr]
expands out negative integer powers and products of sums in *expr*.

```
>> ExpandDenominator[(a + b)^2 / ((c + d)^2 (e + f))]
(a + b)^2 / (c^2e + c^2f + 2cde + 2cdf + d^2e + d^2f)
```

Exponent

Exponent[*expr*, *form*]
 returns the maximum power with which *form* appears in the expanded form of *expr*.
Exponent[*expr*, *form*, *h*]
 applies *h* to the set of exponents with which *form* appears in *expr*.

```
>> Exponent[5 x^2 - 3 x + 7, x]
2
>> Exponent[(x^3 + 1)^2 + 1, x]
6
>> Exponent[x^(n + 1) + Sqrt[x] + 1, x]
Max[1/2, 1 + n]
>> Exponent[x / y, y]
-1
>> Exponent[(x^2 + 1)^3 - 1, x, Min]
2
>> Exponent[1 - 2 x^2 + a x^3, x, List]
{0, 2, 3}
>> Exponent[0, x]
-∞
>> Exponent[1, x]
0
```

Factor

Factor[*expr*]
 factors the polynomial expression *expr*.

```
>> Factor[x^2 + 2 x + 1]
(1 + x)^2
>> Factor[1 / (x^2 + 2x + 1) + 1 / (x^4 + 2x^2 + 1)]
(2 + 2x + 3x^2 + x^4) / ((1 + x)^2 (1 + x^2)^2)
```

FactorTermsList

FactorTermsList[*poly*]
 returns a list of 2 elements. The first element is the numerical factor in *poly*. The second one is the remaining of the polynomial with numerical factor removed
FactorTermsList[*poly*, {*x*₁, *x*₂, ...}]
 returns a list of factors in *poly*. The first element is the numerical factor in *poly*. The next ones are factors that are independent of variables lists which are created by removing each variable *x_i* from right to left. The last one is the remaining of polynomial after dividing *poly* to all previous factors

```
>> FactorTermsList[2 x^2 - 2]
{2, -1 + x^2}
>> FactorTermsList[x^2 - 2 x + 1]
{1, 1 - 2x + x^2}
>> f = 3 (-1 + 2 x) (-1 + y) (1 - a)
3 (-1 + 2x) (-1 + y) (1 - a)
>> FactorTermsList[f]
{-3, -1 + a - 2ax - ay + 2x + y - 2xy + 2axy}
>> FactorTermsList[f, x]
{-3, 1 - a - y + ay, -1 + 2x}
>> FactorTermsList[f, {x, y}]
{-3, -1 + a, -1 + y, -1 + 2x}
```

MinimalPolynomial

MinimalPolynomial[*s*, *x*]
 gives the minimal polynomial in *x* for which the algebraic number *s* is a root.

```
>> MinimalPolynomial[7, x]
-7 + x
>> MinimalPolynomial[Sqrt[2] + Sqrt[3], x]
1 - 10x^2 + x^4
```

```
>> MinimalPolynomial[Sqrt[1 + Sqrt
[3]], x]
-2 - 2x2 + x4

>> MinimalPolynomial[Sqrt[I + Sqrt
[6]], x]
49 - 10x4 + x8
```

Missing

Numerator

`Numerator[expr]`
gives the numerator in *expr*.

```
>> Numerator[a / b]
a

>> Numerator[2 / 3]
2

>> Numerator[a + b]
a + b
```

PolynomialQ

`PolynomialQ[expr, var]`
returns True if *expr* is a polynomial in *var*,
and returns False otherwise.
`PolynomialQ[expr, {var1, ...}]`
tests whether *expr* is a polynomial in the
vari.

```
>> PolynomialQ[x3 - 2 x/y + 3xz, x]
True

>> PolynomialQ[x3 - 2 x/y + 3xz, y]
False

>> PolynomialQ[f[a] + f[a]2, f[a]]
True

>> PolynomialQ[x2 + axy2 - bSin[c], {x, y}]
True
```

```
>> PolynomialQ[x2 + axy2 - bSin[c], {a, b, c}]
False
```

PowerExpand

`PowerExpand[expr]`
expands out powers of the form $(x^y)^z$
and $(x*y)^z$ in *expr*.

```
>> PowerExpand[(a ^ b) ^ c]
abc

>> PowerExpand[(a * b) ^ c]
acbc
```

PowerExpand is not correct without certain assumptions:

```
>> PowerExpand[(x ^ 2) ^ (1/2)]
x
```

Simplify

`Simplify[expr]`
simplifies *expr*.

```
>> Simplify[2*Sin[x]2 + 2*Cos[x]2]
2

>> Simplify[x]
x

>> Simplify[f[x]]
f[x]
```

Together

`Together[expr]`
writes sums of fractions in *expr* together.

```
>> Together[a / c + b / c]

$$\frac{a + b}{c}$$

```

Together operates on lists:

```
>> Together[{x / (y+1)+ x / (y+1)
^2}]
```

$$\left\{ \frac{x (2 + y)}{(1 + y)^2} \right\}$$

But it does not touch other functions:

```
>> Together[f[a / c + b / c]]
```

$$f \left[\frac{a}{c} + \frac{b}{c} \right]$$

UpTo

Variables

`Variables[expr]`

gives a list of the variables that appear in the polynomial *expr*.

```
>> Variables[a x^2 + b x + c]
{a, b, c, x}
```

```
>> Variables[{a + b x, c y^2 + x
/2}]
{a, b, c, x, y}
```

```
>> Variables[x + Sin[y]]
{x, Sin[y]}
```

II. Arithmetic functions

Basic arithmetic functions, including complex number arithmetic.

Contents

Abs	38	I	41	Power (^)	44
Boole	38	Im	41	Product	44
ComplexInfinity	39	Indeterminate	41	Rational	45
Complex	39	InexactNumberQ	42	Re	45
Conjugate	39	Infinity	42	RealNumberQ	45
CubeRoot	39	IntegerQ	42	Real	45
DirectedInfinity	39	Integer	42	Sign	45
Divide (/)	40	MachineNumberQ	42	Sqrt	46
ExactNumberQ	40	Minus (-)	42	Subtract (-)	46
Factorial (!)	40	NumberQ	42	Sum	47
Gamma	41	Piecewise	43	Times (*)	47
HarmonicNumber	41	Plus (+)	43		
		Pochhammer	43		

Abs

Abs[x]
returns the absolute value of x .

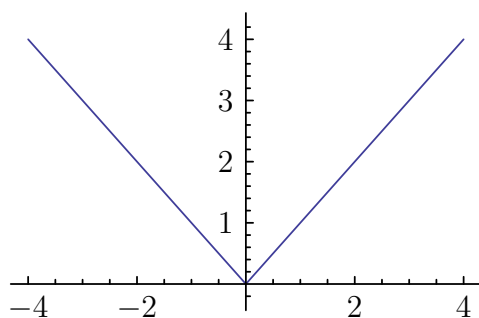
```
>> Abs[-3]
3
```

Abs returns the magnitude of complex numbers:

```
>> Abs[3 + I]
 $\sqrt{10}$ 
```

```
>> Abs[3.0 + I]
3.16228
```

```
>> Plot[Abs[x], {x, -4, 4}]
```



Boole

Boole[expr]
returns 1 if expr is True and 0 if expr is False.

```
>> Boole[2 == 2]
1
```

```
>> Boole[7 < 5]
0
```

```
>> Boole[a == 7]
Boole[a==7]
```

ComplexInfinity

ComplexInfinity
represents an infinite complex quantity of undetermined direction.

```
>> 1 / ComplexInfinity
0
```

```
>> ComplexInfinity * Infinity
ComplexInfinity
```

```
>> FullForm[ComplexInfinity]
DirectedInfinity[]
```

Complex

Complex
is the head of complex numbers.
Complex[*a*, *b*]
constructs the complex number $a + I b$.

```
>> Head[2 + 3*I]
Complex

>> Complex[1, 2/3]
1 +  $\frac{2I}{3}$ 

>> Abs[Complex[3, 4]]
5
```

Conjugate

Conjugate[*z*]
returns the complex conjugate of the complex number *z*.

```
>> Conjugate[3 + 4 I]
3 - 4I

>> Conjugate[3]
3

>> Conjugate[a + b * I]
Conjugate[a] - IConjugate[b]

>> Conjugate[{{1, 2 + I 4, a + I b}, {I}}]
{{1, 2 - 4I, Conjugate[a] - IConjugate[b]}, {-I}}

>> Conjugate[1.5 + 2.5 I]
1.5 - 2.5I
```

CubeRoot

CubeRoot[*n*]
finds the real-valued cube root of the given *n*.

```
>> CubeRoot[16]
 $22^{\frac{1}{3}}$ 
```

DirectedInfinity

DirectedInfinity[*z*]
represents an infinite multiple of the complex number *z*.
DirectedInfinity[]
is the same as **ComplexInfinity**.

```
>> DirectedInfinity[1]
 $\infty$ 

>> DirectedInfinity[]
ComplexInfinity

>> DirectedInfinity[1 + I]
 $\left(\frac{1}{2} + \frac{I}{2}\right) \sqrt{2} \infty$ 

>> 1 / DirectedInfinity[1 + I]
0

>> DirectedInfinity[1] + DirectedInfinity[-1]
Indeterminateexpression
- Infinity + Infinityencountered.
Indeterminate
```

Divide (/)

Divide[*a*, *b*]
a / *b*
represents the division of *a* by *b*.

```
>> 30 / 5
6

>> 1 / 8
 $\frac{1}{8}$ 
```

```
>> Pi / 4
      Pi
      4
```

Use `N` or a decimal point to force numeric evaluation:

```
>> Pi / 4.0
0.785398

>> 1 / 8
      1
      8

>> N[%]
0.125
```

Nested divisions:

```
>> a / b / c
      a
      bc

>> a / (b / c)
      ac
      b

>> a / b / (c / (d / e))
      ad
      bce

>> a / (b ^ 2 * c ^ 3 / e)
      ae
      b^2 c^3
```

ExactNumberQ

`ExactNumberQ[expr]`
returns `True` if `expr` is an exact number, and `False` otherwise.

```
>> ExactNumberQ[10]
True

>> ExactNumberQ[4.0]
False

>> ExactNumberQ[n]
False
```

`ExactNumberQ` can be applied to complex numbers:

```
>> ExactNumberQ[1 + I]
True

>> ExactNumberQ[1 + 1. I]
False
```

Factorial (!)

`Factorial[n]`
`n!`
computes the factorial of `n`.

```
>> 20!
2 432 902 008 176 640 000
```

`Factorial` handles numeric (real and complex) values using the gamma function:

```
>> 10.5!
1.18994 × 107

>> (-3.0+1.5*I)!
0.0427943 - 0.00461565I
```

However, the value at poles is `ComplexInfinity`:

```
>> (-1.)!
ComplexInfinity
```

`Factorial` has the same operator (!) as `Not`, but with higher precedence:

```
>> !a! //FullForm
Not[Factorial[a]]
```

Gamma

`Gamma[z]`
is the gamma function on the complex number `z`.
`Gamma[z, x]`
is the upper incomplete gamma function.
`Gamma[z, x0, x1]`
is equivalent to `Gamma[z, x0] - Gamma[z, x1]`.

`Gamma[z]` is equivalent to $(z - 1)!$:

```
>> Simplify[Gamma[z] - (z - 1)!]
0
```

Exact arguments:

```
>> Gamma[8]
5 040

>> Gamma[1/2]
√Pi

>> Gamma[1, x]
E-x
```



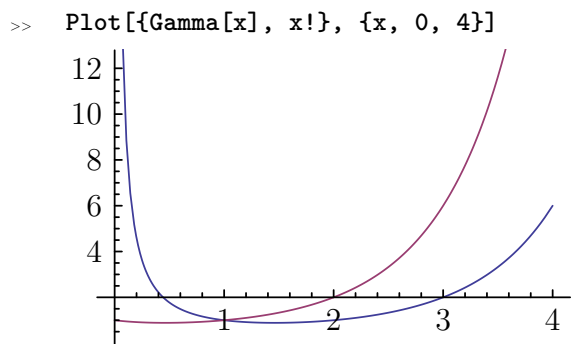
```
>> Gamma[0, x]
ExpIntegralE[1, x]
```

Numeric arguments:

```
>> Gamma[123.78]
4.21078 × 10204
```

```
>> Gamma[1. + I]
0.498016 - 0.15495I
```

Both Gamma and Factorial functions are continuous:



HarmonicNumber

HarmonicNumber[n]
returns the n th harmonic number.

```
>> Table[HarmonicNumber[n], {n, 8}]
{1, 3/2, 11/6, 25/12, 137/60, 49/20, 363/140, 761/280}
```

```
>> HarmonicNumber[3.8]
2.03806
```

I

I
represents the imaginary number $\sqrt{-1}$.

```
>> I^2
-1
```

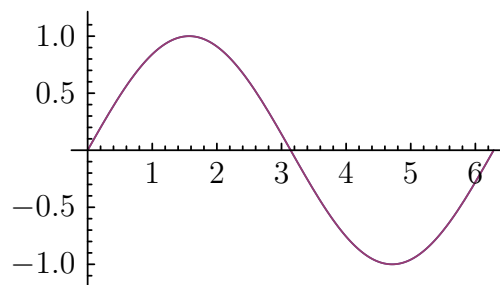
```
>> (3+I)*(3-I)
10
```

Im

Im[z]
returns the imaginary component of the complex number z .

```
>> Im[3+4I]
4
```

```
>> Plot[{Sin[a], Im[E^(I a)]}, {a, 0, 2 Pi}]
```



Indeterminate

Indeterminate
represents an indeterminate result.

```
>> 0^0
Indeterminateexpression00encountered.
Indeterminate
>> Tan[Indeterminate]
Indeterminate
```

InexactNumberQ

InexactNumberQ[expr]
returns True if *expr* is not an exact number, and False otherwise.

```
>> InexactNumberQ[a]
False
>> InexactNumberQ[3.0]
True
>> InexactNumberQ[2/3]
False
```

InexactNumberQ can be applied to complex numbers:

```
>> InexactNumberQ[4.0+I]
True
```

Infinity

`Infinity`
represents an infinite real quantity.

```
>> 1 / Infinity
0
>> Infinity + 100
∞
```

Use `Infinity` in sum and limit calculations:

```
>> Sum[1/x^2, {x, 1, Infinity}]

$$\frac{\pi^2}{6}$$

```

IntegerQ

`IntegerQ[expr]`
returns `True` if *expr* is an integer, and `False` otherwise.

```
>> IntegerQ[3]
True
>> IntegerQ[Pi]
False
```

Integer

`Integer`
is the head of integers.

```
>> Head[5]
Integer
```

MachineNumberQ

`MachineNumberQ[expr]`
returns `True` if *expr* is a machine-precision real or complex number.

```
= True
>> MachineNumberQ
[3.14159265358979324]
False
>> MachineNumberQ[1.5 + 2.3 I]
True
>> MachineNumberQ
[2.71828182845904524 +
3.14159265358979324 I]
False
```

Minus (-)

`Minus[expr]`
is the negation of *expr*.

```
>> -a //FullForm
Times[-1, a]
```

Minus automatically distributes:

```
>> -(x - 2/3)

$$\frac{2}{3} - x$$

```

Minus threads over lists:

```
>> -Range[10]
{-1, -2, -3, -4, -5,
-6, -7, -8, -9, -10}
```

NumberQ

`NumberQ[expr]`
returns `True` if *expr* is an explicit number, and `False` otherwise.

```
>> NumberQ[3+I]
True
>> NumberQ[5!]
True
>> NumberQ[Pi]
False
```

Piecewise

`Piecewise[{{expr1, cond1}, ...}]`
represents a piecewise function.
`Piecewise[{{expr1, cond1}, ...}, expr]`
represents a piecewise function with default `expr`.

Heaviside function

```
>> Piecewise[{{0, x <= 0}}, 1]
Piecewise[{{0, x <= 0}}, 1]

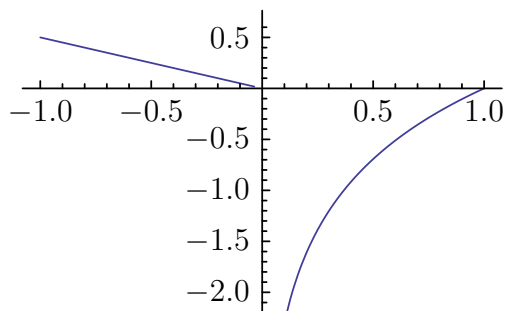
>> Integrate[Piecewise[{{1, x <= 0}, {-1, x > 0}}, x]
Piecewise[{{x, x <= 0}, {-x, True}}]

>> Integrate[Piecewise[{{1, x <= 0}, {-1, x > 0}}, {x, -1, 2}]
-1
```

Piecewise defaults to 0 if no other case is matching.

```
>> Piecewise[{{1, False}}]
0

>> Plot[Piecewise[{{Log[x], x > 0}, {x*-0.5, x < 0}}, {x, -1, 1}]
```



```
>> Piecewise[{{0 ^ 0, False}}, -1]
-1
```

Plus (+)

`Plus[a, b, ...]`
 $a + b + \dots$
represents the sum of the terms a, b, \dots

```
>> 1 + 2
3
```

Plus performs basic simplification of terms:

```
>> a + b + a
2a + b

>> a + a + 3 * a
5a

>> a + b + 4.5 + a + b + a + 2 + 1.5 b
6.5 + 3a + 3.5b
```

Apply Plus on a list to sum up its elements:

```
>> Plus @@ {2, 4, 6}
12
```

The sum of the first 1000 integers:

```
>> Plus @@ Range[1000]
500500
```

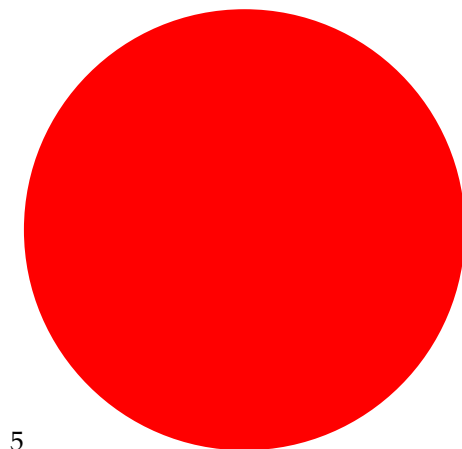
Plus has default value 0:

```
>> DefaultValues[Plus]
{HoldPattern[Default[Plus]] :> 0}

>> a /. n_. + x_ :> {n, x}
{0, a}
```

The sum of 2 red circles and 3 red circles is...

```
>> 2 Graphics[{Red, Disk[]}] + 3
Graphics[{Red, Disk[]}]
```



5

Pochhammer

`Pochhammer[a, n]`
is the Pochhammer symbol $(a)_n$.

```
>> Pochhammer[4, 8]
6652800
```

Power (^)

`Power[a, b]`
 a^b
 represents a raised to the power of b .

```
>> 4 ^ (1/2)
2

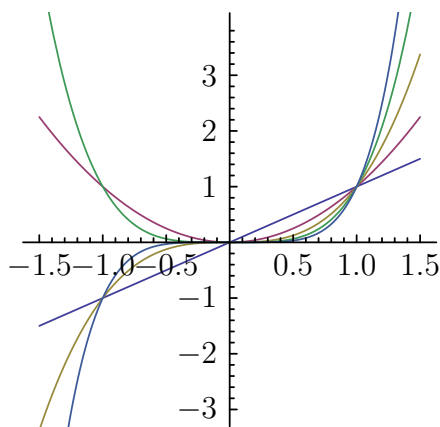
>> 4 ^ (1/3)
2^(2/3)

>> 3^123
48 519 278 097 689 642 681 ~
~155 855 396 759 336 072 ~
~749 841 943 521 979 872 827

>> (y ^ 2) ^ (1/2)
sqrt(y^2)

>> (y ^ 2) ^ 3
y^6

>> Plot[Evaluate[Table[x^y, {y, 1,
5}]], {x, -1.5, 1.5},
AspectRatio -> 1]
```



Use a decimal point to force numeric evaluation:

```
>> 4.0 ^ (1/3)
1.5874
```

Power has default value 1 for its second argument:

```
>> DefaultValues[Power]
{HoldPattern[Default[Power, 2]] :> 1}

>> a /. x_ ^ n_. :> {x, n}
{a, 1}
```

Power can be used with complex numbers:

```
>> (1.5 + 1.0 I) ^ 3.5
-3.68294 + 6.95139 I

>> (1.5 + 1.0 I) ^ (3.5 + 1.5 I)
-3.19182 + 0.645659 I
```

Product

`Product[expr, {i, imin, imax}]`
 evaluates the discrete product of $expr$ with i ranging from $imin$ to $imax$.
`Product[expr, {i, imax}]`
 same as `Product[expr, {i, 1, imax}]`.
`Product[expr, {i, imin, imax, di}]`
 i ranges from $imin$ to $imax$ in steps of di .
`Product[expr, {i, imin, imax}, {j, jmin, jmax}, ...]`
 evaluates $expr$ as a multiple product, with $\{i, \dots\}, \{j, \dots\}, \dots$ being in outermost-to-innermost order.

```
>> Product[k, {k, 1, 10}]
3 628 800

>> 10!
3 628 800

>> Product[x^k, {k, 2, 20, 2}]
x^110

>> Product[2 ^ i, {i, 1, n}]
2^(n/2 + n^2/2)

>> Product[f[i], {i, 1, 7}]
f[1] f[2] f[3] f[4] f[5] f[6] f[7]
```

Symbolic products involving the factorial are evaluated:

```
>> Product[k, {k, 3, n}]
n! / 2
```

Evaluate the n th primorial:

```
>> primorial[0] = 1;

>> primorial[n_Integer] := Product[
Prime[k], {k, 1, n}];

>> primorial[12]
7 420 738 134 810
```

Rational

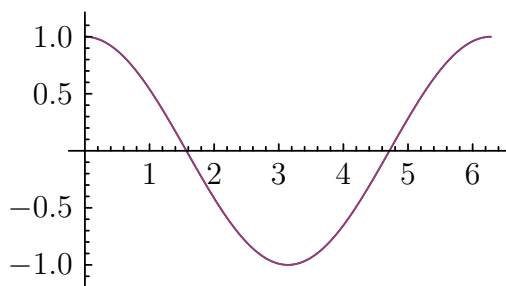
Rational
is the head of rational numbers.
Rational[*a*, *b*]
constructs the rational number a / b .

```
>> Head[1/2]
Rational
>> Rational[1, 2]
 $\frac{1}{2}$ 
```

Re

Re[*z*]
returns the real component of the complex number *z*.

```
>> Re[3+4I]
3
>> Plot[{Cos[a], Re[E^(I a)]}, {a,
0, 2 Pi}]
```



RealNumberQ

RealNumberQ[*expr*]
returns True if *expr* is an explicit number with no imaginary component.

```
>> RealNumberQ[10]
True
>> RealNumberQ[4.0]
True
>> RealNumberQ[1+I]
False
```

```
>> RealNumberQ[0 * I]
True
>> RealNumberQ[0.0 * I]
False
```

Real

Real
is the head of real (inexact) numbers.

```
>> x = 3. ^ -20;
>> InputForm[x]
2.8679719907924413*^-10
>> Head[x]
Real
```

Sign

Sign[*x*]
return -1, 0, or 1 depending on whether *x* is negative, zero, or positive.

```
>> Sign[19]
1
>> Sign[-6]
-1
>> Sign[0]
0
>> Sign[{-5, -10, 15, 20, 0}]
{-1, -1, 1, 1, 0}
>> Sign[3 - 4*I]
 $\frac{3}{5} - \frac{4I}{5}$ 
```

Sqrt

Sqrt[*expr*]
returns the square root of *expr*.

```
>> Sqrt[4]
2
```

```
>> Sqrt[5]
 $\sqrt{5}$ 
```

```
>> Sqrt[5] // N
2.23607
```

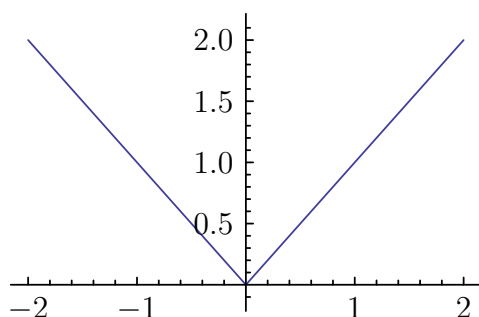
```
>> Sqrt[a]^2
a
```

Complex numbers:

```
>> Sqrt[-4]
2I
```

```
>> I == Sqrt[-1]
True
```

```
>> Plot[Sqrt[a^2], {a, -2, 2}]
```



Subtract (-)

`Subtract[a, b]`

$a - b$

represents the subtraction of b from a .

```
>> 5 - 3
2
```

```
>> a - b // FullForm
Plus[a, Times[-1, b]]
```

```
>> a - b - c
a - b - c
```

```
>> a - (b - c)
a - b + c
```

Sum

`Sum[expr, {i, imin, imax}]`

evaluates the discrete sum of $expr$ with i ranging from $imin$ to $imax$.

`Sum[expr, {i, imax}]`

same as `Sum[expr, {i, 1, imax}]`.

`Sum[expr, {i, imin, imax, di}]`

i ranges from $imin$ to $imax$ in steps of di .

`Sum[expr, {i, imin, imax}, {j, jmin, jmax}, ...]`

evaluates $expr$ as a multiple sum, with $\{i, \dots\}, \{j, \dots\}, \dots$ being in outermost-to-innermost order.

```
>> Sum[k, {k, 1, 10}]
55
```

Double sum:

```
>> Sum[i * j, {i, 1, 10}, {j, 1, 10}]
3025
```

Symbolic sums are evaluated:

```
>> Sum[k, {k, 1, n}]
 $\frac{n(1+n)}{2}$ 
```

```
>> Sum[k, {k, n, 2 n}]
 $\frac{3n(1+n)}{2}$ 
```

```
>> Sum[k, {k, I, I + 1}]
1 + 2I
```

```
>> Sum[1 / k ^ 2, {k, 1, n}]
HarmonicNumber[n, 2]
```

```
>> Sum[f[i], {i, 1, 7}]
f[1] + f[2] + f[3] + f[4] + f[5] + f[6] + f[7]
```

Verify algebraic identities:

```
>> Sum[x ^ 2, {x, 1, y}] - y * (y + 1) * (2 * y + 1) / 6
0
```

```
>> (-1 + a^n) Sum[a^(k n), {k, 0, m - 1}] // Simplify
```

$\text{Piecewise}\left[\left\{\left\{m(-1 + a^n), a^n \neq 1\right\}, \left\{-1 + (a^n)^m, \text{True}\right\}\right\}\right]$

Infinite sums:

```
>> Sum[1 / 2 ^ i, {i, 1, Infinity}]
1

>> Sum[1 / k ^ 2, {k, 1, Infinity}]
Pi^2
6
```

Times (*)

```
Times[a, b, ...]
a * b * ...
a b ...
represents the product of the terms a, b, ...
```

```
>> 10 * 2
20

>> 10 2
20

>> a * a
a^2

>> x ^ 10 * x ^ -2
x^8

>> {1, 2, 3} * 4
{4, 8, 12}

>> Times @@ {1, 2, 3, 4}
24

>> IntegerLength[Times@@Range
[5000]]
16326
```

Times has default value 1:

```
>> DefaultValues[Times]
{HoldPattern[Default[Times]]:>1}

>> a /. n_. * x_ :> {n, x}
{1, a}
```

III. Assignment

Contents

AddTo (+=)	48	Information (??)	52	SubValues	55
Clear	49	LoadModule	52	SubtractFrom (-=)	55
ClearAll	49	Messages	52	TagSet	55
Decrement (--).	49	NValues	53	TagSetDelayed	55
DefaultValues	49	OwnValues	53	TimesBy (*=)	56
Definition	50	PreDecrement (--).	53	Unset (=.)	56
DivideBy (/=)	51	PreIncrement (++)	53	UpSet (^=)	56
DownValues	51	Quit	54	UpSetDelayed (^:=)	57
Increment (++)	51	Set (=)	54	UpValues	57
		SetDelayed (:=)	55		

AddTo (+=)

AddTo[x, dx]
 $x += dx$
 is equivalent to $x = x + dx$.

```
>> a = 10;

>> a += 2
12

>> a
12
```

Clear

Clear[symb1, symb2, ...]
 clears all values of the given symbols.
 The arguments can also be given as
 strings containing symbol names.

```
>> x = 2;

>> Clear[x]

>> x
x

>> x = 2;
```

```
>> y = 3;

>> Clear["Global`*"]

>> x
x

>> y
y
```

ClearAll may not be called for Protected symbols.

```
>> Clear[Sin]
SymbolSinisProtected.
```

The values and rules associated with built-in symbols will not get lost when applying Clear (after unprotecting them):

```
>> Unprotect[Sin]

>> Clear[Sin]

>> Sin[Pi]
0
```

Clear does not remove attributes, messages, options, and default values associated with the symbols. Use ClearAll to do so.

```
>> Attributes[r] = {Flat, Orderless};

>> Clear["r"]
```



```
>> Attributes[r]
{Flat, Orderless}
```

ClearAll

`ClearAll[symb1, symb2, ...]`
clears all values, attributes, messages and options associated with the given symbols. The arguments can also be given as strings containing symbol names.

```
>> x = 2;

>> ClearAll[x]

>> x
x

>> Attributes[r] = {Flat, Orderless};

>> ClearAll[r]

>> Attributes[r]
{}
```

`ClearAll` may not be called for Protected or Locked symbols.

```
>> Attributes[lock] = {Locked};

>> ClearAll[lock]
Symbollockislocked.
```

Decrement (--)

`Decrement[x]`
`x--`
decrements x by 1, returning the original value of x .

```
>> a = 5;

>> a--
5

>> a
4
```

DefaultValues

`DefaultValues[symbol]`
gives the list of default values associated with *symbol*.

```
>> Default[f, 1] = 4
4

>> DefaultValues[f]
{HoldPattern[Default[f, 1]] :> 4}
```

You can assign values to `DefaultValues`:

```
>> DefaultValues[g] = {Default[g] -> 3};

>> Default[g, 1]
3

>> g[x_.] := {x}

>> g[a]
{a}

>> g[]
{3}
```

Definition

`Definition[symbol]`
prints as the user-defined values and rules associated with *symbol*.

`Definition` does not print information for ReadProtected symbols. `Definition` uses `InputForm` to format values.

```
>> a = 2;

>> Definition[a]
a = 2

>> f[x_] := x ^ 2

>> g[f] ^:= 2

>> Definition[f]
f[x_] = x^2
g[f] ^:= 2
```

`Definition` of a rather evolved (though meaningless) symbol:

```
>> Attributes[r] := {Orderless}

>> Format[r[args___]] := Infix[{
  args}, "~"]

>> N[r] := 3.5

>> Default[r, 1] := 2

>> r::msg := "My message"

>> Options[r] := {Opt -> 3}

>> r[arg_., OptionsPattern[r]] := {
  arg, OptionValue[Opt]}
```

Some usage:

```
>> r[z, x, y]
  x ~ y ~ z

>> N[r]
  3.5

>> r[]
  {2, 3}

>> r[5, Opt->7]
  {5, 7}
```

Its definition:

```
>> Definition[r]
  Attributes[r] = {Orderless}
  arg_ ~ OptionsPattern[r]
    = {arg, OptionValue[Opt]}
  N[r, MachinePrecision] = 3.5
  Format[args___, MathMLForm]
    = Infix[{args}, "~"]
  Format[args___,
  OutputForm] = Infix[{args}, "~"]
  Format[args___, StandardForm]
    = Infix[{args}, "~"]
  Format[args___,
  TeXForm] = Infix[{args}, "~"]
  Format[args___, TraditionalForm]
    = Infix[{args}, "~"]
  Default[r, 1] = 2
  Options[r] = {Opt -> 3}
```

For ReadProtected symbols, Definition just prints attributes, default values and options:

```
>> SetAttributes[r, ReadProtected]
```

```
>> Definition[r]
  Attributes[r] = {Orderless,
    ReadProtected}
  Default[r, 1] = 2
  Options[r] = {Opt -> 3}
```

This is the same for built-in symbols:

```
>> Definition[Plus]
  Attributes[Plus] = {Flat, Listable,
    NumericFunction,
    OneIdentity,
    Orderless, Protected}
  Default[Plus] = 0

>> Definition[Level]
  Attributes[Level] = {Protected}
  Options[Level] = {Heads -> False}
```

ReadProtected can be removed, unless the symbol is locked:

```
>> ClearAttributes[r, ReadProtected]
]
```

Clear clears values:

```
>> Clear[r]

>> Definition[r]
  Attributes[r] = {Orderless}
  Default[r, 1] = 2
  Options[r] = {Opt -> 3}
```

ClearAll clears everything:

```
>> ClearAll[r]

>> Definition[r]
  Null
```

If a symbol is not defined at all, Null is printed:

```
>> Definition[x]
  Null
```

DivideBy (/=)

```
DivideBy[x, dx]
x /= dx
  is equivalent to x = x / dx.
```

```
>> a = 10;

>> a /= 2
  5
```

```
>> a
5
```

DownValues

`DownValues[symbol]`
gives the list of downvalues associated with *symbol*.

`DownValues` uses `HoldPattern` and `RuleDelayed` to protect the downvalues from being evaluated. Moreover, it has attribute `HoldAll` to get the specified symbol instead of its value.

```
>> f[x_] := x ^ 2
```

```
>> DownValues[f]
{HoldPattern[f[x_]] :> x^2}
```

Mathics will sort the rules you assign to a symbol according to their specificity. If it cannot decide which rule is more special, the newer one will get higher precedence.

```
>> f[x_Integer] := 2
```

```
>> f[x_Real] := 3
```

```
>> DownValues[f]
{HoldPattern[f[x_Real]] :> 3,
 HoldPattern[f[x_Integer]] :> 2,
 HoldPattern[f[x_]] :> x^2}
```

```
>> f[3]
2
```

```
>> f[3.]
3
```

```
>> f[a]
a^2
```

The default order of patterns can be computed using `Sort` with `PatternsOrderedQ`:

```
>> Sort[{x_, x_Integer},
PatternsOrderedQ]
{x_Integer, x_}
```

By assigning values to `DownValues`, you can override the default ordering:

```
>> DownValues[g] := {g[x_] :> x ^
2, g[x_Integer] :> x}
```

```
>> g[2]
4
```

Fibonacci numbers:

```
>> DownValues[fib] := {fib[0] -> 0,
fib[1] -> 1, fib[n_] :> fib[n -
1] + fib[n - 2]}
```

```
>> fib[5]
5
```

Increment (++)

`Increment[x]`
`x++`
increments *x* by 1, returning the original value of *x*.

```
>> a = 2;
```

```
>> a++
2
```

```
>> a
3
```

Grouping of `Increment`, `PreIncrement` and `Plus`:

```
>> +++a++++2//Hold//FullForm
Hold[Plus[PreIncrement[
PreIncrement[Increment[
Increment[a]]], 2]]
```

Information (??)

`Information[symbol]`
Prints information about a *symbol*

`Information` does not print information for

`ReadProtected` symbols. `Information` uses `InputForm` to format values.

```
>> a = 2;
```

```
>> Information[a]
a = 2
```

```
>> f[x_] := x ^ 2
```

```
>> g[f] ^:= 2
```

```
>> f::usage = "f[x] returns the
square of x";

>> Information[f]
      f[x] returns the square of x
      f[x_] = x2
      g[f]^2

>> ? Table
      'Table[expr, {i, n}]'
      evaluates expr with i ranging from
      1 to n, returning
      a list of the results.
      'Table[expr, {i, start, stop, step}]'
      evaluates expr with i ranging from
      start to stop,
      incrementing by step.
      'Table[expr, {i, {e1, e2, ..., ei}}]'
      evaluates expr with i taking on the
      values e1, e2,
      ..., ei.
```

```
>> Information[Table]
      'Table[expr, {i, n}]'
      evaluates expr with i ranging from
      1 to n, returning
      a list of the results.
      'Table[expr, {i, start, stop, step}]'
      evaluates expr with i ranging from
      start to stop,
      incrementing by step.
      'Table[expr, {i, {e1, e2, ..., ei}}]'
      evaluates expr with i taking on the
      values e1, e2,
      ..., ei.
```

```
Attributes[Table] = {HoldAll,
                    Protected}
```

LoadModule

```
LoadModule[module]
      'Load Maths definitions from the
      python module module
```

```
>> LoadModule["nomodule"]
      Pythonmodule nomodule does not exist.
      $Failed

>> LoadModule["sys"]
      Pythonmodulesys is not a pymathics module.
      $Failed
```

```
# » LoadModule["pymathics.testpymathicsmodule"]
# = pymathics.testpymathicsmodule # »
MyPyTestContext'MyPyTestFunction[a] # =
This is a PyMathics output # » MyPyTest-
Context'MyPyTestSymbol # = 1234 # » ??
MyPyTestContext'MyPyTestFunction # = # .
MyPyTestFunction[m] # . Just an example
function in pymathics module. # . # . At-
tributes[MyPyTestContext'MyPyTestFunction]
= {HoldFirst, OneIdentity, Protected} # »
Quit[] #n» MyPyTestContext'MyPyTestSymbol
# = MyPyTestContext'MyPyTestSymbol # » ??
MyPyTestContext'MyPyTestFunction # = Null
```

Messages

```
Messages[symbol]
      gives the list of messages associated with
      symbol.
```

```
>> a::b = "foo"
      foo

>> Messages[a]
      {HoldPattern[a::b]:>foo}

>> Messages[a] = {a::c:>"bar"};

>> a::c // InputForm
      "bar"

>> Message[a::c]
      bar
```

NValues

```
NValues[symbol]
      gives the list of numerical values associ-
      ated with symbol.
```

```
>> NValues[a]
      {}
```

```
>> N[a] = 3;

>> NValues[a]
{HoldPattern[N[a,
MachinePrecision]]:>3}
```

You can assign values to NValues:

```
>> NValues[b] := {N[b,
MachinePrecision] :> 2}

>> N[b]
2.
```

Be sure to use SetDelayed, otherwise the left-hand side of the transformation rule will be evaluated immediately, causing the head of N to get lost. Furthermore, you have to include the precision in the rules; MachinePrecision will not be inserted automatically:

```
>> NValues[c] := {N[c] :> 3}

>> N[c]
c
```

Mathics will gracefully assign any list of rules to NValues; however, inappropriate rules will never be used:

```
>> NValues[d] = {foo -> bar};

>> NValues[d]
{HoldPattern[foo]:>bar}

>> N[d]
d
```

OwnValues

`OwnValues[symbol]`
gives the list of ownvalues associated with *symbol*.

```
>> x = 3;

>> x = 2;

>> OwnValues[x]
{HoldPattern[x]:>2}

>> x := y

>> OwnValues[x]
{HoldPattern[x]:>y}

>> y = 5;
```

```
>> OwnValues[x]
{HoldPattern[x]:>y}

>> Hold[x] /. OwnValues[x]
Hold[y]

>> Hold[x] /. OwnValues[x] //
ReleaseHold
5
```

PreDecrement (--)

`PreDecrement[x]`
`--x`
decrements *x* by 1, returning the new value of *x*.

`--a` is equivalent to $a = a - 1$:

```
>> a = 2;

>> --a
1

>> a
1
```

PreIncrement (++)

`PreIncrement[x]`
`++x`
increments *x* by 1, returning the new value of *x*.

`++a` is equivalent to $a = a + 1$:

```
>> a = 2;

>> ++a
3

>> a
3
```

Quit

`Quit[]`
removes all user-defined definitions.

```
>> a = 3
3
```

```
>> Quit[]
```

```
>> a
a
```

Quit even removes the definitions of protected and locked symbols:

```
>> x = 5;
```

```
>> Attributes[x] = {Locked,
Protected};
```

```
>> Quit[]
```

```
>> x
x
```

Set (=)

Set[*expr*, *value*]
expr = *value*
evaluates *value* and assigns it to *expr*.
{*s1*, *s2*, *s3*} = {*v1*, *v2*, *v3*}
sets multiple symbols (*s1*, *s2*, ...) to the corresponding values (*v1*, *v2*, ...).

Set can be used to give a symbol a value:

```
>> a = 3
3
>> a
3
```

An assignment like this creates an ownvalue:

```
>> OwnValues[a]
{HoldPattern[a]>3}
```

You can set multiple values at once using lists:

```
>> {a, b, c} = {10, 2, 3}
{10, 2, 3}
>> {a, b, {c, {d}}} = {1, 2, {{c1,
c2}, {a}}}
{1, 2, {{c1, c2}, {10}}}
>> d
10
```

Set evaluates its right-hand side immediately and assigns it to the left-hand side:

```
>> a
1
```

```
>> x = a
1
```

```
>> a = 2
2
```

```
>> x
1
```

Set always returns the right-hand side, which you can again use in an assignment:

```
>> a = b = c = 2;
```

```
>> a == b == c == 2
True
```

Set supports assignments to parts:

```
>> A = {{1, 2}, {3, 4}};
```

```
>> A[[1, 2]] = 5
5
```

```
>> A
{{1, 5}, {3, 4}}
```

```
>> A[[;;, 2]] = {6, 7}
{6, 7}
```

```
>> A
{{1, 6}, {3, 7}}
```

Set a submatrix:

```
>> B = {{1, 2, 3}, {4, 5, 6}, {7,
8, 9}};
```

```
>> B[[1;;2, 2;;-1]] = {{t, u}, {y,
z}};
```

```
>> B
{{1, t, u}, {4, y, z}, {7, 8, 9}}
```

SetDelayed (:=)

SetDelayed[*expr*, *value*]
expr := *value*
assigns *value* to *expr*, without evaluating *value*.

SetDelayed is like Set, except it has attribute HoldAll, thus it does not evaluate the right-hand side immediately, but evaluates it when needed.

```
>> Attributes[SetDelayed]
{HoldAll, Protected, SequenceHold}
```

```
>> a = 1
1
>> x := a
>> x
1
```

Changing the value of a affects x :

```
>> a = 2
2
>> x
2
```

Condition (/;) can be used with SetDelayed to make an assignment that only holds if a condition is satisfied:

```
>> f[x_] := p[x] /; x>0
>> f[3]
p[3]
>> f[-3]
f[-3]
```

SubValues

`SubValues[symbol]`
gives the list of subvalues associated with *symbol*.

```
>> f[1][x_] := x
>> f[2][x_] := x ^ 2
>> SubValues[f]
{HoldPattern[f[2][x_]]:>x^2,
 HoldPattern[f[1][x_]]:>x}
>> Definition[f]
      f[2][x_] = x^2
      f[1][x_] = x
```

SubtractFrom (==)

`SubtractFrom[x, dx]`
 $x \mathrel{==} dx$
is equivalent to $x = x - dx$.

```
>> a = 10;
>> a -= 2
8
>> a
8
```

TagSet

`TagSet[f, expr, value]`
 $f /: \text{expr} = \text{value}$
assigns *value* to *expr*, associating the corresponding rule with the symbol *f*.

Create an upvalue without using UpSet:

```
>> x /: f[x] = 2
2
>> f[x]
2
>> DownValues[f]
{}
>> UpValues[x]
{HoldPattern[f[x]]:>2}
```

The symbol *f* must appear as the ultimate head of *lhs* or as the head of a leaf in *lhs*:

```
>> x /: f[g[x]] = 3;
      Tagxnotfoundortoodeepforanassignedrule.
>> g /: f[g[x]] = 3;
>> f[g[x]]
3
```

TagSetDelayed

`TagSetDelayed[f, expr, value]`
 $f /: \text{expr} := \text{value}$
is the delayed version of TagSet.

TimesBy (*=)

```
TimesBy[x, dx]
x *= dx
is equivalent to x = x * dx.
```

```
>> a = 10;

>> a *= 2
20

>> a
20
```

Unset (=.)

```
Unset[x]
x=.
removes any value belonging to x.
```

```
>> a = 2
2

>> a =.

>> a
a
```

Unsetting an already unset or never defined variable will not change anything:

```
>> a =.

>> b =.
```

Unset can unset particular function values. It will print a message if no corresponding rule is found.

```
>> f[x_] =.
Assignment on f for f[x_] not found.
$Failed

>> f[x_] := x ^ 2

>> f[3]
9

>> f[x_] =.

>> f[3]
f[3]
```

You can also unset OwnValues, DownValues, SubValues, and UpValues directly. This is

equivalent to setting them to {}.

```
>> f[x_] = x; f[0] = 1;

>> DownValues[f] =.

>> f[2]
f[2]
```

Unset threads over lists:

```
>> a = b = 3;

>> {a, {b}} =.
{Null, {Null}}
```

UpSet (^=)

```
f[x] ^= expression
evaluates expression and assigns it to the
value of f[x], associating the value with
x.
```

UpSet creates an upvalue:

```
>> a[b] ^= 3;

>> DownValues[a]
{}

>> UpValues[b]
{HoldPattern[a[b]]:>3}

>> a ^= 3
Nonatomic expression expected.
3
```

You can use UpSet to specify special values like format values. However, these values will not be saved in UpValues:

```
>> Format[r] ^= "custom";

>> r
custom

>> UpValues[r]
{}
```


UpSetDelayed (^:=)

`UpSetDelayed[expression, value]`
expression ^:= *value*
assigns *expression* to the value of *f*[*x*]
(without evaluating *expression*), associating the value with *x*.

```
>> a[b] ^= x
>> x = 2;
>> a[b]
2
>> UpValues[b]
{HoldPattern[a[b]]:>x}
```

UpValues

`UpValues[symbol]`
gives the list of upvalues associated with *symbol*.

```
>> a + b ^= 2
2
>> UpValues[a]
{HoldPattern[a + b]:>2}
>> UpValues[b]
{HoldPattern[a + b]:>2}
```

You can assign values to UpValues:

```
>> UpValues[pi] := {Sin[pi] :> 0}
>> Sin[pi]
0
```

IV. Attributes

There are several builtin-attributes which have a predefined meaning in *Mathics*. However, you can set any symbol as an attribute, in contrast to *Mathematica*®.

Contents

Attributes	58	HoldRest	59	Protect	61
ClearAttributes	59	Listable	60	Protected	61
Constant	59	Locked	60	ReadProtected	61
Flat	59	NHoldAll	60	SequenceHold	62
HoldAll	59	NHoldFirst	60	SetAttributes	62
HoldAllComplete	59	NHoldRest	60	Unprotect	62
HoldFirst	59	OneIdentity	60		
		Orderless	61		

Attributes

`Attributes[symbol]`
returns the attributes of *symbol*.
`Attributes[symbol] = {attr1, attr2}`
sets the attributes of *symbol*, replacing any existing attributes.

```
>> Attributes[Plus]
{Flat, Listable, NumericFunction,
 OneIdentity, Orderless, Protected}
```

Attributes always considers the head of an expression:

```
>> Attributes[a + b + c]
{Flat, Listable, NumericFunction,
 OneIdentity, Orderless, Protected}
```

You can assign values to Attributes to set attributes:

```
>> Attributes[f] = {Flat, Orderless}
{Flat, Orderless}

>> f[b, f[a, c]]
f[a, b, c]
```

Attributes must be symbols:

```
>> Attributes[f] := {a + b}
Argumenta
+ bat position 1 is expected to be a symbol.
$Failed
```

Use Symbol to convert strings to symbols:

```
>> Attributes[f] = Symbol["Listable"]
Listable

>> Attributes[f]
{Listable}
```

ClearAttributes

`ClearAttributes[symbol, attrib]`
removes *attrib* from *symbol*'s attributes.

```
>> SetAttributes[f, Flat]

>> Attributes[f]
{Flat}

>> ClearAttributes[f, Flat]

>> Attributes[f]
{}
```

Attributes that are not even set are simply ig-

nored:

```
>> ClearAttributes[{f}, {Flat}]
```

```
>> Attributes[f]
{}
```

Constant

Constant

is an attribute that indicates that a symbol is a constant.

Mathematical constants like E have attribute **Constant**:

```
>> Attributes[E]
{Constant, Protected, ReadProtected}
```

Constant symbols cannot be used as variables in **Solve** and related functions:

```
>> Solve[x + E == 0, E]
Eisnotavalidvariable.
Solve[E + x == 0, E]
```

Flat

Flat

is an attribute that specifies that nested occurrences of a function should be automatically flattened.

A symbol with the **Flat** attribute represents an associative mathematical operation:

```
>> SetAttributes[f, Flat]
```

```
>> f[a, f[b, c]]
f[a, b, c]
```

Flat is taken into account in pattern matching:

```
>> f[a, b, c] /. f[a, b] -> d
f[d, c]
```

HoldAll

HoldAll

is an attribute specifying that all arguments of a function should be left unevaluated.

HoldAllComplete

HoldAllComplete

is an attribute that includes the effects of **HoldAll** and **SequenceHold**, and also protects the function from being affected by the upvalues of any arguments.

HoldAllComplete even prevents upvalues from being used, and includes **SequenceHold**.

```
>> SetAttributes[f, HoldAllComplete]
```

```
>> f[a] ^= 3;
```

```
>> f[a]
f[a]
```

```
>> f[Sequence[a, b]]
f[Sequence[a, b]]
```

HoldFirst

HoldFirst

is an attribute specifying that the first argument of a function should be left unevaluated.

HoldRest

HoldRest

is an attribute specifying that all but the first argument of a function should be left unevaluated.

Listable

Listable

is an attribute specifying that a function should be automatically applied to each element of a list.

```
>> SetAttributes[f, Listable]
```

```
>> f[{1, 2, 3}, {4, 5, 6}]
{f[1, 4], f[2, 5], f[3, 6]}
```

```
>> f[{1, 2, 3}, 4]
      {f[1,4],f[2,4],f[3,4]}

>> {{1, 2}, {3, 4}} + {5, 6}
      {{6,7},{9,10}}
```

Locked

Locked
is an attribute that prevents attributes on a symbol from being modified.

The attributes of Locked symbols cannot be modified:

```
>> Attributes[lock] = {Flat, Locked}
      ];

>> SetAttributes[lock, {}]
      Symbollockislocked.

>> ClearAttributes[lock, Flat]
      Symbollockislocked.

>> Attributes[lock] = {}
      Symbollockislocked.
      {}

>> Attributes[lock]
      {Flat,Locked}
```

However, their values might be modified (as long as they are not Protected too):

```
>> lock = 3
      3
```

NHoldAll

NHoldAll
is an attribute that protects all arguments of a function from numeric evaluation.

```
>> N[f[2, 3]]
      f[2.,3.]

>> SetAttributes[f, NHoldAll]

>> N[f[2, 3]]
      f[2,3]
```

NHoldFirst

NHoldFirst
is an attribute that protects the first argument of a function from numeric evaluation.

NHoldRest

NHoldRest
is an attribute that protects all but the first argument of a function from numeric evaluation.

OneIdentity

OneIdentity
is an attribute specifying that $f[x]$ should be treated as equivalent to x in pattern matching.

OneIdentity affects pattern matching:

```
>> SetAttributes[f, OneIdentity]

>> a /. f[args___] -> {args}
      {a}
```

It does not affect evaluation:

```
>> f[a]
      f[a]
```

Orderless

Orderless
is an attribute indicating that the leaves in an expression $f[a, b, c]$ can be placed in any order.

The leaves of an Orderless function are automatically sorted:

```
>> SetAttributes[f, Orderless]

>> f[c, a, b, a + b, 3, 1.0]
      f[1.,3,a,b,c,a + b]
```

A symbol with the Orderless attribute represents a commutative mathematical operation.

```
>> f[a, b] == f[b, a]
True

Orderless affects pattern matching:
>> SetAttributes[f, Flat]

>> f[a, b, c] /. f[a, c] -> d
f[b, d]
```

Protect

`Protect[symbol]`
gives *symbol* the attribute Protected.

```
>> A = {1, 2, 3};
>> Protect[A]

>> A[[2]] = 4;
SymbolAisProtected.

>> A
{1, 2, 3}
```

Protected

`Protected`
is an attribute that prevents values on a symbol from being modified.

Values of Protected symbols cannot be modified:

```
>> Attributes[p] = {Protected};

>> p = 2;
SymbolpisProtected.

>> f[p] ^= 3;
Tagpinf[p]isProtected.

>> Format[p] = "text";
SymbolpisProtected.
```

However, attributes might still be set:

```
>> SetAttributes[p, Flat]

>> Attributes[p]
{Flat, Protected}
```

Thus, you can easily remove the attribute Protected:

```
>> Attributes[p] = {};

>> p = 2
2
```

You can also use Protect or Unprotect, resp.

```
>> Protect[p]

>> Attributes[p]
{Protected}

>> Unprotect[p]
```

If a symbol is Protected and Locked, it can never be changed again:

```
>> SetAttributes[p, {Protected, Locked}]

>> p = 2
SymbolpisProtected.
2

>> Unprotect[p]
Symbolpislocked.
```

ReadProtected

`ReadProtected`
is an attribute that prevents values on a symbol from being read.

Values associated with ReadProtected symbols cannot be seen in Definition:

```
>> ClearAll[p]

>> p = 3;

>> Definition[p]
p = 3

>> SetAttributes[p, ReadProtected]

>> Definition[p]
Attributes[p] = {ReadProtected}
```

SequenceHold

`SequenceHold`
is an attribute that prevents `Sequence` objects from being spliced into a function's arguments.

Normally, `Sequence` will be spliced into a function:

```
>> f[Sequence[a, b]]  
f[a, b]
```

It does not for `SequenceHold` functions:

```
>> SetAttributes[f, SequenceHold]  
  
>> f[Sequence[a, b]]  
f[Sequence[a, b]]
```

E.g., `Set` has attribute `SequenceHold` to allow assignment of sequences to variables:

```
>> s = Sequence[a, b];  
  
>> s  
Sequence[a, b]  
  
>> Plus[s]  
a + b
```

SetAttributes

`SetAttributes[symbol, attrib]`
adds *attrib* to *symbol*'s attributes.

```
>> SetAttributes[f, Flat]  
  
>> Attributes[f]  
{Flat}
```

Multiple attributes can be set at the same time using lists:

```
>> SetAttributes[{f, g}, {Flat, Orderless}]  
  
>> Attributes[g]  
{Flat, Orderless}
```

Unprotect

`Unprotect[symbol]`
removes the `Protected` attribute from *symbol*.

V. Calculus functions

Contents

Complexes	63	FindRoot	65	Reals	66
D	64	Integers	65	Root	66
Derivative (')	64	Integrate	66	Solve	67
DiscreteLimit	64	Limit	66		

Complexes

Complexes
is the set of complex numbers.

D

$D[f, x]$
gives the partial derivative of f with respect to x .
 $D[f, x, y, \dots]$
differentiates successively with respect to x, y , etc.
 $D[f, \{x, n\}]$
gives the multiple derivative of order n .
 $D[f, \{x1, x2, \dots\}]$
gives the vector derivative of f with respect to $x1, x2$, etc.

First-order derivative of a polynomial:

```
>> D[x^3 + x^2, x]
2x + 3x^2
```

Second-order derivative:

```
>> D[x^3 + x^2, {x, 2}]
2 + 6x
```

Trigonometric derivatives:

```
>> D[Sin[Cos[x]], x]
-Cos[Cos[x]] Sin[x]

>> D[Sin[x], {x, 2}]
-Sin[x]
```

```
>> D[Cos[t], {t, 2}]
-Cos[t]
```

Unknown variables are treated as constant:

```
>> D[y, x]
0
```

```
>> D[x, x]
1
```

```
>> D[x + y, x]
1
```

Derivatives of unknown functions are represented using `Derivative`:

```
>> D[f[x], x]
f'[x]
```

```
>> D[f[x, x], x]
f^(0,1)[x, x] + f^(1,0)[x, x]
```

```
>> D[f[x, x], x] // InputForm
Derivative[0, 1][f][x, x]
+ Derivative[1, 0][f][x, x]
```

Chain rule:

```
>> D[f[2x+1, 2y, x+y], x]
2f^(1,0,0)[1 + 2x, 2y,
x + y] + f^(0,0,1)[1 + 2x, 2y, x + y]
```

```
>> D[f[x^2, x, 2y], {x,2}, y] //
Expand
```

$$8x f^{(1,1,1)}[x^2, x, 2y] + 8x^2 f^{(2,0,1)}[x^2, x, 2y] + 2f^{(0,2,1)}[x^2, x, 2y] + 4f^{(1,0,1)}[x^2, x, 2y]$$

Compute the gradient vector of a function:

```
>> D[x ^ 3 * Cos[y], {{x, y}}]
{3x^2 Cos[y], -x^3 Sin[y]}
```

Hesse matrix:

```
>> D[Sin[x] * Cos[y], {{x,y}, 2}]
{{{-Cos[y] Sin[x], -Cos[x] Sin[y]}, {-Cos[x] Sin[y], -Cos[y] Sin[x]}}
```

Derivative (')

`Derivative[n][f]`
represents the n th derivative of the function f .
`Derivative[n1, n2, ...][f]`
represents a multivariate derivative.

```
>> Derivative[1][Sin]
Cos[#1]&
```

```
>> Derivative[3][Sin]
-Cos[#1]&
```

```
>> Derivative[2][# ^ 3&]
6#1&
```

Derivative can be entered using ':

```
>> Sin'[x]
Cos[x]
```

```
>> (# ^ 4&)'
12#1^2&
```

```
>> f'[x] // InputForm
Derivative[1][f][x]
```

```
>> Derivative[1][#2 Sin[#1]+Cos
[#2]&]
Cos[#1]#2&
```

```
>> Derivative[1,2][#2^3 Sin[#1]+Cos
[#2]&]
6Cos[#1]#2&
```

Deriving with respect to an unknown parameter yields 0:

```
>> Derivative[1,2,1][#2^3 Sin[#1]+
Cos[#2]&]
0&
```

The 0th derivative of any expression is the expression itself:

```
>> Derivative[0,0,0][a+b+c]
a + b + c
```

You can calculate the derivative of custom functions:

```
>> f[x_] := x ^ 2
>> f'[x]
2x
```

Unknown derivatives:

```
>> Derivative[2, 1][h]
h^(2,1)
>> Derivative[2, 0, 1, 0][h[g]]
h[g]^(2,0,1,0)
```

DiscreteLimit

`DiscreteLimit[f, k->Infinity]`
gives the limit of the sequence f as k tends to infinity.

```
>> DiscreteLimit[n/(n + 1), n ->
Infinity]
1
```

```
>> DiscreteLimit[f[n], n ->
Infinity]
f[∞]
```


FindRoot

```
FindRoot[f, {x, x0}]
  searches for a numerical root of  $f$ , starting
  from  $x=x0$ .
FindRoot[lhs == rhs, {x, x0}]
  tries to solve the equation  $lhs == rhs$ .
```

FindRoot uses Newton's method, so the function of interest should have a first derivative.

```
>> FindRoot[Cos[x], {x, 1}]
{x -> 1.5708}

>> FindRoot[Sin[x] + Exp[x], {x, 0}]
{x -> -0.588533}

>> FindRoot[Sin[x] + Exp[x] == Pi, {
x, 0}]
{x -> 0.866815}
```

FindRoot has attribute HoldAll and effectively uses Block to localize x . However, in the result x will eventually still be replaced by its value.

```
>> x = 3;

>> FindRoot[Tan[x] + Sin[x] == Pi,
{x, 1}]
{3 -> 1.14911}

>> Clear[x]
```

FindRoot stops after 100 iterations:

```
>> FindRoot[x^2 + x + 1, {x, 1}]
The maximum number of iterations was exceeded. The result might be inaccurate.
{x -> -1.}
```

Find complex roots:

```
>> FindRoot[x ^ 2 + x + 1, {x, -I}]
{x -> -0.5 - 0.866025I}
```

The function has to return numerical values:

```
>> FindRoot[f[x] == 0, {x, 0}]
The function value is not a number at x = 0..
FindRoot[f[x] - 0, {x, 0}]
```

The derivative must not be 0:

```
>> FindRoot[Sin[x] == x, {x, 0}]
Encountered a singular derivative at the point x = 0..
FindRoot[Sin[x] - x, {x, 0}]
```

Integers

```
Integers
  is the set of integer numbers.
```

Limit a solution to integer numbers:

```
>> Solve[-4 - 4 x + x^4 + x^5 == 0,
x, Integers]
{{x -> -1}}

>> Solve[x^4 == 4, x, Integers]
{}
```

Integrate

```
Integrate[f, x]
  integrates  $f$  with respect to  $x$ . The result
  does not contain the additive integration
  constant.
Integrate[f, {x, a, b}]
  computes the definite integral of  $f$  with
  respect to  $x$  from  $a$  to  $b$ .
```

Integrate a polynomial:

```
>> Integrate[6 x ^ 2 + 3 x ^ 2 - 4
x + 10, x]
10x - 2x^2 + 3x^3
```

Integrate trigonometric functions:

```
>> Integrate[Sin[x] ^ 5, x]
-Cos[x] -  $\frac{\cos^3[x]}{3} + \frac{2\cos^5[x]}{5}$ 
```

Definite integrals:

```
>> Integrate[x ^ 2 + x, {x, 1, 3}]
 $\frac{38}{3}$ 

>> Integrate[Sin[x], {x, 0, Pi/2}]
1
```

Some other integrals:

```
>> Integrate[1 / (1 - 4 x + x^2), x]
-  $\frac{\sqrt{3}\log[-2 + \sqrt{3} + x]}{6}$ 
+  $\frac{\sqrt{3}\log[-2 - \sqrt{3} + x]}{6}$ 
```

```
>> Integrate[4 Sin[x] Cos[x], x]
2Sin[x]^2
```

Integration in TeX:

```
>> Integrate[f[x], {x, a, b}] //
TeXForm
\int_a^b f(x) dx
```

Sometimes there is a loss of precision during integration. You can check the precision of your result with the following sequence of commands.

```
>> Integrate[Abs[Sin[phi]], {phi,
0, 2Pi}] // N
4.

>> % // Precision
MachinePrecision

>> Integrate[ArcSin[x / 3], x]
x ArcSin[ $\frac{x}{3}$ ] +  $\sqrt{9 - x^2}$ 

>> Integrate[f'[x], {x, a, b}]
f[b] - f[a]
```

Limit

`Limit[expr, x->x0]`
gives the limit of *expr* as *x* approaches *x0*.
`Limit[expr, x->x0, Direction->1]`
approaches *x0* from smaller values.
`Limit[expr, x->x0, Direction->-1]`
approaches *x0* from larger values.

```
>> Limit[x, x->2]
2

>> Limit[Sin[x] / x, x->0]
1

>> Limit[1/x, x->0, Direction->-1]
 $\infty$ 

>> Limit[1/x, x->0, Direction->1]
 $-\infty$ 
```

Reals

`Reals`
is the set of real numbers.

Limit a solution to real numbers:

```
>> Solve[x^3 == 1, x, Reals]
{{x -> 1}}
```

Root

`Root[f, i]`
represents the *i*-th complex root of the polynomial *f*

```
>> Root[#1 ^ 2 - 1 &, 1]
-1

>> Root[#1 ^ 2 - 1 &, 2]
1
```

Roots that can't be represented by radicals:

```
>> Root[#1 ^ 5 + 2 #1 + 1 &, 2]
Root[#1^5 + 2#1 + 1 &, 2]
```

Solve

`Solve[equation, vars]`
attempts to solve *equation* for the variables *vars*.
`Solve[equation, vars, domain]`
restricts variables to *domain*, which can be Complexes or Reals or Integers.

```
>> Solve[x ^ 2 - 3 x == 4, x]
{{x -> -1}, {x -> 4}}

>> Solve[4 y - 8 == 0, y]
{{y -> 2}}
```

Apply the solution:

```
>> sol = Solve[2 x^2 - 10 x - 12 ==
0, x]
{{x -> -1}, {x -> 6}}

>> x /. sol
{-1, 6}
```

Contradiction:

```
>> Solve[x + 1 == x, x]
{}
```

Tautology:

```
>> Solve[x ^ 2 == x ^ 2, x]
{{}}
```

Rational equations:

```
>> Solve[x / (x ^ 2 + 1) == 1, x]
{{x -> 1/2 - I/2 Sqrt[3]},
 {x -> 1/2 + I/2 Sqrt[3]}}

>> Solve[(x^2 + 3 x + 2)/(4 x - 2)
 == 0, x]
{{x -> -2}, {x -> -1}}
```

Transcendental equations:

```
>> Solve[Cos[x] == 0, x]
{{x -> Pi/2}, {x -> 3Pi/2}}
```

Solve can only solve equations with respect to symbols or functions:

```
>> Solve[f[x + y] == 3, f[x + y]]
{{f[x + y] -> 3}}

>> Solve[a + b == 2, a + b]
a + b is not a valid variable.
Solve[a + b == 2, a + b]
```

This happens when solving with respect to an assigned symbol:

```
>> x = 3;

>> Solve[x == 2, x]
3 is not a valid variable.
Solve[False, 3]

>> Clear[x]

>> Solve[a < b, a]
a < b is not a well-formed equation.
Solve[a < b, a]
```

Solve a system of equations:

```
>> eqs = {3 x ^ 2 - 3 y == 0, 3 y ^
 2 - 3 x == 0};
```

```
>> sol = Solve[eqs, {x, y}] //
Simplify
```

$$\left\{ \left\{ x -> 0, y -> 0 \right\}, \left\{ x -> 1, y -> 1 \right\}, \left\{ x -> -\frac{1}{2} + \frac{I}{2}\sqrt{3}, y -> -\frac{1}{2} - \frac{I}{2}\sqrt{3} \right\}, \left\{ x -> \frac{(1 - I\sqrt{3})^2}{4}, y -> -\frac{1}{2} + \frac{I}{2}\sqrt{3} \right\} \right\}$$

```
>> eqs /. sol // Simplify
{{True, True}, {True, True},
 {True, True}, {True, True}}
```

An underdetermined system:

```
>> Solve[x^2 == 1 && z^2 == -1, {x,
 y, z}]
Equations may not give solutions for all "solve" variables.
{{x -> -1, z -> -I},
 {x -> -1, z -> I}, {x -> 1,
 z -> -I}, {x -> 1, z -> I}}
```

Domain specification:

```
>> Solve[x^2 == -1, x, Reals]
{}

>> Solve[x^2 == 1, x, Reals]
{{x -> -1}, {x -> 1}}

>> Solve[x^2 == -1, x, Complexes]
{{x -> -I}, {x -> I}}

>> Solve[4 - 4 * x^2 - x^4 + x^6 ==
 0, x, Integers]
{{x -> -1}, {x -> 1}}
```

VI. Combinatorial

Contents

Binomial	68	MatchingDissimilarity	69	SokalSneathDissimilarity	69
DiceDissimilarity . . .	68	Multinomial	69	Subsets	70
Fibonacci	68	RogersTanimotoDis-similarity	69	YuleDissimilarity . . .	70
JaccardDissimilarity . .	68	RussellRaoDissimilarity	69		

Binomial

`Binomial[n, k]`
gives the binomial coefficient n choose k .

```
>> Binomial[5, 3]
10
```

Binomial supports inexact numbers:

```
>> Binomial[10.5, 3.2]
165.286
```

Some special cases:

```
>> Binomial[10, -2]
0

>> Binomial[-10.5, -3.5]
0.
```

DiceDissimilarity

`DiceDissimilarity[u, v]`
returns the Dice dissimilarity between the two boolean 1-D lists u and v , which is defined as $(c_{tf} + c_{ft}) / (2 * c_{tt} + c_{ft} + c_{tf})$, where n is $\text{len}(u)$ and c_{ij} is the number of occurrences of $u[k]=i$ and $v[k]=j$ for $k < n$.

```
>> DiceDissimilarity[{1, 0, 1, 1,
0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
1/2
```

Fibonacci

`Fibonacci[n]`
computes the n th Fibonacci number.

```
>> Fibonacci[0]
0

>> Fibonacci[1]
1

>> Fibonacci[10]
55

>> Fibonacci[200]
280 571 172 992 510 140 037 ~
~611 932 413 038 677 189 525
```

JaccardDissimilarity

`JaccardDissimilarity[u, v]`
returns the Jaccard-Needham dissimilarity between the two boolean 1-D lists u and v , which is defined as $(c_{tt} + c_{ft} + c_{tf}) / (c_{tt} + c_{ft} + c_{tf})$, where n is $\text{len}(u)$ and c_{ij} is the number of occurrences of $u[k]=i$ and $v[k]=j$ for $k < n$.

```
>> JaccardDissimilarity[{1, 0, 1,
1, 0, 1, 1}, {0, 1, 1, 0, 0, 0,
1}]
2/3
```

MatchingDissimilarity

`MatchingDissimilarity[u, v]`
returns the Matching dissimilarity between the two boolean 1-D lists u and v , which is defined as $(c_tf + c_ft) / n$, where n is $\text{len}(u)$ and c_ij is the number of occurrences of $u[k]=i$ and $v[k]=j$ for $k < n$.

```
>> MatchingDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
4
7
```

Multinomial

`Multinomial[n1, n2, ...]`
gives the multinomial coefficient $(n1 + n2 + \dots) / (n1! n2! \dots)$.

```
>> Multinomial[2, 3, 4, 5]
2522520
```

```
>> Multinomial[]
1
```

Multinomial is expressed in terms of Binomial:

```
>> Multinomial[a, b, c]
Binomial[a, a] Binomial[a + b, b] Binomial[a + b + c, c]
```

`Multinomial[n-k, k]` is equivalent to `Binomial[n, k]`.

```
>> Multinomial[2, 3]
10
```

RogersTanimotoDissimilarity

`RogersTanimotoDissimilarity[u, v]`
returns the Rogers-Tanimoto dissimilarity between the two boolean 1-D lists u and v , which is defined as $R / (c_tt + c_ff + R)$ where n is $\text{len}(u)$, c_ij is the number of occurrences of $u[k]=i$ and $v[k]=j$ for $k < n$, and $R = 2 * (c_tf + c_ft)$.

```
>> RogersTanimotoDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
8
11
```

RussellRaoDissimilarity

`RussellRaoDissimilarity[u, v]`
returns the Russell-Rao dissimilarity between the two boolean 1-D lists u and v , which is defined as $(n - c_tt) / c_tt$ where n is $\text{len}(u)$ and c_ij is the number of occurrences of $u[k]=i$ and $v[k]=j$ for $k < n$.

```
>> RussellRaoDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
5
7
```

SokalSneathDissimilarity

`SokalSneathDissimilarity[u, v]`
returns the Sokal-Sneath dissimilarity between the two boolean 1-D lists u and v , which is defined as $R / (c_tt + R)$ where n is $\text{len}(u)$, c_ij is the number of occurrences of $u[k]=i$ and $v[k]=j$ for $k < n$, and $R = 2 * (c_tf + c_ft)$.

```
>> SokalSneathDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
4
5
```

Subsets

```
Subsets[list]
    finds a list of all possible subsets of list.
Subsets[list, n]
    finds a list of all possible subsets containing at most n elements.
Subsets[list, {n}]
    finds a list of all possible subsets containing exactly n elements.
Subsets[list, {min, max}]
    finds a list of all possible subsets containing between min and max elements.
Subsets[list, spec, n]
    finds a list of the first n possible subsets.
Subsets[list, spec, {n}]
    finds the nth possible subset.
```

All possible subsets (power set):

```
>> Subsets[{a, b, c}]
{{}, {a}, {b}, {c}, {a,
  b}, {a, c}, {b, c}, {a, b, c}}
```

All possible subsets containing up to 2 elements:

```
>> Subsets[{a, b, c, d}, 2]
{{}, {a}, {b}, {c}, {d},
 {a, b}, {a, c}, {a, d},
 {b, c}, {b, d}, {c, d}}
```

Subsets containing exactly 2 elements:

```
>> Subsets[{a, b, c, d}, {2}]
{{a, b}, {a, c}, {a, d},
 {b, c}, {b, d}, {c, d}}
```

The first 5 subsets containing 3 elements:

```
>> Subsets[{a, b, c, d, e}, {3}, 5]
{{a, b, c}, {a, b, d}, {a,
  b, e}, {a, c, d}, {a, c, e}}
```

All subsets with even length:

```
>> Subsets[{a, b, c, d, e}, {0, 5,
  2}]
{{}, {a, b}, {a, c}, {a, d}, {a, e},
 {b, c}, {b, d}, {b, e}, {c, d}, {c,
  e}, {d, e}, {a, b, c, d}, {a, b, c, e},
 {a, b, d, e}, {a, c, d, e}, {b, c, d, e}}
```

The 25th subset:

```
>> Subsets[Range[5], All, {25}]
{{2, 4, 5}}
```

The odd-numbered subsets of {a,b,c,d} in reverse order:

```
>> Subsets[{a, b, c, d}, All, {15,
  1, -2}]
{{b, c, d}, {a, b, d}, {c, d},
 {b, c}, {a, c}, {d}, {b}, {}}
```

YuleDissimilarity

```
YuleDissimilarity[u, v]
    returns the Yule dissimilarity between the two boolean 1-D lists u and v, which is defined as  $R / (c_{tt} * c_{ff} + R / 2)$  where n is len(u), c_ij is the number of occurrences of u[k]=i and v[k]=j for k<n, and  $R = 2 * c_{tf} * c_{ft}$ .
```

```
>> YuleDissimilarity[{1, 0, 1, 1,
  0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
6
5
```

VII. Compilation

Contents

Compile	71	CompiledCodeBox . .	71	CompiledFunction . .	71
-------------------	----	---------------------	----	----------------------	----

Compile

`Compile[{x1, x2, ...}, expr_]`
Compiles *expr* assuming each *xi* is a *Real* number.
`Compile[{x1, t1} {x2, t1} ..., expr_]`
Compiles assuming each *xi* matches type *ti*.

```
>> cf = Compile[{x, y}, x + 2 y]
CompiledFunction[{x, y},
  x + 2y, -CompiledCode-]

>> cf[2.5, 4.3]
11.1

>> cf = Compile[{x, _Real}}, Sin[x]]
CompiledFunction[{x},
  Sin[x], -CompiledCode-]

>> cf[1.4]
0.98545
```

Compile supports basic flow control

```
>> cf = Compile[{x, _Real}, {y,
  _Integer}}, If[x == 0.0 && y <=
  0, 0.0, Sin[x ^ y] + 1 / Min[x,
  0.5]] + 0.5]

CompiledFunction[{x,
  y}, 0.5 + If[x == 0.0 && y <= 0,
  0., Sin[x^y] + 1 / Min[x, 0.5]],
  -CompiledCode-]
```

```
>> cf[3.5, 2]
2.18888
```

Loops and variable assignments are not yet supported

```
>> Compile[{a, _Integer}, {b,
  _Integer}}, While[b != 0, {a, b}
  = {b, Mod[a, b]}; a] (* GCD of
  a, b *)

ExpressionWhile[b!=0, {a,b} = {b,
  Mod[a,b]}; a] could not be compiled.

Compile[{a, _Integer},
  {b, _Integer}}, While[b!=0,
  {a,b} = {b, Mod[a,b]}; a]
```

CompiledCodeBox

CompiledFunction

VIII. Comparison

Contents

BooleanQ	72	LessEqual (<=)	73	SameQ (===)	75
Equal (==)	72	Max	73	SympyComparison	75
Greater (>)	73	Min	74	TrueQ	75
GreaterEqual (>=)	73	Negative	74	Unequal (!=)	75
Inequality	73	NonNegative	74	UnsameQ (!=)	75
Less (<)	73	NonPositive	74	ValueQ	75
		Positive	74		

BooleanQ

`BooleanQ[expr]`
returns True if *expr* is either True or False.

```
>> BooleanQ[True]
True

>> BooleanQ[False]
True

>> BooleanQ[a]
False

>> BooleanQ[1 < 2]
True
```

Equal (==)

`Equal[x, y]`
`x == y`
yields True if *x* and *y* are known to be equal, or False if *x* and *y* are known to be unequal.
`lhs == rhs`
represents the equation *lhs* = *rhs*.

```
>> a==a
True

>> a==b
a==b
```

```
>> 1==1.
True
```

Lists are compared based on their elements:

```
>> {{1}, {2}} == {{1}, {2}}
True

>> {1, 2} == {1, 2, 3}
False
```

Real values are considered equal if they only differ in their last digits:

```
>> 0.739085133215160642 ==
0.739085133215160641
True

>> 0.73908513321516064200000000 ==
0.73908513321516064100000000
False
```

Comparisons are done using the lower precision:

```
>> N[E, 100] == N[E, 150]
True
```

Symbolic constants are compared numerically:

```
>> E > 1
True

>> Pi == 3.14
False
```


Greater (>)

```
Greater[x, y]
x > y
  yields True if  $x$  is known to be greater
  than  $y$ .
lhs > rhs
  represents the inequality  $lhs > rhs$ .
```

```
>> a > b > c //FullForm
Greater[a, b, c]

>> Greater[3, 2, 1]
True
```

GreaterEqual (>=)

```
GreaterEqual[x, y]
x >= y
  yields True if  $x$  is known to be greater
  than or equal to  $y$ .
lhs >= rhs
  represents the inequality  $lhs \geq rhs$ .
```

Inequality

```
Inequality
  is the head of expressions involving dif-
  ferent inequality operators (at least tem-
  porarily). Thus, it is possible to write
  chains of inequalities.
```

```
>> a < b <= c
a < b && b <= c

>> Inequality[a, Greater, b,
LessEqual, c]
a > b && b <= c

>> 1 < 2 <= 3
True

>> 1 < 2 > 0
True

>> 1 < 2 < -1
False
```

Less (<)

```
Less[x, y]
x < y
  yields True if  $x$  is known to be less than  $y$ .
lhs < rhs
  represents the inequality  $lhs < rhs$ .
```

LessEqual (<=)

```
LessEqual[x, y]
x <= y
  yields True if  $x$  is known to be less than
  or equal to  $y$ .
lhs <= rhs
  represents the inequality  $lhs \leq rhs$ .
```

Max

```
Max[e1, e2, ..., ei]
  returns the expression with the greatest
  value among the  $e_i$ .
```

Maximum of a series of numbers:

```
>> Max[4, -8, 1]
4
```

Max flattens lists in its arguments:

```
>> Max[{1,2},3,{-3,3.5,-Infinity},{{1/2}}]
3.5
```

Max with symbolic arguments remains in sym-
bolic form:

```
>> Max[x, y]
Max[x, y]

>> Max[5, x, -3, y, 40]
Max[40, x, y]
```

With no arguments, Max gives -Infinity:

```
>> Max[]
-∞
```

Min

```
Min[e1, e2, ..., ei]  
returns the expression with the lowest  
value among the ei.
```

Minimum of a series of numbers:

```
>> Min[4, -8, 1]  
-8
```

Min flattens lists in its arguments:

```
>> Min[{1,2},3,{-3,3.5,-Infinity  
},{1/2}]  
-∞
```

Min with symbolic arguments remains in symbolic form:

```
>> Min[x, y]  
Min[x, y]  
  
>> Min[5, x, -3, y, 40]  
Min[-3, x, y]
```

With no arguments, Min gives Infinity:

```
>> Min[]  
∞
```

Negative

```
Negative[x]  
returns True if x is a negative real number.
```

```
>> Negative[0]  
False  
  
>> Negative[-3]  
True  
  
>> Negative[10/7]  
False  
  
>> Negative[1+2I]  
False  
  
>> Negative[a + b]  
Negative[a + b]
```

NonNegative

```
NonNegative[x]  
returns True if x is a positive real number  
or zero.
```

```
>> {Positive[0], NonNegative[0]}  
{False, True}
```

NonPositive

```
NonPositive[x]  
returns True if x is a negative real number  
or zero.
```

```
>> {Negative[0], NonPositive[0]}  
{False, True}
```

Positive

```
Positive[x]  
returns True if x is a positive real number.
```

```
>> Positive[1]  
True
```

Positive returns False if x is zero or a complex number:

```
>> Positive[0]  
False  
  
>> Positive[1 + 2 I]  
False
```

SameQ (===)

```
SameQ[x, y]  
x === y  
returns True if x and y are structurally  
identical.
```

Any object is the same as itself:

```
>> a===a  
True
```

Unlike Equal, SameQ only yields True if x and y have the same type:

```
>> {1==1., 1==1.}
{True, False}
```

SympyComparison

TrueQ

```
TrueQ[expr]
returns True if and only if expr is True.
```

```
>> TrueQ[True]
True

>> TrueQ[False]
False

>> TrueQ[a]
False
```

Unequal (!=)

```
Unequal[x, y]
x != y
yields False if x and y are known to be
equal, or True if x and y are known to be
unequal.
lhs == rhs
represents the inequality lhs ≠ rhs.
```

```
>> 1 != 1.
False
```

Lists are compared based on their elements:

```
>> {1} != {2}
True

>> {1, 2} != {1, 2}
False

>> {a} != {a}
False

>> "a" != "b"
True

>> "a" != "a"
False
```

UnsameQ (!=)

```
UnsameQ[x, y]
x != y
returns True if x and y are not structurally
identical.
```

```
>> a!=a
False

>> 1!=1.
True
```

ValueQ

```
ValueQ[expr]
returns True if and only if expr is defined.
```

```
>> ValueQ[x]
False

>> x = 1;

>> ValueQ[x]
True
```

IX. Control statements

Contents

Abort	76	FixedPoint	77	NestList	79
Break	76	FixedPointList	77	NestWhile	79
CompoundExpression (;)	76	For	78	Return	79
Continue	76	If	78	Switch	79
Do	77	Interrupt	78	Which	80
		Nest	78	While	80

Abort

`Abort[]`
aborts an evaluation completely and returns `$Aborted`.

```
>> Print["a"]; Abort[]; Print["b"]
a
$Aborted
```

Break

`Break[]`
exits a `For`, `While`, or `Do` loop.

```
>> n = 0;

>> While[True, If[n>10, Break[]]; n
  =n+1]

>> n
11
```

CompoundExpression (;)

`CompoundExpression[e1, e2, ...]`
`e1; e2; ...`
evaluates its arguments in turn, returning the last result.

```
>> a; b; c; d
d
```

If the last argument is omitted, `Null` is taken:

```
>> a;
```

Continue

`Continue[]`
continues with the next iteration in a `For`, `While`, or `Do` loop.

```
>> For[i=1, i<=8, i=i+1, If[Mod[i
,2] == 0, Continue[]]; Print[i]]
1
3
5
7
```

Do

`Do[expr, {max}]`
evaluates *expr* *max* times.

`Do[expr, {i, max}]`
evaluates *expr* *max* times, substituting *i* in *expr* with values from 1 to *max*.

`Do[expr, {i, min, max}]`
starts with *i* = *max*.

`Do[expr, {i, min, max, step}]`
uses a step size of *step*.

`Do[expr, {i, {i1, i2, ...}}]`
uses values *i1*, *i2*, ... for *i*.

`Do[expr, {i, imin, imax}, {j, jmin, jmax}, ...]`
evaluates *expr* for each *j* from *jmin* to *jmax*, for each *i* from *imin* to *imax*, etc.

```
>> Do[Print[i], {i, 2, 4}]
2
3
4

>> Do[Print[{i, j}], {i, 1, 2}, {j, 3, 5}]
{1, 3}
{1, 4}
{1, 5}
{2, 3}
{2, 4}
{2, 5}
```

You can use `Break[]` and `Continue[]` inside `Do`:

```
>> Do[If[i > 10, Break[], If[Mod[i, 2] == 0, Continue[]]; Print[i]], {i, 5, 20}]
5
7
9
```

FixedPoint

`FixedPoint[f, expr]`
starting with *expr*, iteratively applies *f* until the result no longer changes.

`FixedPoint[f, expr, n]`
performs at most *n* iterations.

```
>> FixedPoint[Cos, 1.0]
0.739085

>> FixedPoint[#+1 &, 1, 20]
21
```

FixedPointList

`FixedPointList[f, expr]`
starting with *expr*, iteratively applies *f* until the result no longer changes, and returns a list of all intermediate results.

`FixedPointList[f, expr, n]`
performs at most *n* iterations.

```
>> FixedPointList[Cos, 1.0, 4]
{1., 0.540302, 0.857~
~553, 0.65429, 0.79348}
```

Observe the convergence of Newton's method for approximating square roots:

```
>> newton[n_] := FixedPointList[.5(# + n/#)&, 1.];

>> newton[9]
{1., 5., 3.4, 3.02353, 3.00009, 3., 3., 3.}
```

Plot the "hailstone" sequence of a number:

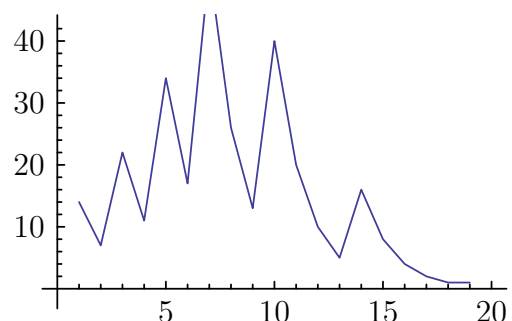
```
>> collatz[1] := 1;

>> collatz[x_ ? EvenQ] := x / 2;

>> collatz[x_] := 3 x + 1;

>> list = FixedPointList[collatz, 14]
{14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 1}

>> ListLinePlot[list]
```



For

`For[start, test, incr, body]`
evaluates *start*, and then iteratively *body*
and *incr* as long as *test* evaluates to True.
`For[start, test, incr]`
evaluates only *incr* and no *body*.
`For[start, test]`
runs the loop without any body.

Compute the factorial of 10 using For:

```
>> n := 1

>> For[i=1, i<=10, i=i+1, n = n * i]

>> n
3 628 800

>> n == 10!
True
```

If

`If[cond, pos, neg]`
returns *pos* if *cond* evaluates to True, and
neg if it evaluates to False.
`If[cond, pos, neg, other]`
returns *other* if *cond* evaluates to neither
True nor False.
`If[cond, pos]`
returns Null if *cond* evaluates to False.

```
>> If[1<2, a, b]
a
```

If the second branch is not specified, Null is taken:

```
>> If[1<2, a]
a

>> If[False, a] //FullForm
Null
```

You might use comments (inside `(*)` and `(*)`) to make the branches of If more readable:

```
>> If[a, (*then*)b, (*else*)c];
```

Interrupt

`Interrupt[]`
Interrupt an evaluation and returns
\$Aborted.

```
>> Print["a"]; Interrupt[]; Print["b"]
a
$Aborted
```

Nest

`Nest[f, expr, n]`
starting with *expr*, iteratively applies *f* *n*
times and returns the final result.

```
>> Nest[f, x, 3]
f[f[f[x]]]

>> Nest[(1+#)^2 &, x, 2]
(1 + (1 + x)2)2
```

NestList

`NestList[f, expr, n]`
starting with *expr*, iteratively applies *f* *n*
times and returns a list of all intermediate results.

```
>> NestList[f, x, 3]
{x, f[x], f[f[x]], f[f[f[x]]]}

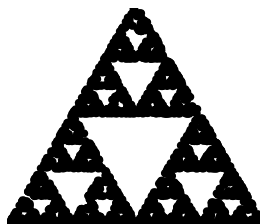
>> NestList[2 # &, 1, 8]
{1, 2, 4, 8, 16, 32, 64, 128, 256}
```

Chaos game rendition of the Sierpinski triangle:

```
>> vertices = {{0,0}, {1,0}, {.5,
.5 Sqrt[3]}};

>> points = NestList[.5(vertices[[
RandomInteger[{1,3}] ] + #)&,
{0.,0.}, 2000];
```

```
>> Graphics[Point[points],
ImageSize->Small]
```



NestWhile

```
NestWhile[f, expr, test]
  applies a function f repeatedly on an ex-
  pression expr, until applying test on the
  result no longer yields True.
NestWhile[f, expr, test, m]
  supplies the last m results to test (default
  value: 1).
NestWhile[f, expr, test, All]
  supplies all results gained so far to test.
```

Divide by 2 until the result is no longer an integer:

```
>> NestWhile[#/2&, 10000, IntegerQ]
625
2
```

Return

```
Return[expr]
  aborts a function call and returns expr.
```

```
>> f[x_] := (If[x < 0, Return[0]];
x)
>> f[-1]
0
>> Do[If[i > 3, Return[]]; Print[i
], {i, 10}]
1
2
3
```

Return only exits from the innermost control flow construct.

```
>> g[x_] := (Do[If[x < 0, Return
[0]], {i, {2, 1, 0, -1}}]; x)
```

```
>> g[-1]
-1
```

Switch

```
Switch[expr, pattern1, value1, pattern2,
value2, ...]
  yields the first value for which expr
  matches the corresponding pattern.
```

```
>> Switch[2, 1, x, 2, y, 3, z]
y
>> Switch[5, 1, x, 2, y]
Switch[5, 1, x, 2, y]
>> Switch[5, 1, x, 2, y, _, z]
z
>> Switch[2, 1]
Switch called with 2 arguments. Switch must be called with an odd number of arguments.
Switch[2, 1]
```

Which

```
Which[cond1, expr1, cond2, expr2, ...]
  yields expr1 if cond1 evaluates to True,
expr2 if cond2 evaluates to True, etc.
```

```
>> n = 5;
>> Which[n == 3, x, n == 5, y]
y
>> f[x_] := Which[x < 0, -x, x ==
0, 0, x > 0, x]
>> f[-3]
3
```

If no test yields True, Which returns Null:

```
>> Which[False, a]
```

If a test does not evaluate to True or False, evaluation stops and a Which expression containing the remaining cases is returned:

```
>> Which[False, a, x, b, True, c]
Which[x, b, True, c]
```

Which must be called with an even number of arguments:

```
>> Which[a, b, c]
Whichcalledwith3arguments.
Which[a, b, c]
```

While

```
While[test, body]
  evaluates body as long as test evaluates to
  True.
While[test]
  runs the loop without any body.
```

Compute the GCD of two numbers:

```
>> {a, b} = {27, 6};

>> While[b != 0, {a, b} = {b, Mod[a
, b]}];

>> a
3
```


X. Date and Time

Contents

<code>AbsoluteTime</code>	81	<code>DateString</code>	83	<code>\$SystemTimeZone</code> . .	83
<code>AbsoluteTiming</code>	81	<code>\$DateStringFormat</code> . .	83	<code>TimeUsed</code>	84
<code>DateDifference</code>	82	<code>EasterSunday</code>	83	<code>\$TimeZone</code>	84
<code>DateList</code>	82	<code>Pause</code>	83	<code>Timing</code>	84
<code>DatePlus</code>	82	<code>SessionTime</code>	83		

AbsoluteTime

`AbsoluteTime[]`
gives the local time in seconds since epoch Jan 1 1900.

`AbsoluteTime[string]`
gives the absolute time specification for a given date string.

`AbsoluteTime[{y, m, d, h, m, s}]`
gives the absolute time specification for a given date list.

`AbsoluteTime[{'string', {e1, e2, ...}}]`
gives the absolute time specification for a given date list with specified elements *ei*.

```
>> AbsoluteTime[]
3.81076 × 109

>> AbsoluteTime[{2000}]
3 155 673 600

>> AbsoluteTime[{"01/02/03", {"Day", "Month", "YearShort"}}]
3 253 046 400

>> AbsoluteTime["6 June 1991"]
2885 155 200

>> AbsoluteTime[{"6-6-91", {"Day", "Month", "YearShort"}}]
2885 155 200
```

AbsoluteTiming

`AbsoluteTiming[expr]`
measures the actual time it takes to evaluate *expr*. It returns a list containing the measured time in seconds and the result of the evaluation.

```
>> AbsoluteTiming[50!]
{0.000219822, 30 414 093 ~
~201 713 378 043 612 608 166 ~
~064 768 844 377 641 568 ~
~960 512 000 000 000 000}

>> Attributes[AbsoluteTiming]
{HoldAll, Protected}
```

DateDifference

`DateDifference[date1, date2]`
returns the difference between *date1* and *date2* in days.

`DateDifference[date1, date2, unit]`
returns the difference in the specified *unit*.

`DateDifference[date1, date2, {unit1, unit2, ...}]`
represents the difference as a list of integer multiples of each *unit*, with any remainder expressed in the smallest unit.

```
>> DateDifference[{2042, 1, 4},
{2057, 1, 1}]
5476

>> DateDifference[{1936, 8, 14},
{2000, 12, 1}, "Year"]
{64.3425, Year}

>> DateDifference[{2010, 6, 1},
{2015, 1, 1}, "Hour"]
{40200, Hour}

>> DateDifference[{2003, 8, 11},
{2003, 10, 19}, {"Week", "Day"}]
{{9, Week}, {6, Day}}
```

DateList

DateList[]
returns the current local time in the form
{year, month, day, hour, minute, second}.

DateList[time]
returns a formatted date for the number
of seconds *time* since epoch Jan 1 1900.

DateList[{y, m, d, h, m, s}]
converts an incomplete date list to the
standard representation.

DateString[string]
returns the formatted date list of a date
string specification.

DateString[string, {e1, e2, ...}]
returns the formatted date list of a *string*
obtained from elements *ei*.

```
>> DateList[0]
{1900, 1, 1, 0, 0, 0.}

>> DateList[3155673600]
{2000, 1, 1, 0, 0, 0.}

>> DateList[{2003, 5, 0.5, 0.1,
0.767}]
{2003, 4, 30, 12, 6, 46.02}

>> DateList[{2012, 1, 300., 10}]
{2012, 10, 26, 10, 0, 0.}

>> DateList["31/10/1991"]
{1991, 10, 31, 0, 0, 0.}
```

```
>> DateList["1/10/1991"]
The interpretation of 1/10/
1991 is ambiguous.
{1991, 1, 10, 0, 0, 0.}

>> DateList[{"31/10/91", {"Day", "
Month", "YearShort"}}]
{1991, 10, 31, 0, 0, 0.}

>> DateList[{"31 10/91", {"Day", "
", "Month", "/"}, "YearShort"}}]
{1991, 10, 31, 0, 0, 0.}
```

If not specified, the current year assumed

```
>> DateList[{"5/18", {"Month", "Day
"}}]
{2020, 5, 18, 0, 0, 0.}
```

DatePlus

DatePlus[date, n]
finds the date *n* days after *date*.

DatePlus[date, {n, "unit"}]
finds the date *n* units after *date*.

**DatePlus[date, {{n1, "unit1"}, {n2, "
unit2"}, ...}]**
finds the date which is *n_i* specified units
after *date*.

DatePlus[n]
finds the date *n* days after the current
date.

DatePlus[offset]
finds the date which is offset from the
current date.

Add 73 days to Feb 5, 2010:

```
>> DatePlus[{2010, 2, 5}, 73]
{2010, 4, 19}
```

Add 8 weeks and 1 day to March 16, 1999:

```
>> DatePlus[{2010, 2, 5}, {{8, "
Week"}, {1, "Day"}}]
{2010, 4, 3}
```

DateString

`DateString[]`
returns the current local time and date as a string.

`DateString[elem]`
returns the time formatted according to *elems*.

`DateString[{e1, e2, ...}]`
concatenates the time formatted according to elements *ei*.

`DateString[time]`
returns the date string of an `AbsoluteTime`.

`DateString[{y, m, d, h, m, s}]`
returns the date string of a date list specification.

`DateString[string]`
returns the formatted date string of a date string specification.

`DateString[spec, elems]`
formats the time in turns of *elems*. Both *spec* and *elems* can take any of the above formats.

The current date and time:

```
>> DateString[];

>> DateString[{1991, 10, 31, 0, 0},
{"Day", " ", "MonthName", " ",
"Year"}]
31 October 1991

>> DateString[{2007, 4, 15, 0}]
Sun 15 Apr 2 007 00:00:00

>> DateString[{1979, 3, 14}, {"
DayName", " ", "Month", "-", "
YearShort"}]
Wednesday 03-79
```

Non-integer values are accepted too:

```
>> DateString[{1991, 6, 6.5}]
Thu 6 Jun 1 991 12:00:00
```

\$DateStringFormat

`$DateStringFormat`
gives the format used for dates generated by `DateString`.

```
>> $DateStringFormat
{DateTimeShort}
```

EasterSunday

`EasterSunday[year]`
returns the date of the Gregorian Easter Sunday as {year, month, day}.

```
>> EasterSunday[2000]
{2000, 4, 23}

>> EasterSunday[2030]
{2030, 4, 21}
```

Pause

`Pause[n]`
pauses for *n* seconds.

```
>> Pause[0.5]
```

SessionTime

`SessionTime[]`
returns the total time in seconds since this session started.

```
>> SessionTime[]
218.37
```

\$SystemTimeZone

`$SystemTimeZone`
gives the current time zone for the computer system on which Mathics is being run.

```
>> $SystemTimeZone
-5.
```

TimeUsed

TimeUsed[]
returns the total CPU time used for this session, in seconds.

```
>> TimeUsed[]  
219.622
```

\$TimeZone

\$TimeZone
gives the current time zone.

```
>> $TimeZone  
-5.
```

Timing

Timing[*expr*]
measures the processor time taken to evaluate *expr*. It returns a list containing the measured time in seconds and the result of the evaluation.

```
>> Timing[50!]  
{0.00022089, 30 414 093 201 713 378 ~  
~043 612 608 166 064 768 844 377 ~  
~641 568 960 512 000 000 000 000}  
  
>> Attributes[Timing]  
{HoldAll, Protected}
```

XI. Differential equation solver functions

Contents

C	85	DSolve	85
-------------	----	------------------	----

C

$C[n]$

represents the n th constant in a solution to a differential equation.

```
>> DSolve[D[y[x, t], t] + 2 D[y[x,
t], x] == 0, y[x, t], {x, t}]
{{y[x, t] -> C[1] [-2t + x]}}
```

DSolve

$\text{DSolve}[eq, y[x], x]$

solves a differential equation for the function $y[x]$.

```
>> DSolve[y''[x] == 0, y[x], x]
{{y[x] -> x C[2] + C[1]}}

>> DSolve[y''[x] == y[x], y[x], x]
{{y[x] -> C[1] E^-x + C[2] E^x}}

>> DSolve[y''[x] == y[x], y, x]
{{y -> (Function[{x},
C[1] E^-x + C[2] E^x])}}
```

DSolve can also solve basic PDE

```
>> DSolve[D[f[x, y], x] / f[x, y] +
3 D[f[x, y], y] / f[x, y] == 2,
f, {x, y}]
{{f -> (Function[{x, y},
E^(x/5 + 3y/5) C[1] [3x - y])}]}
```

```
>> DSolve[D[f[x, y], x] x + D[f[x,
y], y] y == 2, f[x, y], {x, y}]
{{f[x, y] -> 2 Log[x] + C[1] [y/x]}}
```

XII. Evaluation

Contents

Evaluate	86	HoldForm	87	\$RecursionLimit	88
Exit	86	In	87	ReleaseHold	88
\$HistoryLength	86	\$IterationLimit	87	Sequence	89
Hold	87	\$Line	87	Unevaluated	89
HoldComplete	87	Out	88		

Evaluate

`Evaluate[expr]`
forces evaluation of *expr*, even if it occurs inside a held argument or a Hold form.

Create a function *f* with a held argument:

```
>> SetAttributes[f, HoldAll]
```

```
>> f[1 + 2]
      f[1 + 2]
```

Evaluate forces evaluation of the argument, even though *f* has the HoldAll attribute:

```
>> f[Evaluate[1 + 2]]
      f[3]
```

```
>> Hold[Evaluate[1 + 2]]
      Hold[3]
```

```
>> HoldComplete[Evaluate[1 + 2]]
      HoldComplete[Evaluate[1 + 2]]
```

```
>> Evaluate[Sequence[1, 2]]
      Sequence[1, 2]
```

\$HistoryLength

`$HistoryLength`
specifies the maximum number of In and Out entries.

```
>> $HistoryLength
      100
>> $HistoryLength = 1;
>> 42
      42
>> %
      42
>> %%
      %3
>> $HistoryLength = 0;
>> 42
      42
>> %
      %7
```

Exit

`Exit[]`
terminates the Mathics session.
`Exit[n]`
terminates with exit code *n*.

Exit is an alias for Quit.

Hold

`Hold[expr]`
prevents *expr* from being evaluated.

```
>> Attributes[Hold]
      {HoldAll, Protected}
```

HoldComplete

`HoldComplete[expr]`
prevents *expr* from being evaluated, and also prevents Sequence objects from being spliced into argument lists.

```
>> Attributes[HoldComplete]
{HoldAllComplete, Protected}
```

HoldForm

`HoldForm[expr]`
is equivalent to `Hold[expr]`, but prints as *expr*.

```
>> HoldForm[1 + 2 + 3]
1 + 2 + 3
```

HoldForm has attribute HoldAll:

```
>> Attributes[HoldForm]
{HoldAll, Protected}
```

In

`In[k]`
gives the *k*th line of input.

```
>> x = 1
1
>> x = x + 1
2
>> Do[In[2], {3}]
>> x
5
>> In[-1]
5
>> Definition[In]
Attributes[In] = {Protected}
In[6] = Definition[In]
In[5] = In[-1]
In[4] = x
In[3] = Do[In[2], {3}]
In[2] = x = x + 1
In[1] = x = 1
```

\$IterationLimit

`$IterationLimit`
specifies the maximum number of times a reevaluation may happen.

Calculations terminated by `$IterationLimit` return `$Aborted`:

```
>> ClearAll[f]; f[x_] := f[x + 1];
```

```
>> f[x]
Iterationlimit of 1 000 exceeded.
$Aborted
```

```
>> $IterationLimit
1 000
```

```
>> ClearAll[f];
```

```
>> $IterationLimit = x;
```

Cannot set \$IterationLimit to x; value must be an integer between 2

\$Line

`$Line`
holds the current input line number.

```
>> $Line
1
```

```
>> $Line
2
```

```
>> $Line = 12;
```

```
>> 2 * 5
10
```

```
>> Out[13]
10
```

```
>> $Line = -1;
Non-negative integer expected.
```

Out

```
Out[k]
%k
    gives the result of the kth input line.
%, %, etc.
    gives the result of the previous input line,
    of the line before the previous input line,
    etc.

>> 42
    42

>> %
    42

>> 43;

>> %
    43

>> 44
    44

>> %1
    42

>> %%
    44

>> Hold[Out[-1]]
    Hold[%]

>> Hold[%4]
    Hold[%4]

>> Out[0]
    Out[0]
```

\$RecursionLimit

\$RecursionLimit
specifies the maximum allowable recursion depth after which a calculation is terminated.

Calculations terminated by \$RecursionLimit return \$Aborted:

```
>> a = a + a
    Recursiondepthof200exceeded.
    $Aborted

>> $RecursionLimit
    200
```

```
>> $RecursionLimit = x;
    Cannotset$RecursionLimittox;valuemustbeanintegerbetween

>> $RecursionLimit = 512
    512

>> a = a + a
    Recursiondepthof512exceeded.
    $Aborted
```

ReleaseHold

ReleaseHold[expr]
removes any Hold, HoldForm, HoldPattern or HoldComplete head from expr.

```
>> x = 3;

>> Hold[x]
    Hold[x]

>> ReleaseHold[Hold[x]]
    3

>> ReleaseHold[y]
    y
```

Sequence

Sequence[x1, x2, ...]
represents a sequence of arguments to a function.

Sequence is automatically spliced in, except when a function has attribute SequenceHold (like assignment functions).

```
>> f[x, Sequence[a, b], y]
    f[x, a, b, y]

>> Attributes[Set]
    {HoldFirst, Protected, SequenceHold}

>> a = Sequence[b, c];

>> a
    Sequence[b, c]
```

Apply Sequence to a list to splice in arguments:

```
>> list = {1, 2, 3};
```



```
>> f[Sequence @@ list]
      f[1,2,3]
```

Inside Hold or a function with a held argument, Sequence is spliced in at the first level of the argument:

```
>> Hold[a, Sequence[b, c], d]
      Hold[a, b, c, d]
```

If Sequence appears at a deeper level, it is left unevaluated:

```
>> Hold[{a, Sequence[b, c], d}]
      Hold[{a, Sequence[b, c], d}]
```

```
>> g[Unevaluated[Sequence[a, b, c]]]
      g[Unevaluated[Sequence[a, b, c]]]
```

Unevaluated

`Unevaluated[expr]`
temporarily leaves *expr* in an unevaluated form when it appears as a function argument.

Unevaluated is automatically removed when function arguments are evaluated:

```
>> Sqrt[Unevaluated[x]]
       $\sqrt{x}$ 
```

```
>> Length[Unevaluated[1+2+3+4]]
      4
```

Unevaluated has attribute HoldAllComplete:

```
>> Attributes[Unevaluated]
      {HoldAllComplete, Protected}
```

Unevaluated is maintained for arguments to non-executed functions:

```
>> f[Unevaluated[x]]
      f[Unevaluated[x]]
```

Likewise, its kept in flattened arguments and sequences:

```
>> Attributes[f] = {Flat};

>> f[a, Unevaluated[f[b, c]]]
      f[a, Unevaluated[b], Unevaluated[c]]

>> g[a, Sequence[Unevaluated[b],
      Unevaluated[c]]]
      g[a, Unevaluated[b], Unevaluated[c]]
```

However, unevaluated sequences are kept:

XIII. Exponential, trigonometric and hyperbolic functions

Mathics basically supports all important trigonometric and hyperbolic functions. Numerical values and derivatives can be computed; however, most special exact values and simplification rules are not implemented yet.

Contents

AnglePath	91	ArcTan	93	InverseHaversine	95
AngleVector	91	ArcTanh	93	Log	95
ArcCos	91	Cos	93	Log10	95
ArcCosh	92	Cosh	93	Log2	95
ArcCot	92	Cot	93	LogisticSigmoid	95
ArcCoth	92	Coth	93	Pi	96
ArcCsc	92	Csc	94	Sec	96
ArcCsch	92	Csch	94	Sech	96
ArcSec	92	Degree	94	Sin	96
ArcSech	92	E	94	Sinh	96
ArcSin	93	Exp	94	Tan	96
ArcSinh	93	GoldenRatio	94	Tanh	96
		Haversine	95		

AnglePath

AnglePath[*{phi1, phi2, ...}*]
returns the points formed by a turtle starting at {0, 0} and angled at 0 degrees going through the turns given by angles *phi1, phi2, ...* and using distance 1 for each step.

AnglePath[*{r1, phi1}, {r2, phi2}, ...*]
instead of using 1 as distance, use *r1, r2, ...* as distances for the respective steps.

AngleVector[*phi0, {phi1, phi2, ...}*]
returns the points on a path formed by a turtle starting with direction *phi0* instead of 0.

AngleVector[*{x, y}, {phi1, phi2, ...}*]
returns the points on a path formed by a turtle starting at {*x, y*} instead of {0, 0}.

AngleVector[*{x, y}, phi0, {phi1, phi2, ...}*]
specifies initial position {*x, y*} and initial direction *phi0*.

AngleVector[*{x, y}, {dx, dy}, {phi1, phi2, ...}*]
specifies initial position {*x, y*} and a slope {*dx, dy*} that is understood to be the initial direction of the turtle.

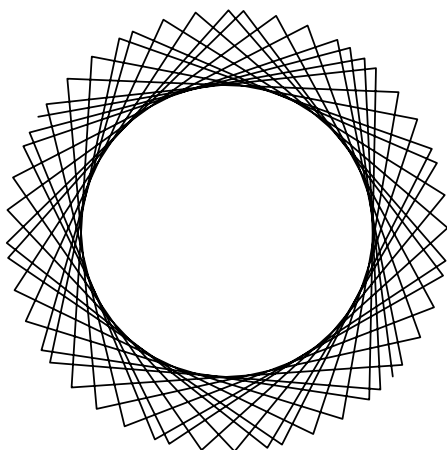
```
>> AnglePath[{90 Degree, 90 Degree,
  90 Degree, 90 Degree}]
{{0,0}, {0,1}, {-1,
  1}, {-1,0}, {0,0}}

>> AnglePath[{{1, 1}, 90 Degree},
  {{1, 90 Degree}, {2, 90 Degree},
  {1, 90 Degree}, {2, 90 Degree
  }}]
{{1,1}, {0,1}, {0,
  -1}, {1, -1}, {1,1}}

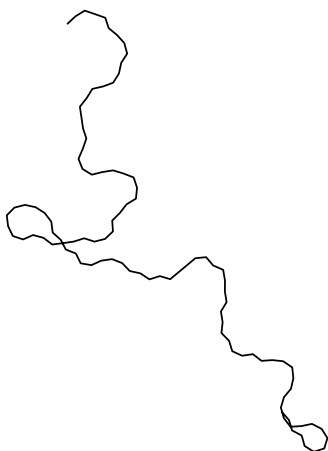
>> AnglePath[{a, b}]
{{0,0}, {Cos[a], Sin[a]}, {Cos[
  a]+Cos[a+b], Sin[a]+Sin[a+b]}}

>> Precision[Part[AnglePath[{N[1/3,
  100], N[2/3, 100]}], 2, 1]]
100.

>> Graphics[Line[AnglePath[Table
  [1.7, {50}]]]]
```



```
>> Graphics[Line[AnglePath[
  RandomReal[{-1, 1}, {100}]]]]
```



AngleVector

AngleVector[ϕ]
returns the point at angle ϕ on the unit circle.

AngleVector[{ r , ϕ }]
returns the point at angle ϕ on a circle of radius r .

AngleVector[{ x , y }, ϕ]
returns the point at angle ϕ on a circle of radius 1 centered at $\{x, y\}$.

AngleVector[{ x , y }, { r , ϕ }]
returns point at angle ϕ on a circle of radius r centered at $\{x, y\}$.

```
>> AngleVector[90 Degree]
{0, 1}

>> AngleVector[{1, 10}, a]
{1 + Cos[a], 10 + Sin[a]}
```

ArcCos

ArcCos[z]
returns the inverse cosine of z .

```
>> ArcCos[1]
0

>> ArcCos[0]
 $\frac{\text{Pi}}{2}$ 

>> Integrate[ArcCos[x], {x, -1, 1}]
Pi
```

ArcCosh

ArcCosh[z]
returns the inverse hyperbolic cosine of z .

```
>> ArcCosh[0]
 $\frac{I}{2}\text{Pi}$ 

>> ArcCosh[0.]
0. + 1.5708I
```



```
>> ArcSin[1]
      Pi
      2
```

ArcSinh

`ArcSinh[z]`
returns the inverse hyperbolic sine of z.

```
>> ArcSinh[0]
0

>> ArcSinh[0.]
0.

>> ArcSinh[1.0]
0.881374
```

ArcTan

`ArcTan[z]`
returns the inverse tangent of z.

```
>> ArcTan[1]
      Pi
      4

>> ArcTan[1.0]
0.785398

>> ArcTan[-1.0]
-0.785398

>> ArcTan[1, 1]
      Pi
      4
```

ArcTanh

`ArcTanh[z]`
returns the inverse hyperbolic tangent of z.

```
>> ArcTanh[0]
0

>> ArcTanh[1]
∞

>> ArcTanh[0]
0
```

```
>> ArcTanh[.5 + 2 I]
0.0964156 + 1.12656 I

>> ArcTanh[2 + I]
ArcTanh[2 + I]
```

Cos

`Cos[z]`
returns the cosine of z.

```
>> Cos[3 Pi]
-1
```

Cosh

`Cosh[z]`
returns the hyperbolic cosine of z.

```
>> Cosh[0]
1
```

Cot

`Cot[z]`
returns the cotangent of z.

```
>> Cot[0]
ComplexInfinity

>> Cot[1.]
0.642093
```

Coth

`Coth[z]`
returns the hyperbolic cotangent of z.

```
>> Coth[0]
ComplexInfinity
```

Csc

`Csc[z]`
returns the cosecant of z.

```
>> Csc[0]
ComplexInfinity

>> Csc[1] (* Csc[1] in Mathematica *)
 $\frac{1}{\sin[1]}$ 

>> Csc[1.]
1.1884
```

Csch

`Csch[z]`
returns the hyperbolic cosecant of z.

```
>> Csch[0]
ComplexInfinity
```

Degree

`Degree`
is the number of radians in one degree.

```
>> Cos[60 Degree]
 $\frac{1}{2}$ 
```

Degree has the value of $\text{Pi} / 180$

```
>> Degree == Pi / 180
True
```

E

`E`
is the constant e.

```
>> N[E]
2.71828
```

```
>> N[E, 50]
2.718281828459045235360287~
~4713526624977572470937000

>> Attributes[E]
{Constant, Protected, ReadProtected}
```

Exp

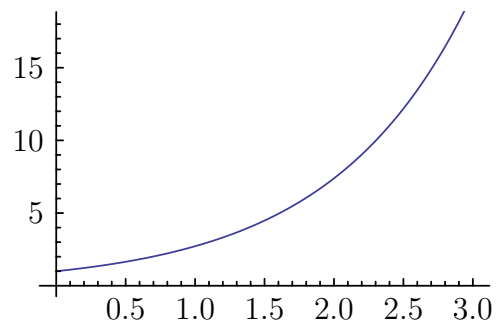
`Exp[z]`
returns the exponential function of z.

```
>> Exp[1]
E

>> Exp[10.0]
22026.5

>> Exp[x] //FullForm
Power[E, x]

>> Plot[Exp[x], {x, 0, 3}]
```



GoldenRatio

`GoldenRatio`
is the golden ratio.

```
>> N[GoldenRatio]
1.61803
```

Haversine

`Haversine[z]`
returns the haversine function of z.

```
>> Haversine[1.5]
0.464631
```

```
>> Haversine[0.5 + 2I]
-1.15082 + 0.869405I
```

InverseHaversine

Haversine[z]
returns the inverse haversine function of z.

```
>> InverseHaversine[0.5]
1.5708

>> InverseHaversine[1 + 2.5 I]
1.76459 + 2.33097I
```

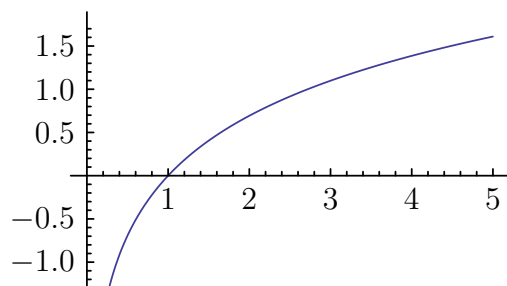
Log

Log[z]
returns the natural logarithm of z.

```
>> Log[{0, 1, E, E * E, E ^ 3, E ^ x}]
{-∞, 0, 1, 2, 3, Log[E^x]}

>> Log[0.]
Indeterminate

>> Plot[Log[x], {x, 0, 5}]
```



Log10

Log10[z]
returns the base-10 logarithm of z.

```
>> Log10[1000]
3

>> Log10[{2., 5.}]
{0.30103, 0.69897}
```

```
>> Log10[E ^ 3]
3
Log[10]
```

Log2

Log2[z]
returns the base-2 logarithm of z.

```
>> Log2[4 ^ 8]
16

>> Log2[5.6]
2.48543

>> Log2[E ^ 2]
2
Log[2]
```

LogisticSigmoid

LogisticSigmoid[z]
returns the logistic sigmoid of z.

```
>> LogisticSigmoid[0.5]
0.622459

>> LogisticSigmoid[0.5 + 2.3 I]
1.06475 + 0.808177I

>> LogisticSigmoid[{-0.2, 0.1, 0.3}]
{0.450166, 0.524979, 0.574443}
```

Pi

Pi
is the constant π .

```
>> N[Pi]
3.14159

>> N[Pi, 50]
3.141592653589793238462643~
~3832795028841971693993751
```

```
>> Attributes[Pi]
{Constant, Protected, ReadProtected}
```

Sec

Sec[z]
returns the secant of z.

```
>> Sec[0]
1

>> Sec[1] (* Sec[1] in Mathematica *)

$$\frac{1}{\cos[1]}$$


>> Sec[1.]
1.85082
```

Sech

Sech[z]
returns the hyperbolic secant of z.

```
>> Sech[0]
1
```

Sin

Sin[z]
returns the sine of z.

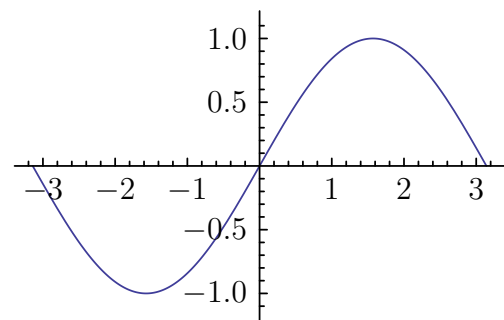
```
>> Sin[0]
0

>> Sin[0.5]
0.479426

>> Sin[3 Pi]
0

>> Sin[1.0 + I]
1.29846 + 0.634964I
```

```
>> Plot[Sin[x], {x, -Pi, Pi}]
```



Sinh

Sinh[z]
returns the hyperbolic sine of z.

```
>> Sinh[0]
0
```

Tan

Tan[z]
returns the tangent of z.

```
>> Tan[0]
0

>> Tan[Pi / 2]
ComplexInfinity
```

Tanh

Tanh[z]
returns the hyperbolic tangent of z.

```
>> Tanh[0]
0
```


XIV. Functional programming

Contents

Composition	97	Identity	98	SlotSequence	98
Function (&)	97	Slot	98		

Composition

`Composition[f, g]`
returns the composition of two functions *f* and *g*.

```
>> Composition[f, g][x]
f[g[x]]

>> Composition[f, g, h][x, y, z]
f[g[h[x, y, z]]]

>> Composition[]
Identity

>> Composition[] [x]
x

>> Attributes[Composition]
{Flat, OneIdentity, Protected}

>> Composition[f, Composition[g, h]]
Composition[f, g, h]
```

Function (&)

`Function[body]`
body &
represents a pure function with parameters #1, #2, etc.
`Function[{x1, x2, ...}, body]`
represents a pure function with parameters *x1*, *x2*, etc.

```
>> f := # ^ 2 &

>> f[3]
9

>> #^3 & /@ {1, 2, 3}
{1, 8, 27}

>> #1 + #2 & [4, 5]
9
```

You can use `Function` with named parameters:

```
>> Function[{x, y}, x * y][2, 3]
6
```

Parameters are renamed, when necessary, to avoid confusion:

```
>> Function[{x}, Function[{y}, f[x, y]]][y]
Function[{y$}, f[y, y$]]

>> Function[{y}, f[x, y]] /. x -> y
Function[{y}, f[y, y]]

>> Function[y, Function[x, y^x]][x]
x^y

>> Function[x, Function[y, x^y]][x]
x^y
```

Slots in inner functions are not affected by outer function application:

```
>> g[#] & [h[#]] & [5]
g[h[5]]
```

Identity

`Identity[x]`
is the identity function, which returns x unchanged.

```
>> Identity[x]
x
>> Identity[x, y]
Identity[x, y]
```

```
>> FullForm[##]
SlotSequence[1]
```

Slot

`# n`
represents the n th argument to a pure function.
`#`
is short-hand for `#1`.
`#0`
represents the pure function itself.

```
>> #
#1
```

Unused arguments are simply ignored:

```
>> {#1, #2, #3}&[1, 2, 3, 4, 5]
{1, 2, 3}
```

Recursive pure functions can be written using `#0`:

```
>> If[#1<=1, 1, #1 #0[#1-1]]& [10]
3628800
```

SlotSequence

`##`
is the sequence of arguments supplied to a pure function.
`## n`
starts with the n th argument.

```
>> Plus[##]& [1, 2, 3]
6
>> Plus[##2]& [1, 2, 3]
5
```

XV. Graphics

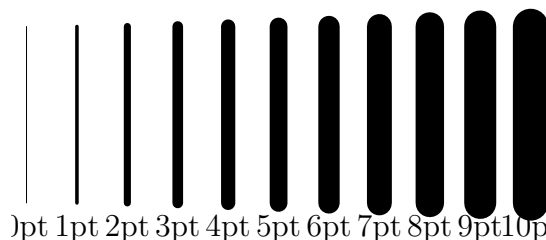
Contents

AbsoluteThickness . . .	99	FaceForm	106	Orange	111
Arrow	100	FilledCurve	106	Point	112
ArrowBox	101	FilledCurveBox	106	PointBox	112
Arrowheads	101	FontColor	106	PointSize	112
Automatic	102	Graphics	107	Polygon	112
BernsteinBasis	102	GraphicsBox	107	PolygonBox	112
BezierCurve	102	Gray	108	Purple	113
BezierCurveBox	102	GrayLevel	108	RGBColor	113
BezierFunction	102	Green	108	Rectangle	114
Black	102	Hue	108	RectangleBox	114
Blend	103	Inset	109	Red	114
Blue	103	InsetBox	109	RegularPolygon	114
CMYKColor	103	LABColor	109	RegularPolygonBox	115
Circle	104	LCHColor	109	Small	115
CircleBox	104	LUVColor	109	Text	115
ColorDistance	104	Large	109	Thick	115
Cyan	104	LightRed	109	Thickness	115
Darker	104	Lighter	110	Thin	115
Directive	105	Line	110	Tiny	115
Disk	105	LineBox	110	White	116
DiskBox	106	Magenta	111	XYZColor	116
EdgeForm	106	Medium	111	Yellow	116
		Offset	111		

AbsoluteThickness

`AbsoluteThickness[p]`
sets the line thickness for subsequent graphics primitives to *p* points.

```
>> Graphics[Table[{
  AbsoluteThickness[t], Line[{{20
t, 10}, {20 t, 80}}], Text[
ToString[t]<"pt", {20 t, 0}],
{t, 0, 10}]]
```



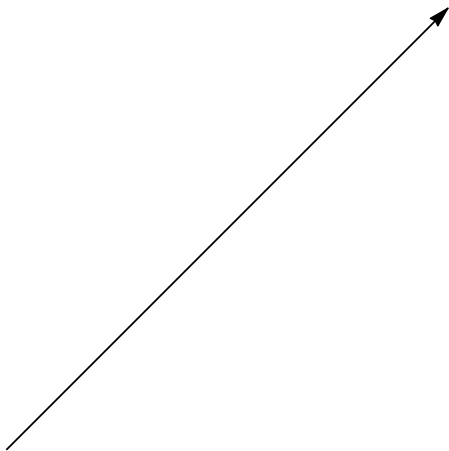
Arrow

`Arrow[{p1, p2}]`
 represents a line from $p1$ to $p2$ that ends with an arrow at $p2$.

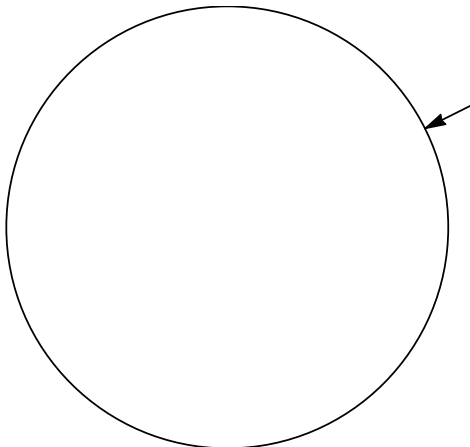
`Arrow[{p1, p2}, s]`
 represents a line with arrow that keeps a distance of s from $p1$ and $p2$.

`Arrow[{point_1, point_2}, {s1, s2}]`
 represents a line with arrow that keeps a distance of $s1$ from $p1$ and a distance of $s2$ from $p2$.

```
>> Graphics[Arrow[{0,0}, {1,1}]]
```

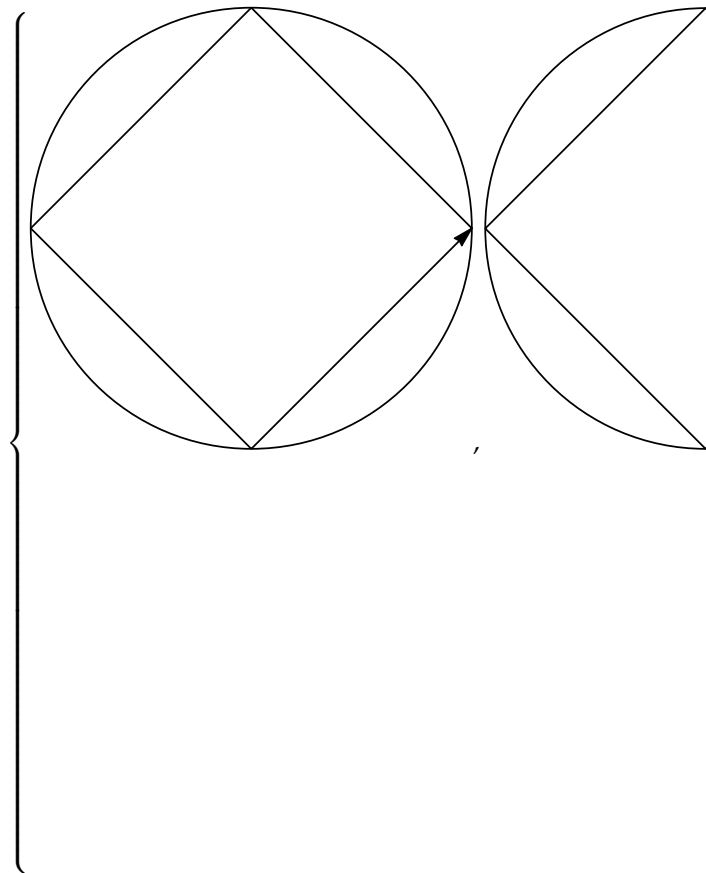


```
>> Graphics[{Circle[], Arrow[{2,1}, {0,0}, 1]}]
```



Keeping distances may happen across multiple segments:

```
>> Table[Graphics[{Circle[], Arrow[
Table[{Cos[phi], Sin[phi]], {phi
,0,2*Pi,Pi/2}], {d, d}]}], {d
,0,2,0.5}]
```



ArrowBox

Arrowheads

`Arrowheads[s]`
specifies that `Arrow[]` draws one arrow of size s (relative to width of image, defaults to 0.04).

`Arrowheads[{spec1, spec2, ..., specn}]`
specifies that `Arrow[]` draws n arrows as defined by $spec1, spec2, \dots, specn$.

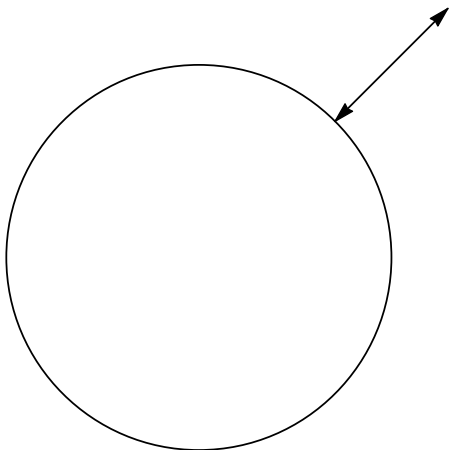
`Arrowheads[{s}]`
specifies that one arrow of size s should be drawn.

`Arrowheads[{s, pos}]`
specifies that one arrow of size s should be drawn at position pos (for the arrow to be on the line, pos has to be between 0, i.e. the start for the line, and 1, i.e. the end of the line).

`Arrowheads[{s, pos, g}]`
specifies that one arrow of size s should be drawn at position pos using Graphics g .

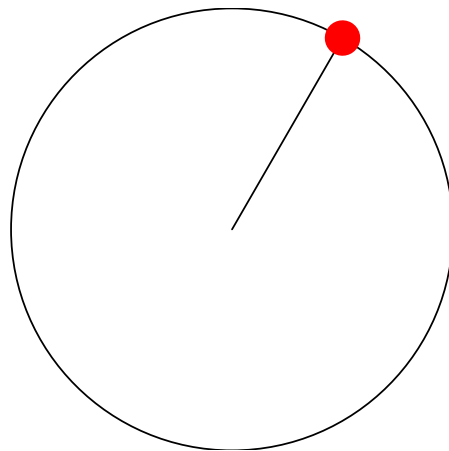
Arrows on both ends can be achieved using negative sizes:

```
>> Graphics[{Circle[], Arrowheads
[{-0.04, 0.04}], Arrow[{0, 0},
{2, 2}], {1, 1}}]
```

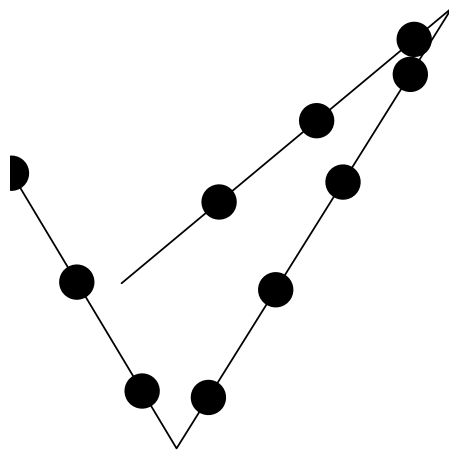


You may also specify our own arrow shapes:

```
>> Graphics[{Circle[], Arrowheads
[{{0.04, 1, Graphics[{Red, Disk
[]}]}}], Arrow[{0, 0}, {Cos[Pi
/3], Sin[Pi/3]}]}]
```



```
>> Graphics[{Arrowheads[Table
[{0.04, i/10, Graphics[Disk
[]]], {i, 1, 10}]], Arrow[{0, 0},
{6, 5}, {1, -3}, {-2, 2}]}]
```



Automatic

`Automatic`
is used to specify an automatically computed option value.

`Automatic` is the default for `PlotRange`, `ImageSize`, and other graphical options:

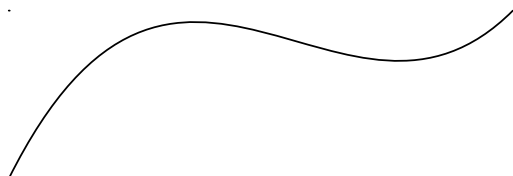
```
>> Cases[Options[Plot], HoldPattern
[_ :> Automatic]]
{Background:>Automatic,
 Exclusions:>Automatic,
 ImageSize:>Automatic,
 MaxRecursion:>Automatic,
 PlotRange:>Automatic,
 PlotRangePadding:>Automatic}
```

BernsteinBasis

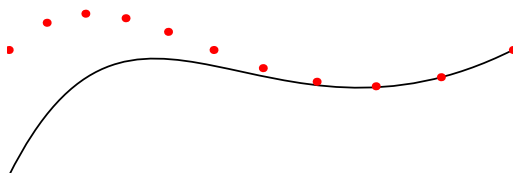
BezierCurve

`BezierCurve[{p1, p2 ...}]`
represents a bezier curve with $p1$, $p2$ as control points.

```
>> Graphics[BezierCurve[{{0, 0},{1,
1},{2, -1},{3, 0}}]]
```



```
>> Module[{p={{0, 0},{1, 1},{2,
-1},{4, 0}}}, Graphics[{
BezierCurve[p], Red, Point[Table
[BezierFunction[p][x], {x, 0, 1,
0.1}]]]]]
```



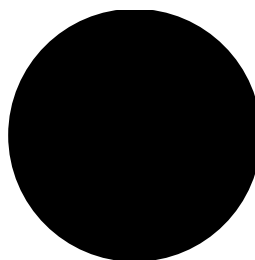
BezierCurveBox

BezierFunction

Black

`Black`
represents the color black in graphics.

```
>> Graphics[{EdgeForm[Black], Black
, Disk[]}, ImageSize->Small]
```



```
>> Black // ToBoxes
StyleBox [GraphicsBox [{EdgeForm [■], ■, RectangleBox [
- > Ignore, AspectRatio
- > Automatic, Axes
- > False, AxesStyle
- > {}, Background
- > Automatic, ImageSize
- > 16, LabelStyle- > {}, PlotRange
- > Automatic, PlotRangePadding
- > Automatic, TicksStyle
- > {}], ImageSizeMultipliers
- > {1,1}]]
```

```
>> Black
```



Blend

`Blend[{c1, c2}]`
represents the color between $c1$ and $c2$.
`Blend[{c1, c2}, x]`
represents the color formed by blending $c1$ and $c2$ with factors $1 - x$ and x respectively.
`Blend[{c1, c2, ..., cn}, x]`
blends between the colors $c1$ to cn according to the factor x .

```
>> Blend[{Red, Blue}]
```



```
>> Blend[{Red, Blue}, 0.3]
```



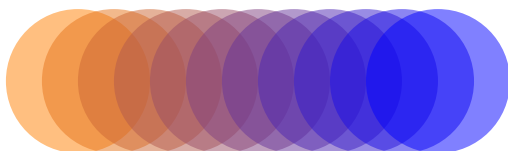
```
>> Blend[{Red, Blue, Green}, 0.75]
```



```
>> Graphics[Table[{Blend[{Red,
Green, Blue}, x], Rectangle[{10
x, 0}]], {x, 0, 1, 1/10}]]
```



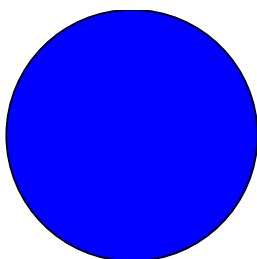
```
>> Graphics[Table[{Blend[{RGBColor
[1, 0.5, 0, 0.5], RGBColor[0, 0,
1, 0.5]}], x], Disk[{5x, 0}]], {
x, 0, 1, 1/10}]]
```



Blue

Blue
represents the color blue in graphics.

```
>> Graphics[{EdgeForm[Black], Blue,
Disk[]}, ImageSize->Small]
```



```
>> Blue // ToBoxes
StyleBox [GraphicsBox [{EdgeForm [■], ■, RectangleBox [{0,0}]}], $OptionSyntax
- > Ignore, AspectRatio
- > Automatic, Axes
- > False, AxesStyle
- > {}, Background
- > Automatic, ImageSize
- > 16, LabelStyle- > {}, PlotRange
- > Automatic, PlotRangePadding
- > Automatic, TicksStyle
- > {}, ImageSizeMultipliers
- > {1,1}]
```

```
>> Blue
■
```

CMYKColor

CMYKColor[*c*, *m*, *y*, *k*]
represents a color with the specified cyan, magenta, yellow and black components.

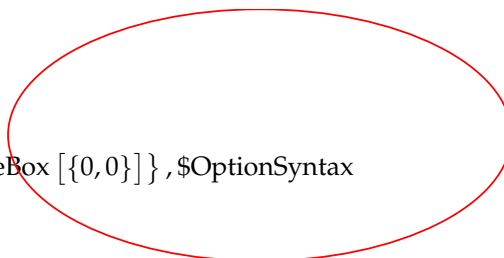
```
>> Graphics[MapIndexed[{CMYKColor
@@ #1, Disk[2*#2 ~Join~{0}]} &,
IdentityMatrix[4]], ImageSize->
Small]
```



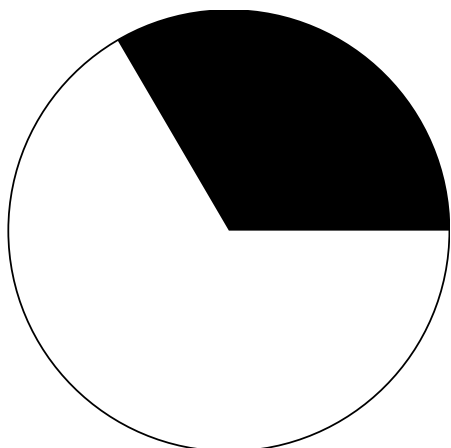
Circle

Circle[{*cx*, *cy*}, *r*]
draws a circle with center (*cx*, *cy*) and radius *r*.
Circle[{*cx*, *cy*}, {*rx*, *ry*}]
draws an ellipse.
Circle[{*cx*, *cy*}]
chooses radius 1.
Circle[]
chooses center (0, 0) and radius 1.

```
>> Graphics[{Red, Circle[{0, 0},
{2, 1}]]]
```



```
>> Graphics[{Circle[], Disk[{0, 0},
{1, 1}, {0, 2.1}]}]
```



CircleBox

ColorDistance

`ColorDistance[c1, c2]`
returns a measure of color distance between the colors *c1* and *c2*.

`ColorDistance[list, c2]`
returns a list of color distances between the colors in *list* and *c2*.

The option `DistanceFunction` specifies the method used to measure the color distance. Available options are:

CIE76: euclidean distance in the LABColor space
CIE94: euclidean distance in the LCH-Color space
CIE2000 or CIEDE2000: CIE94 distance with corrections
CMC: Colour Measurement Committee metric (1984)
DeltaL: difference in the L component of LCHColor
DeltaC: difference in the C component of LCHColor
DeltaH: difference in the H component of LCHColor

It is also possible to specify a custom distance

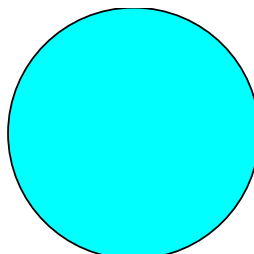
```
>> ColorDistance[Magenta, Green]
2.2507

>> ColorDistance[{Red, Blue}, {
Green, Yellow}, DistanceFunction
-> {"CMC", "Perceptibility"}]
{1.0495, 1.27455}
```

Cyan

Cyan
represents the color cyan in graphics.

```
>> Graphics[{EdgeForm[Black], Cyan,
Disk[]}, ImageSize->Small]
```



```
>> Cyan // ToBoxes
```

```
StyleBox[GraphicsBox[{EdgeForm[Black], Cyan, RectangleBox[
- > Ignore, AspectRatio
- > Automatic, Axes
- > False, AxesStyle
- > {}, Background
- > Automatic, ImageSize
- > 16, LabelStyle- > {}, PlotRange
- > Automatic, PlotRangePadding
- > Automatic, TicksStyle
- > {}], ImageSizeMultipliers
- > {1, 1}]]
```

```
>> Cyan
```



Darker

`Darker[c, f]`
is equivalent to `Blend[{c, Black}, f]`.

`Darker[c]`
is equivalent to `Darker[c, 1/3]`.

```
>> Graphics[Table[{Darker[Yellow, x
], Disk[{12x, 0}]}], {x, 0, 1,
1/6}]]
```

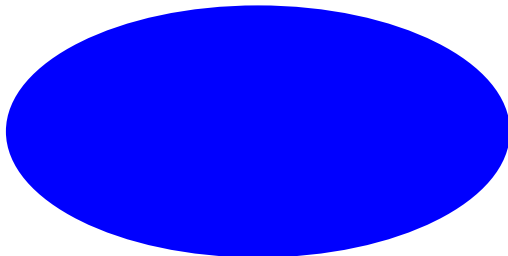


Directive

Disk

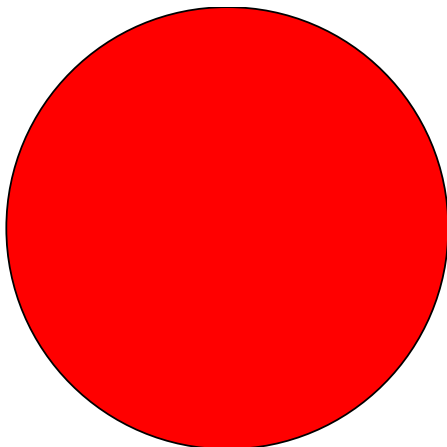
```
Disk[{cx, cy}, r]  
    fills a circle with center (cx, cy) and radius r.  
Disk[{cx, cy}, {rx, ry}]  
    fills an ellipse.  
Disk[{cx, cy}]  
    chooses radius 1.  
Disk[]  
    chooses center (0, 0) and radius 1.  
Disk[{x, y}, ..., {t1, t2}]  
    is a sector from angle t1 to t2.
```

```
>> Graphics[{Blue, Disk[{0, 0}, {2, 1}]}]
```



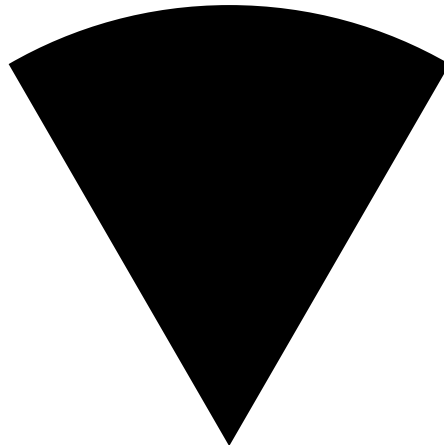
The outer border can be drawn using EdgeForm:

```
>> Graphics[{EdgeForm[Black], Red, Disk[]}]
```

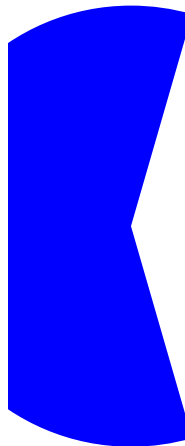


Disk can also draw sectors of circles and ellipses

```
>> Graphics[Disk[{0, 0}, 1, {Pi / 3, 2 Pi / 3}]]
```



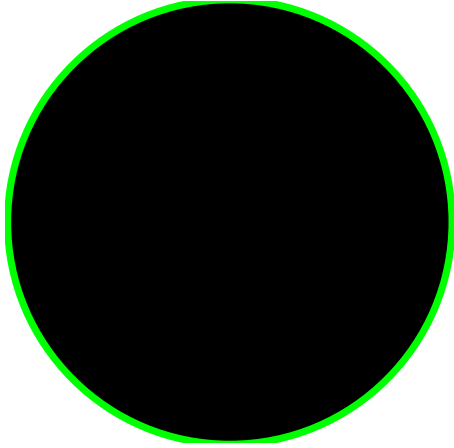
```
>> Graphics[{Blue, Disk[{0, 0}, {1, 2}, {Pi / 3, 5 Pi / 3}]]]
```



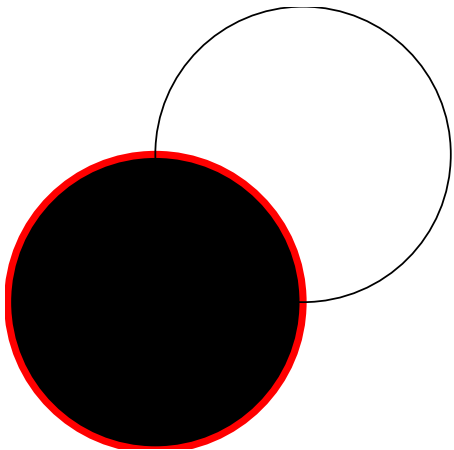
DiskBox

EdgeForm

```
>> Graphics[{EdgeForm[{Thick, Green}], Disk[]}]
```



```
>> Graphics[{Style[Disk[], EdgeForm[{Thick, Red}]], Circle[{1, 1}]}]
```



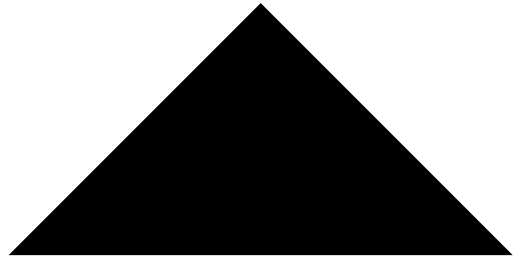
FaceForm

FilledCurve

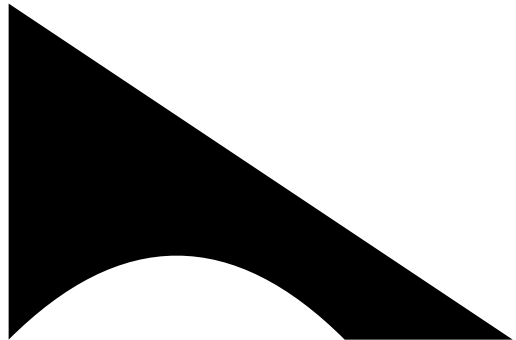
```
FilledCurve[{segment1, segment2 ...}]
```

represents a filled curve.

```
>> Graphics[FilledCurve[{Line[{0, 0}, {1, 1}, {2, 0}]}]]
```



```
>> Graphics[FilledCurve[{BezierCurve[{0, 0}, {1, 1}, {2, 0}], Line[{3, 0}, {0, 2}]}]]
```



FilledCurveBox

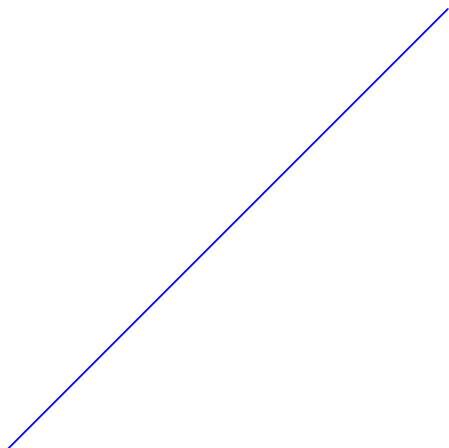
FontColor

FontColor
is an option for Style to set the font color.

Graphics

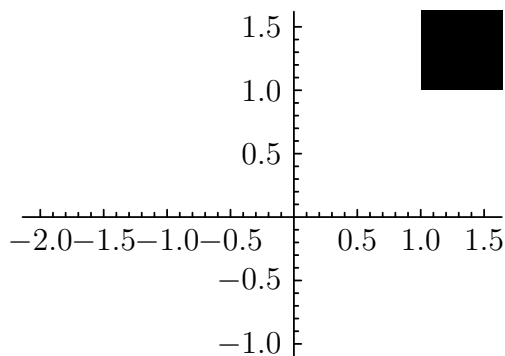
Graphics[primitives, options]
represents a graphic.

```
>> Graphics[{Blue, Line[{{0,0},
{1,1}}]}]
```



Graphics supports PlotRange:

```
>> Graphics[{Rectangle[{1, 1}]},
Axes -> True, PlotRange -> {{-2,
1.5}, {-1, 1.5}}]
```



```
>> Graphics[{Rectangle[], Red, Disk
[{1,0}], PlotRange
->{{0,1},{0,1}}]
```



Graphics produces GraphicsBox boxes:

```
>> Graphics[Rectangle[]] // ToBoxes
// Head
GraphicsBox
```

In TeXForm, Graphics produces Asymptote figures:

```
>> Graphics[Circle[]] // TeXForm

\begin{asy}
usepackage("amsmath");
size(5.8556cm, 5.8333cm);
draw(ellipse((175,175),175,175),
rgb(0, 0, 0)+linewidth(0.66667));
clip(box((-0.33333,0.33333),
(350.33,349.67)));
\end{asy}
```

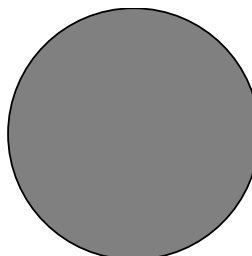
GraphicsBox

Gray

Gray

represents the color gray in graphics.

```
>> Graphics[{EdgeForm[Black], Gray,
Disk[]], ImageSize->Small]
```



```
>> Gray // ToBoxes
```

```
StyleBox [GraphicsBox [{EdgeForm [■], ■, RectangleBox [
- > Ignore, AspectRatio
- > Automatic, Axes
- > False, AxesStyle
- > {}, Background
- > Automatic, ImageSize
- > 16, LabelStyle- > {}, PlotRange
- > Automatic, PlotRangePadding
- > Automatic, TicksStyle
- > {}], ImageSizeMultipliers
- > {1,1}]
```

>> Gray



GrayLevel

GrayLevel[g]

represents a shade of gray specified by g , ranging from 0 (black) to 1 (white).

GrayLevel[g, a]

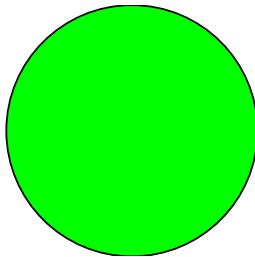
represents a shade of gray specified by g with opacity a .

Green



Green

represents the color green in graphics.

>> Graphics[{EdgeForm[Black], Green, Disk[]}, ImageSize->Small]



>> Green // ToBoxes

StyleBox[GraphicsBox[{EdgeForm[], }, RectangleBox[{0, 0}], \$OptionSyntax

- > Ignore, AspectRatio

- > Automatic, Axes

- > False, AxesStyle

- > {}, Background

- > Automatic, ImageSize

- > 16, LabelStyle- > {}, PlotRange

- > Automatic, PlotRangePadding

- > Automatic, TicksStyle

- > {}, ImageSizeMultipliers

- > {1, 1}]

>> Green



Hue

Hue[h, s, l, a]

represents the color with hue h , saturation s , lightness l and opacity a .

Hue[h, s, l]

is equivalent to Hue[h, s, l, 1].

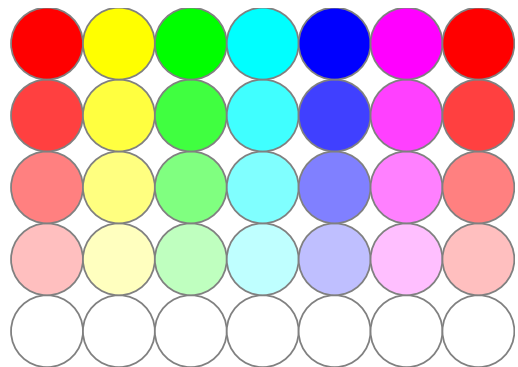
Hue[h, s]

is equivalent to Hue[h, s, 1, 1].

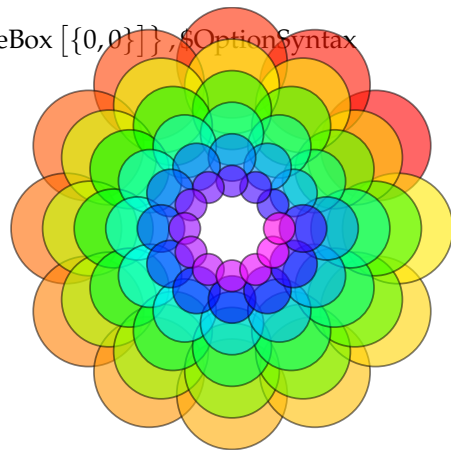
Hue[h]

is equivalent to Hue[h, 1, 1, 1].

>> Graphics[Table[{EdgeForm[Gray], Hue[h, s], Disk[{12h, 8s}]}, {h, 0, 1, 1/6}, {s, 0, 1, 1/4}]]



>> Graphics[Table[{EdgeForm[{GrayLevel[0, 0.5]}], Hue[(-11+q+10r)/72, 1, 1, 0.6], Disk[(8-r){Cos[2Pi q/12], Sin[2Pi q/12]}], (8-r)/3}], {r, 6}, {q, 12}]]



Inset

InsetBox

LABColor

`LABColor[l, a, b]`

represents a color with the specified lightness, red/green and yellow/blue components in the CIE 1976 L*a*b* (CIELAB) color space.

LCHColor

`LCHColor[l, c, h]`

represents a color with the specified lightness, chroma and hue components in the CIE LCh CIE Lab cube color space.

LUVColor

`LCHColor[l, u, v]`

represents a color with the specified components in the CIE 1976 L*u*v* (CIELUV) color space.

Large

`ImageSize -> Large`

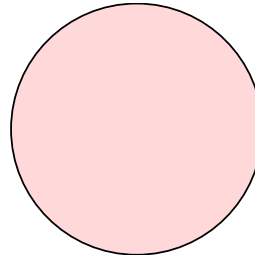
produces a large image.

LightRed

`LightRed`

represents the color light red in graphics.

```
>> Graphics[{EdgeForm[Black],  
LightRed, Disk[]}, ImageSize->  
Small]
```



```
>> LightRed // ToBoxes
```

```
StyleBox [GraphicsBox [{EdgeForm [Black], LightRed, RectangleBox [0, 0, 1, 1]},  
- > Ignore, AspectRatio  
- > Automatic, Axes  
- > False, AxesStyle  
- > {}, Background  
- > Automatic, ImageSize  
- > 16, LabelStyle - > {}, PlotRange  
- > Automatic, PlotRangePadding  
- > Automatic, TicksStyle  
- > {}], ImageSizeMultipliers  
- > {1, 1}]
```

Lighter

`Lighter[c, f]`

is equivalent to `Blend[{c, White}, f]`.

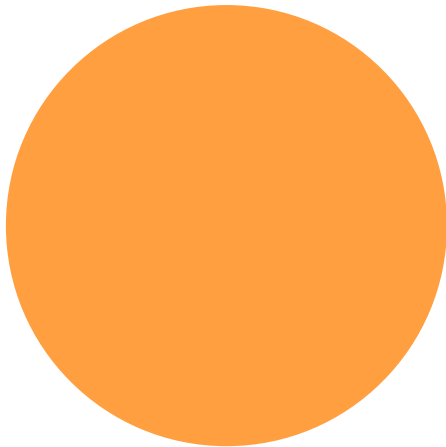
`Lighter[c]`

is equivalent to `Lighter[c, 1/3]`.

```
>> Lighter[Orange, 1/4]
```



```
>> Graphics[{Lighter[Orange, 1/4],
Disk[]}]
```



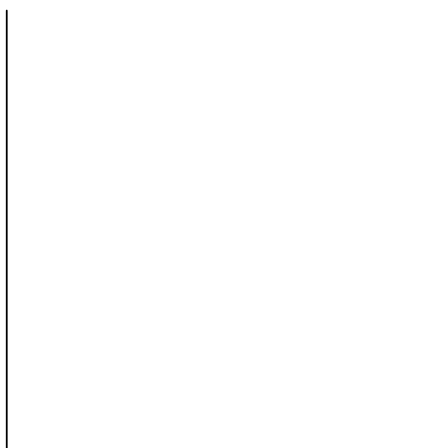
```
>> Graphics[Table[{Lighter[Orange,
x], Disk[{12x, 0}]}], {x, 0, 1,
1/6}]]
```



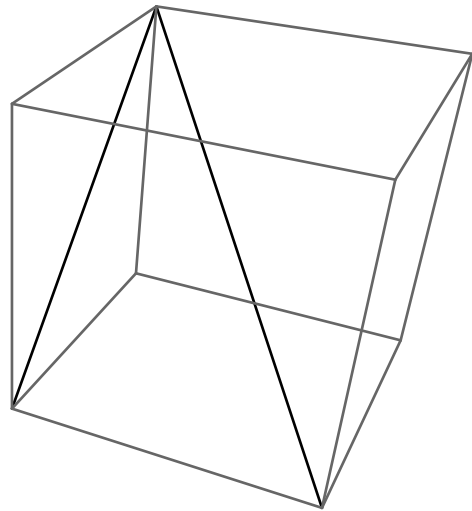
Line

`Line[{point_1, point_2 ...}]`
represents the line primitive.
`Line[{p_11, p_12, ...}, {p_21, p_22,
...}, ...]`
represents a number of line primitives.

```
>> Graphics[Line
[{{0,1},{0,0},{1,0},{1,1}}]]
```



```
>> Graphics3D[Line
[{{0,0,0},{0,1,1},{1,0,0}}]]
```

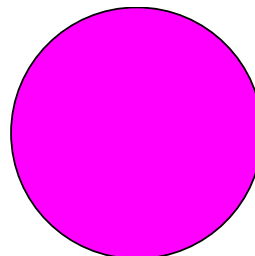


LineBox

Magenta

`Magenta`
represents the color magenta in graphics.

```
>> Graphics[{EdgeForm[Black],
Magenta, Disk[]}, ImageSize->
Small]
```



```
>> Magenta // ToBoxes
```

```
StyleBox [GraphicsBox [{EdgeForm [Black], RectangleBox[80,0], GraphicsBox[{EdgeForm [Magenta], RectangleBox[80,0]}], AspectRatio->Ignore, Axes->Automatic, AxesStyle->False, Background->{ }, ImageSize->Automatic, LabelStyle->16, PlotRange->{ }, PlotRangePadding->Automatic, TicksStyle->Automatic, ImageSizeMultipliers->{1,1}]
```

```
>> Magenta
```



Medium

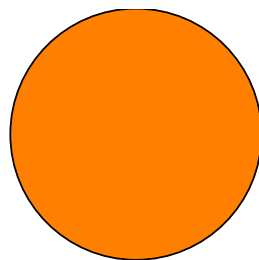
ImageSize -> Medium
produces a medium-sized image.

Offset

Orange

Orange
represents the color orange in graphics.

```
>> Graphics[{EdgeForm[Black],  
Orange, Disk[]}, ImageSize->  
Small]
```



```
>> Orange // ToBoxes
```

```
StyleBox [GraphicsBox [{EdgeForm [Black], RectangleBox[80,0], GraphicsBox[{EdgeForm [Orange], RectangleBox[80,0]}], AspectRatio->Ignore, Axes->Automatic, AxesStyle->False, Background->{ }, ImageSize->Automatic, LabelStyle->16, PlotRange->{ }, PlotRangePadding->Automatic, TicksStyle->Automatic, ImageSizeMultipliers->{1,1}]
```

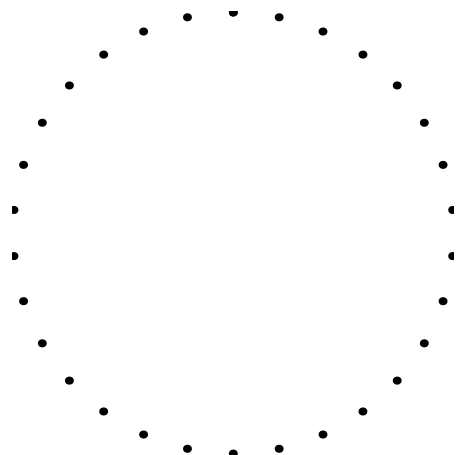
Point

Point[{*point_1*, *point_2* ...}]
represents the point primitive.
Point[{{*p_11*, *p_12*, ...}, {*p_21*, *p_22*, ...}, ...}]
represents a number of point primitives.

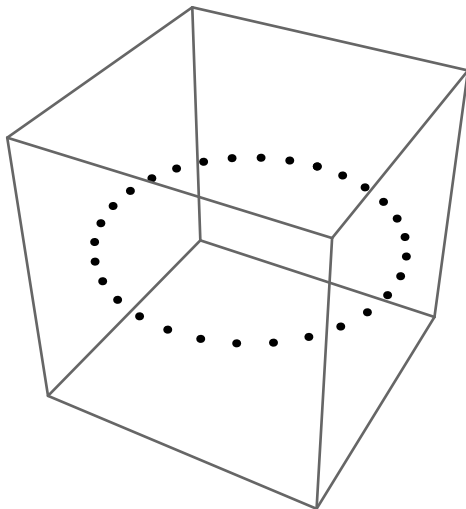
```
>> Graphics[Point[{0,0}]]
```

.

```
>> Graphics[Point[Table[{Sin[t],  
Cos[t]}, {t, 0, 2. Pi, Pi /  
15.}]]]
```



```
>> Graphics3D[Point[Table[{Sin[t],  
Cos[t], 0}, {t, 0, 2. Pi, Pi /  
15.}]]]
```



PointBox

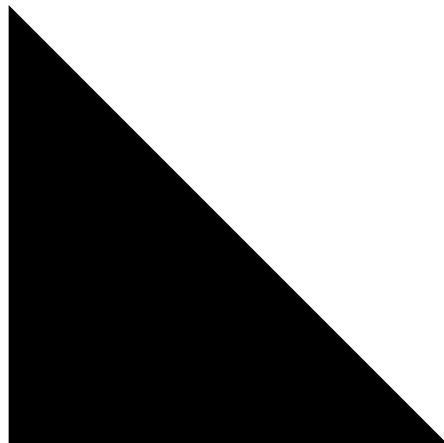
PointSize

`PointSize[t]`
sets the diameter of points to t , which is relative to the overall width.

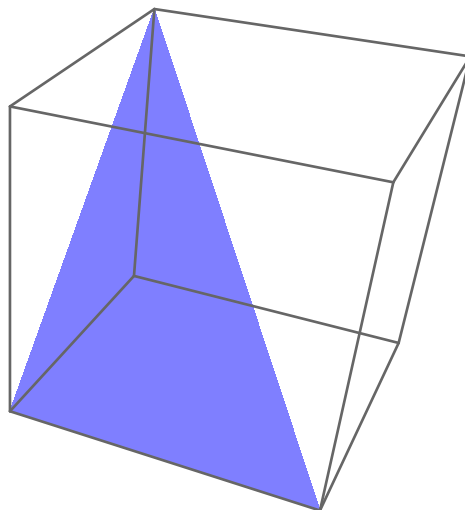
Polygon

`Polygon[{point_1, point_2 ...}]`
represents the filled polygon primitive.
`Polygon[{{p_11, p_12, ...}, {p_21, p_22, ...}, ...}]`
represents a number of filled polygon primitives.

```
>> Graphics[Polygon  
[{{1,0},{0,0},{0,1}}]]
```



```
>> Graphics3D[Polygon  
[{{0,0,0},{0,1,1},{1,0,0}}]]
```

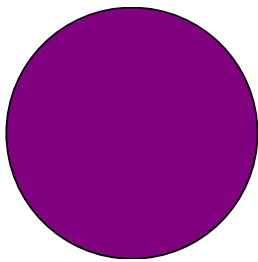


PolygonBox

Purple

`Purple`
represents the color purple in graphics.


```
>> Graphics[{EdgeForm[Black],
Purple, Disk[]}, ImageSize->
Small]
```

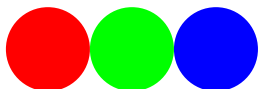


```
>> Purple // ToBoxes
StyleBox [GraphicsBox [{EdgeForm [■], ■, RectangleBox [
- > Ignore, AspectRatio
- > Automatic, Axes
- > False, AxesStyle
- > {}, Background
- > Automatic, ImageSize
- > 16, LabelStyle- > {}, PlotRange
- > Automatic, PlotRangePadding
- > Automatic, TicksStyle
- > {}], ImageSizeMultipliers
- > {1, 1}]]
```

RGBColor

`RGBColor[r, g, b]`
represents a color with the specified red, green and blue components.

```
>> Graphics[MapIndexed[{RGBColor @@
#1, Disk[2*#2 ~Join~{0}]} &,
IdentityMatrix[3]], ImageSize->
Small]
```



```
>> RGBColor[0, 1, 0]
■
```

```
>> RGBColor[0, 1, 0] // ToBoxes
```

```
StyleBox [GraphicsBox [{EdgeForm [■], ■, RectangleBox [
- > Ignore, AspectRatio
- > Automatic, Axes
- > False, AxesStyle
- > {}, Background
- > Automatic, ImageSize
- > 16, LabelStyle- > {}, PlotRange
- > Automatic, PlotRangePadding
- > Automatic, TicksStyle
- > {}], ImageSizeMultipliers
- > {1, 1}]]
```

Rectangle

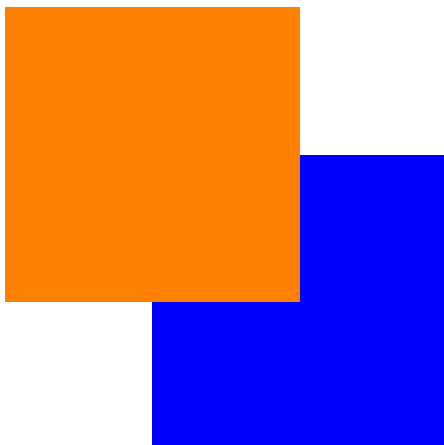
`Rectangle[{xmin, ymin}]`
represents a unit square with bottom-left corner at {xmin, ymin}.

`Rectangle[{xmin, ymin}, {xmax, ymax}]`
is a rectangle extending from {xmin, ymin} to {xmax, ymax}.

```
>> Graphics[Rectangle[]]
```



```
>> Graphics[{Blue, Rectangle[{0.5, 0}], Orange, Rectangle[{0, 0.5}]}]
```



```
>> Red
```



RegularPolygon

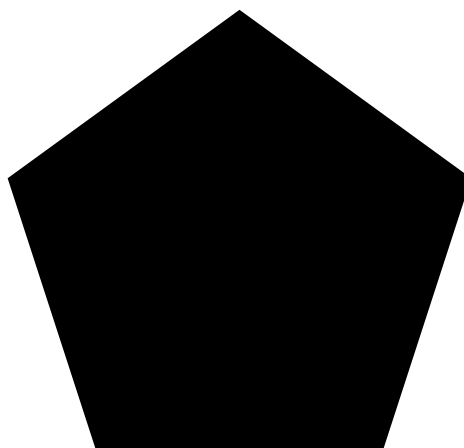
`RegularPolygon[n]`
gives the regular polygon with n edges.

`RegularPolygon[r, n]`
gives the regular polygon with n edges and radius r .

`RegularPolygon[{r, phi}, n]`
gives the regular polygon with radius r with one vertex drawn at angle phi .

`RegularPolygon[{x, y}, r, n]`
gives the regular polygon centered at the position $\{x, y\}$.

```
>> Graphics[RegularPolygon[5]]
```

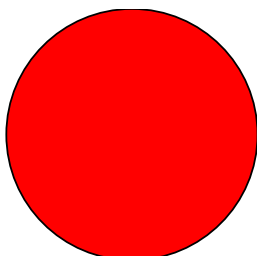


RectangleBox

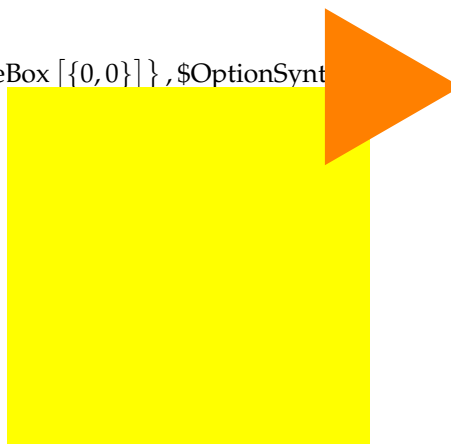
Red

`Red`
represents the color red in graphics.

```
>> Graphics[{EdgeForm[Black], Red, Disk[]}, ImageSize->Small]
```



```
>> Graphics[{Yellow, Rectangle[], Orange, RegularPolygon[{1, 1}, {0.25, 0}, 3]}]
```



```
>> Red // ToBoxes
```

```
StyleBox [GraphicsBox [{EdgeForm [Black], Red, RectangleBox [{0, 0}]}], $OptionSyntax
- > Ignore, AspectRatio
- > Automatic, Axes
- > False, AxesStyle
- > {}, Background
- > Automatic, ImageSize
- > 16, LabelStyle- > {}, PlotRange
- > Automatic, PlotRangePadding
- > Automatic, TicksStyle
- > {}], ImageSizeMultipliers
- > {1, 1}]
```

RegularPolygonBox

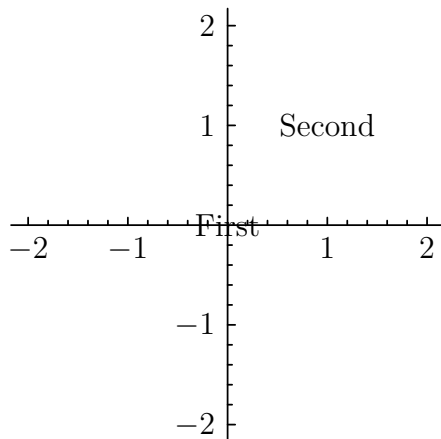
Small

ImageSize -> Small
produces a small image.

Text

Text["text", {x, y}]
draws *text* centered on position {x, y}.

```
>> Graphics[{Text["First", {0, 0}],  
Text["Second", {1, 1}], Axes->  
True, PlotRange->{{-2, 2}, {-2,  
2}}]
```



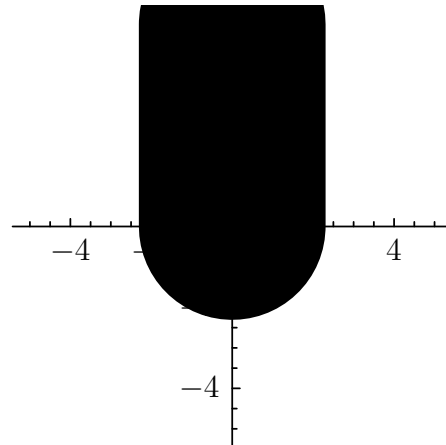
Thick

Thick
sets the line width for subsequent graphics primitives to 2pt.

Thickness

Thickness[t]
sets the line thickness for subsequent graphics primitives to *t* times the size of the plot area.

```
>> Graphics[{Thickness[0.2], Line  
[{{0, 0}, {0, 5}}]}, Axes->True,  
PlotRange->{{-5, 5}, {-5, 5}}]
```



Thin

Thin
sets the line width for subsequent graphics primitives to 0.5pt.

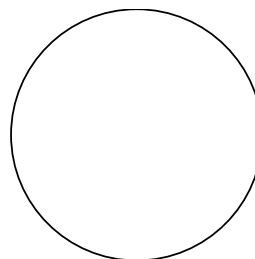
Tiny

ImageSize -> Tiny
produces a tiny image.

White

White
represents the color white in graphics.

```
>> Graphics[{EdgeForm[Black], White  
, Disk[]}, ImageSize->Small]
```



```

>> White // ToBoxes
StyleBox [GraphicsBox [{EdgeForm [Black], RectangleBox[{0,0},{1,1}], PlotRange->Automatic, AspectRatio->False, AxesStyle->Automatic, Axes->False, Background->Automatic, ImageSize->Automatic, LabelStyle->Automatic, PlotRangePadding->Automatic, TicksStyle->Automatic, ImageSizeMultipliers->{1,1}}], ImageSizeMultipliers->{1,1}]

>> Yellow // ToBoxes
StyleBox [GraphicsBox [{EdgeForm [Black], RectangleBox[{0,0},{1,1}], PlotRange->Automatic, AspectRatio->False, AxesStyle->Automatic, Axes->False, Background->Automatic, ImageSize->Automatic, LabelStyle->Automatic, PlotRangePadding->Automatic, TicksStyle->Automatic, ImageSizeMultipliers->{1,1}}], ImageSizeMultipliers->{1,1}]

>> White
□

>> Yellow
■

```

XYZColor

`XYZColor[x, y, z]`
 represents a color with the specified components in the CIE 1931 XYZ color space.

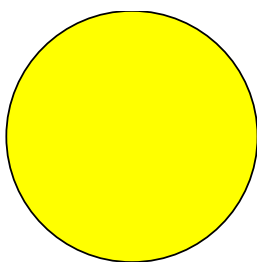
Yellow

`Yellow`
 represents the color yellow in graphics.

```

>> Graphics[{EdgeForm[Black],
Yellow, Disk[]}, ImageSize->Small]

```



XVI. Graphics (3D)

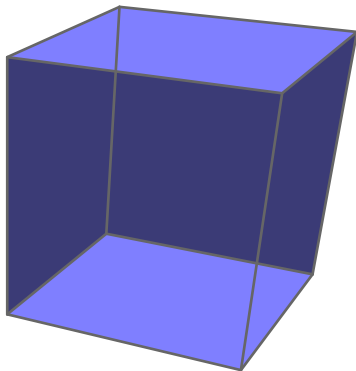
Contents

Cuboid	118	Line3DBox	119	Sphere	120
Graphics3D	119	Point3DBox	119	Sphere3DBox	120
Graphics3DBox	119	Polygon3DBox	119		

Cuboid

`Cuboid[{xmin, ymin, zmin}]`
is a unit cube.
`Cuboid[{xmin, ymin, zmin}, {xmax,
ymax, zmax}]`
represents a cuboid extending from
{*xmin*, *ymin*, *zmin*} to {*xmax*, *y**max*, *z**max*}.

```
>> Graphics3D[Cuboid[{0, 0, 1}]]
```



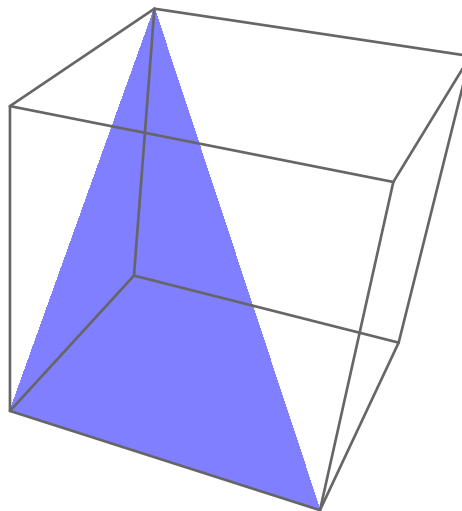
```
>> Graphics3D[{Red, Cuboid[{0, 0,
0}, {1, 1, 0.5}], Blue, Cuboid
[{0.25, 0.25, 0.5}, {0.75, 0.75,
1}]]}
```

```
Graphics3DBox[List[StyleBox[Graphics[List[Edge
RGBColor[1, 0, 0], Rectangle[List[0,
0]]], Rule[ImageSize, 16]],
Rule[ImageSizeMultipliers, List[1,
1]]], Polygon3DBox[List[List[List[0.,
0., 0.], List[0., 1., 0.], List[0., 1., 0.5]],
List[List[0., 0., 0.], List[0., 0., 0.5],
List[0., 1., 0.5]], List[List[1., 0.,
0.], List[1., 1., 0.], List[1., 1., 0.5]],
List[List[1., 0., 0.], List[1., 0., 0.5],
List[1., 1., 0.5]], List[List[0., 0.,
0.], List[1., 0., 0.], List[1., 0., 0.5]],
List[List[0., 0., 0.], List[0., 0., 0.5],
List[1., 0., 0.5]], List[List[0., 1.,
0.], List[1., 1., 0.], List[1., 1., 0.5]],
List[List[0., 1., 0.], List[0., 1., 0.5],
List[1., 1., 0.5]], List[List[0., 0.,
0.], List[0., 1., 0.], List[1., 1., 0.]],
List[List[0., 0., 0.], List[1., 0., 0.], List[1.,
1., 0.]], List[List[0., 0., 0.5], List[0., 1.,
0.5], List[1., 1., 0.5]], List[List[0., 0.,
0.5], List[1., 0., 0.5], List[1., 1., 0.5]]],
StyleBox[Graphics[List[EdgeForm[GrayLevel[0]],
RGBColor[0, 0, 1], Rectangle[List[0,
0]]], Rule[ImageSize, 16]],
Rule[ImageSizeMultipliers, List[1, 1]]],
Polygon3DBox[List[List[List[0.25, 0.25
, 0.5], List[0.25, 0.75, 0.5], List[0.25, 0.75
, 1.]], List[List[0.25, 0.25, 0.5], List[0.25,
0.25, 1.], List[0.25, 0.75, 1.]], List[List[
0.75, 0.25, 0.5], List[0.75, 0.75, 0.5], List[
0.75, 0.75, 1.]], List[List[0.75, 0.25, 0.5],
List[0.75, 0.25, 1.], List[0.75, 0.75, 1.]],
List[List[0.25, 0.25, 0.5], List[0.75, 0.25,
0.5], List[0.75, 0.25, 1.]], List[List[0.25,
0.25, 0.5], List[0.25, 0.25, 1.], List[0.75,
0.25, 1.]], List[List[0.25, 0.75, 0.5], List[
0.75, 0.75, 0.5], List[0.75, 0.75, 1.]],
List[List[0.25, 0.75, 0.5], List[0.25, 0.75,
1.], List[0.75, 0.75, 1.]], List[List[0.25
, 0.25, 0.5], List[0.25, 0.75, 0.5], List[
0.75, 0.75, 0.5]], List[List[0.25, 0.25,
0.5], List[0.75, 0.25, 0.5], List[0.75, 0.75,
0.5]], List[List[0.25, 0.25, 1.], List[0.25,
0.75, 1.], List[0.75, 0.75, 1.]], List[List[
0.25, 0.25, 1.], List[0.75, 0.25, 1.], List[
0.75, 0.75, 1.]]], Rule[$OptionSyntax,
Ignore], Rule[AspectRatio, Automatic],
Rule[Axes, False], Rule[AxesStyle,
List[]], Rule[Background, Automatic],
Rule[BoxRatios, Automatic],
Rule[ImageSize, Automatic]
```

Graphics3D

`Graphics3D[primitives, options]`
represents a three-dimensional graphic.

```
>> Graphics3D[Polygon[{0,0,0},
{0,1,1}, {1,0,0}]]
```



In TeXForm, Graphics3D creates Asymptote figures:

```
>> Graphics3D[Sphere[]] // TeXForm

\begin{asy}
import three;
import solids;
size(6.6667cm, 6.6667cm);
currentprojection=perspective(2.6,-4.8,4.0);
currentlight=light(rgb(0.5,0.5,1),
specular=red, (2,0,2), (2,2,2), (0,2,2));
draw(surface(sphere((0, 0, 0), 1)),
rgb(1,1,1));
draw((( -1,-1,-1)-(1,-1,-1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw((( -1,1,-1)-(1,1,-1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw((( -1,-1,1)-(1,-1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw((( -1,1,1)-(1,1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw((( -1,-1,-1)-(-1,1,-1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw((( -1,-1,-1)-(-1,-1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw((( -1,-1,-1)-(-1,1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw((( -1,-1,-1)-(-1,-1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw((( -1,1,-1)-(-1,1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw((( 1,-1,-1)-(1,1,-1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
\end{asy}
```

Graphics3DBox

Line3DBox

Point3DBox

Polygon3DBox

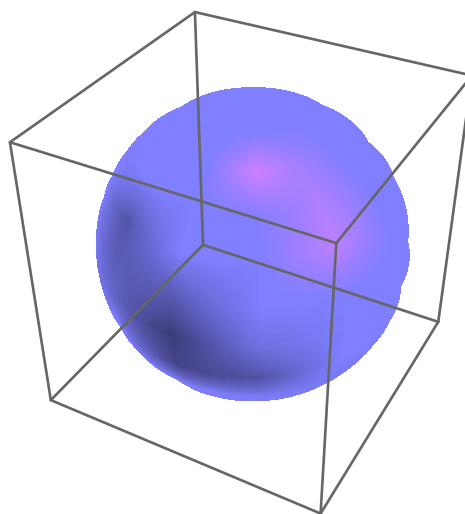
Sphere

`Sphere[{x, y, z}]`
is a sphere of radius 1 centered at the point {x, y, z}.

`Sphere[{x, y, z}, r]`
is a sphere of radius r centered at the point {x, y, z}.

`Sphere[{ {x1, y1, z1}, {x2, y2, z2}, ... }, r]`
is a collection spheres of radius r centered at the points {x1, y2, z2}, {x2, y2, z2}, ...

```
>> Graphics3D[Sphere[{0, 0, 0}, 1]]
```



```
>> Graphics3D[{{Yellow, Sphere[{{-1,
  0, 0}, {1, 0, 0}, {0, 0, Sqrt
[3.]}}, 1]]}

Graphics3DBox[List[StyleBox[Graphics[List[EdgeForm[GrayLevel[0]],
RGBColor[1, 1, 0], Rectangle[List[0,
0]]], Rule[ImageSize, 16]],
Rule[ImageSizeMultipliers, List[1,
1]]], Sphere3DBox[List[List[-1, 0, 0],
List[1, 0, 0], List[0, 0, 1.732050807~
~5688772]], 1]], Rule[$OptionSyntax,
Ignore], Rule[AspectRatio, Automatic],
Rule[Axes, False], Rule[AxesStyle,
List[]], Rule[Background, Automatic],
Rule[BoxRatios, Automatic],
Rule[ImageSize, Automatic],
Rule[LabelStyle, List[]], Rule[Lighting,
Automatic], Rule[PlotRange,
Automatic], Rule[PlotRangePadding,
Automatic], Rule[TicksStyle,
List[]], Rule[ViewPoint, List[1.3,
- 2.4, 2.]]]isnotavalidboxstructure.
```

Sphere3DBox

XVII. Image[] and image related functions.

Note that you (currently) need scikit-image installed in order for this module to work.

Contents

Binarize	121	ImageAdd	125	ImageSubtract	128
BinaryImageQ	121	ImageAdjust	125	ImageTake	128
Blur	122	ImageAspectRatio	125	ImageType	128
BoxMatrix	122	Image	125	MaxFilter	128
Closing	122	ImageBox	125	MedianFilter	128
ColorCombine	122	ImageChannels	125	MinFilter	128
ColorConvert	122	ImageColorSpace	125	MorphologicalCompo- nents	129
ColorNegate	122	ImageConvolve	125	Opening	129
ColorQuantize	122	ImageData	126	PillowImageFilter	129
ColorSeparate	122	ImageDimensions	126	PixelValue	129
Colorize	123	ImageExport	126	PixelValuePositions	129
DiamondMatrix	123	ImageImport	126	RandomImage	129
Dilation	123	ImageMultiply	126	Sharpen	129
DiskMatrix	123	ImagePartition	127	TextRecognize	130
DominantColors	124	ImageQ	127	Threshold	130
EdgeDetect	124	ImageReflect	127	WordCloud	130
Erosion	124	ImageResize	127		
GaussianFilter	124	ImageRotate	128		

Binarize

`Binarize[image]`
gives a binarized version of *image*, in which each pixel is either 0 or 1.

`Binarize[image, t]`
map values $x > t$ to 1, and values $x \leq t$ to 0.

`Binarize[image, {t1, t2}]`
map $t1 < x < t2$ to 1, and all other values to 0.

```
>> img = Import["ExampleData/lena.tif"];
>> Binarize[img]
-Image-
>> Binarize[img, 0.7]
-Image-
```

```
>> Binarize[img, {0.2, 0.6}]
-Image-
```

BinaryImageQ

`BinaryImageQ[$image]`
returns True if the pixels of *\$image* are binary bit values, and False otherwise.

```
>> img = Import["ExampleData/lena.tif"];
>> BinaryImageQ[img]
False
>> BinaryImageQ[Binarize[img]]
True
```

Blur

`Blur[image]`
gives a blurred version of *image*.
`Blur[image, r]`
blurs *image* with a kernel of size *r*.

```
>> lena = Import["ExampleData/lena.tif"];  
  
>> Blur[lena]  
-Image-  
  
>> Blur[lena, 5]  
-Image-
```

BoxMatrix

`BoxMatrix[$s]`
Gives a box shaped kernel of size $2s + 1$.

```
>> BoxMatrix[3]  
{{1,1,1,1,1,1,1}, {1,1,1,1,1,1,1},  
 1,1}, {1,1,1,1,1,1,1}, {1,1,1,  
 1,1,1,1}, {1,1,1,1,1,1,1}, {1,  
 1,1,1,1,1,1}, {1,1,1,1,1,1,1}}
```

Closing

`Closing[image, ker]`
Gives the morphological closing of *image* with respect to structuring element *ker*.

```
>> ein = Import["ExampleData/Einstein.jpg"];  
  
>> Closing[ein, 2.5]  
-Image-
```

ColorCombine

`ColorCombine[channels, colorspace]`
Gives an image with *colorspace* and the respective components described by the given channels.

```
>> ColorCombine[{{1, 0}, {0,  
 0.75}}, {{0, 1}, {0, 0.25}},  
 {{0, 0}, {1, 0.5}}}, "RGB"]  
-Image-
```

ColorConvert

`ColorConvert[c, colorspace]`
returns the representation of *c* in the color space *colorspace*. *c* may be a color or an image.

Valid values for *colorspace* are:

CMYK: convert to CMYKColor Grayscale: convert to GrayLevel HSB: convert to Hue LAB: convert to LABColor LCH: convert to LCHColor LUV: convert to LUVColor RGB: convert to RGBColor XYZ: convert to XYZColor

ColorNegate

`ColorNegate[image]`
Gives a version of *image* with all colors negated.

ColorQuantize

`ColorQuantize[image, n]`
gives a version of *image* using only *n* colors.

```
>> img = Import["ExampleData/lena.tif"];  
  
>> ColorQuantize[img, 6]  
-Image-
```

ColorSeparate

`ColorSeparate[image]`
Gives each channel of *image* as a separate grayscale image.

Colorize

`Colorize[values]`

returns an image where each number in the rectangular matrix *values* is a pixel and each occurrence of the same number is displayed in the same unique color, which is different from the colors of all non-identical numbers.

`Colorize[image]`

gives a colorized version of *image*.

```
>> Colorize[{{1.3, 2.1, 1.5}, {1.3,
  1.3, 2.1}, {1.3, 2.1, 1.5}}]
-Image-

>> Colorize[{{1, 2}, {2, 2}, {2,
  3}}, ColorFunction -> (Blend[{
  White, Blue}, #]&)]
-Image-
```

DiamondMatrix

`DiamondMatrix[$s]`

Gives a diamond shaped kernel of size $2s + 1$.

```
>> DiamondMatrix[3]
{{0,0,0,1,0,0,0}, {0,0,1,1,1,
  0,0}, {0,1,1,1,1,1,0}, {1,1,1,
  1,1,1,1}, {0,1,1,1,1,1,0}, {0,
  0,1,1,1,0,0}, {0,0,0,1,0,0,0}}
```

Dilation

`Dilation[image, ker]`

Gives the morphological dilation of *image* with respect to structuring element *ker*.

```
>> ein = Import["ExampleData/
  Einstein.jpg"];

>> Dilation[ein, 2.5]
-Image-
```

DiskMatrix

`DiskMatrix[$s]`

Gives a disk shaped kernel of size $2s + 1$.

```
>> DiskMatrix[3]
{{0,0,1,1,1,0,0}, {0,1,1,1,1,
  1,0}, {1,1,1,1,1,1,1}, {1,1,1,
  1,1,1,1}, {1,1,1,1,1,1,1}, {0,
  1,1,1,1,1,0}, {0,0,1,1,1,0,0}}
```

DominantColors

`DominantColors[image]`

gives a list of colors which are dominant in the given image.

`DominantColors[image, n]`

returns at most *n* colors.

`DominantColors[image, n, prop]`

returns the given property *prop*, which may be "Color" (return RGB colors), "LABColor" (return LAB colors), "Count" (return the number of pixels a dominant color covers), "Coverage" (return the fraction of the image a dominant color covers), or "CoverageImage" (return a black and white image indicating with white the parts that are covered by a dominant color).

The option "ColorCoverage" specifies the minimum amount of coverage needed to include a dominant color in the result.

The option "MinColorDistance" specifies the distance (in LAB color space) up to which colors are merged and thus regarded as belonging to the same dominant color.

```
>> img = Import["ExampleData/lena.
  tif"]
-Image-

>> DominantColors[img]
{{Red, Red, Blue, Red, Red, Red}}



>> DominantColors[img, 3]
{{Red, Red, Blue}}
```



```
>> DominantColors[img, 3, "Coverage
"]




$$\left\{ \frac{28\,579}{131\,072}, \frac{751}{4\,096}, \frac{23\,841}{131\,072} \right\}$$


>> DominantColors[img, 3, "
CoverageImage"]
{-Image-, -Image-, -Image-}

>> DominantColors[img, 3, "Count"]
{57 158, 48 064, 47 682}

>> DominantColors[img, 2, "LABColor
"]
{, }

>> DominantColors[img,
MinColorDistance -> 0.5]
{, }

>> DominantColors[img,
ColorCoverage -> 0.15]
{, , }
```

EdgeDetect

`EdgeDetect[image]`
returns an image showing the edges in *image*.

```
>> lena = Import["ExampleData/lena.
.tif"];

>> EdgeDetect[lena]
-Image-

>> EdgeDetect[lena, 5]
-Image-

>> EdgeDetect[lena, 4, 0.5]
-Image-
```

Erosion

`Erosion[image, ker]`
Gives the morphological erosion of *image* with respect to structuring element *ker*.

```
>> ein = Import["ExampleData/
Einstein.jpg"];

>> Erosion[ein, 2.5]
-Image-
```

GaussianFilter

`GaussianFilter[image, r]`
blurs *image* using a Gaussian blur filter of radius *r*.

```
>> lena = Import["ExampleData/lena.
.tif"];

>> GaussianFilter[lena, 2.5]
-Image-
```

ImageAdd

`ImageAdd[image, expr_1, expr_2, ...]`
adds all *expr_i* to *image* where each *expr_i* must be an image or a real number.

```
>> i = Image[{{0, 0.5, 0.2, 0.1,
0.9}, {1.0, 0.1, 0.3, 0.8,
0.6}}];

>> ImageAdd[i, 0.5]
-Image-

>> ImageAdd[i, i]
-Image-

>> ein = Import["ExampleData/
Einstein.jpg"];

>> noise = RandomImage[{-0.1, 0.1},
ImageDimensions[ein]];

>> ImageAdd[noise, ein]
-Image-

>> lena = Import["ExampleData/lena.
.tif"];

>> noise = RandomImage[{-0.2, 0.2},
ImageDimensions[lena],
ColorSpace -> "RGB"];
```

```
>> ImageAdd[noise, lena]
      -Image-
```

ImageAdjust

`ImageAdjust[image]`
adjusts the levels in *image*.
`ImageAdjust[image, c]`
adjusts the contrast in *image* by *c*.
`ImageAdjust[image, {c, b}]`
adjusts the contrast *c*, and brightness *b* in *image*.
`ImageAdjust[image, {c, b, g}]`
adjusts the contrast *c*, brightness *b*, and gamma *g* in *image*.

```
>> lena = Import["ExampleData/lena.tif"];

>> ImageAdjust[lena]
      -Image-
```

ImageAspectRatio

`ImageAspectRatio[image]`
gives the aspect ratio of *image*.

```
>> img = Import["ExampleData/lena.tif"];

>> ImageAspectRatio[img]
1

>> ImageAspectRatio[Image[{{0, 1}, {1, 0}, {1, 1}}]]
3
2
```

Image

ImageBox

ImageChannels

`ImageChannels[image]`
gives the number of channels in *image*.

```
>> ImageChannels[Image[{{0, 1}, {1, 0}}]]
1

>> img = Import["ExampleData/lena.tif"];

>> ImageChannels[img]
3
```

ImageColorSpace

`ImageColorSpace[image]`
gives *image*'s color space, e.g. "RGB" or "CMYK".

```
>> img = Import["ExampleData/lena.tif"];

>> ImageColorSpace[img]
RGB
```

ImageConvolve

`ImageConvolve[image, kernel]`
Computes the convolution of *image* using *kernel*.

```
>> img = Import["ExampleData/lena.tif"];

>> ImageConvolve[img, DiamondMatrix[5] / 61]
      -Image-

>> ImageConvolve[img, DiskMatrix[5] / 97]
      -Image-

>> ImageConvolve[img, BoxMatrix[5] / 121]
      -Image-
```

ImageData

`ImageData[image]`
gives a list of all color values of *image* as a matrix.

`ImageData[image, stype]`
gives a list of color values in type *stype*.

```
>> img = Image[{{0.2, 0.4}, {0.9, 0.6}, {0.5, 0.8}}];
>> ImageData[img]
{{0.2, 0.4}, {0.9, 0.6}, {0.5, 0.8}}
>> ImageData[img, "Byte"]
{{51, 102}, {229, 153}, {127, 204}}
>> ImageData[Image[{{0, 1}, {1, 0}, {1, 1}}], "Bit"]
{{0, 1}, {1, 0}, {1, 1}}
```

ImageDimensions

`ImageDimensions[image]`
Returns the dimensions of *image* in pixels.

```
>> lena = Import["ExampleData/lena.tif"];
>> ImageDimensions[lena]
{512, 512}
>> ImageDimensions[RandomImage[1, {50, 70}]]
{50, 70}
```

ImageExport

ImageImport

```
>> Import["ExampleData/Einstein.jpg"]
-Image-
>> Import["ExampleData/MadTeaParty.gif"]
-Image-
```

```
>> Import["ExampleData/moon.tif"]
-Image-
```

ImageMultiply

`ImageMultiply[image, expr_1, expr_2, ...]`
multiplies all *expr_i* with *image* where each *expr_i* must be an image or a real number.

```
>> i = Image[{{0, 0.5, 0.2, 0.1, 0.9}, {1.0, 0.1, 0.3, 0.8, 0.6}}];
>> ImageMultiply[i, 0.2]
-Image-
>> ImageMultiply[i, i]
-Image-
>> ein = Import["ExampleData/Einstein.jpg"];
>> noise = RandomImage[{0.7, 1.3}, ImageDimensions[ein]];
>> ImageMultiply[noise, ein]
-Image-
```

ImagePartition

`ImagePartition[image, s]`
Partitions an image into an array of *s* x *s* pixel subimages.

`ImagePartition[image, {w, h}]`
Partitions an image into an array of *w* x *h* pixel subimages.

```
>> lena = Import["ExampleData/lena.tif"];
>> ImageDimensions[lena]
{512, 512}
>> ImagePartition[lena, 256]
{{-Image-, -Image-},
 {-Image-, -Image-}}
```

```
>> ImagePartition[lena, {512, 128}]
{{{-Image-}}, {-Image-}},
{{{-Image-}}, {-Image-}}}
```

ImageQ

`ImageQ[Image[$pixels]]`
returns True if \$pixels has dimensions from which an Image can be constructed, and False otherwise.

```
>> ImageQ[Image[{{0, 1}, {1, 0}}]]
True
>> ImageQ[Image[{{{0, 0, 0}, {0, 1, 0}}, {{0, 1, 0}, {0, 1, 1}}]]]
True
>> ImageQ[Image[{{{0, 0, 0}, {0, 1}}, {{0, 1, 0}, {0, 1, 1}}]]]
False
>> ImageQ[Image[{{1, 0, 1}}]]
False
>> ImageQ["abc"]
False
```

ImageReflect

`ImageReflect[image]`
Flips *image* top to bottom.
`ImageReflect[image, side]`
Flips *image* so that *side* is interchanged with its opposite.
`ImageReflect[image, side_1 -> side_2]`
Flips *image* so that *side_1* is interchanged with *side_2*.

```
>> ein = Import["ExampleData/Einstein.jpg"];
>> ImageReflect[ein]
-Image-
>> ImageReflect[ein, Left]
-Image-
>> ImageReflect[ein, Left -> Top]
-Image-
```

ImageResize

`ImageResize[image, width]`
`ImageResize[image, {width, height}]`

```
>> ein = Import["ExampleData/Einstein.jpg"];
>> ImageDimensions[ein]
{615, 768}
>> ImageResize[ein, {400, 600}]
-Image-
>> ImageResize[ein, 256]
-Image-
>> ImageDimensions[%]
{256, 320}
The default sampling method is Bicubic
>> ImageResize[ein, 256, Resampling -> "Bicubic"]
-Image-
>> ImageResize[ein, 256, Resampling -> "Nearest"]
-Image-
>> ImageResize[ein, 256, Resampling -> "Gaussian"]
-Image-
```

ImageRotate

`ImageRotate[image]`
Rotates *image* 90 degrees counterclockwise.
`ImageRotate[image, theta]`
Rotates *image* by a given angle *theta*

```
>> ein = Import["ExampleData/Einstein.jpg"];
>> ImageRotate[ein]
-Image-
>> ImageRotate[ein, 45 Degree]
-Image-
```

```
>> ImageRotate[ein, Pi / 2]
-Image-
```

ImageSubtract

`ImageSubtract[image, expr_1, expr_2, ...]`
 subtracts all *expr_i* from *image* where each *expr_i* must be an image or a real number.

```
>> i = Image[{{0, 0.5, 0.2, 0.1, 0.9}, {1.0, 0.1, 0.3, 0.8, 0.6}}];
>> ImageSubtract[i, 0.2]
-Image-
>> ImageSubtract[i, i]
-Image-
```

ImageTake

`ImageTake[image, n]`
 gives the first *n* rows of *image*.
`ImageTake[image, -n]`
 gives the last *n* rows of *image*.
`ImageTake[image, {r1, r2}]`
 gives rows *r1*, ..., *r2* of *image*.
`ImageTake[image, {r1, r2}, {c1, c2}]`
 gives a cropped version of *image*.

ImageType

`ImageType[image]`
 gives the interval storage type of *image*, e.g. "Real", "Bit32", or "Bit".

```
>> img = Import["ExampleData/lena.tif"];
>> ImageType[img]
Byte
>> ImageType[Image[{{0, 1}, {1, 0}}]]
Real
```

```
>> ImageType[Binarize[img]]
Bit
```

MaxFilter

`MaxFilter[image, r]`
 gives *image* with a maximum filter of radius *r* applied on it. This always picks the largest value in the filter's area.

```
>> lena = Import["ExampleData/lena.tif"];
>> MaxFilter[lena, 5]
-Image-
```

MedianFilter

`MedianFilter[image, r]`
 gives *image* with a median filter of radius *r* applied on it. This always picks the median value in the filter's area.

```
>> lena = Import["ExampleData/lena.tif"];
>> MedianFilter[lena, 5]
-Image-
```

MinFilter

`MinFilter[image, r]`
 gives *image* with a minimum filter of radius *r* applied on it. This always picks the smallest value in the filter's area.

```
>> lena = Import["ExampleData/lena.tif"];
>> MinFilter[lena, 5]
-Image-
```


MorphologicalComponents

Opening

`Opening[image, ker]`

Gives the morphological opening of *image* with respect to structuring element *ker*.

```
>> ein = Import["ExampleData/  
Einstein.jpg"];
```

```
>> Opening[ein, 2.5]  
—Image—
```

PillowImageFilter

PixelValue

`PixelValue[image, {x, y}]`

gives the value of the pixel at position {x, y} in *image*.

```
>> lena = Import["ExampleData/lena.  
tif"];
```

```
>> PixelValue[lena, {1, 1}]  
{0.321569, 0.0862745, 0.223529}
```

PixelValuePositions

`PixelValuePositions[image, val]`

gives the positions of all pixels in *image* that have value *val*.

```
>> PixelValuePositions[Image[{{0,  
1}, {1, 0}, {1, 1}}, 1]  
{{1,1}, {1,2}, {2,1}, {2,3}}  
  
>> PixelValuePositions[Image[{{0.2,  
0.4}, {0.9, 0.6}, {0.3, 0.8}},  
0.5, 0.15]  
{{2,2}, {2,3}}  
  
>> img = Import["ExampleData/lena.  
tif"];
```

```
>> PixelValuePositions[img, 3 /  
255, 0.5 / 255]  
{{180,192,2}, {181,192,2},  
 {181,193,2}, {188,204,2},  
 {265,314,2}, {364,77,2}, {365,  
 72,2}, {365,73,2}, {365,77,  
 2}, {366,70,2}, {367,65,2}}  
  
>> PixelValue[img, {180, 192}]  
{0.25098, 0.0117647, 0.215686}
```

RandomImage

`RandomImage[max]`

creates an image of random pixels with values 0 to *max*.

`RandomImage[{min, max}]`

creates an image of random pixels with values *min* to *max*.

`RandomImage[... , size]`

creates an image of the given *size*.

```
>> RandomImage[1, {100, 100}]  
—Image—
```

Sharpen

`Sharpen[image]`

gives a sharpened version of *image*.

`Sharpen[image, r]`

sharpens *image* with a kernel of size *r*.

```
>> lena = Import["ExampleData/lena.  
tif"];  
  
>> Sharpen[lena]  
—Image—  
  
>> Sharpen[lena, 5]  
—Image—
```

TextRecognize

`TextRecognize[{image}]`
Recognizes text in *image* and returns it as string.

Threshold

`Threshold[image]`
gives a value suitable for binarizing *image*.

The option "Method" may be "Cluster" (use Otsu's threshold), "Median", or "Mean".

```
>> img = Import["ExampleData/lena.tif"];

>> Threshold[img]
0.456739

>> Binarize[img, %]
-Image-

>> Threshold[img, Method -> "Mean"]
0.486458

>> Threshold[img, Method -> "Median"]
0.504726
```

WordCloud

`WordCloud[{word1, word2, ...}]`
Gives a word cloud with the given list of words.

`WordCloud[{weight1 -> word1, weight2 -> word2, ...}]`
Gives a word cloud with the words weighted using the given weights.

`WordCloud[{weight1, weight2, ...} -> {word1, word2, ...}]`
Also gives a word cloud with the words weighted using the given weights.

`WordCloud[{word1, weight1}, {word2, weight2}, ...}]`
Gives a word cloud with the words weighted using the given weights.

```
>> WordCloud[StringSplit[Import["ExampleData/EinsteinSzilLetter.txt"]]]
-Image-

>> WordCloud[Range[50] -> ToString /@ Range[50]]
-Image-
```

XVIII. Input and Output

Contents

BaseForm	131	Message	134	StandardForm	137
Center	131	MessageName (: :) . .	134	StringForm	137
Check	132	NonAssociative	134	Style	137
Format	132	NumberForm	134	Subscript	137
FullForm	132	Off	135	SubscriptBox	137
General	132	On	135	Subsuperscript	137
Grid	132	OutputForm	135	SubsuperscriptBox . .	137
GridBox	133	Postfix (/)	135	Superscript	137
Infix	133	Precedence	136	SuperscriptBox	137
InputForm	133	Prefix (@)	136	Syntax	138
Left	133	Print	136	TableForm	138
MakeBoxes	133	Quiet	136	TeXForm	138
MathMLForm	134	Right	137	ToBoxes	139
MatrixForm	134	Row	137	\$UseSansSerif	139
		RowBox	137		

BaseForm

`BaseForm[expr, n]`
prints numbers in *expr* in base *n*.

```
>> BaseForm[33, 2]
100 0012

>> BaseForm[234, 16]
ea16

>> BaseForm[12.3, 2]
1 100.010011001100110012

>> BaseForm[-42, 16]
-2a16

>> BaseForm[x, 2]
x

>> BaseForm[12, 3] // FullForm
BaseForm[12, 3]
```

Bases must be between 2 and 36:

```
>> BaseForm[12, -3]
Positivemachine
— sized integer expected at position 2 in BaseForm[12,
— 3].
MakeBoxes[BaseForm[12, -3],
StandardForm] is not a valid box structure.

>> BaseForm[12, 100]
Requested base 100 must be between 2 and 36.
MakeBoxes[BaseForm[12, 100],
StandardForm] is not a valid box structure.
```

Center

`Center`
is used with the `ColumnAlignments` option to `Grid` or `TableForm` to specify a centered column.

Check

```
Check[expr, failexpr]
  evaluates expr, and returns the result, unless
  messages were generated, in which case it
  evaluates and failexpr will be returned.
Check[expr, failexpr, {s1::t1,s2::t2,...}]
  checks only for the specified messages.
```

Return err when a message is generated:

```
>> Check[1/0, err]
      Infiniteexpression1/0encountered.
      err
```

Check only for specific messages:

```
>> Check[Sin[0^0], err, Sin::argx]
      Indeterminateexpression0^0encountered.
      Indeterminate
>> Check[1/0, err, Power::infy]
      Infiniteexpression1/0encountered.
      err
```

Format

```
Format[expr]
  holds values specifying how expr should be
  printed.
```

Assign values to Format to control how particular expressions should be formatted when printed to the user.

```
>> Format[f[x___]] := Infix[{x}, "~"]
>> f[1, 2, 3]
      1 ~ 2 ~ 3
>> f[1]
      1
```

Raw objects cannot be formatted:

```
>> Format[3] = "three";
      Cannotassigntorawobject3.
```

Format types must be symbols:

```
>> Format[r, a + b] = "r";
      Formattypea + bisnotasymbol.
```

Formats must be attached to the head of an expression:

```
>> f /: Format[g[f]] = "my f";
      Tagfnotfoundortoodeepforanassignedrule.
```

FullForm

```
FullForm[expr]
  displays the underlying form of expr.
```

```
>> FullForm[a + b * c]
      Plus[a, Times[b, c]]
>> FullForm[2/3]
      Rational[2, 3]
>> FullForm["A string"]
      "A string"
```

General

```
General
  is a symbol to which all general-purpose
  messages are assigned.
```

```
>> General::argr
      '1' called with 1 argument;
      '2' arguments are expected.
>> Message[Rule::argr, Rule, 2]
      Rulecalledwith1argument;2argumentsareexpected.
```

Grid

```
Grid[{{a1, a2, ...}, {b1, b2, ...}, ...}]
  formats several expressions inside a
  GridBox.
```

```
>> Grid[{{a, b}, {c, d}}]
      a  b
      c  d
```

GridBox

`GridBox[{{...}, {...}}]`
is a box construct that represents a sequence of boxes arranged in a grid.

Infix

`Infix[expr, oper, prec, assoc]`
displays *expr* with the infix operator *oper*, with precedence *prec* and associativity *assoc*.

`Infix` can be used with `Format` to display certain forms with user-defined infix notation:

```
>> Format[g[x_, y_]] := Infix[{x, y}, "#", 350, Left]

>> g[a, g[b, c]]
a#(b#c)

>> g[g[a, b], c]
a#b#c

>> g[a + b, c]
(a + b)#c

>> g[a * b, c]
a#b#c

>> g[a, b] + c
c + a#b

>> g[a, b] * c
c(a#b)

>> Infix[{a, b, c}, {"+", "-"}]
a + b - c
```

InputForm

`InputForm[expr]`
displays *expr* in an unambiguous form suitable for input.

```
>> InputForm[a + b * c]
a + b * c
```

```
>> InputForm["A string"]
"A string"

>> InputForm[f' [x]]
Derivative[1] [f] [x]

>> InputForm[Derivative[1, 0] [f] [x]]
Derivative[1, 0] [f] [x]
```

Left

`Left`
is used with operator formatting constructs to specify a left-associative operator.

MakeBoxes

`MakeBoxes[expr]`
is a low-level formatting primitive that converts *expr* to box form, without evaluating it.

```
\( ... \)
```

directly inputs box objects.

String representation of boxes

```
>> \(\mathbf{x}^2\)
SuperscriptBox[x, 2]

>> \(\mathbf{x}_2\)
SubscriptBox[x, 2]

>> \(\mathbf{a}^{\mathbf{b}} \mathbf{c}\)
UnderoverscriptBox[a, b, c]

>> \(\mathbf{a}^{\mathbf{b}} \mathbf{c}\)
UnderoverscriptBox[a, c, b]

>> \(\mathbf{x}^{\mathbf{y}}\)
OverscriptBox[x, y]

>> \(\mathbf{x}_y\)
UnderscriptBox[x, y]
```

MathMLForm

`MathMLForm[expr]`
displays *expr* as a MathML expression.

```
>> MathMLForm[HoldForm[Sqrt[a^3]]]
<math display="block">
  <msqrt><msup><mi>a</mi>
  <mn>3</mn></msup>
</msqrt></math>
```

MatrixForm

`MatrixForm[m]`
displays a matrix *m*, hiding the underlying list structure.

```
>> Array[a,{4,3}]/MatrixForm
( a [1,1]  a [1,2]  a [1,3] )
( a [2,1]  a [2,2]  a [2,3] )
( a [3,1]  a [3,2]  a [3,3] )
( a [4,1]  a [4,2]  a [4,3] )
```

Message

`Message[symbol::msg, expr1, expr2, ...]`
displays the specified message, replacing placeholders in the message text with the corresponding expressions.

```
>> a::b = "Hello world!"
Hello world!

>> Message[a::b]
Hello world!

>> a::c := "Hello '1', Mr 00'2'!"

>> Message[a::c, "you", 3 + 4]
Hello you, Mr007!
```

MessageName (::)

`MessageName[symbol, tag]`
symbol::tag
identifies a message.

`MessageName` is the head of message IDs of the form *symbol::tag*.

```
>> FullForm[a::b]
MessageName[a,"b"]
```

The second parameter *tag* is interpreted as a string.

```
>> FullForm[a::"b"]
MessageName[a,"b"]
```

NonAssociative

`NonAssociative`
is used with operator formatting constructs to specify a non-associative operator.

NumberForm

`NumberForm[expr, n]`
prints a real number *expr* with *n*-digits of precision.
`NumberForm[expr, {n, f}]`
prints with *n*-digits and *f* digits to the right of the decimal point.

```
>> NumberForm[N[Pi], 10]
3.141592654

>> NumberForm[N[Pi], {10, 5}]
3.14159
```

Off

`Off[symbol::tag]`
turns a message off so it is no longer printed.

```
>> Off[Power::infy]
```

```
>> 1 / 0
ComplexInfinity

>> Off[Power::indet, Syntax::com]

>> {0 ^ 0,}
{Indeterminate, Null}
```

On

`On[symbol::tag]`
turns a message on for printing.

```
>> Off[Power::infy]

>> 1 / 0
ComplexInfinity

>> On[Power::infy]

>> 1 / 0
Infiniteexpression1/0encountered.
ComplexInfinity
```

OutputForm

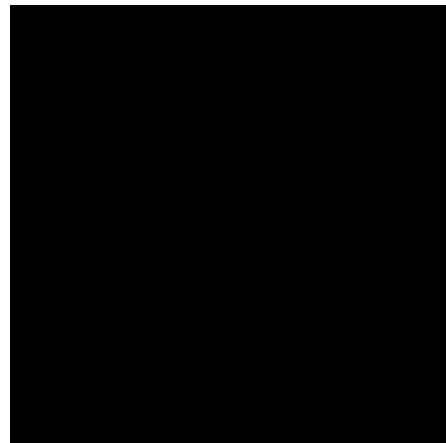
`OutputForm[expr]`
displays *expr* in a plain-text form.

```
>> OutputForm[f' [x]]
f' [x]

>> OutputForm[Derivative[1, 0] [f] [x]]
Derivative[1,0] [f] [x]

>> OutputForm["A string"]
A string
```

```
>> OutputForm[Graphics[Rectangle
[]]]
```



Postfix (//)

$x // f$
is equivalent to $f[x]$.

```
>> b // a
a[b]

>> c // b // a
a[b[c]]
```

The postfix operator `//` is parsed to an expression before evaluation:

```
>> Hold[x // a // b // c // d // e
// f]
Hold[f[e[d[c[b[a[x]]]]]]]
```

Precedence

`Precedence[op]`
returns the precedence of the built-in operator *op*.

```
>> Precedence[Plus]
310.

>> Precedence[Plus] < Precedence[
Times]
True
```

Unknown symbols have precedence 670:

```
>> Precedence[f]
670.
```

Other expressions have precedence 1000:

```
>> Precedence[a + b]
1000.
```

Prefix (@)

$f @ x$
is equivalent to $f[x]$.

```
>> a @ b
a[b]

>> a @ b @ c
a[b[c]]

>> Format[p[x_]] := Prefix[{x},
"~"]

>> p[3]
*3

>> Format[q[x_]] := Prefix[{x}, "~
", 350]

>> q[a+b]
~ (a + b)

>> q[a*b]
~ ab

>> q[a]+b
b+ ~ a
```

The prefix operator @ is parsed to an expression before evaluation:

```
>> Hold[a @ b @ c @ d @ e @ f @ x]
Hold[a[b[c[d[e[f[x]]]]]]]
```

Print

`Print[expr, ...]`
prints each *expr* in string form.

```
>> Print["Hello world!"]
Hello world!
```

```
>> Print["The answer is ", 7 * 6,
"."]
```

The answer is 42.

Quiet

`Quiet[expr, {s1::t1, ...}]`
evaluates *expr*, without messages {s1::t1, ...} being displayed.

`Quiet[expr, All]`
evaluates *expr*, without any messages being displayed.

`Quiet[expr, None]`
evaluates *expr*, without all messages being displayed.

`Quiet[expr, off, on]`
evaluates *expr*, with messages *off* being suppressed, but messages *on* being displayed.

```
>> a::b = "Hello";

>> Quiet[x+x, {a::b}]
2x

>> Quiet[Message[a::b]; x+x, {a::b
}]
2x

>> Message[a::b]; y=Quiet[Message[a
::b]; x+x, {a::b}]; Message[a::b
]; y
Hello
Hello
2x

>> Quiet[expr, All, All]
Arguments2and3ofQuiet[expr,
All, All]shouldnotbothbeAll.
Quiet[expr, All, All]

>> Quiet[x + x, {a::b}, {a::b}]
InQuiet[x + x, {a::b},
{a::b}]themessagename(s){a::b}appearinboththelistofofmessage
Quiet[x + x, {a::b}, {a::b}]
```


Right

`Right`

is used with operator formatting constructs to specify a right-associative operator.

Row

`Row[{expr, ...}]`

formats several expressions inside a `RowBox`.

RowBox

`RowBox[{...}]`

is a box construct that represents a sequence of boxes arranged in a horizontal row.

StandardForm

`StandardForm[expr]`

displays *expr* in the default form.

```
>> StandardForm[a + b * c]
a + bc
```

```
>> StandardForm["A string"]
A string
```

`StandardForm` is used by default:

```
>> "A string"
A string
```

```
>> f'[x]
f'[x]
```

StringForm

`StringForm[str, expr1, expr2, ...]`

displays the string *str*, replacing placeholders in *str* with the corresponding expressions.

```
>> StringForm["'1' bla '2' blub ' '
    bla '2'", a, b, c]
a bla b blub c bla b
```

Style

Subscript

`Subscript[a, i]`
displays as a_i .

```
>> Subscript[x,1,2,3] // TeXForm
x_{1,2,3}
```

SubscriptBox

Subsuperscript

`Subsuperscript[a, b, c]`
displays as a_b^c .

```
>> Subsuperscript[a, b, c] //
TeXForm
a_b^c
```

SubsuperscriptBox

Superscript

`Superscript[x, y]`
displays as x^y .

```
>> Superscript[x,3] // TeXForm
x^3
```

SuperscriptBox

Syntax

`Syntax`

is a symbol to which all syntax messages are assigned.

```
>> 1 +
>> Sin[1]
>> ^ 2
>> 1.5''
```

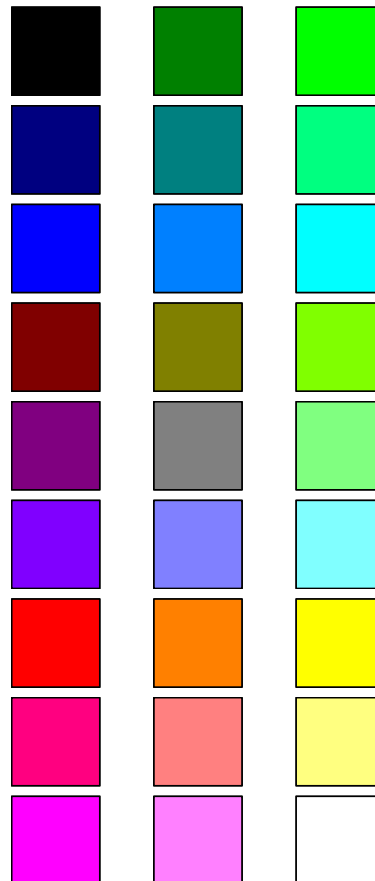
TableForm

`TableForm[expr]`
displays *expr* as a table.

```
>> TableForm[Array[a, {3,2}],
TableDepth->1]
{a[1,1],a[1,2]}
{a[2,1],a[2,2]}
{a[3,1],a[3,2]}
```

A table of Graphics:

```
>> Table[Style[Graphics[{EdgeForm[{
Black}], RGBColor[r,g,b],
Rectangle[]}],
ImageSizeMultipliers->{0.2, 1}],
{r,0,1,1/2}, {g,0,1,1/2}, {b
,0,1,1/2}] // TableForm
```



TeXForm

`TeXForm[expr]`
displays *expr* using TeX math mode commands.

```
>> TeXForm[HoldForm[Sqrt[a^3]]]
\sqrt{a^3}
```

ToBoxes

`ToBoxes[expr]`
evaluates *expr* and converts the result to box form.

Unlike `MakeBoxes`, `ToBoxes` evaluates its argument:

```
>> ToBoxes[a + a]
      RowBox[{2, a}]

>> ToBoxes[a + b]
      RowBox[{a, +, b}]

>> ToBoxes[a ^ b] // FullForm
      SuperscriptBox["a", "b"]
```

`$UseSansSerif`

`$UseSansSerif`
specifies the font of the web interface.

When `True`, the output in `MathMLForm` uses SansSerif fonts instead of the standard ones...

XIX. Integer functions

Contents

BitLength	140	Floor	141	IntegerLength	142
Ceiling	140	FromDigits	141	IntegerReverse	142
DigitCount	140	IntegerDigits	141	IntegerString	142

BitLength

BitLength $[x]$
gives the number of bits needed to represent the integer x . x 's sign is ignored.

```
>> BitLength[1023]
10
>> BitLength[100]
7
>> BitLength[-5]
3
>> BitLength[0]
0
```

Ceiling

Ceiling $[x]$
gives the first integer greater than x .

```
>> Ceiling[1.2]
2
>> Ceiling[3/2]
2
```

For complex x , take the ceiling of real and imaginary parts.

```
>> Ceiling[1.3 + 0.7 I]
2 + I
```

DigitCount

DigitCount $[n, b, d]$
returns the number of times digit d occurs in the base b representation of n .

DigitCount $[n, b]$
returns a list indicating the number of times each digit occurs in the base b representation of n .

DigitCount $[n]$
returns a list indicating the number of times each digit occurs in the decimal representation of n .

```
>> DigitCount[1022]
{1, 2, 0, 0, 0, 0, 0, 0, 1}
>> DigitCount[Floor[Pi * 10^100]]
{8, 12, 12, 10, 8, 9, 8, 12, 14, 8}
>> DigitCount[1022, 2]
{9, 1}
>> DigitCount[1022, 2, 1]
9
```

Floor

Floor $[x]$
gives the smallest integer less than or equal to x .

Floor $[x, a]$
gives the smallest multiple of a less than or equal to x .

```
>> Floor[10.4]
10
>> Floor[10/3]
3
>> Floor[10]
10
>> Floor[21, 2]
20
>> Floor[2.6, 0.5]
2.5
>> Floor[-10.4]
-11
```

For complex x , take the floor of real and imaginary parts.

```
>> Floor[1.5 + 2.7 I]
1 + 2I
```

For negative a , the smallest multiple of a greater than or equal to x is returned.

```
>> Floor[10.4, -1]
11
>> Floor[-10.4, -1]
-10
```

FromDigits

FromDigits[l]
returns the integer corresponding to the decimal representation given by l . l can be a list of digits or a string.

FromDigits[l, b]
returns the integer corresponding to the base b representation given by l . l can be a list of digits or a string.

```
>> FromDigits["123"]
123
>> FromDigits[{1, 2, 3}]
123
>> FromDigits[{1, 0, 1}, 1000]
1 000 001
```

FromDigits can handle symbolic input:

```
>> FromDigits[{a, b, c}, 5]
c + 5 (5a + b)
```

Note that FromDigits does not automatically detect if you are providing a non-decimal representation:

```
>> FromDigits["a0"]
100
>> FromDigits["a0", 16]
160
```

FromDigits on empty lists or strings returns 0:

```
>> FromDigits[{}]
0
>> FromDigits[""]
0
```

IntegerDigits

IntegerDigits[n]
returns the decimal representation of integer x as list of digits. x 's sign is ignored.

IntegerDigits[n, b]
returns the base b representation of integer x as list of digits. x 's sign is ignored.

IntegerDigits[$n, b, length$]
returns a list of length $length$. If the number is too short, the list gets padded with 0 on the left. If the number is too long, the $length$ least significant digits are returned.

```
>> IntegerDigits[12345]
{1, 2, 3, 4, 5}
>> IntegerDigits[-500]
{5, 0, 0}
>> IntegerDigits[12345, 10, 8]
{0, 0, 0, 1, 2, 3, 4, 5}
>> IntegerDigits[12345, 10, 3]
{3, 4, 5}
>> IntegerDigits[11, 2]
{1, 0, 1, 1}
>> IntegerDigits[123, 8]
{1, 7, 3}
>> IntegerDigits[98765, 20]
{12, 6, 18, 5}
```

IntegerLength

`IntegerLength[x]`
gives the number of digits in the base-10 representation of x .
`IntegerLength[x, b]`
gives the number of base- b digits in x .

```
>> IntegerLength[123456]
6
>> IntegerLength[10^10000]
10001
>> IntegerLength[-10^1000]
1001
```

IntegerLength with base 2:

```
>> IntegerLength[8, 2]
4
```

Check that IntegerLength is correct for the first 100 powers of 10:

```
>> IntegerLength /@ (10 ^ Range
[100]) == Range[2, 101]
True
```

The base must be greater than 1:

```
>> IntegerLength[3, -2]
Base - 2 is not an integer greater than 1.
IntegerLength[3, -2]
```

0 is a special case:

```
>> IntegerLength[0]
0
```

IntegerReverse

`IntegerReverse[n]`
returns the integer that has the reverse decimal representation of x without sign.
`IntegerReverse[n, b]`
returns the integer that has the reverse base b representation of x without sign.

```
>> IntegerReverse[1234]
4321
>> IntegerReverse[1022, 2]
511
```

```
>> IntegerReverse[-123]
321
```

IntegerString

`IntegerString[n]`
returns the decimal representation of integer x as string. x 's sign is ignored.
`IntegerString[n, b]`
returns the base b representation of integer x as string. x 's sign is ignored.
`IntegerString[n, b, length]`
returns a string of length *length*. If the number is too short, the string gets padded with 0 on the left. If the number is too long, the *length* least significant digits are returned.

For bases > 10 , alphabetic characters a, b, ... are used to represent digits 11, 12, Note that base must be an integer in the range from 2 to 36.

```
>> IntegerString[12345]
12345
>> IntegerString[-500]
500
>> IntegerString[12345, 10, 8]
00012345
>> IntegerString[12345, 10, 3]
345
>> IntegerString[11, 2]
1011
>> IntegerString[123, 8]
173
>> IntegerString[32767, 16]
7fff
>> IntegerString[98765, 20]
c6i5
```

XX. Iohooks

Contents

\$Post	143	\$PrePrint	143	\$SyntaxHandler	143
\$Pre	143	\$PreRead	143		

\$Post

\$Post
is a global variable whose value, if set, is applied to every output expression.

```
>> $Pre = .; $Post = .; $PrePrint =  
    .; $EnlapsedTime = .;  
    [Processinginput...]  
  
>> 2 + 2  
    4
```

\$Pre

\$Pre
is a global variable whose value, if set, is applied to every input expression.

Set *Timing* as the **\$Pre** function, stores the elapsed time in a variable, stores just the result in `Out[$Line]` and print a formatted version showing the elapsed time

```
>> $Pre := (Print["[Processing  
    input...]"; #1]&  
  
>> $Post := (Print["[Storing result  
    ...]"; #1]&  
    [Processinginput...]  
    [Storingresult...]  
  
>> $PrePrint := (Print["The result  
    is:"]; {TimeUsed[], #1})&  
    [Processinginput...]  
    [Storingresult...]  
  
>> 2 + 2  
    [Processinginput...]  
    [Storingresult...]  
    Theresultis :  
    {386.71,4}
```

\$PrePrint

\$PrePrint
is a global variable whose value, if set, is applied to every output expression before it is printed.

\$PreRead

\$PreRead
is a global variable whose value, if set, is applied to the text or box form of every input expression before it is fed to the parser.
(Not implemented yet)

\$SyntaxHandler

\$SyntaxHandler
is a global variable whose value, if set, is applied to any input string that is found to contain a syntax error.
(Not implemented yet)

XXI. Linear algebra

Contents

BrayCurtisDistance . . .	144	FittedModel	146	NullSpace	148
CanberraDistance . . .	144	Inverse	146	PseudoInverse	149
ChessboardDistance . .	144	LeastSquares	146	QRDecomposition . . .	149
CosineDistance	145	LinearModelFit	147	RowReduce	149
Cross	145	LinearSolve	147	SingularValueDecom- position	149
DesignMatrix	145	ManhattanDistance . .	147	SquaredEuclideanDis- tance	149
Det	145	MatrixExp	147	Tr	150
Eigensystem	145	MatrixPower	148	VectorAngle	150
Eigenvalues	145	MatrixRank	148		
Eigenvectors	146	Norm	148		
EuclideanDistance . .	146	Normalize	148		

BrayCurtisDistance

`BrayCurtisDistance[u, v]`
returns the Bray Curtis distance between u and v .

```
>> BrayCurtisDistance[-7, 5]
6

>> BrayCurtisDistance[{-1, -1},
{10, 10}]
11
9
```

CanberraDistance

`CanberraDistance[u, v]`
returns the canberra distance between u and v , which is a weighted version of the Manhattan distance.

```
>> CanberraDistance[-7, 5]
1

>> CanberraDistance[{-1, -1}, {1,
1}]
2
```

ChessboardDistance

`ChessboardDistance[u, v]`
returns the chessboard distance (also known as Chebyshev distance) between u and v , which is the number of moves a king on a chessboard needs to get from square u to square v .

```
>> ChessboardDistance[-7, 5]
12

>> ChessboardDistance[{-1, -1}, {1,
1}]
2
```

CosineDistance

`CosineDistance[u, v]`
returns the cosine distance between u and v .

```
>> N[CosineDistance[{7, 9}, {71,
89}]]
0.0000759646
```



```
>> CosineDistance[{a, b}, {c, d}]
1
+ 
$$\frac{-ac - bd}{\sqrt{\text{Abs}[a]^2 + \text{Abs}[b]^2} \sqrt{\text{Abs}[c]^2 + \text{Abs}[d]^2}}$$

```

Cross

`Cross[a, b]`
computes the vector cross product of a and b .

```
>> Cross[{x1, y1, z1}, {x2, y2, z2}]
{y1z2 - y2z1,
 -x1z2 + x2z1, x1y2 - x2y1}
>> Cross[{x, y}]
{-y, x}
>> Cross[{1, 2}, {3, 4, 5}]
The arguments are expected to be vectors of equal length,
and the number of arguments is expected to be less than 10.
Cross[{1, 2}, {3, 4, 5}]
```

DesignMatrix

`DesignMatrix[m, f, x]`
returns the design matrix.

```
>> DesignMatrix[{{2, 1}, {3, 4}, {5, 3}, {7, 6}}, x, x]
{{1, 2}, {1, 3}, {1, 5}, {1, 7}}
>> DesignMatrix[{{2, 1}, {3, 4}, {5, 3}, {7, 6}}, f[x], x]
{{1, f[2]}, {1, f[3]}, {1, f[5]}, {1, f[7]}}
```

Det

`Det[m]`
computes the determinant of the matrix m .

```
>> Det[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}]
-2
```

Symbolic determinant:

```
>> Det[{{a, b, c}, {d, e, f}, {g, h, i}}]
aei - afh - bdi + bfg + cdh - ceg
```

Eigensystem

`Eigensystem[m]`
returns the list {Eigenvalues[m], Eigenvectors[m]}.

```
>> Eigensystem[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}]
{{2, -1, 1}, {{1, 1, 1}, {1, -2, 1}, {-1, 0, 1}}}
```

Eigenvalues

`Eigenvalues[m]`
computes the eigenvalues of the matrix m .

```
>> Eigenvalues[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}] // Sort
{-1, 1, 2}
>> Eigenvalues[{{Cos[theta], Sin[theta], 0}, {-Sin[theta], Cos[theta], 0}, {0, 0, 1}}] // Sort
{1, Cos[theta] +  $\sqrt{-1 + \text{Cos}[\text{theta}]^2}$ , Cos[theta] -  $\sqrt{-1 + \text{Cos}[\text{theta}]^2}$ }
>> Eigenvalues[{{7, 1}, {-4, 3}}]
{5, 5}
```

Eigenvectors

Eigenvectors $[m]$
computes the eigenvectors of the matrix m .

```
>> Eigenvectors[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}]
{{1, 1, 1}, {1, -2, 1}, {-1, 0, 1}}

>> Eigenvectors[{{1, 0, 0}, {0, 1, 0}, {0, 0, 0}}]
{{0, 1, 0}, {1, 0, 0}, {0, 0, 1}}

>> Eigenvectors[{{2, 0, 0}, {0, -1, 0}, {0, 0, 0}}]
{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}

>> Eigenvectors[{{0.1, 0.2}, {0.8, 0.5}}]
{{-0.355518, -1.15048}, {-0.62896, 0.777438}}
```

EuclideanDistance

EuclideanDistance $[u, v]$
returns the euclidean distance between u and v .

```
>> EuclideanDistance[-7, 5]
12

>> EuclideanDistance[{-1, -1}, {1, 1}]
2√2

>> EuclideanDistance[{a, b}, {c, d}]
√Abs[a - c]² + Abs[b - d]²
```

FittedModel

Inverse

Inverse $[m]$
computes the inverse of the matrix m .

```
>> Inverse[{{1, 2, 0}, {2, 3, 0}, {3, 4, 1}}]
{{-3, 2, 0}, {2, -1, 0}, {1, -2, 1}}

>> Inverse[{{1, 0}, {0, 0}}]
The matrix {{1, 0}, {0, 0}} is singular.
Inverse[{{1, 0}, {0, 0}}]

>> Inverse[{{1, 0, 0}, {0, Sqrt[3]/2, 1/2}, {0, -1/2, Sqrt[3]/2}}]
{{1, 0, 0}, {0, √3/2, -1/2}, {1/2, √3/2, 0}}
```

LeastSquares

LeastSquares $[m, b]$
computes the least squares solution to $m x = b$, finding an x that solves for b optimally.

```
>> LeastSquares[{{1, 2}, {2, 3}, {5, 6}}, {1, 5, 3}]
{{28/13, 31/13}}

>> Simplify[LeastSquares[{{1, 2}, {2, 3}, {5, 6}}, {1, x, 3}]]
{{12/13 - 8x/13, -4/13 + 7x/13}}

>> LeastSquares[{{1, 1, 1}, {1, 1, 2}}, {1, 3}]
Solving for underdetermined system not implemented.
LeastSquares[{{1, 1, 1}, {1, 1, 2}}, {1, 3}]
```

LinearModelFit

LinearModelFit $[m, f, x]$
returns the design matrix.

```
>> m = LinearModelFit[{{2, 1}, {3, 4}, {5, 3}, {7, 6}}, x, x];
```

```

>> m["BasisFunctions"]
{1, x}

>> m["BestFit"]
0.186441 + 0.779661x

>> m["BestFitParameters"]
{0.186441, 0.779661}

>> m["DesignMatrix"]
{{1, 2}, {1, 3}, {1, 5}, {1, 7}}

>> m["Function"]
0.186441 + 0.779661#1&

>> m["Response"]
{1, 4, 3, 6}

>> m["FitResiduals"]
{-0.745763, 1.47458, -1.08475, 0.355932}

>> m = LinearModelFit[{{2, 2, 1}, {3, 2, 4}, {5, 6, 3}, {7, 9, 6}}, {Sin[x], Cos[y]}, {x, y}];

>> m["BasisFunctions"]
{1, Sin[x], Cos[y]}

>> m["Function"]
3.33077 - 5.65221Cos[
  #2] - 5.01042Sin[#1]&

>> m = LinearModelFit[{{1, 4}, {1, 5}, {1, 7}}, {1, 2, 3}];

>> m["BasisFunctions"]
{#1, #2}

>> m["FitResiduals"]
{-0.142857, 0.214286, -0.0714286}

```

LinearSolve

`LinearSolve[matrix, right]`
solves the linear equation system *matrix* · *x* = *right* and returns one corresponding solution *x*.

```

>> LinearSolve[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}, {1, 2, 3}]
{0, 1, 2}

```

Test the solution:

```

>> {{1, 1, 0}, {1, 0, 1}, {0, 1, 1}} . {0, 1, 2}
{1, 2, 3}

```

If there are several solutions, one arbitrary solution is returned:

```

>> LinearSolve[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}, {1, 1, 1}]
{-1, 1, 0}

```

Infeasible systems are reported:

```

>> LinearSolve[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}, {1, -2, 3}]

```

Linearequationencounteredthathasnosolution.

```

LinearSolve[{1, 2, 3}, {4, 5, 6}, {7, 8, 9}], {1, -2, 3}]

```

ManhattanDistance

`ManhattanDistance[u, v]`
returns the Manhattan distance between *u* and *v*, which is the number of horizontal or vertical moves in the gridlike Manhattan city layout to get from *u* to *v*.

```

>> ManhattanDistance[-7, 5]
12

>> ManhattanDistance[{-1, -1}, {1, 1}]
4

```

MatrixExp

`MatrixExp[m]`
computes the exponential of the matrix *m*.

```

>> MatrixExp[{{0, 2}, {0, 1}}]
{{1, -2 + 2E}, {0, E}}

>> MatrixExp[{{1.5, 0.5}, {0.5, 2.0}}]
{{5.16266, 3.02952}, {3.02952, 8.19218}}

```

MatrixPower

`MatrixPower[m, n]`
computes the n th power of a matrix m .

```
>> MatrixPower[{{1, 2}, {1, 1}},  
10]  
{ {3 363, 4 756}, {2 378, 3 363} }  
  
>> MatrixPower[{{1, 2}, {2, 5}},  
-3]  
{ {169, - 70}, {-70, 29} }
```

MatrixRank

`MatrixRank[matrix]`
returns the rank of $matrix$.

```
>> MatrixRank[{{1, 2, 3}, {4, 5,  
6}, {7, 8, 9}}]  
2  
  
>> MatrixRank[{{1, 1, 0}, {1, 0,  
1}, {0, 1, 1}}]  
3  
  
>> MatrixRank[{{a, b}, {3 a, 3 b}}]  
1
```

Norm

`Norm[m, l]`
computes the l -norm of matrix m (currently only works for vectors!).

`Norm[m]`
computes the 2-norm of matrix m (currently only works for vectors!).

```
>> Norm[{1, 2, 3, 4}, 2]  
 $\sqrt{30}$   
  
>> Norm[{10, 100, 200}, 1]  
310  
  
>> Norm[{a, b, c}]  
 $\sqrt{\text{Abs}[a]^2 + \text{Abs}[b]^2 + \text{Abs}[c]^2}$ 
```

```
>> Norm[{-100, 2, 3, 4}, Infinity]  
100  
  
>> Norm[1 + I]  
 $\sqrt{2}$ 
```

Normalize

`Normalize[v]`
calculates the normalized vector v .

`Normalize[z]`
calculates the normalized complex number z .

```
>> Normalize[{1, 1, 1, 1}]  
 $\left\{ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\}$   
  
>> Normalize[1 + I]  
 $\left( \frac{1}{2} + \frac{I}{2} \right) \sqrt{2}$ 
```

NullSpace

`NullSpace[matrix]`
returns a list of vectors that span the nullspace of $matrix$.

```
>> NullSpace[{{1, 2, 3}, {4, 5, 6},  
{7, 8, 9}}]  
{ {1, - 2, 1} }  
  
>> A = {{1, 1, 0}, {1, 0, 1}, {0,  
1, 1}};  
  
>> NullSpace[A]  
{ }  
  
>> MatrixRank[A]  
3
```

PseudoInverse

`PseudoInverse[m]`
computes the Moore-Penrose pseudoinverse of the matrix m . If m is invertible, the pseudoinverse equals the inverse.

```
>> PseudoInverse[{{1, 2}, {2, 3},
{3, 4}}]

$$\left\{ \left\{ -\frac{11}{6}, -\frac{1}{3}, \frac{7}{6} \right\}, \left\{ \frac{4}{3}, \frac{1}{3}, -\frac{2}{3} \right\} \right\}$$

>> PseudoInverse[{{1, 2, 0}, {2, 3,
0}, {3, 4, 1}}]

$$\left\{ \{-3, 2, 0\}, \{2, -1, 0\}, \{1, -2, 1\} \right\}$$

>> PseudoInverse[{{1.0, 2.5}, {2.5,
1.0}}]

$$\left\{ \{-0.190476, 0.47619\}, \{0.47619, -0.190476\} \right\}$$

```

QRDecomposition

QRDecomposition $[m]$
computes the QR decomposition of the matrix m .

```
>> QRDecomposition[{{1, 2}, {3, 4},
{5, 6}}]

$$\left\{ \left\{ \left\{ \frac{\sqrt{35}}{35}, \frac{3\sqrt{35}}{35}, \frac{\sqrt{35}}{7} \right\}, \left\{ \frac{13\sqrt{210}}{210}, \frac{2\sqrt{210}}{105}, -\frac{\sqrt{210}}{42} \right\} \right\}, \left\{ \left\{ \sqrt{35}, \frac{44\sqrt{35}}{35} \right\}, \left\{ 0, \frac{2\sqrt{210}}{35} \right\} \right\} \right\}$$

```

RowReduce

RowReduce $[matrix]$
returns the reduced row-echelon form of $matrix$.

```
>> RowReduce[{{1, 0, a}, {1, 1, b}
}]

$$\left\{ \{1, 0, a\}, \{0, 1, -a + b\} \right\}$$

```

```
>> RowReduce[{{1, 2, 3}, {4, 5, 6},
{7, 8, 9}}] // MatrixForm

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

```

SingularValueDecomposition

SingularValueDecomposition $[m]$
calculates the singular value decomposition for the matrix m .

SingularValueDecomposition returns u , s , w such that $m=usv$, $uu=1$, $vv=1$, and s is diagonal.

```
>> SingularValueDecomposition
[{{1.5, 2.0}, {2.5, 3.0}}]

$$\left\{ \left\{ \{0.538954, 0.842335\}, \{0.842335, -0.538954\} \right\}, \left\{ \{4.63555, 0\}, \{0., 0.107862\} \right\}, \left\{ \{0.628678, 0.777666\}, \{-0.777666, 0.628678\} \right\} \right\}$$

```

SquaredEuclideanDistance

SquaredEuclideanDistance $[u, v]$
returns squared the euclidean distance between u and v .

```
>> SquaredEuclideanDistance[-7, 5]
144
>> SquaredEuclideanDistance[{-1,
-1}, {1, 1}]
8
```

Tr

Tr $[m]$
computes the trace of the matrix m .

```
>> Tr[{{1, 2, 3}, {4, 5, 6}, {7, 8,
9}}]
15
```

Symbolic trace:

```
>> Tr[{a, b, c}, {d, e, f}, {g, h,
    i}]
a + e + i
```

VectorAngle

`VectorAngle[u , v]`
gives the angles between vectors u and v

```
>> VectorAngle[{1, 0}, {0, 1}]
Pi
2

>> VectorAngle[{1, 2}, {3, 1}]
Pi
4

>> VectorAngle[{1, 1, 0}, {1, 0,
1}]
Pi
3
```

XXII. List functions

Contents

Accumulate	151	IntersectingQ	158	RankedMax	166
All	151	Intersection	158	RankedMin	166
Append	152	Join	159	Reap	167
AppendTo	152	Keys	159	ReplacePart	167
Array	152	Kurtosis	159	Rest	167
Association	153	Last	159	Reverse	168
AssociationQ	153	LeafCount	159	Riffle	168
Cases	153	Length	160	RotateLeft	168
Catenate	153	Level	160	RotateRight	168
CentralMoment	153	LevelQ	161	Select	169
ClusteringComponents	153	List	161	Skewness	169
Complement	154	ListQ	161	Sow	169
ConstantArray	154	Mean	161	Span (;;)	169
ContainsOnly	154	Median	161	Split	169
Correlation	154	MemberQ	161	SplitBy	169
Count	154	Most	162	StandardDeviation	170
Covariance	154	Nearest	162	SubsetQ	170
Delete	155	None	162	Table	170
DeleteCases	155	NotListQ	162	Take	171
DeleteDuplicates	156	PadLeft	163	TakeLargest	171
DisjointQ	156	PadRight	163	TakeLargestBy	171
Drop	156	Part	164	TakeSmallest	171
Extract	156	Partition	164	TakeSmallestBy	171
FindClusters	157	Permutations	165	Tally	172
First	157	Pick	165	Total	172
FirstPosition	157	Position	165	Tuples	172
Fold	157	Prepend	165	Union	173
FoldList	158	PrependTo	165	UnitVector	173
Gather	158	Quantile	166	Values	173
GatherBy	158	Quartiles	166	Variance	173
		Range	166		

Accumulate

`Accumulate[list]`
accumulates the values of *list*, returning a new list.

```
>> Accumulate[{1, 2, 3}]
{1, 3, 6}
```

All

`All`
is a possible value for `Span` and `Quiet`.

Append

`Append[expr, item]`
returns *expr* with *item* appended to its leaves.

```
>> Append[{1, 2, 3}, 4]
      {1, 2, 3, 4}
```

Append works on expressions with heads other than List:

```
>> Append[f[a, b], c]
      f[a, b, c]
```

Unlike Join, Append does not flatten lists in *item*:

```
>> Append[{a, b}, {c, d}]
      {a, b, {c, d}}
```

AppendTo

`AppendTo[s, item]`
append *item* to value of *s* and sets *s* to the result.

```
>> s = {};
>> AppendTo[s, 1]
      {1}
>> s
      {1}
```

Append works on expressions with heads other than List:

```
>> y = f[];
>> AppendTo[y, x]
      f[x]
>> y
      f[x]
```

Array

`Array[f, n]`
returns the *n*-element list $\{f[1], \dots, f[n]\}$.
`Array[f, n, a]`
returns the *n*-element list $\{f[a], \dots, f[a + n]\}$.
`Array[f, {n, m}, {a, b}]`
returns an *n*-by-*m* matrix created by applying *f* to indices ranging from (*a*, *b*) to (*a* + *n*, *b* + *m*).
`Array[f, dims, origins, h]`
returns an expression with the specified dimensions and index origins, with head *h* (instead of List).

```
>> Array[f, 4]
      {f[1], f[2], f[3], f[4]}
>> Array[f, {2, 3}]
      {{f[1, 1], f[1, 2], f[1, 3]},
       {f[2, 1], f[2, 2], f[2, 3]}}
>> Array[f, {2, 3}, 3]
      {{f[3, 3], f[3, 4], f[3, 5]},
       {f[4, 3], f[4, 4], f[4, 5]}}
>> Array[f, {2, 3}, {4, 6}]
      {{f[4, 6], f[4, 7], f[4, 8]},
       {f[5, 6], f[5, 7], f[5, 8]}}
>> Array[f, {2, 3}, 1, Plus]
      f[1, 1] + f[1, 2] + f[1, 3] + f[2, 1] + f[2, 2] + f[2, 3]
```

Association

`Association[key1 -> val1, key2 -> val2, ...]`
`<|key1 -> val1, key2 -> val2, ...|>`
represents an association between keys and values.

Association is the head of associations:

```
>> Head[<|a -> x, b -> y, c -> z|>]
      Association
>> <|a -> x, b -> y|>
      <|a -> x, b -> y|>
```



```
>> Association[{a -> x, b -> y}]
<|a -> x, b -> y|>
```

Associations can be nested:

```
>> <|a -> x, b -> y, <|a -> z, d ->
t|>|>
<|a -> z, b -> y, d -> t|>
```

AssociationQ

AssociationQ[*expr*]
return True if *expr* is a valid Association object, and False otherwise.

```
>> AssociationQ[<|a -> 1, b -> 2|>]
True
>> AssociationQ[<|a, b|>]
False
```

Cases

Cases[*list*, *pattern*]
returns the elements of *list* that match *pattern*.
Cases[*list*, *pattern*, *ls*]
returns the elements matching at level-spec *ls*.

```
>> Cases[{a, 1, 2.5, "string"},
_Integer|_Real]
{1, 2.5}
>> Cases[_Complex] [{1, 2I, 3, 4-I,
5}]
{2I, 4 - I}
```

Catenate

Catenate[{*l1*, *l2*, ...}]
concatenates the lists *l1*, *l2*, ...

```
>> Catenate[{ {1, 2, 3}, {4, 5} }]
{1, 2, 3, 4, 5}
```

CentralMoment

CentralMoment[*list*, *r*]
gives the the *r*th central moment (i.e. the *r*th moment about the mean) of *list*.

```
>> CentralMoment[{1.1, 1.2, 1.4,
2.1, 2.4}, 4]
0.100845
```

ClusteringComponents

ClusteringComponents[*list*]
forms clusters from *list* and returns a list of cluster indices, in which each element shows the index of the cluster in which the corresponding element in *list* ended up.

ClusteringComponents[*list*, *k*]
forms *k* clusters from *list* and returns a list of cluster indices, in which each element shows the index of the cluster in which the corresponding element in *list* ended up.

For more detailed documentation regarding options and behavior, see **FindClusters**[].

```
>> ClusteringComponents[{1, 2, 3,
1, 2, 10, 100}]
{1, 1, 1, 1, 1, 1, 2}
>> ClusteringComponents[{10, 100,
20}, Method -> "KMeans"]
{1, 0, 1}
```

Complement

Complement[*all*, *e1*, *e2*, ...]
returns an expression containing the elements in the set *all* that are not in any of *e1*, *e2*, etc.

Complement[*all*, *e1*, *e2*, ..., *SameTest* -> *test*]
applies *test* to the elements in *all* and each of the *ei* to determine equality.

The sets *all*, *e1*, etc can have any head, which must all match. The returned expression has the

same head as the input expressions. The expression will be sorted and each element will only occur once.

```
>> Complement[{a, b, c}, {a, c}]
      {b}

>> Complement[{a, b, c}, {a, c}, {b}]
      {}

>> Complement[f[z, y, x, w], f[x],
      f[x, z]]
      f[w, y]

>> Complement[{c, b, a}]
      {a, b, c}
```

ConstantArray

`ConstantArray[expr, n]`
returns a list of n copies of *expr*.

```
>> ConstantArray[a, 3]
      {a, a, a}

>> ConstantArray[a, {2, 3}]
      {{a, a, a}, {a, a, a}}
```

ContainsOnly

`ContainsOnly[list1, list2]`
yields True if *list1* contains only elements that appear in *list2*.

```
>> ContainsOnly[{b, a, a}, {a, b, c}]
      True
```

The first list contains elements not present in the second list:

```
>> ContainsOnly[{b, a, d}, {a, b, c}]
      False

>> ContainsOnly[{}, {a, b, c}]
      True
```

Use `Equal` as the comparison function to have numerical tolerance:

```
>> ContainsOnly[{a, 1.0}, {1, a, b}, {SameTest -> Equal}]
      True
```

Correlation

`Correlation[a, b]`
computes Pearson's correlation of two equal-sized vectors *a* and *b*.

An example from Wikipedia:

```
>> Correlation[{10, 8, 13, 9, 11,
      14, 6, 4, 12, 7, 5}, {8.04,
      6.95, 7.58, 8.81, 8.33, 9.96,
      7.24, 4.26, 10.84, 4.82, 5.68}]
      0.816421
```

Count

`Count[list, pattern]`
returns the number of times *pattern* appears in *list*.
`Count[list, pattern, ls]`
counts the elements matching at level-spec *ls*.

```
>> Count[{3, 7, 10, 7, 5, 3, 7,
      10}, 3]
      2

>> Count[{{a, a}, {a, a, a}, a}, a, {2}]
      5
```

Covariance

`Covariance[a, b]`
computes the covariance between the equal-sized vectors *a* and *b*.

```
>> Covariance[{0.2, 0.3, 0.1},
      {0.3, 0.3, -0.2}]
      0.025
```

Delete

```
Delete[expr, i]
  deletes the element at position i in expr.
  The position is counted from the end if i
  is negative.
Delete[expr, {m, n, ...}]
  deletes the element at position {m, n, ...}.
Delete[expr, {{m1, n1, ...}, {m2,
n2, ...}, ...}]
  deletes the elements at several positions.
```

Delete the element at position 3:

```
>> Delete[{a, b, c, d}, 3]
{a, b, d}
```

Delete at position 2 from the end:

```
>> Delete[{a, b, c, d}, -2]
{a, b, d}
```

Delete at positions 1 and 3:

```
>> Delete[{a, b, c, d}, {{1}, {3}}]
{b, d}
```

Delete in a 2D array:

```
>> Delete[{{a, b}, {c, d}}, {2, 1}]
{{a, b}, {d}}
```

Deleting the head of a whole expression gives a Sequence object:

```
>> Delete[{a, b, c}, 0]
Sequence[a, b, c]
```

Delete in an expression with any head:

```
>> Delete[f[a, b, c, d], 3]
f[a, b, d]
```

Delete a head to splice in its arguments:

```
>> Delete[f[a, b, u + v, c], {3, 0}]
f[a, b, u, v, c]
```

```
>> Delete[{a, b, c}, 0]
Sequence[a, b, c]
```

Delete without the position:

```
>> Delete[{a, b, c, d}]
Delete called with 1 argument; 2 arguments are expected.
Delete[{a, b, c, d}]
```

Delete with many arguments:

```
>> Delete[{a, b, c, d}, 1, 2]
Delete called with 3 arguments; 2 arguments are expected.
Delete[{a, b, c, d}, 1, 2]
```

Delete the element out of range:

```
>> Delete[{a, b, c, d}, 5]
Part{5} of {a, b, c, d} does not exist.
Delete[{a, b, c, d}, 5]
```

Delete the position not integer:

```
>> Delete[{a, b, c, d}, {1, n}]
Position specification n in {a, b, c, d} is not a machine
– sized integer or a list of machine
– sized integers.
Delete[{a, b, c, d}, {1, n}]
```

DeleteCases

```
DeleteCases[list, pattern]
  returns the elements of list that do not
  match pattern.
```

```
>> DeleteCases[{a, 1, 2.5, "string"}, _Integer | _Real]
{a, string}
>> DeleteCases[{a, b, 1, c, 2, 3}, _Symbol]
{1, 2, 3}
```

DeleteDuplicates

```
DeleteDuplicates[list]
  deletes duplicates from list.
DeleteDuplicates[list, test]
  deletes elements from list based on
  whether the function test yields True on
  pairs of elements. DeleteDuplicates does
  not change the order of the remaining el-
  ements.
```

```
>> DeleteDuplicates[{1, 7, 8, 4, 3, 4, 1, 9, 9, 2, 1}]
{1, 7, 8, 4, 3, 9, 2}
```

```
>> DeleteDuplicates[{3,2,1,2,3,4},
Less]
{3,2,1}
```

```
>> Extract[{a, b}, {c, d}], {{1},
{2, 2}}]
{{a,b},d}
```

DisjointQ

`DisjointQ[a, b]`
gives True if \$a and \$b are disjoint, or False if \$a and \$b have any common elements.

Drop

`Drop[expr, n]`
returns *expr* with the first *n* leaves removed.

```
>> Drop[{a, b, c, d}, 3]
{d}
>> Drop[{a, b, c, d}, -2]
{a,b}
>> Drop[{a, b, c, d, e}, {2, -2}]
{a,e}
```

Drop a submatrix:

```
>> A = Table[i*10 + j, {i, 4}, {j, 4}]
{{11,12,13,14}, {21,22,23,24},
{31,32,33,34}, {41,42,43,44}}
>> Drop[A, {2, 3}, {2, 3}]
{{11,14}, {41,44}}
```

Extract

`Extract[expr, list]`
extracts parts of *expr* specified by *list*.
`Extract[expr, {list1, list2, ...}]`
extracts a list of parts.

`Extract[expr, i, j, ...]` is equivalent to `Part[expr, {i, j, ...}]`.

```
>> Extract[a + b + c, {2}]
b
```

FindClusters

`FindClusters[list]`
returns a list of clusters formed from the elements of *list*. The number of cluster is determined automatically.
`FindClusters[list, k]`
returns a list of *k* clusters formed from the elements of *list*.

```
>> FindClusters[{1, 2, 20, 10, 11, 40, 19, 42}]
{{1,2,20,10,11,19}, {40,42}}
>> FindClusters[{25, 100, 17, 20}]
{{25,17,20}, {100}}
>> FindClusters[{3, 6, 1, 100, 20, 5, 25, 17, -10, 2}]
{{3,6,1,5, -10,2}, {100}, {20,25,17}}
>> FindClusters[{1, 2, 10, 11, 20, 21}]
{{1,2}, {10,11}, {20,21}}
>> FindClusters[{1, 2, 10, 11, 20, 21}, 2]
{{1,2,10,11}, {20,21}}
>> FindClusters[{1 -> a, 2 -> b, 10 -> c}]
{{a,b}, {c}}
>> FindClusters[{1, 2, 5} -> {a, b, c}]
{{a,b}, {c}}
>> FindClusters[{1, 2, 3, 1, 2, 10, 100}, Method -> "Agglomerate"]
{{1,2,3,1,2,10}, {100}}
>> FindClusters[{1, 2, 3, 10, 17, 18}, Method -> "Agglomerate"]
{{1,2,3}, {10}, {17,18}}
```

```
>> FindClusters[{{1}, {5, 6}, {7},
{2, 4}}, DistanceFunction -> (
Abs[Length[#1] - Length[#2]]&)]
{{{1}, {7}}, {{5, 6}, {2, 4}}}

>> FindClusters[{"meep", "heap", "
deep", "weep", "sheep", "leap",
"keep"}, 3]
{{meep, deep, weep, keep},
{heap, leap}, {sheep}}
```

FindClusters' automatic distance function detection supports scalars, numeric tensors, boolean vectors and strings.

The Method option must be either "Agglomerate" or "Optimize". If not specified, it defaults to "Optimize". Note that the Agglomerate and Optimize methods usually produce different clusterings.

The runtime of the Agglomerate method is quadratic in the number of clustered points n , builds the clustering from the bottom up, and is exact (no element of randomness). The Optimize method's runtime is linear in n , Optimize builds the clustering from top down, and uses random sampling.

First

First[*expr*]
returns the first element in *expr*.

First[*expr*] is equivalent to *expr*[[1]].

```
>> First[{a, b, c}]
a

>> First[a + b + c]
a

>> First[x]
Nonatomic expression expected.
First[x]
```

FirstPosition

FirstPosition[*expr*, *pattern*]
gives the position of the first element in *expr* that matches *pattern*, or Missing["NotFound"] if no such element is found.

FirstPosition[*expr*, *pattern*, *default*]
gives default if no element matching *pattern* is found.

FirstPosition[*expr*, *pattern*, *default*, *levelspec*]
finds only objects that appear on levels specified by *levelspec*.

```
>> FirstPosition[{a, b, a, a, b, c,
b}, b]
{2}

>> FirstPosition[{{a, a, b}, {b, a,
a}, {a, b, a}}, b]
{1, 3}

>> FirstPosition[{x, y, z}, b]
Missing[NotFound]
```

Find the first position at which x^2 appears:

```
>> FirstPosition[{1 + x^2, 5, x^4,
a + (1 + x^2)^2}, x^2]
{1, 2}
```

Fold

Fold[*f*, *x*, *list*]
returns the result of iteratively applying the binary operator *f* to each element of *list*, starting with *x*.

Fold[*f*, *list*]
is equivalent to **Fold**[*f*, **First**[*list*], **Rest**[*list*]].

```
>> Fold[Plus, 5, {1, 1, 1}]
8

>> Fold[f, 5, {1, 2, 3}]
f[f[f[5, 1], 2], 3]
```

FoldList

`FoldList[f, x, list]`

returns a list starting with x , where each element is the result of applying the binary operator f to the previous result and the next element of $list$.

`FoldList[f, list]`

is equivalent to `FoldList[f, First[list], Rest[list]]`.

```
>> FoldList[f, x, {1, 2, 3}]
```

```
{x, f[x, 1], f[f[x, 1], 2], f[f[f[x, 1], 2], 3]}
```

```
>> FoldList[Times, {1, 2, 3}]
```

```
{1, 2, 6}
```

```
>> GatherBy[{{1, 3}, {2, 2}, {1, 1}}, Total]
```

```
{{{1, 3}, {2, 2}}, {{1, 1}}}
```

```
>> GatherBy[{"xy", "abc", "ab"}, StringLength]
```

```
{{xy, ab}, {abc}}
```

```
>> GatherBy[{{2, 0}, {1, 5}, {1, 0}}, Last]
```

```
{{{2, 0}, {1, 0}}, {{1, 5}}}
```

```
>> GatherBy[{{1, 2}, {2, 1}, {3, 5}, {5, 1}, {2, 2, 2}}, {Total, Length}]
```

```
{{{{1, 2}, {2, 1}}}, {{{3, 5}}}, {{{5, 1}}, {{2, 2, 2}}}}
```

Gather

`Gather[list, test]`

gathers leaves of $list$ into sub lists of items that are the same according to $test$.

`Gather[list]`

gathers leaves of $list$ into sub lists of items that are the same.

The order of the items inside the sub lists is the same as in the original list.

```
>> Gather[{1, 7, 3, 7, 2, 3, 9}]
```

```
{{1}, {7, 7}, {3, 3}, {2}, {9}}
```

```
>> Gather[{1/3, 2/6, 1/9}]
```

```
{{{1/3, 1/3}, {1/9}}, {{2/6}}}
```

GatherBy

`GatherBy[list, f]`

gathers leaves of $list$ into sub lists of items whose image under f identical.

`GatherBy[list, {f, g, ...}]`

gathers leaves of $list$ into sub lists of items whose image under f identical. Then, gathers these sub lists again into sub sub lists, that are identical under g .

IntersectingQ

`IntersectingQ[a, b]`

gives True if there are any common elements in a and b , or False if a and b are disjoint.

Intersection

`Intersection[a, b, ...]`

gives the intersection of the sets. The resulting list will be sorted and each element will only occur once.

```
>> Intersection[{1000, 100, 10, 1}, {1, 5, 10, 15}]
```

```
{1, 10}
```

```
>> Intersection[{{a, b}, {x, y}}, {{x, x}, {x, y}, {x, z}}]
```

```
{{x, y}}
```

```
>> Intersection[{c, b, a}]
```

```
{a, b, c}
```

```
>> Intersection[{1, 2, 3}, {2, 3, 4}, SameTest->Less]
```

```
{3}
```

Join

```
Join[l1, l2]
concatenates the lists l1 and l2.
```

Join concatenates lists:

```
>> Join[{a, b}, {c, d, e}]
      {a, b, c, d, e}

>> Join[{{a, b}, {c, d}}, {{1, 2},
      {3, 4}}]
      {{a, b}, {c, d}, {1, 2}, {3, 4}}
```

The concatenated expressions may have any head:

```
>> Join[a + b, c + d, e + f]
      a + b + c + d + e + f
```

However, it must be the same for all expressions:

```
>> Join[a + b, c * d]
      HeadsPlusandTimesareexpectedtobethesame.
Join[a + b, cd]
```

Keys

```
Keys[<|key1 -> val1, key2 -> val2,
...|>]
return a list of the keys keyi in an association.
Keys[{key1 -> val1, key2 -> val2, ...}]
return a list of the keyi in a list of rules.
```

```
>> Keys[<|a -> x, b -> y|>]
      {a, b}

>> Keys[{a -> x, b -> y}]
      {a, b}
```

Keys automatically threads over lists:

```
>> Keys[{<|a -> x, b -> y|>, {w ->
      z, {}}}]
      {{a, b}, {w, {}}}
```

Keys are listed in the order of their appearance:

```
>> Keys[{c -> z, b -> y, a -> x}]
      {c, b, a}
```

Kurtosis

```
Kurtosis[list]
gives the Pearson measure of kurtosis for
list (a measure of existing outliers).
```

```
>> Kurtosis[{1.1, 1.2, 1.4, 2.1,
      2.4}]
      1.42098
```

Last

```
Last[expr]
returns the last element in expr.
```

Last[expr] is equivalent to expr[[-1]] .

```
>> Last[{a, b, c}]
      c

>> Last[x]
      Nonatomicexpressionexpected.
Last[x]
```

LeafCount

```
LeafCount[expr]
returns the total number of indivisible
subexpressions in expr.
```

```
>> LeafCount[1 + x + y^a]
      6

>> LeafCount[f[x, y]]
      3

>> LeafCount[{1 / 3, 1 + I}]
      7

>> LeafCount[Sqrt[2]]
      5

>> LeafCount[100!]
      1
```

Length

`Length[expr]`
returns the number of leaves in *expr*.

Length of a list:

```
>> Length[{1, 2, 3}]
3
```

Length operates on the FullForm of expressions:

```
>> Length[Exp[x]]
2
```

```
>> FullForm[Exp[x]]
Power[E, x]
```

The length of atoms is 0:

```
>> Length[a]
0
```

Note that rational and complex numbers are atoms, although their FullForm might suggest the opposite:

```
>> Length[1/3]
0
```

```
>> FullForm[1/3]
Rational[1, 3]
```

Level

`Level[expr, levelspec]`
gives a list of all subexpressions of *expr* at the level(s) specified by *levspec*.

Level uses standard level specifications:

n
levels 1 through *n*
Infinity
all levels from level 1
{*n*}
level *n* only
{*m*, *n*}
levels *m* through *n*

Level 0 corresponds to the whole expression.
A negative level $-n$ consists of parts with depth *n*.

Level -1 is the set of atoms in an expression:

```
>> Level[a + b ^ 3 * f[2 x ^ 2],
{-1}]
{a, b, 3, 2, x, 2}
```

```
>> Level[{{{a}}}], 3]
{{a}, {{a}}, {{{a}}}}
```

```
>> Level[{{{a}}}], -4]
{{{a}}}
```

```
>> Level[{{{a}}}], -5]
{}
```

```
>> Level[h0[h1[h2[h3[a]]]], {0,
-1}]
{a, h3[a], h2[h3[a]], h1[h2[
h3[a]]], h0[h1[h2[h3[a]]]]}
```

Use the option Heads -> True to include heads:

```
>> Level[{{{a}}}], 3, Heads ->
True]
{List, List, List, {a}, {{a}}, {{{a}}}}
```

```
>> Level[x^2 + y^3, 3, Heads ->
True]
{Plus, Power, x, 2, x^2, Power, y, 3, y^3}
```

```
>> Level[a ^ 2 + 2 * b, {-1}, Heads
-> True]
{Plus, Power, a, 2, Times, 2, b}
```

```
>> Level[f[g[h]] [x], {-1}, Heads ->
True]
{f, g, h, x}
```

```
>> Level[f[g[h]] [x], {-2, -1},
Heads -> True]
{f, g, h, g[h], x, f[g[h]] [x]}
```

LevelQ

`LevelQ[expr]`
tests whether *expr* is a valid level specification.

```
>> LevelQ[2]
True
```

```
>> LevelQ[{2, 4}]
True
```



```
>> LevelQ[Infinity]
True

>> LevelQ[a + b]
False
```

List

`List[e1, e2, ..., ei]`
`{e1, e2, ..., ei}`
 represents a list containing the elements $e1...ei$.

List is the head of lists:

```
>> Head[{1, 2, 3}]
List
```

Lists can be nested:

```
>> {{a, b, {c, d}}}
{{a, b, {c, d}}}
```

ListQ

`ListQ[expr]`
 tests whether *expr* is a List.

```
>> ListQ[{1, 2, 3}]
True

>> ListQ[{{1, 2}, {3, 4}}]
True

>> ListQ[x]
False
```

Mean

`Mean[list]`
 returns the statistical mean of *list*.

```
>> Mean[{26, 64, 36}]
42

>> Mean[{1, 1, 2, 3, 5, 8}]
 $\frac{10}{3}$ 
```

```
>> Mean[{a, b}]
 $\frac{a + b}{2}$ 
```

Median

`Median[list]`
 returns the median of *list*.

```
>> Median[{26, 64, 36}]
36
```

For lists with an even number of elements, Median returns the mean of the two middle values:

```
>> Median[{-11, 38, 501, 1183}]
 $\frac{539}{2}$ 
```

Passing a matrix returns the medians of the respective columns:

```
>> Median[{{100, 1, 10, 50}, {-1,
1, -2, 2}}]
 $\left\{\frac{99}{2}, 1, 4, 26\right\}$ 
```

MemberQ

`MemberQ[list, pattern]`
 returns True if *pattern* matches any element of *list*, or False otherwise.

```
>> MemberQ[{a, b, c}, b]
True

>> MemberQ[{a, b, c}, d]
False

>> MemberQ[{"a", b, f[x]}, _?
NumericQ]
False

>> MemberQ[_List][{{}}]
True
```

Most

`Most[expr]`
returns *expr* with the last element removed.

`Most[expr]` is equivalent to `expr[;;-2]`.

```
>> Most[{a, b, c}]
{a, b}

>> Most[a + b + c]
a + b

>> Most[x]
Nonatomicexpressionexpected.
Most[x]
```

Nearest

`Nearest[list, x]`
returns the one item in *list* that is nearest to *x*.

`Nearest[list, x, n]`
returns the *n* nearest items.

`Nearest[list, x, {n, r}]`
returns up to *n* nearest items that are not farther from *x* than *r*.

`Nearest[{p1 -> q1, p2 -> q2, ...}, x]`
returns *q1, q2, ...* but measures the distances using *p1, p2, ...*

`Nearest[{p1, p2, ...} -> {q1, q2, ...}, x]`
returns *q1, q2, ...* but measures the distances using *p1, p2, ...*

```
>> Nearest[{5, 2.5, 10, 11, 15, 8.5, 14}, 12]
{11}
```

Return all items within a distance of 5:

```
>> Nearest[{5, 2.5, 10, 11, 15, 8.5, 14}, 12, {All, 5}]
{11, 10, 14}

>> Nearest[{Blue -> "blue", White -> "white", Red -> "red", Green -> "green"}, {Orange, Gray}]
{{red}, {white}}
```

```
>> Nearest[{{0, 1}, {1, 2}, {2, 3}} -> {a, b, c}, {1.1, 2}]
{b}
```

None

`None`
is a possible value for `Span` and `Quiet`.

NotListQ

`NotListQ[expr]`
returns true if *expr* is not a list.

PadLeft

`PadLeft[list, n]`
pads *list* to length *n* by adding 0 on the left.

`PadLeft[list, n, x]`
pads *list* to length *n* by adding *x* on the left.

`PadLeft[list, {n1, $n2, ...}, x]`
pads *list* to lengths *n1, n2* at levels 1, 2, ... respectively by adding *x* on the left.

`PadLeft[list, n, x, m]`
pads *list* to length *n* by adding *x* on the left and adding a margin of *m* on the right.

`PadLeft[list, n, x, {m1, m2, ...}]`
pads *list* to length *n* by adding *x* on the left and adding margins of *m1, m2, ...* on levels 1, 2, ... on the right.

`PadLeft[list]`
turns the ragged list *list* into a regular list by adding 0 on the left.

```
>> PadLeft[{1, 2, 3}, 5]
{0, 0, 1, 2, 3}

>> PadLeft[x[a, b, c], 5]
x[0, 0, a, b, c]

>> PadLeft[{1, 2, 3}, 2]
{2, 3}
```

```
>> PadLeft[{}, {1, 2}, {1, 2, 3}]
{{0,0,0}, {0,1,2}, {1,2,3}}

>> PadLeft[{1, 2, 3}, 10, {a, b, c}, 2]
{b,c,a,b,c,1,2,3,a,b}

>> PadLeft[{{1, 2, 3}}, {5, 2}, x, 1]
{{x,x}, {x,x}, {x,x}, {3,x}, {x,x}}
```

PadRight

`PadRight[list, n]`
pads *list* to length *n* by adding 0 on the right.

`PadRight[list, n, x]`
pads *list* to length *n* by adding *x* on the right.

`PadRight[list, {n1, $n2, ...}, x]`
pads *list* to lengths *n1*, *n2* at levels 1, 2, ... respectively by adding *x* on the right.

`PadRight[list, n, x, m]`
pads *list* to length *n* by adding *x* on the left and adding a margin of *m* on the left.

`PadRight[list, n, x, {m1, m2, ...}]`
pads *list* to length *n* by adding *x* on the right and adding margins of *m1*, *m2*, ... on levels 1, 2, ... on the left.

`PadRight[list]`
turns the ragged list *list* into a regular list by adding 0 on the right.

```
>> PadRight[{1, 2, 3}, 5]
{1,2,3,0,0}

>> PadRight[x[a, b, c], 5]
x[a,b,c,0,0]

>> PadRight[{1, 2, 3}, 2]
{1,2}

>> PadRight[{{}}, {1, 2}, {1, 2, 3}]
{{0,0,0}, {1,2,0}, {1,2,3}}

>> PadRight[{1, 2, 3}, 10, {a, b, c}, 2]
{b,c,1,2,3,a,b,c,a,b}
```

```
>> PadRight[{{1, 2, 3}}, {5, 2}, x, 1]
{{x,x}, {x,1}, {x,x}, {x,x}, {x,x}}
```

Part

`Part[expr, i]`
returns part *i* of *expr*.

Extract an element from a list:

```
>> A = {a, b, c, d};

>> A[[3]]
c
```

Negative indices count from the end:

```
>> {a, b, c}][[-2]]
b
```

Part can be applied on any expression, not necessarily lists.

```
>> (a + b + c)[[2]]
b
```

`expr[[0]]` gives the head of *expr*:

```
>> (a + b + c)[[0]]
Plus
```

Parts of nested lists:

```
>> M = {{a, b}, {c, d}};

>> M[[1, 2]]
b
```

You can use `Span` to specify a range of parts:

```
>> {1, 2, 3, 4}][[2;;4]]
{2,3,4}

>> {1, 2, 3, 4}][[2;;-1]]
{2,3,4}
```

A list of parts extracts elements at certain indices:

```
>> {a, b, c, d}][[{1, 3, 3}]]
{a,c,c}
```

Get a certain column of a matrix:

```
>> B = {{a, b, c}, {d, e, f}, {g, h, i}};

>> B[[2]]
{b,c,i}
```

```
>> B[;;, 2]]
{b,e,h}
```

Extract a submatrix of 1st and 3rd row and the two last columns:

```
>> B = {{1, 2, 3}, {4, 5, 6}, {7,
8, 9}};
```

```
>> B[{{1, 3}, -2;;-1]]
{{2,3}, {8,9}}
```

Further examples:

```
>> (a+b+c+d)[[-1;;-2]]
0
```

```
>> x[[2]]
Part specification is longer than depth of object.
x[[2]]
```

Assignments to parts are possible:

```
>> B[;;, 2]] = {10, 11, 12}
{10,11,12}
```

```
>> B
{{1,10,3}, {4,11,6}, {7,12,9}}
```

```
>> B[;;, 3]] = 13
13
```

```
>> B
{{1,10,13}, {4,11,13}, {7,12,13}}
```

```
>> B[[1;;-2]] = t;
```

```
>> B
{t,t, {7,12,13}}
```

```
>> F = Table[i*j*k, {i, 1, 3}, {j,
1, 3}, {k, 1, 3}];
```

```
>> F[;;, All, 2 ;; 3, 2]] = t;
```

```
>> F
{{{1,2,3}, {2,t,6}, {3,t,9}},
 {{2,4,6}, {4,t,12}, {6,t,18}},
 {{3,6,9}, {6,t,18}, {9,t,27}}}
```

```
>> F[;;, All, 1 ;; 2, 3 ;; 3]] = k;
```

```
>> F
{{{1,2,k}, {2,t,k}, {3,t,9}},
 {{2,4,k}, {4,t,k}, {6,t,18}},
 {{3,6,k}, {6,t,k}, {9,t,27}}}
```

Of course, part specifications have precedence

over most arithmetic operations:

```
>> A[[1]] + B[[2]] + C[[3]] // Hold
// FullForm
Hold[Plus[Part[A,1],
Part[B,2],Part[C,3]]]
```

Partition

`Partition[list, n]`
partitions *list* into sublists of length *n*.
`Partition[list, n, d]`
partitions *list* into sublists of length *n*
which overlap *d* indices.

```
>> Partition[{a, b, c, d, e, f}, 2]
{{a,b}, {c,d}, {e,f}}
```

```
>> Partition[{a, b, c, d, e, f}, 3,
1]
{{a,b,c}, {b,c,d}, {c,d,e}, {d,e,f}}
```

Permutations

`Permutations[list]`
gives all possible orderings of the items
in *list*.
`Permutations[list, n]`
gives permutations up to length *n*.
`Permutations[list, {n}]`
gives permutations of length *n*.

```
>> Permutations[{y, 1, x}]
{{y,1,x}, {y,x,1}, {1,y,x},
 {1,x,y}, {x,y,1}, {x,1,y}}
```

Elements are differentiated by their position in *list*, not their value.

```
>> Permutations[{a, b, b}]
{{a,b,b}, {a,b,b}, {b,a,b},
 {b,b,a}, {b,a,b}, {b,b,a}}
```

```
>> Permutations[{1, 2, 3}, 2]
{{}, {1}, {2}, {3}, {1,2}, {1,
3}, {2,1}, {2,3}, {3,1}, {3,2}}
```

```
>> Permutations[{1, 2, 3}, {2}]
{{1,2}, {1,3}, {2,1},
 {2,3}, {3,1}, {3,2}}
```

Pick

```
Pick[list, sel]
  returns those items in list that are True in
  sel.
Pick[list, sel, patt]
  returns those items in list that match patt
  in sel.
```

```
>> Pick[{a, b, c}, {False, True,
False}]
{b}

>> Pick[f[g[1, 2], h[3, 4]], {{True
, False}, {False, True}}]
f[g[1], h[4]]

>> Pick[{a, b, c, d, e}, {1, 2,
3.5, 4, 5.5}, _Integer]
{a, b, d}
```

Position

```
Position[expr, patt]
  returns the list of positions for which expr
  matches patt.
Position[expr, patt, ls]
  returns the positions on levels specified
  by levelspec ls.
```

```
>> Position[{1, 2, 2, 1, 2, 3, 2},
2]
{{2}, {3}, {5}, {7}}
```

Find positions upto 3 levels deep

```
>> Position[{1 + Sin[x], x, (Tan[x]
- y)^2}, x, 3]
{{1, 2, 1}, {2}}
```

Find all powers of x

```
>> Position[{1 + x^2, x y ^ 2, 4 y,
x ^ z}, x^_]
{{1, 2}, {4}}
```

Use Position as an operator

```
>> Position[_Integer][{1.5, 2,
2.5}]
{{2}}
```

Prepend

```
Prepend[expr, item]
  returns expr with item prepended to its
  leaves.
```

Prepend is similar to Append, but adds *item* to the beginning of *expr*:

```
>> Prepend[{2, 3, 4}, 1]
{1, 2, 3, 4}
```

Prepend works on expressions with heads other than List:

```
>> Prepend[f[b, c], a]
f[a, b, c]
```

Unlike Join, Prepend does not flatten lists in *item*:

```
>> Prepend[{c, d}, {a, b}]
{{a, b}, c, d}
```

PrependTo

```
PrependTo[s, item]
  prepends item to value of s and sets s to
  the result.
```

Assign *s* to a list

```
>> s = {1, 2, 4, 9}
{1, 2, 4, 9}
```

Add a new value at the beginning of the list:

```
>> PrependTo[s, 0]
{0, 1, 2, 4, 9}
```

The value assigned to *s* has changed:

```
>> s
{0, 1, 2, 4, 9}
```

PrependTo works with a head other than List:

```
>> y = f[a, b, c];
```

```
>> PrependTo[y, x]
f[x, a, b, c]
```

```
>> y
f[x, a, b, c]
```

Quantile

`Quantile[list, q]`
returns the q th quantile of *list*.

```
>> Quantile[Range[11], 1/3]
4
>> Quantile[Range[16], 1/4]
5
```

Quartiles

`Quartiles[list]`
returns the 1/4, 1/2, and 3/4 quantiles of *list*.

```
>> Quartiles[Range[25]]
{ 27/4, 13, 77/4 }
```

Range

`Range[n]`
returns a list of integers from 1 to n .
`Range[a, b]`
returns a list of integers from a to b .

```
>> Range[5]
{1, 2, 3, 4, 5}
>> Range[-3, 2]
{-3, -2, -1, 0, 1, 2}
>> Range[0, 2, 1/3]
{0, 1/3, 2/3, 1, 4/3, 5/3, 2}
```

RankedMax

`RankedMax[list, n]`
returns the n th largest element of *list* (with $n = 1$ yielding the largest element, $n = 2$ yielding the second largest element, and so on).

```
>> RankedMax[{482, 17, 181, -12},
2]
181
```

RankedMin

`RankedMin[list, n]`
returns the n th smallest element of *list* (with $n = 1$ yielding the smallest element, $n = 2$ yielding the second smallest element, and so on).

```
>> RankedMin[{482, 17, 181, -12},
2]
17
```

Reap

`Reap[expr]`
gives the result of evaluating *expr*, together with all values sown during this evaluation. Values sown with different tags are given in different lists.

`Reap[expr, pattern]`
only yields values sown with a tag matching *pattern*. `Reap[expr]` is equivalent to `Reap[expr, _]`.

`Reap[expr, {pattern1, pattern2, ...}]`
uses multiple patterns.

`Reap[expr, pattern, f]`
applies *f* on each tag and the corresponding values sown in the form $f[\text{tag}, \{e1, e2, \dots\}]$.

```
>> Reap[Sow[3]; Sow[1]]
{1, {{3, 1}}}
>> Reap[Sow[2, {x, x, x}]; Sow[3, x]
; Sow[4, y]; Sow[4, 1], {
_Symbol, _Integer, x}, f]
{4, {{f[x, {2, 2, 2, 3}], f[
y, {4}]}, {f[1, {4}]},
{f[x, {2, 2, 2, 3}]}}}
```

Find the unique elements of a list, keeping their order:

```
>> Reap[Sow[Null, {a, a, b, d, c, a
}], _, # &][[2]]
{a,b,d,c}
```

Sown values are reaped by the innermost matching Reap:

```
>> Reap[Reap[Sow[a, x]; Sow[b, 1],
_Symbol, Print["Inner: ",
#1]&];, _, f]
Inner: x
{Null, {f[1, {b}]}}
```

When no value is sown, an empty list is returned:

```
>> Reap[x]
{x, {}}
```

ReplacePart

```
ReplacePart[expr, i -> new]
  replaces part i in expr with new.
ReplacePart[expr, {{i, j} -> e1, {k,
l} -> e2}]
  replaces parts i and j with e1, and parts k
and l with e2.
```

```
>> ReplacePart[{a, b, c}, 1 -> t]
{t,b,c}

>> ReplacePart[{{a, b}, {c, d}},
{2, 1} -> t]
{{a,b}, {t,d}}

>> ReplacePart[{{a, b}, {c, d}},
{{2, 1} -> t, {1, 1} -> t}]
{{t,b}, {t,d}}

>> ReplacePart[{a, b, c}, {{1},
{2}} -> t]
{t,t,c}
```

Delayed rules are evaluated once for each replacement:

```
>> n = 1;

>> ReplacePart[{a, b, c, d}, {{1},
{3}} :> n++]
{1,b,2,d}
```

Non-existing parts are simply ignored:

```
>> ReplacePart[{a, b, c}, 4 -> t]
{a,b,c}
```

You can replace heads by replacing part 0:

```
>> ReplacePart[{a, b, c}, 0 ->
Times]
abc
```

(This is equivalent to Apply.)

Negative part numbers count from the end:

```
>> ReplacePart[{a, b, c}, -1 -> t]
{a,b,t}
```

Rest

```
Rest[expr]
  returns expr with the first element re-
  moved.
```

Rest[expr] is equivalent to expr[[2;]].

```
>> Rest[{a, b, c}]
{b,c}

>> Rest[a + b + c]
b + c

>> Rest[x]
Nonatomic expression expected.
Rest[x]
```

Reverse

```
Reverse[expr]
  reverses the order of expr's items (on the
  top level)
Reverse[expr, n]
  reverses the order of items in expr on level
  n
Reverse[expr, {n1, n2, ...}]
  reverses the order of items in expr on lev-
  els n1, n2, ...
```

```
>> Reverse[{1, 2, 3}]
{3,2,1}

>> Reverse[x[a, b, c]]
x[c,b,a]
```

```
>> Reverse[{{1, 2}, {3, 4}}, 1]
{{3,4}, {1,2}}

>> Reverse[{{1, 2}, {3, 4}}, 2]
{{2,1}, {4,3}}

>> Reverse[{{1, 2}, {3, 4}}, {1,
2}]
{{4,3}, {2,1}}
```

Riffle

`Riffle[list, x]`
inserts a copy of *x* between each element of *list*.

`Riffle[{a1, a2, ...}, {b1, b2, ...}]`
interleaves the elements of both lists, returning {a1, b1, a2, b2, ...}.

```
>> Riffle[{a, b, c}, x]
{a,x,b,x,c}

>> Riffle[{a, b, c}, {x, y, z}]
{a,x,b,y,c,z}

>> Riffle[{a, b, c, d, e, f}, {x, y,
z}]
{a,x,b,y,c,z,d,x,e,y,f}
```

RotateLeft

`RotateLeft[expr]`
rotates the items of *expr* by one item to the left.

`RotateLeft[expr, n]`
rotates the items of *expr* by *n* items to the left.

`RotateLeft[expr, {n1, n2, ...}]`
rotates the items of *expr* by *n1* items to the left at the first level, by *n2* items to the left at the second level, and so on.

```
>> RotateLeft[{1, 2, 3}]
{2,3,1}

>> RotateLeft[Range[10], 3]
{4,5,6,7,8,9,10,1,2,3}
```

```
>> RotateLeft[x[a, b, c], 2]
x[c,a,b]

>> RotateLeft[{{a, b, c}, {d, e, f
}, {g, h, i}}, {1, 2}]
{{f,d,e}, {i,g,h}, {c,a,b}}
```

RotateRight

`RotateRight[expr]`
rotates the items of *expr* by one item to the right.

`RotateRight[expr, n]`
rotates the items of *expr* by *n* items to the right.

`RotateRight[expr, {n1, n2, ...}]`
rotates the items of *expr* by *n1* items to the right at the first level, by *n2* items to the right at the second level, and so on.

```
>> RotateRight[{1, 2, 3}]
{3,1,2}

>> RotateRight[Range[10], 3]
{8,9,10,1,2,3,4,5,6,7}

>> RotateRight[x[a, b, c], 2]
x[b,c,a]

>> RotateRight[{{a, b, c}, {d, e, f
}, {g, h, i}}, {1, 2}]
{{h,i,g}, {b,c,a}, {e,f,d}}
```

Select

`Select[{e1, e2, ...}, f]`
returns a list of the elements *ei* for which *f[ei]* returns True.

Find numbers greater than zero:

```
>> Select[{-3, 0, 1, 3, a}, #>0&]
{1,3}
```

Select works on an expression with any head:

```
>> Select[f[a, 2, 3], NumberQ]
f[2,3]
```



```
>> Select[a, True]
Nonatomicexpressionexpected.
Select[a, True]
```

Skewness

Skewness[*list*]
gives Pearson's moment coefficient of skewness for *list* (a measure for estimating the symmetry of a distribution).

```
>> Skewness[{1.1, 1.2, 1.4, 2.1, 2.4}]
0.407041
```

Sow

Sow[*e*]
sends the value *e* to the innermost Reap.

Sow[*e*, *tag*]
sows *e* using *tag*. Sow[*e*] is equivalent to Sow[*e*, Null].

Sow[*e*, {*tag1*, *tag2*, ...}]
uses multiple tags.

Span (; ;)

Span
is the head of span ranges like 1 ;; 3.

```
>> ;; // FullForm
Span[1, All]

>> 1 ;; 4 ;; 2 // FullForm
Span[1, 4, 2]

>> 2 ;; -2 // FullForm
Span[2, -2]

>> ;; 3 // FullForm
Span[1, 3]
```

Split

Split[*list*]
splits *list* into collections of consecutive identical elements.

Split[*list*, *test*]
splits *list* based on whether the function *test* yields True on consecutive elements.

```
>> Split[{x, x, x, y, x, y, y, z}]
{{x, x, x}, {y}, {x}, {y, y}, {z}}
```

Split into increasing or decreasing runs of elements

```
>> Split[{1, 5, 6, 3, 6, 1, 6, 3, 4, 5, 4}, Less]
{{1, 5, 6}, {3, 6}, {1, 6}, {3, 4, 5}, {4}}

>> Split[{1, 5, 6, 3, 6, 1, 6, 3, 4, 5, 4}, Greater]
{{1}, {5}, {6, 3}, {6, 1}, {6, 3}, {4}, {5, 4}}
```

Split based on first element

```
>> Split[{x -> a, x -> y, 2 -> a, z -> c, z -> a}, First[#1] === First[#2] &]
{{x -> a, x -> y}, {2 -> a}, {z -> c, z -> a}}
```

SplitBy

SplitBy[*list*, *f*]
splits *list* into collections of consecutive elements that give the same result when *f* is applied.

```
>> SplitBy[Range[1, 3, 1/3], Round]
{{1, 4/3}, {5/3, 2, 7/3}, {8/3, 3}}

>> SplitBy[{1, 2, 1, 1.2}, {Round, Identity}]
{{{1}}, {{2}}, {{1}, {1.2}}}
```

StandardDeviation

`StandardDeviation[list]`

computes the standard deviation of *list*.
list may consist of numerical values or symbols. Numerical values may be real or complex.

`StandardDeviation[{a1, a2, ...}, {b1, b2, ...}, ...]` will yield `{StandardDeviation[{a1, b1, ...}, StandardDeviation[{a2, b2, ...}], ...}`.

```
>> StandardDeviation[{1, 2, 3}]
1

>> StandardDeviation[{7, -5, 101,
100}]

$$\frac{\sqrt{13297}}{2}$$


>> StandardDeviation[{a, a}]
0

>> StandardDeviation[{{1, 10}, {-1,
20}}]

$$\{\sqrt{2}, 5\sqrt{2}\}$$

```

SubsetQ

`SubsetQ[list1, list2]`

returns True if *list2* is a subset of *list1*, and False otherwise.

```
>> SubsetQ[{1, 2, 3}, {3, 1}]
True
```

The empty list is a subset of every list:

```
>> SubsetQ[{}, {}]
True
```

```
>> SubsetQ[{1, 2, 3}, {}]
True
```

Every list is a subset of itself:

```
>> SubsetQ[{1, 2, 3}, {1, 2, 3}]
True
```

Table

`Table[expr, {i, n}]`

evaluates *expr* with *i* ranging from 1 to *n*, returning a list of the results.

`Table[expr, {i, start, stop, step}]`

evaluates *expr* with *i* ranging from *start* to *stop*, incrementing by *step*.

`Table[expr, {i, {e1, e2, ..., ei}}]`

evaluates *expr* with *i* taking on the values *e1, e2, ..., ei*.

```
>> Table[x, {4}]
{x, x, x, x}

>> n = 0;

>> Table[n = n + 1, {5}]
{1, 2, 3, 4, 5}

>> Table[i, {i, 4}]
{1, 2, 3, 4}

>> Table[i, {i, 2, 5}]
{2, 3, 4, 5}

>> Table[i, {i, 2, 6, 2}]
{2, 4, 6}

>> Table[i, {i, Pi, 2 Pi, Pi / 2}]
{Pi,  $\frac{3Pi}{2}$ , 2Pi}

>> Table[x^2, {x, {a, b, c}}]
{a^2, b^2, c^2}
```

Table supports multi-dimensional tables:

```
>> Table[{i, j}, {i, {a, b}}, {j,
1, 2}]
{{{a, 1}, {a, 2}}, {{b, 1}, {b, 2}}}
```

Take

`Take[expr, n]`

returns *expr* with all but the first *n* leaves removed.

```
>> Take[{a, b, c, d}, 3]
{a, b, c}
```

```
>> Take[{a, b, c, d}, -2]
      {c, d}

>> Take[{a, b, c, d, e}, {2, -2}]
      {b, c, d}
```

Take a submatrix:

```
>> A = {{a, b, c}, {d, e, f}};

>> Take[A, 2, 2]
      {{a, b}, {d, e}}
```

Take a single column:

```
>> Take[A, All, {2}]
      {{b}, {e}}
```

TakeLargest

`TakeLargest[list, f, n]`
returns the a sorted list of the n largest items in *list*.

```
>> TakeLargest[{100, -1, 50, 10},
                2]
      {100, 50}
```

None, Null, Indeterminate and expressions with head Missing are ignored by default:

```
>> TakeLargest[{-8, 150, Missing[
  abc]}, 2]
      {150, -8}
```

You may specify which items are ignored using the option `ExcludedForms`:

```
>> TakeLargest[{-8, 150, Missing[
  abc]}, 2, ExcludedForms -> {}]
      {Missing[abc], 150}
```

TakeLargestBy

`TakeLargestBy[list, f, n]`
returns the a sorted list of the n largest items in *list* using *f* to retrieve the items' keys to compare them.

For details on how to use the `ExcludedForms` option, see `TakeLargest[]`.

```
>> TakeLargestBy[{{1, -1}, {10,
  100}, {23, 7, 8}, {5, 1}}, Total
  , 2]
      {{10, 100}, {23, 7, 8}}

>> TakeLargestBy[{"abc", "ab", "x
  "}, StringLength, 1]
      {abc}
```

TakeSmallest

`TakeSmallest[list, f, n]`
returns the a sorted list of the n smallest items in *list*.

For details on how to use the `ExcludedForms` option, see `TakeLargest[]`.

```
>> TakeSmallest[{100, -1, 50, 10},
                2]
      {-1, 10}
```

TakeSmallestBy

`TakeSmallestBy[list, f, n]`
returns the a sorted list of the n smallest items in *list* using *f* to retrieve the items' keys to compare them.

For details on how to use the `ExcludedForms` option, see `TakeLargest[]`.

```
>> TakeSmallestBy[{{1, -1}, {10,
  100}, {23, 7, 8}, {5, 1}}, Total
  , 2]
      {{1, -1}, {5, 1}}

>> TakeSmallestBy[{"abc", "ab", "x
  "}, StringLength, 1]
      {x}
```

Tally

Tally[*list*]

counts and returns the number of occurrences of objects and returns the result as a list of pairs {object, count}.

Tally[*list*, *test*]

counts the number of occurrences of objects and uses *\$test* to determine if two objects should be counted in the same bin.

```
>> Tally[{a, b, c, b, a}]
{{a,2}, {b,2}, {c,1}}
```

Tally always returns items in the order as they first appear in *list*:

```
>> Tally[{b, b, a, a, a, d, d, d, d, c}]
{{b,2}, {a,3}, {d,4}, {c,1}}
```

Total

Total[*list*]

adds all values in *list*.

Total[*list*, *n*]

adds all values up to level *n*.

Total[*list*, {*n*}]

totals only the values at level {*n*}.

Total[*list*, {*n*₁, *n*₂}]

totals at levels {*n*₁, *n*₂}.

```
>> Total[{1, 2, 3}]
6

>> Total[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}]
{12,15,18}
```

Total over rows and columns

```
>> Total[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}, 2]
45
```

Total over rows instead of columns

```
>> Total[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}, {2}]
{6,15,24}
```

Tuples

Tuples[*list*, *n*]

returns a list of all *n*-tuples of elements in *list*.

Tuples[{*list*₁, *list*₂, ...}]

returns a list of tuples with elements from the given lists.

```
>> Tuples[{a, b, c}, 2]
{{a,a}, {a,b}, {a,c}, {b,a}, {b,b}, {b,c}, {c,a}, {c,b}, {c,c}}

>> Tuples[{}, 2]
{}

>> Tuples[{a, b, c}, 0]
{{}}

>> Tuples[{{a, b}, {1, 2, 3}}]
{{a,1}, {a,2}, {a,3}, {b,1}, {b,2}, {b,3}}
```

The head of *list* need not be List:

```
>> Tuples[f[a, b, c], 2]
{f[a,a], f[a,b], f[a,c], f[b,a], f[b,b], f[b,c], f[c,a], f[c,b], f[c,c]}
```

However, when specifying multiple expressions, List is always used:

```
>> Tuples[{f[a, b], g[c, d]]}
{{a,c}, {a,d}, {b,c}, {b,d}}
```

Union

Union[*a*, *b*, ...]

gives the union of the given set or sets. The resulting list will be sorted and each element will only occur once.

```
>> Union[{5, 1, 3, 7, 1, 8, 3}]
{1,3,5,7,8}

>> Union[{a, b, c}, {c, d, e}]
{a,b,c,d,e}

>> Union[{c, b, a}]
{a,b,c}
```

```
>> Union[{{a, 1}, {b, 2}}, {{c, 1},
      {d, 3}}, SameTest->(SameQ[Last
      [#1], Last[#2]]&)]
{{b, 2}, {c, 1}, {d, 3}}
>> Union[{1, 2, 3}, {2, 3, 4},
      SameTest->Less]
{1, 2, 2, 3, 4}
```

UnitVector

`UnitVector[n, k]`
 returns the n -dimensional unit vector
 with a 1 in position k .
`UnitVector[k]`
 is equivalent to `UnitVector[2, k]`.

```
>> UnitVector[2]
{0, 1}
>> UnitVector[4, 3]
{0, 0, 1, 0}
```

Values

`Values[<|key1 -> val1, key2 -> val2, ...|>]`
 return a list of the values *vali* in an asso-
 ciation.
`Values[{key1 -> val1, key2 -> val2, ...}]`
 return a list of the *vali* in a list of rules.

```
>> Values[<|a -> x, b -> y|>]
{x, y}
>> Values[{a -> x, b -> y}]
{x, y}
```

Values automatically threads over lists:

```
>> Values[{<|a -> x, b -> y|>, {c
-> z, {}}}]
{{x, y}, {z, {}}}
```

Values are listed in the order of their appearance:

```
>> Values[{c -> z, b -> y, a -> x}]
{z, y, x}
```

Variance

`Variance[list]`
 computes the variance of \$list. *list* may
 consist of numerical values or symbols.
 Numerical values may be real or com-
 plex.
`Variance[{a1, a2, ...}, {b1, b2, ...}, ...]` will yield
`{Variance[a1, b1, ...], Variance[a2, b2, ...], ...}`.

```
>> Variance[{1, 2, 3}]
1
>> Variance[{7, -5, 101, 3}]
 $\frac{7475}{3}$ 
>> Variance[{1 + 2I, 3 - 10I}]
74
>> Variance[{a, a}]
0
>> Variance[{{1, 3, 5}, {4, 10,
100}}]
 $\left\{\frac{9}{2}, \frac{49}{2}, \frac{9025}{2}\right\}$ 
```

XXIII. Logic

Contents

AllTrue	174	False	175	Or ()	175
And (&&)	174	Implies (=>)	175	True	175
AnyTrue	174	NoneTrue	175	Xor (xor)	176
Equivalent (==)	175	Not (!)	175		

AllTrue

AllTrue[{*expr1*, *expr2*, ...}, *test*]
returns True if all applications of *test* to *expr1*, *expr2*, ... evaluate to True.

AllTrue[*list*, *test*, *level*]
returns True if all applications of *test* to items of *list* at *level* evaluate to True.

AllTrue[*test*]
gives an operator that may be applied to expressions.

```
>> AllTrue[{2, 4, 6}, EvenQ]
True
>> AllTrue[{2, 4, 7}, EvenQ]
False
```

And (&&)

And[*expr1*, *expr2*, ...]
expr1 && *expr2* && ...
evaluates each expression in turn, returning False as soon as an expression evaluates to False. If all expressions evaluate to True, And returns True.

```
>> True && True && False
False
```

If an expression does not evaluate to True or False, And returns a result in symbolic form:

```
>> a && b && True && c
a&& b&& c
```

AnyTrue

AnyTrue[{*expr1*, *expr2*, ...}, *test*]
returns True if any application of *test* to *expr1*, *expr2*, ... evaluates to True.

AnyTrue[*list*, *test*, *level*]
returns True if any application of *test* to items of *list* at *level* evaluates to True.

AnyTrue[*test*]
gives an operator that may be applied to expressions.

```
>> AnyTrue[{1, 3, 5}, EvenQ]
False
>> AnyTrue[{1, 4, 5}, EvenQ]
True
```

Equivalent (==)

Equivalent[*expr1*, *expr2*, ...]
expr1 == *expr2* == ...
is equivalent to (*expr1* && *expr2* && ...) || (!*expr1* && !*expr2* && ...)

```
>> Equivalent[True, True, False]
False
```

If all expressions do not evaluate to True or False, Equivalent returns a result in symbolic form:

```
>> Equivalent[a, b, c]
abc
```

Otherwise, `Equivalent` returns a result in DNF

```
>> Equivalent[a, b, True, c]
a&&b&&c
```

False

```
False
  represents the Boolean false value.
```

Implies (=>)

```
Implies[expr1, expr2]
expr1 => expr2
  evaluates each expression in turn, returning True as soon as the first expression evaluates to False. If the first expression evaluates to True, Implies returns the second expression.
```

```
>> Implies[False, a]
True

>> Implies[True, a]
a
```

If an expression does not evaluate to True or False, `Implies` returns a result in symbolic form:

```
>> Implies[a, Implies[b, Implies[
  True, c]]]
abc
```

NoneTrue

```
NoneTrue[{expr1, expr2, ...}, test]
  returns True if no application of test to expr1, expr2, ... evaluates to True.
NoneTrue[list, test, level]
  returns True if no application of test to items of list at level evaluates to True.
NoneTrue[test]
  gives an operator that may be applied to expressions.
```

```
>> NoneTrue[{1, 3, 5}, EvenQ]
True
```

```
>> NoneTrue[{1, 4, 5}, EvenQ]
False
```

Not (!)

```
Not[expr]
!expr
  negates the logical expression expr.
```

```
>> !True
False

>> !False
True

>> !b
!b
```

Or (||)

```
Or[expr1, expr2, ...]
expr1 || expr2 || ...
  evaluates each expression in turn, returning True as soon as an expression evaluates to True. If all expressions evaluate to False, Or returns False.
```

```
>> False || True
True
```

If an expression does not evaluate to True or False, `Or` returns a result in symbolic form:

```
>> a || False || b
a||b
```

True

```
True
  represents the Boolean true value.
```

Xor (xor)

```
Xor[expr1, expr2, ...]  
expr1 xor expr2 xor...  
    evaluates each expression in turn, return-  
    ing True as soon as not all expressions  
    evaluate to the same value. If all expres-  
    sions evaluate to the same value, Xor re-  
    turns False.
```

```
>> Xor[False, True]  
True
```

```
>> Xor[True, True]  
False
```

If an expression does not evaluate to True or False, Xor returns a result in symbolic form:

```
>> Xor[a, False, b]  
ab
```


XXIV. Manipulate

Contents

Manipulate	177	Sys-	
		tem'Private'ManipulateParameter	177

Manipulate

`Manipulate[expr1, {u, u_min, u_max}]`
interactively compute and display an expression with different values of u .
`Manipulate[expr1, {u, u_min, u_max, du}]`
allows u to vary between u_{min} and u_{max} in steps of du .
`Manipulate[expr1, {{u, u_init}, u_min, u_max, ...}]`
starts with initial value of u_{init} .
`Manipulate[expr1, {{u, u_init, u_lbl}, ...}]`
labels the u controll by u_{lbl} .
`Manipulate[expr1, {u, {u_1, u_2, ...}}]`
sets u to take discrete values u_1, u_2, \dots .
`Manipulate[expr1, {u, ...}, {v, ...}, ...]`
control each of u, v, \dots .

```
>> Manipulate[N[Sin[y]], {y, 1, 20, 2}]
Manipulate[N[Sin[y]], {y, 1, 20, 2}]

>> Manipulate[i^3, {i, {2, x^4, a}}]
Manipulate[i^3, {i, {2, x^4, a}}]

>> Manipulate[x^y, {x, 1, 20}, {y, 1, 3}]
Manipulate[x^y, {x, 1, 20}, {y, 1, 3}]
```

```
>> Manipulate[N[1 / x], {{x, 1}, 0, 2}]
Manipulate[N[1 / x], {{x, 1}, 0, 2}]

>> Manipulate[N[1 / x], {{x, 1}, 0, 2, 0.1}]
Manipulate[N[1 / x], {{x, 1}, 0, 2, 0.1}]
```

System'Private'ManipulateParameter

XXV. Quantities

Contents

KnownUnitQ	178	QuantityMagnitude . .	178	QuantityUnit	179
Quantity	178	QuantityQ	178	UnitConvert	179

KnownUnitQ

KnownUnitQ[*unit*]
returns True if *unit* is a canonical unit,
and False otherwise.

```
>> KnownUnitQ["Feet"]
True

>> KnownUnitQ["Foo"]
False
```

Quantity

Quantity[*magnitude*, *unit*]
represents a quantity with size *magnitude*
and unit specified by *unit*.
Quantity[*unit*]
assumes the magnitude of the specified
unit to be 1.

```
>> Quantity["Kilogram"]
1kilogram

>> Quantity[10, "Meters"]
10meter

>> Quantity[{10,20}, "Meters"]
{10meter,20meter}
```

QuantityMagnitude

QuantityMagnitude[*quantity*]
gives the amount of the specified *quantity*.
QuantityMagnitude[*quantity*, *unit*]
gives the value corresponding to *quantity*
when converted to *unit*.

```
>> QuantityMagnitude[Quantity["
Kilogram"]]
1

>> QuantityMagnitude[Quantity[10, "
Meters"]]
10

>> QuantityMagnitude[Quantity
[{10,20}, "Meters"]]
{10,20}
```

QuantityQ

QuantityQ[*expr*]
return True if *expr* is a valid Association
object, and False otherwise.

```
>> QuantityQ[Quantity[3, "Meters"]]
True

>> QuantityQ[Quantity[3, "Maters"]]
UnabletointerpretunitspecificationMaters.
False
```

QuantityUnit

`QuantityUnit[quantity]`
returns the unit associated with the specified *quantity*.

```
>> QuantityUnit[Quantity["Kilogram
"]]
kilogram

>> QuantityUnit[Quantity[10, "
Meters"]]
meter

>> QuantityUnit[Quantity[{10,20}, "
Meters"]]
{meter, meter}
```

UnitConvert

`UnitConvert[quantity, targetunit]`
converts the specified *quantity* to the specified *targetunit*.
`UnitConvert[quantity]`
converts the specified *quantity* to its "SI Base" units.

Convert from miles to kilometers:

```
>> UnitConvert[Quantity[5.2, "miles
"], "kilometers"]
8.36859kilometer
```

Convert a Quantity object to the appropriate SI base units:

```
>> UnitConvert[Quantity[3.8, "
Pounds"]]
1.72365kilogram
```

XXVI. Number theoretic functions

Contents

CoprimeQ	180	LCM	181	PrimePi	183
Divisors	180	MantissaExponent . . .	181	PrimePowerQ	183
EvenQ	180	Mod	182	PrimeQ	183
FactorInteger	181	NextPrime	182	Quotient	183
FractionalPart	181	OddQ	182	QuotientRemainder . .	183
GCD	181	PowerMod	182	RandomPrime	184
IntegerExponent	181	Prime	182		

CoprimeQ

`CoprimeQ[x, y]`
tests whether x and y are coprime by computing their greatest common divisor.

```
>> CoprimeQ[7, 9]
```

```
True
```

```
>> CoprimeQ[-4, 9]
```

```
True
```

```
>> CoprimeQ[12, 15]
```

```
False
```

`CoprimeQ` also works for complex numbers

```
>> CoprimeQ[1+2I, 1-I]
```

```
True
```

```
>> CoprimeQ[4+2I, 6+3I]
```

```
False
```

```
>> CoprimeQ[2, 3, 5]
```

```
True
```

```
>> CoprimeQ[2, 4, 5]
```

```
False
```

Divisors

`Divisors[n]`
returns a list of the integers that divide n .

```
>> Divisors[96]
```

```
{1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96}
```

```
>> Divisors[704]
```

```
{1, 2, 4, 8, 11, 16, 22, 32,
 44, 64, 88, 176, 352, 704}
```

```
>> Divisors[{87, 106, 202, 305}]
```

```
{{1, 3, 29, 87}, {1, 2, 53, 106},
 {1, 2, 101, 202}, {1, 5, 61, 305}}
```

EvenQ

`EvenQ[x]`
returns True if x is even, and False otherwise.

```
>> EvenQ[4]
```

```
True
```

```
>> EvenQ[-3]
```

```
False
```

```
>> EvenQ[n]
```

```
False
```

FactorInteger

`FactorInteger[n]`
returns the factorization of n as a list of factors and exponents.

```
>> factors = FactorInteger[2010]
{{2,1},{3,1},{5,1},{67,1}}
```

To get back the original number:

```
>> Times @@ Power @@@ factors
2010
```

`FactorInteger` factors rationals using negative exponents:

```
>> FactorInteger[2010 / 2011]
{{2,1},{3,1},{5,1},
 {67,1},{2011,-1}}
```

FractionalPart

`FractionalPart[n]`
finds the fractional part of n .

```
>> FractionalPart[4.1]
0.1
```

```
>> FractionalPart[-5.25]
-0.25
```

GCD

`GCD[n1, n2, ...]`
computes the greatest common divisor of the given integers.

```
>> GCD[20, 30]
10
```

```
>> GCD[10, y]
GCD[10, y]
```

GCD is Listable:

```
>> GCD[4, {10, 11, 12, 13, 14}]
{2,1,4,1,2}
```

GCD does not work for rational numbers and Gaussian integers yet.

IntegerExponent

`IntegerExponent[n, b]`
gives the highest exponent of b that divides n .

```
>> IntegerExponent[16, 2]
4
```

```
>> IntegerExponent[-510000]
4
```

```
>> IntegerExponent[10, b]
IntegerExponent[10, b]
```

LCM

`LCM[n1, n2, ...]`
computes the least common multiple of the given integers.

```
>> LCM[15, 20]
60
```

```
>> LCM[20, 30, 40, 50]
600
```

MantissaExponent

`MantissaExponent[n]`
finds a list containing the mantissa and exponent of a given number n .
`MantissaExponent[n, b]`
finds the base b mantissa and exponent of n .

```
>> MantissaExponent[2.5*10^20]
{0.25,21}
```

```
>> MantissaExponent[125.24]
{0.12524,3}
```

```
>> MantissaExponent[125., 2]
{0.976563,7}
```

```
>> MantissaExponent[10, b]
MantissaExponent[10, b]
```

Mod

`Mod[x, m]`
returns x modulo m .

```
>> Mod[14, 6]
2
>> Mod[-3, 4]
1
>> Mod[-3, -4]
-3
>> Mod[5, 0]
The argument 0 should be nonzero.
Mod[5, 0]
```

NextPrime

`NextPrime[n]`
gives the next prime after n .
`NextPrime[n, k]`
gives the k th prime after n .

```
>> NextPrime[10000]
10007
>> NextPrime[100, -5]
73
>> NextPrime[10, -5]
-2
>> NextPrime[100, 5]
113
>> NextPrime[5.5, 100]
563
>> NextPrime[5, 10.5]
NextPrime[5, 10.5]
```

OddQ

`OddQ[x]`
returns True if x is odd, and False otherwise.

```
>> OddQ[-3]
True
```

```
>> OddQ[0]
False
```

PowerMod

`PowerMod[x, y, m]`
computes x^y modulo m .

```
>> PowerMod[2, 10000000, 3]
1
>> PowerMod[3, -2, 10]
9
>> PowerMod[0, -1, 2]
0 is not invertible modulo 2.
PowerMod[0, -1, 2]
>> PowerMod[5, 2, 0]
The argument 0 should be nonzero.
PowerMod[5, 2, 0]
```

PowerMod does not support rational coefficients (roots) yet.

Prime

`Prime[n]`
returns the n th prime number.

```
>> Prime[1]
2
>> Prime[167]
991
```

PrimePi

`PrimePi[x]`
gives the number of primes less than or equal to x .

```
>> PrimePi[100]
25
>> PrimePi[-1]
0
>> PrimePi[3.5]
2
```

```
>> PrimePi[E]
1
```

PrimePowerQ

`PrimePowerQ[n]`
returns True if n is a power of a prime number.

```
>> PrimePowerQ[9]
True

>> PrimePowerQ[52142]
False

>> PrimePowerQ[-8]
True

>> PrimePowerQ[371293]
True
```

PrimeQ

`PrimeQ[n]`
returns True if n is a prime number.

For very large numbers, PrimeQ uses probabilistic prime testing, so it might be wrong sometimes (a number might be composite even though PrimeQ says it is prime). The algorithm might be changed in the future.

```
>> PrimeQ[2]
True

>> PrimeQ[-3]
True

>> PrimeQ[137]
True

>> PrimeQ[2 ^ 127 - 1]
True
```

All prime numbers between 1 and 100:

```
>> Select[Range[100], PrimeQ]
{2, 3, 5, 7, 11, 13, 17, 19, 23,
 29, 31, 37, 41, 43, 47, 53, 59,
 61, 67, 71, 73, 79, 83, 89, 97}
```

PrimeQ has attribute Listable:

```
>> PrimeQ[Range[20]]
{False, True, True, False, True,
 False, True, False, False, False,
 True, False, True, False, False,
 False, True, False, True, False}
```

Quotient

`Quotient[m, n]`
computes the integer quotient of m and n .

```
>> Quotient[23, 7]
3
```

QuotientRemainder

`QuotientRemainder[m, n]`
computes a list of the quotient and remainder from division of m by n .

```
>> QuotientRemainder[23, 7]
{3, 2}
```

RandomPrime

`RandomPrime[{imin, $imax}]`
gives a random prime between $imin$ and $imax$.
`RandomPrime[imax]`
gives a random prime between 2 and $imax$.
`RandomPrime[range, n]`
gives a list of n random primes in $range$.

```
>> RandomPrime[{14, 17}]
17

>> RandomPrime[{14, 16}, 1]
There are no primes in the specified interval.
RandomPrime[{14, 16}, 1]

>> RandomPrime[{8, 12}, 3]
{11, 11, 11}
```

```
>> RandomPrime[{10,30}, {2,5}]  
{ {23,23,23,23,23} ,  
  {23,23,23,23,23} }
```


XXVII. Numeric evaluation

Support for numeric evaluation with arbitrary precision is just a proof-of-concept. Precision is not “guarded” through the evaluation process. Only integer precision is supported. However, things like `N[Pi, 100]` should work as expected.

Contents

<code>Chop</code>	185	<code>\$MaxPrecision</code>	186	<code>RealDigits</code>	189
<code>Hash</code>	185	<code>\$MinPrecision</code>	187	<code>Inter-</code>	
<code>IntegerDigits</code>	186	<code>N</code>	188	<code>nal'RealValuedNumberQ</code>	189
<code>\$MachineEpsilon</code>	186	<code>NumericQ</code>	188	<code>Inter-</code>	
<code>MachinePrecision</code>	186	<code>Precision</code>	188	<code>nal'RealValuedNumericQ</code>	189
<code>\$MachinePrecision</code>	186	<code>Rationalize</code>	188	<code>Round</code>	189

Chop

`Chop[expr]`
replaces floating point numbers close to 0 by 0.

`Chop[expr, delta]`
uses a tolerance of *delta*. The default tolerance is 10^{-10} .

```
>> Chop[10.0 ^ -16]
0
>> Chop[10.0 ^ -9]
1. × 10-9
>> Chop[10 ^ -11 I]
 $\frac{I}{100\,000\,000\,000}$ 
>> Chop[0. + 10 ^ -11 I]
0
```

Hash

`Hash[expr]`
returns an integer hash for the given *expr*.

`Hash[expr, type]`
returns an integer hash of the specified *type* for the given *expr*.
The types supported are “MD5”, “Adler32”, “CRC32”, “SHA”, “SHA224”, “SHA256”, “SHA384”, and “SHA512”.

```
> Hash["The Adventures of Huckleberry Finn"]
= 213425047836523694663619736686226550816
> Hash["The Adventures of Huckleberry Finn",
"SHA256"] = 950926495945903842880571834086092549189343518
> Hash[1/3] = 56073172797010645108327809727054836008
> Hash[{a, b, {c, {d, e, f}}}] = 135682164776235407777080772547528
> Hash[SomeHead[3.1415]] = 5804231647347187731544201546970
>> Hash[{a, b, c}, "xyzstr"]
Hash[{a, b, c}, xyzstr]
```

IntegerDigits

`IntegerDigits[n]`
returns a list of the base-10 digits in the integer n .
`IntegerDigits[n, base]`
returns a list of the base- $base$ digits in n .
`IntegerDigits[n, base, length]`
returns a list of length $length$, truncating or padding with zeroes on the left as necessary.

```
>> IntegerDigits[76543]
{7, 6, 5, 4, 3}

The sign of  $n$  is discarded:
>> IntegerDigits[-76543]
{7, 6, 5, 4, 3}

>> IntegerDigits[15, 16]
{15}

>> IntegerDigits[1234, 16]
{4, 13, 2}

>> IntegerDigits[1234, 10, 5]
{0, 1, 2, 3, 4}
```

\$MachineEpsilon

`$MachineEpsilon`
is the distance between 1.0 and the next nearest representable machine-precision number.

```
>> $MachineEpsilon
2.22045  $\times 10^{-16}$ 

>> x = 1.0 + {0.4, 0.5, 0.6}
$MachineEpsilon;

>> x - 1
{0., 0., 2.22045  $\times 10^{-16}$ }
```

MachinePrecision

`MachinePrecision`
represents the precision of machine precision numbers.

```
>> N[MachinePrecision]
15.9546

>> N[MachinePrecision, 30]
15.9545897701910033463281614204
```

\$MachinePrecision

`$MachinePrecision`
is the number of decimal digits of precision for machine-precision numbers.

```
>> $MachinePrecision
15.9546
```

\$MaxPrecision

`$MaxPrecision`
represents the maximum number of digits of precision permitted in arbitrary-precision numbers.

```
>> $MaxPrecision
 $\infty$ 

>> $MaxPrecision = 10;

>> N[Pi, 11]
Requested precision 11 is larger than $MaxPrecision. Using current
= Infinity specifies that any precision should be allowed.
3.141592654
```

\$MinPrecision

`$MinPrecision`
represents the minimum number of digits of precision permitted in arbitrary-precision numbers.

```
>> $MinPrecision
0
```

```
>> $MinPrecision = 10;
```

```
>> N[Pi, 9]
```

Requested precision 9 is smaller than \$MinPrecision. Using current \$MinPrecision of 10 instead.

```
3.141592654
```

N

`N[expr, prec]`
evaluates *expr* numerically with a precision of *prec* digits.

```
>> N[Pi, 50]
```

```
3.141592653589793238462643~  
~3832795028841971693993751
```

```
>> N[1/7]
```

```
0.142857
```

```
>> N[1/7, 5]
```

```
0.14286
```

You can manually assign numerical values to symbols. When you do not specify a precision, `MachinePrecision` is taken.

```
>> N[a] = 10.9
```

```
10.9
```

```
>> a
```

```
a
```

`N` automatically threads over expressions, except when a symbol has attributes `NHoldAll`, `NHoldFirst`, or `NHoldRest`.

```
>> N[a + b]
```

```
10.9 + b
```

```
>> N[a, 20]
```

```
a
```

```
>> N[a, 20] = 11;
```

```
>> N[a + b, 20]
```

```
11.000000000000000000 + b
```

```
>> N[f[a, b]]
```

```
f[10.9, b]
```

```
>> SetAttributes[f, NHoldAll]
```

```
>> N[f[a, b]]
```

```
f[a, b]
```

The precision can be a pattern:

```
>> N[c, p_?(#>10&)] := p
```

```
>> N[c, 3]
```

```
>> N[c, 11]
```

```
11.000000000
```

You can also use `UpSet` or `TagSet` to specify values for `N`:

```
>> N[d] ^= 5;
```

However, the value will not be stored in `UpValues`, but in `NValues` (as for `Set`):

```
>> UpValues[d]
```

```
{}
```

```
>> NValues[d]
```

```
{HoldPattern[N[d,  
MachinePrecision]]:>5}
```

```
>> e /: N[e] = 6;
```

```
>> N[e]
```

```
6.
```

Values for `N[expr]` must be associated with the head of *expr*:

```
>> f /: N[e[f]] = 7;
```

Tag f not found or too deep for an assigned rule.

You can use `Condition`:

```
>> N[g[x_, y_], p_] := x + y * Pi  
/; x + y > 3
```

```
>> SetAttributes[g, NHoldRest]
```

```
>> N[g[1, 1]]
```

```
g[1, 1]
```

```
>> N[g[2, 2]] // InputForm
```

```
8.283185307179586
```

The precision of the result is no higher than the precision of the input

```
>> N[Exp[0.1], 100]
```

```
1.10517
```

```
>> % // Precision
```

```
MachinePrecision
```

```
>> N[Exp[1/10], 100]
1.105170918075647624811707~
~826490246668224547194737~
~518718792863289440967966~
~747654302989143318970748654

>> % // Precision
100.

>> N[Exp[1.0^20], 100]
2.7182818284590452354

>> % // Precision
20.
```

NumericQ

`NumericQ[expr]`
tests whether *expr* represents a numeric quantity.

```
>> NumericQ[2]
True

>> NumericQ[Sqrt[Pi]]
True

>> NumberQ[Sqrt[Pi]]
False
```

Precision

`Precision[expr]`
examines the number of significant digits of *expr*.

This is rather a proof-of-concept than a full im-

plementation. Precision of compound expression is not supported yet.

```
>> Precision[1]
∞

>> Precision[1/2]
∞

>> Precision[0.5]
MachinePrecision
```

Rationalize

`Rationalize[x]`
converts a real number *x* to a nearby rational number.
`Rationalize[x, dx]`
finds the rational number within *dx* of *x* with the smallest denominator.

```
>> Rationalize[2.2]
 $\frac{11}{5}$ 
```

Not all numbers can be well approximated.

```
>> Rationalize[N[Pi]]
3.14159
```

Find the exact rational representation of `N[Pi]`

```
>> Rationalize[N[Pi], 0]
 $\frac{245850922}{78256779}$ 
```

RealDigits

`RealDigits[n]`
returns the decimal representation of the real number *n* as list of digits, together with the number of digits that are to the left of the decimal point.
`RealDigits[n, b]`
returns a list of base-*b* representation of the real number *n*.
`RealDigits[n, b, len]`
returns a list of *len* digits.
`RealDigits[n, b, len, p]`
return *len* digits starting with the coefficient of b^p

Return the list of digits and exponent:

```
>> RealDigits[123.55555]
{{1, 2, 3, 5, 5, 5, 5, 5}, {3}}

>> RealDigits[0.000012355555]
{{1, 2, 3, 5, 5, 5, 5, 5}, {-4}}

>> RealDigits[-123.55555]
{{1, 2, 3, 5, 5, 5, 5, 5}, {3}}
```

Return 25 digits of in base 10:

```
>> RealDigits[Pi, 10, 25]
{{3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9,
  7, 9, 3, 2, 3, 8, 4, 6, 2, 6, 4, 3}, 1}
```

Return an explicit recurring decimal form:

```
>> RealDigits[19 / 7]
{{2, {7, 1, 4, 2, 8, 5}}, 1}
```

20 digits starting with the coefficient of 10^{-5} :

```
>> RealDigits[Pi, 10, 20, -5]
{{9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3,
  2, 3, 8, 4, 6, 2, 6, 4, 3}, -4}
```

The 10000th digit of is an 8:

```
>> RealDigits[Pi, 10, 1, -10000]
{{8}, -9999}
```

RealDigits gives Indeterminate if more digits than the precision are requested:

```
>> RealDigits[123.45, 10, 18]
{{1, 2, 3, 4, 5, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, Indeterminate,
  Indeterminate}, 3}
```

Round to exact value

```
>> Round[2.6, 1/3]
 $\frac{8}{3}$ 
```

```
>> Round[10, Pi]
3Pi
```

Round complex numbers

```
>> Round[6/(2 + 3 I)]
1 - I
```

```
>> Round[1 + 2 I, 2 I]
2I
```

Round Negative numbers too

```
>> Round[-1.4]
-1
```

Expressions other than numbers remain unevaluated:

```
>> Round[x]
Round[x]
```

```
>> Round[1.5, k]
Round[1.5, k]
```

Internal'RealValuedNumberQ

Internal'RealValuedNumericQ

Round

`Round[expr]`
rounds *expr* to the nearest integer.
`Round[expr, k]`
rounds *expr* to the closest multiple of *k*.

```
>> Round[10.6]
11
```

```
>> Round[0.06, 0.1]
0.1
```

```
>> Round[0.04, 0.1]
0.
```

Constants can be rounded too

```
>> Round[Pi, .5]
3.
```

```
>> Round[Pi^2]
10
```

XXVIII. Options and default arguments

Contents

Default	190	NotOptionQ	190	OptionValue	191
FilterRules	190	OptionQ	191	Options	192

Default

`Default[f]`
gives the default value for an omitted parameter of `f`.
`Default[f, k]`
gives the default value for a parameter on the k th position.
`Default[f, k, n]`
gives the default value for the k th parameter out of n .

Assign values to `Default` to specify default values.

```
>> Default[f] = 1
1
>> f[x_.] := x ^ 2
>> f[]
1
```

Default values are stored in `DefaultValues`:

```
>> DefaultValues[f]
{HoldPattern[Default[f]] :> 1}
```

You can use patterns for k and n :

```
>> Default[h, k_, n_] := {k, n}
```

Note that the position of a parameter is relative to the pattern, not the matching expression:

```
>> h[] /. h[___, ___, x_., y_., ___] -> {x, y}
{{3, 5}, {4, 5}}
```

FilterRules

`FilterRules[rules, pattern]`
gives those *rules* that have a left side that matches *pattern*.
`FilterRules[rules, {pattern1, pattern2, ...}]`
gives those *rules* that have a left side that match at least one of *pattern1*, *pattern2*, ...

```
>> FilterRules[{x -> 100, y -> 1000}, x]
{x -> 100}
>> FilterRules[{x -> 100, y -> 1000, z -> 10000}, {a, b, x, z}]
{x -> 100, z -> 10000}
```

NotOptionQ

`NotOptionQ[expr]`
returns `True` if *expr* does not have the form of a valid option specification.

```
>> NotOptionQ[x]
True
>> NotOptionQ[2]
True
>> NotOptionQ["abc"]
True
>> NotOptionQ[a -> True]
False
```

OptionQ

`OptionQ[expr]`
returns True if *expr* has the form of a valid option specification.

Examples of option specifications:

```
>> OptionQ[a -> True]
True
>> OptionQ[a :> True]
True
>> OptionQ[{a -> True}]
True
>> OptionQ[{a :> True}]
True
```

OptionQ returns False if its argument is not a valid option specification:

```
>> OptionQ[x]
False
```

OptionValue

`OptionValue[name]`
gives the value of the option *name* as specified in a call to a function with `OptionsPattern`.

```
>> f[a->3] /. f[OptionsPattern[{}]]
-> {OptionValue[a]}
{3}
```

Unavailable options generate a message:

```
>> f[a->3] /. f[OptionsPattern[{}]]
-> {OptionValue[b]}
Optionnamebnotfound.
{OptionValue[b]}
```

The argument of `OptionValue` must be a symbol:

```
>> f[a->3] /. f[OptionsPattern[{}]]
-> {OptionValue[a+b]}
Argumenta
+ batposition1isexpectedtobeasymbol.
{OptionValue[a + b]}
```

However, it can be evaluated dynamically:

```
>> f[a->5] /. f[OptionsPattern[{}]]
-> {OptionValue[Symbol["a"]]}
{5}
```

Options

`Options[f]`
gives a list of optional arguments to *f* and their default values.

You can assign values to `Options` to specify options.

```
>> Options[f] = {n -> 2}
{n -> 2}
>> Options[f]
{n:>2}
>> f[x_, OptionsPattern[f]] := x ^
OptionValue[n]
>> f[x]
x2
>> f[x, n -> 3]
x3
```

Delayed option rules are evaluated just when the corresponding `OptionValue` is called:

```
>> f[a :> Print["value"]] /. f[
OptionsPattern[{}]] :> (
OptionValue[a]; Print["between
"]; OptionValue[a]);
value
between
value
```

In contrast to that, normal option rules are evaluated immediately:

```
>> f[a -> Print["value"]] /. f[
OptionsPattern[{}]] :> (
OptionValue[a]; Print["between
"]; OptionValue[a]);
value
between
```

Options must be rules or delayed rules:

```
>> Options[f] = {a}
{a}isnotavalidlistofoptionrules.
{a}
```

A single rule need not be given inside a list:

```
>> Options[f] = a -> b  
a -> b
```

```
>> Options[f]  
{a:>b}
```

Options can only be assigned to symbols:

```
>> Options[a + b] = {a -> b}  
Argumenta  
+ bat position 1 is expected to be a symbol.  
{a -> b}
```


XXIX. Patterns and rules

Some examples:

```
>> a + b + c /. a + b -> t
      c + t

>> a + 2 + b + c + x * y /.
      n_Integer + s__Symbol + rest_ ->
      {n, s, rest}
      {2, a, b + c + xy}

>> f[a, b, c, d] /. f[first_,
      rest__] -> {first, {rest}}
      {a, {b, c, d}}
```

Tests and Conditions:

```
>> f[4] /. f[x_?(# > 0&)] -> x ^ 2
      16

>> f[4] /. f[x_] /; x > 0 -> x ^ 2
      16
```

Leaves in the beginning of a pattern rather

match fewer leaves:

```
>> f[a, b, c, d] /. f[start__,
      end__] -> {{start}, {end}}
      {{a}, {b, c, d}}
```

Optional arguments using Optional:

```
>> f[a] /. f[x_, y_:3] -> {x, y}
      {a, 3}
```

Options using OptionsPattern and OptionValue:

```
>> f[y, a->3] /. f[x_,
      OptionsPattern[{a->2, b->5}]] ->
      {x, OptionValue[a], OptionValue
      [b]}
      {y, 3, 5}
```

The attributes Flat, Orderless, and OneIdentity affect pattern matching.

Contents

Alternatives ()	193	MatchQ	195	ReplaceAll (/.)	198
Blank	194	Optional (:)	196	ReplaceList	198
BlankNullSequence	194	OptionsPattern	196	ReplaceRepeated (//.)	199
BlankSequence	194	PatternTest (?)	196	RuleDelayed (:>)	199
Condition (/;)	195	Pattern	197	Rule (->)	199
Except	195	Repeated (..)	197	Shortest	199
HoldPattern	195	RepeatedNull (...)	197	Verbatim	199
Longest	195	Replace	198		

Alternatives (|)

Alternatives[*p*₁, *p*₂, ..., *p*_{*i*}]
*p*₁ | *p*₂ | ... | *p*_{*i*}
 is a pattern that matches any of the patterns '*p*₁, *p*₂, ..., *p*_{*i*}'.

```
>> a+b+c+d/.(a|b)->t
      c + d + 2t
```

Alternatives can also be used for string expressions

```
>> StringReplace["0123 3210", "1" |
      "2" -> "X"]
      0XX3 3XX0
```

Blank

```
Blank[]
-
  represents any single expression in a pattern.
Blank[h]
_h
  represents any expression with head h.
```

```
>> MatchQ[a + b, _]
True
```

Patterns of the form `_h` can be used to test the types of objects:

```
>> MatchQ[42, _Integer]
True

>> MatchQ[1.0, _Integer]
False

>> {42, 1.0, x} /. {_Integer -> "integer", _Real -> "real"} //
InputForm
{"integer", "real", x}
```

Blank only matches a single expression:

```
>> MatchQ[f[1, 2], f[_]]
False
```

BlankNullSequence

```
BlankNullSequence[]
---
  represents any sequence of expression leaves in a pattern, including an empty sequence.
```

BlankNullSequence is like BlankSequence, except it can match an empty sequence:

```
>> MatchQ[f[], f[___]]
True
```

BlankSequence

```
BlankSequence[]
--
  represents any non-empty sequence of expression leaves in a pattern.
BlankSequence[h]
__h
  represents any sequence of leaves, all of which have head h.
```

Use a BlankSequence pattern to stand for a non-empty sequence of arguments:

```
>> MatchQ[f[1, 2, 3], f[___]]
True

>> MatchQ[f[], f[___]]
False
```

`__h` will match only if all leaves have head *h*:

```
>> MatchQ[f[1, 2, 3], f[__Integer]]
True

>> MatchQ[f[1, 2.0, 3], f[__Integer]]
False
```

The value captured by a named BlankSequence pattern is a Sequence object:

```
>> f[1, 2, 3] /. f[x__] -> x
Sequence[1, 2, 3]
```

Condition (/;)

```
Condition[pattern, expr]
pattern /; expr
  places an additional constraint on pattern that only allows it to match if expr evaluates to True.
```

The controlling expression of a Condition can use variables from the pattern:

```
>> f[3] /. f[x_] /; x>0 -> t
t

>> f[-3] /. f[x_] /; x>0 -> t
f[-3]
```

Condition can be used in an assignment:

```
>> f[x_] := p[x] /; x>0
```

```
>> f[3]
p[3]

>> f[-3]
f[-3]
```

Except

`Except[c]`
represents a pattern object that matches any expression except those matching *c*.

`Except[c, p]`
represents a pattern object that matches *p* but not *c*.

```
>> Cases[{x, a, b, x, c}, Except[x]]
{a, b, c}

>> Cases[{a, 0, b, 1, c, 2, 3}, Except[1, _Integer]]
{0, 2, 3}
```

Except can also be used for string expressions:

```
>> StringReplace["Hello world!", Except[LetterCharacter] -> ""]
Helloworld
```

HoldPattern

`HoldPattern[expr]`
is equivalent to *expr* for pattern matching, but maintains it in an unevaluated form.

```
>> HoldPattern[x + x]
HoldPattern[x + x]

>> x /. HoldPattern[x] -> t
t
```

HoldPattern has attribute HoldAll:

```
>> Attributes[HoldPattern]
{HoldAll, Protected}
```

Longest

```
>> StringCases["aabaab", Longest["a" ~~__ ~~"b"]]
{aabaab}

>> StringCases["aabaab", Longest[RegularExpression["a+b"]]]
{aab, aab}
```

MatchQ

`MatchQ[expr, form]`
tests whether *expr* matches *form*.

```
>> MatchQ[123, _Integer]
True

>> MatchQ[123, _Real]
False

>> MatchQ[_Integer][123]
True
```

Optional (:)

`Optional[patt, default]`
patt : *default*
is a pattern which matches *patt*, which if omitted should be replaced by *default*.

```
>> f[x_, y_:1] := {x, y}

>> f[1, 2]
{1, 2}

>> f[a]
{a, 1}
```

Note that *symb* : *patt* represents a Pattern object. However, there is no disambiguity, since *symb* has to be a symbol in this case.

```
>> x:_ // FullForm
Pattern[x, Blank[]]

>> _:d // FullForm
Optional[Blank[], d]
```

```
>> x:_+y_:d // FullForm
Pattern[x, Plus[Blank[],
Optional[Pattern[y, Blank[]], d]]]
```

`s_.` is equivalent to `Optional[s_]` and represents an optional parameter which, if omitted, gets its value from `Default`.

```
>> FullForm[s_.]
Optional[Pattern[s, Blank[]]]

>> Default[h, k_] := k

>> h[a] /. h[x_, y_.] -> {x, y}
{a, 2}
```

OptionsPattern

`OptionsPattern[f]`
is a pattern that stands for a sequence of options given to a function, with default values taken from `Options[f]`. The options can be of the form `opt->value` or `opt:>value`, and might be in arbitrarily nested lists.

`OptionsPattern[{opt1->value1, ...}]`
takes explicit default values from the given list. The list may also contain symbols `f`, for which `Options[f]` is taken into account; it may be arbitrarily nested. `OptionsPattern[{}]` does not use any default values.

The option values can be accessed using `OptionValue`.

```
>> f[x_, OptionsPattern[{n->2}]] :=
  x ^ OptionValue[n]

>> f[x]
x^2

>> f[x, n->3]
x^3
```

Delayed rules as options:

```
>> e = f[x, n:>a]
x^a

>> a = 5;

>> e
x^5
```

Options might be given in nested lists:

```
>> f[x, {{{n->4}}}]
x^4
```

PatternTest (?)

`PatternTest[pattern, test]`
`pattern ? test`
constrains `pattern` to match `expr` only if the evaluation of `test[expr]` yields `True`.

```
>> MatchQ[3, _Integer?(#>0&)]
True

>> MatchQ[-3, _Integer?(#>0&)]
False
```

Pattern

`Pattern[symb, patt]`
`symb : patt`
assigns the name `symb` to the pattern `patt`.

`symb_head`
is equivalent to `symb : _head` (accordingly with `_` and `__`).

`symb : patt : default`
is a pattern with name `symb` and default value `default`, equivalent to `Optional[patt : symb, default]`.

```
>> FullForm[a_b]
Pattern[a, Blank[b]]

>> FullForm[a:_:b]
Optional[Pattern[a, Blank[]], b]
```

`Pattern` has attribute `HoldFirst`, so it does not evaluate its name:

```
>> x = 2
2

>> x_
x_
```

Nested `Pattern` assign multiple names to the same pattern. Still, the last parameter is the default value.

```
>> f[y] /. f[a:b, _:d] -> {a, b}
f[y]
```

This is equivalent to:

```
>> f[a] /. f[a:_:b] -> {a, b}
      {a, b}
```

FullForm:

```
>> FullForm[a:b:c:d:e]
      Optional[Pattern[a, b],
      Optional[Pattern[c, d], e]]

>> f[] /. f[a:_:b] -> {a, b}
      {b, b}
```

Repeated (..)

`Repeated[pattern]`
matches one or more occurrences of *pattern*.

```
>> a_Integer.. // FullForm
      Repeated[Pattern[a, Blank[Integer]]]

>> 0..1//FullForm
      Repeated[0]

>> {{}, {a}, {a, b}, {a, a, a}, {a,
      a, a, a}} /. {Repeated[x : a |
      b, 3]} -> x
      {{}, a, {a, b}, a, {a, a, a, a}}

>> f[x, 0, 0, 0] /. f[x, s:0..] ->
      s
      Sequence[0, 0, 0]
```

RepeatedNull (...)

`RepeatedNull[pattern]`
matches zero or more occurrences of *pattern*.

```
>> a___Integer...//FullForm
      RepeatedNull[Pattern[a,
      BlankNullSequence[Integer]]]

>> f[x] /. f[x, 0...] -> t
      t
```

Replace

`Replace[expr, x -> y]`
yields the result of replacing *expr* with *y* if it matches the pattern *x*.
`Replace[expr, x -> y, levelspec]`
replaces only subexpressions at levels specified through *levelspec*.
`Replace[expr, {x -> y, ...}]`
performs replacement with multiple rules, yielding a single result expression.
`Replace[expr, {{a -> b, ...}, {c -> d, ...}, ...}]`
returns a list containing the result of performing each set of replacements.

```
>> Replace[x, {x -> 2}]
      2
```

By default, only the top level is searched for matches

```
>> Replace[1 + x, {x -> 2}]
      1 + x

>> Replace[x, {{x -> 1}, {x -> 2}}]
      {1, 2}
```

Replace stops after the first replacement

```
>> Replace[x, {x -> {}, _List -> y}]
      {}
```

Replace replaces the deepest levels first

```
>> Replace[x[1], {x[1] -> y, 1 ->
      2}, All]
      x[2]
```

By default, heads are not replaced

```
>> Replace[x[x[y]], x -> z, All]
      x[x[y]]
```

Heads can be replaced using the Heads option

```
>> Replace[x[x[y]], x -> z, All,
      Heads -> True]
      z[z[y]]
```

Note that heads are handled at the level of leaves

```
>> Replace[x[x[y]], x -> z, {1},
      Heads -> True]
      z[x[y]]
```

You can use Replace as an operator

```
>> Replace[{x_ -> x + 1}][10]
11
```

ReplaceAll (/.)

`ReplaceAll[expr, x -> y]`
`expr /. x -> y`
 yields the result of replacing all subexpressions of `expr` matching the pattern `x` with `y`.
`expr /. {x -> y, ...}`
 performs replacement with multiple rules, yielding a single result expression.
`expr /. {{a -> b, ...}, {c -> d, ...}, ...}`
 returns a list containing the result of performing each set of replacements.

```
>> a+b+c /. c->d
a + b + d

>> g[a+b+c,a] /. g[x_+y_,x_]->{x,y}
{a,b + c}
```

If `rules` is a list of lists, a list of all possible respective replacements is returned:

```
>> {a, b} /. {{a->x, b->y}, {a->u, b->v}}
{{x,y}, {u,v}}
```

The list can be arbitrarily nested:

```
>> {a, b} /. {{{a->x, b->y}, {a->w, b->z}}, {a->u, b->v}}
{{{x,y}, {w,z}}, {u,v}}
```

```
>> {a, b} /. {{{a->x, b->y}, a->w, b->z}, {a->u, b->v}}
```

Elementsof{{a->x,b->y},a->w,b->z}areamixtureoflistsandnonlists.

```
{{a,b} /. {{a->x,b->y}, a->w,b->z}, {u,v}}
```

`ReplaceAll` also can be used as an operator:

```
>> ReplaceAll[{a -> 1}][{a, b}]
{1,b}
```

`ReplaceAll` replaces the shallowest levels first:

```
>> ReplaceAll[x[1], {x[1] -> y, 1 -> 2}]
y
```

ReplaceList

`ReplaceList[expr, rules]`
 returns a list of all possible results of applying `rules` to `expr`.

Get all subsequences of a list:

```
>> ReplaceList[{a, b, c}, {___, x_, ___} -> {x}]
{{a}, {a,b}, {a,b,c}, {b}, {b,c}, {c}}
```

You can specify the maximum number of items:

```
>> ReplaceList[{a, b, c}, {___, x_, ___} -> {x}, 3]
{{a}, {a,b}, {a,b,c}}
```

```
>> ReplaceList[{a, b, c}, {___, x_, ___} -> {x}, 0]
{}
```

If no rule matches, an empty list is returned:

```
>> ReplaceList[a, b->x]
{}
```

Like in `ReplaceAll`, `rules` can be a nested list:

```
>> ReplaceList[{a, b, c}, {{{___, x_, ___} -> {x}}, {{a, b, c} -> t}}, 2]
{{{a}, {a,b}}, {t}}
```

```
>> ReplaceList[expr, {}, -1]
Non
-negativeintegerorInfinityexpectedatposition3.
ReplaceList[expr, {}, -1]
```

Possible matches for a sum:

```
>> ReplaceList[a + b + c, x_ + y_ -> {x, y}]
{{a,b + c}, {b,a + c}, {c,a + b}, {a + b,c}, {a + c,b}, {b + c,a}}
```

ReplaceRepeated (//.)

`ReplaceRepeated[expr, x -> y]`
`expr //. x -> y`
 repeatedly applies the rule `x -> y` to `expr` until the result no longer changes.

```
>> a+b+c //. c->d
a + b + d
```

Simplification of logarithms:

```
>> logrules = {Log[x_ * y_] :> Log[
x] + Log[y], Log[x_ ^ y_] :> y *
Log[x]};

>> Log[a * (b * c)^ d ^ e * f] //.
logrules

Log[a] + Log[f] + (Log[b] + Log[c]) d^e
```

ReplaceAll just performs a single replacement:

```
>> Log[a * (b * c)^ d ^ e * f] /.
logrules

Log[a] + Log[f (bc)^d^e]
```

RuleDelayed (:>)

```
RuleDelayed[x, y]
x :> y
represents a rule replacing x with y, with
y held unevaluated.
```

```
>> Attributes[RuleDelayed]
{HoldRest, Protected, SequenceHold}
```

Rule (->)

```
Rule[x, y]
x -> y
represents a rule replacing x with y.
```

```
>> a+b+c /. c->d
a + b + d

>> {x, x^2, y} /. x->3
{3, 9, y}
```

Shortest

Verbatim

```
Verbatim[expr]
prevents pattern constructs in expr from
taking effect, allowing them to match
themselves.
```

Create a pattern matching Blank:

```
>> _ /. Verbatim[_]->t
t

>> x /. Verbatim[_]->t
x
```

Without Verbatim, Blank has its normal effect:

```
>> x /. _->t
t
```

XXX. Plotting

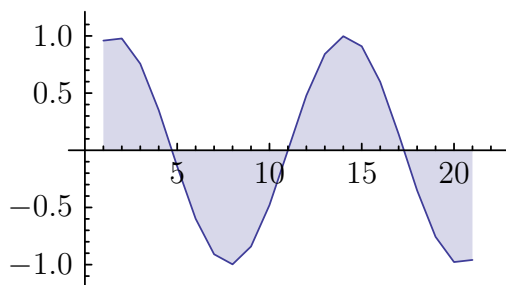
Contents

Axis	200	Full	202	PieChart	206
BarChart	201	Histogram	203	Plot	207
Bottom	201	ListLinePlot	203	Plot3D	208
ColorData	201	ListPlot	203	PolarPlot	208
ColorDataFunction . .	201	Mesh	204	Top	209
DensityPlot	202	ParametricPlot	205		

Axis

Axis
is a possible value for the Filling option.

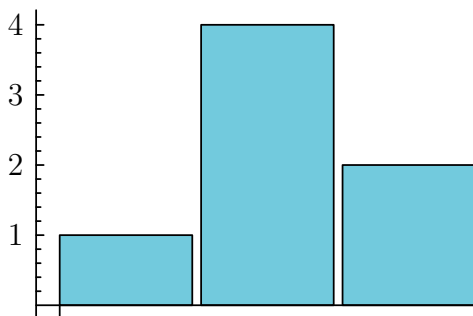
```
>> ListLinePlot[Table[Sin[x], {x, -5, 5, 0.5}], Filling->Axis]
```



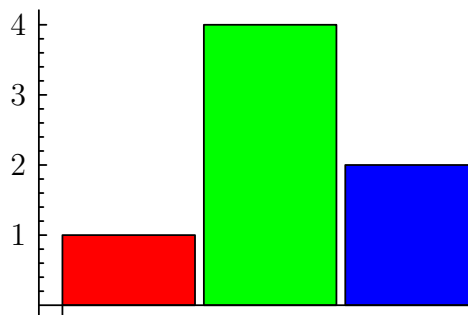
BarChart

BarChart[{p1, p2 ...}]
draws a bar chart.

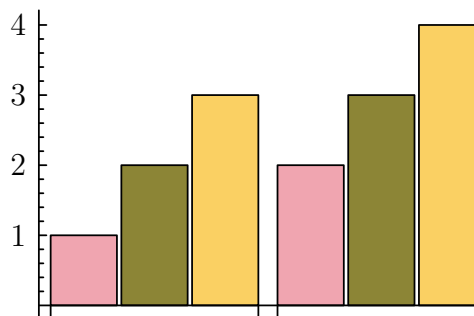
```
>> BarChart[{1, 4, 2}]
```



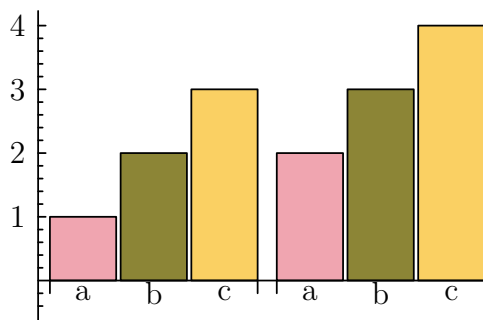
```
>> BarChart[{1, 4, 2}, ChartStyle -> {Red, Green, Blue}]
```



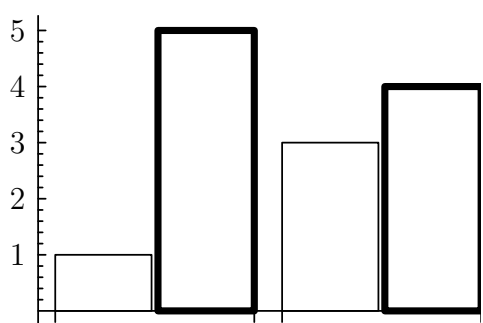
```
>> BarChart[{{1, 2, 3}, {2, 3, 4}}]
```




```
>> BarChart[{{1, 2, 3}, {2, 3, 4}},
  ChartLabels -> {"a", "b", "c"}]
```



```
>> BarChart[{{1, 5}, {3, 4}},
  ChartStyle -> {{EdgeForm[Thin],
  White}, {EdgeForm[Thick], White
  }}]
```

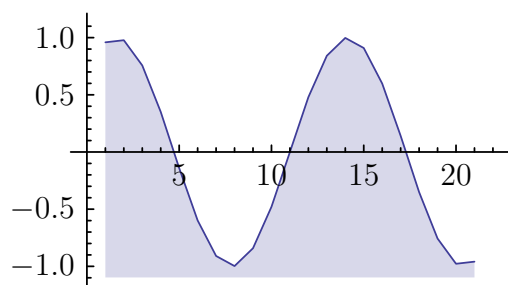


Bottom

Bottom

is a possible value for the Filling option.

```
>> ListLinePlot[Table[Sin[x], {x,
-5, 5, 0.5}], Filling->Bottom]
```



ColorData

ColorData["name"]

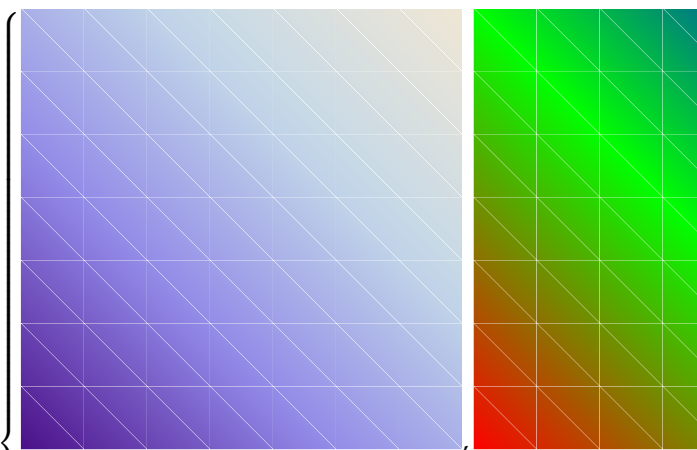
returns a color function with the given name.

Define a user-defined color function:

```
>> Unprotect[ColorData]; ColorData
["test"] := ColorDataFunction["
test", "Gradients", {0, 1},
Blend[{Red, Green, Blue}, #1
&]; Protect[ColorData]
```

Compare it to the default color function, LakeColors:

```
>> {DensityPlot[x + y, {x, -1, 1},
{y, -1, 1}], DensityPlot[x + y,
{x, -1, 1}, {y, -1, 1},
ColorFunction->"test"]}
```



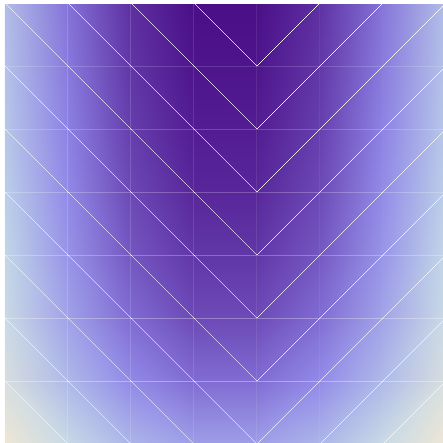
ColorDataFunction

DensityPlot

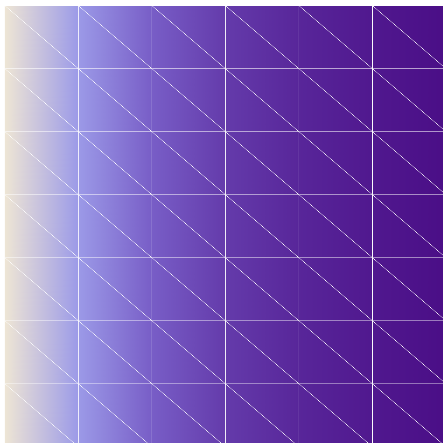
DensityPlot[f, {x, xmin, xmax}, {y, ymin, ymax}]

plots a density plot of f with x ranging from $xmin$ to $xmax$ and y ranging from $ymin$ to $ymax$.

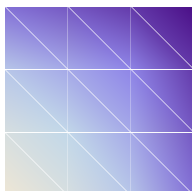
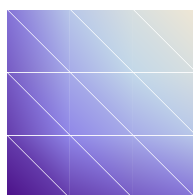
```
>> DensityPlot[x ^ 2 + 1 / y, {x,
-1, 1}, {y, 1, 4}]
```



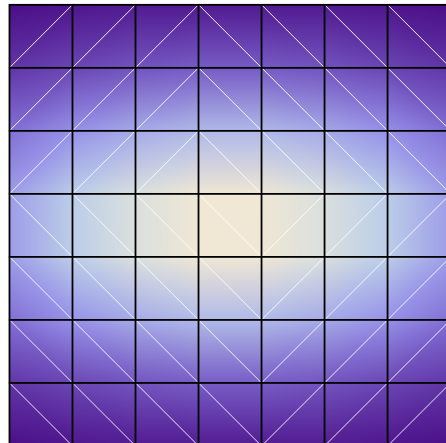
```
>> DensityPlot[1 / x, {x, 0, 1}, {y
, 0, 1}]
```



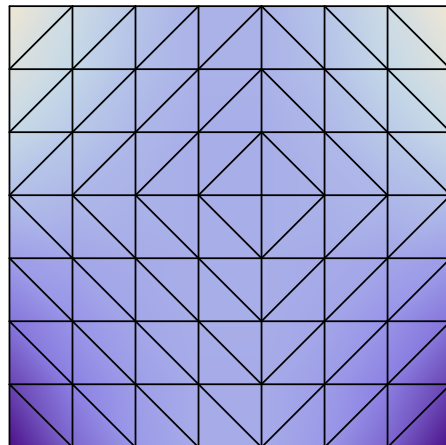
```
>> DensityPlot[Sqrt[x * y], {x, -1,
1}, {y, -1, 1}]
```



```
>> DensityPlot[1/(x^2 + y^2 + 1), {
x, -1, 1}, {y, -2,2}, Mesh->Full
]
```



```
>> DensityPlot[x^2 y, {x, -1, 1}, {
y, -1, 1}, Mesh->All]
```



Full

Full

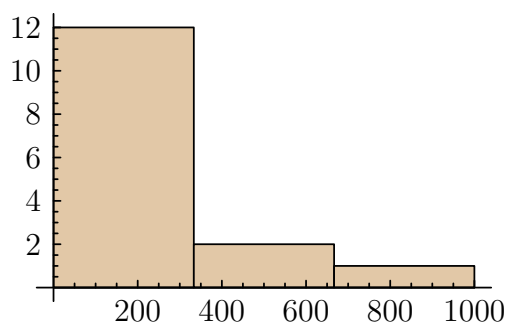
is a possible value for the Mesh and PlotRange options.

Histogram

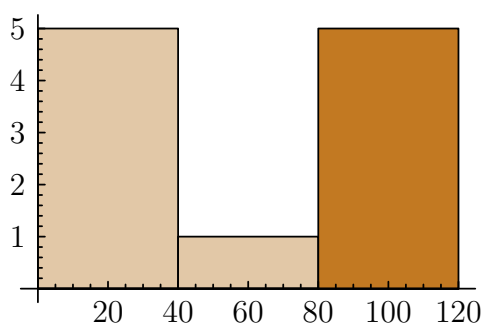
Histogram[{x1, x2 ...}]

gives a histogram using the values x1, x2,

```
>> Histogram[{3, 8, 10, 100, 1000,
500, 300, 200, 10, 20, 200, 100,
200, 300, 500}]
```



```
>> Histogram[{{1, 2, 10, 5, 50,
20}, {90, 100, 101, 120, 80}}]
```



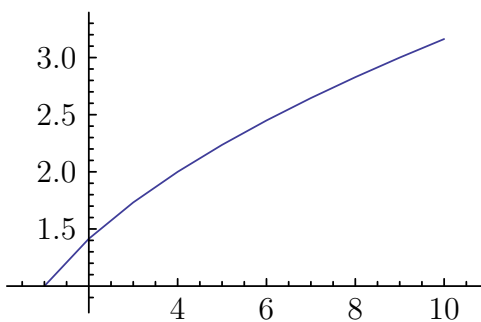
ListLinePlot

`ListLinePlot[{y1, y2, ...}]`
plots a line through a list of y -values, assuming integer x -values 1, 2, 3, ...

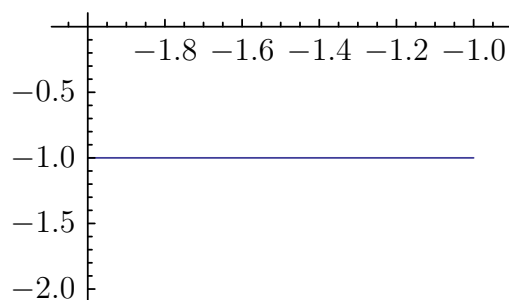
`ListLinePlot[{x1, y1}, {x2, y2}, ...]`
plots a line through a list of x, y pairs.

`ListLinePlot[{list1, list2, ...}]`
plots several lines.

```
>> ListLinePlot[Table[{n, n ^ 0.5},
{n, 10}]]
```



```
>> ListLinePlot[{{-2, -1}, {-1,
-1}}]
```



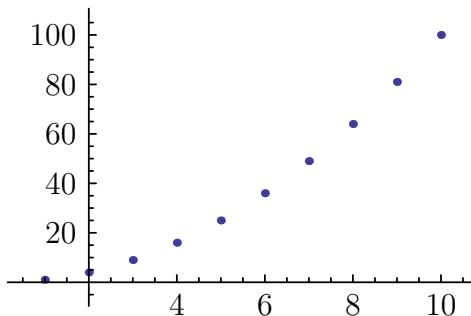
ListPlot

`ListPlot[{y1, y2, ...}]`
plots a list of y -values, assuming integer x -values 1, 2, 3, ...

`ListPlot[{x1, y1}, {x2, y2}, ...]`
plots a list of x, y pairs.

`ListPlot[{list1, list2, ...}]`
plots several lists of points.

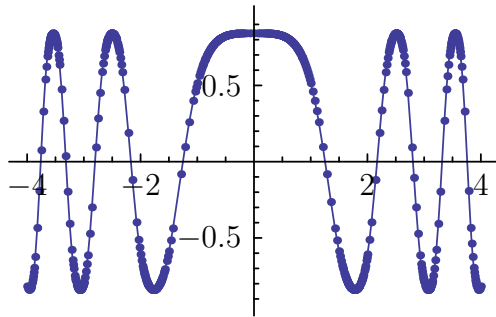
```
>> ListPlot[Table[n ^ 2, {n, 10}]]
```



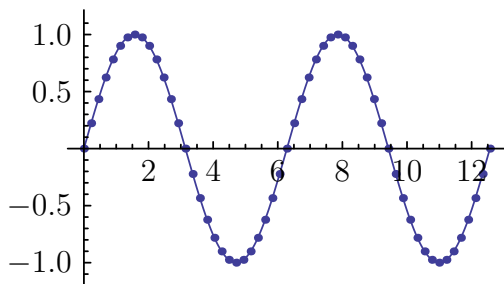
Mesh

`Mesh`
is an option for `Plot` that specifies the mesh to be drawn. The default is `Mesh->None`.

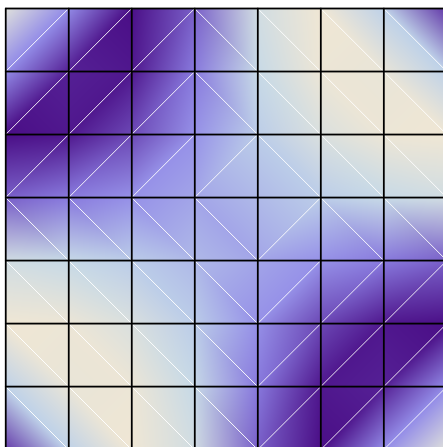
```
>> Plot[Sin[Cos[x^2]],{x,-4,4},Mesh->All]
```



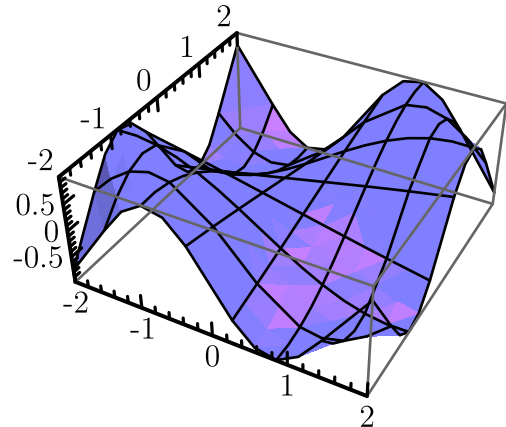
```
>> Plot[Sin[x], {x,0,4 Pi}, Mesh->Full]
```



```
>> DensityPlot[Sin[x y], {x, -2, 2}, {y, -2, 2}, Mesh->Full]
```



```
>> Plot3D[Sin[x y], {x, -2, 2}, {y, -2, 2}, Mesh->Full]
```



ParametricPlot

```
ParametricPlot[{f_x, f_y}, {u, umin, umax}]
```

plots a parametric function f with the parameter u ranging from $umin$ to $umax$.

```
ParametricPlot[{f_x, f_y}, {g_x, g_y}, ..., {u, umin, umax}]
```

plots several parametric functions f, g, \dots

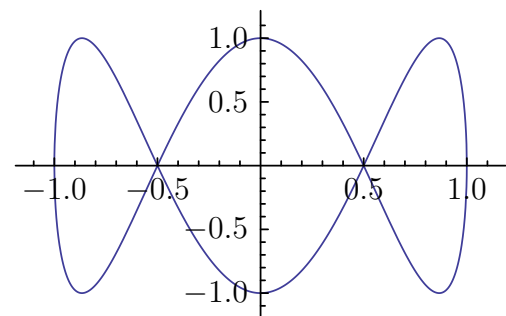
```
ParametricPlot[{f_x, f_y}, {u, umin, umax}, {v, vmin, vmax}]
```

plots a parametric area.

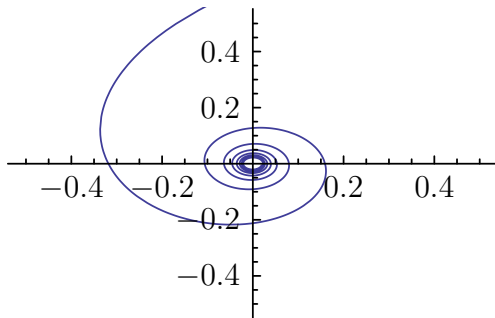
```
ParametricPlot[{f_x, f_y}, {g_x, g_y}, ..., {u, umin, umax}, {v, vmin, vmax}]
```

plots several parametric areas.

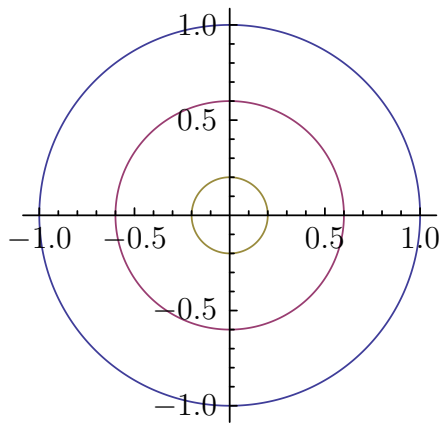
```
>> ParametricPlot[{Sin[u], Cos[3 u]}, {u, 0, 2 Pi}]
```



```
>> ParametricPlot[{Cos[u] / u, Sin[u] / u}, {u, 0, 50}, PlotRange -> 0.5]
```



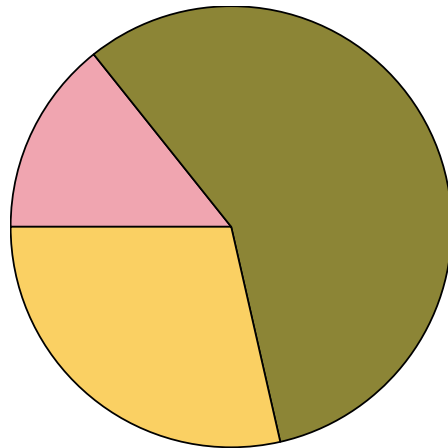
```
>> ParametricPlot[{{Sin[u], Cos[u]}, {0.6 Sin[u], 0.6 Cos[u]}, {0.2 Sin[u], 0.2 Cos[u]}}, {u, 0, 2 Pi}, PlotRange -> 1, AspectRatio -> 1]
```



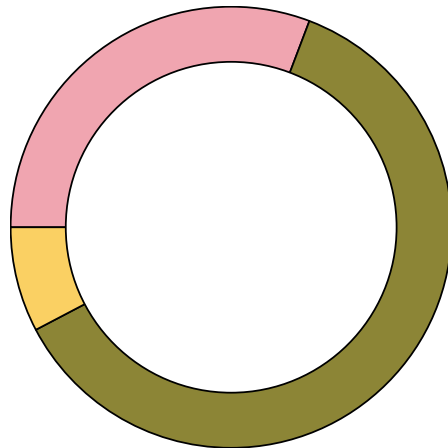
PieChart

`PieChart[{p1, p2 ...}]`
draws a pie chart.

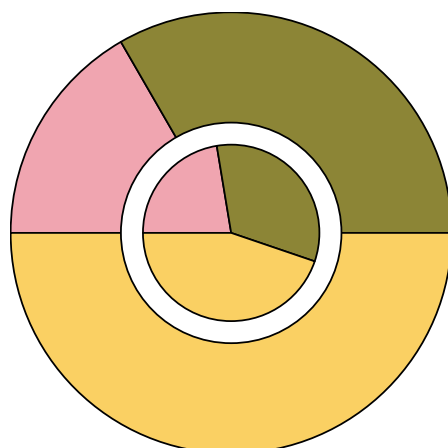
```
>> PieChart[{1, 4, 2}]
```



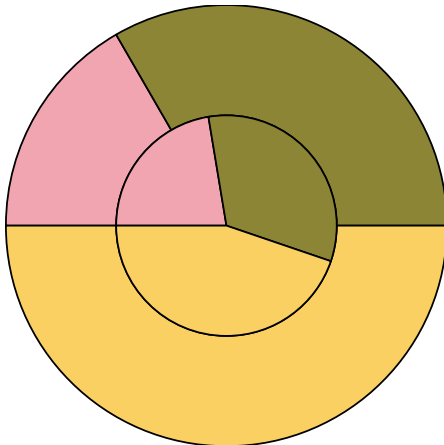
```
>> PieChart[{8, 16, 2}, SectorOrigin -> {Automatic, 1.5}]
```



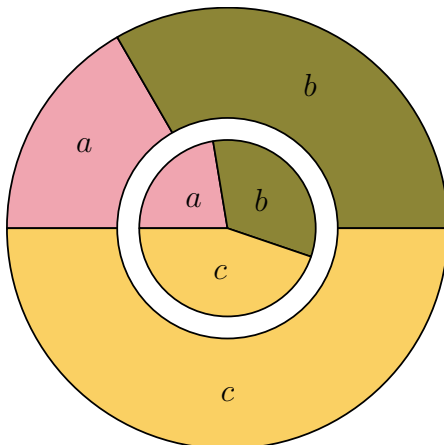
```
>> PieChart[{{10, 20, 30}, {15, 22, 30}}]
```



```
>> PieChart[{{10, 20, 30}, {15, 22, 30}}, SectorSpacing -> None]
```

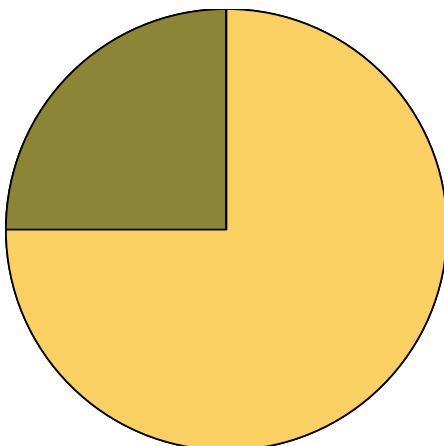


```
>> PieChart[{{10, 20, 30}, {15, 22, 30}}, ChartLabels -> {a, b, c}]
```



Negative values are clipped to 0.

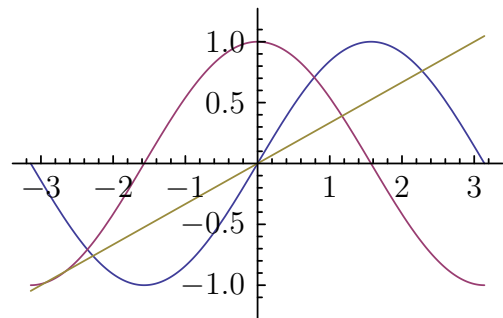
```
>> PieChart[{1, -1, 3}]
```



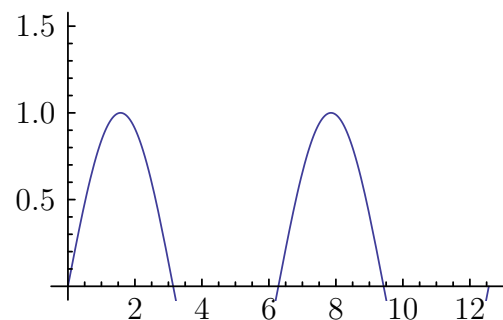
Plot

```
Plot[f, {x, xmin, xmax}]
plots f with x ranging from xmin to xmax.
Plot[{f1, f2, ...}, {x, xmin, xmax}]
plots several functions f1, f2, ...
```

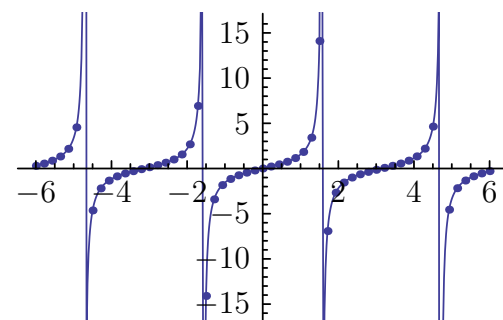
```
>> Plot[{Sin[x], Cos[x], x / 3}, {x, -Pi, Pi}]
```



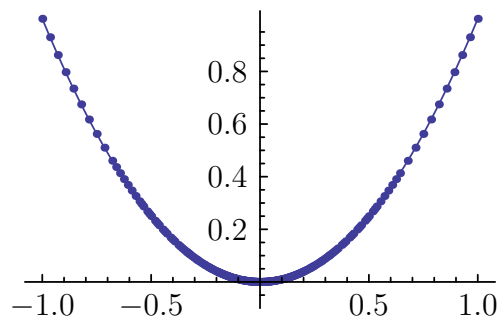
```
>> Plot[Sin[x], {x, 0, 4 Pi},
PlotRange->{{0, 4 Pi}, {0, 1.5}}]
```



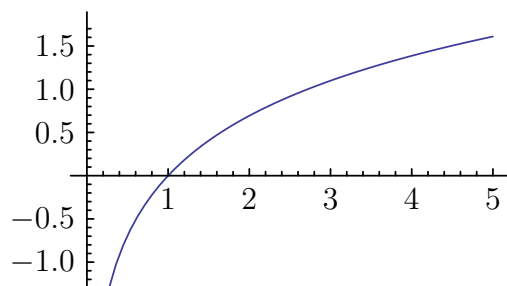
```
>> Plot[Tan[x], {x, -6, 6}, Mesh->Full]
```



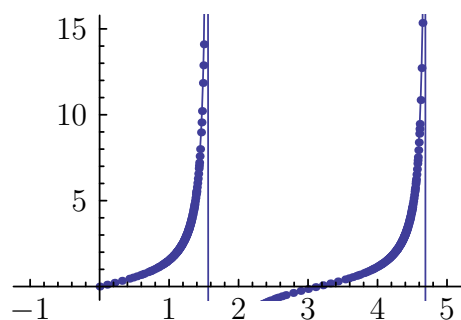
```
>> Plot[x^2, {x, -1, 1},
MaxRecursion->5, Mesh->All]
```



```
>> Plot[Log[x], {x, 0, 5},
MaxRecursion->0]
```

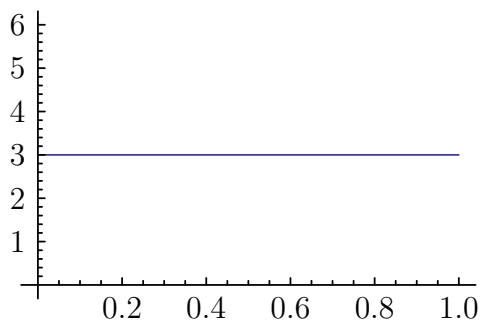


```
>> Plot[Tan[x], {x, 0, 6}, Mesh->
All, PlotRange->{{-1, 5}, {0,
15}}, MaxRecursion->10]
```



A constant function:

```
>> Plot[3, {x, 0, 1}]
```

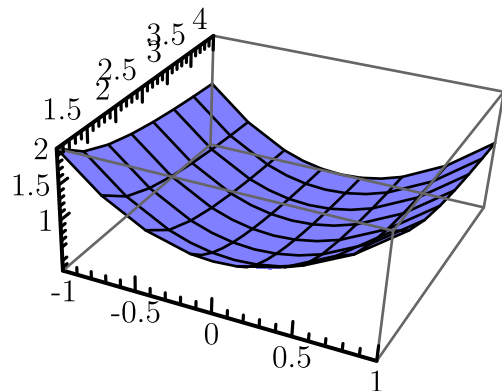


Plot3D

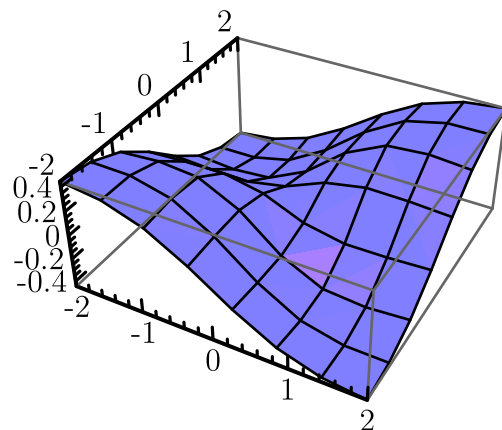
```
Plot3D[f, {x, xmin, xmax}, {y, ymin,
ymax}]
```

creates a three-dimensional plot of f with x ranging from $xmin$ to $xmax$ and y ranging from $ymin$ to $ymax$.

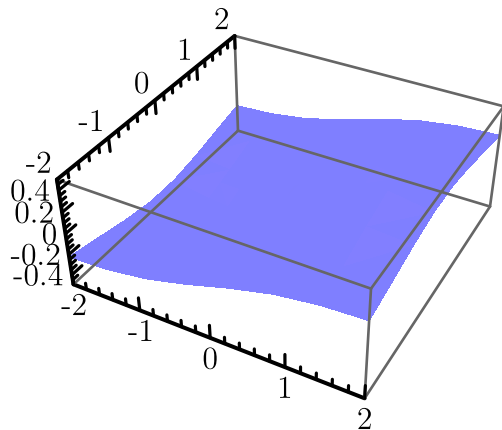
```
>> Plot3D[x ^ 2 + 1 / y, {x, -1,
1}, {y, 1, 4}]
```



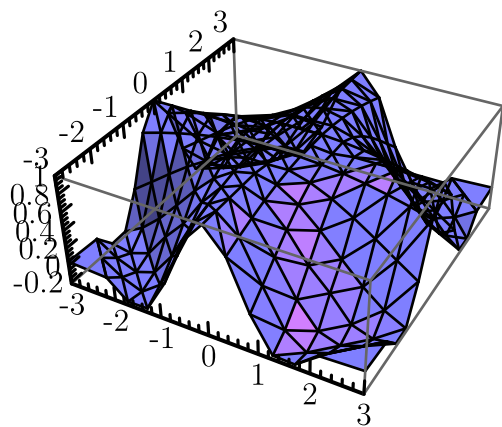
```
>> Plot3D[x y / (x ^ 2 + y ^ 2 + 1)
, {x, -2, 2}, {y, -2, 2}]
```



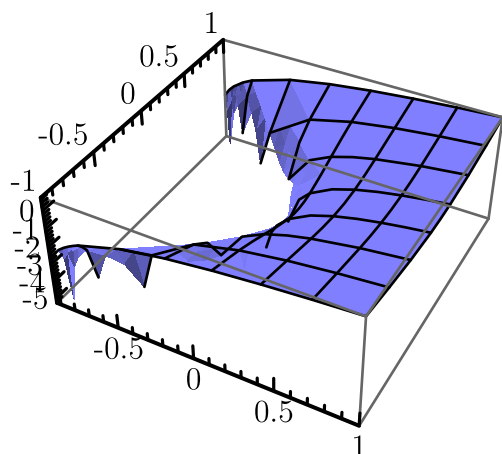
```
>> Plot3D[x / (x ^ 2 + y ^ 2 + 1),
{x, -2, 2}, {y, -2, 2}, Mesh->
None]
```



```
>> Plot3D[Sin[x y] / (x y), {x, -3,
3}, {y, -3, 3}, Mesh->All]
```



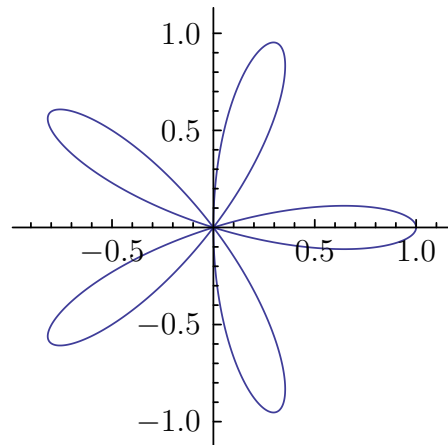
```
>> Plot3D[Log[x + y^2], {x, -1, 1},
{y, -1, 1}]
```



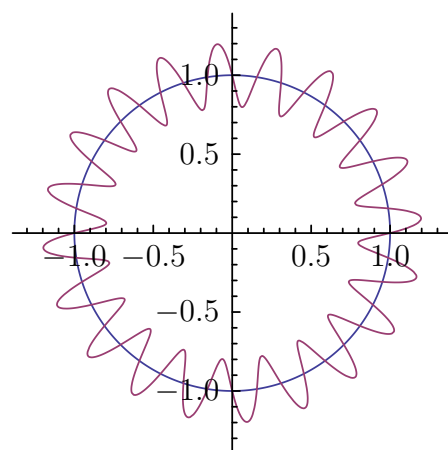
PolarPlot

`PolarPlot[r, {t, tmin, tmax}]`
creates a polar plot of r with angle t ranging from $tmin$ to $tmax$.

```
>> PolarPlot[Cos[5t], {t, 0, Pi}]
```



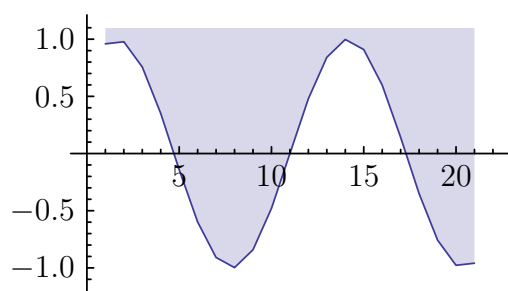
```
>> PolarPlot[{1, 1 + Sin[20 t] /
5}, {t, 0, 2 Pi}]
```



Top

`Top`
is a possible value for the `Filling` option.


```
>> ListLinePlot[Table[Sin[x], {x,  
-5, 5, 0.5}], Filling->Top]
```



XXXI. Physical and Chemical data

Contents

ElementData 211

ElementData

`ElementData["name", "property"]`
gives the value of the *property* for the chemical specified by *name*.
`ElementData[n, "property"]`
gives the value of the *property* for the *n*th chemical element.

```
>> ElementData[74]
Tungsten

>> ElementData["He", "
AbsoluteBoilingPoint"]
4.22

>> ElementData["Carbon", "
IonizationEnergies"]
{1 086.5, 2 352.6, 4 620.5
, 6 222.7, 37 831, 47 277.}

>> ElementData[16, "
ElectronConfigurationString"]
[Ne] 3s2 3p4

>> ElementData[73, "
ElectronConfiguration"]
{{2}, {2, 6}, {2, 6, 10}, {2,
6, 10, 14}, {2, 6, 3}, {2}}
```

The number of known elements:

```
>> Length[ElementData[All]]
118
```

Some properties are not appropriate for certain elements:

```
>> ElementData["He", "
ElectroNegativity"]
Missing[NotApplicable]
```

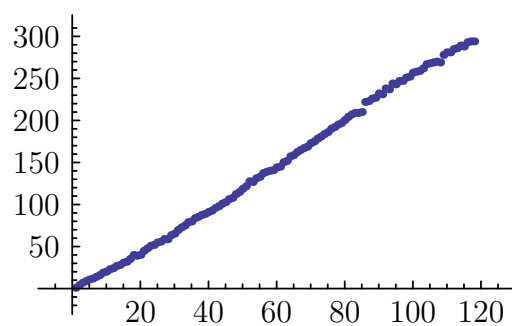
Some data is missing:

```
>> ElementData["Tc", "SpecificHeat
"]
Missing[NotAvailable]
```

All the known properties:

```
>> ElementData["Properties"]
{Abbreviation,
AbsoluteBoilingPoint,
AbsoluteMeltingPoint,
AtomicNumber, AtomicRadius,
AtomicWeight, Block, BoilingPoint,
BrinellHardness, BulkModulus,
CovalentRadius, CrustAbundance,
Density, DiscoveryYear,
ElectroNegativity, ElectronAffinity,
ElectronConfiguration,
ElectronConfigurationString,
ElectronShellConfiguration,
FusionHeat, Group,
IonizationEnergies, LiquidDensity,
MeltingPoint, MohsHardness,
Name, Period, PoissonRatio,
Series, ShearModulus,
SpecificHeat, StandardName,
ThermalConductivity,
VanDerWaalsRadius,
VaporizationHeat,
VickersHardness, YoungModulus}
```

```
>> ListPlot[Table[ElementData[z, "AtomicWeight"], {z, 118}]]
```



XXXII. Random number generation

Random numbers are generated using the Mersenne Twister.

Contents

Random	212	RandomInteger	213	\$RandomState	214
RandomChoice	212	RandomReal	214	SeedRandom	215
RandomComplex	213	RandomSample	214		

Random

Legacy function. Superseded by RandomReal, RandomInteger and RandomComplex.

RandomChoice

RandomChoice[items]
randomly picks one item from *items*.

RandomChoice[items, n]
randomly picks *n* items from *items*. Each pick in the *n* picks happens from the given set of *items*, so each item can be picked any number of times.

RandomChoice[items, {n1, n2, ...}]
randomly picks items from *items* and arranges the picked items in the nested list structure described by {*n1*, *n2*, ...}.

RandomChoice[weights -> items, n]
randomly picks *n* items from *items* and uses the corresponding numeric values in *weights* to determine how probable it is for each item in *items* to get picked (in the long run, items with higher weights will get picked more often than ones with lower weight).

RandomChoice[weights -> items]
randomly picks one items from *items* using weights *weights*.

RandomChoice[weights -> items, {n1, n2, ...}]
randomly picks a structured list of items from *items* using weights *weights*.

```
>> RandomChoice[{a, b, c}]
      {c}

>> SeedRandom[42]

>> RandomChoice[{a, b, c}, 20]
      {c,a,c,c,a,a,c,b,c,c,
        c,c,a,c,b,a,b,b,b,b}

>> SeedRandom[42]

>> RandomChoice[{"a", {1, 2}, x, {}}, 10]
      {x, {}, a,x,x, {}, a,a,x, {1,2}}

>> SeedRandom[42]

>> RandomChoice[{a, b, c}, {5, 2}]
      {{c,a}, {c,c}, {a,a}, {c,b}, {c,c}}

>> SeedRandom[42]

>> RandomChoice[{1, 100, 5} -> {a, b, c}, 20]
      {b,b,b,b,b,b,b,b,b,b,
        b,c,b,b,b,b,b,b,b,b}
```

```
>> SeedRandom[42]
```

RandomComplex

`RandomComplex[{z_min, z_max}]`
yields a pseudorandom complex number in the rectangle with complex corners `z_min` and `z_max`.

`RandomComplex[z_max]`
yields a pseudorandom complex number in the rectangle with corners at the origin and at `z_max`.

`RandomComplex[]`
yields a pseudorandom complex number with real and imaginary parts from 0 to 1.

`RandomComplex[range, n]`
gives a list of `n` pseudorandom complex numbers.

`RandomComplex[range, {n1, n2, ...}]`
gives a nested list of pseudorandom complex numbers.

```
>> RandomComplex[]
0.648764 + 0.464368I

>> RandomComplex[{1+I, 5+5I}]
1.14471 + 1.87619I

>> RandomComplex[1+I, 5]
{0.678702 + 0.796872I, 0.085~
~4039 + 0.802309I, 0.0935271 +
0.662646I, 0.0344949 + 0.700~
~559I, 0.810801 + 0.153145I}

>> RandomComplex[{1+I, 2+2I}, {2,
2}]
{{1.68744 + 1.90658I, 1.680~
~64 + 1.03828I}, {1.88193 +
1.72737I, 1.41043 + 1.69971I}}
```

RandomInteger

`RandomInteger[{min, max}]`
yields a pseudorandom integer in the range from `min` to `max` inclusive.

`RandomInteger[max]`
yields a pseudorandom integer in the range from 0 to `max` inclusive.

`RandomInteger[]`
gives 0 or 1.

`RandomInteger[range, n]`
gives a list of `n` pseudorandom integers.

`RandomInteger[range, {n1, n2, ...}]`
gives a nested list of pseudorandom integers.

```
>> RandomInteger[{1, 5}]
2

>> RandomInteger[100, {2, 3}] //
TableForm
    26  71  38
    55   0  33
```

Calling `RandomInteger` changes `$RandomState`:

```
>> previousState = $RandomState;

>> RandomInteger[]
0

>> $RandomState != previousState
True
```

RandomReal

`RandomReal[{min, max}]`
yields a pseudorandom real number in the range from `min` to `max`.

`RandomReal[max]`
yields a pseudorandom real number in the range from 0 to `max`.

`RandomReal[]`
yields a pseudorandom real number in the range from 0 to 1.

`RandomReal[range, n]`
gives a list of `n` pseudorandom real numbers.

`RandomReal[range, {n1, n2, ...}]`
gives a nested list of pseudorandom real numbers.

```
>> RandomReal[]
0.348099

>> RandomReal[{1, 5}]
1.68381
```

RandomSample

`RandomSample[items]`
randomly picks one item from *items*.

`RandomSample[items, n]`
randomly picks *n* items from *items*. Each pick in the *n* picks happens after the previous items picked have been removed from *items*, so each item can be picked at most once.

`RandomSample[items, {n1, n2, ...}]`
randomly picks items from *items* and arranges the picked items in the nested list structure described by {*n1*, *n2*, ...}. Each item gets picked at most once.

`RandomSample[weights -> items, n]`
randomly picks *n* items from *items* and uses the corresponding numeric values in *weights* to determine how probable it is for each item in *items* to get picked (in the long run, items with higher weights will get picked more often than ones with lower weight). Each item gets picked at most once.

`RandomSample[weights -> items]`
randomly picks one items from *items* using weights *weights*. Each item gets picked at most once.

`RandomSample[weights -> items, {n1, n2, ...}]`
randomly picks a structured list of items from *items* using weights *weights*. Each item gets picked at most once.

```
>> SeedRandom[42]

>> RandomSample[{a, b, c}]
{a}

>> SeedRandom[42]

>> RandomSample[{a, b, c, d, e, f, g, h}, 7]
{b, f, a, h, c, e, d}

>> SeedRandom[42]
```

```
>> RandomSample[{"a", {1, 2}, x, {}], 3]
{{1, 2}, {}, a}

>> SeedRandom[42]

>> RandomSample[Range[100], {2, 3}]
{{84, 54, 71}, {46, 45, 40}}

>> SeedRandom[42]

>> RandomSample[Range[100] -> Range[100], 5]
{62, 98, 86, 78, 40}
```

\$RandomState

`$RandomState`
is a long number representing the internal state of the pseudorandom number generator.

```
>> Mod[$RandomState, 10^100]
1 156 557 101 271 990 351 193 260 ~
~314 310 678 720 104 592 502 193 ~
~096 091 636 467 718 455 179 373 ~
~939 855 678 876 750 624 706 334 766

>> IntegerLength[$RandomState]
6466
```

So far, it is not possible to assign values to `$RandomState`.

```
>> $RandomState = 42
It is not possible to change the random state.
42
```

Not even to its own value:

```
>> $RandomState = $RandomState;
It is not possible to change the random state.
```

SeedRandom

`SeedRandom[n]`
resets the pseudorandom generator with seed *n*.

`SeedRandom[]`
uses the current date and time as the seed.

SeedRandom can be used to get reproducible random numbers:

```
>> SeedRandom[42]

>> RandomInteger[100]
51

>> RandomInteger[100]
92

>> SeedRandom[42]

>> RandomInteger[100]
51

>> RandomInteger[100]
92
```

String seeds are supported as well:

```
>> SeedRandom["Mathics"]

>> RandomInteger[100]
27
```

Calling SeedRandom without arguments will seed the random number generator to a random state:

```
>> SeedRandom[]

>> RandomInteger[100]
35
```

XXXIII. Recurrence relation solvers

Contents

RSolve 216

RSolve

`RSolve[eqn, a[n], n]`
solves a recurrence equation for the function $a[n]$.

```
>> RSolve[a[n] == a[n+1], a[n], n]
{{a[n] -> C[0]}}
```

No boundary conditions gives two general parameters:

```
>> RSolve[{a[n + 2] == a[n]}, a, n]
{{a -> (Function[{n},
  C[0] + C[1] - 1^n])}}
```

One boundary condition:

```
>> RSolve[{a[n + 2] == a[n], a[0] == 1}, a, n]
{{a -> (Function[{n},
  C[0] + (1 - C[0]) - 1^n])}}
```

Two boundary conditions:

```
>> RSolve[{a[n + 2] == a[n], a[0] == 1, a[1] == 4}, a, n]
{{a -> (Function[{n},
  5/2 - (3 - 1^n)/2])}}
```


XXXIV. Special functions

Contents

AiryAi	217	ChebyshevU	220	KelvinBei	222
AiryAiPrime	218	Erf	220	KelvinBer	222
AiryAiZero	218	Erfc	220	KelvinKei	223
AiryBi	218	ExpIntegralE	220	KelvinKer	223
AiryBiPrime	218	ExpIntegralEi	221	LaguerreL	223
AiryBiZero	218	FresnelC	221	LegendreP	224
AngerJ	218	FresnelS	221	LegendreQ	224
BesselI	219	GegenbauerC	221	ProductLog	224
BesselJ	219	HankelH1	221	SphericalHarmonicY	224
BesselJZero	219	HankelH2	221	StruveH	224
BesselK	219	HermiteH	221	StruveL	225
BesselY	219	InverseErf	222	WeberE	225
BesselYZero	219	InverseErfc	222	Zeta	225
ChebyshevT	220	JacobiP	222		

AiryAi

AiryAi[x]
returns the Airy function Ai(x).

Exact values:

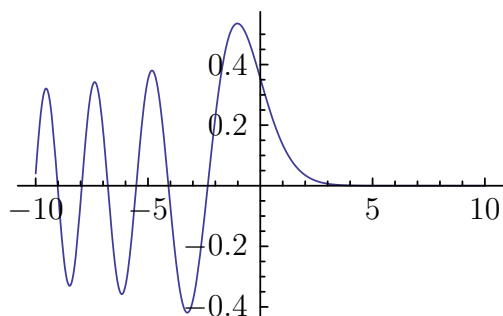
```
>> AiryAi[0]
      31/3
-----
3Gamma[2/3]
```

AiryAi can be evaluated numerically:

```
>> AiryAi[0.5]
0.231694

>> AiryAi[0.5 + I]
0.157118 - 0.24104I
```

```
>> Plot[AiryAi[x], {x, -10, 10}]
```



AiryAiPrime

AiryAiPrime[x]
returns the derivative of the Airy function AiryAi[x].

Exact values:

```
>> AiryAiPrime[0]
      32/3
-----
3Gamma[1/3]
```

Numeric evaluation:

```
>> AiryAiPrime[0.5]
-0.224911
```

AiryAiZero

`AiryAiZero[k]`
returns the k th zero of the Airy function $\text{Ai}(z)$.

```
>> N[AiryAiZero[1]]
-2.33811
```

AiryBi

`AiryBi[x]`
returns the Airy function of the second kind $\text{Bi}(x)$.

Exact values:

```
>> AiryBi[0]

$$\frac{3^{\frac{5}{6}}}{3\Gamma\left[\frac{2}{3}\right]}$$

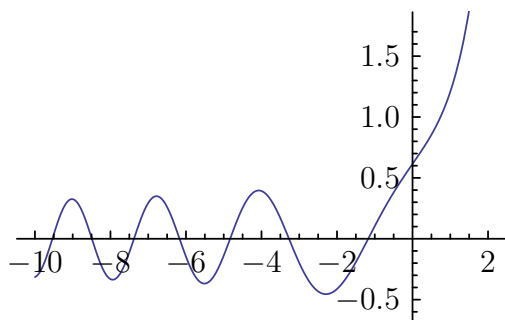
```

Numeric evaluation:

```
>> AiryBi[0.5]
0.854277

>> AiryBi[0.5 + I]
0.688145 + 0.370815I

>> Plot[AiryBi[x], {x, -10, 2}]
```



AiryBiPrime

`AiryBiPrime[x]`
returns the derivative of the Airy function of the second kind $\text{Bi}(x)$.

Exact values:

```
>> AiryBiPrime[0]

$$\frac{3^{\frac{1}{6}}}{\Gamma\left[\frac{1}{3}\right]}$$

```

Numeric evaluation:

```
>> AiryBiPrime[0.5]
0.544573
```

AiryBiZero

`AiryBiZero[k]`
returns the k th zero of the Airy function $\text{Bi}(z)$.

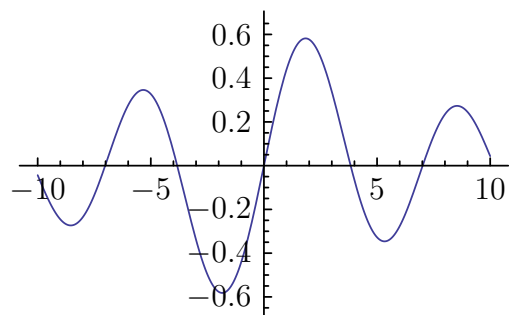
```
>> N[AiryBiZero[1]]
-1.17371
```

AngerJ

`AngerJ[n, z]`
returns the Anger function $J_n(z)$.

```
>> AngerJ[1.5, 3.5]
0.294479

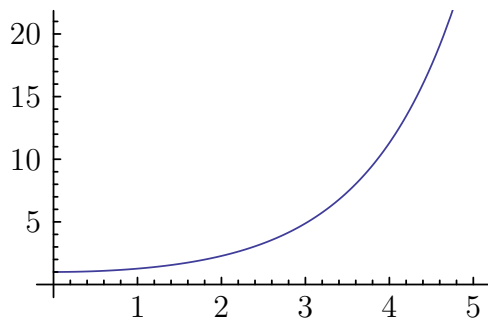
>> Plot[AngerJ[1, x], {x, -10, 10}]
```



BesselI

BesselI[n , z]
returns the modified Bessel function of the first kind $I_n(z)$.

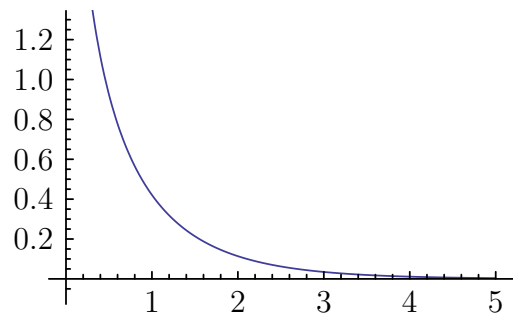
```
>> BesselI[1.5, 4]  
8.17263  
  
>> Plot[BesselI[0, x], {x, 0, 5}]
```



BesselK

BesselK[n , z]
returns the modified Bessel function of the second kind $K_n(z)$.

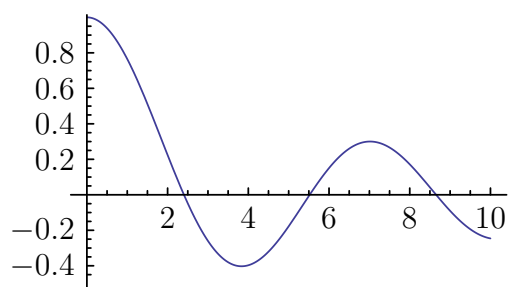
```
>> BesselK[1.5, 4]  
0.014347  
  
>> Plot[BesselK[0, x], {x, 0, 5}]
```



BesselJ

BesselJ[n , z]
returns the Bessel function of the first kind $J_n(z)$.

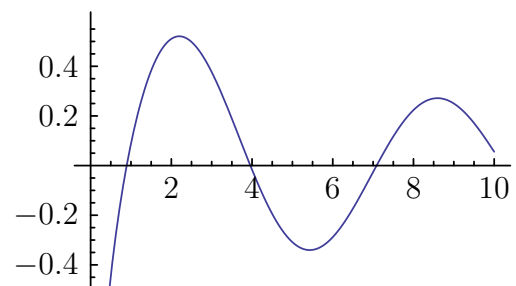
```
>> BesselJ[0, 5.2]  
-0.11029  
  
>> Plot[BesselJ[0, x], {x, 0, 10}]
```



BesselY

BesselY[n , z]
returns the Bessel function of the second kind $Y_n(z)$.

```
>> BesselY[1.5, 4]  
0.367112  
  
>> Plot[BesselY[0, x], {x, 0, 10}]
```



BesselJZero

BesselJZero[n , k]
returns the k th zero of the Bessel function of the first kind $J_n(z)$.

```
>> N[BesselJZero[0, 1]]  
2.40483
```

BesselYZero

BesselYZero[n , k]
returns the k th zero of the Bessel function of the second kind $Y_n(z)$.

```
>> N[BesselYZero[0, 1]]  
0.893577
```

ChebyshevT

ChebyshevT $[n, x]$
returns the Chebyshev polynomial of the first kind $T_n(x)$.

```
>> ChebyshevT[8, x]
1 - 32x2 + 160x4 - 256x6 + 128x8

>> ChebyshevT[1 - I, 0.5]
0.800143 + 1.08198I
```

ChebyshevU

ChebyshevU $[n, x]$
returns the Chebyshev polynomial of the second kind $U_n(x)$.

```
>> ChebyshevU[8, x]
1 - 40x2 + 240x4 - 448x6 + 256x8

>> ChebyshevU[1 - I, 0.5]
1.60029 + 0.721322I
```

Erf

Erf $[z]$
returns the error function of z .
Erf $[z0, z1]$
returns the result of $\text{Erf}[z1] - \text{Erf}[z0]$.

$\text{Erf}[x]$ is an odd function:

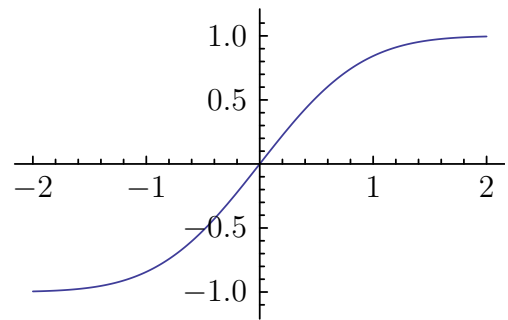
```
>> Erf[-x]
-Erf[x]

>> Erf[1.0]
0.842701

>> Erf[0]
0

>> {Erf[0, x], Erf[x, 0]}
{Erf[x], -Erf[x]}
```

```
>> Plot[Erf[x], {x, -2, 2}]
```



Erfc

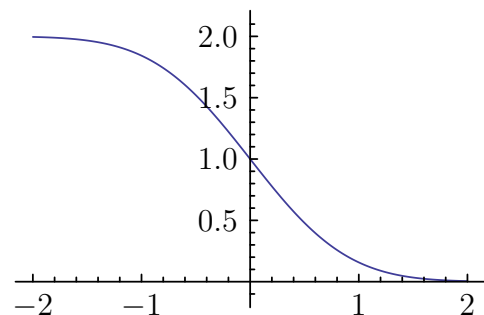
Erfc $[z]$
returns the complementary error function of z .

```
>> Erfc[-x] / 2
(2 - Erfc[x]) / 2

>> Erfc[1.0]
0.157299

>> Erfc[0]
1

>> Plot[Erfc[x], {x, -2, 2}]
```



ExpIntegralE

ExpIntegralE $[n, z]$
returns the exponential integral function $E_n(z)$.

```
>> ExpIntegralE[2.0, 2.0]
0.0375343
```

ExpIntegralEi

`ExpIntegralEi[z]`
returns the exponential integral function $\text{Ei}(z)$.

```
>> ExpIntegralEi[2.0]
4.95423
```

FresnelC

`FresnelC[z]`
is the Fresnel C integral $C(z)$.

```
>> FresnelC[{0, Infinity}]
{0, 1/2}

>> Integrate[Cos[x^2 Pi/2], {x, 0,
z}]
FresnelC[z] Gamma[1/4]
-----
4 Gamma[5/4]
```

FresnelS

`FresnelS[z]`
is the Fresnel S integral $S(z)$.

```
>> FresnelS[{0, Infinity}]
{0, 1/2}

>> Integrate[Sin[x^2 Pi/2], {x, 0,
z}]
3 FresnelS[z] Gamma[3/4]
-----
4 Gamma[7/4]
```

GegenbauerC

`GegenbauerC[n, m, x]`
returns the Gegenbauer polynomial $C_n^{(m)}(x)$.

```
>> GegenbauerC[6, 1, x]
-1 + 24x^2 - 80x^4 + 64x^6

>> GegenbauerC[4 - I, 1 + 2 I, 0.7]
-3.2621 - 24.9739I
```

HankelH1

`HankelH1[n, z]`
returns the Hankel function of the first kind $H_n^{(1)}(z)$.

```
>> HankelH1[1.5, 4]
0.185286 + 0.367112I
```

HankelH2

`HankelH2[n, z]`
returns the Hankel function of the second kind $H_n^{(2)}(z)$.

```
>> HankelH2[1.5, 4]
0.185286 - 0.367112I
```

HermiteH

`HermiteH[n, x]`
returns the Hermite polynomial $H_n(x)$.

```
>> HermiteH[8, x]
1 680 - 13 440x^2 + 13 ~
~ 440x^4 - 3 584x^6 + 256x^8

>> HermiteH[3, 1 + I]
-28 + 4I

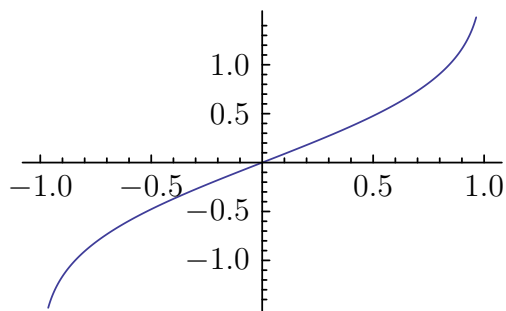
>> HermiteH[4.2, 2]
77.5291
```

InverseErf

`InverseErf[z]`
returns the inverse error function of z .

```
>> InverseErf /@ {-1, 0, 1}
{-∞, 0, ∞}
```

```
>> Plot[InverseErf[x], {x, -1, 1}]
```



InverseErf[z] only returns numeric values for $-1 \leq z \leq 1$:

```
>> InverseErf /@ {0.9, 1.0, 1.1}
{1.16309, ∞, InverseErf[1.1]}
```

InverseErfc

InverseErfc[z]
returns the inverse complementary error function of z.

```
>> InverseErfc /@ {0, 1, 2}
{∞, 0, -∞}
```

JacobiP

JacobiP[n, a, b, x]
returns the Jacobi polynomial $P_n^{(a,b)}(x)$.

```
>> JacobiP[1, a, b, z]
 $\frac{a}{2} - \frac{b}{2} + z \left( 1 + \frac{a}{2} + \frac{b}{2} \right)$ 

>> JacobiP[3.5 + I, 3, 2, 4 - I]
1410.02 + 5797.3I
```

KelvinBei

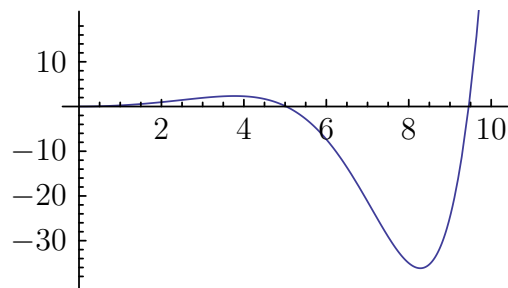
KelvinBei[z]
returns the Kelvin function bei(z).
KelvinBei[n, z]
returns the Kelvin function bei_n(z).

```
>> KelvinBei[0.5]
0.0624932
```

```
>> KelvinBei[1.5 + I]
0.326323 + 0.755606I
```

```
>> KelvinBei[0.5, 0.25]
0.370153
```

```
>> Plot[KelvinBei[x], {x, 0, 10}]
```



KelvinBer

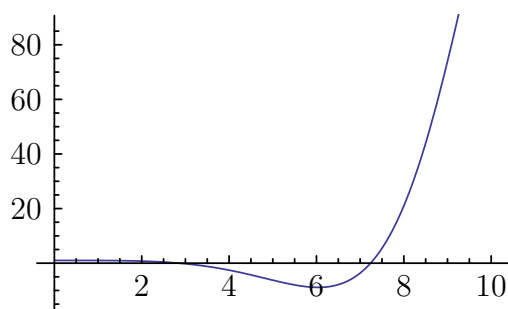
KelvinBer[z]
returns the Kelvin function ber(z).
KelvinBer[n, z]
returns the Kelvin function ber_n(z).

```
>> KelvinBer[0.5]
0.999023
```

```
>> KelvinBer[1.5 + I]
1.1162 - 0.117944I
```

```
>> KelvinBer[0.5, 0.25]
0.148824
```

```
>> Plot[KelvinBer[x], {x, 0, 10}]
```



KelvinKei

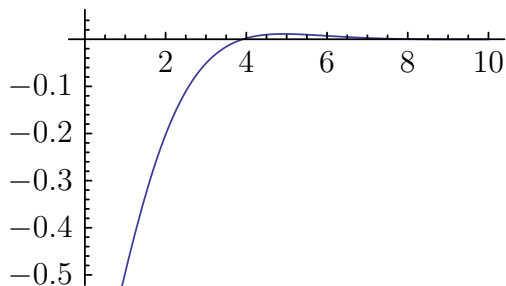
KelvinKei[z]
returns the Kelvin function kei(z).
KelvinKei[n, z]
returns the Kelvin function kei_n(z).

```
>> KelvinKei[0.5]
-0.671582

>> KelvinKei[1.5 + I]
-0.248994 + 0.303326I

>> KelvinKei[0.5, 0.25]
-2.0517

>> Plot[KelvinKei[x], {x, 0, 10}]
```



KelvinKer

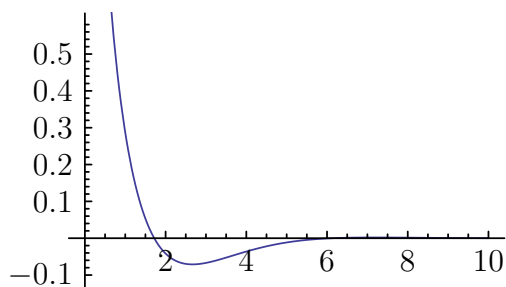
`KelvinKer[z]`
returns the Kelvin function $\ker(z)$.
`KelvinKer[n, z]`
returns the Kelvin function $\ker_n(z)$.

```
>> KelvinKer[0.5]
0.855906

>> KelvinKer[1.5 + I]
-0.167162 - 0.184404I

>> KelvinKer[0.5, 0.25]
0.450023

>> Plot[KelvinKer[x], {x, 0, 10}]
```



LaguerreL

`LaguerreL[n, x]`
returns the Laguerre polynomial $L_n(x)$.
`LaguerreL[n, a, x]`
returns the generalised Laguerre polynomial $L_n^a(x)$.

```
>> LaguerreL[8, x]
1 - 8x + 14x^2 - 28x^3/3 + 35x^4/12
- 7x^5/15 + 7x^6/180 - x^7/630 + x^8/40320

>> LaguerreL[3/2, 1.7]
-0.947134

>> LaguerreL[5, 2, x]
21 - 35x + 35x^2/2 - 7x^3/2 + 7x^4/24 - x^5/120
```

LegendreP

`LegendreP[n, x]`
returns the Legendre polynomial $P_n(x)$.
`LegendreP[n, m, x]`
returns the associated Legendre polynomial $P_n^m(x)$.

```
>> LegendreP[4, x]
3/8 - 15x^2/4 + 35x^4/8

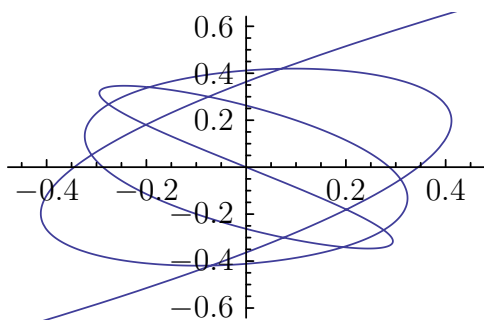
>> LegendreP[5/2, 1.5]
4.17762

>> LegendreP[1.75, 1.4, 0.53]
-1.32619

>> LegendreP[1.6, 3.1, 1.5]
-0.303998 - 1.91937I
```

`LegendreP` can be used to draw generalized Lis-sajous figures:

```
>> ParametricPlot[ {LegendreP[7, x], LegendreP[5, x]}, {x, -1, 1}]
```



LegendreQ

LegendreQ[n, x]
returns the Legendre function of the second kind $Q_n(x)$.

LegendreQ[n, m, x]
returns the associated Legendre function of the second kind $Q_n^m(x)$.

```
>> LegendreQ[5/2, 1.5]
0.036211 - 6.56219I

>> LegendreQ[1.75, 1.4, 0.53]
2.05499

>> LegendreQ[1.6, 3.1, 1.5]
-1.71931 - 7.70273I
```

ProductLog

ProductLog[z]
returns the value of the Lambert W function at z.

The defining equation:

```
>> z == ProductLog[z] * E ^
ProductLog[z]
True
```

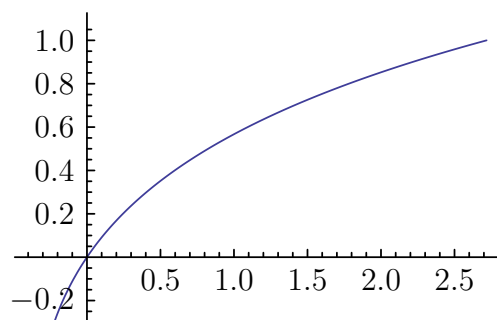
Some special values:

```
>> ProductLog[0]
0

>> ProductLog[E]
1
```

The graph of ProductLog:

```
>> Plot[ProductLog[x], {x, -1/E, E}]
```



SphericalHarmonicY

SphericalHarmonicY[l, m, theta, phi]
returns the spherical harmonic function $Y_l^m(\theta, \phi)$.

```
>> SphericalHarmonicY[3/4, 0.5, Pi/5, Pi/3]
0.254247 + 0.14679I

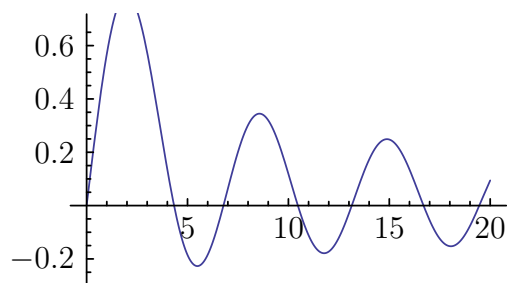
>> SphericalHarmonicY[3, 1, theta, phi]
sqrt(21) (1 - 5Cos[theta]^2) E^Iphi Sin[theta] / (8 sqrt(Pi))
```

StruveH

StruveH[n, z]
returns the Struve function $H_n(z)$.

```
>> StruveH[1.5, 3.5]
1.13192

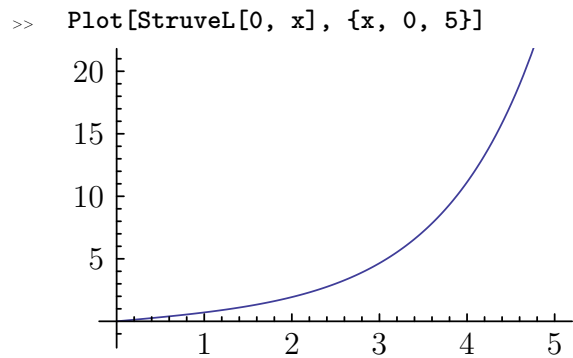
>> Plot[StruveH[0, x], {x, 0, 20}]
```



StruveL

`StruveL[n, z]`
returns the modified Struve function $L_n(z)$.

```
>> StruveL[1.5, 3.5]  
4.41126
```

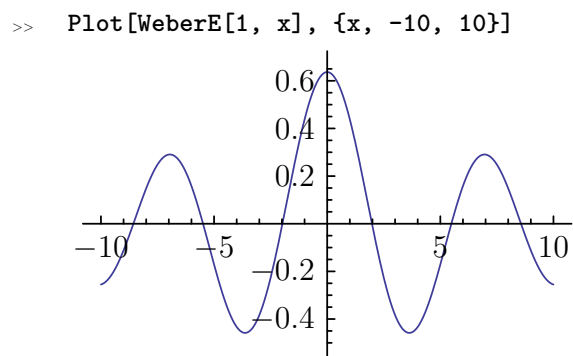


```
>> Zeta[-2.5 + I]  
0.0235936 + 0.0014078I
```

WeberE

`WeberE[n, z]`
returns the Weber function $E_n(z)$.

```
>> WeberE[1.5, 3.5]  
-0.397256
```



Zeta

`Zeta[z]`
returns the Riemann zeta function of z .

```
>> Zeta[2]  
 $\frac{\pi^2}{6}$ 
```

XXXV. Scoping

Contents

Begin	226	Sys-	End	228
BeginPackage	226	tem'Private'\$ContextPathStack 227	EndPackage	228
Block	227	Sys-	Module	228
Context	227	tem'Private'\$ContextStack 227	\$ModuleNumber . . .	228
\$ContextPath	227	\$Context	Unique	229
		Contexts		

Begin

Begin[*context*]
temporarily sets the current context to *context*.

```
>> Begin["test'"]
test'
>> {$Context, $ContextPath}
{test', {Global', System'}}
>> Context[newsymbol]
test'
>> End[]
test'
>> End[]
Nopreviouscontextdefined.
Global'
```

BeginPackage

BeginPackage[*context*]
starts the package given by *context*.

The *context* argument must be a valid context name. BeginPackage changes the values of \$Context and \$ContextPath, setting the current context to *context*.

```
>> {$Context, $ContextPath}
{Global', {Global', System'}}
```

```
>> BeginPackage["test'"]
test'
>> {$Context, $ContextPath}
{test', {test', System'}}
>> Context[newsymbol]
test'
>> EndPackage[]
>> {$Context, $ContextPath}
{Global', {test', Global', System'}}
>> EndPackage[]
Nopreviouscontextdefined.
```

Block

Block[{*x*, *y*, ...}, *expr*]
temporarily removes the definitions of the given variables, evaluates *expr*, and restores the original definitions afterwards.

Block[{*x*=*x0*, *y*=*y0*, ...}, *expr*]
assigns temporary values to the variables during the evaluation of *expr*.

```
>> n = 10
10
>> Block[{n = 5}, n ^ 2]
25
```

```
>> n
10
```

Values assigned to block variables are evaluated at the beginning of the block. Keep in mind that the result of Block is evaluated again, so a returned block variable will get its original value.

```
>> Block[{x = n+2, n}, {x, n}]
{12, 10}
```

If the variable specification is not of the described form, an error message is raised:

```
>> Block[{x + y}, x]
Local variablespecificationcontainsx+y,
whichisnotasymboloranassignmenttoasymbol.
x
```

Variable names may not appear more than once:

```
>> Block[{x, x}, x]
Duplicatelocalvariablexfoundinlocalvariablespecification
x
```

Context

Context[symbol]
yields the name of the context where symbol is defined in.

Context[]
returns the value of \$Context.

```
>> Context[a]
Global'

>> Context[b'c]
b'

>> Context[Sin] // InputForm
"System"

>> InputForm[Context[]]
"Global"
```

\$ContextPath

\$ContextPath
is the search path for contexts.

```
>> $ContextPath // InputForm
{"Global", "System"}
```

System'Private'\$ContextPathStack

System'Private'\$ContextPathStack
is an internal variable tracking the values of \$ContextPath saved by Begin and BeginPackage.

System'Private'\$ContextStack

System'Private'\$ContextStack
is an internal variable tracking the values of \$Context saved by Begin and BeginPackage.

\$Context

\$Context
is the current context.

```
>> $Context
Global'
```

Contexts

Contexts[]
yields a list of all contexts.

```
>> x = 5;

>> Contexts[] // InputForm
{"Combinatorica",
 "Global", "ImportExport",
 "Internal", "System",
 "System'Convert'B64Dump",
 "System'Convert'Image",
 "System'Convert'JSONDump",
 "System'Convert'TableDump",
 "System'Convert'TextDump",
 "System'Private",
 "XML", "XML'Parser"}
```

End

```
End[]
  ends a context started by Begin.
```

EndPackage

```
EndPackage[]
  marks the end of a package, undoing a
  previous BeginPackage.
```

After EndPackage, the values of \$Context and \$ContextPath at the time of the BeginPackage call are restored, with the new package's context prepended to \$ContextPath.

Module

```
Module[{vars}, expr]
  localizes variables by giving them a tem-
  porary name of the form name$number,
  where number is the current value of
  $ModuleNumber. Each time a module
  is evaluated, $ModuleNumber is incre-
  mented.
```

```
>> x = 10;

>> Module[{x=x}, x=x+1; x]
11
>> x
10
>> t === Module[{t}, t]
False
```

Initial values are evaluated immediately:

```
>> Module[{t=x}, x = x + 1; t]
10
>> x
11
```

Variables inside other scoping constructs are not affected by the renaming of Module:

```
>> Module[{a}, Block[{a}, a]]
a
>> Module[{a}, Block[{}, a]]
a$5
```

\$ModuleNumber

```
$ModuleNumber
  is the current "serial number" to be used
  for local module variables.
```

```
>> Unprotect[$ModuleNumber]

>> $ModuleNumber = 20;

>> Module[{x}, x]
x$20

>> $ModuleNumber = x;
Cannot set $ModuleNumber to x; value must be a positive integer.
```

Unique

```
Unique[]
  generates a new symbol and gives a name
  of the form $number.
Unique[x]
  generates a new symbol and gives a name
  of the form x$number.
Unique[{x, y, ...}]
  generates a list of new symbols.
Unique['xxx']
  generates a new symbol and gives a name
  of the form xxxnumber.
```

Create a unique symbol with no particular name:

```
>> Unique[]
$12

>> Unique[sym]
sym$1
```

Create a unique symbol whose name begins with x:

```
>> Unique["x"]
x13
```

Each use of Unique[symbol] increments \$ModuleNumber:

```
>> {$ModuleNumber, Unique[x],
  $ModuleNumber}
{4, x$4, 5}
```

Unique[symbol] creates symbols in the same way Module does:

```
>> {Module[{x}, x], Unique[x]}
     {x$5,x$6}
```

Unique with more arguments

```
>> Unique[{x, "s"}, Flat ^ Listable
     ^ Orderless]

FlatListableObjectlessisnotknownattribute.
Unique  $\left[ \{x, s\}, \text{Flat}^{\text{Listable}^{\text{Orderless}}} \right]$ 
```

Unique call without symbol argument

```
>> Unique[x + y]

x + yisnotasymboloravalidsymbolname.
Unique  $[x + y]$ 
```

XXXVI. String functions

Contents

<code>\$CharacterEncoding</code>	230	<code>NumberString</code>	233	<code>StringReplace</code>	238
<code>\$CharacterEncodings</code>	230	<code>RegularExpression</code>	233	<code>StringRiffle</code>	238
<code>CharacterRange</code>	230	<code>RemoveDiacritics</code>	234	<code>StringSplit</code>	239
<code>Characters</code>	230	<code>StartOfLine</code>	234	<code>StringTake</code>	239
<code>DamerauLevenshteinDistance</code>	231	<code>StartOfString</code>	234	<code>StringTrim</code>	239
<code>DigitCharacter</code>	231	<code>StringCases</code>	235	<code>String</code>	239
<code>DigitQ</code>	231	<code>StringContainsQ</code>	235	<code>ToCharacterCode</code>	240
<code>EditDistance</code>	232	<code>StringDrop</code>	235	<code>ToExpression</code>	240
<code>EndOfLine</code>	232	<code>StringExpression (~~)</code>	235	<code>ToLowerCase</code>	240
<code>EndOfString</code>	232	<code>StringFreeQ</code>	236	<code>ToString</code>	240
<code>FromCharacterCode</code>	232	<code>StringInsert</code>	237	<code>ToUpperCase</code>	241
<code>HammingDistance</code>	233	<code>StringJoin (<>)</code>	237	<code>Transliterate</code>	241
<code>HexadecimalCharacter</code>	233	<code>StringLength</code>	237	<code>UpperCaseQ</code>	241
<code>LetterCharacter</code>	233	<code>StringMatchQ</code>	237	<code>Whitespace</code>	241
<code>LetterQ</code>	233	<code>StringPosition</code>	237	<code>WhitespaceCharacter</code>	241
<code>LowerCaseQ</code>	233	<code>StringQ</code>	238	<code>WordBoundary</code>	241
		<code>StringRepeat</code>	238	<code>WordCharacter</code>	242

\$CharacterEncoding

`CharacterEncoding`
specifies the default character encoding to use if no other encoding is specified.

\$CharacterEncodings

CharacterRange

`CharacterRange["a", "b"]`
returns a list of the Unicode characters from *a* to *b* inclusive.

```
>> CharacterRange["a", "e"]
{a, b, c, d, e}

>> CharacterRange["b", "a"]
{}
```

Characters

`Characters["string"]`
returns a list of the characters in *string*.

```
>> Characters["abc"]
{a, b, c}
```

DamerauLevenshteinDistance

`DamerauLevenshteinDistance[a, b]`
returns the Damerau-Levenshtein distance of *a* and *b*, which is defined as the minimum number of transpositions, insertions, deletions and substitutions needed to transform one into the other. In contrast to `EditDistance`, `DamerauLevenshteinDistance` counts transposition of adjacent items (e.g. "ab" into "ba") as one operation of change.


```
>> EditDistance[{1, E, 2, Pi}, {1,
  E, Pi, 2}]
2
```

EndOfLine

EndOfString
represents the end of a line in a string.

```
>> StringReplace["aba\nbba\na\nab",
  "a" ~~EndOfLine -> "c"]

abc
  bbc
    c
  ab

>> StringSplit["abc\ndef\nhij",
  EndOfLine]

{abc,
  def,
  hij}
```

EndOfString

EndOfString
represents the end of a string.

Test whether strings end with “e”:

```
>> StringMatchQ[#, __ ~~"e" ~~
  EndOfString] &/@ {"apple", "
  banana", "artichoke"}

{True, False, True}

>> StringReplace["aab\nabb", "b" ~~
  EndOfString -> "c"]

aab
  abc
```

FromCharCode

FromCharCode[*n*]
returns the character corresponding to
Unicode codepoint *n*.
FromCharCode[{*n*₁, *n*₂, ...}]
returns a string with characters corre-
sponding to *n*_{*i*}.
FromCharCode[{{*n*₁₁, *n*₁₂, ...}, {
*n*₂₁, *n*₂₂, ...}, ...}]
returns a list of strings.

```
>> FromCharCode[100]
d

>> FromCharCode[228, "ISO8859
-1"]
ä

>> FromCharCode[{100, 101,
  102}]
def

>> ToCharCode[%]
{100,101,102}

>> FromCharCode[{{97, 98, 99},
  {100, 101, 102}}]
{abc,def}

>> ToCharCode["abc 123"] //
  FromCharCode
abc 123
```

HammingDistance

HammingDistance[*u*, *v*]
returns the Hamming distance between
u and *v*, i.e. the number of different el-
ements. *u* and *v* may be lists or strings.

```
>> HammingDistance[{1, 0, 1, 0},
  {1, 0, 0, 1}]
2

>> HammingDistance["time", "dime"]
1

>> HammingDistance["TIME", "dime",
  IgnoreCase -> True]
1
```


HexidecimalCharacter

HexidecimalCharacter
represents the characters 0-9, a-f and A-F.

```
>> StringMatchQ[#,  
  HexidecimalCharacter] & /@ {"a",  
  "1", "A", "x", "H", " ", "."}  
  
{True, True, True, False,  
  False, False, False}
```

LetterCharacter

LetterCharacter
represents letters.

```
>> StringMatchQ[#, LetterCharacter]  
  & /@ {"a", "1", "A", " ", "."}  
  
{True, False, True, False, False}
```

LetterCharacter also matches unicode characters.

```
>> StringMatchQ["\[Lambda]",  
  LetterCharacter]  
  
True
```

LetterQ

LetterQ[*string*] yields True if all the characters in the *string* are letters, and yields False otherwise.

```
>> LetterQ["m"]  
True  
  
>> LetterQ["9"]  
False  
  
>> LetterQ["Mathics"]  
True  
  
>> LetterQ["Welcome to Mathics"]  
False
```

LowerCaseQ

LowerCaseQ[*s*]
returns True if *s* consists wholly of lower case characters.

```
>> LowerCaseQ["abc"]  
True
```

An empty string returns True.

```
>> LowerCaseQ[""]  
True
```

NumberString

NumberString
represents the characters in a number.

```
>> StringMatchQ["1234",  
  NumberString]  
  
True  
  
>> StringMatchQ["1234.5",  
  NumberString]  
  
True  
  
>> StringMatchQ["1.2'20",  
  NumberString]  
  
False
```

RegularExpression

RegularExpression[‘‘*regex*’']
represents the regex specified by the string \$“*regex*”\$.

```
>> StringSplit["1.23, 4.56 7.89",  
  RegularExpression["(\\s|,)+"]]  
  
{1.23, 4.56, 7.89}
```

RemoveDiacritics

RemoveDiacritics[*s*]
returns a version of *s* with all diacritics removed.

```
>> RemoveDiacritics["en prononçant
pêcher et pêcher"]
en prononcant pecher et pecher

>> RemoveDiacritics["piñata"]
pinata
```

StartOfLine

StartOfString
represents the start of a line in a string.

```
>> StringReplace["aba\nbba\na\nab",
  StartOfLine ~~"a" -> "c"]

cba
  bba
    c
  cb

>> StringSplit["abc\ndef\nhij",
  StartOfLine]

{abc
 ,def
 ,hij}
```

StartOfString

StartOfString
represents the start of a string.

Test whether strings start with "a":

```
>> StringMatchQ[#, StartOfString ~~
"a" ~~__] &/@ {"apple", "banana", "artichoke"}

{True, False, True}

>> StringReplace["aba\nabb",
  StartOfString ~~"a" -> "c"]

cba
  abb
```

StringCases

StringCases["string", *pattern*]
gives all occurrences of *pattern* in *string*.
StringReplace["string", *pattern* -> *form*]
gives all instances of *form* that stem from
occurrences of *pattern* in *string*.
StringCases["string", {*pattern1*, *pattern2*,
...}]
gives all occurrences of *pattern1*, *pattern2*,
....
StringReplace["string", *pattern*, *n*]
gives only the first *n* occurrences.
StringReplace[{"string1", "string2",
...}, *pattern*]
gives occurrences in *string1*, *string2*, ...

```
>> StringCases["axbaxxb", "a" ~~x_
~~"b"]

{axb}

>> StringCases["axbaxxb", "a" ~~x_
~~"b"]

{axbaxxb}

>> StringCases["axbaxxb", Shortest
["a" ~~x_ ~~"b"]]

{axb, axxb}

>> StringCases["-abc- def -uvw- xyz", Shortest["-" ~~x_ ~~"-"] ->
x]

{abc, uvw}

>> StringCases["-öhi- -abc- -. -",
  "- " ~~x : WordCharacter .. ~~"- "
  -> x]

{öhi, abc}

>> StringCases["abc-abc xyz-uvw",
  Shortest[x : WordCharacter .. ~~
  "- " ~~x_] -> x]

{abc}

>> StringCases["abba", {"a" -> 10,
  "b" -> 20}, 2]

{10, 20}

>> StringCases["a#ä_123",
  WordCharacter]

{a, ä, 1, 2, 3}
```

```
>> StringCases["a#ä_123",
LetterCharacter]

{a,ä}
```

StringContainsQ

`StringContainsQ["string", patt]`
returns True if any part of *string* matches *patt*, and returns False otherwise.
`StringContainsQ[{‘s1’, “s2”, ...}, patt]`
returns the list of results for each element of string list.
`StringContainsQ[patt]`
represents an operator form of `StringContainsQ` that can be applied to an expression.

```
>> StringContainsQ["mathics", "m" ~
~__ ~~"s"]
True

>> StringContainsQ["mathics", "a" ~
~__ ~~"m"]
False

>> StringContainsQ["Mathics", "MA"
, IgnoreCase -> True]
True

>> StringContainsQ[{"g", "a", "laxy",
, "universe", "sun"}, "u"]
{False, False, False, True, True}

>> StringContainsQ["e" ~~__ ~~"u"]
/@ {"The Sun", "Mercury", "
Venus", "Earth", "Mars", "
Jupiter", "Saturn", "Uranus", "
Neptune"}
{True, True, True, False, False,
False, False, False, True}
```

StringDrop

`StringDrop["string", n]`
gives *string* with the first *n* characters dropped.
`StringDrop["string", -n]`
gives *string* with the last *n* characters dropped.
`StringDrop["string", {n}]`
gives *string* with the *n*th character dropped.
`StringDrop["string", {m, n}]`
gives *string* with the characters *m* through *n* dropped.

```
>> StringDrop["abcde", 2]
cde

>> StringDrop["abcde", -2]
abc

>> StringDrop["abcde", {2}]
acde

>> StringDrop["abcde", {2,3}]
ade

>> StringDrop["abcd",{3,2}]
abcd

>> StringDrop["abcd",0]
abcd
```

StringExpression (~~)

`StringExpression[s_1, s_2, ...]`
represents a sequence of strings and symbolic string objects *s_i*.

```
>> "a" ~~ "b" // FullForm
"ab"
```

StringFreeQ

`StringFreeQ["string", patt]`
returns True if no substring in *string* matches the string expression *patt*, and returns False otherwise.

`StringFreeQ[{'s1', "s2", ...}, patt]`
returns the list of results for each element of string list.

`StringFreeQ['string', {p1, p2, ...}]`
returns True if no substring matches any of the *pi*.

`StringFreeQ[patt]`
represents an operator form of `StringFreeQ` that can be applied to an expression.

```
>> StringFreeQ["mathics", "m" ~~__
~~"s"]
False

>> StringFreeQ["mathics", "a" ~~__
~~"m"]
True

>> StringFreeQ["Mathics", "MA" ,
IgnoreCase -> True]
False

>> StringFreeQ[{"g", "a", "laxy", "
universe", "sun"}, "u"]
{True, True, True, False, False}

>> StringFreeQ["e" ~~__ ~~"u"] /@
{"The Sun", "Mercury", "Venus",
"Earth", "Mars", "Jupiter", "
Saturn", "Uranus", "Neptune"}
{False, False, False, True,
True, True, True, True, False}

>> StringFreeQ[{"A", "Galaxy", "Far
", "Far", "Away"}, {"F" ~~__ ~~"
r", "aw" ~~__}, IgnoreCase ->
True]
{True, True, False, False, False}
```

StringInsert

`StringInsert["strsource", "strnew", pos]`
returns a string with *strnew* inserted starting at position *pos* in *strsource*.

`StringInsert["strsource", "strnew", -pos]`
returns a string with *strnew* inserted at position *pos* from the end of *strsource*.

`StringInsert["strsource", "strnew", {pos_1, pos_2, ...}]`
returns a string with *strnew* inserted at each position *pos_i* in *strsource*, the *pos_i* are taken before any insertion is done.

`StringInsert[{str_1, str_2, ...}, "strnew", pos]`
inserts *strnew* to each of *s_i* at the position *pos*

```
>> StringInsert["abcdefghijklm", "X
", 4]
abcXdefghijklm

>> StringInsert["abcdefghijklm", "X
", 1]
Xabcdefghijklm

>> StringInsert["abcdefghijklm", "X
", 14]
abcdefghijklmX
```

(watch the empty line).

```
>> StringInsert["abcdefghijklm", "X
", -1]
abcdefghijklmX

>> StringInsert["abcdefghijklm", "X
", -14]
Xabcdefghijklm
```

(watch the empty line).

```
>> StringInsert["abcdefghijklm", "X
", {1, 4, 9}]
XabcXdefghXijklm
```

(watch the empty line).

```
>> StringInsert[{"abcdefghijklm", "
Mathics"}, "X", 4]
{abcXdefghijklm, MatXhics}

>> StringInsert["1234567890123456",
".", Range[-16, -4, 3]]
1.234.567.890.123.456
```

StringJoin (<>)

```
StringJoin["s1'", "s2", ...]  
returns the concatenation of the strings  
s1, s2, .
```

```
>> StringJoin["a", "b", "c"]  
abc  
  
>> "a" <> "b" <> "c" // InputForm  
"abc"  
  
StringJoin flattens lists out:  
>> StringJoin[{"a", "b"}] //  
InputForm  
"ab"  
  
>> Print[StringJoin[{"Hello", " ",  
{"world"}}, {"!"}]  
  
Helloworld!
```

StringLength

```
StringLength["string"]  
gives the length of string.
```

```
>> StringLength["abc"]  
3  
  
StringLength is listable:  
>> StringLength[{"a", "bc"}]  
{1, 2}  
  
>> StringLength[x]  
StringExpected.  
StringLength[x]
```

StringMatchQ

```
>> StringMatchQ["abc", "abc"]  
True  
  
>> StringMatchQ["abc", "abd"]  
False  
  
>> StringMatchQ["15a94xcZ6", (  
DigitCharacter | LetterCharacter  
)..]  
True
```

Use StringMatchQ as an operator

```
>> StringMatchQ[LetterCharacter] ["a"  
"]  
True
```

StringPosition

```
StringPosition["string", patt]  
gives a list of starting and ending posi-  
tions where patt matches "string".  
StringPosition["string", patt, n]  
returns the first n matches only.  
StringPosition["string", {patt1, patt2,  
...}, n]  
matches multiple patterns.  
StringPosition[{s1, s2, ...}, patt]  
returns a list of matches for multiple  
strings.
```

```
>> StringPosition["123  
ABCxyABCzzzABCABC", "ABC"]  
{ {4, 6}, {9, 11}, {15, 17}, {18, 20} }  
  
>> StringPosition["123  
ABCxyABCzzzABCABC", "ABC", 2]  
{ {4, 6}, {9, 11} }
```

StringPosition can be useful for searching through text.

```
>> data = Import["ExampleData/  
EinsteinSzilLetter.txt"];  
  
>> StringPosition[data, "uranium"]  
{ {299, 305}, {870, 876}, {1538, 1~  
~544}, {1671, 1677}, {2300, 2306  
}, {2784, 2790}, {3093, 3099} }
```

StringQ

```
StringQ[expr]  
returns True if expr is a String, or False  
otherwise.
```

```
>> StringQ["abc"]  
True  
  
>> StringQ[1.5]  
False
```

```
>> Select[{12, 1, 3, 5, "yz", x,
y}, StringQ]
{12, yz}
```

StringRepeat

```
StringRepeat["string", n]
  gives string repeated n times.
StringRepeat["string", n, max]
  gives string repeated n times, but not
  more than max characters.
```

```
>> StringRepeat["abc", 3]
abcabcabc

>> StringRepeat["abc", 10, 7]
abcabca
```

StringReplace

```
StringReplace["string", "a" -> "b"]
  replaces each occurrence of old with new
  in string.
StringReplace["string", {"s1" -> "sp1",
"s2" -> "sp2"}]
  performs multiple replacements of each
  si by the corresponding spi in string.
StringReplace["string", srules, n]
  only performs the first n replacements.
StringReplace[{string1, string2,
...}, srules]
  performs the replacements specified by
  srules on a list of strings.
```

StringReplace replaces all occurrences of one substring with another:

```
>> StringReplace["xyxyxyxyxyxyxy",
"xy" -> "A"]
AAxyxyxyA
```

Multiple replacements can be supplied:

```
>> StringReplace["xyzwxyzwxyzxyzw",
{"xyz" -> "A", "w" -> "BCD"}]
ABCDABCDxABCBD
```

Only replace the first 2 occurrences:

```
>> StringReplace["xyxyxyxyxyxyxy",
"xy" -> "A", 2]
AAxyxyxyxyxy
```

Also works for multiple rules:

```
>> StringReplace["abba", {"a" -> "A",
"b" -> "B"}, 2]
ABba
```

StringReplace acts on lists of strings too:

```
>> StringReplace[{"xyxyxyxy", "xyxyxyxyxyxy"},
"xy" -> "A"]
{AAxA, yAAxxAyA}
```

StringReplace also can be used as an operator:

```
>> StringReplace["y" -> "ies"]["city"]
cities
```

StringRiffle

```
StringRiffle[{s1, s2, s3, ...}]
  returns a new string by concatenating
  all the si, with spaces inserted between
  them.
StringRiffle[list, sep]
  inserts the separator sep between all ele-
  ments in list.
StringRiffle[list, {"left", "sep",
"right"}]
  use left and right as delimiters after con-
  catenation.
```

```
>> StringRiffle[{"a", "b", "c", "d",
"e"}]
a b c d e

>> StringRiffle[{"a", "b", "c", "d",
"e"}, " "]
a, b, c, d, e

>> StringRiffle[{"a", "b", "c", "d",
"e"}, {"(", " ", ")"}]
(a b c d e)
```

StringSplit

```
StringSplit["s"]
  splits the string s at whitespace, discarding
  the whitespace and returning a list of
  strings.
StringSplit["s'", "d"]
  splits s at the delimiter d.
StringSplit[s, {"d1'", "d2", ...}]
  splits s using multiple delimiters.
```

```
>> StringSplit["abc,123", ","]
{abc,123}

>> StringSplit["abc 123"]
{abc,123}

>> StringSplit["abc,123.456", {"",",",
  "."}]
{abc,123,456}

>> StringSplit["a b c",
  RegularExpression[" +"]]
{a,b,c}
```

StringTake

```
StringTake["string", n]
  gives the first n characters in string.
StringTake["string", -n]
  gives the last n characters in string.
StringTake["string", {n}]
  gives the nth character in string.
StringTake["string", {m, n}]
  gives characters m through n in string.
StringTake["string", {m, n, s}]
  gives characters m through n in steps of s.
```

```
>> StringTake["abcde", 2]
ab

>> StringTake["abcde", 0]

(watch the empty line).
>> StringTake["abcde", -2]
de

>> StringTake["abcde", {2}]
b
```

```
>> StringTake["abcd", {2,3}]
bc

>> StringTake["abcdefgh", {1, 5,
  2}]
ace
```

StringTake also supports standard sequence specifications

```
>> StringTake["abcdef", All]
abcdef
```

StringTrim

```
StringTrim[s]
  returns a version of s with whitespace removed
  from start and end.
```

```
>> StringJoin["a", StringTrim[" \tb
  \n "], "c"]
abc

>> StringTrim["ababaxababyaabab",
  RegularExpression["(ab)+"]]
axababya
```

String

```
String
  is the head of strings.
```

```
>> Head["abc"]
String

>> "abc"
abc
```

Use InputForm to display quotes around strings:

```
>> InputForm["abc"]
"abc"
```

FullForm also displays quotes:

```
>> FullForm["abc" + 2]
Plus[2, "abc"]
```

ToCharacterCode

`ToCharacterCode["string"]`
converts the string to a list of character codes (Unicode codepoints).
`ToCharacterCode[{"string1", "string2", ...}]`
converts a list of strings to character codes.

```
>> ToCharacterCode["abc"]
{97, 98, 99}

>> FromCharacterCode[%]
abc

>> ToCharacterCode["\[Alpha]\[Beta]\[Gamma]"]
{945, 946, 947}

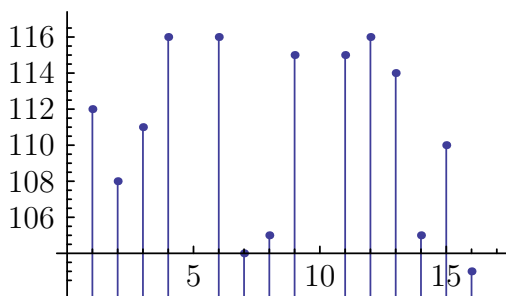
>> ToCharacterCode["ä", "UTF8"]
{195, 164}

>> ToCharacterCode["ä", "ISO8859-1"]
{228}

>> ToCharacterCode[{"ab", "c"}]
{{97, 98}, {99}}

>> ToCharacterCode[{"ab", x}]
String or list of strings expected at position 1 in ToCharacterCode[{ab, x}].
ToCharacterCode[{ab, x}]

>> ListPlot[ToCharacterCode["plot this string"], Filling -> Axis]
```



ToExpression

`ToExpression[input]`
interprets a given string as Mathics input.
`ToExpression[input, form]`
reads the given input in the specified form.
`ToExpression[input, form, h]`
applies the head *h* to the expression before evaluating it.

```
>> ToExpression["1 + 2"]
3

>> ToExpression["{2, 3, 1}",
InputForm, Max]
3
```

ToLowerCase

`ToLowerCase[s]`
returns *s* in all lower case.

```
>> ToLowerCase["New York"]
new york
```

ToString

`ToString[expr]`
returns a string representation of *expr*.

```
>> ToString[2]
2

>> ToString[2] // InputForm
"2"

>> ToString[a+b]
a + b

>> "U" <> 2
String expected.
U<>2

>> "U" <> ToString[2]
U2
```


ToUpperCase

```
ToUpperCase[s]  
returns s in all upper case.
```

```
>> ToUpperCase["New York"]  
NEW YORK
```

Transliterate

```
Transliterate[s]  
transliterates a text in some script into an  
ASCII string.
```

```
# The following examples were taken from  
# https://en.wikipedia.org/wiki/Iliad, #  
https://en.wikipedia.org/wiki/Russian\_language,  
and # https://en.wikipedia.org/wiki/Hiragana
```

UpperCaseQ

```
UpperCaseQ[s]  
returns True if s consists wholly of upper  
case characters.
```

```
>> UpperCaseQ["ABC"]  
True
```

An empty string returns True.

```
>> UpperCaseQ[""]  
True
```

Whitespace

```
Whitespace  
represents a sequence of whitespace char-  
acters.
```

```
>> StringMatchQ["\r\n", Whitespace  
]  
True
```

```
>> StringSplit["a\nb\r\ncd",  
Whitespace]  
{a,b,c,d}
```

```
>> StringReplace[" this has leading  
and trailing whitespace\n", (  
StartOfString ~~Whitespace)| (  
Whitespace ~~EndOfString)-> ""]  
<> " removed" // FullForm
```

```
"this has leading and trailing  
whitespace removed"
```

WhitespaceCharacter

```
WhitespaceCharacter  
represents a single whitespace character.
```

```
>> StringMatchQ["\n",  
WhitespaceCharacter]  
True  
>> StringSplit["a\nb\r\nc\r d",  
WhitespaceCharacter]  
{a,b,c,d}
```

For sequences of whitespace characters use
Whitespace:

```
>> StringMatchQ["\n",  
WhitespaceCharacter]  
False  
>> StringMatchQ["\n", Whitespace]  
True
```

WordBoundary

```
WordBoundary  
represents the boundary between words.
```

```
>> StringReplace["apple banana  
orange artichoke", "e" ~~  
WordBoundary -> "E"]  
applE banana orangE artichokE
```

WordCharacter

```
WordCharacter  
represents a single letter or digit charac-  
ter.
```

```
>> StringMatchQ[#, WordCharacter]
    &/@ {"1", "a", "A", ",", " "}
    {True, True, True, False, False}
```

Test whether a string is alphanumeric:

```
>> StringMatchQ["abc123DEF",
    WordCharacter..]
    True

>> StringMatchQ["$b;123",
    WordCharacter..]
    False
```

XXXVII. Structure

Contents

Apply (@@)	243	Head	245	Scan	247
ApplyLevel (@@@)	243	Map (/@)	245	Sort	248
AtomQ	244	MapIndexed	246	SortBy	248
Combinator- ica'BinarySearch	244	MapThread	246	SymbolName	248
ByteCount	244	Null	246	SymbolQ	248
Depth	244	Operate	247	Symbol	248
Flatten	245	Order	247	Thread	248
FreeQ	245	OrderedQ	247	Through	249
		PatternsOrderedQ	247		

Apply (@@)

`Apply[f, expr]`
`f @@ expr`
replaces the head of `expr` with `f`.
`Apply[f, expr, levelspec]`
applies `f` on the parts specified by `levelspec`.

```
>> f @@ {1, 2, 3}
f[1,2,3]
```

```
>> Plus @@ {1, 2, 3}
6
```

The head of `expr` need not be `List`:

```
>> f @@ (a + b + c)
f[a,b,c]
```

Apply on level 1:

```
>> Apply[f, {a + b, g[c, d, e * f],
3}, {1}]
{f[a,b], f[c,d,ef], 3}
```

The default level is 0:

```
>> Apply[f, {a, b, c}, {0}]
f[a,b,c]
```

Range of levels, including negative level (counting from bottom):

```
>> Apply[f, {{{{a}}}}, {2, -3}]
{{f[f[{a}]]}}
```

Convert all operations to lists:

```
>> Apply[List, a + b * c ^ e * f[g
], {0, Infinity}]
{a, {b, {g}}, {c,e}}
```

ApplyLevel (@@@)

`ApplyLevel[f, expr]`
`f @@@ expr`
is equivalent to `Apply[f, expr, {1}]`.

```
>> f @@@ {{a, b}, {c, d}}
{f[a,b], f[c,d]}
```

AtomQ

`AtomQ[x]`
is true if `x` is an atom (an object such as a number or string, which cannot be divided into subexpressions using `Part`).

```
>> AtomQ[x]
True
```

```
>> AtomQ[1.2]
True

>> AtomQ[2 + 1]
True

>> AtomQ[2 / 3]
True

>> AtomQ[x + y]
False
```

Combinatorica'BinarySearch

`Combinatorica'BinarySearch[l, k]`
searches the list *l*, which has to be sorted, for key *k* and returns its index in *l*. If *k* does not exist in *l*, `BinarySearch` returns $(a + b) / 2$, where *a* and *b* are the indices between which *k* would have to be inserted in order to maintain the sorting order in *l*. Please note that *k* and the elements in *l* need to be comparable under a strict total order (see https://en.wikipedia.org/wiki/Total_order).

`Combinatorica'BinarySearch[l, k, f]`
the index of *k* in the elements of *l* if *f* is applied to the latter prior to comparison. Note that *f* needs to yield a sorted sequence if applied to the elements of *l*.

```
>> Combinatorica'BinarySearch[{3,
4, 10, 100, 123}, 100]
4

>> Combinatorica'BinarySearch[{2,
3, 9}, 7] // N
2.5

>> Combinatorica'BinarySearch[{2,
7, 9, 10}, 3] // N
1.5

>> Combinatorica'BinarySearch[{-10,
5, 8, 10}, -100] // N
0.5

>> Combinatorica'BinarySearch[{-10,
5, 8, 10}, 20] // N
4.5
```

```
>> Combinatorica'BinarySearch[{{a,
1}, {b, 7}}, 7, #[[2]]&]
2
```

ByteCount

`ByteCount[expr]`
gives the internal memory space used by *expr*, in bytes.

The results may heavily depend on the Python implementation in use.

Depth

`Depth[expr]`
gives the depth of *expr*.

The depth of an expression is defined as one plus the maximum number of `Part` indices required to reach any part of *expr*, except for heads.

```
>> Depth[x]
1

>> Depth[x + y]
2

>> Depth[{{{x}}}]
5
```

Complex numbers are atomic, and hence have depth 1:

```
>> Depth[1 + 2 I]
1
```

Depth ignores heads:

```
>> Depth[f[a, b][c]]
2
```

Flatten

`Flatten[expr]`
flattens out nested lists in *expr*.
`Flatten[expr, n]`
stops flattening at level *n*.
`Flatten[expr, n, h]`
flattens expressions with head *h* instead of `List`.

```
>> Flatten[{{a, b}, {c, {d}, e}, {f, {g, h}}}]
{a, b, c, d, e, f, g, h}

>> Flatten[{{a, b}, {c, {e}, e}, {f, {g, h}}}, 1]
{a, b, c, {e}, e, f, {g, h}}

>> Flatten[f[a, f[b, f[c, d]], e], Infinity, f]
f[a, b, c, d, e]

>> Flatten[{{a, b}, {c, d}}, {{2}, {1}}]
{{a, c}, {b, d}}

>> Flatten[{{a, b}, {c, d}}, {{1, 2}}]
{a, b, c, d}
```

Flatten also works in irregularly shaped arrays

```
>> Flatten[{{1, 2, 3}, {4}, {6, 7}, {8, 9, 10}}, {{2}, {1}}]
{{1, 4, 6, 8}, {2, 7, 9}, {3, 10}}
```

FreeQ

FreeQ[*expr*, *x*]
returns True if *expr* does not contain the expression *x*.

```
>> FreeQ[y, x]
True

>> FreeQ[a+b+c, a+b]
False

>> FreeQ[{1, 2, a^(a+b)}, Plus]
False

>> FreeQ[a+b, x_+y_+z_]
True

>> FreeQ[a+b+c, x_+y_+z_]
False

>> FreeQ[x_+y_+z_] [a+b]
True
```

Head

Head[*expr*]
returns the head of the expression or atom *expr*.

```
>> Head[a * b]
Times

>> Head[6]
Integer

>> Head[x]
Symbol
```

Map (/@)

Map[*f*, *expr*] or ***f* /@ *expr***
applies *f* to each part on the first level of *expr*.
Map[*f*, *expr*, *levelspec*]
applies *f* to each level specified by *levelspec* of *expr*.

```
>> f /@ {1, 2, 3}
{f[1], f[2], f[3]}

>> #^2 & /@ {1, 2, 3, 4}
{1, 4, 9, 16}
```

Map *f* on the second level:

```
>> Map[f, {{a, b}, {c, d, e}}, {2}]
{{f[a], f[b]}, {f[c], f[d], f[e]}}
```

Include heads:

```
>> Map[f, a + b + c, Heads->True]
f[Plus] [f[a], f[b], f[c]]
```

MapIndexed

MapIndexed[*f*, *expr*]
applies *f* to each part on the first level of *expr*, including the part positions in the call to *f*.
MapIndexed[*f*, *expr*, *levelspec*]
applies *f* to each level specified by *levelspec* of *expr*.

```
>> MapIndexed[f, {a, b, c}]
      {f[a, {1}], f[b, {2}], f[c, {3}]}
```

Include heads (index 0):

```
>> MapIndexed[f, {a, b, c}, Heads->
      True]
      f[List, {0}] [f[a, {1}],
                    f[b, {2}], f[c, {3}]]
```

Map on levels 0 through 1 (outer expression gets index {}):

```
>> MapIndexed[f, a + b + c * d, {0,
      1}]
      f[f[a, {1}] + f[b,
          {2}] + f[cd, {3}], {}]
```

Get the positions of atoms in an expression (convert operations to List first to disable Listable functions):

```
>> expr = a + b * f[g] * c ^ e;

>> listified = Apply[List, expr,
      {0, Infinity}];

>> MapIndexed[#2 &, listified,
      {-1}]
      {{1}, {{2, 1}, {{2, 2, 1}},
          {{2, 3, 1}, {2, 3, 2}}}}
```

Replace the heads with their positions, too:

```
>> MapIndexed[#2 &, listified,
      {-1}, Heads -> True]
      {0} [ {1}, {2, 0} [ {2, 1},
          {2, 2, 0} [ {2, 2, 1}], {2, 3,
          0} [ {2, 3, 1}, {2, 3, 2}]]]
```

The positions are given in the same format as used by Extract. Thus, mapping Extract on the indices given by MapIndexed re-constructs the original expression:

```
>> MapIndexed[Extract[expr, #2] &,
      listified, {-1}, Heads -> True]
      a + b f[g] c^e
```

MapThread

MapThread[f, {{a1, a2, ...}, {b1, b2, ...}, ...}]
returns {f[a1, b1, ...], f[a2, b2, ...], ...}.
MapThread[f, {expr1, expr2, ...}, n]
applies f at level n.

```
>> MapThread[f, {{a, b, c}, {1, 2,
      3}}]
      {f[a, 1], f[b, 2], f[c, 3]}

>> MapThread[f, {{a, b}, {c, d}},
      {{e, f}, {g, h}}, 2]
      {{f[a, e], f[b, f]},
        {f[c, g], f[d, h]}}
```

Null

Null
is the implicit result of expressions that do not yield a result.

```
>> FullForm[a:=b]
      Null
```

It is not displayed in StandardForm,

```
>> a:=b
```

in contrast to the empty string:

```
>> ""
```

(watch the empty line).

Operate

Operate[p, expr]
applies p to the head of expr.
Operate[p, expr, n]
applies p to the nth head of expr.

```
>> Operate[p, f[a, b]]
      p[f][a, b]
```

The default value of n is 1:

```
>> Operate[p, f[a, b], 1]
      p[f][a, b]
```

With $n=0$, `Operate` acts like `Apply`:

```
>> Operate[p, f[a][b][c], 0]
p[f[a][b][c]]
```

Order

`Order[x, y]`

returns a number indicating the canonical ordering of x and y . 1 indicates that x is before y , -1 that y is before x . 0 indicates that there is no specific ordering. Uses the same order as `Sort`.

```
>> Order[7, 11]
1
>> Order[100, 10]
-1
>> Order[x, z]
1
>> Order[x, x]
0
```

OrderedQ

`OrderedQ[a, b]`

is True if a sorts before b according to canonical ordering.

```
>> OrderedQ[a, b]
True
>> OrderedQ[b, a]
False
```

PatternsOrderedQ

`PatternsOrderedQ[patt1, patt2]`

returns True if pattern $patt1$ would be applied before $patt2$ according to canonical pattern ordering.

```
>> PatternsOrderedQ[x_, x_]
False
```

```
>> PatternsOrderedQ[x_, x_]
True
>> PatternsOrderedQ[b, a]
True
```

Scan

`Scan[f, expr]`

applies f to each element of $expr$ and returns Null.

`'Scan[f, expr, levelspec]`

applies f to each level specified by *levelspec* of $expr$.

```
>> Scan[Print, {1, 2, 3}]
1
2
3
```

Sort

`Sort[list]`

sorts $list$ (or the leaves of any other expression) according to canonical ordering.

`Sort[list, p]`

sorts using p to determine the order of two elements.

```
>> Sort[{4, 1.0, a, 3+I}]
{1., 3 + I, 4, a}
```

`Sort` uses `OrderedQ` to determine ordering by default. You can sort patterns according to their precedence using `PatternsOrderedQ`:

```
>> Sort[{items___, item_,
OptionsPattern[], item_symbol,
item_?test}, PatternsOrderedQ]
{item_symbol, item_?test, item_,
items___, OptionsPattern[]}
```

When sorting patterns, values of atoms do not matter:

```
>> Sort[{a, b;/t}, PatternsOrderedQ]
{b;/t, a}
```

```
>> Sort[{2+c_, 1+b__},
PatternsOrderedQ]
{2 + c_, 1 + b__}

>> Sort[{x_ + n_*y_, x_ + y_},
PatternsOrderedQ]
{x_ + n_y_, x_ + y_}
```

SortBy

`SortBy[list, f]`
sorts *list* (or the leaves of any other expression) according to canonical ordering of the keys that are extracted from the *list*'s elements using *f*. Chunks of leaves that appear the same under *f* are sorted according to their natural order (without applying *f*).

`SortBy[f]`
creates an operator function that, when applied, sorts by *f*.

```
>> SortBy[{{5, 1}, {10, -1}}, Last]
{{10, - 1}, {5, 1}}

>> SortBy[Total][{{5, 1}, {10, -9}}]
{{10, - 9}, {5, 1}}
```

SymbolName

`SymbolName[s]`
returns the name of the symbol *s* (without any leading context name).

```
>> SymbolName[x] // InputForm
"x"
```

SymbolQ

`SymbolQ[x]`
is `True` if *x* is a symbol, or `False` otherwise.

```
>> SymbolQ[a]
True
```

```
>> SymbolQ[1]
False

>> SymbolQ[a + b]
False
```

Symbol

`Symbol`
is the head of symbols.

```
>> Head[x]
Symbol
```

You can use `Symbol` to create symbols from strings:

```
>> Symbol["x"] + Symbol["x"]
2x
```

Thread

`Thread[f[args]]`
threads *f* over any lists that appear in *args*.
`Thread[f[args], h]`
threads over any parts with head *h*.

```
>> Thread[f[{a, b, c}]]
{f[a], f[b], f[c]}

>> Thread[f[{a, b, c}, t]]
{f[a, t], f[b, t], f[c, t]}

>> Thread[f[a + b + c], Plus]
f[a] + f[b] + f[c]
```

Functions with attribute `Listable` are automatically threaded over lists:

```
>> {a, b, c} + {d, e, f} + g
{a + d + g, b + e + g, c + f + g}
```

Through

`Through[p[f] [x]]`
gives *p[f[x]]*.


```
>> Through[f[g][x]]  
f[g[x]]  
  
>> Through[p[f, g][x]]  
p[f[x], g[x]]
```

XXXVIII. System functions

Contents

<code>\$Aborted</code>	250	<code>\$Machine</code>	251	<code>\$ProcessorType</code>	251
<code>\$ByteOrdering</code>	250	<code>\$MachineName</code>	251	<code>\$ScriptCommandLine</code>	251
<code>\$CommandLine</code>	250	<code>Names</code>	251	<code>\$SystemID</code>	252
<code>Environment</code>	250	<code>\$Packages</code>	251	<code>\$SystemWordLength</code>	252
<code>\$Failed</code>	250	<code>\$ParentProcessID</code>	251	<code>\$Version</code>	252
<code>GetEnvironment</code>	250	<code>\$ProcessID</code>	251		

\$Aborted

`$Aborted`
is returned by a calculation that has been aborted.

\$ByteOrdering

`$ByteOrdering`
returns the native ordering of bytes in binary data on your computer system.

```
>> $ByteOrdering == -1 ||  
    $ByteOrdering == 1  
True
```

\$CommandLine

`$CommandLine`
is a list of strings passed on the command line to launch the Mathics session.

```
>> $CommandLine  
{mathics/test.py, -o}
```

Environment

`Environment[var]`
gives the value of an operating system environment variable.

Example:

```
<pre> In[1] = Environment["HOME"]  
Out[1] = /home/rocky </pre>
```

\$Failed

`$Failed`
is returned by some functions in the event of an error.

```
>> Get["nonexistent_file.m"]  
Cannotopennonexistent_file.m.  
$Failed
```

GetEnvironment

`GetEnvironment["var$"]`
gives the setting corresponding to the variable “var” in the operating system environment.

Example:

```
<pre> In[1] = GetEnvironment["HOME"] Out[1] = HOME -> /home-  
/rocky </pre>
```

\$Machine

\$Machine
returns a string describing the type of computer system on which the Mathics is being run.

Example:

```
In[1] = $Machine Out[1] = linux
```

\$MachineName

\$MachineName
returns a string that gives the assigned name of the computer on which Mathics is being run, if such a name is defined.

Example:

```
In[1] = $MachineName Out[1] = buster
```

Names

Names["pattern"]
returns the list of names matching *pattern*.

```
>> Names["List"]  
{List}
```

The wildcard * matches any character:

```
>> Names["List*"]  
{List, ListLinePlot,  
ListPlot, ListQ, Listable}
```

The wildcard @ matches only lowercase characters:

```
>> Names["List@"]  
{Listable}
```

```
>> x = 5;
```

```
>> Names["Global`*"]  
{x}
```

The number of built-in symbols:

```
>> Length[Names["System`*"]]  
1027
```

\$Packages

\$Packages
returns a list of the contexts corresponding to all packages which have been loaded into Mathics.

```
>> MemberQ[$Packages, "System"] = True
```

\$ParentProcessID

\$ParentProcessID
gives the ID assigned to the process which invokes the Mathics by the operating system under which it is run.

```
>> Head[$ParentProcessID] == Integer = True
```

\$ProcessID

\$ProcessID
gives the ID assigned to the Mathics process by the operating system under which it is run.

```
>> Head[$ProcessID] == Integer = True
```

\$ProcessorType

\$ProcessorType
gives a string giving the architecture of the processor on which the Mathics is being run.

Example:

```
In[1] = $ProcessorType Out[1] = x86_64
```

\$ScriptCommandLine

\$ScriptCommandLine
is a list of string arguments when running the kernel in script mode.

```
>> $ScriptCommandLine  
{}
```

\$SystemID

\$SystemID
returns a short string that identifies the type of computer system on which the Mathics is being run.

Example:

```
In[1] = $SystemID Out[1] =  
linux
```

\$SystemWordLength

\$SystemWordLength
gives the effective number of bits in raw machine words on the computer system where Mathics is running.

Example:

```
In[1] = $SystemWordLength  
Out[1] = 64
```

```
>> Head[$SystemWordLength] ==  
Integer  
True
```

\$Version

\$Version
returns a string with the current Mathics version and the versions of relevant libraries.

```
>> $Version  
Mathics 1.1.dev0 on CPython 3.7.9  
(default, Aug 30 2020, 12:42:20)  
using SymPy 1.6.2, mpmath 1.1.0
```

XXXIX. Tensor functions

Contents

ArrayDepth	253	Dot (.)	254	Outer	255
ArrayQ	253	IdentityMatrix	254	Transpose	255
DiagonalMatrix	253	Inner	254	VectorQ	255
Dimensions	254	MatrixQ	254		

ArrayDepth

`ArrayDepth[a]`
returns the depth of the non-ragged array *a*, defined as `Length[Dimensions[a]]`.

```
>> ArrayDepth[{{a,b},{c,d}}]
2
>> ArrayDepth[x]
0
```

ArrayQ

`ArrayQ[expr]`
tests whether *expr* is a full array.
`ArrayQ[expr, pattern]`
also tests whether the array depth of *expr* matches *pattern*.
`ArrayQ[expr, pattern, test]`
furthermore tests whether *test* yields `True` for all elements of *expr*. `ArrayQ[expr]` is equivalent to `ArrayQ[expr, _, True&]`.

```
>> ArrayQ[a]
False
>> ArrayQ[{a}]
True
>> ArrayQ[{{a}},{{b,c}}]
False
```

```
>> ArrayQ[{{a, b}, {c, d}}, 2,
SymbolQ]
True
```

DiagonalMatrix

`DiagonalMatrix[list]`
gives a matrix with the values in *list* on its diagonal and zeroes elsewhere.

```
>> DiagonalMatrix[{1, 2, 3}]
{{1,0,0},{0,2,0},{0,0,3}}
```

```
>> MatrixForm[%]

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

```

Dimensions

`Dimensions[expr]`
returns a list of the dimensions of the expression *expr*.

A vector of length 3:

```
>> Dimensions[{a, b, c}]
{3}
```

A 3x2 matrix:

```
>> Dimensions[{{a, b}, {c, d}, {e, f}}]
{3,2}
```

Ragged arrays are not taken into account:

```
>> Dimensions[{{a, b}, {b, c}, {c,
    d, e}}]
{3}
```

The expression can have any head:

```
>> Dimensions[f[f[a, b, c]]]
{1,3}
```

Dot (.)

```
Dot[x, y]
x . y
    computes the vector dot product or matrix product  $x \cdot y$ .
```

Scalar product of vectors:

```
>> {a, b, c} . {x, y, z}
ax + by + cz
```

Product of matrices and vectors:

```
>> {{a, b}, {c, d}} . {x, y}
{ax + by, cx + dy}
```

Matrix product:

```
>> {{a, b}, {c, d}} . {{r, s}, {t,
    u}}
{{ar + bt, as + bu}, {cr + dt, cs + du}}
```

```
>> a . b
a.b
```

IdentityMatrix

```
IdentityMatrix[n]
    gives the identity matrix with  $n$  rows and columns.
```

```
>> IdentityMatrix[3]
{{1,0,0}, {0,1,0}, {0,0,1}}
```

Inner

```
Inner[f, x, y, g]
    computes a generalised inner product of  $x$  and  $y$ , using a multiplication function  $f$  and an addition function  $g$ .
```

```
>> Inner[f, {a, b}, {x, y}, g]
g[f[a, x], f[b, y]]
```

Inner can be used to compute a dot product:

```
>> Inner[Times, {a, b}, {c, d},
    Plus] == {a, b} . {c, d}
True
```

The inner product of two boolean matrices:

```
>> Inner[And, {{False, False}, {
    False, True}}, {{True, False}, {
    True, True}}, Or]
{{False, False}, {True, True}}
```

Inner works with tensors of any depth:

```
>> Inner[f, {{{a, b}}, {{c, d}}},
    {{1}, {2}}, g]
{{{g[f[a, 1], f[b, 2]]}},
 {{g[f[c, 1], f[d, 2]]}}}
```

MatrixQ

```
MatrixQ[m]
    returns True if  $m$  is a list of equal-length lists.
MatrixQ[m, f]
    only returns True if  $f[x]$  returns True for each element  $x$  of the matrix  $m$ .
```

```
>> MatrixQ[{{1, 3}, {4.0, 3/2}},
    NumberQ]
True
```

Outer

```
Outer[f, x, y]
    computes a generalised outer product of  $x$  and  $y$ , using the function  $f$  in place of multiplication.
```

```
>> Outer[f, {a, b}, {1, 2, 3}]
{{f[a, 1], f[a, 2], f[a, 3]},
 {f[b, 1], f[b, 2], f[b, 3]}}
```

Outer product of two matrices:

```
>> Outer[Times, {{a, b}, {c, d}},
{{1, 2}, {3, 4}}]
{{{a, 2a}, {3a, 4a}}, {{b,
 2b}, {3b, 4b}}}, {{c, 2c}, {3c,
 4c}}, {{d, 2d}, {3d, 4d}}}]
```

Outer of multiple lists:

```
>> Outer[f, {a, b}, {x, y, z}, {1,
 2}]
{{{f[a, x, 1], f[a, x, 2]}, {f[
  a, y, 1], f[a, y, 2]}, {f[a, z, 1],
  f[a, z, 2]}}, {{f[b, x, 1], f[
  b, x, 2]}, {f[b, y, 1], f[b, y,
  2]}, {f[b, z, 1], f[b, z, 2]}}}
```

Arrays can be ragged:

```
>> Outer[Times, {{1, 2}}, {{a, b},
{c, d, e}}]
{{{a, b}, {c, d, e}},
 {{2a, 2b}, {2c, 2d, 2e}}}]
```

Word combinations:

```
>> Outer[StringJoin, {"", "re", "un"}, {"cover", "draw", "wind"}, {"", "ing", "s"}] // InputForm
{{{ "cover", "covering", "covers"},
 { "draw", "drawing", "draws"},
 { "wind", "winding", "winds"}},
 {{ "recover", "recovering",
 "recovers"}, { "redraw",
 "redrawing", "redraws"},
 { "rewind", "rewinding",
 "rewinds"}}, { {"uncover",
 "uncovering", "uncovers"},
 { "undraw", "undrawing",
 "undraws"}, { "unwind",
 "unwinding", "unwinds"}}}
```

Compositions of trigonometric functions:

```
>> trigs = Outer[Composition, {Sin,
  Cos, Tan}, {ArcSin, ArcCos,
  ArcTan}]
{{Composition[Sin, ArcSin],
  Composition[Sin, ArcCos],
  Composition[Sin, ArcTan]},
 {Composition[Cos, ArcSin],
  Composition[Cos, ArcCos],
  Composition[Cos, ArcTan]},
 {Composition[Tan, ArcSin],
  Composition[Tan, ArcCos],
  Composition[Tan, ArcTan]}}
```

Evaluate at 0:

```
>> Map[# [0] &, trigs, {2}]
{{0, 1, 0}, {1, 0, 1}, {0,
  ComplexInfinity, 0}}
```

Transpose

Transpose $[m]$

transposes rows and columns in the matrix m .

```
>> Transpose[{{1, 2, 3}, {4, 5,
  6}}]
{{1, 4}, {2, 5}, {3, 6}}
```

```
>> MatrixForm[%]
```

$$\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

VectorQ

VectorQ $[v]$

returns True if v is a list of elements which are not themselves lists.

VectorQ $[v, f]$

returns True if v is a vector and $f[x]$ returns True for each element x of v .

```
>> VectorQ[{a, b, c}]
True
```

XL. XML

Contents

XML'PlaintextImport .	256	XMLElement	256	XMLObject	256
XML'TagsImport . . .	256	XML'Parser'XMLGet .	256	XML'XMLObjectImport	256
		XML'Parser'XMLGetString	256		

XML'PlaintextImport

```
>> StringReplace[StringTake[Import
["ExampleData/InventionNo1.xml",
"Plaintext"],31],
FromCharacterCode[10]->"/"]
MuseScore 1.2/2012-09-12/5.7/40
```

XML'TagsImport

```
>> Take[Import["ExampleData/
InventionNo1.xml", "Tags"], 10]
{accidental, alter, arpeggiate,
articulations, attributes, backup,
bar-style, barline, beam, beat-type}
```

XMLElement

XML'Parser'XMLGet

XML'Parser'XMLGetString

```
>> Head[XML'Parser'XMLGetString["<a
></a>"]]
XMLObject[Document]
```

XMLObject

XML'XMLObjectImport

```
>> Part[Import["ExampleData/
InventionNo1.xml", "XMLObject"],
2, 3, 1]
XMLElement[identification,
{ }, {XMLElement[encoding,
{ }, {XMLElement[software,
{ }, {MuseScore 1.2}]}],
XMLElement[encoding-date,
{ }, {2012-09-12}]}]}]}
>> Part[Import["ExampleData/
Namespaces.xml"], 2]
XMLElement[book,
{{http://www.w3.org/2000/xmlns/,
xmlns}->urn:loc.gov:books},
{XMLElement[title, { }, {Cheaper
by the Dozen}]}, XMLElement[
{urn:ISBN:0-395-36341-6, number},
{ }, {1568491379}], XMLElement[
notes, { }, {XMLElement[p,
{{http://www.w3.org/2000/xmlns/,
xmlns}->http://www.w3.org/1999/xhtml},
{This is a, XMLElement[i,
{ }, {funny, book!}]}]}]}]}]}]
```


XLI. Optimization

Contents

Maximize	257	Minimize	257
--------------------	-----	--------------------	-----

Maximize

`Maximize[f, x]`
compute the maximum of f respect x that
change between a and b

```
>> Maximize[-2 x^2 - 3 x + 5, x]

$$\left\{ \left\{ \frac{49}{8}, \left\{ x - > -\frac{3}{4} \right\} \right\} \right\}$$

#» Maximize[1 - (x y - 3)^2, {x, y}] = {{1, {x -> 3, y -> 1}}}
#» Maximize[{x - 2 y, x^2 + y^2 <= 1}, {x, y}] =
{{Sqrt[5], {x -> Sqrt[5] / 5, y -> -2 Sqrt[5] / 5}}}
```

Minimize

`Minimize[f, x]`
compute the minimum of f respect x that
change between a and b

```
>> Minimize[2 x^2 - 3 x + 5, x]

$$\left\{ \left\{ \frac{31}{8}, \left\{ x - > \frac{3}{4} \right\} \right\} \right\}$$

#» Minimize[(x y - 3)^2 + 1, {x, y}] = {{1, {x -> 3, y -> 1}}}
#» Minimize[{x - 2 y, x^2 + y^2 <= 1}, {x, y}] =
{{-Sqrt[5], {x -> -Sqrt[5] / 5, y -> 2 Sqrt[5] / 5}}}
```

XLII. File Operations

Contents

AbsoluteFileName . . .	258	FileHash	263	\$Path	267
BinaryRead	259	FileInformation	263	\$PathnameSeparator . .	267
BinaryWrite	260	FileNameDepth	263	Put (>>)	267
Byte	260	FileNameJoin	263	PutAppend (>>>) . . .	268
Character	260	FileNameSplit	263	Read	268
Close	260	FilePrint	264	ReadList	268
Compress	260	FileType	264	Record	268
CopyDirectory	260	Find	264	RenameDirectory . . .	268
CopyFile	260	FindFile	264	RenameFile	269
CreateDirectory	261	FindList	265	ResetDirectory	269
DeleteDirectory	261	Get (<<)	265	\$RootDirectory	269
DeleteFile	261	\$HomeDirectory	265	SetDirectory	269
Directory	261	\$InitialDirectory	265	SetFileDate	269
DirectoryName	261	\$Input	265	SetStreamPosition . . .	269
DirectoryQ	261	\$InputFileName	265	Skip	270
DirectoryStack	261	InputStream	265	StreamPosition	270
EndOfFile	262	\$InstallationDirectory .	266	Streams	270
ExpandFileName	262	Needs	266	StringToStream	270
Expression	262	Number	266	\$TemporaryDirectory .	271
File	262	OpenAppend	266	ToFileName	271
FileBaseName	262	OpenRead	266	Uncompress	271
FileByteCount	262	OpenWrite	266	Word	271
FileDate	262	\$OperatingSystem . . .	266	Write	271
FileExistsQ	262	OutputStream	266	WriteString	271
FileExtension	263	ParentDirectory	267		

AbsoluteFileName

`AbsoluteFileName["name"]`
returns the absolute version of the given filename.

```
>> AbsoluteFileName["ExampleData/sunflowers.jpg"]
/src/external-vcs/github/mathics/Mathics/matl
```

BinaryRead

`BinaryRead[stream]`
reads one byte from the stream as an integer from 0 to 255.

`BinaryRead[stream, type]`
reads one object of specified type from the stream.

`BinaryRead[stream, {type1, type2, ...}]`
reads a sequence of objects of specified types.

```
>> strm = OpenWrite[BinaryFormat ->
    True]
    OutputStream [
        /tmp/tmpo70wbelj, 329]
>> BinaryWrite[strm, {97, 98, 99}]
    OutputStream [
        /tmp/tmpo70wbelj, 329]
>> Close[strm]
    /tmp/tmpo70wbelj
>> strm = OpenRead[%, BinaryFormat
    -> True]
    InputStream [
        /tmp/tmpo70wbelj, 330]
>> BinaryRead[strm, {"Character8",
    "Character8", "Character8"}]
    {a,b,c}
>> Close[strm];
```

BinaryWrite

BinaryWrite[channel, b]
writes a single byte given as an integer
from 0 to 255.

BinaryWrite[channel, {b1, b2, ...}]
writes a sequence of byte.

BinaryWrite[channel, 'string']
writes the raw characters in a string.

BinaryWrite[channel, x, type]
writes x as the specified type.

BinaryWrite[channel, {x1, x2, ...},
type]
writes a sequence of objects as the speci-
fied type.

BinaryWrite[channel, {x1, x2, ...}, {
type1, type2, ...}]
writes a sequence of objects using a se-
quence of specified types.

```
>> strm = OpenWrite[BinaryFormat ->
    True]
    OutputStream [
        /tmp/tmpzpj83xm, 731]
```

```
>> BinaryWrite[strm, {39, 4, 122}]
    OutputStream [
        /tmp/tmpzpj83xm, 731]
>> Close[strm]
    /tmp/tmpzpj83xm
>> strm = OpenRead[%, BinaryFormat
    -> True]
    InputStream [
        /tmp/tmpzpj83xm, 732]
>> BinaryRead[strm]
    39
>> BinaryRead[strm, "Byte"]
    4
>> BinaryRead[strm, "Character8"]
    z
>> Close[strm];
```

Write a String

```
>> strm = OpenWrite[BinaryFormat ->
    True]
    OutputStream [
        /tmp/tmpadad4f8h, 733]
>> BinaryWrite[strm, "abc123"]
    OutputStream [
        /tmp/tmpadad4f8h, 733]
>> Close[%]
    /tmp/tmpadad4f8h
```

Read as Bytes

```
>> strm = OpenRead[%, BinaryFormat
    -> True]
    InputStream [
        /tmp/tmpadad4f8h, 734]
>> BinaryRead[strm, {"Character8",
    "Character8", "Character8", "
    Character8", "Character8", "
    Character8", "Character8"}]
    {a,b,c,1,2,3,EndOfFile}
>> Close[strm]
    /tmp/tmpadad4f8h
```

Read as Characters

```
>> strm = OpenRead[%, BinaryFormat
-> True]

InputStream [
  /tmp/tmpadad4f8h,735]

>> BinaryRead[strm, {"Byte", "Byte",
  "Byte", "Byte", "Byte", "Byte",
  "Byte"}]

{97,98,99,49,50,51,EndOfFile}

>> Close[strm]
/tmp/tmpadad4f8h
```

Write Type

```
>> strm = OpenWrite[BinaryFormat ->
True]

OutputStream [
  /tmp/tmp7rnt2oy4,736]

>> BinaryWrite[strm, 97, "Byte"]

OutputStream [
  /tmp/tmp7rnt2oy4,736]

>> BinaryWrite[strm, {97, 98, 99},
{"Byte", "Byte", "Byte"}]

OutputStream [
  /tmp/tmp7rnt2oy4,736]

>> Close[%]
/tmp/tmp7rnt2oy4

>> strm = OpenWrite["/dev/full",
BinaryFormat -> True]

OutputStream [/dev/full,857]

>> BinaryWrite[strm, {39, 4, 122}]
Nospaceleftondevice.

OutputStream [/dev/full,857]

>> Close[strm]
Nospaceleftondevice.
/dev/full
```

Byte

Byte
is a data type for Read.

Character

Character
is a data type for Read.

Close

Close[*stream*]
closes an input or output stream.

```
>> Close[StringToStream["123abc"]]
String

>> Close[OpenWrite[]]
/tmp/tmp6wbjd4hn
```

Compress

Compress[*expr*]
gives a compressed string representation
of *expr*.

```
>> Compress[N[Pi, 10]]
eJwz1jM0MTS1NDIzNQEADRsCNw==
```

CopyDirectory

CopyDirectory["*dir1*", "*dir2*"]
copies directory *dir1* to *dir2*.

CopyFile

CopyFile["*file1*", "*file2*"]
copies *file1* to *file2*.

```
>> CopyFile["ExampleData/sunflowers
.jpg", "MathicsSunflowers.jpg"]
MathicsSunflowers.jpg

>> DeleteFile["MathicsSunflowers.
.jpg"]
```

CreateDirectory

```
CreateDirectory["dir"]
  creates a directory called dir.
CreateDirectory[]
  creates a temporary directory.
```

```
>> dir = CreateDirectory[]
      /tmp/mrbpp996s
```

DeleteDirectory

```
DeleteDirectory["dir"]
  deletes a directory called dir.
```

```
>> dir = CreateDirectory[]
      /tmp/modmbuztw
>> DeleteDirectory[dir]
>> DirectoryQ[dir]
      False
```

DeleteFile

```
Delete["file"]
  deletes file.
Delete[{"file1", "file2", ...}]
  deletes a list of files.
```

```
>> CopyFile["ExampleData/sunflowers
.jpg", "MathicsSunflowers.jpg"];
>> DeleteFile["MathicsSunflowers.
.jpg"]
>> CopyFile["ExampleData/sunflowers
.jpg", "MathicsSunflowers1.jpg
"];
>> CopyFile["ExampleData/sunflowers
.jpg", "MathicsSunflowers2.jpg
"];
>> DeleteFile[{"MathicsSunflowers1.
.jpg", "MathicsSunflowers2.jpg"}]
```

Directory

```
Directory[]
  returns the current working directory.
```

```
>> Directory[]
      /src/external-vcs/github/mathics/Mathics
```

DirectoryName

```
DirectoryName["name"]
  extracts the directory name from a file-
  name.
```

```
>> DirectoryName["a/b/c"]
      a/b
>> DirectoryName["a/b/c", 2]
      a
```

DirectoryQ

```
DirectoryQ["name"]
  returns True if the directory called name
  exists and False otherwise.
```

```
>> DirectoryQ["ExampleData/"]
      True
>> DirectoryQ["ExampleData/
MythicalSubdir/"]
      False
```

DirectoryStack

```
DirectoryStack[]
  returns the directory stack.
```

```
>> DirectoryStack[]
      {/src/external-vcs/github/mathics/Mathics}
```

EndOfFile

EndOfFile
is returned by Read when the end of an input stream is reached.

ExpandFileName

ExpandFileName["*name*"]
expands *name* to an absolute filename for your system.

```
>> ExpandFileName["ExampleData/sunflowers.jpg"]
/src/external-vcs/github/mathics/Mathics/ExampleData/sunflowers.jpg
```

Expression

Expression
is a data type for Read.

File

FileNameBase

FileNameBase["*file*"]
gives the base name for the specified file name.

```
>> FileNameBase["file.txt"]
file
>> FileNameBase["file.tar.gz"]
file.tar
```

FileByteCount

FileByteCount["*file*"]
returns the number of bytes in *file*.

```
>> FileByteCount["ExampleData/sunflowers.jpg"]
142286
```

FileDate

FileDate["*file*", *types*]
returns the time and date at which the file was last modified.

```
>> FileDate["ExampleData/sunflowers.jpg"]
{2020,9,6,7,16,33.2822}
```

```
>> FileDate["ExampleData/sunflowers.jpg", "Access"]
{2020,10,4,1,19,56.4569}
```

```
>> FileDate["ExampleData/sunflowers.jpg", "Creation"]
Missing[NotApplicable]
```

```
>> FileDate["ExampleData/sunflowers.jpg", "Change"]
{2020,9,6,7,16,33.2822}
```

```
>> FileDate["ExampleData/sunflowers.jpg", "Modification"]
{2020,9,6,7,16,33.2822}
```

```
>> FileDate["ExampleData/sunflowers.jpg", "Rules"]
{Access->{2020,10,4,1,19,56.4569},Creation->Missing[NotApplicable],Change->{2020,9,6,7,16,33.2822},Modification->{2020,9,6,7,16,33.2822}}
```

FileExistsQ

FileExistsQ["*file*"]
returns True if *file* exists and False otherwise.

```
>> FileExistsQ["ExampleData/sunflowers.jpg"]
True
```

```
>> FileExistsQ["ExampleData/sunflowers.png"]
False
```

FileExtension

```
FileExtension["file"]
  gives the extension for the specified file
  name.
```

```
>> FileExtension["file.txt"]
txt

>> FileExtension["file.tar.gz"]
gz
```

FileHash

```
FileHash[file]
  returns an integer hash for the given file.
FileHash[file, type]
  returns an integer hash of the specified
  type for the given file.
  The types supported are "MD5",
  "Adler32", "CRC32", "SHA", "SHA224",
  "SHA256", "SHA384", and "SHA512".
```

```
>> FileHash["ExampleData/sunflowers
.jpg"]
109 937 059 621 979 839 ~
~952 736 809 235 486 742 106

>> FileHash["ExampleData/sunflowers
.jpg", "MD5"]
109 937 059 621 979 839 ~
~952 736 809 235 486 742 106

>> FileHash["ExampleData/sunflowers
.jpg", "Adler32"]
1 607 049 478

>> FileHash["ExampleData/sunflowers
.jpg", "SHA256"]
111 619 807 552 579 450 300 684 600 ~
~241 129 773 909 359 865 098 672 ~
~286 468 229 443 390 003 894 913 065
```

FileInformation

```
FileInformation["file"]
  returns information about file.
```

This function is totally undocumented in MMA!

```
>> FileInformation["ExampleData/
sunflowers.jpg"]
{File
 - > /src/external-vcs/github/mathics/Mathics/ExampleD
FileType - > File, ByteCount - >
142 286, Date - > 3.80837 × 109}
```

FileNameDepth

```
FileNameDepth["name"]
  gives the number of path parts in the
  given filename.
```

```
>> FileNameDepth["a/b/c"]
3

>> FileNameDepth["a/b/c/"]
3
```

FileNameJoin

```
FileNameJoin[{"dir_1", "dir_2", ...}]
  joins the dir_i together into one path.
```

```
>> FileNameJoin[{"dir1", "dir2", "
dir3"}]
dir1/dir2/dir3

>> FileNameJoin[{"dir1", "dir2", "
dir3"}, OperatingSystem -> "Unix
"]
dir1/dir2/dir3
```

FileNameSplit

```
FileNameSplit["filenams"]
  splits a filename into a list of parts.
```

```
>> FileNameSplit["example/path/file
.txt"]
{example, path, file.txt}
```

FilePrint

```
FilePrint[file]  
prints the raw contents of file.
```

FileType

```
FileType["file"]  
returns the type of a file, from File,  
Directory or None.
```

```
>> FileType["ExampleData/sunflowers  
.jpg"]  
File  
  
>> FileType["ExampleData"]  
Directory  
  
>> FileType["ExampleData/  
nonexistent"]  
None
```

Find

```
Find[stream, text]  
find the first line in stream that contains  
text.
```

```
>> str = OpenRead["ExampleData/  
EinsteinSzilLetter.txt"];  
  
>> Find[str, "uranium"]  
in manuscript, leads me  
to expect that the element  
uranium may be turned into  
  
>> Find[str, "uranium"]  
become possible to set up  
a nuclear chain reaction in  
a large mass of uranium,  
  
>> Close[str]  
ExampleData/EinsteinSzilLetter.txt  
  
>> str = OpenRead["ExampleData/  
EinsteinSzilLetter.txt"];
```

```
>> Find[str, {"energy", "power"} ]  
a new and important source  
of energy in the immediate  
future. Certain aspects  
  
>> Find[str, {"energy", "power"} ]  
by which vast amounts of  
power and large quantities  
of new radium-like  
  
>> Close[str]  
ExampleData/EinsteinSzilLetter.txt
```

FindFile

```
FindFile[name]  
searches $Path for the given filename.
```

```
>> FindFile["ExampleData/sunflowers  
.jpg"]  
/src/external-vcs/github/mathics/Mathics/mathics/data/  
  
>> FindFile["VectorAnalysis"]  
/src/external-vcs/github/mathics/Mathics/mathics/pack  
  
>> FindFile["VectorAnalysis"  
VectorAnalysis"]  
/src/external-vcs/github/mathics/Mathics/mathics/pack
```

FindList

```
FindList[file, text]  
returns a list of all lines in file that contain  
text.  
FindList[file, {text1, text2, ...}]  
returns a list of all lines in file that contain  
any of the specified string.  
FindList[{file1, file2, ...}, ...]  
returns a list of all lines in any of the filei  
that contain the specified strings.
```

```
>> str = FindList["ExampleData/  
EinsteinSzilLetter.txt", "  
uranium"];
```



```
>> FindList["ExampleData/
EinsteinSzilLetter.txt", "
uranium", 1]

{in manuscript, leads me
 to expect that the element
 uranium may be turned into}
```

Get (<<)

```
<<name
  reads a file and evaluates each expres-
  sion, returning only the last one.

>> Put[x + y, "example_file"]

>> <<"example_file"
    "x"cannotbefollowedby"
    text{+}y"(line1of"./example_file").

>> Put[x + y, 2x^2 + 4z!, Cos[x] +
    I Sin[x], "example_file"]

>> <<"example_file"
    "x"cannotbefollowedby"
    text{+}y"(line1of"./example_file").

>> 40! >> "fourtyfactorial"

>> FilePrint["fourtyfactorial"]
    815 915 283 247 897 734 345 611 ~
    ~269 596 115 894 272 000 000 000

>> <<"fourtyfactorial"
    815 915 283 247 897 734 345 611 ~
    ~269 596 115 894 272 000 000 000
```

\$HomeDirectory

```
$HomeDirectory
  returns the users HOME directory.

>> $HomeDirectory
    /home/rocky
```

\$InitialDirectory

```
$InitialDirectory
  returns the directory from which Mathics
  was started.
```

```
>> $InitialDirectory
    /src/external-vcs/github/mathics/Mathics
```

\$Input

```
$Input
  is the name of the stream from which in-
  put is currently being read.
```

```
>> $Input
```

\$InputFileName

```
$InputFileName
  is the name of the file from which input is
  currently being read.
```

While in interactive mode, \$InputFileName is "".

```
>> $InputFileName
```

InputStream

```
InputStream[name, n]
  represents an input stream.
```

```
>> str = StringToStream["Mathics is
    cool!"]

    InputStream[String, 944]

>> Close[str]
    String
```

\$InstallationDirectory

`$InstallationDirectory`
returns the directory in which *Mathics* was installed.

```
>> $InstallationDirectory
/src/external-vcs/github/mathics/Mathics/mathics
```

Needs

`Needs["context"]`
loads the specified context if not already in `$Packages`.

```
>> Needs["VectorAnalysis"]
```

Number

`Number`
is a data type for `Read`.

OpenAppend

`OpenAppend["file"]`
opens a file and returns an `OutputStream` to which writes are appended.

```
>> OpenAppend[]
OutputStream [
/tmp/tmpqryli2rk, 967]
```

OpenRead

`OpenRead["file"]`
opens a file and returns an `InputStream`.

```
>> OpenRead["ExampleData/
EinsteinSzilLetter.txt"]

InputStream [
ExampleData/EinsteinSzilLetter.txt,
975]
```

```
>> OpenRead["https://raw.
githubusercontent.com/mathics/
Mathics/master/README.rst"]
```

```
InputStream [
https://raw.githubusercontent.com/mathics/Mathics/mas
976]
```

OpenWrite

`OpenWrite["file"]`
opens a file and returns an `OutputStream`.

```
>> OpenWrite[]
OutputStream [
/tmp/tmpprs7c283h, 982]
```

\$OperatingSystem

`$OperatingSystem`
gives the type of operating system running *Mathics*.

```
>> $OperatingSystem
Unix
```

OutputStream

`OutputStream[name, n]`
represents an output stream.

```
>> OpenWrite[]
OutputStream [
/tmp/tmp2p4aq8x2, 986]

>> Close[%]
/tmp/tmp2p4aq8x2
```

ParentDirectory

`ParentDirectory []`
returns the parent of the current working directory.
`ParentDirectory ["dir"]`
returns the parent *dir*.

```
>> ParentDirectory []  
/src/external-vcs/github/mathics
```

\$Path

`$Path`
returns the list of directories to search when looking for a file.

```
>> $Path  
{., /home/rocky,  
/src/external-vcs/github/mathics/Mathics/matl  
/src/external-vcs/github/mathics/Mathics/mathics/packages}
```

\$PathnameSeparator

`$PathnameSeparator`
returns a string for the separator in paths.

```
>> $PathnameSeparator  
/
```

Put (>>)

expr >> *filename*
write *expr* to a file.
`Put[expr1, expr2, ..., '$'filename']`
write a sequence of expressions to a file.

```
>> 40! >> "fourtyfactorial"  
  
>> FilePrint["fourtyfactorial"]  
815 915 283 247 897 734 345 611 ~  
~269 596 115 894 272 000 000 000  
  
>> Put[50!, "fiftyfactorial"]
```

```
>> FilePrint["fiftyfactorial"]  
30 414 093 201 713 378 043 612 ~  
~608 166 064 768 844 377 641 ~  
~568 960 512 000 000 000 000  
  
>> Put[10!, 20!, 30!, "factorials"]  
  
>> FilePrint["factorials"]  
3 628 800  
2 432 902 008 176 640 000  
265 252 859 812 191 ~  
~058 636 308 480 000 000  
  
=
```

PutAppend (>>>)

expr >>> *filename*
append *expr* to a file.
`PutAppend[expr1, expr2, ..., '$'filename']`
write a sequence of expressions to a file.

```
>> Put[50!, "factorials"]  
  
>> FilePrint["factorials"]  
30 414 093 201 713 378 043 612 ~  
~608 166 064 768 844 377 641 ~  
~568 960 512 000 000 000 000  
  
>> PutAppend[10!, 20!, 30!, "  
factorials"]  
  
>> FilePrint["factorials"]  
30 414 093 201 713 378 043 612 ~  
~608 166 064 768 844 377 641 ~  
~568 960 512 000 000 000 000  
3 628 800  
2 432 902 008 176 640 000  
265 252 859 812 191 ~  
~058 636 308 480 000 000  
  
>> 60! >>> "factorials"
```

```
>> FilePrint["factorials"]
30 414 093 201 713 378 043 612 ~
~608 166 064 768 844 377 641 ~
~568 960 512 000 000 000 000
3 628 800
2 432 902 008 176 640 000
265 252 859 812 191 ~
~058 636 308 480 000 000
8 320 987 112 741 390 144 ~
~276 341 183 223 364 380 754 ~
~172 606 361 245 952 449 277 ~
~696 409 600 000 000 000 000

>> "string" >>> factorials

>> FilePrint["factorials"]
30 414 093 201 713 378 043 612 ~
~608 166 064 768 844 377 641 ~
~568 960 512 000 000 000 000
3 628 800
2 432 902 008 176 640 000
265 252 859 812 191 ~
~058 636 308 480 000 000
8 320 987 112 741 390 144 ~
~276 341 183 223 364 380 754 ~
~172 606 361 245 952 449 277 ~
~696 409 600 000 000 000 000
"string"
```

Read

`Read[stream]`
reads the input stream and returns one expression.

`Read[stream, type]`
reads the input stream and returns an object of the given type.

```
>> str = StringToStream["abc123"];

>> Read[str, String]
abc123

>> str = StringToStream["abc 123"];

>> Read[str, Word]
abc

>> Read[str, Word]
123
```

```
>> str = StringToStream["123, 4"];

>> Read[str, Number]
123

>> Read[str, Number]
4

>> str = StringToStream["123 abc"];

>> Read[str, {Number, Word}]
{123, abc}
```

ReadList

`ReadList["file"]`
Reads all the expressions until the end of file.

`ReadList["file", type]`
Reads objects of a specified type until the end of file.

`ReadList["file", {type1, type2, ...}]`
Reads a sequence of specified types until the end of file.

```
>> ReadList[StringToStream["a 1 b
2"], {Word, Number}]
{{a, 1}, {b, 2}}

>> str = StringToStream["abc123"];

>> ReadList[str]
{abc123}

>> InputForm[%]
{"abc123"}
```

Record

`Record`
is a data type for `Read`.

RenameDirectory

`RenameDirectory["dir1", "dir2"]`
renames directory *dir1* to *dir2*.

RenameFile

```
RenameFile["file1" ' ', "file2"]
renames file1 to file2.

>> CopyFile["ExampleData/sunflowers
.jpg", "MathicsSunflowers.jpg"]
MathicsSunflowers.jpg

>> RenameFile["MathicsSunflowers.
.jpg", "MathicsSunnyFlowers.jpg"]
MathicsSunnyFlowers.jpg

>> DeleteFile["MathicsSunnyFlowers.
.jpg"]
```

ResetDirectory

```
ResetDirectory[]
pops a directory from the directory stack
and returns it.

>> ResetDirectory[]
Directorystackisempty.
/src/external-vcs/github/mathics/Mathics
```

\$RootDirectory

```
$RootDirectory
returns the system root directory.

>> $RootDirectory
/
```

SetDirectory

```
SetDirectory[dir]
sets the current working directory to dir.

>> SetDirectory[]
/home/rocky
```

SetFileDate

```
SetFileDate["file"]
set the file access and modification dates
of file to the current date.
SetFileDate["file", date]
set the file access and modification dates
of file to the specified date list.
SetFileDate["file", date, "type"]
set the file date of file to the specified date
list. The "type" can be one of "Access",
"Creation", "Modification", or All.
```

Create a temporary file (for example purposes)

```
>> tmpfilename =
$TemporaryDirectory <> "/tmp0";

>> Close[OpenWrite[tmpfilename]];

>> SetFileDate[tmpfilename, {2000,
1, 1, 0, 0, 0.}, "Access"];

>> FileDate[tmpfilename, "Access"]
{2000,1,1,0,0,0.}
```

SetStreamPosition

```
SetStreamPosition[stream, n]
sets the current position in a stream.
```

```
>> str = StringToStream["Mathics is
cool!"]
InputStream[String,1100]

>> SetStreamPosition[str, 8]
8

>> Read[str, Word]
is

>> SetStreamPosition[str, Infinity]
16
```

Skip

`Skip[stream, type]`
skips ahead in an input stream by one object of the specified *type*.
`Skip[stream, type, n]`
skips ahead in an input stream by *n* objects of the specified *type*.

```
>> str = StringToStream["a b c d"];
>> Read[str, Word]
a
>> Skip[str, Word]
>> Read[str, Word]
c
>> str = StringToStream["a b c d"];
>> Read[str, Word]
a
>> Skip[str, Word, 2]
>> Read[str, Word]
d
```

StreamPosition

`StreamPosition[stream]`
returns the current position in a stream as an integer.

```
>> str = StringToStream["Mathics is cool!"]
InputStream [String, 1 109]
>> Read[str, Word]
Mathics
>> StreamPosition[str]
7
```

Streams

`Streams[]`
returns a list of all open streams.

```
>> Streams[]
{InputStream [<stdin>, 0],
 OutputStream [<stdout>,
 1], OutputStream [<stderr>,
 2], OutputStream [
MathicsNonExampleFile,
 964], OutputStream [
MathicsNonExampleFile,
 966], OutputStream [
MathicsNonExampleFile,
 968], InputStream [String,
 1 048], InputStream [String,
 1 062], InputStream [String,
 1 076], InputStream [String,
 1 086], InputStream [String,
 1 088], InputStream [String,
 1 089], InputStream [String,
 1 091], InputStream [String,
 1 092], InputStream [String,
 1 094], InputStream [String,
 1 098], InputStream [String,
 1 099], InputStream [String,
 1 100], InputStream [String,
 1 107], InputStream [String,
 1 108], InputStream [String,
 1 109], OutputStream [
/tmp/tmpnu84r4ot, 1 ~
~110], OutputStream [
/tmp/tmpj2d42qfe, 1 111]}
```

StringToStream

`StringToStream[string]`
converts a *string* to an open input stream.

```
>> strm = StringToStream["abc 123"]
InputStream [String, 1 115]
```

\$TemporaryDirectory

`$TemporaryDirectory`
returns the directory used for temporary files.

```
>> $TemporaryDirectory
/tmp
```

ToFileName

```
ToFileName[{"dir_1'", "dir_2", ...}]
joins the dir_i together into one path.
```

ToFileName has been superseded by FileNameJoin.

```
>> ToFileName[{"dir1", "dir2"}, "
file"]
dir1/dir2/file
>> ToFileName["dir1", "file"]
dir1/file
>> ToFileName[{"dir1", "dir2", "
dir3"}]
dir1/dir2/dir3
```

Uncompress

```
Uncompress["string"]
recovers an expression from a string generated by Compress.
```

```
>> Compress["Mathics is cool"]
eJxT8k0sychMLlbILFZlzs/PUQIANFwF1w==
>> Uncompress[%]
Mathics is cool
>> a = x ^ 2 + y Sin[x] + 10 Log
[15];
>> b = Compress[a];
>> Uncompress[b]
x2 + y Sin[x] + 10 Log[15]
```

Word

```
Word
is a data type for Read.
```

Write

```
Write[channel, expr1, expr2, ...]
writes the expressions to the output channel followed by a newline.
```

```
>> str = OpenWrite[]
OutputStream [
/tmp/tmpfzz4qert, 1120]
>> Write[str, 10 x + 15 y ^ 2]
>> Write[str, 3 Sin[z]]
>> Close[str]
/tmp/tmpfzz4qert
>> str = OpenRead[%];
>> ReadList[str]
{10 x + 15 y ^ 2, 3 Sin[z]}
```

WriteString

```
WriteString[stream, $str1, str2, ... ]
writes the strings to the output stream.
```

```
>> str = OpenWrite[];
>> WriteString[str, "This is a test
1"]
>> WriteString[str, "This is also a
test 2"]
>> Close[str]
/tmp/tmpiquubvzu
>> FilePrint[%]
Thisisatest1Thisisalsoatest2
>> str = OpenWrite[];
>> WriteString[str, "This is a test
1", "This is also a test 2"]
>> Close[str]
/tmp/tmp2qt_8hj_
>> FilePrint[%]
Thisisatest1Thisisalsoatest2
```

XLIII. Importing and Exporting

Contents

	\$ExportFormats	273		ImportString	274
Sys-	ExportString	273		ImportEx-	
tem‘Convert‘B64Dump‘B64Decode	FetchURL	273		port‘RegisterExport	275
Sys-	FileFormat	273		ImportEx-	
tem‘Convert‘B64Dump‘B64Encode	Import	274		port‘RegisterImport	276
Export	\$ImportFormats	274			

System‘Convert‘B64Dump‘B64Decode

System‘Convert‘B64Dump‘B64Decode [*string*]
Decode *string* in Base64 coding to an expression.

```
>> System‘Convert‘B64Dump‘B64Decode
["R!="]
String"R!
="isnotavalidb64encodedstring.
$Failed
```

System‘Convert‘B64Dump‘B64Decode [%]

Integrate[f[x], {x, 0, 2}]

```
>> System‘Convert‘B64Dump‘B64Encode
["Sf dx"]
4oirIGYg752MIHg=
```

```
>> System‘Convert‘B64Dump‘B64Decode
[%]
f dx
```

Export

System‘Convert‘B64Dump‘B64Encode

System‘Convert‘B64Dump‘B64Encode [*expr*]
Encodes *expr* in Base64 coding

```
>> System‘Convert‘B64Dump‘B64Encode
["Hello world"]
SGVsbG8gd29ybGQ=

>> System‘Convert‘B64Dump‘B64Decode
[%]
Hello world

>> System‘Convert‘B64Dump‘B64Encode
[Integrate[f[x], {x, 0, 2}]]
SW50ZWdyYXRlW2ZbeF0sIHt4LCAwLCAyfV0=
```

Export ["file.ext", *expr*]
exports *expr* to a file, using the extension *ext* to determine the format.
Export ["file", *expr*, "format"]
exports *expr* to a file in the specified format.
Export ["file", *exprs*, *elems*]
exports *exprs* to a file as elements specified by *elems*.

\$ExportFormats

\$ExportFormats
returns a list of file formats supported by Export.


```
>> $ExportFormats
{BMP, Base64, CSV, GIF, JPEG,
 JPEG2000, PBM, PCX, PGM,
 PNG, PPM, SVG, TIFF, Text}
```

ExportString

```
ExportString[expr, form]
  exports expr to a string, in the format
  form.
Export["file", exprs, elems]
  exports exprs to a string as elements spec-
  ified by elems.
```

```
>> ExportString
[{{1,2,3,4},{3},{2},{4}}, "CSV"]
1,2,3,4
3,
2,
4,

>> ExportString[{1,2,3,4}, "CSV"]
1,
2,
3,
4,

>> ExportString[Integrate[f[x],{x
,0,2}], "SVG"]
<svg><mrow><msubsup>
  <mo></mo> <mn>0</mn>
  <mn>2</mn></msubsup>
  <mrow><mi>f</mi>
  <mo></mo> <mi>x</mi>
  <mo></mo></mrow> <mo>
  form="prefix" lspace="0"
  rspace="0.2em"></mo> <mrow>
  <mtext></mtext> <mi>x</mi>
  </mrow></mrow></svg>
```

FetchURL

FileFormat

```
FileFormat["name"]
  attempts to determine what format
  Import should use to import specified
  file.
```

```
>> FileFormat["ExampleData/
sunflowers.jpg"]
JPEG

>> FileFormat["ExampleData/
EinsteinSzilLetter.txt"]
Text

>> FileFormat["ExampleData/lena.tif"]
TIFF
```

Import

```
Import["file"]
  imports data from a file.
Import["file", elements]
  imports the specified elements from a file.
Import["http://url", ...] and Import["
ftp://url", ...]
  imports from a URL.
```

```
>> Import["ExampleData/ExampleData.
txt", "Elements"]
{Data, Lines, Plaintext, String, Words}

>> Import["ExampleData/ExampleData.
txt", "Lines"]
{Example File Format, Created
 by Angus, 0.629452 0.586355,
 0.711009 0.687453, 0.246540
 0.433973, 0.926871 0.887255,
 0.825141 0.940900, 0.847035
 0.127464, 0.054348 0.296494,
 0.838545 0.247025, 0.838697
 0.436220, 0.309496 0.833591}
```

```
>> Import["ExampleData/colors.json"]

{colorsArray
  - > {{colorName- > black,
    rgbValue- > (0, 0, 0),
    hexValue- > #000 000} ,
    {colorName- > red,
    rgbValue- > (255, 0, 0),
    hexValue- > #FF0 000} ,
    {colorName- > green,
    rgbValue- > (0, 255, 0),
    hexValue- > #00FF00} ,
    {colorName- > blue,
    rgbValue- > (0, 0, 255),
    hexValue- > #0 000FF} ,
    {colorName- > yellow,
    rgbValue- > (255, 255, 0),
    hexValue- > #FFFF00} ,
    {colorName- > cyan,
    rgbValue- > (0, 255, 255),
    hexValue- > #00FFFF} ,
    {colorName- > magenta,
    rgbValue- > (255, 0, 255),
    hexValue- > #FF00FF} ,
    {colorName- > white,
    rgbValue- > (255, 255, 255),
    hexValue- > #FFFFFF}}}
```

\$ImportFormats

`$ImportFormats`
returns a list of file formats supported by Import.

```
>> $ImportFormats
{BMP, Base64, CSV, GIF,
 ICO, JPEG, JPEG2 000, JSON,
 PBM, PCX, PGM, PNG, PPM,
 Package, TGA, TIFF, Text, XML}
```

ImportString

`ImportString["data" , "format"]`
imports data in the specified format from a string.

`ImportString["file" , elements]`
imports the specified elements from a string.

`ImportString["data"]`
attempts to determine the format of the string from its content.

```
>> str = "Hello!\n This is a
testing text\n";

>> ImportString[str, "Elements"]
{Data, Lines, Plaintext, String, Words}

>> ImportString[str, "Lines"]
{Hello!, This is a testing text}
```

ImportExport‘RegisterExport

`RegisterExport["format" , func]`
register *func* as the default function used when exporting from a file of type "*format*".

Simple text exporter

```
>> ExampleExporter1[filename_,
  data_, opts___] := Module[{strm
  = OpenWrite[filename], char =
  data}, WriteString[strm, char];
  Close[strm]]

>> ImportExport‘RegisterExport["
ExampleFormat1",
ExampleExporter1]

>> Export["sample.txt", "Encode
this string!", "ExampleFormat1
"];

>> FilePrint["sample.txt"]
Encodethisstring!
```

Very basic encrypted text exporter

```
>> ExampleExporter2[filename_,
  data_, opts___] := Module[{strm
= OpenWrite[filename], char}, (*
  TODO: Check data *)char =
  FromCharacterCode[Mod[
  ToCharacterCode[data] - 84, 26]
+ 97]; WriteString[strm, char];
  Close[strm]]

>> ImportExport`RegisterExport["
ExampleFormat2",
ExampleExporter2]

>> Export["sample.txt", "
encodethisstring", "
ExampleFormat2"];

>> FilePrint["sample.txt"]
  rapbqrguvffgevat
```

ImportExport`RegisterImport

```
RegisterImport["format", defaultFunction]
  register defaultFunction as the default
  function used when importing from a file
  of type "format".
RegisterImport["format", {"elem1" :>
conditionalFunction1, "elem2" :> conditional-
Function2, ..., defaultFunction}]
  registers multiple elements (elem1, ...)
  and their corresponding converter func-
  tions (conditionalFunction1, ...) in addition
  to the defaultFunction.
RegisterImport["format", {"
conditionalFunctions, defaultFunction,
"elem3" :> postFunction3, "elem4" :>
postFunction4, ...}]
  also registers additional elements (elem3,
  ...) whose converters (postFunction3, ...)
  act on output from the low-level func-
  tions.
```

First, define the default function used to import the data.

```
>> ExampleFormat1Import[
  filename_String] := Module[{
  stream, head, data}, stream =
  OpenRead[filename]; head =
  ReadList[stream, String, 2];
  data = Partition[ReadList[stream
  , Number], 2]; Close[stream]; {"
Header" -> head, "Data" -> data
}]
```

RegisterImport is then used to register the above function to a new data format.

```
>> ImportExport`RegisterImport["
ExampleFormat1",
ExampleFormat1Import]
```

```
>> FilePrint["ExampleData/
ExampleData.txt"]
```

ExampleFileFormat

CreatedbyAngus

0.6294520.586355

0.7110090.687453

0.2465400.433973

0.9268710.887255

0.8251410.940900

0.8470350.127464

0.0543480.296494

0.8385450.247025

0.8386970.436220

0.3094960.833591

```
>> Import["ExampleData/ExampleData.
txt", {"ExampleFormat1", "
Elements"}]
```

{Data, Header}

```
>> Import["ExampleData/ExampleData.
txt", {"ExampleFormat1", "Header
"}]
```

{Example File Format,
Created by Angus}

Conditional Importer:

```
>> ExampleFormat2DefaultImport[
  filename_String] := Module[{
  stream, head}, stream = OpenRead
[filename]; head = ReadList[
stream, String, 2]; Close[stream
]; {"Header" -> head}]
```

```

>> ExampleFormat2DataImport[
  filename_String] := Module[{
    stream, data}, stream = OpenRead
    [filename]; Skip[stream, String,
    2]; data = Partition[ReadList[
    stream, Number], 2]; Close[
    stream]; {"Data" -> data}]

>> ImportExport`RegisterImport["
  ExampleFormat2", {"Data" :>
  ExampleFormat2DataImport,
  ExampleFormat2DefaultImport}]

>> Import["ExampleData/ExampleData.
  txt", {"ExampleFormat2", "
  Elements"}]

  {Data, Header}

>> Import["ExampleData/ExampleData.
  txt", {"ExampleFormat2", "Header
  "}

  {Example File Format,
   Created by Angus}

>> Import["ExampleData/ExampleData.
  txt", {"ExampleFormat2", "Data
  "} // Grid

0.629452 0.586355
0.711009 0.687453
0.24654 0.433973
0.926871 0.887255
0.825141 0.9409
0.847035 0.127464
0.054348 0.296494
0.838545 0.247025
0.838697 0.43622
0.309496 0.833591

```

Part III.

License

A. GNU General Public License

Version 3, 29 June 2007

Copyright © 2007 Free Software Foundation, Inc. <http://fsf.org/>

Everyone is permitted to copy and distribute verbatim copies of this license document, but changing it is not allowed.

Preamble

The GNU General Public License is a free, copyleft license for software and other kinds of works.

The licenses for most software and other practical works are designed to take away your freedom to share and change the works. By contrast, the GNU General Public License is intended to guarantee your freedom to share and change all versions of a program—to make sure it remains free software for all its users. We, the Free Software Foundation, use the GNU General Public License for most of our software; it applies also to any other work released this way by its authors. You can apply it to your programs, too.

When we speak of free software, we are referring to freedom, not price. Our General Public Licenses are designed to make sure that you have the freedom to distribute copies of free software (and charge for them if you wish), that you receive source code or can get it if you want it, that you can change the software or use pieces of it in new free programs, and that you know you can do these things.

To protect your rights, we need to prevent others from denying you these rights or asking you to surrender the rights. Therefore, you have certain responsibilities if you distribute copies of the software, or if you modify it: responsibilities to respect the freedom of others.

For example, if you distribute copies of such a program, whether gratis or for a fee, you must pass on to the recipients the same freedoms that you received. You must make sure that they, too, receive or can get the source code. And you must show them these terms so they know their rights.

Developers that use the GNU GPL protect your rights with two steps: (1) assert copyright on the software, and (2) offer you this License giving you legal permission to copy, distribute and/or modify it.

For the developers and authors protection, the GPL clearly explains that there is no warranty for this free software. For both users and authors sake, the GPL requires that modified versions be marked as changed, so that their problems will not be attributed erroneously to authors of previous versions.

Some devices are designed to deny users access to install or run modified versions of the software inside them, although the manufacturer can do so. This is fundamentally incompatible with the aim of protecting users' freedom to change the software. The systematic pattern of such abuse occurs in the area of products for individuals to use, which is precisely where it is most unacceptable. Therefore, we have designed this version of the GPL to prohibit the practice for those products. If such problems arise substantially in other domains, we stand ready to extend this provision to those domains in future versions of the GPL, as needed to protect the freedom of users.

Finally, every program is threatened constantly by software patents. States should not allow patents to restrict development and use of software on general-purpose computers, but in those that do, we wish to avoid the special danger that patents applied to a free program could make it effectively proprietary. To prevent this, the GPL assures that patents cannot be used to render the program non-free.

The precise terms and conditions for copying, distribution and modification follow.

TERMS AND CONDITIONS

0. Definitions.

This License refers to version 3 of the GNU General Public License.

Copyright also means copyright-like laws that apply to other kinds of works, such as semiconductor masks.

The Program refers to any copyrightable work licensed under this License. Each licensee is addressed as you. Licensees and recipients may be individuals or organizations.

To modify a work means to copy from or adapt all or part of the work in a fashion requiring copyright permission, other than the making of an exact copy. The resulting work is called a modified version of the earlier work or a work based on the earlier work.

A covered work means either the unmodified Program or a work based on the Program.

To propagate a work means to do anything with it that, without permission, would make you directly or secondarily liable for infringement under applicable copyright law, except executing it on a computer or modifying a private copy. Propagation includes copying, distribution (with or without modification), making available to the public, and in some countries other activities as well.

To convey a work means any kind of propagation that enables other parties to make or receive copies. Mere interaction with a user through a computer network, with no transfer of a copy, is not conveying.

An interactive user interface displays Appropriate Legal Notices to the extent that it includes a convenient and prominently visible feature that (1) displays an appropriate copyright notice, and (2) tells the user that there is no warranty for the work (except to the extent that warranties are provided), that licensees may convey the work under this License, and how to view a copy of this License. If the interface presents a list of user commands or options, such as a menu, a prominent item in the list meets this criterion.

1. Source Code.

The source code for a work means the preferred form of the work for making modifications to it. Object code means any non-source form of a work.

A Standard Interface means an interface that either is an official standard defined by a recognized standards body, or, in the case of interfaces specified for a particular programming language, one that is widely used among developers working in that language.

The System Libraries of an executable work include anything, other than the work as a whole, that (a) is included in the normal form of packaging a Major Component, but which is not part of that Major Component, and (b) serves only to enable use of the work with that Major Component, or to implement a Standard Interface for which an implementation is available to the public in source code form. A Major Component, in this context, means a major essential component (kernel, window system, and so on) of the specific operating system (if any) on which the executable work runs, or a compiler used to produce the work, or an object code interpreter used to run it.

The Corresponding Source for a work in object code form means all the source code needed to generate, install, and (for an executable work) run the object code and to modify the work, including scripts to control those activities. However, it does not include the work's System Libraries, or general-purpose tools or generally available free programs which are used unmodified in performing those activities but which are not part of the work. For example, Corresponding Source includes interface definition files associated with source files for the work, and the source code for shared libraries and dynamically linked subprograms that the work is specifically designed to require, such as by intimate data communication or control flow between those subprograms and other parts of the work.

The Corresponding Source need not include anything that users can regenerate automatically from other parts of the Corresponding Source.

The Corresponding Source for a work in source code form is that same work.

2. Basic Permissions.

All rights granted under this License are granted for the term of copyright on the Program, and are irrevocable provided the stated conditions are met. This License explicitly affirms your unlimited permission to run the unmodified Program. The output from running a covered work is covered by this License only if the output, given its content, constitutes a covered work. This License acknowledges your rights of fair use or other equivalent, as provided by copyright law.

You may make, run and propagate covered works that you do not convey, without conditions so long as your license otherwise remains in force. You may convey covered works to others for the sole purpose of having them make modifications exclusively for you, or provide you with facilities for running those works, provided that you comply with the terms of this License in conveying all material for which you do not control copyright. Those thus making or running the covered works for you must do so exclusively on your behalf, under your direction and control, on terms that prohibit them from making any copies of your copyrighted material outside their relationship with you.

Conveying under any other circumstances is permitted solely under the conditions stated below. Sub-licensing is not allowed; section 10 makes it unnecessary.

3. Protecting Users' Legal Rights From Anti-Circumvention Law.

No covered work shall be deemed part of an effective technological measure under any applicable law fulfilling obligations under article 11 of the WIPO copyright treaty adopted on 20 December 1996, or similar laws prohibiting or restricting circumvention of such measures.

When you convey a covered work, you waive any legal power to forbid circumvention of technological measures to the extent such circumvention is effected by exercising rights under this License with respect to the covered work, and you disclaim any intention to limit operation or modification of the work as a means of enforcing, against the works users, your or third parties legal rights to forbid circumvention of technological measures.

4. Conveying Verbatim Copies.

You may convey verbatim copies of the Program's source code as you receive it, in any medium, provided that you conspicuously and appropriately publish on each copy an appropriate copyright notice; keep intact all notices stating that this License and any non-permissive terms added in accord with section 7 apply to the code; keep intact all notices of the absence of any warranty; and give all recipients a copy of this License along with the Program.

You may charge any price or no price for each copy that you convey, and you may offer support or warranty protection for a fee.

5. Conveying Modified Source Versions.

You may convey a work based on the Program, or the modifications to produce it from the Program, in the form of source code under the terms of section 4, provided that you also meet all of these conditions:

- a) The work must carry prominent notices stating that you modified it, and giving a relevant date.
- b) The work must carry prominent notices stating that it is released under this License and any conditions added under section 7. This requirement modifies the requirement in section 4 to keep intact all notices.
- c) You must license the entire work, as a whole, under this License to anyone who comes into possession of a copy. This License will therefore apply, along with any applicable section 7 additional terms, to the whole of the work, and all its parts, regardless of how they are packaged. This License gives no permission to license the work in any other way, but it does not invalidate such permission if you have separately received it.
- d) If the work has interactive user interfaces, each must display Appropriate Legal Notices; however, if the Program has interactive interfaces that do not display Appropriate Legal Notices, your work need not make them do so.

A compilation of a covered work with other separate and independent works, which are not by their nature extensions of the covered work, and which are not combined with it such as to form a larger program, in or on a volume of a storage or distribution medium, is called an aggregate if the compilation and its resulting copyright are not used to limit the access or legal rights of the compilation's users beyond what the individual works permit. Inclusion of a covered work in an aggregate does not cause this License to apply to the other parts of the aggregate.

6. Conveying Non-Source Forms.

You may convey a covered work in object code form under the terms of sections 4 and 5, provided that you also convey the machine-readable Corresponding Source under the terms of this License, in one of these ways:

- a) Convey the object code in, or embodied in, a physical product (including a physical distribution medium), accompanied by the Corresponding Source fixed on a durable physical medium customarily used for software interchange.
- b) Convey the object code in, or embodied in, a physical product (including a physical distribution medium), accompanied by a written offer, valid for at least three years and valid for as long as you offer spare parts or customer support for that product model, to give anyone who possesses the object code either (1) a copy of the Corresponding Source for all the software in the product that is covered by this License, on a durable physical medium customarily used for software interchange, for a price no more than your reasonable cost of physically performing this conveying of source, or (2) access to copy the Corresponding Source from a network server at no charge.
- c) Convey individual copies of the object code with a copy of the written offer to provide the Corresponding Source. This alternative is allowed only occasionally and noncommercially, and only if you received the object code with such an offer, in accord with subsection 6b.
- d) Convey the object code by offering access from a designated place (gratis or for a charge), and offer equivalent access to the Corresponding Source in the same way through the same place at no further charge. You need not require recipients to copy the Corresponding Source along with the object code. If the place to copy the object code is a network server, the Corresponding Source may be on a different server (operated by you or a third party) that supports equivalent copying facilities, provided you maintain clear directions next to the object code saying where to find the Corresponding Source. Regardless of what server hosts the Corresponding Source, you remain obligated to ensure that it is available for as long as needed to satisfy these requirements.
- e) Convey the object code using peer-to-peer transmission, provided you inform other peers where the object code and Corresponding Source of the work are being offered to the general public at no charge under subsection 6d.

A separable portion of the object code, whose source code is excluded from the Corresponding Source as a System Library, need not be included in conveying the object code work.

A User Product is either (1) a consumer product, which means any tangible personal property which is normally used for personal, family, or household purposes, or (2) anything designed or sold for incorporation into a dwelling. In determining whether a product is a consumer product, doubtful cases shall be resolved in favor of coverage. For a particular product received by a particular user, normally used refers to a typical or common use of that class of product, regardless of the status of the particular user or of the way in which the particular user actually uses, or expects or is expected to use, the product. A product is a consumer product regardless of whether the product has substantial commercial, industrial or non-consumer uses, unless such uses represent the only significant mode of use of the product.

Installation Information for a User Product means any methods, procedures, authorization keys, or other information required to install and execute modified versions of a covered work in that User Product from a modified version of its Corresponding Source. The information must suffice to ensure that the continued functioning of the modified object code is in no case prevented or interfered with solely because modification has been made.

If you convey an object code work under this section in, or with, or specifically for use in, a User Product, and the conveying occurs as part of a transaction in which the right of possession and use of the User

Product is transferred to the recipient in perpetuity or for a fixed term (regardless of how the transaction is characterized), the Corresponding Source conveyed under this section must be accompanied by the Installation Information. But this requirement does not apply if neither you nor any third party retains the ability to install modified object code on the User Product (for example, the work has been installed in ROM).

The requirement to provide Installation Information does not include a requirement to continue to provide support service, warranty, or updates for a work that has been modified or installed by the recipient, or for the User Product in which it has been modified or installed. Access to a network may be denied when the modification itself materially and adversely affects the operation of the network or violates the rules and protocols for communication across the network.

Corresponding Source conveyed, and Installation Information provided, in accord with this section must be in a format that is publicly documented (and with an implementation available to the public in source code form), and must require no special password or key for unpacking, reading or copying.

7. Additional Terms.

Additional permissions are terms that supplement the terms of this License by making exceptions from one or more of its conditions. Additional permissions that are applicable to the entire Program shall be treated as though they were included in this License, to the extent that they are valid under applicable law. If additional permissions apply only to part of the Program, that part may be used separately under those permissions, but the entire Program remains governed by this License without regard to the additional permissions.

When you convey a copy of a covered work, you may at your option remove any additional permissions from that copy, or from any part of it. (Additional permissions may be written to require their own removal in certain cases when you modify the work.) You may place additional permissions on material, added by you to a covered work, for which you have or can give appropriate copyright permission.

Notwithstanding any other provision of this License, for material you add to a covered work, you may (if authorized by the copyright holders of that material) supplement the terms of this License with terms:

- a) Disclaiming warranty or limiting liability differently from the terms of sections 15 and 16 of this License; or
 - b) Requiring preservation of specified reasonable legal notices or author attributions in that material or in the Appropriate Legal Notices displayed by works containing it; or
 - c) Prohibiting misrepresentation of the origin of that material, or requiring that modified versions of such material be marked in reasonable ways as different from the original version; or
 - d) Limiting the use for publicity purposes of names of licensors or authors of the material; or
 - e) Declining to grant rights under trademark law for use of some trade names, trademarks, or service marks; or
 - f) Requiring indemnification of licensors and authors of that material by anyone who conveys the material (or modified versions of it) with contractual assumptions of liability to the recipient, for any liability that these contractual assumptions directly impose on those licensors and authors.
- All other non-permissive additional terms are considered further restrictions within the meaning of section 10. If the Program as you received it, or any part of it, contains a notice stating that it is governed by this License along with a term that is a further restriction, you may remove that term. If a license document contains a further restriction but permits relicensing or conveying under this License, you may add to a covered work material governed by the terms of that license document, provided that the further restriction does not survive such relicensing or conveying.

If you add terms to a covered work in accord with this section, you must place, in the relevant source files, a statement of the additional terms that apply to those files, or a notice indicating where to find the applicable terms.

Additional terms, permissive or non-permissive, may be stated in the form of a separately written license, or stated as exceptions; the above requirements apply either way.

8. Termination.

You may not propagate or modify a covered work except as expressly provided under this License. Any attempt otherwise to propagate or modify it is void, and will automatically terminate your rights under this License (including any patent licenses granted under the third paragraph of section 11).

However, if you cease all violation of this License, then your license from a particular copyright holder is reinstated (a) provisionally, unless and until the copyright holder explicitly and finally terminates your license, and (b) permanently, if the copyright holder fails to notify you of the violation by some reasonable means prior to 60 days after the cessation.

Moreover, your license from a particular copyright holder is reinstated permanently if the copyright holder notifies you of the violation by some reasonable means, this is the first time you have received notice of violation of this License (for any work) from that copyright holder, and you cure the violation prior to 30 days after your receipt of the notice.

Termination of your rights under this section does not terminate the licenses of parties who have received copies or rights from you under this License. If your rights have been terminated and not permanently reinstated, you do not qualify to receive new licenses for the same material under section 10.

9. Acceptance Not Required for Having Copies.

You are not required to accept this License in order to receive or run a copy of the Program. Ancillary propagation of a covered work occurring solely as a consequence of using peer-to-peer transmission to receive a copy likewise does not require acceptance. However, nothing other than this License grants you permission to propagate or modify any covered work. These actions infringe copyright if you do not accept this License. Therefore, by modifying or propagating a covered work, you indicate your acceptance of this License to do so.

10. Automatic Licensing of Downstream Recipients.

Each time you convey a covered work, the recipient automatically receives a license from the original licensors, to run, modify and propagate that work, subject to this License. You are not responsible for enforcing compliance by third parties with this License.

An entity transaction is a transaction transferring control of an organization, or substantially all assets of one, or subdividing an organization, or merging organizations. If propagation of a covered work results from an entity transaction, each party to that transaction who receives a copy of the work also receives whatever licenses to the work the party's predecessor in interest had or could give under the previous paragraph, plus a right to possession of the Corresponding Source of the work from the predecessor in interest, if the predecessor has it or can get it with reasonable efforts.

You may not impose any further restrictions on the exercise of the rights granted or affirmed under this License. For example, you may not impose a license fee, royalty, or other charge for exercise of rights granted under this License, and you may not initiate litigation (including a cross-claim or counterclaim in a lawsuit) alleging that any patent claim is infringed by making, using, selling, offering for sale, or importing the Program or any portion of it.

11. Patents.

A contributor is a copyright holder who authorizes use under this License of the Program or a work on which the Program is based. The work thus licensed is called the contributor's contributor version.

A contributor's essential patent claims are all patent claims owned or controlled by the contributor, whether already acquired or hereafter acquired, that would be infringed by some manner, permitted by this License, of making, using, or selling its contributor version, but do not include claims that would be infringed only as a consequence of further modification of the contributor version. For purposes of this definition, control includes the right to grant patent sublicenses in a manner consistent with the requirements of this License.

Each contributor grants you a non-exclusive, worldwide, royalty-free patent license under the contributor's essential patent claims, to make, use, sell, offer for sale, import and otherwise run, modify and propagate the contents of its contributor version.

In the following three paragraphs, a patent license is any express agreement or commitment, however denominated, not to enforce a patent (such as an express permission to practice a patent or covenant not to sue for patent infringement). To grant such a patent license to a party means to make such an agreement or commitment not to enforce a patent against the party.

If you convey a covered work, knowingly relying on a patent license, and the Corresponding Source of the work is not available for anyone to copy, free of charge and under the terms of this License, through a publicly available network server or other readily accessible means, then you must either (1) cause the Corresponding Source to be so available, or (2) arrange to deprive yourself of the benefit of the patent license for this particular work, or (3) arrange, in a manner consistent with the requirements of this License, to extend the patent license to downstream recipients. Knowingly relying means you have actual knowledge that, but for the patent license, your conveying the covered work in a country, or your recipient's use of the covered work in a country, would infringe one or more identifiable patents in that country that you have reason to believe are valid.

If, pursuant to or in connection with a single transaction or arrangement, you convey, or propagate by procuring conveyance of, a covered work, and grant a patent license to some of the parties receiving the covered work authorizing them to use, propagate, modify or convey a specific copy of the covered work, then the patent license you grant is automatically extended to all recipients of the covered work and works based on it.

A patent license is discriminatory if it does not include within the scope of its coverage, prohibits the exercise of, or is conditioned on the non-exercise of one or more of the rights that are specifically granted under this License. You may not convey a covered work if you are a party to an arrangement with a third party that is in the business of distributing software, under which you make payment to the third party based on the extent of your activity of conveying the work, and under which the third party grants, to any of the parties who would receive the covered work from you, a discriminatory patent license (a) in connection with copies of the covered work conveyed by you (or copies made from those copies), or (b) primarily for and in connection with specific products or compilations that contain the covered work, unless you entered into that arrangement, or that patent license was granted, prior to 28 March 2007.

Nothing in this License shall be construed as excluding or limiting any implied license or other defenses to infringement that may otherwise be available to you under applicable patent law.

12. No Surrender of Others' Freedom.

If conditions are imposed on you (whether by court order, agreement or otherwise) that contradict the conditions of this License, they do not excuse you from the conditions of this License. If you cannot convey a covered work so as to satisfy simultaneously your obligations under this License and any other pertinent obligations, then as a consequence you may not convey it at all. For example, if you agree to terms that obligate you to collect a royalty for further conveying from those to whom you convey the Program, the only way you could satisfy both those terms and this License would be to refrain entirely from conveying the Program.

13. Use with the GNU Affero General Public License.

Notwithstanding any other provision of this License, you have permission to link or combine any covered work with a work licensed under version 3 of the GNU Affero General Public License into a single combined work, and to convey the resulting work. The terms of this License will continue to apply to the part which is the covered work, but the special requirements of the GNU Affero General Public License, section 13, concerning interaction through a network will apply to the combination as such.

14. Revised Versions of this License.

The Free Software Foundation may publish revised and/or new versions of the GNU General Public License from time to time. Such new versions will be similar in spirit to the present version, but may differ in detail to address new problems or concerns.

Each version is given a distinguishing version number. If the Program specifies that a certain numbered version of the GNU General Public License or any later version applies to it, you have the option of following the terms and conditions either of that numbered version or of any later version published by the Free Software Foundation. If the Program does not specify a version number of the GNU General Public License, you may choose any version ever published by the Free Software Foundation.

If the Program specifies that a proxy can decide which future versions of the GNU General Public License can be used, that proxy's public statement of acceptance of a version permanently authorizes you to choose that version for the Program.

Later license versions may give you additional or different permissions. However, no additional obligations are imposed on any author or copyright holder as a result of your choosing to follow a later version.

15. Disclaimer of Warranty.

THERE IS NO WARRANTY FOR THE PROGRAM, TO THE EXTENT PERMITTED BY APPLICABLE LAW. EXCEPT WHEN OTHERWISE STATED IN WRITING THE COPYRIGHT HOLDERS AND/OR OTHER PARTIES PROVIDE THE PROGRAM AS IS WITHOUT WARRANTY OF ANY KIND, EITHER EXPRESSED OR IMPLIED, INCLUDING, BUT NOT LIMITED TO, THE IMPLIED WARRANTIES OF MERCHANTABILITY AND FITNESS FOR A PARTICULAR PURPOSE. THE ENTIRE RISK AS TO THE QUALITY AND PERFORMANCE OF THE PROGRAM IS WITH YOU. SHOULD THE PROGRAM PROVE DEFECTIVE, YOU ASSUME THE COST OF ALL NECESSARY SERVICING, REPAIR OR CORRECTION.

16. Limitation of Liability.

IN NO EVENT UNLESS REQUIRED BY APPLICABLE LAW OR AGREED TO IN WRITING WILL ANY COPYRIGHT HOLDER, OR ANY OTHER PARTY WHO MODIFIES AND/OR CONVEYS THE PROGRAM AS PERMITTED ABOVE, BE LIABLE TO YOU FOR DAMAGES, INCLUDING ANY GENERAL, SPECIAL, INCIDENTAL OR CONSEQUENTIAL DAMAGES ARISING OUT OF THE USE OR INABILITY TO USE THE PROGRAM (INCLUDING BUT NOT LIMITED TO LOSS OF DATA OR DATA BEING RENDERED INACCURATE OR LOSSES SUSTAINED BY YOU OR THIRD PARTIES OR A FAILURE OF THE PROGRAM TO OPERATE WITH ANY OTHER PROGRAMS), EVEN IF SUCH HOLDER OR OTHER PARTY HAS BEEN ADVISED OF THE POSSIBILITY OF SUCH DAMAGES.

17. Interpretation of Sections 15 and 16.

If the disclaimer of warranty and limitation of liability provided above cannot be given local legal effect according to their terms, reviewing courts shall apply local law that most closely approximates an absolute waiver of all civil liability in connection with the Program, unless a warranty or assumption of liability accompanies a copy of the Program in return for a fee.

END OF TERMS AND CONDITIONS

How to Apply These Terms to Your New Programs

If you develop a new program, and you want it to be of the greatest possible use to the public, the best way to achieve this is to make it free software which everyone can redistribute and change under these terms.

To do so, attach the following notices to the program. It is safest to attach them to the start of each source file to most effectively state the exclusion of warranty; and each file should have at least the copyright line and a pointer to where the full notice is found.

```
<one line to give the program's name and a brief idea of what it does.>
  Copyright (C) <year>  <name of author>
```

```

  This program is free software: you can redistribute it and/or
    modify
  it under the terms of the GNU General Public License as published
    by
  the Free Software Foundation, either version 3 of the License, or
  (at your option) any later version.
```

```

  This program is distributed in the hope that it will be useful,
  but WITHOUT ANY WARRANTY; without even the implied warranty of
  MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
  GNU General Public License for more details.
```

```

  You should have received a copy of the GNU General Public License
  along with this program.  If not, see <http://www.gnu.org/licenses/>.
/>.
```

Also add information on how to contact you by electronic and paper mail.

If the program does terminal interaction, make it output a short notice like this when it starts in an interactive mode:

```
<program>  Copyright (C) <year>  <name of author>
  This program comes with ABSOLUTELY NO WARRANTY; for details type `
    show w`.
  This is free software, and you are welcome to redistribute it
  under certain conditions; type `show c' for details.
```

The hypothetical commands 'show w' and 'show c' should show the appropriate parts of the General Public License. Of course, your program's commands might be different; for a GUI interface, you would use an about box.

You should also get your employer (if you work as a programmer) or school, if any, to sign a copyright disclaimer for the program, if necessary. For more information on this, and how to apply and follow the GNU GPL, see <http://www.gnu.org/licenses/>.

The GNU General Public License does not permit incorporating your program into proprietary programs. If your program is a subroutine library, you may consider it more useful to permit linking proprietary applications with the library. If this is what you want to do, use the GNU Lesser General Public License instead of this License. But first, please read <http://www.gnu.org/philosophy/why-not-lgpl.html>.

B. Included software and data

Included data

Mathics includes data from Wikipedia that is published under the Creative Commons Attribution-Sharealike 3.0 Unported License and the GNU Free Documentation License contributed by the respective authors that are listed on the websites specified in "data/elements.txt".

scriptaculous

Copyright © 2005-2008 Thomas Fuchs (<http://script.aculo.us>, <http://mir.aculo.us>)

Permission is hereby granted, free of charge, to any person obtaining a copy of this software and associated documentation files (the Software), to deal in the Software without restriction, including without limitation the rights to use, copy, modify, merge, publish, distribute, sublicense, and/or sell copies of the Software, and to permit persons to whom the Software is furnished to do so, subject to the following conditions:

The above copyright notice and this permission notice shall be included in all copies or substantial portions of the Software.

THE SOFTWARE IS PROVIDED AS IS, WITHOUT WARRANTY OF ANY KIND, EXPRESS OR IMPLIED, INCLUDING BUT NOT LIMITED TO THE WARRANTIES OF MERCHANTABILITY, FITNESS FOR A PARTICULAR PURPOSE AND NONINFRINGEMENT. IN NO EVENT SHALL THE AUTHORS OR COPYRIGHT HOLDERS BE LIABLE FOR ANY CLAIM, DAMAGES OR OTHER LIABILITY, WHETHER IN AN ACTION OF CONTRACT, TORT OR OTHERWISE, ARISING FROM, OUT OF OR IN CONNECTION WITH THE SOFTWARE OR THE USE OR OTHER DEALINGS IN THE SOFTWARE.

Prototype

Copyright © 2005-2010 Sam Stephenson

Permission is hereby granted, free of charge, to any person obtaining a copy of this software and associated documentation files (the "Software"), to deal in the Software without restriction, including without limitation the rights to use, copy, modify, merge, publish, distribute, sublicense, and/or sell copies of the Software, and to permit persons to whom the Software is furnished to do so, subject to the following conditions:

THE SOFTWARE IS PROVIDED "AS IS", WITHOUT WARRANTY OF ANY KIND, EXPRESS OR IMPLIED, INCLUDING BUT NOT LIMITED TO THE WARRANTIES OF MERCHANTABILITY, FITNESS FOR A PARTICULAR PURPOSE AND NONINFRINGEMENT. IN NO EVENT SHALL THE AUTHORS OR COPYRIGHT HOLDERS BE LIABLE FOR ANY CLAIM, DAMAGES OR OTHER LIABILITY, WHETHER IN AN ACTION OF CONTRACT, TORT OR OTHERWISE, ARISING FROM, OUT OF OR IN CONNECTION WITH THE SOFTWARE OR THE USE OR OTHER DEALINGS IN THE SOFTWARE.

MathJax

Copyright © 2009-2010 Design Science, Inc.

Licensed under the Apache License, Version 2.0 (the “License”); you may not use this file except in compliance with the License. You may obtain a copy of the License at <http://www.apache.org/licenses/LICENSE-2.0>

Unless required by applicable law or agreed to in writing, software distributed under the License is distributed on an “AS IS” BASIS, WITHOUT WARRANTIES OR CONDITIONS OF ANY KIND, either express or implied. See the License for the specific language governing permissions and limitations under the License.

Three.js

Copyright I 2010-2020 Three.js authors.

Permission is hereby granted, free of charge, to any person obtaining a copy of this software and associated documentation files (the “Software”), to deal in the Software without restriction, including without limitation the rights to use, copy, modify, merge, publish, distribute, sublicense, and/or sell copies of the Software, and to permit persons to whom the Software is furnished to do so, subject to the following conditions:

The above copyright notice and this permission notice shall be included in all copies or substantial portions of the Software.

THE SOFTWARE IS PROVIDED “AS IS”, WITHOUT WARRANTY OF ANY KIND, EXPRESS OR IMPLIED, INCLUDING BUT NOT LIMITED TO THE WARRANTIES OF MERCHANTABILITY, FITNESS FOR A PARTICULAR PURPOSE AND NONINFRINGEMENT. IN NO EVENT SHALL THE AUTHORS OR COPYRIGHT HOLDERS BE LIABLE FOR ANY CLAIM, DAMAGES OR OTHER LIABILITY, WHETHER IN AN ACTION OF CONTRACT, TORT OR OTHERWISE, ARISING FROM, OUT OF OR IN CONNECTION WITH THE SOFTWARE OR THE USE OR OTHER DEALINGS IN THE SOFTWARE.

pymimemagic

Copyright (c) 2009, Xiaohai Lu All rights reserved.

Redistribution and use in source and binary forms, with or without modification, are permitted provided that the following conditions are met:

Redistributions of source code must retain the above copyright notice, this list of conditions and the following disclaimer.

Redistributions in binary form must reproduce the above copyright notice, this list of conditions and the following disclaimer in the documentation and/or other materials provided with the distribution.

Neither the name of the <ORGANIZATION> nor the names of its contributors may be used to endorse or promote products derived from this software without specific prior written permission.

THIS SOFTWARE IS PROVIDED BY THE COPYRIGHT HOLDERS AND CONTRIBUTORS “AS IS” AND ANY EXPRESS OR IMPLIED WARRANTIES, INCLUDING, BUT NOT LIMITED TO, THE IMPLIED WARRANTIES OF MERCHANTABILITY AND FITNESS FOR A PARTICULAR PURPOSE ARE DISCLAIMED. IN NO EVENT SHALL THE COPYRIGHT HOLDER OR CONTRIBUTORS BE LIABLE FOR ANY DIRECT, INDIRECT, INCIDENTAL, SPECIAL, EXEMPLARY, OR CONSEQUENTIAL DAMAGES (INCLUDING, BUT NOT LIMITED TO, PROCUREMENT OF SUBSTITUTE GOODS OR SERVICES; LOSS OF USE, DATA, OR PROFITS; OR BUSINESS INTERRUPTION) HOWEVER CAUSED AND ON ANY THEORY OF LIABILITY, WHETHER IN CONTRACT, STRICT LIABILITY, OR TORT (INCLUDING NEGLIGENCE OR OTHERWISE) ARISING IN ANY WAY OUT OF THE USE OF THIS SOFTWARE, EVEN IF ADVISED OF THE POSSIBILITY OF SUCH DAMAGE.

SciPy

Copyright I 2001, 2002 Enthought, Inc. All rights reserved.

Copyright © 2003-2019 SciPy Developers. All rights reserved. Redistribution and use in source and binary forms, with or without modification, are permitted provided that the following conditions are met:

Redistributions of source code must retain the above copyright notice, this list of conditions and the following disclaimer.

Redistributions in binary form must reproduce the above copyright notice, this list of conditions and the following disclaimer in the documentation and/or other materials provided with the distribution.

Neither the name of Enthought nor the names of the SciPy Developers may be used to endorse or promote products derived from this software without specific prior written permission.

THIS SOFTWARE IS PROVIDED BY THE COPYRIGHT HOLDERS AND CONTRIBUTORS AS IS AND ANY EXPRESS OR IMPLIED WARRANTIES, INCLUDING, BUT NOT LIMITED TO, THE IMPLIED WARRANTIES OF MERCHANTABILITY AND FITNESS FOR A PARTICULAR PURPOSE ARE DISCLAIMED. IN NO EVENT SHALL THE REGENTS OR CONTRIBUTORS BE LIABLE FOR ANY DIRECT, INDIRECT, INCIDENTAL, SPECIAL, EXEMPLARY, OR CONSEQUENTIAL DAMAGES (INCLUDING, BUT NOT LIMITED TO, PROCUREMENT OF SUBSTITUTE GOODS OR SERVICES; LOSS OF USE, DATA, OR PROFITS; OR BUSINESS INTERRUPTION) HOWEVER CAUSED AND ON ANY THEORY OF LIABILITY, WHETHER IN CONTRACT, STRICT LIABILITY, OR TORT (INCLUDING NEGLIGENCE OR OTHERWISE) ARISING IN ANY WAY OUT OF THE USE OF THIS SOFTWARE, EVEN IF ADVISED OF THE POSSIBILITY OF SUCH DAMAGE.

Index

\$Aborted, 250
\$ByteOrdering, 250
\$CharacterEncoding, 230
\$CharacterEncodings, 230
\$CommandLine, 250
\$Context, 227
\$ContextPath, 227
\$DateStringFormat, 83
\$ExportFormats, 272
\$Failed, 250
\$HistoryLength, 86
\$HomeDirectory, 265
\$ImportFormats, 274
\$InitialDirectory, 265
\$Input, 265
\$InputFileName, 265
\$InstallationDirectory, 266
\$IterationLimit, 87
\$Line, 87
\$Machine, 251
\$MachineEpsilon, 186
\$MachineName, 251
\$MachinePrecision, 186
\$MaxPrecision, 186
\$MinPrecision, 186
\$ModuleNumber, 228
\$OperatingSystem, 266
\$Packages, 251
\$ParentProcessID, 251
\$Path, 267
\$PathnameSeparator, 267
\$Post, 143
\$Pre, 143
\$PrePrint, 143
\$PreRead, 143
\$ProcessID, 251
\$ProcessorType, 251
\$RandomState, 214
\$RecursionLimit, 88
\$RootDirectory, 269
\$ScriptCommandLine, 251
\$SyntaxHandler, 143
\$SystemID, 252
\$SystemTimeZone, 83
\$SystemWordLength, 252
\$TemporaryDirectory, 270
\$TimeZone, 84
\$UseSansSerif, 139
\$Version, 252

Abort, 76
Abs, 38
AbsoluteFileName, 258
AbsoluteThickness, 99
AbsoluteTime, 81
AbsoluteTiming, 81
Accumulate, 151
AddTo, 48
AiryAi, 217
AiryAiPrime, 217
AiryAiZero, 218
AiryBi, 218
AiryBiPrime, 218
AiryBiZero, 218
All, 151
AllTrue, 174
Alternatives, 193
And, 174
AngerJ, 218
AnglePath, 90
AngleVector, 91
AnyTrue, 174
Apart, 32
Append, 152
AppendTo, 152
Apply, 243
ApplyLevel, 243
ArcCos, 91
ArcCosh, 91
ArcCot, 92
ArcCoth, 92
ArcCsc, 92
ArcCsch, 92
ArcSec, 92
ArcSech, 92
ArcSin, 92
ArcSinh, 93
ArcTan, 93
ArcTanh, 93
Array, 152
ArrayDepth, 253

ArrayQ, 253
 Arrow, 100
 ArrowBox, 101
 Arrowheads, 101
 Association, 152
 AssociationQ, 153
 AtomQ, 243
 Attributes, 58
 Automatic, 101
 Axis, 200

 BarChart, 200
 BaseForm, 131
 Begin, 226
 BeginPackage, 226
 BernsteinBasis, 102
 BesselI, 219
 BesselJ, 219
 BesselJZero, 219
 BesselK, 219
 BesselY, 219
 BesselYZero, 219
 BezierCurve, 102
 BezierCurveBox, 102
 BezierFunction, 102
 Binarize, 121
 BinaryImageQ, 121
 BinaryRead, 258
 BinaryWrite, 259
 Binomial, 68
 BitLength, 140
 Black, 102
 Blank, 194
 BlankNullSequence, 194
 BlankSequence, 194
 Blend, 102
 Block, 226
 Blue, 103
 Blur, 122
 Boole, 38
 BooleanQ, 72
 Bottom, 201
 BoxMatrix, 122
 BrayCurtisDistance, 144
 Break, 76
 Byte, 260
 ByteCount, 244

 C, 85
 CanberraDistance, 144
 Cancel, 32
 Cases, 153
 Catenate, 153
 Ceiling, 140
 Center, 131

 CentralMoment, 153
 Character, 260
 CharacterRange, 230
 Characters, 230
 ChebyshevT, 220
 ChebyshevU, 220
 Check, 132
 ChessboardDistance, 144
 Chop, 185
 Circle, 103
 CircleBox, 104
 Clear, 48
 ClearAll, 49
 ClearAttributes, 58
 Close, 260
 Closing, 122
 ClusteringComponents, 153
 CMYKColor, 103
 Coefficient, 32
 CoefficientList, 33
 ColorCombine, 122
 ColorConvert, 122
 ColorData, 201
 ColorDataFunction, 201
 ColorDistance, 104
 Colorize, 123
 ColorNegate, 122
 ColorQuantize, 122
 ColorSeparate, 122
 Combinatorica'BinarySearch, 244
 Compile, 71
 CompiledCodeBox, 71
 CompiledFunction, 71
 Complement, 153
 Complex, 39
 Complexes, 63
 ComplexInfinity, 38
 Composition, 97
 CompoundExpression, 76
 Compress, 260
 Condition, 194
 Conjugate, 39
 Constant, 59
 ConstantArray, 154
 ContainsOnly, 154
 Context, 227
 Contexts, 227
 Continue, 76
 CoprimeQ, 180
 CopyDirectory, 260
 CopyFile, 260
 Correlation, 154
 Cos, 93
 Cosh, 93
 CosineDistance, 144

Cot, 93
 Coth, 93
 Count, 154
 Covariance, 154
 CreateDirectory, 261
 Cross, 145
 Csc, 94
 Csch, 94
 CubeRoot, 39
 Cuboid, 117
 Cyan, 104

 D, 63
 DamerauLevenshteinDistance, 230
 Darker, 104
 DateDifference, 81
 DateList, 82
 DatePlus, 82
 DateString, 83
 Decrement, 49
 Default, 190
 DefaultValues, 49
 Definition, 49
 Degree, 94
 Delete, 155
 DeleteCases, 155
 DeleteDirectory, 261
 DeleteDuplicates, 155
 DeleteFile, 261
 Denominator, 33
 DensityPlot, 201
 Depth, 244
 Derivative, 64
 DesignMatrix, 145
 Det, 145
 DiagonalMatrix, 253
 DiamondMatrix, 123
 DiceDissimilarity, 68
 DigitCharacter, 231
 DigitCount, 140
 DigitQ, 231
 Dilation, 123
 Dimensions, 253
 DirectedInfinity, 39
 Directive, 105
 Directory, 261
 DirectoryName, 261
 DirectoryQ, 261
 DirectoryStack, 261
 DiscreteLimit, 64
 DisjointQ, 156
 Disk, 105
 DiskBox, 106
 DiskMatrix, 123
 Divide, 39
 DivideBy, 50
 Divisors, 180
 Do, 77
 DominantColors, 123
 Dot, 254
 DownValues, 51
 Drop, 156
 DSolve, 85

 E, 94
 EasterSunday, 83
 EdgeDetect, 124
 EdgeForm, 106
 EditDistance, 231
 Eigensystem, 145
 Eigenvalues, 145
 Eigenvectors, 146
 ElementData, 210
 End, 228
 EndOfFile, 262
 EndOfLine, 232
 EndOfString, 232
 EndPackage, 228
 Environment, 250
 Equal, 72
 Equivalent, 174
 Erf, 220
 Erfc, 220
 Erosion, 124
 EuclideanDistance, 146
 Evaluate, 86
 EvenQ, 180
 ExactNumberQ, 40
 Except, 195
 Exit, 86
 Exp, 94
 Expand, 34
 ExpandAll, 34
 ExpandDenominator, 34
 ExpandFileName, 262
 ExpIntegralE, 220
 ExpIntegralEi, 221
 Exponent, 35
 Export, 272
 ExportString, 273
 Expression, 262
 Extract, 156

 FaceForm, 106
 Factor, 35
 Factorial, 40
 FactorInteger, 181
 FactorTermsList, 35
 False, 175
 FetchURL, 273

Fibonacci, 68
 File, 262
 FileBaseName, 262
 FileByteCount, 262
 FileDate, 262
 FileExistsQ, 262
 FileExtension, 263
 FileFormat, 273
 FileHash, 263
 FileInformation, 263
 FileNameDepth, 263
 FileNameJoin, 263
 FileNameSplit, 263
 FilePrint, 264
 FileType, 264
 FilledCurve, 106
 FilledCurveBox, 106
 FilterRules, 190
 Find, 264
 FindClusters, 156
 FindFile, 264
 FindList, 264
 FindRoot, 65
 First, 157
 FirstPosition, 157
 FittedModel, 146
 FixedPoint, 77
 FixedPointList, 77
 Flat, 59
 Flatten, 244
 Floor, 140
 Fold, 157
 FoldList, 158
 FontColor, 106
 For, 78
 Format, 132
 FractionalPart, 181
 FreeQ, 245
 FresnelC, 221
 FresnelS, 221
 FromCharacterCode, 232
 FromDigits, 141
 Full, 202
 FullForm, 132
 Function, 97

 Gamma, 40
 Gather, 158
 GatherBy, 158
 GaussianFilter, 124
 GCD, 181
 GegenbauerC, 221
 General, 132
 Get, 265
 GetEnvironment, 250

 GoldenRatio, 94
 Graphics, 106
 Graphics3D, 118
 Graphics3DBox, 119
 GraphicsBox, 107
 Gray, 107
 GrayLevel, 108
 Greater, 73
 GreaterEqual, 73
 Green, 108
 Grid, 132
 GridBox, 133

 HammingDistance, 232
 HankelH1, 221
 HankelH2, 221
 HarmonicNumber, 41
 Hash, 185
 Haversine, 94
 Head, 245
 HermiteH, 221
 HexadecimalCharacter, 233
 Histogram, 202
 Hold, 86
 HoldAll, 59
 HoldAllComplete, 59
 HoldComplete, 87
 HoldFirst, 59
 HoldForm, 87
 HoldPattern, 195
 HoldRest, 59
 Hue, 108

 I, 41
 Identity, 98
 IdentityMatrix, 254
 If, 78
 Im, 41
 Image, 125
 ImageAdd, 124
 ImageAdjust, 125
 ImageAspectRatio, 125
 ImageBox, 125
 ImageChannels, 125
 ImageColorSpace, 125
 ImageConvolve, 125
 ImageData, 126
 ImageDimensions, 126
 ImageExport, 126
 ImageImport, 126
 ImageMultiply, 126
 ImagePartition, 126
 ImageQ, 127
 ImageReflect, 127
 ImageResize, 127

ImageRotate, 127
 ImageSubtract, 128
 ImageTake, 128
 ImageType, 128
 Implies, 175
 Import, 273
 ImportExport'RegisterExport, 274
 ImportExport'RegisterImport, 275
 ImportString, 274
 In, 87
 Increment, 51
 Indeterminate, 41
 Inequality, 73
 InexactNumberQ, 41
 Infinity, 42
 Infix, 133
 Information, 51
 Inner, 254
 InputForm, 133
 InputStream, 265
 Inset, 109
 InsetBox, 109
 Integer, 42
 IntegerDigits, 141, 186
 IntegerExponent, 181
 IntegerLength, 142
 IntegerQ, 42
 IntegerReverse, 142
 Integers, 65
 IntegerString, 142
 Integrate, 65
 Internal'RealValuedNumberQ, 189
 Internal'RealValuedNumericQ, 189
 Interrupt, 78
 IntersectingQ, 158
 Intersection, 158
 Inverse, 146
 InverseErf, 221
 InverseErfc, 222
 InverseHaversine, 95

 JaccardDissimilarity, 68
 JacobiP, 222
 Join, 159

 KelvinBei, 222
 KelvinBer, 222
 KelvinKei, 222
 KelvinKer, 223
 Keys, 159
 KnownUnitQ, 178
 Kurtosis, 159

 LABColor, 109
 LaguerreL, 223

 Large, 109
 Last, 159
 LCHColor, 109
 LCM, 181
 LeafCount, 159
 LeastSquares, 146
 Left, 133
 LegendreP, 223
 LegendreQ, 224
 Length, 160
 Less, 73
 LessEqual, 73
 LetterCharacter, 233
 LetterQ, 233
 Level, 160
 LevelQ, 160
 Lighter, 109
 LightRed, 109
 Limit, 66
 Line, 110
 Line3DBox, 119
 LinearModelFit, 146
 LinearSolve, 147
 LineBox, 110
 List, 161
 Listable, 59
 ListLinePlot, 203
 ListPlot, 203
 ListQ, 161
 LoadModule, 52
 Locked, 60
 Log, 95
 Log10, 95
 Log2, 95
 LogisticSigmoid, 95
 Longest, 195
 LowerCaseQ, 233
 LUVColor, 109

 MachineNumberQ, 42
 MachinePrecision, 186
 Magenta, 110
 MakeBoxes, 133
 ManhattanDistance, 147
 Manipulate, 177
 MantissaExponent, 181
 Map, 245
 MapIndexed, 245
 MapThread, 246
 MatchingDissimilarity, 69
 MatchQ, 195
 MathMLForm, 134
 MatrixExp, 147
 MatrixForm, 134
 MatrixPower, 148

- MatrixQ, 254
- MatrixRank, 148
- Max, 73
- MaxFilter, 128
- Maximize, 257
- Mean, 161
- Median, 161
- MedianFilter, 128
- Medium, 111
- MemberQ, 161
- Mesh, 203
- Message, 134
- MessageName, 134
- Messages, 52
- Min, 74
- MinFilter, 128
- MinimalPolynomial, 35
- Minimize, 257
- Minus, 42
- Missing, 36
- Mod, 182
- Module, 228
- MorphologicalComponents, 129
- Most, 162
- Multinomial, 69

- N, 187
- Names, 251
- Nearest, 162
- Needs, 266
- Negative, 74
- Nest, 78
- NestList, 78
- NestWhile, 79
- NextPrime, 182
- NHoldAll, 60
- NHoldFirst, 60
- NHoldRest, 60
- NonAssociative, 134
- None, 162
- NoneTrue, 175
- NonNegative, 74
- NonPositive, 74
- Norm, 148
- Normalize, 148
- Not, 175
- NotListQ, 162
- NotOptionQ, 190
- Null, 246
- NullSpace, 148
- Number, 266
- NumberForm, 134
- NumberQ, 42
- NumberString, 233
- Numerator, 36

- NumericQ, 188
- NValues, 52

- OddQ, 182
- Off, 134
- Offset, 111
- On, 135
- OneIdentity, 60
- OpenAppend, 266
- Opening, 129
- OpenRead, 266
- OpenWrite, 266
- Operate, 246
- Optional, 195
- OptionQ, 191
- Options, 191
- OptionsPattern, 196
- OptionValue, 191
- Or, 175
- Orange, 111
- Order, 247
- OrderedQ, 247
- Orderless, 60
- Out, 88
- Outer, 254
- OutputForm, 135
- OutputStream, 266
- OwnValues, 53

- PadLeft, 162
- PadRight, 163
- ParametricPlot, 204
- ParentDirectory, 267
- Part, 163
- Partition, 164
- Pattern, 196
- PatternsOrderedQ, 247
- PatternTest, 196
- Pause, 83
- Permutations, 164
- Pi, 95
- Pick, 165
- Piecewise, 43
- PieChart, 205
- PillowImageFilter, 129
- PixelValue, 129
- PixelValuePositions, 129
- Plot, 206
- Plot3D, 207
- Plus, 43
- Pochhammer, 43
- Point, 111
- Point3DBox, 119
- PointBox, 112
- PointSize, 112

PolarPlot, 208
 Polygon, 112
 Polygon3DBox, 119
 PolygonBox, 112
 PolynomialQ, 36
 Position, 165
 Positive, 74
 Postfix, 135
 Power, 44
 PowerExpand, 36
 PowerMod, 182
 Precedence, 135
 Precision, 188
 PreDecrement, 53
 Prefix, 136
 PreIncrement, 53
 Prepend, 165
 PrependTo, 165
 Prime, 182
 PrimePi, 182
 PrimePowerQ, 183
 PrimeQ, 183
 Print, 136
 Product, 44
 ProductLog, 224
 Protect, 61
 Protected, 61
 PseudoInverse, 148
 Purple, 112
 Put, 267
 PutAppend, 267

 QRDecomposition, 149
 Quantile, 166
 Quantity, 178
 QuantityMagnitude, 178
 QuantityQ, 178
 QuantityUnit, 179
 Quartiles, 166
 Quiet, 136
 Quit, 53
 Quotient, 183
 QuotientRemainder, 183

 Random, 212
 RandomChoice, 212
 RandomComplex, 213
 RandomImage, 129
 RandomInteger, 213
 RandomPrime, 183
 RandomReal, 213
 RandomSample, 214
 Range, 166
 RankedMax, 166
 RankedMin, 166

 Rational, 45
 Rationalize, 188
 Re, 45
 Read, 268
 ReadList, 268
 ReadProtected, 61
 Real, 45
 RealDigits, 188
 RealNumberQ, 45
 Reals, 66
 Reap, 166
 Record, 268
 Rectangle, 113
 RectangleBox, 114
 Red, 114
 RegularExpression, 233
 RegularPolygon, 114
 RegularPolygonBox, 115
 ReleaseHold, 88
 RemoveDiacritics, 233
 RenameDirectory, 268
 RenameFile, 269
 Repeated, 197
 RepeatedNull, 197
 Replace, 197
 ReplaceAll, 198
 ReplaceList, 198
 ReplacePart, 167
 ReplaceRepeated, 198
 ResetDirectory, 269
 Rest, 167
 Return, 79
 Reverse, 167
 RGBColor, 113
 Riffle, 168
 Right, 137
 RogersTanimotoDissimilarity, 69
 Root, 66
 RotateLeft, 168
 RotateRight, 168
 Round, 189
 Row, 137
 RowBox, 137
 RowReduce, 149
 RSolve, 216
 Rule, 199
 RuleDelayed, 199
 RussellRaoDissimilarity, 69

 SameQ, 74
 Scan, 247
 Sec, 96
 Sech, 96
 SeedRandom, 214
 Select, 168

Sequence, 88
 SequenceHold, 62
 SessionTime, 83
 Set, 54
 SetAttributes, 62
 SetDelayed, 54
 SetDirectory, 269
 SetFileDate, 269
 SetStreamPosition, 269
 Sharpen, 129
 Shortest, 199
 Sign, 45
 Simplify, 36
 Sin, 96
 SingularValueDecomposition, 149
 Sinh, 96
 Skewness, 169
 Skip, 270
 Slot, 98
 SlotSequence, 98
 Small, 115
 SokalSneathDissimilarity, 69
 Solve, 66
 Sort, 247
 SortBy, 248
 Sow, 169
 Span, 169
 Sphere, 119
 Sphere3DBox, 120
 SphericalHarmonicY, 224
 Split, 169
 SplitBy, 169
 Sqrt, 45
 SquaredEuclideanDistance, 149
 StandardDeviation, 170
 StandardForm, 137
 StartOfLine, 234
 StartOfString, 234
 StreamPosition, 270
 Streams, 270
 String, 239
 StringCases, 234
 StringContainsQ, 235
 StringDrop, 235
 StringExpression, 235
 StringForm, 137
 StringFreeQ, 236
 StringInsert, 236
 StringJoin, 237
 StringLength, 237
 StringMatchQ, 237
 StringPosition, 237
 StringQ, 237
 StringRepeat, 238
 StringReplace, 238
 StringRiffle, 238
 StringSplit, 239
 StringTake, 239
 StringToStream, 270
 StringTrim, 239
 StruveH, 224
 StruveL, 225
 Style, 137
 Subscript, 137
 SubscriptBox, 137
 SubsetQ, 170
 Subsets, 70
 Subsuperscript, 137
 SubsuperscriptBox, 137
 Subtract, 46
 SubtractFrom, 55
 SubValues, 55
 Sum, 46
 Superscript, 137
 SuperscriptBox, 137
 Switch, 79
 Symbol, 248
 SymbolName, 248
 SymbolQ, 248
 SymPyComparison, 75
 Syntax, 137
 System'Convert'B64Dump'B64Decode, 272
 System'Convert'B64Dump'B64Encode, 272
 System'Private'\$ContextPathStack, 227
 System'Private'\$ContextStack, 227
 System'Private'ManipulateParameter, 177
 Table, 170
 TableForm, 138
 TagSet, 55
 TagSetDelayed, 55
 Take, 170
 TakeLargest, 171
 TakeLargestBy, 171
 TakeSmallest, 171
 TakeSmallestBy, 171
 Tally, 172
 Tan, 96
 Tanh, 96
 TeXForm, 138
 Text, 115
 TextRecognize, 130
 Thick, 115
 Thickness, 115
 Thin, 115
 Thread, 248
 Threshold, 130
 Through, 248
 Times, 47
 TimesBy, 56

TimeUsed, 84
 Timing, 84
 Tiny, 115
 ToBoxes, 138
 ToCharCode, 240
 ToExpression, 240
 ToFileName, 271
 Together, 36
 ToLowerCase, 240
 Top, 208
 ToString, 240
 Total, 172
 ToUpperCase, 241
 Tr, 149
 Transliterate, 241
 Transpose, 255
 True, 175
 TrueQ, 75
 Tuples, 172

 Uncompress, 271
 Unequal, 75
 Unevaluated, 89
 Union, 172
 Unique, 228
 UnitConvert, 179
 UnitVector, 173
 Unprotect, 62
 UnsameQ, 75
 Unset, 56
 UpperCaseQ, 241
 UpSet, 56
 UpSetDelayed, 57
 UpTo, 37
 UpValues, 57

 ValueQ, 75
 Values, 173
 Variables, 37
 Variance, 173
 VectorAngle, 150
 VectorQ, 255
 Verbatim, 199

 WeberE, 225
 Which, 79
 While, 80
 White, 115
 Whitespace, 241
 WhitespaceCharacter, 241
 Word, 271
 WordBoundary, 241
 WordCharacter, 241
 WordCloud, 130
 Write, 271

 WriteString, 271

 XML'Parser'XMLGet, 256
 XML'Parser'XMLGetString, 256
 XML'PlaintextImport, 256
 XML'TagsImport, 256
 XML'XMLObjectImport, 256
 XMLElement, 256
 XMLObject, 256
 Xor, 176
 XYZColor, 116

 Yellow, 116
 YuleDissimilarity, 70

 Zeta, 225