Mathics

A free, open-source alternative to Mathematica

The Mathics Team

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Contents

Part I.

Manual

1. Introduction

Mathics—to be pronounced like "Mathematics" without the "emat"—is a general-purpose computer algebra system (CAS). It is meant to be a free, open-source alternative to *Mathematica*®. It is free both as in "free beer" and as in "freedom". Mathics can be run *Mathics* locally, and to facilitate installation of the vast amount of software need to run this, there is a docker image available. See `https://hub.docker.com/r/mathicsorg/mathics`.

The programming language of *Mathics* is meant to resemble *Wolfram*'s famous *Mathematica*® as much as possible. However, *Mathics* is in no way affiliated or supported by *Wolfram*. *Mathics* will probably never have the power to compete with *Mathematica*® in industrial applications; yet, it is an alternative for educational purposes. It also invites community development at all levels.

See `https://mathics-development-guide.readthedocs.io/en/latest/installing/index.html` for the most recent instructions for installing from PyPI, source, or from *docker*.

For implementation details see `https://mathics-development-guide.readthedocs.io/en/latest/`.

Contents

Why yet another CAS?

Mathematica® is great, but it a couple of disadvantages.

- It is not open source.
- Its development is tightly controled and centralized.

The last point some may find and advantage.

Even if you are willing to pay hundreds of dollars for the software, would will not be able to see what's going on "inside" the program if that is your interest. That's what free, open-source, and community-supported software is for!

Mathics aims at combining the best of both worlds: the beauty of *Mathematica*® backed by a free, extensible Python core which includes a rich set of Python tools including:

- mpmath `https://mpmath.org/` for floating-point arithmetic with arbitrary precision,
- numpy `https://numpy.org/numpy` for numeric computation,
- sympy `https://sympy.org` for symbolic mathematics, and
- optionally scipy `https://www.scipy.org/` for Scientific calculations.

Performance of *Mathics* is not, right now, practical in large-scale projects and calculations. However can be used as a tool for quick explorations and to educate people who might later switch to *Mathematica*®.

What does it offer?

Some of the features of *Mathics* are:

- a powerful functional programming language,
- a system driven by pattern matching and rules application,
- rationals, complex numbers, and arbitrary-precision arithmetic,
- lots of list and structure manipulation routines,
- an interactive graphical user interface right in the Web browser using MathML (apart from a command line interface),
- creation of graphics (e.g. plots) and display in the browser using SVG for 2D

graphics and three.js for 3D graphics,
- export of results to LaTeX (using Asymptote for graphics),
- an easy way of defining new functions in Python and which hooks into Python libraries
- an integrated documentation and testing system.

What is missing?

There are lots of ways in which *Mathics* could still be improved.

Most notably, performance is still slow. Although there are various ways to speed up Python, some serious work is need in *Mathics*, to speed it up. This will be addressed in the future.

Apart from performance issues, *Mathics* has about about half of the features and libraries of *Mathematica*®.

Graphics has always been lagging and in the future we intend to decouple Graphics better so that the rich set of graphics packages that are out there can be more easily used.

Who is behind it?

Mathics was created by Jan Pöschk in 2011. From 2013 to about 2017 it had been maintained mostly by Angus Griffith and Ben Jones. Since then, a number of others have been people involved in *Mathics*; the list can be found in the `AUTHORS.txt` file, `https://github.com/mathics/Mathics/blob/master/AUTHORS.txt`.

If you have any ideas on how to improve *Mathics* or even want to help out yourself, please contact us!

Welcome to *Mathics*, have fun!

2. Language Tutorials

The following sections are introductions to the basic principles of the language of *Mathics*. A few examples and functions are presented. Only their most common usages are listed; for a full description of a Symbols possible arguments, options, etc., see its entry in the Reference of Built-in Symbols.
However if you google for "Mathematica Tutorials" you will find easily dozens of other tutorials which are applicable. Be warned though that *Mathics* does not yet offer the full range and features and capabilities of *Mathematica*®.

Contents

Basic calculations

Mathics can be used to calculate basic stuff:

```
>> 1 + 2
   3
```

To submit a command to *Mathics*, press `Shift+Return` in the Web interface or `Return` in the console interface. The result will be printed in a new line below your query.
Mathics understands all basic arithmetic operators and applies the usual operator precedence. Use parentheses when needed:

```
>> 1 - 2 * (3 + 5)/ 4
   −3
```

The multiplication can be omitted:

```
>> 1 - 2 (3 + 5)/ 4
   −3

>> 2 4
   8
```

Powers can be entered using `^`:

```
>> 3 ^ 4
   81
```

Integer divisions yield rational numbers:

```
>> 6 / 4
```

$\frac{3}{2}$

To convert the result to a floating point number, apply the function `N`:

```
>> N[6 / 4]
   1.5
```

As you can see, functions are applied using square braces `[` and `]`, in contrast to the common notation of `(` and `)`. At first hand, this might seem strange, but this distinction between function application and precedence change is necessary to allow some general syntax structures, as you will see later.
Mathics provides many common mathematical functions and constants, e.g.:

```
>> Log[E]
   1

>> Sin[Pi]
   0

>> Cos[0.5]
   0.877583
```

When entering floating point numbers in your query, *Mathics* will perform a numerical evaluation and present a numerical result, pretty much like if you had applied `N`.

Of course, *Mathics* has complex numbers:

```
>> Sqrt[-4]
```
$2I$

```
>> I ^ 2
```
-1

```
>> (3 + 2 I)^ 4
```
$-119 + 120I$

```
>> (3 + 2 I)^ (2.5 - I)
```
$43.663 + 8.28556I$

```
>> Tan[I + 0.5]
```
$0.195577 + 0.842966I$

`Abs` calculates absolute values:

```
>> Abs[-3]
```
3

```
>> Abs[3 + 4 I]
```
5

Mathics can operate with pretty huge numbers:

```
>> 100!
```
93 326 215 443 944 152 681 699 ˜
˜238 856 266 700 490 715 968 ˜
˜264 381 621 468 592 963 895 ˜
˜217 599 993 229 915 608 941 ˜
˜463 976 156 518 286 253 697 920 ˜
˜827 223 758 251 185 210 916 864 ˜
˜000 000 000 000 000 000 000 000

(`!` denotes the factorial function.) The precision of numerical evaluation can be set:

```
>> N[Pi, 100]
```
3.1415926535897932384626433˜
˜83279502884197169399375˜
˜10582097494459230781640628˜
˜6208998628034825342117068

Division by zero is forbidden:

```
>> 1 / 0
```
Infiniteexpression1/0encountered.

ComplexInfinity

Other expressions involving `Infinity` are evaluated:

```
>> Infinity + 2 Infinity
```
∞

In contrast to combinatorial belief, `0^0` is undefined:

```
>> 0 ^ 0
```
Indeterminateexpression0^0*encountered.*

Indeterminate

The result of the previous query to *Mathics* can be accessed by `%`:

```
>> 3 + 4
```
7

```
>> % ^ 2
```
49

Symbols and Assignments

Symbols need not be declared in *Mathics*, they can just be entered and remain variable:

```
>> x
```
x

Basic simplifications are performed:

```
>> x + 2 x
```
$3x$

Symbols can have any name that consists of characters and digits:

```
>> iAm1Symbol ^ 2
```
iAm1Symbol^2

You can assign values to symbols:

```
>> a = 2
```
2

```
>> a ^ 3
```
8

```
>> a = 4
```
4

```
>> a ^ 3
```
64

Assigning a value returns that value. If you want to suppress the output of any result, add a `;` to the end of your query:

```
>> a = 4;
```

Values can be copied from one variable to another:

```
>> b = a;
```

Now changing `a` does not affect `b`:

```
>> a = 3;
```

```
>> b
```
4

Such a dependency can be achieved by using "delayed assignment" with the `:=` operator (which does not return anything, as the right side is not even evaluated):

```
>> b := a ^ 2
```

```
>> b
```

9

```
>> a = 5;
```

```
>> b
```

25

Comparisons and Boolean Logic

Values can be compared for equality using the operator `==`:

```
>> 3 == 3
```

True

```
>> 3 == 4
```

False

The special symbols `True` and `False` are used to denote truth values. Naturally, there are inequality comparisons as well:

```
>> 3 > 4
```

False

Inequalities can be chained:

```
>> 3 < 4 >= 2 != 1
```

True

Truth values can be negated using `!` (logical *not*) and combined using `&&` (logical *and*) and `||` (logical *or*):

```
>> !True
```

False

```
>> !False
```

True

```
>> 3 < 4 && 6 > 5
```

True

`&&` has higher precedence than `||`, i.e. it binds stronger:

```
>> True && True || False && False
```

True

```
>> True && (True || False)&& False
```

False

Strings

Strings can be entered with `"` as delimiters:

```
>> "Hello world!"
```

Hello world!

As you can see, quotation marks are not printed in the output by default. This can be changed by using `InputForm`:

```
>> InputForm["Hello world!"]
```

"Hello world!"

Strings can be joined using `<>`:

```
>> "Hello" <> " " <> "world!"
```

Hello world!

Numbers cannot be joined to strings:

```
>> "Debian" <> 6
```

Stringexpected.

Debian<>6

They have to be converted to strings using `ToString` first:

```
>> "Debian" <> ToString[6]
```

Debian6

Lists

Lists can be entered in *Mathics* with curly braces `{` and `}`:

```
>> mylist = {a, b, c, d}
```

$\{a, b, c, d\}$

There are various functions for constructing lists:

```
>> Range[5]
```

$\{1, 2, 3, 4, 5\}$

```
>> Array[f, 4]
```

$\{f[1], f[2], f[3], f[4]\}$

```
>> ConstantArray[x, 4]
```

$\{x, x, x, x\}$

```
>> Table[n ^ 2, {n, 2, 5}]
```

$\{4, 9, 16, 25\}$

The number of elements of a list can be determined with `Length`:

```
>> Length[mylist]
```

4

Elements can be extracted using double square braces:

```
>> mylist[[3]]
```
c

Negative indices count from the end:

```
>> mylist[[-3]]
```
b

Lists can be nested:

```
>> mymatrix = {{1, 2}, {3, 4}, {5,
   6}};
```

There are alternate forms to display lists:

```
>> TableForm[mymatrix]
```

1	2
3	4
5	6

```
>> MatrixForm[mymatrix]
```

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

There are various ways of extracting elements from a list:

```
>> mymatrix[[2, 1]]
```
3

```
>> mymatrix[[;;, 2]]
```
$\{2,4,6\}$

```
>> Take[mylist, 3]
```
$\{a,b,c\}$

```
>> Take[mylist, -2]
```
$\{c,d\}$

```
>> Drop[mylist, 2]
```
$\{c,d\}$

```
>> First[mymatrix]
```
$\{1,2\}$

```
>> Last[mylist]
```
d

```
>> Most[mylist]
```
$\{a,b,c\}$

```
>> Rest[mylist]
```
$\{b,c,d\}$

Lists can be used to assign values to multiple variables at once:

```
>> {a, b} = {1, 2};
```

```
>> a
```
1

```
>> b
```
2

Many operations, like addition and multiplication, "thread" over lists, i.e. lists are combined element-wise:

```
>> {1, 2, 3} + {4, 5, 6}
```
$\{5,7,9\}$

```
>> {1, 2, 3} * {4, 5, 6}
```
$\{4,10,18\}$

It is an error to combine lists with unequal lengths:

```
>> {1, 2} + {4, 5, 6}
```
Objectsofunequallengthcannotbecombined.

$\{1,2\} + \{4,5,6\}$

The Structure of Things

Every expression in *Mathics* is built upon the same principle: it consists of a *head* and an arbitrary number of *children*, unless it is an *atom*, i.e. it can not be subdivided any further. To put it another way: everything is a function call. This can be best seen when displaying expressions in their "full form":

```
>> FullForm[a + b + c]
```
Plus $[a,b,c]$

Nested calculations are nested function calls:

```
>> FullForm[a + b * (c + d)]
```
Plus $[a,$ Times $[b,$ Plus $[c,d]]]$

Even lists are function calls of the function `List`:

```
>> FullForm[{1, 2, 3}]
```
List $[1,2,3]$

The head of an expression can be determined with `Head`:

```
>> Head[a + b + c]
```
Plus

The children of an expression can be accessed like list elements:

```
>> (a + b + c)[[2]]
```
b

The head is the 0th element:

```
>> (a + b + c)[[0]]
```

Plus

The head of an expression can be exchanged using the function `Apply`:

```
>> Apply[g, f[x, y]]
```

$g[x, y]$

```
>> Apply[Plus, a * b * c]
```

$a + b + c$

`Apply` can be written using the operator `@@`:

```
>> Times @@ {1, 2, 3, 4}
```

24

(This exchanges the head `List` of `{1, 2, 3, 4}` with `Times`, and then the expression `Times[1, 2, 3, 4]` is evaluated, yielding 24.) `Apply` can also be applied on a certain *level* of an expression:

```
>> Apply[f, {{1, 2}, {3, 4}}, {1}]
```

$\{f[1,2], f[3,4]\}$

Or even on a range of levels:

```
>> Apply[f, {{1, 2}, {3, 4}}, {0,
   2}]
```

$f[f[1,2], f[3,4]]$

`Apply` is similar to `Map` (`/@`):

```
>> Map[f, {1, 2, 3, 4}]
```

$\{f[1], f[2], f[3], f[4]\}$

```
>> f /@ {{1, 2}, {3, 4}}
```

$\{f[\{1,2\}], f[\{3,4\}]\}$

The atoms of *Mathics* are numbers, symbols, and strings. `AtomQ` tests whether an expression is an atom:

```
>> AtomQ[5]
```

True

```
>> AtomQ[a + b]
```

False

The full form of rational and complex numbers looks like they were compound expressions:

```
>> FullForm[3 / 5]
```

Rational [3, 5]

```
>> FullForm[3 + 4 I]
```

Complex [3, 4]

However, they are still atoms, thus unaffected by applying functions, for instance:

```
>> f @@ Complex[3, 4]
```

$3 + 4I$

Nevertheless, every atom has a head:

```
>> Head /@ {1, 1/2, 2.0, I, "a
   string", x}
```

{Integer, Rational, Real, Complex, String, Symbol}

The operator `===` tests whether two expressions are the same on a structural level:

```
>> 3 === 3
```

True

```
>> 3 == 3.0
```

True

But

```
>> 3 === 3.0
```

False

because `3` (an `Integer`) and `3.0` (a `Real`) are structurally different.

Functions and Patterns

Functions can be defined in the following way:

```
>> f[x_] := x ^ 2
```

This tells *Mathics* to replace every occurrence of `f` with one (arbitrary) parameter `x` with `x ^ 2`.

```
>> f[3]
```

9

```
>> f[a]
```

a^2

The definition of `f` does not specify anything for two parameters, so any such call will stay unevaluated:

```
>> f[1, 2]
```

$f[1,2]$

In fact, *functions* in *Mathics* are just one aspect of *patterns*: `f[x_]` is a pattern that *matches* expressions like `f[3]` and `f[a]`. The following patterns are available:

_ or Blank[]
: matches one expression.

Pattern[x, p]
: matches the pattern p and stores the value in x.

x_ or Pattern[x, Blank[]]
: matches one expression and stores it in x.

__ or BlankSequence[]
: matches a sequence of one or more expressions.

___ or BlankNullSequence[]
: matches a sequence of zero or more expressions.

_h or Blank[h]
: matches one expression with head h.

x_h or Pattern[x, Blank[h]]
: matches one expression with head h and stores it in x.

p | q or Alternatives[p, q]
: matches either pattern p or q.

p ? t or PatternTest[p, t]
: matches p if the test t[p] yields True.

p /; c or Condition[p, c]
: matches p if condition c holds.

Verbatim[p]
: matches an expression that equals p, without regarding patterns inside p.

As before, patterns can be used to define functions:

```
>> g[s___] := Plus[s] ^ 2

>> g[1, 2, 3]
   36
```

MatchQ[e, p] tests whether e matches p:

```
>> MatchQ[a + b, x_ + y_]
   True

>> MatchQ[6, _Integer]
   True
```

ReplaceAll (/.) replaces all occurrences of a pattern in an expression using a Rule given by ->:

```
>> {2, "a", 3, 2.5, "b", c} /. x_Integer -> x ^ 2
```

$\{4, \mathrm{a}, 9, 2.5, \mathrm{b}, c\}$

You can also specify a list of rules:

```
>> {2, "a", 3, 2.5, "b", c} /. {x_Integer -> x ^ 2.0, y_String -> 10}
```

$\{4., 10, 9., 2.5, 10, c\}$

ReplaceRepeated (//.) applies a set of rules repeatedly, until the expression doesn't change anymore:

```
>> {2, "a", 3, 2.5, "b", c} //. {x_Integer -> x ^ 2.0, y_String -> 10}
```

$\{4., 100., 9., 2.5, 100., c\}$

There is a "delayed" version of Rule which can be specified by :> (similar to the relation of := to =):

```
>> a :> 1 + 2
```

a:>1 + 2

```
>> a -> 1 + 2
```

$a- > 3$

This is useful when the right side of a rule should not be evaluated immediately (before matching):

```
>> {1, 2} /. x_Integer -> N[x]
```

$\{1, 2\}$

Here, N is applied to x before the actual matching, simply yielding x. With a delayed rule this can be avoided:

```
>> {1, 2} /. x_Integer :> N[x]
```

$\{1., 2.\}$

While ReplaceAll and ReplaceRepeated simply take the first possible match into account, ReplaceList returns a list of all possible matches. This can be used to get all subsequences of a list, for instance:

```
>> ReplaceList[{a, b, c}, {___, x__, ___} -> {x}]
```

$\{\{a\}, \{a, b\}, \{a, b, c\}, \{b\}, \{b, c\}, \{c\}\}$

ReplaceAll would just return the first expression:

```
>> ReplaceAll[{a, b, c}, {___, x__, ___} -> {x}]
```

$\{a\}$

In addition to defining functions as rules for certain patterns, there are *pure* functions that can be defined using the & postfix operator, where everything before it is treated as the function body and # can be used as argument placeholder:

```
>> h = # ^ 2 &;

>> h[3]
   9
```

Multiple arguments can simply be indexed:

```
>> sum = #1 + #2 &;
```

```
>> sum[4, 6]
```

10

It is also possible to name arguments using `Function`:

```
>> prod = Function[{x, y}, x * y];
```

```
>> prod[4, 6]
```

24

Pure functions are very handy when functions are used only locally, e.g., when combined with operators like `Map`:

```
>> # ^ 2 & /@ Range[5]
```

$\{1, 4, 9, 16, 25\}$

Sort according to the second part of a list:

```
>> Sort[{{x, 10}, {y, 2}, {z, 5}}, #1[[2]] < #2[[2]] &]
```

$\{\{y, 2\}, \{z, 5\}, \{x, 10\}\}$

Functions can be applied using prefix or postfix notation, in addition to using `[]`:

```
>> h @ 3
```

9

```
>> 3 // h
```

9

Control Statements

Like most programming languages, *Mathics* has common control statements for conditions, loops, etc.:

`If[`*cond*`,` *pos*`,` *neg*`]`
: returns *pos* if *cond* evaluates to `True`, and *neg* if it evaluates to `False`.

`Which[`*cond1*`,` *expr1*`,` *cond2*`,` *expr2*`, ...]`
: yields *expr1* if *cond1* evaluates to `True`, *expr2* if *cond2* evaluates to `True`, etc.

`Do[`*expr*`, {`*i*`,` *max*`}]`
: evaluates *expr* *max* times, substituting *i* in *expr* with values from 1 to *max*.

`For[`*start*`,` *test*`,` *incr*`,` *body*`]`
: evaluates *start*, and then iteratively *body* and *incr* as long as *test* evaluates to `True`.

`While[`*test*`,` *body*`]`
: evaluates *body* as long as *test* evaluates to `True`.

`Nest[`*f*`,` *expr*`,` *n*`]`
: returns an expression with *f* applied *n* times to *expr*.

`NestWhile[`*f*`,` *expr*`,` *test*`]`
: applies a function *f* repeatedly on an expression *expr*, until applying *test* on the result no longer yields `True`.

`FixedPoint[`*f*`,` *expr*`]`
: starting with *expr*, repeatedly applies *f* until the result no longer changes.

```
>> If[2 < 3, a, b]
```

a

```
>> x = 3; Which[x < 2, a, x > 4, b, x < 5, c]
```

c

Compound statements can be entered with `;`. The result of a compound expression is its last part or `Null` if it ends with a `;`.

```
>> 1; 2; 3
```

3

```
>> 1; 2; 3;
```

Inside `For`, `While`, and `Do` loops, `Break[]` exits the loop and `Continue[]` continues to the next iteration.

```
>> For[i = 1, i <= 5, i++, If[i == 4, Break[]]; Print[i]]
```

1
2
3

Scoping

By default, all symbols are "global" in *Mathics*, i.e. they can be read and written in any part of your program. However, sometimes "local" variables are needed in order not to disturb the global namespace. *Mathics* provides two ways to support this:

- *lexical scoping* by `Module`, and
- *dynamic scoping* by `Block`.

`Module[{`*vars*`}, `*expr*`]`
: localizes variables by giving them a temporary name of the form `name$number`, where number is the current value of `$ModuleNumber`. Each time a module is evaluated, `$ModuleNumber` is incremented.

`Block[{`*vars*`}, `*expr*`]`
: temporarily stores the definitions of certain variables, evaluates *expr* with reset values and restores the original definitions afterwards.

Both scoping constructs shield inner variables from affecting outer ones:

```
>> t = 3;

>> Module[{t}, t = 2]
   2

>> Block[{t}, t = 2]
   2

>> t
   3
```

`Module` creates new variables:

```
>> y = x ^ 3;

>> Module[{x = 2}, x * y]
```

$2x^3$

`Block` does not:

```
>> Block[{x = 2}, x * y]
   16
```

Thus, `Block` can be used to temporarily assign a value to a variable:

```
>> expr = x ^ 2 + x;

>> Block[{x = 3}, expr]
   12

>> x
```

x

`Block` can also be used to temporarily change the value of system parameters:

```
>> Block[{$RecursionLimit = 30}, x
   = 2 x]
```

Recursiondepthof30exceeded.

$Aborted

```
>> f[x_] := f[x + 1]; Block[{
   $IterationLimit = 30}, f[1]]
```

Iterationlimitof30exceeded.

$Aborted

It is common to use scoping constructs for function definitions with local variables:

```
>> fac[n_] := Module[{k, p}, p = 1;
    For[k = 1, k <= n, ++k, p *= k
   ]; p]

>> fac[10]
   3 628 800

>> 10!
   3 628 800
```

Formatting Output

The way results are formatted for output in *Mathics* is rather sophisticated, as compatibility to the way *Mathematica*® does things is one of the design goals. It can be summed up in the following procedure:

1. The result of the query is calculated.
2. The result is stored in `Out` (which `%` is a shortcut for).
3. Any `Format` rules for the desired output form are applied to the result. In the console version of *Mathics*, the result is formatted as `OutputForm`; `MathMLForm` for the `StandardForm` is used in the interactive Web version; and `TeXForm` for the `StandardForm` is used to generate the LaTeX version of this documentation.
4. `MakeBoxes` is applied to the formatted result, again given either `OutputForm`, `MathMLForm`, or `TeXForm` depending on the execution context of *Mathics*. This yields a new expression consisting of "box constructs".
5. The boxes are turned into an ordinary string and displayed in the console, sent to the browser, or written to the documentation LaTeX file.

As a consequence, there are various ways to implement your own formatting strategy for custom objects.
You can specify how a symbol shall be formatted by assigning values to `Format`:

```
>> Format[x] = "y";

>> x
   y
```

This will apply to `MathMLForm`, `OutputForm`, `StandardForm`, `TeXForm`, and `TraditionalForm`.

```
>> x // InputForm
   x
```

You can specify a specific form in the assignment to `Format`:

```
>> Format[x, TeXForm] = "z";

>> x // TeXForm
   \text{z}
```

Special formats might not be very relevant for individual symbols, but rather for custom functions (objects):

```
>> Format[r[args___]] = "<an r
   object>";

>> r[1, 2, 3]
   <an r object>
```

You can use several helper functions to format expressions:

`Infix[`*expr*`, `*op*`]`
: formats the arguments of *expr* with infix operator *op*.

`Prefix[`*expr*`, `*op*`]`
: formats the argument of *expr* with prefix operator *op*.

`Postfix[`*expr*`, `*op*`]`
: formats the argument of *expr* with postfix operator *op*.

`StringForm[`*form*`, `*arg1*`, `*arg2*`, ...]`
: formats arguments using a format string.

```
>> Format[r[args___]] = Infix[{args
   }, "~"];

>> r[1, 2, 3]
   1 ~ 2 ~ 3

>> StringForm["`1` and `2`", n, m]
   n and m
```

There are several methods to display expressions in 2-D:

`Row[{...}]`
: displays expressions in a row.

`Grid[{{...}}]`
: displays a matrix in two-dimensional form.

`Subscript[`*expr*`, `*i1*`, `*i2*`, ...]`
: displays *expr* with subscript indices *i1*, *i2*, ...

`Superscript[`*expr*`, `*exp*`]`
: displays *expr* with superscript (exponent) *exp*.

```
>> Grid[{{a, b}, {c, d}}]
   a  b
   c  d

>> Subscript[a, 1, 2] // TeXForm
   a_{1,2}
```

If you want even more low-level control of how expressions are displayed, you can override `MakeBoxes`:

```
>> MakeBoxes[b, StandardForm] = "c
   ";

>> b
   c
```

This will even apply to `TeXForm`, because `TeXForm` implies `StandardForm`:

```
>> b // TeXForm
   c
```

Except some other form is applied first:

```
>> b // OutputForm // TeXForm
   b
```

`MakeBoxes` for another form:

```
>> MakeBoxes[b, TeXForm] = "d";

>> b // TeXForm
   d
```

You can cause a much bigger mess by overriding `MakeBoxes` than by sticking to `Format`, e.g. generate invalid XML:

```
>> MakeBoxes[c, MathMLForm] = "<not
    closed";

>> c // MathMLForm
   <not closed
```

However, this will not affect formatting of expressions involving `c`:

```
>> c + 1 // MathMLForm
```

<math display="block"><mrow>
<mn>1</mn> <mo>+</mo>
<mi>c</mi></mrow></math>

That's because `MathMLForm` will, when not overridden for a special case, call `StandardForm` first. `Format` will produce escaped output:

```
>> Format[d, MathMLForm] = "<not
   closed";
```

```
>> d // MathMLForm
```

<math display="block">
<mtext><not closed</mtext>
</math>

```
>> d + 1 // MathMLForm
```

<math display="block"><mrow>
<mn>1</mn> <mo>+</mo>
<mtext><not closed</mtext>
</mrow></math>

For instance, you can override `MakeBoxes` to format lists in a different way:

```
>> MakeBoxes[{items___},
   StandardForm] := RowBox[{"[",
   Sequence @@ Riffle[MakeBoxes /@
   {items}, " "], "]"}]
```

```
>> {1, 2, 3}
```

[123]

However, this will not be accepted as input to *Mathics* anymore:

```
>> [1 2 3]
```

```
>> Clear[MakeBoxes]
```

By the way, `MakeBoxes` is the only built-in symbol that is not protected by default:

```
>> Attributes[MakeBoxes]
```

{HoldAllComplete}

`MakeBoxes` must return a valid box construct:

```
>> MakeBoxes[squared[args___],
   StandardForm] := squared[args] ^
    2
```

```
>> squared[1, 2]
```

Power[*squared*[1, 2], 2]*isnotavalidboxstructure*.

```
>> squared[1, 2] // TeXForm
```

Power[*squared*[1, 2], 2]*isnotavalidboxstructure*.

= The desired effect can be achieved in the following way:

```
>> MakeBoxes[squared[args___],
   StandardForm] := SuperscriptBox[
   RowBox[{MakeBoxes[squared], "[",
    RowBox[Riffle[MakeBoxes[#]& /@
   {args}, ","]], "]"}], 2]
```

```
>> squared[1, 2]
```

$\text{squared}\,[1,2]^2$

You can view the box structure of a formatted expression using `ToBoxes`:

```
>> ToBoxes[m + n]
```

RowBox [{m, +, n}]

The list elements in this `RowBox` are strings, though string delimiters are not shown in the default output form:

```
>> InputForm[%]
```

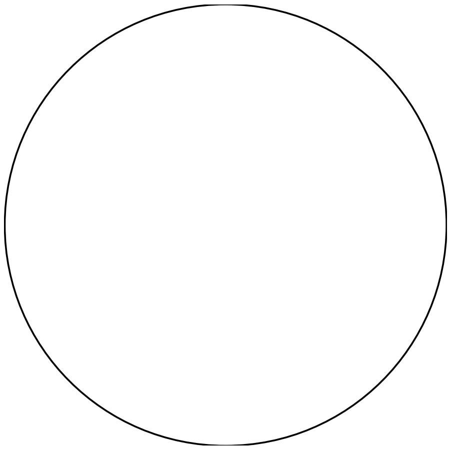
RowBox [{"m", "+", "n"}]

Graphics Introduction Examples

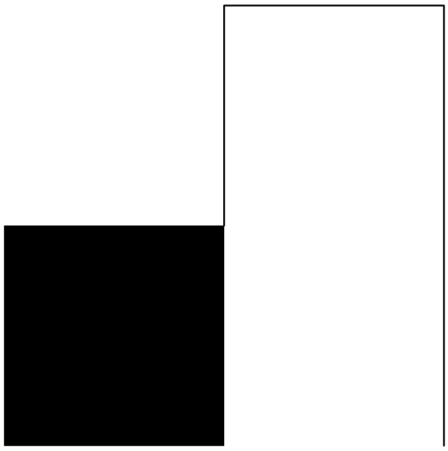
Two-dimensional graphics can be created using the function `Graphics` and a list of graphics primitives. For three-dimensional graphics see the following section. The following primitives are available:

`Circle[{`*x*`, `*y*`}, `*r*`]`
  draws a circle.
`Disk[{`*x*`, `*y*`}, `*r*`]`
  draws a filled disk.
`Rectangle[{`*x1*`, `*y1*`}, {`*x2*`, `*y2*`}]`
  draws a filled rectangle.
`Polygon[{{`*x1*`, `*y1*`}, {`*x2*`, `*y2*`}, ...}]`
  draws a filled polygon.
`Line[{{`*x1*`, `*y1*`}, {`*x2*`, `*y2*`}, ...}]`
  draws a line.
`Text[`*text*`, {`*x*`, `*y*`}]`
  draws text in a graphics.

```
>> Graphics[{Circle[{0, 0}, 1]}]
```



```
>> Graphics[{Line[{{0, 0}, {0, 1},
   {1, 1}, {1, -1}}], Rectangle[{0,
    0}, {-1, -1}]}]
```



Colors can be added in the list of graphics primitives to change the drawing color. The following ways to specify colors are supported:

`RGBColor[`*r*`, `*g*`, `*b*`]`
: specifies a color using red, green, and blue.

`CMYKColor[`*c*`, `*m*`, `*y*`, `*k*`]`
: specifies a color using cyan, magenta, yellow, and black.

`Hue[`*h*`, `*s*`, `*b*`]`
: specifies a color using hue, saturation, and brightness.

`GrayLevel[`*l*`]`
: specifies a color using a gray level.

All components range from 0 to 1. Each color function can be supplied with an additional argument specifying the desired opacity ("alpha") of the color. There are many predefined colors, such as `Black`, `White`, `Red`, `Green`, `Blue`, etc.

```
>> Graphics[{Red, Disk[]}]
```

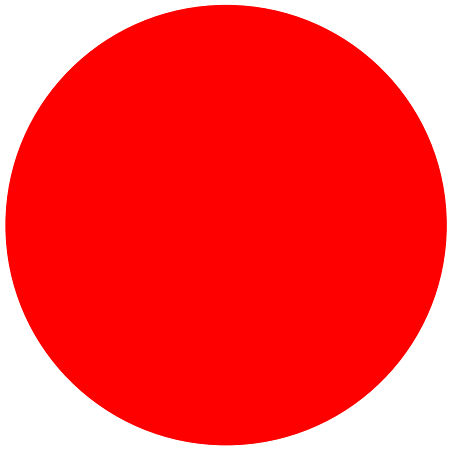
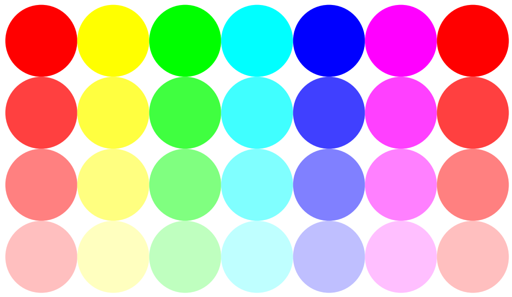


Table of hues:

```
>> Graphics[Table[{Hue[h, s], Disk
   [{12h, 8s}]}, {h, 0, 1, 1/6}, {s
   , 0, 1, 1/4}]]
```



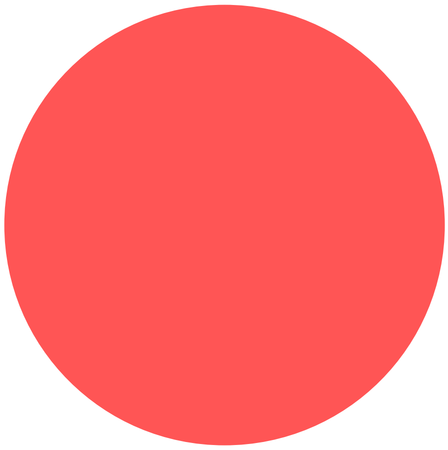
Colors can be mixed and altered using the following functions:

`Blend[{`*color1*`, `*color2*`}, `*ratio*`]`
: mixes *color1* and *color2* with *ratio*, where a ratio of 0 returns *color1* and a ratio of 1 returns *color2*.

`Lighter[`*color*`]`
: makes *color* lighter (mixes it with `White`).

`Darker[`*color*`]`
: makes *color* darker (mixes it with `Black`).

```
>> Graphics[{Lighter[Red], Disk[]}]
```



`Graphics` produces a `GraphicsBox`:

```
>> Head[ToBoxes[Graphics[{Circle
   []}]]]
```

GraphicsBox

3D Graphics

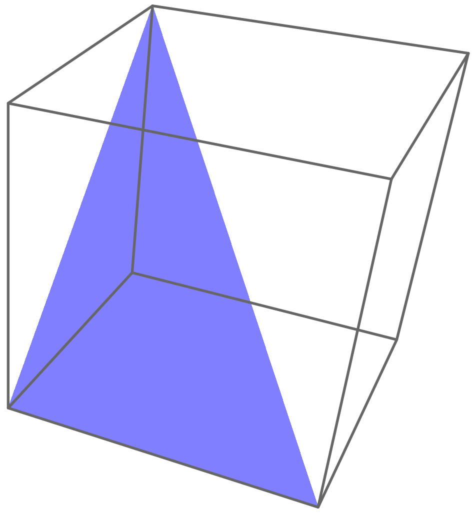
Three-dimensional graphics are created using the function `Graphics3D` and a list of 3D primitives. The following primitives are supported so far:

`Polygon[{{`*x1*`, `*y1*`, `*z1*`}, {`*x2*`, `*y2*`, `*z3*`}, ...}]`
draws a filled polygon.

`Line[{{`*x1*`, `*y1*`, `*z1*`}, {`*x2*`, `*y2*`, `*z3*`}, ...}]`
draws a line.

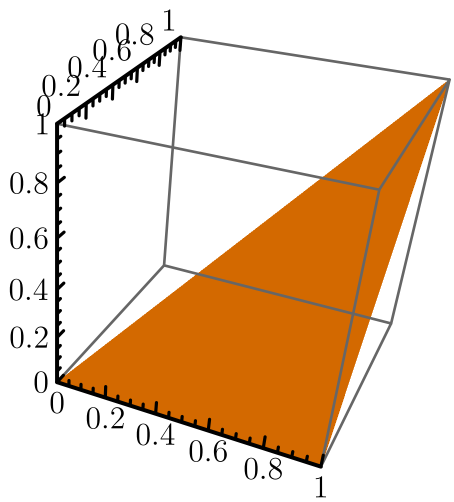
`Point[{`*x1*`, `*y1*`, `*z1*`}]`
draws a point.

```
>> Graphics3D[Polygon[{{0,0,0},
   {0,1,1}, {1,0,0}}]]
```



Colors can also be added to three-dimensional primitives.

```
>> Graphics3D[{Orange, Polygon
   [{{0,0,0}, {1,1,1}, {1,0,0}}]},
   Axes->True]
```



`Graphics3D` produces a `Graphics3DBox`:

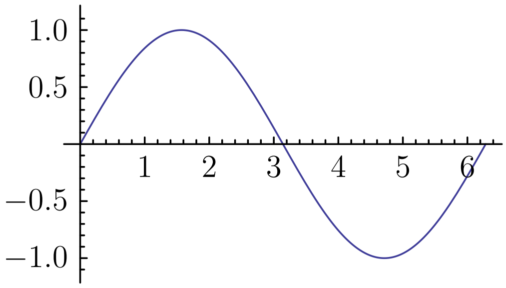
```
>> Head[ToBoxes[Graphics3D[{Polygon
   []}]]]
```

Graphics3DBox

Plotting Introduction Examples

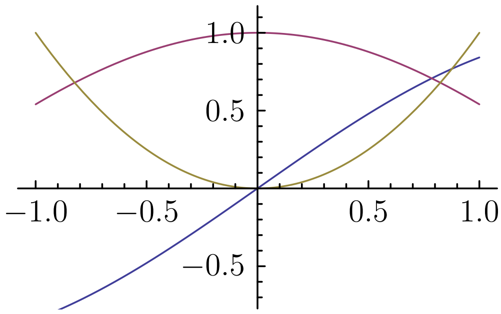
Mathics can plot functions:

```
>> Plot[Sin[x], {x, 0, 2 Pi}]
```



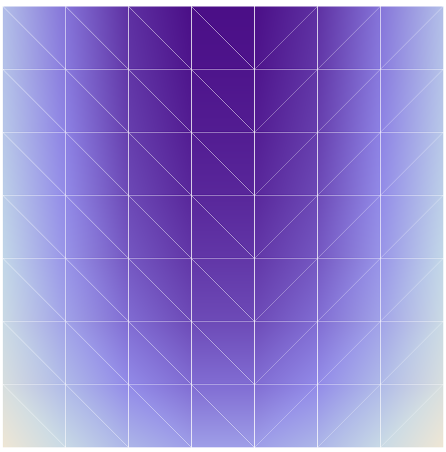
You can also plot multiple functions at once:

```
>> Plot[{Sin[x], Cos[x], x ^ 2}, {x
   , -1, 1}]
```



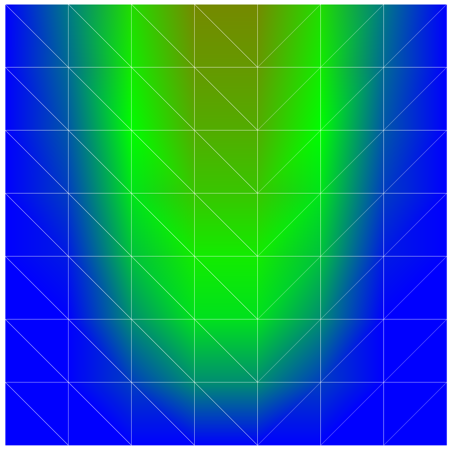
Two-dimensional functions can be plotted using `DensityPlot`:

```
>> DensityPlot[x ^ 2 + 1 / y, {x,
   -1, 1}, {y, 1, 4}]
```



You can use a custom coloring function:

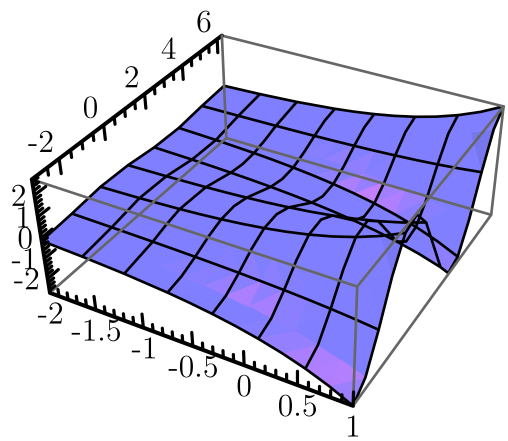
```
>> DensityPlot[x ^ 2 + 1 / y, {x,
   -1, 1}, {y, 1, 4}, ColorFunction
    -> (Blend[{Red, Green, Blue},
   #]&)]
```



One problem with `DensityPlot` is that it's still very slow, basically due to function evaluation being pretty slow in general—and `DensityPlot` has to evaluate a lot of functions.

Three-dimensional plots are supported as well:

```
>> Plot3D[Exp[x] Cos[y], {x, -2,
   1}, {y, -Pi, 2 Pi}]
```



3. Examples

Contents

Curve sketching

Let's sketch the function

```
>> f[x_] := 4 x / (x ^ 2 + 3 x + 5)
```

The derivatives are

```
>> {f'[x], f''[x], f'''[x]} //
   Together
```

$$\left\{\frac{-4\left(-5+x^2\right)}{\left(5+3x+x^2\right)^2}, \frac{8\left(-15-15x+x^3\right)}{\left(5+3x+x^2\right)^3}, \frac{-24\left(-20-60x-30x^2+x^4\right)}{\left(5+3x+x^2\right)^4}\right\}$$

To get the extreme values of `f`, compute the zeroes of the first derivatives:

```
>> extremes = Solve[f'[x] == 0, x]
```

$$\left\{\left\{x->-\sqrt{5}\right\},\left\{x->\sqrt{5}\right\}\right\}$$

And test the second derivative:

```
>> f''[x] /. extremes // N
```

$\{1.65086, -0.064079\}$

Thus, there is a local maximum at `x = Sqrt[5]` and a local minimum at `x = -Sqrt[5]`. Compute the inflection points numerically, chopping imaginary parts close to 0:

```
>> inflections = Solve[f''[x] == 0,
    x] // N // Chop
```

$\{\{x->-1.0852\},\{x->-3.21463\},\{x->4.29983\}\}$

Insert into the third derivative:

```
>> f'''[x] /. inflections
```

$\{-3.67683, 0.694905, 0.00671894\}$

Being different from 0, all three points are actual inflection points. `f` is not defined where its denominator is 0:

```
>> Solve[Denominator[f[x]] == 0, x]
```

$$\left\{\left\{x->-\frac{3}{2}-\frac{I}{2}\sqrt{11}\right\},\left\{x->-\frac{3}{2}+\frac{I}{2}\sqrt{11}\right\}\right\}$$

These are non-real numbers, consequently `f` is defined on all real numbers. The behaviour of `f` at the boundaries of its definition:

```
>> Limit[f[x], x -> Infinity]
```

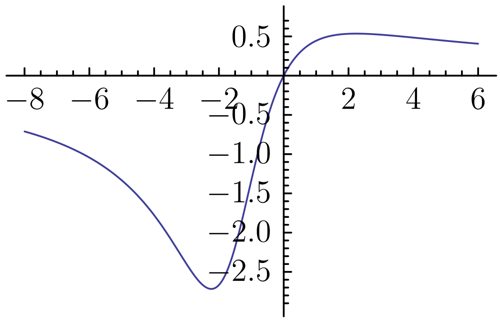
0

```
>> Limit[f[x], x -> -Infinity]
```

0

Finally, let's plot `f`:

```
>> Plot[f[x], {x, -8, 6}]
```



Linear algebra

Let's consider the matrix

```
>> A = {{1, 1, 0}, {1, 0, 1}, {0,
   1, 1}};
```

```
>> MatrixForm[A]
```

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

We can compute its eigenvalues and eigenvectors:

```
>> Eigenvalues[A]
```

$\{2, -1, 1\}$

```
>> Eigenvectors[A]
```

$\{\{1,1,1\}, \{1, -2, 1\}, \{-1, 0, 1\}\}$

This yields the diagonalization of `A`:

```
>> T = Transpose[Eigenvectors[A]];
   MatrixForm[T]
```

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & -2 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

```
>> Inverse[T] . A . T // MatrixForm
```

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
>> % == DiagonalMatrix[Eigenvalues[
   A]]
```

True

We can solve linear systems:

```
>> LinearSolve[A, {1, 2, 3}]
```

$\{0, 1, 2\}$

```
>> A . %
```

$\{1, 2, 3\}$

In this case, the solution is unique:

```
>> NullSpace[A]
```

$\{\}$

Let's consider a singular matrix:

```
>> B = {{1, 2, 3}, {4, 5, 6}, {7,
   8, 9}};
```

```
>> MatrixRank[B]
```

2

```
>> s = LinearSolve[B, {1, 2, 3}]
```

$\left\{-\frac{1}{3}, \frac{2}{3}, 0\right\}$

```
>> NullSpace[B]
```

$\{\{1, -2, 1\}\}$

```
>> B . (RandomInteger[100] * %[[1]]
    + s)
```

$\{1, 2, 3\}$

Dice

Let's play with dice in this example. A `Dice` object shall represent the outcome of a series of rolling a dice with six faces, e.g.:

```
>> Dice[1, 6, 4, 4]
```

Dice [1, 6, 4, 4]

Like in most games, the ordering of the individual throws does not matter. We can express this by making `Dice` `Orderless`:

```
>> SetAttributes[Dice, Orderless]
```

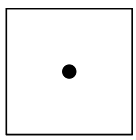
```
>> Dice[1, 6, 4, 4]
```

Dice [1, 4, 4, 6]

A dice object shall be displayed as a rectangle with the given number of points in it, positioned like on a traditional dice:

```
>> Format[Dice[n_Integer?(1 <= # <=
    6 &)]] := Block[{p = 0.2, r =
   0.05}, Graphics[{EdgeForm[Black
   ], White, Rectangle[], Black,
   EdgeForm[], If[OddQ[n], Disk
   [{0.5, 0.5}, r]], If[MemberQ[{2,
    3, 4, 5, 6}, n], Disk[{p, p}, r
   ]], If[MemberQ[{2, 3, 4, 5, 6},
   n], Disk[{1 - p, 1 - p}, r]], If
   [MemberQ[{4, 5, 6}, n], Disk[{p,
    1 - p}, r]], If[MemberQ[{4, 5,
   6}, n], Disk[{1 - p, p}, r]], If
   [n === 6, {Disk[{p, 0.5}, r],
   Disk[{1 - p, 0.5}, r]}]},
   ImageSize -> Tiny]]
```

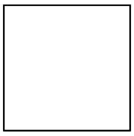
```
>> Dice[1]
```



The empty series of dice shall be displayed as an empty dice:

```
>> Format[Dice[]] := Graphics[{
   EdgeForm[Black], White,
   Rectangle[]}, ImageSize -> Tiny]
```

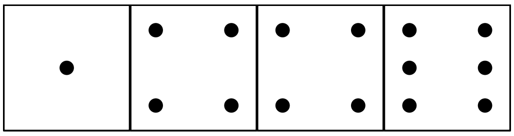
```
>> Dice[]
```



Any non-empty series of dice shall be displayed as a row of individual dice:

```
>> Format[Dice[d___Integer?(1 <= #
   <= 6 &)]] := Row[Dice /@ {d}]
```

```
>> Dice[1, 6, 4, 4]
```



Note that *Mathics* will automatically sort the given format rules according to their "generality", so the rule for the empty dice does not get overridden by the rule for a series of dice. We can still see the original form by using `InputForm`:

```
>> Dice[1, 6, 4, 4] // InputForm
```

Dice [1, 4, 4, 6]

We want to combine `Dice` objects using the + operator:

```
>> Dice[a___] + Dice[b___] ^:= Dice
   [Sequence @@ {a, b}]
```

The `^:=` (`UpSetDelayed`) tells *Mathics* to associate this rule with `Dice` instead of `Plus`, which is protected—we would have to unprotect it first:

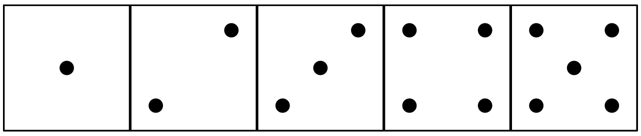
```
>> Dice[a___] + Dice[b___] := Dice[
   Sequence @@ {a, b}]
```

TagPlusinDice[a___]
+ Dice[b___]isProtected.

$Failed

We can now combine dice:

```
>> Dice[1, 5] + Dice[3, 2] + Dice
   [4]
```



Let's write a function that returns the sum of the rolled dice:

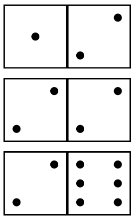
```
>> DiceSum[Dice[d___]] := Plus @@ {
   d}
```

```
>> DiceSum @ Dice[1, 2, 5]
```

8

And now let's put some dice into a table:

```
>> Table[{Dice[Sequence @@ d],
   DiceSum @ Dice[Sequence @@ d]},
   {d, {{1, 2}, {2, 2}, {2, 6}}}]
   // TableForm
```



3

4

8

It is not very sophisticated from a mathematical point of view, but it's beautiful.

4. Django-based Web Interface

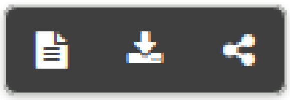
In the future, we plan on providing an interface to Jupyter as a separate package.

However currently as part *Mathics*, we distribute a browser-based interface using long-term-release (LTS) Django 3.2.

Since a Jupyter-based interface seems preferable to the home-grown interface described here, it is doubtful whether there will be future improvements to the this interface.

When you enter Mathics in the top after the Mathics logo and the word “Mathics” you’ll see a *menubar*.

It looks like this:



Contents

URIs

For the most part, the application is a single-page application. Assuming your are running locally or on a host called `localhost` using the default port, 8000, here are some URLs and what they do:

`http://localhost:8000`
: The single-page application; the main page.

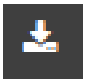
`http://localhost:8000/about`
: A page giving:

- the software versions of this package and version information of important software this uses.
- directory path information for the current setup
- machine information
- system information

`http://localhost:8000/doc`
: An on-line formatted version of the documentation, which include this text. You can see this as a right side frame of the main page, when clicking "?" on the right-hand upper corner.

Saving, Loading, and Deleting Worksheets

<subsection title="Saving Worksheets"> Worksheets exist in the browser window only and are not stored on the server, by default. To save all your queries and results, use the *Save* button which is the middle graphic of the menu bar. It looks like this:



Depending on browser, desktop, and OS-settings, the "Ctrl+S" key combination may do the same thing.

<subsection title="Loading and Deleting Worksheets">

Saved worksheets can be loaded or deleted using the *File Open* button which is the left-most button in the menu bar. It looks like this:

Depending on browser, desktop, and OS-settings, the "Ctrl+O" key combination may do the same thing.

A popup menu should appear with the list of saved worksheets with an option to either load or delete the worksheet.

Persistence of Mathics Definitions in a Session

When you use the Django-based Web interface of *Mathics*, a browser session is created. Cookies have to be enabled to allow this. Your session holds a key which is used to access your definitions that are stored in a database on the server. As long as you don't clear the cookies in your browser, your definitions will remain even when you close and re-open the browser.

This implies that you should not store sensitive, private information in *Mathics* variables when using the online Web interface. In addition to their values being stored in a database on the server, your queries might be saved for debugging purposes. However, the fact that they are transmitted over plain HTTP should make you aware that you should not transmit any sensitive information. When you want to do calculations with that kind of stuff, simply install *Mathics* locally!

If you are using a public terminal, to erase all your definitions and close the browser window. When you use *Mathics* in a browser, use the command `Quit[]` or its alias, `Exit[]`.

Normally, when you reload the current page in a browser using the default url, e.g `http:localhost:8000`, all of the previous input and output disappears, even though definitions as described above do not, unless `Quit[]` or `Exit[]` is entered as described above.

However if you want a URL that will that records the input entered the *Generate Input Hash* button does this. The button looks like this:

For example, assuming you have a *Mathics* server running at port 8000 on `localhost`, and you enter the url `http://localhost:8000/#cXVlcmllcz14`, you should see a single line of input containing `x` entered.

Of course, what the value of this is when evaluated depends on whether `x` has been previously defined.

Keyboard Commands

There are some keyboard commands you can use in the Django-based Web interface of *Mathics*.

`Shift+Return`
: This evaluates the current cell (the most important one, for sure). On the right-hand side you may also see an "=" button which can be clicked to do the same thing.

`Ctrl+D`
: This moves the cursor over to the documentation pane on the right-hand side. From here you can preform a search for a pre-defined *Mathics* function, or symbol. Clicking on the "?" symbol on the right-hand side does the same thing.

`Ctrl+C`
: This moves the cursor back to document code pane area where you type *Mathics* expressions

`Ctrl+S`
: Save worksheet

`Ctrl+O`
: Open worksheet

`Right Click` on MathML output
: Opens MathJax Menu

Of special note is the last item on the list: right-click to open the MathJax menu. Under "Math Setting"/"Zoom Trigger", if the zoom trigger is set to a value other then "No Zoom", then when that trigger is applied on MathML formatted output, the MathML formula pop up a window for the formula. The window can show the formula larger. Also, this is a way to see output that is too large to fit on the display since the window allows for scrolling.

Keyboard commands behavior depends the browser used, the operating system, desktop settings, and customization. We hook into the desktop "Open the current document" and "Save the current document" functions that many desktops provide. For example see: `https://help.ubuntu.com/community/KeyboardShortcuts#Finding_keyboard_shortcuts`

Often, these shortcut keyboard command are only recognized when a text field has focus; otherwise,the browser might do some browser-specific actions, like setting a bookmark etc.

Part II.

Reference of Built-in Symbols

1. Date and Time

Dates and times are represented symbolically; computations can be performed on them.
Date object can also input and output dates and times in a wide range of formats, as well as handle calendars.

Contents

AbsoluteTime

`AbsoluteTime[]`
: gives the local time in seconds since epoch January 1, 1900, in your time zone.

`AbsoluteTime[{`*y*`, `*m*`, `*d*`, `*h*`, `*m*`, `*s*`}]`
: gives the absolute time specification corresponding to a date list.

`AbsoluteTime["`*string*`"]`
: gives the absolute time specification for a given date string.

`AbsoluteTime[{"`*string*`",{`*e1*`, `*e2*`, ...}}]`
: takgs the date string to contain the elements "*ei*".

```
>> AbsoluteTime[]
```
3.83429×10^9

```
>> AbsoluteTime[{2000}]
```
3 155 673 600

```
>> AbsoluteTime[{"01/02/03", {"Day
", "Month", "YearShort"}}]
```
3 253 046 400

```
>> AbsoluteTime["6 June 1991"]
```
2 885 155 200

```
>> AbsoluteTime[{"6-6-91", {"Day",
"Month", "YearShort"}}]
```
2 885 155 200

AbsoluteTiming

`AbsoluteTiming[`*expr*`]`
: evaluates *expr*, returning a list of the absolute number of seconds in real time that have elapsed, together with the result obtained.

```
>> AbsoluteTiming[50!]
```
{0.000205278, 30 414 093~
~201 713 378 043 612 608 166~
~064 768 844 377 641 568~
~960 512 000 000 000 000}

```
>> Attributes[AbsoluteTiming]
```
{HoldAll, Protected}

DateDifference

`DateDifference[`*date1*`, `*date2*`]`
: returns the difference between *date1* and *date2* in days.

`DateDifference[`*date1*`, `*date2*`, `*unit*`]`
: returns the difference in the specified *unit*.

`DateDifference[`*date1*`, `*date2*`, {`*unit1*`, `*unit2*`, ...}]`
: represents the difference as a list of integer multiples of each *unit*, with any remainder expressed in the smallest unit.

```
>> DateDifference[{2042, 1, 4}, {2057, 1, 1}]
```
5 476

```
>> DateDifference[{1936, 8, 14}, {2000, 12, 1}, "Year"]
```
{64.3425, Year}

```
>> DateDifference[{2010, 6, 1}, {2015, 1, 1}, "Hour"]
```
{40 200, Hour}

```
>> DateDifference[{2003, 8, 11}, {2003, 10, 19}, {"Week", "Day"}]
```
{{9, Week} , {6, Day}}

DateList

`DateList[]`
: returns the current local time in the form {*year*, *month*, *day*, *hour*, *minute*, *second*}.

`DateList[`*time*`]`
: returns a formatted date for the number of seconds *time* since epoch Jan 1 1900.

`DateList[{`*y*`, `*m*`, `*d*`, `*h*`, `*m*`, `*s*`}]`
: converts an incomplete date list to the standard representation.

`DateString[`*string*`]`
: returns the formatted date list of a date string specification.

`DateString[`*string*`, {`*e1*`, `*e2*`, ...}]`
: returns the formatted date list of a *string* obtained from elements *ei*.

```
>> DateList[0]
```
{1 900, 1, 1, 0, 0, 0.}

```
>> DateList[3155673600]
```
{2 000, 1, 1, 0, 0, 0.}

```
>> DateList[{2003, 5, 0.5, 0.1, 0.767}]
```
{2 003, 4, 30, 12, 6, 46.02}

```
>> DateList[{2012, 1, 300., 10}]
```
{2 012, 10, 26, 10, 0, 0.}

```
>> DateList["31/10/1991"]
```
{1 991, 10, 31, 0, 0, 0.}

```
>> DateList["1/10/1991"]
```
Theinterpretationof1/10/1991isambiguous.

{1 991, 1, 10, 0, 0, 0.}

```
>> DateList[{"31/10/91", {"Day", "Month", "YearShort"}}]
```
{1 991, 10, 31, 0, 0, 0.}

```
>> DateList[{"31 10/91", {"Day", " ", "Month", "/", "YearShort"}}]
```
{1 991, 10, 31, 0, 0, 0.}

If not specified, the current year assumed

```
>> DateList[{"5/18", {"Month", "Day"}}]
```
{2 021, 5, 18, 0, 0, 0.}

DateObject

`DateObject[...]`
: Returns an object codifiyng DateList....

```
>> DateObject[{2020, 4, 15}]
```
[Wed 15 Apr 2 020 00:00:00 GTM − 5]

DatePlus

`DatePlus[`*date*`, `*n*`]`
: finds the date *n* days after *date*.

`DatePlus[`*date*`, {`*n*`, "`*unit*`"}]`
: finds the date *n* units after *date*.

`DatePlus[`*date*`, {{`*n1*`, "`*unit1*`"}, {`*n2*`, "`*unit2*`"}, ...}]`
: finds the date which is *n_i* specified units after *date*.

`DatePlus[`*n*`]`
: finds the date *n* days after the current date.

`DatePlus[`*offset*`]`
: finds the date which is offset from the current date.

Add 73 days to Feb 5, 2010:

```
>> DatePlus[{2010, 2, 5}, 73]
```

{2 010, 4, 19}

Add 8 weeks and 1 day to March 16, 1999:

```
>> DatePlus[{2010, 2, 5}, {{8, "Week"}, {1, "Day"}}]
```

{2 010, 4, 3}

DateString

`DateString[]`
: returns the current local time and date as a string.

`DateString[`*elem*`]`
: returns the time formatted according to *elems*.

`DateString[{`*e1*`, `*e2*`, ...}]`
: concatinates the time formatted according to elements *ei*.

`DateString[`*time*`]`
: returns the date string of an AbsoluteTime.

`DateString[{`*y*`, `*m*`, `*d*`, `*h*`, `*m*`, `*s*`}]`
: returns the date string of a date list specification.

`DateString[`*string*`]`
: returns the formatted date string of a date string specification.

`DateString[`*spec*`, `*elems*`]`
: formats the time in turns of *elems*. Both *spec* and *elems* can take any of the above formats.

The current date and time:

```
>> DateString[];
```

```
>> DateString[{1991, 10, 31, 0, 0}, {"Day", " ", "MonthName", " ", "Year"}]
```

31 October 1 991

```
>> DateString[{2007, 4, 15, 0}]
```

Sun 15 Apr 2 007 00:00:00

```
>> DateString[{1979, 3, 14}, {"DayName", " ", "Month", "-", "YearShort"}]
```

Wednesday 03-79

Non-integer values are accepted too:

```
>> DateString[{1991, 6, 6.5}]
```

Thu 6 Jun 1 991 12:00:00

$DateStringFormat

`$DateStringFormat`
: gives the format used for dates generated by `DateString`.

```
>> $DateStringFormat
```

{DateTimeShort}

EasterSunday

`EasterSunday[`*year*`]`
: returns the date of the Gregorian Easter Sunday as {year, month, day}.

```
>> EasterSunday[2000]
```

{2 000, 4, 23}

```
>> EasterSunday[2030]
```

{2 030, 4, 21}

Now

`Now`
: gives the current time on the system.

```
>> Now
```
[Sat 3 Jul 2 021 09:35:47 GTM − 5]

Pause

`Pause[n]`
pauses for n seconds.

```
>> Pause[0.5]
```

SessionTime

`SessionTime[]`
returns the total time in seconds since this session started.

```
>> SessionTime[]
```
36.9798

$SystemTimeZone

`$SystemTimeZone`
gives the current time zone for the computer system on which Mathics is being run.

```
>> $SystemTimeZone
```
−5.

TimeConstrained

`TimeConstrained[expr, t]`
evaluates *expr*, stopping after t seconds.

`TimeConstrained[expr, t, failexpr]`
returns *failexpr* if the time constraint is not met.

```
>> TimeConstrained[Integrate[Sin[x
   ]^1000000,x],1]
```
$Aborted

```
>> TimeConstrained[Integrate[Sin[x
   ]^1000000,x], 1, Integrate[Cos[x
   ],x]]
```
$\text{Sin}\left[x\right]$

```
>> s=TimeConstrained[Integrate[Sin[
   x] ^ 3, x], a]
```
Numberofsecondsaisnotapositivemachine − sizednumberorInfinity.

$\text{TimeConstrained}\left[\int \text{Sin}\left[x\right]^3 dx, a\right]$

```
>> a=1; s
```
$\frac{\text{Cos}\left[x\right]\left(-5+\text{Cos}\left[2x\right]\right)}{6}$

Possible issues: for certain time-consuming functions (like simplify) which are based on sympy or other libraries, it is possible that the evaluation continues after the timeout. However, at the end of the evaluation, the function will return $
Aborted and the results will not affect the state of the mathics kernel.

TimeRemaining

`TimeRemaining[]`
Gives the number of seconds remaining until the earliest enclosing `TimeConstrained` will request the current computation to stop.

`TimeConstrained[expr, t, failexpr]`
returns *failexpr* if the time constraint is not met.

If TimeConstrained is called out of a TimeConstrained expression, returns ‘Infinity‘

```
>> TimeRemaining[]
```
∞

```
>> TimeConstrained[1+2; Print[
   TimeRemaining[]], 0.9]
```
0.89935

TimeUsed

`TimeUsed[]`
returns the total CPU time used for this session, in seconds.

>> `TimeUsed[]`

96.4546

$TimeZone

`$TimeZone`
: gives the current time zone to assume for dates and times.

>> `$TimeZone`

−5.

Timing

`Timing[`*expr*`]`
: measures the processor time taken to evaluate *expr*. It returns a list containing the measured time in seconds and the result of the evaluation.

>> `Timing[50!]`

{0.00020343, 30 414 093 201 713 378 ˜
˜043 612 608 166 064 768 844 377 ˜
˜641 568 960 512 000 000 000 000}

>> `Attributes[Timing]`

{HoldAll, Protected}

2. Input and Output

Contents

BaseForm

`BaseForm[`*expr*`, `*n*`]`
prints numbers in *expr* in base *n*.

\>> `BaseForm[33, 2]`
$100\,001_2$

\>> `BaseForm[234, 16]`
ea_{16}

\>> `BaseForm[12.3, 2]`
$1\,100.010011001100110011_2$

\>> `BaseForm[-42, 16]`
$-2a_{16}$

\>> `BaseForm[x, 2]`
x

\>> `BaseForm[12, 3] // FullForm`
BaseForm [12, 3]

Bases must be between 2 and 36:

\>> `BaseForm[12, -3]`
Positivemachine −sizedintegerexpectedatposition2inBaseForm[12, − 3].
MakeBoxes[*BaseForm*[12, − 3], *StandardForm*]*isnotavalidboxstructure.*

\>> `BaseForm[12, 100]`
*Requestedbase*100*mustbebetween2and*36.
MakeBoxes[*BaseForm*[12, 100], *StandardForm*]*isnotavalidboxstructure.*

BoxData

`BoxData[...]`
is a low-level representation of the contents of a typesetting cell.

Center

`Center`
: is used with the `ColumnAlignments` option to `Grid` or `TableForm` to specify a centered column.

Check

`Check`[*expr*, *failexpr*]
: evaluates *expr*, and returns the result, unless messages were generated, in which case it evaluates and *failexpr* will be returned.

`Check`[*expr*, *failexpr*, `{s1::t1,s2::t2,...}`]
: checks only for the specified messages.

Return err when a message is generated:

```
>> Check[1/0, err]
```

Infiniteexpression1/0encountered.

err

Check only for specific messages:

```
>> Check[Sin[0^0], err, Sin::argx]
```

Indeterminateexpression0^0*encountered.*

Indeterminate

```
>> Check[1/0, err, Power::infy]
```

Infiniteexpression1/0encountered.

err

Format

`Format`[*expr*]
: holds values specifying how *expr* should be printed.

Assign values to `Format` to control how particular expressions should be formatted when printed to the user.

```
>> Format[f[x___]] := Infix[{x}, "~"]
```

```
>> f[1, 2, 3]
```

$1 \sim 2 \sim 3$

```
>> f[1]
```

1

Raw objects cannot be formatted:

```
>> Format[3] = "three";
```

Cannotassigntorawobject3.

Format types must be symbols:

```
>> Format[r, a + b] = "r";
```

Formattypea + bisnotasymbol.

Formats must be attached to the head of an expression:

```
>> f /: Format[g[f]] = "my f";
```

Tagfnotfoundortoodeepforanassignedrule.

FullForm

`FullForm`[*expr*]
: displays the underlying form of *expr*.

```
>> FullForm[a + b * c]
```

Plus [*a*, Times [*b*, *c*]]

```
>> FullForm[2/3]
```

Rational [2, 3]

```
>> FullForm["A string"]
```

"A string"

General

`General`
: is a symbol to which all general-purpose messages are assigned.

```
>> General::argr
```

'1' called with 1 argument;
'2' arguments are expected.

```
>> Message[Rule::argr, Rule, 2]
```

Rulecalledwith1argument;2argumentsareexpected.

Grid

`Grid[{{`*a1*, *a2*, `...}, {`*b1*, *b2*, `...}, ...}]`
: formats several expressions inside a `GridBox`.

```
>> Grid[{{a, b}, {c, d}}]
```

a	b
c	d

Infix

`Infix[`*expr*`, `*oper*`, `*prec*`, `*assoc*`]`
: displays *expr* with the infix operator *oper*, with precedence *prec* and associativity *assoc*.

`Infix` can be used with `Format` to display certain forms with user-defined infix notation:

```
>> Format[g[x_, y_]] := Infix[{x, y
   }, "#", 350, Left]
```

```
>> g[a, g[b, c]]
```

$a\#(b\#c)$

```
>> g[g[a, b], c]
```

$a\#b\#c$

```
>> g[a + b, c]
```

$(a+b)\#c$

```
>> g[a * b, c]
```

$ab\#c$

```
>> g[a, b] + c
```

$c + a\#b$

```
>> g[a, b] * c
```

$c\,(a\#b)$

```
>> Infix[{a, b, c}, {"+", "-"}]
```

$a + b - c$

InputForm

`InputForm[`*expr*`]`
: displays *expr* in an unambiguous form suitable for input.

```
>> InputForm[a + b * c]
```

$a + b * c$

```
>> InputForm["A string"]
```

"A string"

```
>> InputForm[f'[x]]
```

Derivative $[1]\left[f\right][x]$

```
>> InputForm[Derivative[1, 0][f][x
   ]]
```

Derivative $[1,0]\left[f\right][x]$

Left

`Left`
: is used with operator formatting constructs to specify a left-associative operator.

MakeBoxes

`MakeBoxes[`*expr*`]`
: is a low-level formatting primitive that converts *expr* to box form, without evaluating it.

`\( ... \)`
: directly inputs box objects.

String representation of boxes

```
>> \(x \^ 2\)
```

SuperscriptBox [x, 2]

```
>> \(x \_ 2\)
```

SubscriptBox [x, 2]

```
>> \( a \+ b \% c\)
```

UnderoverscriptBox [a, b, c]

```
>> \( a \& b \% c\)
```

UnderoverscriptBox [a, c, b]

```
>> \(x \& y \)
```

OverscriptBox [x, y]

```
>> \(x \+ y \)
```

UnderscriptBox [x, y]

MathMLForm

`MathMLForm[`*expr*`]`
: displays *expr* as a MathML expression.

>> `MathMLForm[HoldForm[Sqrt[a^3]]]`

<math display="block">
<msqrt><msup><mi>a</mi>
<mn>3</mn></msup>
</msqrt></math>

>> `MathMLForm[\[Mu]]`

<math display="block">
<mi></mi></math>

This can causes the TeX to fail # » MathMLForm[Graphics[Text["μ"]]] # = ...
= ...

MatrixForm

`MatrixForm[`*m*`]`
displays a matrix *m*, hiding the underlying list structure.

>> `Array[a,{4,3}]//MatrixForm`

$$\begin{pmatrix} a[1,1] & a[1,2] & a[1,3] \\ a[2,1] & a[2,2] & a[2,3] \\ a[3,1] & a[3,2] & a[3,3] \\ a[4,1] & a[4,2] & a[4,3] \end{pmatrix}$$

Message

`Message[`*symbol*`::`*msg*`, `*expr1*`, `*expr2*`, ...]`
displays the specified message, replacing placeholders in the message text with the corresponding expressions.

>> `a::b = "Hello world!"`

Hello world!

>> `Message[a::b]`

Helloworld!

>> `a::c := "Hello ‘1‘, Mr 00‘2‘!"`

>> `Message[a::c, "you", 3 + 4]`

Helloyou, *Mr*007!

MessageName (::)

`MessageName[`*symbol*`, `*tag*`]`
symbol`::`*tag*
identifies a message.

`MessageName` is the head of message IDs of the form `symbol::tag`.

>> `FullForm[a::b]`

MessageName [*a*, "b"]

The second parameter `tag` is interpreted as a string.

>> `FullForm[a::"b"]`

MessageName [*a*, "b"]

NonAssociative

`NonAssociative`
is used with operator formatting constructs to specify a non-associative operator.

NumberForm

`NumberForm[`*expr*`, `*n*`]`
prints a real number *expr* with *n*-digits of precision.
`NumberForm[`*expr*`, {`*n*`, `*f*`}]`
prints with *n*-digits and *f* digits to the right of the decimal point.

>> `NumberForm[N[Pi], 10]`

3.141592654

>> `NumberForm[N[Pi], {10, 5}]`

3.14159

Off

`Off[`*symbol*`::`*tag*`]`
turns a message off so it is no longer printed.

>> `Off[Power::infy]`

`>>` `1 / 0`

ComplexInfinity

`>>` `Off[Power::indet, Syntax::com]`

`>>` `{0 ^ 0,}`

{Indeterminate, Null}

On

`On[`*symbol*`::`*tag*`]`
turns a message on for printing.

`>>` `Off[Power::infy]`

`>>` `1 / 0`

ComplexInfinity

`>>` `On[Power::infy]`

`>>` `1 / 0`

Infiniteexpression1/0encountered.

ComplexInfinity

OutputForm

`OutputForm[`*expr*`]`
displays *expr* in a plain-text form.

`>>` `OutputForm[f'[x]]`

$f'[x]$

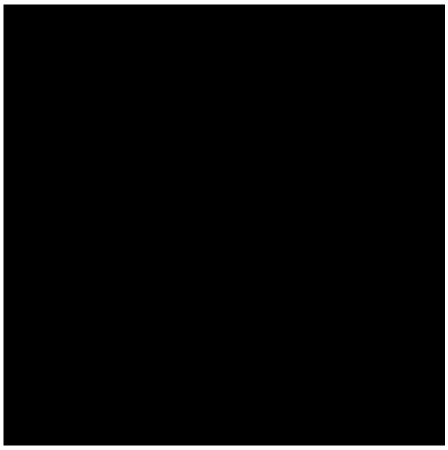
`>>` `OutputForm[Derivative[1, 0][f][x]]`

Derivative $[1,0]$ $[f]$ $[x]$

`>>` `OutputForm["A string"]`

A string

`>>` `OutputForm[Graphics[Rectangle[]]]`



Postfix (//)

x `//` *f*
is equivalent to $f[x]$.

`>>` `b // a`

$a[b]$

`>>` `c // b // a`

$a[b[c]]$

The postfix operator // is parsed to an expression before evaluation:

`>>` `Hold[x // a // b // c // d // e // f]`

Hold $[f[e[d[c[b[a[x]]]]]]]$

Precedence

`Precedence[`*op*`]`
returns the precedence of the built-in operator *op*.

`>>` `Precedence[Plus]`

310.

`>>` `Precedence[Plus] < Precedence[Times]`

True

Unknown symbols have precedence 670:

```
>> Precedence[f]
```
670.

Other expressions have precedence 1000:

```
>> Precedence[a + b]
```
1 000.

Prefix (@)

`f @ x`
: is equivalent to $f[x]$.

```
>> a @ b
```
$a[b]$

```
>> a @ b @ c
```
$a[b[c]]$

```
>> Format[p[x_]] := Prefix[{x}, "*"]
```

```
>> p[3]
```
$*3$

```
>> Format[q[x_]] := Prefix[{x}, "~ ", 350]
```

```
>> q[a+b]
```
$\sim (a + b)$

```
>> q[a*b]
```
$\sim ab$

```
>> q[a]+b
```
$b+ \sim a$

The prefix operator `@` is parsed to an expression before evaluation:

```
>> Hold[a @ b @ c @ d @ e @ f @ x]
```
Hold $[a[b[c[d[e[f[x]]]]]]]$

Print

`Print[expr, ...]`
: prints each *expr* in string form.

```
>> Print["Hello world!"]
```
Helloworld!

```
>> Print["The answer is ", 7 * 6, "."]
```
Theansweris42.

PythonForm

`PythonForm[expr]`
: returns an approximate equivalent of *expr* in Python, when that is possible. We assume that Python has sympy imported. No explicit import will be include in the result.

```
>> PythonForm[Infinity]
```
math.inf

```
>> PythonForm[Pi]
```
sympy.pi

```
>> E // PythonForm
```
sympy.E

```
>> {1, 2, 3} // PythonForm
```
[1, 2, 3]

Quiet

`Quiet[expr, {s1::t1, ...}]`
: evaluates *expr*, without messages `{s1::t1, ...}` being displayed.

`Quiet[expr, All]`
: evaluates *expr*, without any messages being displayed.

`Quiet[expr, None]`
: evaluates *expr*, without all messages being displayed.

`Quiet[expr, off, on]`
: evaluates *expr*, with messages *off* being suppressed, but messages *on* being displayed.

Evaluate without generating messages:

```
>> Quiet[1/0]
```
ComplexInfinity

Same as above:

```
>> Quiet[1/0, All]
```
ComplexInfinity

```
>> a::b = "Hello";
```

```
>> Quiet[x+x, {a::b}]
```

$2x$

```
>> Quiet[Message[a::b]; x+x, {a::b
   }]
```

$2x$

```
>> Message[a::b]; y=Quiet[Message[a
   ::b]; x+x, {a::b}]; Message[a::b
   ]; y
```

Hello

Hello

$2x$

```
>> Quiet[x + x, {a::b}, {a::b}]
```

InQuiet[x + x, {a :: b}, {a :: b}]themessagename(s){a :: b}appearinboththelistofmessagestoswitchoffandthelistofmessagestoswitchon.

Quiet $[x + x, \{a::b\}, \{a::b\}]$

Right

`Right`
: is used with operator formatting constructs to specify a right-associative operator.

Row

`Row[{expr, ...}]`
: formats several expressions inside a `RowBox`.

StandardForm

`StandardForm[expr]`
: displays *expr* in the default form.

```
>> StandardForm[a + b * c]
```

$a + bc$

```
>> StandardForm["A string"]
```

A string

`StandardForm` is used by default:

```
>> "A string"
```

A string

```
>> f'[x]
```

$f'[x]$

StringForm

`StringForm[str, expr1, expr2, ...]`
: displays the string *str*, replacing placeholders in *str* with the corresponding expressions.

```
>> StringForm["`1` bla `2` blub ``
   bla `2`", a, b, c]
```

a bla b blub c bla b

Subscript

`Subscript[a, i]`
: displays as *a_i*.

```
>> Subscript[x,1,2,3] // TeXForm
```

x_{1,2,3}

Subsuperscript

`Subsuperscript[a, b, c]`
: displays as a_b^c.

```
>> Subsuperscript[a, b, c] //
   TeXForm
```

a_b^c

Superscript

`Superscript[x, y]`
: displays as *x^y*.

```
>> Superscript[x,3] // TeXForm
```

x^3

SympyForm

`SympyForm[`*expr*`]`
: returns an Sympy *expr* in Python. Sympy is used internally to implement a number of Mathics functions, like Simplify.

```
>> SympyForm[Pi^2]
```
pi**2

```
>> E^2 + 3E // SympyForm
```
exp(2) + 3*E

Syntax

`Syntax`
: is a symbol to which all syntax messages are assigned.

```
>> 1 +
```

```
>> Sin[1)
```

```
>> ^ 2
```

```
>> 1.5``
```

TableForm

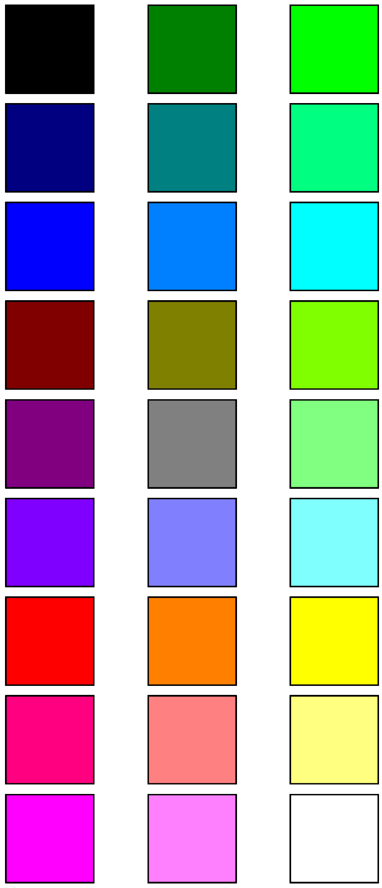
`TableForm[`*expr*`]`
: displays *expr* as a table.

```
>> TableForm[Array[a, {3,2}],
TableDepth->1]
```

$\{a[1,1], a[1,2]\}$
$\{a[2,1], a[2,2]\}$
$\{a[3,1], a[3,2]\}$

A table of Graphics:

```
>> Table[Style[Graphics[{EdgeForm[{
Black}], RGBColor[r,g,b],
Rectangle[]}],
ImageSizeMultipliers->{0.2, 1}],
{r,0,1,1/2}, {g,0,1,1/2}, {b
,0,1,1/2}] // TableForm
```



TeXForm

`TeXForm[`*expr*`]`
: displays *expr* using TeX math mode commands.

```
>> TeXForm[HoldForm[Sqrt[a^3]]]
```
\sqrt{a^3}

TextData

`TextData[...]`
: is a low-level representation of the contents of a textual cell.

ToBoxes

`ToBoxes[`*expr*`]`
: evaluates *expr* and converts the result to box form.

Unlike `MakeBoxes`, `ToBoxes` evaluates its argument:

```
>> ToBoxes[a + a]
```
RowBox [{2, , a}]

```
>> ToBoxes[a + b]
```
RowBox [{a, +, b}]

```
>> ToBoxes[a ^ b] // FullForm
```
SuperscriptBox ["a", "b"]

3. Procedural Programming

Procedural programming is a programming paradigm, derived from imperative programming, based on the concept of the procedure call. This term is sometimes compared and contrasted with Functional Programming.

Procedures (a type of routine or subroutine) simply contain a series of computational steps to be carried out. Any given procedure might be called at any point during a program's execution, including by other procedures or itself.

Procedural functions are integrated into Mathics symbolic programming environment.

Contents

Abort

`Abort[]`
: aborts an evaluation completely and returns `$Aborted`.

```
>> Print["a"]; Abort[]; Print["b"]
   a
   $Aborted
```

Break

`Break[]`
: exits a `For`, `While`, or `Do` loop.

```
>> n = 0;
```

```
>> While[True, If[n>10, Break[]]; n
   =n+1]
```

```
>> n
   11
```

Catch

`Catch[`*expr*`]`
: returns the argument of the first `Throw` generated in the evaluation of *expr*.

`Catch[`*expr*`, `*form*`]`
: returns value from the first `Throw[`*value*`, `*tag*`]` for which *form* matches *tag*.

`Catch[`*expr*`, `*form*`, `*f*`]`
: returns *f*[*value*, *tag*].

Exit to the enclosing `Catch` as soon as `Throw` is evaluated:

```
>> Catch[r; s; Throw[t]; u; v]
   t
```

Define a function that can "throw an exception":

```
>> f[x_] := If[x > 12, Throw[
   overflow], x!]
```

The result of `Catch` is just what is thrown by `Throw`:

```
>> Catch[f[1] + f[15]]
   overflow
```

```
>> Catch[f[1] + f[4]]
   25
```

CompoundExpression (;)

`CompoundExpression[`*e1*`, `*e2*`, ...]`
e1`; `*e2*`; ...`
evaluates its arguments in turn, returning the last result.

```
>> a; b; c; d
```
d

If the last argument is omitted, `Null` is taken:

```
>> a;
```

Continue

`Continue[]`
continues with the next iteration in a `For`, `While`, or `Do` loop.

```
>> For[i=1, i<=8, i=i+1, If[Mod[i
   ,2] == 0, Continue[]]; Print[i]]
```
1
3
5
7

Do

`Do[`*expr*`, {`*max*`}]`
evaluates *expr* *max* times.
`Do[`*expr*`, {`*i*`, `*max*`}]`
evaluates *expr* *max* times, substituting *i* in *expr* with values from 1 to *max*.
`Do[`*expr*`, {`*i*`, `*min*`, `*max*`}]`
starts with *i* `=` *max*`.`
`Do[`*expr*`, {`*i*`, `*min*`, `*max*`, `*step*`}]`
uses a step size of *step*.
`Do[`*expr*`, {`*i*`, {`*i1*`, `*i2*`, ...}}]`
uses values *i1*, *i2*, ... for *i*.
`Do[`*expr*`, {`*i*`, `*imin*`, `*imax*`}, {`*j*`, `*jmin*`, `*jmax*`}, ...]`
evaluates *expr* for each *j* from *jmin* to *jmax*, for each *i* from *imin* to *imax*, etc.

```
>> Do[Print[i], {i, 2, 4}]
```
2
3
4

```
>> Do[Print[{i, j}], {i,1,2}, {j
   ,3,5}]
```
{1,3}
{1,4}
{1,5}
{2,3}
{2,4}
{2,5}

You can use `Break[]` and `Continue[]` inside `Do`:

```
>> Do[If[i > 10, Break[], If[Mod[i,
    2] == 0, Continue[]]; Print[i
   ]], {i, 5, 20}]
```
5
7
9

FixedPoint

`FixedPoint[`*f*`, `*expr*`]`
starting with *expr*, iteratively applies *f* until the result no longer changes.
`FixedPoint[`*f*`, `*expr*`, `*n*`]`
performs at most *n* iterations. The same that using $MaxIterations->n$

```
>> FixedPoint[Cos, 1.0]
```
0.739085

```
>> FixedPoint[#+1 &, 1, 20]
```
21

FixedPointList

`FixedPointList[`*f*`, `*expr*`]`
starting with *expr*, iteratively applies *f* until the result no longer changes, and returns a list of all intermediate results.
`FixedPointList[`*f*`, `*expr*`, `*n*`]`
performs at most *n* iterations.

```
>> FixedPointList[Cos, 1.0, 4]
```
{1., 0.540302, 0.857~
~553, 0.65429, 0.79348}

Observe the convergence of Newton's method for approximating square roots:

```
>> newton[n_] := FixedPointList
   [.5(# + n/#)&, 1.];
```

```
>> newton[9]
```
$\{1., 5., 3.4, 3.02353, 3.00009, 3., 3., 3.\}$

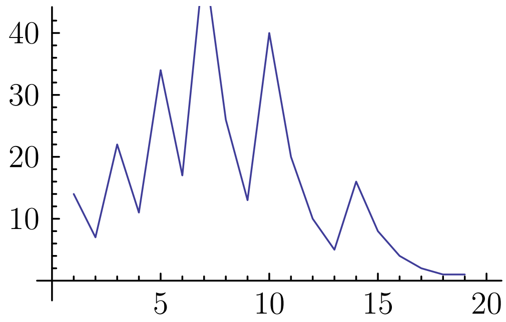
Plot the “hailstone” sequence of a number:

```
>> collatz[1] := 1;
```

```
>> collatz[x_ ? EvenQ] := x / 2;
```

```
>> collatz[x_] := 3 x + 1;
```

```
>> list = FixedPointList[collatz,
   14]
```
$\{14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 1\}$

```
>> ListLinePlot[list]
```


For

`For[start, test, incr, body]`
: evaluates *start*, and then iteratively *body* and *incr* as long as *test* evaluates to `True`.

`For[start, test, incr]`
: evaluates only *incr* and no *body*.

`For[start, test]`
: runs the loop without any body.

Compute the factorial of 10 using `For`:

```
>> n := 1
```

```
>> For[i=1, i<=10, i=i+1, n = n * i
   ]
```

```
>> n
```
3 628 800

```
>> n == 10!
```
True

If

`If[cond, pos, neg]`
: returns *pos* if *cond* evaluates to `True`, and *neg* if it evaluates to `False`.

`If[cond, pos, neg, other]`
: returns *other* if *cond* evaluates to neither `True` nor `False`.

`If[cond, pos]`
: returns `Null` if *cond* evaluates to `False`.

```
>> If[1<2, a, b]
```
a

If the second branch is not specified, `Null` is taken:

```
>> If[1<2, a]
```
a

```
>> If[False, a] //FullForm
```
Null

You might use comments (inside `(*` and `*)`) to make the branches of `If` more readable:

```
>> If[a, (*then*)b, (*else*)c];
```

Interrupt

`Interrupt[]`
: Interrupt an evaluation and returns `$Aborted`.

```
>> Print["a"]; Interrupt[]; Print["
   b"]
```
a

$Aborted

Nest

`Nest[f, expr, n]`
: starting with *expr*, iteratively applies *f* *n* times and returns the final result.

```
>> Nest[f, x, 3]
```
$f[f[f[x]]]$

```
>> Nest[(1+#)^ 2 &, x, 2]
```
$\left(1+(1+x)^2\right)^2$

NestList

`NestList[`*f*`, `*expr*`, `*n*`]`
: starting with *expr*, iteratively applies *f* *n* times and returns a list of all intermediate results.

```
>> NestList[f, x, 3]
```
$\{x, f[x], f[f[x]], f[f[f[x]]]\}$

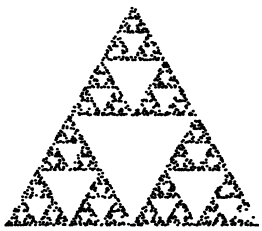
```
>> NestList[2 # &, 1, 8]
```
$\{1, 2, 4, 8, 16, 32, 64, 128, 256\}$

Chaos game rendition of the Sierpinski triangle:

```
>> vertices = {{0,0}, {1,0}, {.5, .5 Sqrt[3]}};
```

```
>> points = NestList[.5(vertices[[RandomInteger[{1,3}] ]] + #)&, {0.,0.}, 2000];
```

```
>> Graphics[Point[points], ImageSize->Small]
```



NestWhile

`NestWhile[`*f*`, `*expr*`, `*test*`]`
: applies a function *f* repeatedly on an expression *expr*, until applying *test* on the result no longer yields `True`.

`NestWhile[`*f*`, `*expr*`, `*test*`, `*m*`]`
: supplies the last *m* results to *test* (default value: 1).

`NestWhile[`*f*`, `*expr*`, `*test*`, All]`
: supplies all results gained so far to *test*.

Divide by 2 until the result is no longer an integer:

```
>> NestWhile[#/2&, 10000, IntegerQ]
```
$\frac{625}{2}$

Return

`Return[`*expr*`]`
: aborts a function call and returns *expr*.

```
>> f[x_] := (If[x < 0, Return[0]]; x)
```

```
>> f[-1]
```
0

```
>> Do[If[i > 3, Return[]]; Print[i], {i, 10}]
```
1
2
3

`Return` only exits from the innermost control flow construct.

```
>> g[x_] := (Do[If[x < 0, Return[0]], {i, {2, 1, 0, -1}}]; x)
```

```
>> g[-1]
```
-1

Switch

`Switch[`*expr*`, `*pattern1*`, `*value1*`, `*pattern2*`, `*value2*`, ...]`
: yields the first *value* for which *expr* matches the corresponding *pattern*.

```
>> Switch[2, 1, x, 2, y, 3, z]
```
y

```
>> Switch[5, 1, x, 2, y]
```
$\text{Switch}[5, 1, x, 2, y]$

```
>> Switch[5, 1, x, 2, a, _, b]
```
b

```
>> Switch[2, 1]
```
Switchcalledwith2arguments.Switchmustbecalledwithanoddn

$\text{Switch}[2, 1]$

Throw

`Throw['value']`
: stops evaluation and returns 'value' as the value of the nearest enclosing Catch.

`Catch['value', 'tag']`
: is caught only by 'Catch[expr,form]', where tag matches form.

Using Throw can affect the structure of what is returned by a function:

```
>> NestList[#^2 + 1 &, 1, 7]
```

{1, 2, 5, 26, 677, 458 330, 210 066 388 901, 44 127~ ~887 745 906 175 987 802}

```
>> Catch[NestList[If[# > 1000, Throw[#], #^2 + 1] &, 1, 7]]
```

458 330

```
>> Throw[1]
```

UncaughtHold[Throw[1]]returnedtotoplevel.

Hold [Throw [1]]

Which

`Which[cond1, expr1, cond2, expr2, ...]`
: yields *expr1* if *cond1* evaluates to `True`, *expr2* if *cond2* evaluates to `True`, etc.

```
>> n = 5;
```

```
>> Which[n == 3, x, n == 5, y]
```

y

```
>> f[x_] := Which[x < 0, -x, x == 0, 0, x > 0, x]
```

```
>> f[-3]
```

3

If no test yields `True`, `Which` returns `Null`:

```
>> Which[False, a]
```

If a test does not evaluate to `True` or `False`, evaluation stops and a `Which` expression containing the remaining cases is returned:

```
>> Which[False, a, x, b, True, c]
```

Which $[x, b, \text{True}, c]$

`Which` must be called with an even number of arguments:

```
>> Which[a, b, c]
```

Whichcalledwith3arguments.

Which $[a, b, c]$

While

`While[test, body]`
: evaluates *body* as long as *test* evaluates to `True`.

`While[test]`
: runs the loop without any body.

Compute the GCD of two numbers:

```
>> {a, b} = {27, 6};
```

```
>> While[b != 0, {a, b} = {b, Mod[a, b]}];
```

```
>> a
```

3

4. Global System Information

Contents

$Aborted

`$Aborted`
is returned by a calculation that has been aborted.

$ByteOrdering

`$ByteOrdering`
returns the native ordering of bytes in binary data on your computer system.

```
>> $ByteOrdering
```
-1

$CommandLine

`$CommandLine`
is a list of strings passed on the command line to launch the Mathics session.

```
>> $CommandLine
```
{mathics/docpipeline.py, -ot, -k}

Environment

`Environment[`*var*`]`
gives the value of an operating system environment variable.

```
>> Environment["HOME"]
```
/home/rocky

$Failed

`$Failed`
is returned by some functions in the event of an error.

GetEnvironment

`GetEnvironment["`*var*`"]`
gives the setting corresponding to the variable "var" in the operating system environment.

```
>> GetEnvironment["HOME"]
```
HOME− > /home/rocky

$Machine

`$Machine`
: returns a string describing the type of computer system on which the Mathics is being run.

```
>> $Machine
```
linux

$MachineName

`$MachineName`
: is a string that gives the assigned name of the computer on which Mathics is being run, if such a name is defined.

```
>> $MachineName
```
muffin

MathicsVersion

`MathicsVersion`
: this string is the version of Mathics we are running.

```
>> MathicsVersion
```
3.1.0

MemoryAvailable

`MemoryAvailable`
: Returns the amount of the available physical memory.

```
>> MemoryAvailable[]
```
4 667 019 264

The relationship between $SystemMemory, MemoryAvailable, and MemoryInUse:

```
>> $SystemMemory > MemoryAvailable
   [] > MemoryInUse[]
```
True

MemoryInUse

`MemoryInUse[]`
: Returns the amount of memory used by the definitions object.

```
>> MemoryInUse[]
```
56

Names

`Names["`*pattern*`"]`
: returns the list of names matching *pattern*.

```
>> Names["List"]
```
{List}

The wildcard `*` matches any character:

```
>> Names["List*"]
```
{List, ListLinePlot, ListPlot, ListQ, Listable}

The wildcard `@` matches only lowercase characters:

```
>> Names["List@"]
```
{Listable}

```
>> x = 5;
```

```
>> Names["Global`*"]
```
{x}

The number of built-in symbols:

```
>> Length[Names["System`*"]]
```
1 109

$Packages

`$Packages`
: returns a list of the contexts corresponding to all packages which have been loaded into Mathics.

```
>> $Packages
```
{ImportExport`, XML`, Internal`, System`, Global`}

$ParentProcessID

`$ParentProcesID`
: gives the ID assigned to the process which invokes the *Mathics* by the operating system under which it is run.

```
>> $ParentProcessID
3 883 171
```

$ProcessID

`$ProcessID`
: gives the ID assigned to the *Mathics* process by the operating system under which it is run.

```
>> $ProcessID
3 883 173
```

$ProcessorType

`$ProcessorType`
: gives a string giving the architecture of the processor on which the *Mathics* is being run.

```
>> $ProcessorType
x86_64
```

Run

`Run[`*command*`]`
: runs command as an external operating system command, returning the exit code obtained.

```
>> Run["date"]
0
```

$ScriptCommandLine

`$ScriptCommandLine`
: is a list of string arguments when running the kernel is script mode.

```
>> $ScriptCommandLine
{}
```

Share

`Share[]`
: Tries to reduce the amount of memory required to store definitions, by reducing duplicated definitions. Now it just do nothing.

`Share[Symbol]`
: Tries to reduce the amount of memory required to store definitions associated to *Symbol*.

```
>> Share[]
0
```

$SystemID

`$SystemID`
: is a short string that identifies the type of computer system on which the *Mathics* is being run.

```
>> $SystemID
linux
```

$SystemMemory

`$SystemMemory`
: Returns the total amount of physical memory.

```
>> $SystemMemory
33 691 602 944
```

$SystemWordLength

`$SystemWordLength`
: gives the effective number of bits in raw machine words on the computer system where *Mathics* is running.

>> `$SystemWordLength`

64

$UserName

$UserName
: returns a string describing the type of computer system on which *Mathics* is being run.

>> `$UserName`

rocky

$Version

`$Version`
: returns a string with the current Mathics version and the versions of relevant libraries.

>> `$Version`

Mathics 3.1.0 on CPython 3.6.13 (default, Feb 23 2 021, 10:08:17) using SymPy 1.8, mpmath 1.2.1, numpy 1.19.5, cython 0.29.22

$VersionNumber

`$VersionNumber`
: is a real number which gives the current Wolfram Language version that *Mathics* tries to be compatible with.

>> `$VersionNumber`

10.

5. SparseArray Functions

Contents

SparseArray

`SparseArray[`*rules*`]`
: Builds a sparse array acording to the list of *rules*.

`SparseArray[`*rules*`, `*dims*`]`
: Builds a sparse array of dimensions *dims* acording to the *rules*.

`SparseArray[`*list*`]`
: Builds a sparse representation of *list*.

```
>> SparseArray[{{1, 2} -> 1, {2, 1}
   -> 1}]
```

SparseArray [Automatic, {2,2}, 0, {{1,2} − > 1, {2,1} − > 1}]

```
>> SparseArray[{{1, 2} -> 1, {2, 1}
   -> 1}, {3, 3}]
```

SparseArray [Automatic, {3,3}, 0, {{1,2} − > 1, {2,1} − > 1}]

```
>> M=SparseArray[{{0, a}, {b, 0}}]
```

SparseArray [Automatic, {2,2}, 0, {{1,2} − > a, {2,1} − > b}]

```
>> M //Normal
```

{{0, a}, {b, 0}}

6. Solving Recurrence Equations

Contents

RSolve

`RSolve[`*eqn*`, `*a*`[`*n*`], `*n*`]`
    solves a recurrence equation for the function *a*[*n*].

Solve a difference equation:

```
>> RSolve[a[n] == a[n+1], a[n], n]
```

$\{\{a[n]->C[0]\}\}$

No boundary conditions gives two general paramaters:

```
>> RSolve[{a[n + 2] == a[n]}, a, n]
```

$\{\{a->(\text{Function}[\{n\}, C[0]+C[1]-1^\wedge n])\}\}$

Include one boundary condition:

```
>> RSolve[{a[n + 2] == a[n], a[0]
   == 1}, a, n]
```

$\{\{a->(\text{Function}[\{n\},C[0]+(1-C[0])-1^\wedge n])\}\}$

Geta "pure function" solution for a with two boundary conditions:

```
>> RSolve[{a[n + 2] == a[n], a[0]
   == 1, a[1] == 4}, a, n]
```

$$\left\{\left\{a->\left(\text{Function}\left[\{n\},\frac{5}{2}-\frac{3-1^\wedge n}{2}\right]\right)\right\}\right\}$$

7. Patterns and Rules

Some examples:

```
>> a + b + c /. a + b -> t
```
$c + t$

```
>> a + 2 + b + c + x * y /.
   n_Integer + s__Symbol + rest_ ->
    {n, s, rest}
```
$\{2, a, b + c + xy\}$

```
>> f[a, b, c, d] /. f[first_,
   rest___] -> {first, {rest}}
```
$\{a, \{b, c, d\}\}$

Tests and Conditions:

```
>> f[4] /. f[x_?(# > 0&)] -> x ^ 2
```
16

```
>> f[4] /. f[x_] /; x > 0 -> x ^ 2
```
16

Leaves in the beginning of a pattern rather match fewer leaves:

```
>> f[a, b, c, d] /. f[start__,
   end__] -> {{start}, {end}}
```
$\{\{a\}, \{b, c, d\}\}$

Optional arguments using `Optional`:

```
>> f[a] /. f[x_, y_:3] -> {x, y}
```
$\{a, 3\}$

Options using `OptionsPattern` and `OptionValue`:

```
>> f[y, a->3] /. f[x_,
   OptionsPattern[{a->2, b->5}]] ->
    {x, OptionValue[a], OptionValue
   [b]}
```
$\{y, 3, 5\}$

The attributes `Flat`, `Orderless`, and `OneIdentity` affect pattern matching.

Contents

Alternatives (|)

`Alternatives[`*p1*`,` *p2*`, ...,` *p_i*`]`
p1 `|` *p2* `| ... |` *p_i*
: is a pattern that matches any of the patterns '*p1, p2, ..., p_i*'.

```
>> a+b+c+d/.(a|b)->t
```
$c + d + 2t$

Alternatives can also be used for string expressions

```
>> StringReplace["0123 3210", "1" |
    "2" -> "X"]
```
0XX3 3XX0

Blank

`Blank[]`
`_`
: represents any single expression in a pattern.

`Blank[`*h*`]`
`_`*h*
: represents any expression with head *h*.

```
>> MatchQ[a + b, _]
   True
```

Patterns of the form _*h* can be used to test the types of objects:

```
>> MatchQ[42, _Integer]
   True
```

```
>> MatchQ[1.0, _Integer]
   False
```

```
>> {42, 1.0, x} /. {_Integer -> "
   integer", _Real -> "real"} //
   InputForm
```

{"integer", "real", x}

`Blank` only matches a single expression:

```
>> MatchQ[f[1, 2], f[_]]
   False
```

BlankNullSequence

`BlankNullSequence[]`
`___`
: represents any sequence of expression leaves in a pattern, including an empty sequence.

`BlankNullSequence` is like `BlankSequence`, except it can match an empty sequence:

```
>> MatchQ[f[], f[___]]
   True
```

BlankSequence

`BlankSequence[]`
`__`
: represents any non-empty sequence of expression leaves in a pattern.

`BlankSequence[`*h*`]`
`__`*h*
: represents any sequence of leaves, all of which have head *h*.

Use a `BlankSequence` pattern to stand for a non-empty sequence of arguments:

```
>> MatchQ[f[1, 2, 3], f[__]]
   True
```

```
>> MatchQ[f[], f[__]]
   False
```

__*h* will match only if all leaves have head *h*:

```
>> MatchQ[f[1, 2, 3], f[__Integer]]
   True
```

```
>> MatchQ[f[1, 2.0, 3], f[__Integer
   ]]
   False
```

The value captured by a named `BlankSequence` pattern is a `Sequence` object:

```
>> f[1, 2, 3] /. f[x__] -> x
```

Sequence $[1, 2, 3]$

Condition (/;)

`Condition[`*pattern*`, `*expr*`]`
pattern `/;` *expr*
: places an additional constraint on *pattern* that only allows it to match if *expr* evaluates to `True`.

The controlling expression of a `Condition` can use variables from the pattern:

```
>> f[3] /. f[x_] /; x>0 -> t
```

t

```
>> f[-3] /. f[x_] /; x>0 -> t
```

$f[-3]$

`Condition` can be used in an assignment:

```
>> f[x_] := p[x] /; x>0
```

```
>> f[3]
```
$p[3]$

```
>> f[-3]
```
$f[-3]$

Dispatch

`Dispatch[`*rulelist*`]`
: Introduced for compatibility. Currently, it just return *rulelist*. In the future, it should return an optimized DispatchRules atom, containing an optimized set of rules.

Except

`Except[`*c*`]`
: represents a pattern object that matches any expression except those matching *c*.

`Except[`*c*`, `*p*`]`
: represents a pattern object that matches *p* but not *c*.

```
>> Cases[{x, a, b, x, c}, Except[x]]
```
$\{a, b, c\}$

```
>> Cases[{a, 0, b, 1, c, 2, 3}, Except[1, _Integer]]
```
$\{0, 2, 3\}$

Except can also be used for string expressions:

```
>> StringReplace["Hello world!", Except[LetterCharacter] -> ""]
```
Helloworld

HoldPattern

`HoldPattern[`*expr*`]`
: is equivalent to *expr* for pattern matching, but maintains it in an unevaluated form.

```
>> HoldPattern[x + x]
```
HoldPattern $[x + x]$

```
>> x /. HoldPattern[x] -> t
```
t

`HoldPattern` has attribute `HoldAll`:

```
>> Attributes[HoldPattern]
```
{HoldAll, Protected}

Longest

```
>> StringCases["aabaaab", Longest["a" ~~__ ~~"b"]]
```
{aabaaab}

```
>> StringCases["aabaaab", Longest[RegularExpression["a+b"]]]
```
{aab, aaab}

MatchQ

`MatchQ[`*expr*`, `*form*`]`
: tests whether *expr* matches *form*.

```
>> MatchQ[123, _Integer]
```
True

```
>> MatchQ[123, _Real]
```
False

```
>> MatchQ[_Integer][123]
```
True

```
>> MatchQ[3, Pattern[3]]
```
FirstelementinpatternPattern[3]*isnotavalidpatternname.*

False

Optional (:)

`Optional[`*patt*`, `*default*`]`
patt `:` *default*
: is a pattern which matches *patt*, which if omitted should be replaced by *default*.

```
>> f[x_, y_:1] := {x, y}
```

```
>> f[1, 2]
```
{1, 2}

```
>> f[a]
```
$\{a, 1\}$

Note that *symb* : *patt* represents a `Pattern` object. However, there is no disambiguity, since *symb* has to be a symbol in this case.

```
>> x:_ // FullForm
```
Pattern [x, Blank []]

```
>> _:d // FullForm
```
Optional [Blank [] , d]

```
>> x:_+y_:d // FullForm
```
Pattern [x, Plus [Blank [] ,
 Optional [Pattern [y, Blank []] , d]]]

`s_.` is equivalent to `Optional[s_]` and represents an optional parameter which, if omitted, gets its value from `Default`.

```
>> FullForm[s_.]
```
Optional [Pattern [s, Blank []]]

```
>> Default[h, k_] := k
```

```
>> h[a] /. h[x_, y_.] -> {x, y}
```
$\{a, 2\}$

OptionsPattern

`OptionsPattern[`*f*`]`
: is a pattern that stands for a sequence of options given to a function, with default values taken from `Options[`*f*`]`. The options can be of the form *opt*`->`*value* or *opt*`:>`*value*, and might be in arbitrarily nested lists.

`OptionsPattern[{`*opt1*`->`*value1*`, ...}]`
: takes explicit default values from the given list. The list may also contain symbols *f*, for which `Options[`*f*`]` is taken into account; it may be arbitrarily nested. `OptionsPattern[{}]` does not use any default values.

The option values can be accessed using `OptionValue`.

```
>> f[x_, OptionsPattern[{n->2}]] :=
    x ^ OptionValue[n]
```

```
>> f[x]
```
x^2

```
>> f[x, n->3]
```
x^3

Delayed rules as options:

```
>> e = f[x, n:>a]
```
x^a

```
>> a = 5;
```

```
>> e
```
x^5

Options might be given in nested lists:

```
>> f[x, {{{n->4}}}]
```
x^4

PatternTest (?)

`PatternTest[`*pattern*`, ` *test*`]`
pattern `?` *test*
: constrains *pattern* to match *expr* only if the evaluation of *test*`[`*expr*`]` yields `True`.

```
>> MatchQ[3, _Integer?(#>0&)]
```
True

```
>> MatchQ[-3, _Integer?(#>0&)]
```
False

```
>> MatchQ[3, Pattern[3]]
```
FirstelementinpatternPattern[3]*isnotavalidpatternname.*

False

Pattern

`Pattern[`*symb*`, ` *patt*`]`
symb `:` *patt*
: assigns the name *symb* to the pattern *patt*.

symb_head
: is equivalent to *symb* : *_head* (accordingly with __ and ___).

symb `:` *patt* `:` *default*
: is a pattern with name *symb* and default value *default*, equivalent to `Optional[`*patt* : *symb*`, ` *default*`]`.

```
>> FullForm[a_b]
```
Pattern [a, Blank [b]]

```
>> FullForm[a:_:b]
   Optional[Pattern[a, Blank[]], b]
```

`Pattern` has attribute `HoldFirst`, so it does not evaluate its name:

```
>> x = 2
   2
>> x_
   x_
```

Nested `Pattern` assign multiple names to the same pattern. Still, the last parameter is the default value.

```
>> f[y] /. f[a:b,_:d] -> {a, b}
   f[y]
```

This is equivalent to:

```
>> f[a] /. f[a:_:b] -> {a, b}
   {a, b}
```

`FullForm`:

```
>> FullForm[a:b:c:d:e]
   Optional[Pattern[a, b],
     Optional[Pattern[c, d], e]]
>> f[] /. f[a:_:b] -> {a, b}
   {b, b}
```

Repeated (..)

`Repeated[`*pattern*`]`
matches one or more occurrences of *pattern*.

```
>> a_Integer.. // FullForm
   Repeated[Pattern[a, Blank[Integer]]]
>> 0..1//FullForm
   Repeated[0]
>> {{}, {a}, {a, b}, {a, a, a}, {a,
    a, a, a}} /. {Repeated[x : a |
   b, 3]} -> x
   {{}, a, {a, b}, a, {a, a, a, a}}
>> f[x, 0, 0, 0] /. f[x, s:0..] ->
   s
   Sequence[0, 0, 0]
```

RepeatedNull (...)

`RepeatedNull[`*pattern*`]`
matches zero or more occurrences of *pattern*.

```
>> a___Integer...//FullForm
   RepeatedNull[Pattern[a,
     BlankNullSequence[Integer]]]
>> f[x] /. f[x, 0...] -> t
   t
```

Replace

`Replace[`*expr*`, `*x*` -> `*y*`]`
yields the result of replacing *expr* with *y* if it matches the pattern *x*.

`Replace[`*expr*`, `*x*` -> `*y*`, `*levelspec*`]`
replaces only subexpressions at levels specified through *levelspec*.

`Replace[`*expr*`, {`*x*` -> `*y*`, ...}]`
performs replacement with multiple rules, yielding a single result expression.

`Replace[`*expr*`, {{`*a*` -> `*b*`, ...}, {`*c*` -> `*d*`, ...}, ...}]`
returns a list containing the result of performing each set of replacements.

```
>> Replace[x, {x -> 2}]
   2
```

By default, only the top level is searched for matches

```
>> Replace[1 + x, {x -> 2}]
   1 + x
>> Replace[x, {{x -> 1}, {x -> 2}}]
   {1, 2}
```

Replace stops after the first replacement

```
>> Replace[x, {x -> {}, _List -> y
   }]
   {}
```

Replace replaces the deepest levels first

```
>> Replace[x[1], {x[1] -> y, 1 ->
   2}, All]
   x[2]
```

By default, heads are not replaced

```
>> Replace[x[x[y]], x -> z, All]
```

$x[x[y]]$

Heads can be replaced using the Heads option

```
>> Replace[x[x[y]], x -> z, All,
   Heads -> True]
```

$z[z[y]]$

Note that heads are handled at the level of leaves

```
>> Replace[x[x[y]], x -> z, {1},
   Heads -> True]
```

$z[x[y]]$

You can use Replace as an operator

```
>> Replace[{x_ -> x + 1}][10]
```

11

ReplaceAll (/.)

`ReplaceAll[`*expr*`, `*x*` -> `*y*`]`
expr `/.` *x* `->` *y*
: yields the result of replacing all subexpressions of *expr* matching the pattern *x* with *y*.

expr `/. {`*x*` -> `*y*`, ...}`
: performs replacement with multiple rules, yielding a single result expression.

expr `/. {{`*a*` -> `*b*`, ...}, {`*c*` -> `*d*`, ...}, ...}`
: returns a list containing the result of performing each set of replacements.

```
>> a+b+c /. c->d
```

$a + b + d$

```
>> g[a+b+c,a]/.g[x_+y_,x_]->{x,y}
```

$\{a, b + c\}$

If *rules* is a list of lists, a list of all possible respective replacements is returned:

```
>> {a, b} /. {{a->x, b->y}, {a->u,
   b->v}}
```

$\{\{x, y\}, \{u, v\}\}$

The list can be arbitrarily nested:

```
>> {a, b} /. {{{a->x, b->y}, {a->w,
    b->z}}, {a->u, b->v}}
```

$\{\{\{x, y\}, \{w, z\}\}, \{u, v\}\}$

```
>> {a, b} /. {{{a->x, b->y}, a->w,
   b->z}, {a->u, b->v}}
```

$Elementsof\{\{a->x, b->y\}, a->w, b->z\}areamixtureoflistsandnonlists.$

$\{\{a, b\} /. \{\{a->x, b->y\}, a->w, b->z\}, \{u, v\}\}$

ReplaceAll also can be used as an operator:

```
>> ReplaceAll[{a -> 1}][{a, b}]
```

$\{1, b\}$

ReplaceAll replaces the shallowest levels first:

```
>> ReplaceAll[x[1], {x[1] -> y, 1
   -> 2}]
```

y

ReplaceList

`ReplaceList[`*expr*`, `*rules*`]`
: returns a list of all possible results of applying *rules* to *expr*.

Get all subsequences of a list:

```
>> ReplaceList[{a, b, c}, {___, x__
   , ___} -> {x}]
```

$\{\{a\}, \{a, b\}, \{a, b, c\}, \{b\}, \{b, c\}, \{c\}\}$

You can specify the maximum number of items:

```
>> ReplaceList[{a, b, c}, {___, x__
   , ___} -> {x}, 3]
```

$\{\{a\}, \{a, b\}, \{a, b, c\}\}$

```
>> ReplaceList[{a, b, c}, {___, x__
   , ___} -> {x}, 0]
```

$\{\}$

If no rule matches, an empty list is returned:

```
>> ReplaceList[a, b->x]
```

$\{\}$

Like in `ReplaceAll`, *rules* can be a nested list:

```
>> ReplaceList[{a, b, c}, {{{___,
   x__, ___} -> {x}}, {{a, b, c} ->
    t}}, 2]
```

$\{\{\{a\}, \{a, b\}\}, \{t\}\}$

```
>> ReplaceList[expr, {}, -1]
```
Non −negativeintegerorInfinityexpectedatposition3.

ReplaceList $[\text{expr}, \{\}, -1]$

Possible matches for a sum:

```
>> ReplaceList[a + b + c, x_ + y_
   -> {x, y}]
```
$\{\{a, b+c\}, \{b, a+c\}, \{c, a+b\}, \{a+b, c\}, \{a+c, b\}, \{b+c, a\}\}$

ReplaceRepeated (//.)

`ReplaceRepeated[`*expr*`, `*x*` -> `*y*`]`
expr `//. ` *x* ` -> ` *y*
: repeatedly applies the rule *x* `->` *y* to *expr* until the result no longer changes.

```
>> a+b+c //. c->d
```
$a+b+d$

```
>> f = ReplaceRepeated[c->d];
```

```
>> f[a+b+c]
```
$a+b+d$

```
>> Clear[f];
```

Simplification of logarithms:

```
>> logrules = {Log[x_ * y_] :> Log[
   x] + Log[y], Log[x_ ^ y_] :> y *
    Log[x]};
```

```
>> Log[a * (b * c)^ d ^ e * f] //.
   logrules
```
$\mathrm{Log}[a] + \mathrm{Log}[f] + (\mathrm{Log}[b] + \mathrm{Log}[c])\, d^e$

`ReplaceAll` just performs a single replacement:

```
>> Log[a * (b * c)^ d ^ e * f] /.
   logrules
```
$\mathrm{Log}[a] + \mathrm{Log}\left[f\,(bc)^{d^e}\right]$

RuleDelayed (:>)

`RuleDelayed[`*x*`, `*y*`]`
x ` :> ` *y*
: represents a rule replacing *x* with *y*, with *y* held unevaluated.

```
>> Attributes[RuleDelayed]
```
$\{\mathrm{HoldRest}, \mathrm{Protected}, \mathrm{SequenceHold}\}$

Rule (->)

`Rule[`*x*`, `*y*`]`
x ` -> ` *y*
: represents a rule replacing *x* with *y*.

```
>> a+b+c /. c->d
```
$a+b+d$

```
>> {x,x^2,y} /. x->3
```
$\{3, 9, y\}$

Verbatim

`Verbatim[`*expr*`]`
: prevents pattern constructs in *expr* from taking effect, allowing them to match themselves.

Create a pattern matching `Blank`:

```
>> _ /. Verbatim[_]->t
```
t

```
>> x /. Verbatim[_]->t
```
x

Without `Verbatim`, `Blank` has its normal effect:

```
>> x /. _->t
```
t

8. Mathematical Functions

Basic arithmetic functions, including complex number arithmetic.

Contents

Abs

`Abs[`*x*`]`
: returns the absolute value of *x*.

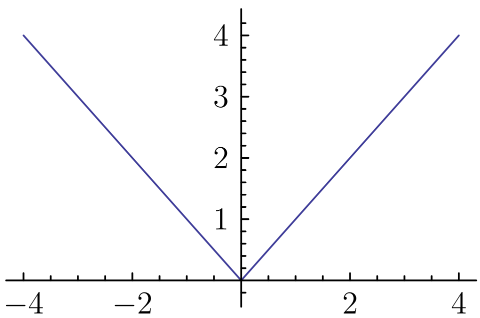
```
>> Abs[-3]
3
```

`Abs` returns the magnitude of complex numbers:

```
>> Abs[3 + I]
```
$\sqrt{10}$

```
>> Abs[3.0 + I]
3.16228
```

```
>> Plot[Abs[x], {x, -4, 4}]
```



Arg

`Arg[`*z*, *method_option*`]`
: returns the argument of a complex value *z*.</dd>

- `Arg[`*z*`]` is left unevaluated if *z* is not a numeric quantity.
- `Arg[`*z*`]` gives the phase angle of *z* in radians.
- The result from `Arg[`*z*`]` is always between -Pi and +Pi.
- `Arg[`*z*`]` has a branch cut discontinuity in the complex *z* plane running from -Infinity to 0.
- `Arg[0]` is 0.

```
>> Arg[-3]
Pi
```

Same as above using sympy's method:

```
>> Arg[-3, Method->"sympy"]
Pi
```

```
>> Arg[1-I]
```
$-\frac{\text{Pi}}{4}$

Arg evaluate the direction of DirectedInfinity quantities by the Arg of they arguments:

```
>> Arg[DirectedInfinity[1+I]]
```

$\frac{\mathrm{Pi}}{4}$

```
>> Arg[DirectedInfinity[]]
```

1

Arg for 0 is assumed to be 0:

```
>> Arg[0]
```

0

Assuming

`Assuming[`*cond*`, `*expr*`]`
: Evaluates *expr* assuming the conditions *cond*.

```
>> $Assumptions = { x > 0 }
```

$\{x > 0\}$

```
>> Assuming[y>0,
   ConditionalExpression[y x^2, y
   >0]//Simplify]
```

$x^2 y$

```
>> Assuming[Not[y>0],
   ConditionalExpression[y x^2, y
   >0]//Simplify]
```

Undefined

```
>> ConditionalExpression[y x ^ 2, y
    > 0]//Simplify
```

ConditionalExpression $\left[x^2 y, y > 0\right]$

$Assumptions

`$Assumptions`
: is the default setting for the Assumptions option used in such functions as Simplify, Refine, and Integrate.

Boole

`Boole[expr]`
: returns 1 if expr is True and 0 if expr is False.

```
>> Boole[2 == 2]
```

1

```
>> Boole[7 < 5]
```

0

```
>> Boole[a == 7]
```

Boole $[a==7]$

Complex

`Complex`
: is the head of complex numbers.

`Complex[`*a*`, `*b*`]`
: constructs the complex number *a* `+ I` *b*.

```
>> Head[2 + 3*I]
```

Complex

```
>> Complex[1, 2/3]
```

$1 + \frac{2I}{3}$

```
>> Abs[Complex[3, 4]]
```

5

ConditionalExpression

`ConditionalExpression[`*expr*`, `*cond*`]`
: returns *expr* if *cond* evaluates to *True*, *Undefined* if *cond* evaluates to *False*.

```
>> ConditionalExpression[x^2, True]
```

x^2

```
>> ConditionalExpression[x^2, False
   ]
```

Undefined

```
>> f = ConditionalExpression[x^2, x
   >0]
```

ConditionalExpression $\left[x^2, x > 0\right]$

```
>> f /. x -> 2
```
4

```
>> f /. x -> -2
```
Undefined

`ConditionalExpression` uses assumptions to evaluate the condition:

```
>> $Assumptions = x > 0;
```

```
>> ConditionalExpression[x ^ 2, x
>0]//Simplify
```
x^2

```
>> $Assumptions = True;
```

» ConditionalExpression[ConditionalExpression[s,x>a], x<b] # = ConditionalExpression[s, And[x>a, x<b]]

Conjugate

`Conjugate[`*z*`]`
: returns the complex conjugate of the complex number *z*.

```
>> Conjugate[3 + 4 I]
```
$3 - 4I$

```
>> Conjugate[3]
```
3

```
>> Conjugate[a + b * I]
```
$\text{Conjugate}[a] - I\text{Conjugate}[b]$

```
>> Conjugate[{{1, 2 + I 4, a + I b
}, {I}}]
```
$\{\{1, 2 - 4I, \text{Conjugate}[a] - I\text{Conjugate}[b]\}, \{-I\}\}$

```
>> Conjugate[1.5 + 2.5 I]
```
$1.5 - 2.5I$

DirectedInfinity

`DirectedInfinity[`*z*`]`
: represents an infinite multiple of the complex number *z*.

`DirectedInfinity[]`
: is the same as `ComplexInfinity`.

```
>> DirectedInfinity[1]
```
∞

```
>> DirectedInfinity[]
```
ComplexInfinity

```
>> DirectedInfinity[1 + I]
```
$\left(\frac{1}{2} + \frac{I}{2}\right)\sqrt{2}\infty$

```
>> 1 / DirectedInfinity[1 + I]
```
0

```
>> DirectedInfinity[1] +
DirectedInfinity[-1]
```
Indeterminateexpression − Infinity + Infinityencountered.

Indeterminate

```
>> DirectedInfinity[0]
```
Indeterminateexpression0Infinityencountered.

Indeterminate

ExactNumberQ

`ExactNumberQ[`*expr*`]`
: returns `True` if *expr* is an exact number, and `False` otherwise.

```
>> ExactNumberQ[10]
```
True

```
>> ExactNumberQ[4.0]
```
False

```
>> ExactNumberQ[n]
```
False

`ExactNumberQ` can be applied to complex numbers:

```
>> ExactNumberQ[1 + I]
```
True

```
>> ExactNumberQ[1 + 1. I]
```
False

Factorial (!)

`Factorial[`*n*`]`
n`!`
    computes the factorial of *n*.

>> `20!`
2 432 902 008 176 640 000

`Factorial` handles numeric (real and complex) values using the gamma function:

>> `10.5!`
1.18994×10^7

>> `(-3.0+1.5*I)!`
$0.0427943 - 0.00461565I$

However, the value at poles is `ComplexInfinity`:

>> `(-1.)!`
ComplexInfinity

`Factorial` has the same operator (`!`) as `Not`, but with higher precedence:

>> `!a! //FullForm`
Not [Factorial [*a*]]

Factorial2 (!!)

`Factorial2[`*n*`]`
n`!!`
    computes the double factorial of *n*.

The double factorial or semifactorial of a number *n*, is the product of all the integers from 1 up to n that have the same parity (odd or even) as *n*.

>> `5!!`
15.

>> `Factorial2[-3]`
$-1.$

`Factorial2` accepts Integers, Rationals, Reals, or Complex Numbers:

>> `I!! + 1`
$3.71713 + 0.279527I$

Irrationals can be handled by using numeric approximation:

>> `N[Pi!!, 6]`
3.35237

I

`I`
    represents the imaginary number `Sqrt[-1]`.

>> `I^2`
-1

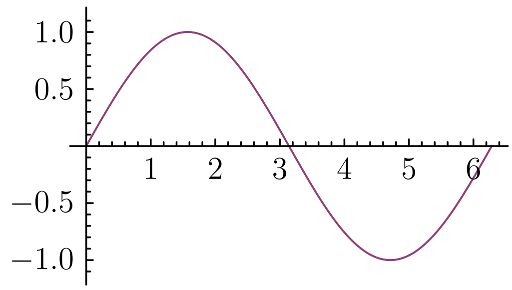
>> `(3+I)*(3-I)`
10

Im

`Im[`*z*`]`
    returns the imaginary component of the complex number *z*.

>> `Im[3+4I]`
4

>> `Plot[{Sin[a], Im[E^(I a)]}, {a, 0, 2 Pi}]`



InexactNumberQ

`InexactNumberQ[`*expr*`]`
    returns `True` if *expr* is not an exact number, and `False` otherwise.

>> `InexactNumberQ[a]`
False

>> `InexactNumberQ[3.0]`
True

>> `InexactNumberQ[2/3]`
False

`InexactNumberQ` can be applied to complex numbers:

```
>> InexactNumberQ[4.0+I]
```
True

IntegerQ

`IntegerQ[`*expr*`]`
: returns `True` if *expr* is an integer, and `False` otherwise.

```
>> IntegerQ[3]
```
True

```
>> IntegerQ[Pi]
```
False

Integer

`Integer`
: is the head of integers.

```
>> Head[5]
```
Integer

MachineNumberQ

`MachineNumberQ[`*expr*`]`
: returns `True` if *expr* is a machine-precision real or complex number.

= True

```
>> MachineNumberQ
[3.14159265358979324]
```
False

```
>> MachineNumberQ[1.5 + 2.3 I]
```
True

```
>> MachineNumberQ
[2.71828182845904524 +
3.14159265358979324 I]
```
False

NumberQ

`NumberQ[`*expr*`]`
: returns `True` if *expr* is an explicit number, and `False` otherwise.

```
>> NumberQ[3+I]
```
True

```
>> NumberQ[5!]
```
True

```
>> NumberQ[Pi]
```
False

Piecewise

`Piecewise[{{expr1, cond1}, ...}]`
: represents a piecewise function.

`Piecewise[{{expr1, cond1}, ...}, expr]`
: represents a piecewise function with default `expr`.

Heaviside function

```
>> Piecewise[{{0, x <= 0}}, 1]
```
Piecewise $[\{\{0, x{<=}0\}\}, 1]$

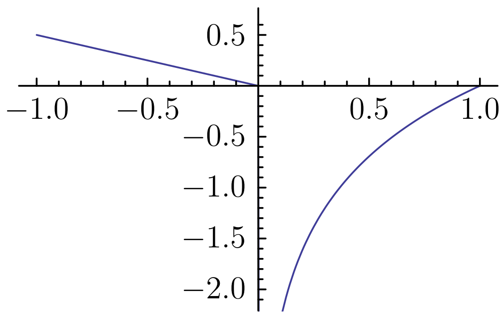
```
>> Integrate[Piecewise[{{1, x <=
0}, {-1, x > 0}}], x]
```
Piecewise $[\{\{x, x{<=}0\}\}, -x]$

```
>> Integrate[Piecewise[{{1, x <=
0}, {-1, x > 0}}], {x, -1, 2}]
```
-1

Piecewise defaults to 0 if no other case is matching.

```
>> Piecewise[{{1, False}}]
```
0

```
>> Plot[Piecewise[{{Log[x], x > 0},
 {x*-0.5, x < 0}}], {x, -1, 1}]
```



```
>> Piecewise[{{0 ^ 0, False}}, -1]
```
-1

PossibleZeroQ

`PossibleZeroQ[`*expr*`]`
: returns `True` if basic symbolic and numerical methods suggest that expr has value zero, and `False` otherwise.

Test whether a numeric expression is zero:

```
>> PossibleZeroQ[E^(I Pi/4)- (-1)
   ^(1/4)]
```
True

The determination is approximate.
Test whether a symbolic expression is likely to be identically zero:

```
>> PossibleZeroQ[(x + 1)(x - 1)- x
   ^2 + 1]
```
True

```
>> PossibleZeroQ[(E + Pi)^2 - E^2 -
    Pi^2 - 2 E Pi]
```
True

Show that a numeric expression is nonzero:

```
>> PossibleZeroQ[E^Pi - Pi^E]
```
False

```
>> PossibleZeroQ[1/x + 1/y - (x + y
   )/(x y)]
```
True

Decide that a numeric expression is zero, based on approximate computations:

```
>> PossibleZeroQ[2^(2 I)- 2^(-2 I)-
    2 I Sin[Log[4]]]
```
True

```
>> PossibleZeroQ[Sqrt[x^2] - x]
```
False

Product

`Product[`*expr*`, {`*i*`, `*imin*`, `*imax*`}]`
: evaluates the discrete product of *expr* with *i* ranging from *imin* to *imax*.

`Product[`*expr*`, {`*i*`, `*imax*`}]`
: same as `Product[`*expr*`, {`*i*`, 1, `*imax*`}]`.

`Product[`*expr*`, {`*i*`, `*imin*`, `*imax*`, `*di*`}]`
: *i* ranges from *imin* to *imax* in steps of *di*.

`Product[`*expr*`, {`*i*`, `*imin*`, `*imax*`}, {`*j*`, `*jmin*`, `*jmax*`}, ...]`
: evaluates *expr* as a multiple product, with {*i*, ...}, {*j*, ...}, ... being in outermost-to-innermost order.

```
>> Product[k, {k, 1, 10}]
```
3 628 800

```
>> 10!
```
3 628 800

```
>> Product[x^k, {k, 2, 20, 2}]
```
x^{110}

```
>> Product[2 ^ i, {i, 1, n}]
```
$2^{\frac{n}{2}+\frac{n^2}{2}}$

```
>> Product[f[i], {i, 1, 7}]
```
$f[1] f[2] f[3] f[4] f[5] f[6] f[7]$

Symbolic products involving the factorial are evaluated:

```
>> Product[k, {k, 3, n}]
```
$\frac{n!}{2}$

Evaluate the nth primorial:

```
>> primorial[0] = 1;
```

```
>> primorial[n_Integer] := Product[
   Prime[k], {k, 1, n}];
```

```
>> primorial[12]
```
7 420 738 134 810

Rational

`Rational`
: is the head of rational numbers.

`Rational[`*a*`, `*b*`]`
: constructs the rational number *a* / *b*.

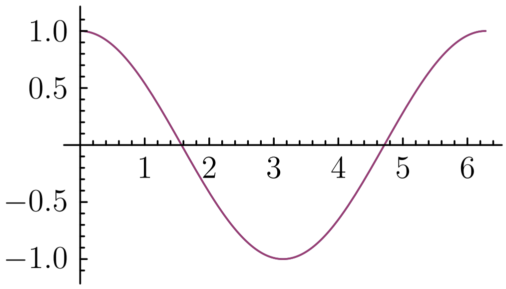
```
>> Head[1/2]
```
Rational

```
>> Rational[1, 2]
```
$\frac{1}{2}$

Re

`Re[z]`
: returns the real component of the complex number z.

```
>> Re[3+4I]
```
3

```
>> Plot[{Cos[a], Re[E^(I a)]}, {a, 0, 2 Pi}]
```


RealNumberQ

`RealNumberQ[expr]`
: returns `True` if *expr* is an explicit number with no imaginary component.

```
>> RealNumberQ[10]
```
True

```
>> RealNumberQ[4.0]
```
True

```
>> RealNumberQ[1+I]
```
False

```
>> RealNumberQ[0 * I]
```
True

```
>> RealNumberQ[0.0 * I]
```
True

Real

`Real`
: is the head of real (inexact) numbers.

```
>> x = 3. ^ -20;
```

```
>> InputForm[x]
```
2.8679719907924413*^ − 10

```
>> Head[x]
```
Real

Sign

`Sign[x]`
: return -1, 0, or 1 depending on whether x is negative, zero, or positive.

```
>> Sign[19]
```
1

```
>> Sign[-6]
```
-1

```
>> Sign[0]
```
0

```
>> Sign[{-5, -10, 15, 20, 0}]
```
$\{-1, -1, 1, 1, 0\}$

```
>> Sign[3 - 4*I]
```
$\frac{3}{5} - \frac{4I}{5}$

Sum

`Sum[expr, {i, imin, imax}]`
: evaluates the discrete sum of *expr* with i ranging from *imin* to *imax*.

`Sum[expr, {i, imax}]`
: same as `Sum[expr, {i, 1, imax}]`.

`Sum[expr, {i, imin, imax, di}]`
: i ranges from *imin* to *imax* in steps of *di*.

`Sum[expr, {i, imin, imax}, {j, jmin, jmax}, ...]`
: evaluates *expr* as a multiple sum, with {*i*, ...}, {*j*, ...}, ... being in outermost-to-innermost order.

A sum that Gauss in elementary school was asked to do to kill time:

```
>> Sum[k, {k, 1, 10}]
```

55

The symbolic form he used:

```
>> Sum[k, {k, 1, n}]
```

$\frac{n(1+n)}{2}$

A Geometric series with a finite limit:

```
>> Sum[1 / 2 ^ i, {i, 1, k}]
```

$1-2^{-k}$

A Geometric series using Infinity:

```
>> Sum[1 / 2 ^ i, {i, 1, Infinity}]
```

1

Leibniz forumla used in computing Pi:

```
>> Sum[1 / ((-1)^k (2k + 1)), {k,
   0, Infinity}]
```

$\frac{\text{Pi}}{4}$

A table of double sums to compute squares:

```
>> Table[ Sum[i * j, {i, 0, n}, {j,
    0, n}], {n, 0, 4} ]
```

$\{0,1,9,36,100\}$

Computing Harmonic using a sum

```
>> Sum[1 / k ^ 2, {k, 1, n}]
```

$\text{HarmonicNumber}[n,2]$

Other symbolic sums:

```
>> Sum[k, {k, n, 2 n}]
```

$\frac{3n(1+n)}{2}$

A sum with Complex-number iteration values

```
>> Sum[k, {k, I, I + 1}]
```

$1+2I$

```
>> Sum[f[i], {i, 1, 7}]
```

$f[1]+f[2]+f[3]+f[$
$4]+f[5]+f[6]+f[7]$

Verify algebraic identities:

```
>> Sum[x ^ 2, {x, 1, y}] - y * (y +
    1)* (2 * y + 1)/ 6
```

0

9. Functional Programming

Functional programming is a programming paradigm where programs are constructed by applying and composing functions. This is term is often used in contrast to Procedural programming.
It is made richer by expressions like $f[x]$ being treating as symbolic data.

Contents

Composition

`Composition[`*f*`, `*g*`]`
: returns the composition of two functions f and g.

```
>> Composition[f, g][x]
```
$f[g[x]]$

```
>> Composition[f, g, h][x, y, z]
```
$f[g[h[x,y,z]]]$

```
>> Composition[]
```
Identity

```
>> Composition[][x]
```
x

```
>> Attributes[Composition]
```
{Flat, OneIdentity, Protected}

```
>> Composition[f, Composition[g, h
]]
```
Composition $[f,g,h]$

Function (&)

`Function[`*body*`]`
body `&`
: represents a pure function with parameters `#1`, `#2`, etc.

`Function[{`*x1*`, `*x2*`, ...}, `*body*`]`
: represents a pure function with parameters *x1*, *x2*, etc.

`Function[{`*x1*`, `*x2*`, ...}, `*body*`, `*attr*`]`
: assume that the function has the attributes *attr*.

```
>> f := # ^ 2 &
```

```
>> f[3]
```
9

```
>> #^3& /@ {1, 2, 3}
```
{1, 8, 27}

```
>> #1+#2&[4, 5]
```
9

You can use `Function` with named parameters:

```
>> Function[{x, y}, x * y][2, 3]
```
6

Parameters are renamed, when necessary, to avoid confusion:

```
>> Function[{x}, Function[{y}, f[x,
 y]]][y]
```
Function [{y$}, f [y, y$]]

```
>> Function[{y}, f[x, y]] /. x->y
```
Function $[\{y\}, f[y,y]]$

>> `Function[y, Function[x, y^x]][x][y]`

x^y

>> `Function[x, Function[y, x^y]][x][y]`

x^y

Slots in inner functions are not affected by outer function application:

>> `g[#] & [h[#]] & [5]`

$g[h[5]]$

Identity

`Identity[`*x*`]`
: is the identity function, which returns *x* unchanged.

>> `Identity[x]`

x

>> `Identity[x, y]`

Identity $[x, y]$

Slot

`#`*n*
: represents the *n*th argument to a pure function.

`#`
: is short-hand for `#1`.

`#0`
: represents the pure function itself.

>> `#`

#1

Unused arguments are simply ignored:

>> `{#1, #2, #3}&[1, 2, 3, 4, 5]`

$\{1, 2, 3\}$

Recursive pure functions can be written using `#0`:

>> `If[#1<=1, 1, #1 #0[#1-1]]& [10]`

3 628 800

SlotSequence

`##`
: is the sequence of arguments supplied to a pure function.

`##`*n*
: starts with the *n*th argument.

>> `Plus[##]& [1, 2, 3]`

6

>> `Plus[##2]& [1, 2, 3]`

5

>> `FullForm[##]`

SlotSequence [1]

10. Code Compilation

Code compilation allows Mathics functions to be run faster.
When LLVM and Python libraries are available, compilation produces LLVM code.

Contents

Compile

`Compile[{`*x1*`, `*x2*`, ...}, `*expr*`]`
: Compiles *expr* assuming each *xi* is a *Real* number.

`Compile[{{`*x1*`, `*t1*`} {`*x2*`, `*t1*`} ...}, `*expr*`]`
: Compiles assuming each *xi* matches type *ti*.

Compilation is performed using llvmlite , or Python's builtin "compile" function.

```
>> cf = Compile[{x, y}, x + 2 y]
```

CompiledFunction $\left[\{x,y\}, x+2y, -\mathit{CompiledCode}-\right]$

```
>> cf[2.5, 4.3]
```

11.1

```
>> cf = Compile[{{x, _Real}}, Sin[x
]]
```

CompiledFunction $\left[\{x\}, \mathrm{Sin}[x], -\mathit{CompiledCode}-\right]$

```
>> cf[1.4]
```

0.98545

Compile supports basic flow control:

```
>> cf = Compile[{{x, _Real}, {y,
_Integer}}, If[x == 0.0 && y <=
0, 0.0, Sin[x ^ y] + 1 / Min[x,
0.5]] + 0.5]
```

CompiledFunction $\left[\{x, y\}, 0.5+\mathrm{If}\left[x{=}{=}0.\&\&y{<}{=}0, 0., \mathrm{Sin}\left[x^{y}\right]+\frac{1}{\mathrm{Min}[x,0.5]}\right], -\mathit{CompiledCode}-\right]$

```
>> cf[3.5, 2]
```

2.18888

Loops and variable assignments are supported usinv Python builtin "compile" function:

```
>> Compile[{{a, _Integer}, {b,
_Integer}}, While[b != 0, {a, b}
 = {b, Mod[a, b]}]; a] (* GCD of
 a, b *)
```

CompiledFunction $\left[\{a, b\}, a, -\mathit{PythonizedCode}-\right]$

CompiledFunction

,

`CompiledFunction[`*args*`...]`
: represents compiled code for evaluating a compiled function.

```
>> sqr = Compile[{x}, x x]
```

CompiledFunction $\left[\{x\},\right.$
$\left.x^2, -CompiledCode-\right]$

```
>> Head[sqr]
```

CompiledFunction

```
>> sqr[2]
```

4.

11. Options and Default Arguments

Contents

Default

`Default[`*f*`]`
: gives the default value for an omitted paramter of *f*.

`Default[`*f*`, `*k*`]`
: gives the default value for a parameter on the *k*th position.

`Default[`*f*`, `*k*`, `*n*`]`
: gives the default value for the *k*th parameter out of *n*.

Assign values to `Default` to specify default values.

```
>> Default[f] = 1
```
1

```
>> f[x_.] := x ^ 2
```

```
>> f[]
```
1

Default values are stored in `DefaultValues`:

```
>> DefaultValues[f]
```
$\{\text{HoldPattern}\left[\text{Default}\left[f\right]\right] :> 1\}$

You can use patterns for *k* and *n*:

```
>> Default[h, k_, n_] := {k, n}
```

Note that the position of a parameter is relative to the pattern, not the matching expression:

```
>> h[] /. h[___, ___, x_., y_., ___
   ] -> {x, y}
```
$\{\{3,5\},\{4,5\}\}$

FilterRules

`FilterRules[`*rules*`, `*pattern*`]`
: gives those *rules* that have a left side that matches *pattern*.

`FilterRules[`*rules*`, {`*pattern1*`, `*pattern2*`, ...}]`
: gives those *rules* that have a left side that match at least one of *pattern1*, *pattern2*, ...

```
>> FilterRules[{x -> 100, y ->
   1000}, x]
```
$\{x- > 100\}$

```
>> FilterRules[{x -> 100, y ->
   1000, z -> 10000}, {a, b, x, z}]
```
$\{x- > 100, z- > 10\,000\}$

NotOptionQ

`NotOptionQ[`*expr*`]`
: returns `True` if *expr* does not have the form of a valid option specification.

```
>> NotOptionQ[x]
```
True

```
>> NotOptionQ[2]
```
True

```
>> NotOptionQ["abc"]
```
True

```
>> NotOptionQ[a -> True]
```
False

OptionQ

`OptionQ[`*expr*`]`
: returns `True` if *expr* has the form of a valid option specification.

Examples of option specifications:

```
>> OptionQ[a -> True]
```
True

```
>> OptionQ[a :> True]
```
True

```
>> OptionQ[{a -> True}]
```
True

```
>> OptionQ[{a :> True}]
```
True

Options lists are flattened when are applyied, so

```
>> OptionQ[{a -> True, {b->1, "c
   "->2}}]
```
True

```
>> OptionQ[{a -> True, {b->1, c}}]
```
False

```
>> OptionQ[{a -> True, F[b->1,c
   ->2]}]
```
False

`OptionQ` returns `False` if its argument is not a valid option specification:

```
>> OptionQ[x]
```
False

OptionValue

`OptionValue[`*name*`]`
: gives the value of the option *name* as specified in a call to a function with `OptionsPattern`.

`OptionValue[`*f*`, `*name*`]`
: recover the value of the option *name* associated to the symbol *f*.

`OptionValue[`*f*`, `*optvals*`, `*name*`]`
: recover the value of the option *name* associated to the symbol *f*, extracting the values from *optvals* if available.

`OptionValue[..., `*list*`]`
: recover the value of the options in *list* .

```
>> f[a->3] /. f[OptionsPattern[{}]]
   -> {OptionValue[a]}
```
$\{3\}$

Unavailable options generate a message:

```
>> f[a->3] /. f[OptionsPattern[{}]]
   -> {OptionValue[b]}
```
Optionnamebnotfound.

$\{b\}$

The argument of `OptionValue` must be a symbol:

```
>> f[a->3] /. f[OptionsPattern[{}]]
   -> {OptionValue[a+b]}
```
Argumenta
+ batposition1isexpectedtobeasymbol.

$\{\text{OptionValue}[a+b]\}$

However, it can be evaluated dynamically:

```
>> f[a->5] /. f[OptionsPattern[{}]]
   -> {OptionValue[Symbol["a"]]}
```
$\{5\}$

Options

`Options[`*f*`]`
: gives a list of optional arguments to *f* and their default values.

You can assign values to `Options` to specify options.

```
>> Options[f] = {n -> 2}
```
$\{n- > 2\}$

```
>> Options[f]
```
$\{n{:>}2\}$

```
>> f[x_, OptionsPattern[f]] := x ^
   OptionValue[n]
```

```
>> f[x]
```
x^2

```
>> f[x, n -> 3]
```
x^3

Delayed option rules are evaluated just when the corresponding `OptionValue` is called:

```
>> f[a :> Print["value"]] /. f[
   OptionsPattern[{}]] :> (
   OptionValue[a]; Print["between
   "]; OptionValue[a]);
```

value
between
value

In contrast to that, normal option rules are evaluated immediately:

```
>> f[a -> Print["value"]] /. f[
   OptionsPattern[{}]] :> (
   OptionValue[a]; Print["between
   "]; OptionValue[a]);
```

value
between

Options must be rules or delayed rules:

```
>> Options[f] = {a}
```

$\{a\}isnotavalidlistofoptionrules.$

$\{a\}$

A single rule need not be given inside a list:

```
>> Options[f] = a -> b
```

$a- > b$

```
>> Options[f]
```

$\{a{:>}b\}$

Options can only be assigned to symbols:

```
>> Options[a + b] = {a -> b}
```

$Argumenta$
$+ batposition1isexpectedtobeasymbol.$

$\{a- > b\}$

12. Attributes

There are several builtin-attributes which have a predefined meaning in *Mathics*. However, you can set any symbol as an attribute, in contrast to *Mathematica®*.

Contents

Attributes

`Attributes`[*symbol*]
returns the attributes of *symbol*.

`Attributes`["*string*"]
returns the attributes of `Symbol`["*string*"].

`Attributes`[*symbol*] = {*attr1*, *attr2*}
sets the attributes of *symbol*, replacing any existing attributes.

```
>> Attributes[Plus]
```
{Flat, Listable, NumericFunction, OneIdentity, Orderless, Protected}

```
>> Attributes["Plus"]
```
{Flat, Listable, NumericFunction, OneIdentity, Orderless, Protected}

`Attributes` always considers the head of an expression:

```
>> Attributes[a + b + c]
```
{Flat, Listable, NumericFunction, OneIdentity, Orderless, Protected}

You can assign values to `Attributes` to set attributes:

```
>> Attributes[f] = {Flat, Orderless}
```
{Flat, Orderless}

```
>> f[b, f[a, c]]
```
$f[a, b, c]$

Attributes must be symbols:

```
>> Attributes[f] := {a + b}
```
Argumenta
+ batposition1isexpectedtobeasymbol.

$Failed

Use `Symbol` to convert strings to symbols:

```
>> Attributes[f] = Symbol["Listable"]
```
Listable

```
>> Attributes[f]
```
{Listable}

ClearAttributes

`ClearAttributes`[*symbol*, *attrib*]
removes *attrib* from *symbol*'s attributes.

```
>> SetAttributes[f, Flat]
```

```
>> Attributes[f]
```
{Flat}

```
>> ClearAttributes[f, Flat]
```

```
>> Attributes[f]
```

{}

Attributes that are not even set are simply ignored:

```
>> ClearAttributes[{f}, {Flat}]
```

```
>> Attributes[f]
```

{}

Constant

`Constant`
: is an attribute that indicates that a symbol is a constant.

Mathematical constants like `E` have attribute `Constant`:

```
>> Attributes[E]
```

{Constant, Protected, ReadProtected}

Constant symbols cannot be used as variables in `Solve` and related functions:

```
>> Solve[x + E == 0, E]
```

Eisnotavalidvariable.

Solve $[E + x{==}0, E]$

Flat

`Flat`
: is an attribute that specifies that nested occurrences of a function should be automatically flattened.

A symbol with the `Flat` attribute represents an associative mathematical operation:

```
>> SetAttributes[f, Flat]
```

```
>> f[a, f[b, c]]
```

$f[a, b, c]$

`Flat` is taken into account in pattern matching:

```
>> f[a, b, c] /. f[a, b] -> d
```

$f[d, c]$

HoldAll

`HoldAll`
: is an attribute specifying that all arguments of a function should be left unevaluated.

```
>> Attributes[Function]
```

{HoldAll, Protected}

HoldAllComplete

`HoldAllComplete`
: is an attribute that includes the effects of `HoldAll` and `SequenceHold`, and also protects the function from being affected by the upvalues of any arguments.

`HoldAllComplete` even prevents upvalues from being used, and includes `SequenceHold`.

```
>> SetAttributes[f, HoldAllComplete
   ]
```

```
>> f[a] ^= 3;
```

```
>> f[a]
```

$f[a]$

```
>> f[Sequence[a, b]]
```

$f\,[\text{Sequence}\,[a, b]]$

HoldFirst

`HoldFirst`
: is an attribute specifying that the first argument of a function should be left unevaluated.

```
>> Attributes[Set]
```

{HoldFirst, Protected, SequenceHold}

HoldRest

`HoldRest`
: is an attribute specifying that all but the first argument of a function should be left unevaluated.

```
>> Attributes[If]
```
{HoldRest, Protected}

Listable

`Listable`
is an attribute specifying that a function should be automatically applied to each element of a list.

```
>> SetAttributes[f, Listable]
```

```
>> f[{1, 2, 3}, {4, 5, 6}]
```
$\{f[1,4], f[2,5], f[3,6]\}$

```
>> f[{1, 2, 3}, 4]
```
$\{f[1,4], f[2,4], f[3,4]\}$

```
>> {{1, 2}, {3, 4}} + {5, 6}
```
$\{\{6,7\}, \{9,10\}\}$

Locked

`Locked`
is an attribute that prevents attributes on a symbol from being modified.

The attributes of `Locked` symbols cannot be modified:

```
>> Attributes[lock] = {Flat, Locked
};
```

```
>> SetAttributes[lock, {}]
```
Symbollockislocked.

```
>> ClearAttributes[lock, Flat]
```
Symbollockislocked.

```
>> Attributes[lock] = {}
```
Symbollockislocked.

{}

```
>> Attributes[lock]
```
{Flat, Locked}

However, their values might be modified (as long as they are not `Protected` too):

```
>> lock = 3
```
3

NHoldAll

`NHoldAll`
is an attribute that protects all arguments of a function from numeric evaluation.

```
>> N[f[2, 3]]
```
$f[2.,3.]$

```
>> SetAttributes[f, NHoldAll]
```

```
>> N[f[2, 3]]
```
$f[2,3]$

NHoldFirst

`NHoldFirst`
is an attribute that protects the first argument of a function from numeric evaluation.

NHoldRest

`NHoldRest`
is an attribute that protects all but the first argument of a function from numeric evaluation.

OneIdentity

`OneIdentity`
is an attribute specifying that $f[x]$ should be treated as equivalent to x in pattern matching.

`OneIdentity` affects pattern matching:

```
>> SetAttributes[f, OneIdentity]
```

```
>> a /. f[args___] -> {args}
```
$\{a\}$

It does not affect evaluation:

```
>> f[a]
```
$f[a]$

Orderless

`Orderless`
: is an attribute that can be assigned to a symbol *f* to indicate that the elements *ei* in expressions of the form *f*[*e1*, *e2*, ...] should automatically be sorted into canonical order. This property is accounted for in pattern matching.

The leaves of an `Orderless` function are automatically sorted:

```
>> SetAttributes[f, Orderless]

>> f[c, a, b, a + b, 3, 1.0]
```
$f[1., 3, a, b, c, a + b]$

A symbol with the `Orderless` attribute represents a commutative mathematical operation.

```
>> f[a, b] == f[b, a]
```
True

`Orderless` affects pattern matching:

```
>> SetAttributes[f, Flat]

>> f[a, b, c] /. f[a, c] -> d
```
$f[b, d]$

Protect

`Protect[`*s1*, *s2*, ...`]`
: sets the attribute `Protected` for the symbols *si*.

`Protect[`*str1*, *str2*, ...`]`
: protects all symbols whose names textually match *stri*.

```
>> A = {1, 2, 3};

>> Protect[A]

>> A[[2]] = 4;
```
Symbol A is Protected.

```
>> A
```
$\{1, 2, 3\}$

Protected

`Protected`
: is an attribute that prevents values on a symbol from being modified.

Values of `Protected` symbols cannot be modified:

```
>> Attributes[p] = {Protected};

>> p = 2;
```
Symbol p is Protected.

```
>> f[p] ^= 3;
```
Tag p in f[p] is Protected.

```
>> Format[p] = "text";
```
Symbol p is Protected.

However, attributes might still be set:

```
>> SetAttributes[p, Flat]

>> Attributes[p]
```
{Flat, Protected}

Thus, you can easily remove the attribute `Protected`:

```
>> Attributes[p] = {};

>> p = 2
```
2

You can also use `Protect` or `Unprotect`, resp.

```
>> Protect[p]

>> Attributes[p]
```
{Protected}

```
>> Unprotect[p]
```

If a symbol is `Protected` and `Locked`, it can never be changed again:

```
>> SetAttributes[p, {Protected,
   Locked}]

>> p = 2
```
Symbol p is Protected.

2

```
>> Unprotect[p]
```
Symbol p is locked.

ReadProtected

`ReadProtected`
: is an attribute that prevents values on a symbol from being read.

Values associated with `ReadProtected` symbols cannot be seen in `Definition`:

```
>> ClearAll[p]
```

```
>> p = 3;
```

```
>> Definition[p]
```

$$p = 3$$

```
>> SetAttributes[p, ReadProtected]
```

```
>> Definition[p]
```

$$\text{Attributes}\left[p\right] = \{\text{ReadProtected}\}$$

SequenceHold

`SequenceHold`
: is an attribute that prevents `Sequence` objects from being spliced into a function's arguments.

Normally, `Sequence` will be spliced into a function:

```
>> f[Sequence[a, b]]
```

$f\,[a, b]$

It does not for `SequenceHold` functions:

```
>> SetAttributes[f, SequenceHold]
```

```
>> f[Sequence[a, b]]
```

$f\left[\text{Sequence}\,[a, b]\right]$

E.g., `Set` has attribute `SequenceHold` to allow assignment of sequences to variables:

```
>> s = Sequence[a, b];
```

```
>> s
```

$\text{Sequence}\,[a, b]$

```
>> Plus[s]
```

$a + b$

SetAttributes

`SetAttributes[`*symbol*, *attrib*`]`
: adds *attrib* to the list of *symbol*'s attributes.

```
>> SetAttributes[f, Flat]
```

```
>> Attributes[f]
```

$\{\text{Flat}\}$

Multiple attributes can be set at the same time using lists:

```
>> SetAttributes[{f, g}, {Flat,
   Orderless}]
```

```
>> Attributes[g]
```

$\{\text{Flat}, \text{Orderless}\}$

Unprotect

`Unprotect[`*s1*, *s2*, ...`]`
: removes the attribute `Protected` for the symbols *si*.

`Unprotect[`*str*`]`
: unprotects symbols whose names textually match *str*.

13. Tensors

In mathematics, a tensor is an algebraic object that describes a (multilinear) relationship between sets of algebraic objects related to a vector space. Objects that tensors may map between include vectors and scalars, and even other tensors.

There are many types of tensors, including scalars and vectors (which are the simplest tensors), dual vectors, multilinear maps between vector spaces, and even some operations such as the dot product. Tensors are defined independent of any basis, although they are often referred to by their components in a basis related to a particular coordinate system.

Mathics represents tensors of vectors and matrices as lists; tensors of any rank can be handled.

Contents

ArrayDepth

`ArrayDepth[`*a*`]`
: returns the depth of the non-ragged array *a*, defined as `Length[Dimensions[`*a*`]]`.

>> `ArrayDepth[{{a,b},{c,d}}]`

2

>> `ArrayDepth[x]`

0

ArrayQ

`ArrayQ[`*expr*`]`
: tests whether *expr* is a full array.

`ArrayQ[`*expr*`, `*pattern*`]`
: also tests whether the array depth of *expr* matches *pattern*.

`ArrayQ[`*expr*`, `*pattern*`, `*test*`]`
: furthermore tests whether *test* yields `True` for all elements of *expr*. `ArrayQ[`*expr*`]` is equivalent to `ArrayQ[`*expr*`, _, True&]`.

>> `ArrayQ[a]`

False

>> `ArrayQ[{a}]`

True

>> `ArrayQ[{{{a}},{{b,c}}}]`

False

>> `ArrayQ[{{a, b}, {c, d}}, 2, SymbolQ]`

True

DiagonalMatrix

`DiagonalMatrix[`*list*`]`
: gives a matrix with the values in *list* on its diagonal and zeroes elsewhere.

>> `DiagonalMatrix[{1, 2, 3}]`

$\{\{1,0,0\},\{0,2,0\},\{0,0,3\}\}$

```
>> MatrixForm[%]
```
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Dimensions

`Dimensions[`*expr*`]`
: returns a list of the dimensions of the expression *expr*.

A vector of length 3:
```
>> Dimensions[{a, b, c}]
```
$\{3\}$

A 3x2 matrix:
```
>> Dimensions[{{a, b}, {c, d}, {e, f}}]
```
$\{3,2\}$

Ragged arrays are not taken into account:
```
>> Dimensions[{{a, b}, {b, c}, {c, d, e}}]
```
$\{3\}$

The expression can have any head:
```
>> Dimensions[f[f[a, b, c]]]
```
$\{1,3\}$

Dot (.)

`Dot[`*x*`, `*y*`]`
x `.` *y*
: computes the vector dot product or matrix product *x* . *y*.

Scalar product of vectors:
```
>> {a, b, c} . {x, y, z}
```
$ax + by + cz$

Product of matrices and vectors:
```
>> {{a, b}, {c, d}} . {x, y}
```
$\{ax + by, cx + dy\}$

Matrix product:
```
>> {{a, b}, {c, d}} . {{r, s}, {t, u}}
```
$\{\{ar + bt, as + bu\}, \{cr + dt, cs + du\}\}$

```
>> a . b
```
$a.b$

IdentityMatrix

`IdentityMatrix[`*n*`]`
: gives the identity matrix with *n* rows and columns.

```
>> IdentityMatrix[3]
```
$\{\{1,0,0\}, \{0,1,0\}, \{0,0,1\}\}$

Inner

`Inner[`*f*`, `*x*`, `*y*`, `*g*`]`
: computes a generalised inner product of *x* and *y*, using a multiplication function *f* and an addition function *g*.

```
>> Inner[f, {a, b}, {x, y}, g]
```
$g\left[f[a,x], f[b,y]\right]$

Inner can be used to compute a dot product:
```
>> Inner[Times, {a, b}, {c, d}, Plus] == {a, b} . {c, d}
```
True

The inner product of two boolean matrices:
```
>> Inner[And, {{False, False}, {False, True}}, {{True, False}, {True, True}}, Or]
```
{{False, False}, {True, True}}

Inner works with tensors of any depth:
```
>> Inner[f, {{{a, b}}, {{c, d}}}, {{1}, {2}}, g]
```
$\{\{\{g\left[f[a,1], f[b,2]\right]\}\}, \{\{g\left[f[c,1], f[d,2]\right]\}\}\}$

MatrixQ

`MatrixQ[m]`
: returns `True` if m is a list of equal-length lists.

`MatrixQ[m, f]`
: only returns `True` if $f[x]$ returns `True` for each element x of the matrix m.

```
>> MatrixQ[{{1, 3}, {4.0, 3/2}}, NumberQ]
```

True

Outer

`Outer[f, x, y]`
: computes a generalised outer product of x and y, using the function f in place of multiplication.

```
>> Outer[f, {a, b}, {1, 2, 3}]
```

$\{\{f[a,1], f[a,2], f[a,3]\}, \{f[b,1], f[b,2], f[b,3]\}\}$

Outer product of two matrices:

```
>> Outer[Times, {{a, b}, {c, d}}, {{1, 2}, {3, 4}}]
```

$\{\{\{\{a,2a\}, \{3a,4a\}\}, \{\{b, 2b\}, \{3b,4b\}\}\}, \{\{\{c,2c\}, \{3c, 4c\}\}, \{\{d,2d\}, \{3d,4d\}\}\}\}$

`Outer` of multiple lists:

```
>> Outer[f, {a, b}, {x, y, z}, {1, 2}]
```

$\{\{\{f[a,x,1], f[a,x,2]\}, \{f[a,y,1], f[a,y,2]\}, \{f[a,z,1], f[a,z,2]\}\}, \{\{f[b,x,1], f[b,x,2]\}, \{f[b,y,1], f[b,y,2]\}, \{f[b,z,1], f[b,z,2]\}\}\}$

Arrays can be ragged:

```
>> Outer[Times, {{1, 2}}, {{a, b}, {c, d, e}}]
```

$\{\{\{\{a,b\}, \{c,d,e\}\}, \{\{2a,2b\}, \{2c,2d,2e\}\}\}\}$

Word combinations:

```
>> Outer[StringJoin, {"", "re", "un"}, {"cover", "draw", "wind"}, {"", "ing", "s"}] // InputForm
```

{{{"cover", "covering", "covers"}, {"draw", "drawing", "draws"}, {"wind", "winding", "winds"}}, {{"recover", "recovering", "recovers"}, {"redraw", "redrawing", "redraws"}, {"rewind", "rewinding", "rewinds"}}, {{"uncover", "uncovering", "uncovers"}, {"undraw", "undrawing", "undraws"}, {"unwind", "unwinding", "unwinds"}}}

Compositions of trigonometric functions:

```
>> trigs = Outer[Composition, {Sin, Cos, Tan}, {ArcSin, ArcCos, ArcTan}]
```

{{Composition[Sin, ArcSin], Composition[Sin, ArcCos], Composition[Sin, ArcTan]}, {Composition[Cos, ArcSin], Composition[Cos, ArcCos], Composition[Cos, ArcTan]}, {Composition[Tan, ArcSin], Composition[Tan, ArcCos], Composition[Tan, ArcTan]}}

Evaluate at 0:

```
>> Map[#[0] &, trigs, {2}]
```

{{0, 1, 0}, {1, 0, 1}, {0, ComplexInfinity, 0}}

Transpose

`Tranpose[m]`
: transposes rows and columns in the matrix m.

```
>> Transpose[{{1, 2, 3}, {4, 5, 6}}]
```

$\{\{1,4\}, \{2,5\}, \{3,6\}\}$

```
>> MatrixForm[%]
```

$$\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

VectorQ

`VectorQ[`*v*`]`
: returns `True` if *v* is a list of elements which are not themselves lists.

`VectorQ[`*v*`, `*f*`]`
: returns `True` if *v* is a vector and *f*`[`*x*`]` returns `True` for each element *x* of *v*.

```
>> VectorQ[{a, b, c}]
```

True

14. Structural Operations

Contents

Apply (@@)

`Apply[`*f*`, `*expr*`]`
f `@@` *expr*
replaces the head of *expr* with *f*.
`Apply[`*f*`, `*expr*`, `*levelspec*`]`
applies *f* on the parts specified by *levelspec*.

```
>> f @@ {1, 2, 3}
```
$f[1,2,3]$

```
>> Plus @@ {1, 2, 3}
```
6

The head of *expr* need not be `List`:
```
>> f @@ (a + b + c)
```
$f[a,b,c]$

Apply on level 1:
```
>> Apply[f, {a + b, g[c, d, e * f], 3}, {1}]
```
$\{f[a,b], f[c,d,ef], 3\}$

The default level is 0:
```
>> Apply[f, {a, b, c}, {0}]
```
$f[a,b,c]$

Range of levels, including negative level (counting from bottom):
```
>> Apply[f, {{{{{a}}}}}, {2, -3}]
```
$\{\{f[f[\{a\}]]\}\}$

Convert all operations to lists:
```
>> Apply[List, a + b * c ^ e * f[g], {0, Infinity}]
```
$\{a, \{b, \{g\}, \{c,e\}\}\}$

ApplyLevel (@@@)

`ApplyLevel[`*f*`, `*expr*`]`
f `@@@` *expr*
is equivalent to `Apply[`*f*`, `*expr*`, {1}]`.

```
>> f @@@ {{a, b}, {c, d}}
```
$\{f[a,b], f[c,d]\}$

AtomQ

`AtomQ[`*x*`]`
is true if *x* is an atom (an object such as a number or string, which cannot be divided into subexpressions using `Part`).

```
>> AtomQ[x]
```
True

```
>> AtomQ[1.2]
```
True

```
>> AtomQ[2 + I]
```
True

```
>> AtomQ[2 / 3]
```
True

```
>> AtomQ[x + y]
```
False

CombinatoricaOld‘BinarySearch

`CombinatoricaOld‘BinarySearch[`*l*`, `*k*`]`
: searches the list *l*, which has to be sorted, for key *k* and returns its index in *l*. If *k* does not exist in *l*, `BinarySearch` returns (a + b) / 2, where a and b are the indices between which *k* would have to be inserted in order to maintain the sorting order in *l*. Please note that *k* and the elements in *l* need to be comparable under a strict total order (see https://en.wikipedia.org/wiki/Total_order).

`CombinatoricaOld‘BinarySearch[`*l*`, `*k*`, `*f*`]`
: the index of $k in the elements of *l* if *f* is applied to the latter prior to comparison. Note that *f* needs to yield a sorted sequence if applied to the elements of $l.

```
>> CombinatoricaOld‘BinarySearch
   [{3, 4, 10, 100, 123}, 100]
```
4

```
>> CombinatoricaOld‘BinarySearch
   [{2, 3, 9}, 7] // N
```
2.5

```
>> CombinatoricaOld‘BinarySearch
   [{2, 7, 9, 10}, 3] // N
```
1.5

```
>> CombinatoricaOld‘BinarySearch
   [{-10, 5, 8, 10}, -100] // N
```
0.5

```
>> CombinatoricaOld‘BinarySearch
   [{-10, 5, 8, 10}, 20] // N
```
4.5

```
>> CombinatoricaOld‘BinarySearch[{{
   a, 1}, {b, 7}}, 7, #[[2]]&]
```
2

ByteCount

`ByteCount[`*expr*`]`
: gives the internal memory space used by *expr*, in bytes.

The results may heavily depend on the Python implementation in use.

Depth

`Depth[`*expr*`]`
: gives the depth of *expr*.

The depth of an expression is defined as one plus the maximum number of `Part` indices required to reach any part of *expr*, except for heads.

```
>> Depth[x]
```
1

```
>> Depth[x + y]
```
2

```
>> Depth[{{{{x}}}}]
```
5

Complex numbers are atomic, and hence have depth 1:

```
>> Depth[1 + 2 I]
```
1

`Depth` ignores heads:

```
>> Depth[f[a, b][c]]
```
2

Flatten

`Flatten[`*expr*`]`
: flattens out nested lists in *expr*.

`Flatten[`*expr*`, `*n*`]`
: stops flattening at level *n*.

`Flatten[`*expr*`, `*n*`, `*h*`]`
: flattens expressions with head *h* instead of `List`.

```
>> Flatten[{{a, b}, {c, {d}, e}, {f
   , {g, h}}}]
```

$\{a,b,c,d,e,f,g,h\}$

```
>> Flatten[{{a, b}, {c, {e}, e}, {f
   , {g, h}}}, 1]
```

$\{a,b,c,\{e\},e,f,\{g,h\}\}$

```
>> Flatten[f[a, f[b, f[c, d]], e],
   Infinity, f]
```

$f[a,b,c,d,e]$

```
>> Flatten[{{a, b}, {c, d}}, {{2},
   {1}}]
```

$\{\{a,c\},\{b,d\}\}$

```
>> Flatten[{{a, b}, {c, d}}, {{1,
   2}}]
```

$\{a,b,c,d\}$

Flatten also works in irregularly shaped arrays

```
>> Flatten[{{1, 2, 3}, {4}, {6, 7},
    {8, 9, 10}}, {{2}, {1}}]
```

$\{\{1,4,6,8\},\{2,7,9\},\{3,10\}\}$

FreeQ

`FreeQ[`*expr*`, `*x*`]`
: returns `True` if *expr* does not contain the expression *x*.

```
>> FreeQ[y, x]
```

True

```
>> FreeQ[a+b+c, a+b]
```

False

```
>> FreeQ[{1, 2, a^(a+b)}, Plus]
```

False

```
>> FreeQ[a+b, x_+y_+z_]
```

True

```
>> FreeQ[a+b+c, x_+y_+z_]
```

False

```
>> FreeQ[x_+y_+z_][a+b]
```

True

Head

`Head[`*expr*`]`
: returns the head of the expression or atom *expr*.

```
>> Head[a * b]
```

Times

```
>> Head[6]
```

Integer

```
>> Head[x]
```

Symbol

Map (/@)

`Map[`*f*`, `*expr*`]` or *f* `/@` *expr*
: applies *f* to each part on the first level of *expr*.

`Map[`*f*`, `*expr*`, `*levelspec*`]`
: applies *f* to each level specified by *levelspec* of *expr*.

```
>> f /@ {1, 2, 3}
```

$\{f[1],f[2],f[3]\}$

```
>> #^2& /@ {1, 2, 3, 4}
```

$\{1,4,9,16\}$

Map *f* on the second level:

```
>> Map[f, {{a, b}, {c, d, e}}, {2}]
```

$\{\{f[a],f[b]\},\{f[c],f[d],f[e]\}\}$

Include heads:

```
>> Map[f, a + b + c, Heads->True]
```

$f[\text{Plus}][f[a],f[b],f[c]]$

MapAt

`MapAt[`*f*`, `*expr*`, `*n*`]`
: applies *f* to the element at position *n* in *expr*. If *n* is negative, the position is counted from the end.

`MapAt[f, `*expr*`, {`*i*`, `*j*` ...}]`
: applies *f* to the part of *expr* at position {*i*, *j*, ...}.

`MapAt[`*f*`,`*pos*`]`
: represents an operator form of MapAt that can be applied to an expression.

Map *f* onto the part at position 2:

```
>> MapAt[f, {a, b, c, d}, 2]
```

$\{a, f[b], c, d\}$

Map *f* onto multiple parts:

```
>> MapAt[f, {a, b, c, d}, {{1}, {4}}]
```

$\{f[a], b, c, f[d]\}$

Map *f* onto the at the end:

```
>> MapAt[f, {a, b, c, d}, -1]
```

$\{a, b, c, f[d]\}$

Map *f* onto an association:

```
>> MapAt[f, <|"a" -> 1, "b" -> 2, "c" -> 3, "d" -> 4, "e" -> 5|>, 3]
```

$\{\mathrm{a}->1, \mathrm{b}->2, \mathrm{c}->f[3], \mathrm{d}->4, \mathrm{e}->5\}$

Use negative position in an association:

```
>> MapAt[f, <|"a" -> 1, "b" -> 2, "c" -> 3, "d" -> 4|>, -3]
```

$\{\mathrm{a}->1, \mathrm{b}->f[2], \mathrm{c}->3, \mathrm{d}->4\}$

Use the operator form of MapAt:

```
>> MapAt[f, 1][{a, b, c, d}]
```

$\{f[a], b, c, d\}$

MapIndexed

`MapIndexed[`*f*`, `*expr*`]`
: applies *f* to each part on the first level of *expr*, including the part positions in the call to *f*.

`MapIndexed[`*f*`, `*expr*`, `*levelspec*`]`
: applies *f* to each level specified by *levelspec* of *expr*.

```
>> MapIndexed[f, {a, b, c}]
```

$\{f[a, \{1\}], f[b, \{2\}], f[c, \{3\}]\}$

Include heads (index 0):

```
>> MapIndexed[f, {a, b, c}, Heads->True]
```

$f[\mathrm{List}, \{0\}][f[a, \{1\}], f[b, \{2\}], f[c, \{3\}]]$

Map on levels 0 through 1 (outer expression gets index `{}`):

```
>> MapIndexed[f, a + b + c * d, {0, 1}]
```

$f[f[a, \{1\}] + f[b, \{2\}] + f[cd, \{3\}], \{\}]$

Get the positions of atoms in an expression (convert operations to `List` first to disable `Listable` functions):

```
>> expr = a + b * f[g] * c ^ e;
```

```
>> listified = Apply[List, expr, {0, Infinity}];
```

```
>> MapIndexed[#2 &, listified, {-1}]
```

$\{\{1\}, \{\{2,1\}, \{\{2,2,1\}\}, \{\{2,3,1\}, \{2,3,2\}\}\}\}$

Replace the heads with their positions, too:

```
>> MapIndexed[#2 &, listified, {-1}, Heads -> True]
```

$\{0\}[\{1\}, \{2,0\}[\{2,1\}, \{2,2,0\}[\{2,2,1\}], \{2,3,0\}[\{2,3,1\}, \{2,3,2\}]]]$

The positions are given in the same format as used by `Extract`. Thus, mapping `Extract` on the indices given by `MapIndexed` re-constructs the original expression:

```
>> MapIndexed[Extract[expr, #2] &,
   listified, {-1}, Heads -> True]
```

$a + bf[g]c^e$

MapThread

'MapThread[f, {{$a1$, $a2$, ...}, {$b1$, $b2$, ...}, ...}]
: returns {f[$a1$, $b1$, ...], f[$a2$, $b2$, ...], ...}.

`MapThread[`f`, {`$expr1$`, `$expr2$`, ...}, `n`]`
: applies f at level n.

```
>> MapThread[f, {{a, b, c}, {1, 2,
   3}}]
```

$\{f[a,1], f[b,2], f[c,3]\}$

```
>> MapThread[f, {{{a, b}, {c, d}},
   {{e, f}, {g, h}}}, 2]
```

$\{\{f[a,e], f[b,f]\}, \{f[c,g], f[d,h]\}\}$

Null

`Null`
: is the implicit result of expressions that do not yield a result.

```
>> FullForm[a:=b]
```

Null

It is not displayed in StandardForm,

```
>> a:=b
```

in contrast to the empty string:

```
>> ""
```

Operate

`Operate[`p`, `$expr$`]`
: applies p to the head of $expr$.

`Operate[`p`, `$expr$`, `n`]`
: applies p to the nth head of $expr$.

```
>> Operate[p, f[a, b]]
```

$p[f][a,b]$

The default value of n is 1:

```
>> Operate[p, f[a, b], 1]
```

$p[f][a,b]$

With n=0, `Operate` acts like `Apply`:

```
>> Operate[p, f[a][b][c], 0]
```

$p[f[a][b][c]]$

Order

`Order[`x`, `y`]`
: returns a number indicating the canonical ordering of x and y. 1 indicates that x is before y, -1 that y is before x. 0 indicates that there is no specific ordering. Uses the same order as `Sort`.

```
>> Order[7, 11]
```

1

```
>> Order[100, 10]
```

-1

```
>> Order[x, z]
```

1

```
>> Order[x, x]
```

0

OrderedQ

`OrderedQ[`a`, `b`]`
: is `True` if a sorts before b according to canonical ordering.

```
>> OrderedQ[a, b]
```

True

```
>> OrderedQ[b, a]
```

False

PatternsOrderedQ

`PatternsOrderedQ[`$patt1$`, `$patt2$`]`
: returns `True` if pattern $patt1$ would be applied before $patt2$ according to canonical pattern ordering.

>> `PatternsOrderedQ[x__, x_]`
False

>> `PatternsOrderedQ[x_, x__]`
True

>> `PatternsOrderedQ[b, a]`
True

Scan

`Scan[`*f*`, `*expr*`]`
: applies *f* to each element of *expr* and returns `Null`.

'Scan[*f*, *expr*, *levelspec*]
: applies *f* to each level specified by *levelspec* of *expr*.

>> `Scan[Print, {1, 2, 3}]`
1
2
3

Sort

`Sort[`*list*`]`
: sorts *list* (or the leaves of any other expression) according to canonical ordering.

`Sort[`*list*`, `*p*`]`
: sorts using *p* to determine the order of two elements.

>> `Sort[{4, 1.0, a, 3+I}]`
$\{1., 3 + I, 4, a\}$

Sort uses `OrderedQ` to determine ordering by default. You can sort patterns according to their precedence using `PatternsOrderedQ`:

>> `Sort[{items___, item_, OptionsPattern[], item_symbol, item_?test}, PatternsOrderedQ]`
{item_symbol, item_? test, item_, items___, OptionsPattern []}

When sorting patterns, values of atoms do not matter:

>> `Sort[{a, b/;t}, PatternsOrderedQ]`
$\{b/;t, a\}$

>> `Sort[{2+c_, 1+b__}, PatternsOrderedQ]`
{2 + c_, 1 + b__}

>> `Sort[{x_ + n_*y_, x_ + y_}, PatternsOrderedQ]`
{x_ + n_y_, x_ + y_}

SortBy

`SortBy[`*list*`, `*f*`]`
: sorts *list* (or the leaves of any other expression) according to canonical ordering of the keys that are extracted from the *list*'s elements using $f. Chunks of leaves that appear the same under $f are sorted according to their natural order (without applying $f).

`SortBy[`*f*`]`
: creates an operator function that, when applied, sorts by $f.

>> `SortBy[{{5, 1}, {10, -1}}, Last]`
$\{\{10, -1\}, \{5, 1\}\}$

>> `SortBy[Total][{{5, 1}, {10, -9}}]`
$\{\{10, -9\}, \{5, 1\}\}$

SymbolName

`SymbolName[`*s*`]`
: returns the name of the symbol *s* (without any leading context name).

>> `SymbolName[x] // InputForm`
"x"

SymbolQ

`SymbolQ[`*x*`]`
: is `True` if *x* is a symbol, or `False` otherwise.

>> `SymbolQ[a]`
True

```
>> SymbolQ[1]
```
False

```
>> SymbolQ[a + b]
```
False

Symbol

`Symbol`
: is the head of symbols.

```
>> Head[x]
```
Symbol

You can use `Symbol` to create symbols from strings:

```
>> Symbol["x"] + Symbol["x"]
```
$2x$

Thread

`Thread[`*f*`[`*args*`]]`
: threads *f* over any lists that appear in *args*.

`Thread[`*f*`[`*args*`], ` *h*`]`
: threads over any parts with head *h*.

```
>> Thread[f[{a, b, c}]]
```
$\{f[a], f[b], f[c]\}$

```
>> Thread[f[{a, b, c}, t]]
```
$\{f[a,t], f[b,t], f[c,t]\}$

```
>> Thread[f[a + b + c], Plus]
```
$f[a] + f[b] + f[c]$

Functions with attribute `Listable` are automatically threaded over lists:

```
>> {a, b, c} + {d, e, f} + g
```
$\{a + d + g, b + e + g, c + f + g\}$

Through

`Through[`*p*`[`*f*`][`*x*`]]`
: gives $p[f[x]]$.

```
>> Through[f[g][x]]
```
$f[g[x]]$

```
>> Through[p[f, g][x]]
```
$p[f[x], g[x]]$

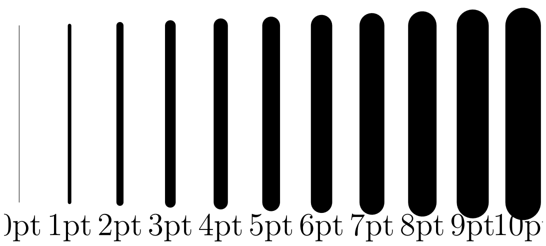
15. Drawing Graphics

Contents

AbsoluteThickness

`AbsoluteThickness[`*p*`]`
sets the line thickness for subsequent graphics primitives to *p* points.

```
>> Graphics[Table[{
   AbsoluteThickness[t], Line[{{20
   t, 10}, {20 t, 80}}], Text[
   ToString[t]<>"pt", {20 t, 0}]},
   {t, 0, 10}]]
```



Arrow

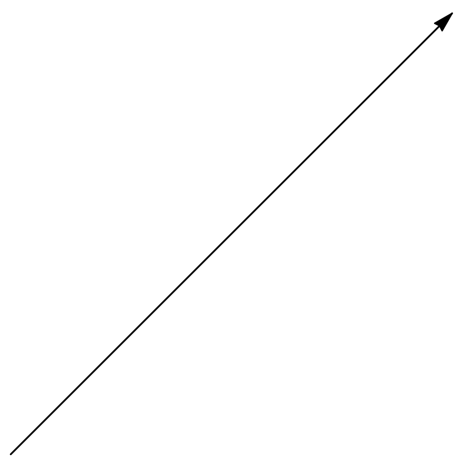
`Arrow[{`*p1*`, `*p2*`}]`
represents a line from *p1* to *p2* that ends with an arrow at *p2*.

`Arrow[{`*p1*`, `*p2*`}, `*s*`]`
represents a line with arrow that keeps a distance of *s* from *p1* and *p2*.

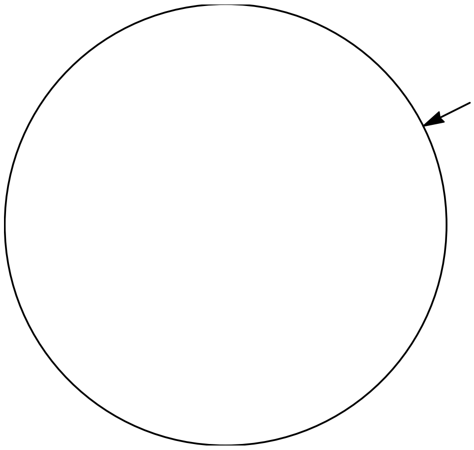
`Arrow[{`*point_1*`, `*point_2*`}, {`*s1*`, `*s2*`}]`
represents a line with arrow that keeps a distance of *s1* from *p1* and a distance of *s2* from *p2*.

`Arrow[{`*point_1*`, `*point_2*`}, {`*s1*`, `*s2*`}]`
represents a line with arrow that keeps a distance of *s1* from *p1* and a distance of *s2* from *p2*.

```
>> Graphics[Arrow[{{0,0}, {1,1}}]]
```

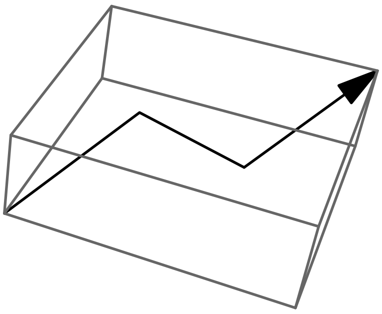


```
>> Graphics[{Circle[], Arrow[{{2,
   1}, {0, 0}}, 1]}]
```



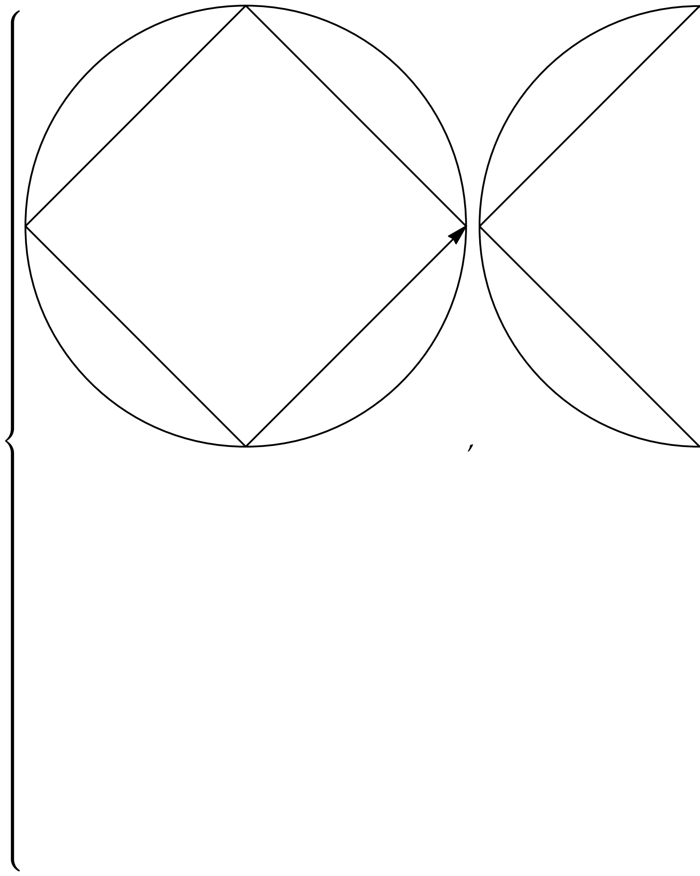
Arrows can also be drawn in 3D by giving poing in three dimensions:

```
>> Graphics3D[Arrow[{{1, 1, -1},
   {2, 2, 0}, {3, 3, -1}, {4, 4,
   0}}]]
```



Keeping distances may happen across multiple segments:

```
>> Table[Graphics[{Circle[], Arrow[
   Table[{Cos[phi],Sin[phi]},{phi
   ,0,2*Pi,Pi/2}],{d, d}]}],{d
   ,0,2,0.5}]
```



Arrowheads

`Arrowheads[`*s*`]`
: specifies that Arrow[] draws one arrow of size *s* (relative to width of image, defaults to 0.04).

`Arrowheads[{`*spec1*`, `*spec2*`, ..., `*specn*`}]`
: specifies that Arrow[] draws n arrows as defined by *spec1*, *spec2*, ... *specn*.

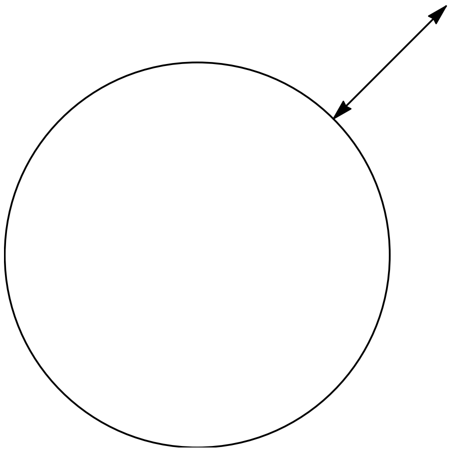
`Arrowheads[{{`*s*`}}]`
: specifies that one arrow of size *s* should be drawn.

`Arrowheads[{{`*s*`, `*pos*`}}]`
: specifies that one arrow of size *s* should be drawn at position *pos* (for the arrow to be on the line, *pos* has to be between 0, i.e. the start for the line, and 1, i.e. the end of the line).

`Arrowheads[{{`*s*`, `*pos*`, `*g*`}}]`
: specifies that one arrow of size *s* should be drawn at position *pos* using Graphics *g*.

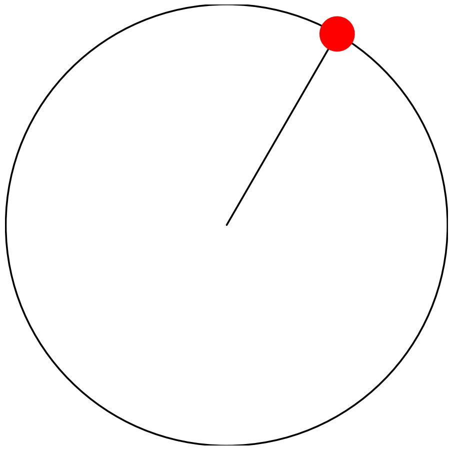
Arrows on both ends can be achieved using negative sizes:

```
>> Graphics[{Circle[],Arrowheads
   [{-0.04, 0.04}], Arrow[{{0, 0},
   {2, 2}}, {1,1}]}]
```

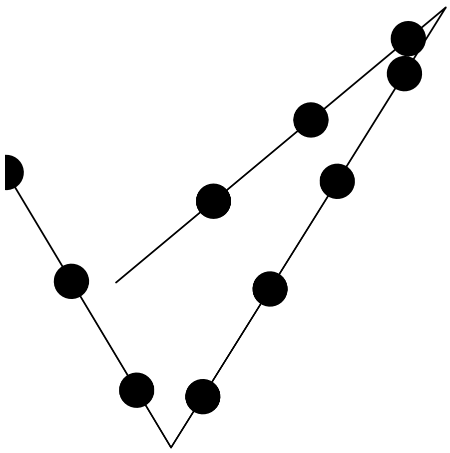


You may also specify our own arrow shapes:

```
>> Graphics[{Circle[], Arrowheads
   [{{0.04, 1, Graphics[{Red, Disk
   []}]}}], Arrow[{{0, 0}, {Cos[Pi
   /3],Sin[Pi/3]}}]}]
```



```
>> Graphics[{Arrowheads[Table
   [{0.04, i/10, Graphics[Disk
   []]},{i,1,10}]], Arrow[{{0, 0},
   {6, 5}, {1, -3}, {-2, 2}}]}]
```



Circle

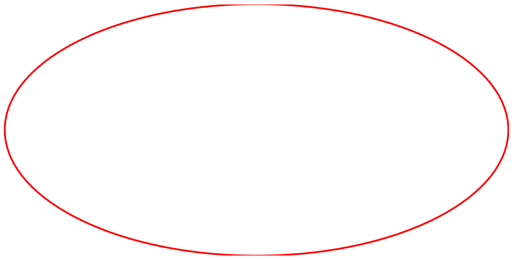
`Circle[{`*cx*`, `*cy*`}, `*r*`]`
: draws a circle with center (*cx*, *cy*) and radius *r*.

`Circle[{`*cx*`, `*cy*`}, {`*rx*`, `*ry*`}]`
: draws an ellipse.

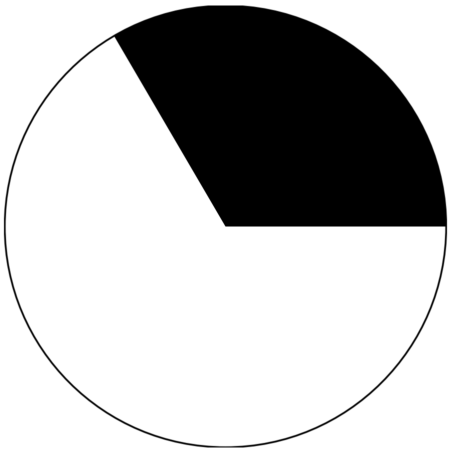
`Circle[{`*cx*`, `*cy*`}]`
: chooses radius 1.

`Circle[]`
: chooses center `(0, 0)` and radius 1.

```
>> Graphics[{Red, Circle[{0, 0}, {2, 1}]}]
```

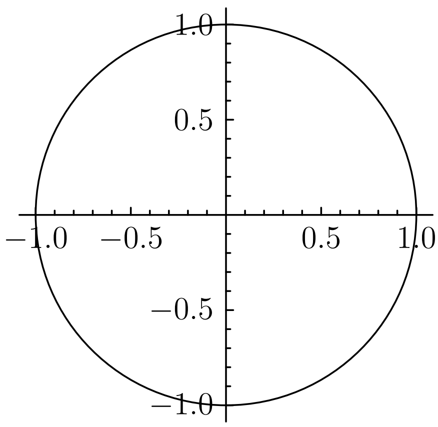


```
>> Graphics[{Circle[], Disk[{0, 0}, {1, 1}, {0, 2.1}]}]
```



Target practice:

```
>> Graphics[Circle[], Axes-> True]
```



Disk

`Disk[{cx, cy}, r]`
: fills a circle with center (*cx*, *cy*) and radius *r*.

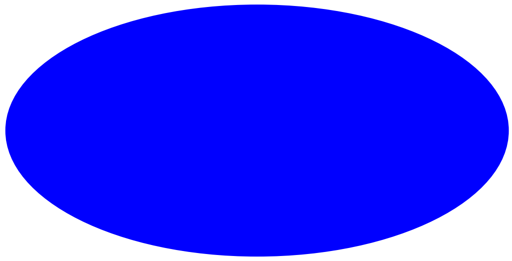
`Disk[{cx, cy}, {rx, ry}]`
: fills an ellipse.

`Disk[{cx, cy}]`
: chooses radius 1.

`Disk[]`
: chooses center `(0, 0)` and radius 1.

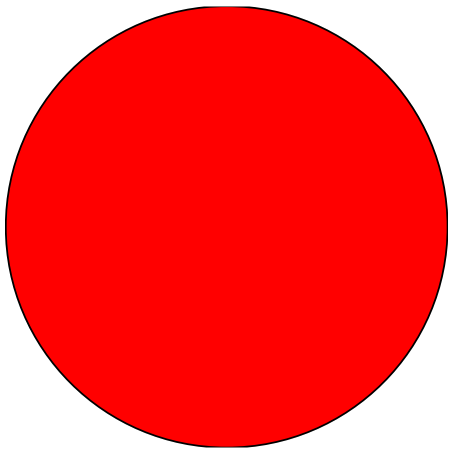
`Disk[{x, y}, ..., {t1, t2}]`
: is a sector from angle *t1* to *t2*.

```
>> Graphics[{Blue, Disk[{0, 0}, {2, 1}]}]
```



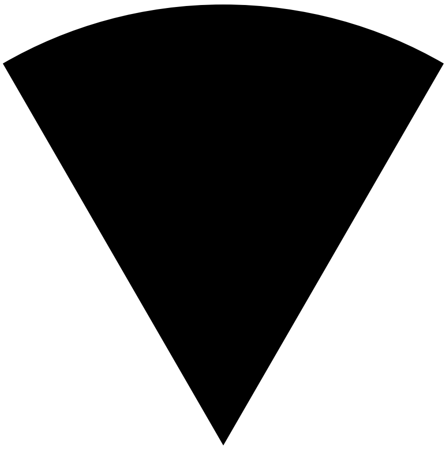
The outer border can be drawn using `EdgeForm`:

```
>> Graphics[{EdgeForm[Black], Red, Disk[]}]
```

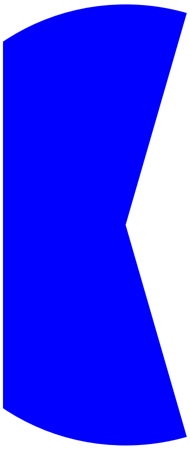


Disk can also draw sectors of circles and ellipses

```
>> Graphics[Disk[{0, 0}, 1, {Pi /
   3, 2 Pi / 3}]]
```

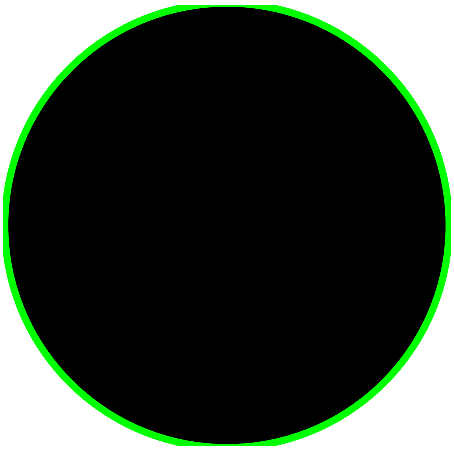


```
>> Graphics[{Blue, Disk[{0, 0}, {1,
    2}, {Pi / 3, 5 Pi / 3}]}]
```

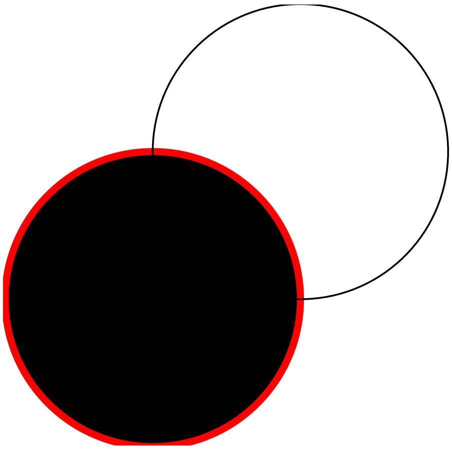


EdgeForm

```
>> Graphics[{EdgeForm[{Thick, Green
   }], Disk[]}]
```



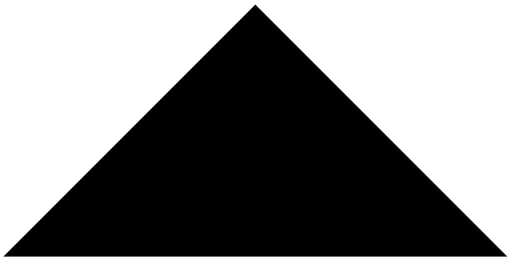
```
>> Graphics[{Style[Disk[],EdgeForm
   [{Thick,Red}]], Circle[{1,1}]}]
```



FilledCurve

`FilledCurve[{`*segment1*`, `*segment2* `...}]`
    represents a filled curve.

```
>> Graphics[FilledCurve[{Line[{{0,
   0}, {1, 1}, {2, 0}}]}]]
```



```
>> Graphics[FilledCurve[{
   BezierCurve[{{0, 0}, {1, 1}, {2,
    0}}], Line[{{3, 0}, {0, 2}}]}]]
```

FontColor

`FontColor`
: is an option for Style to set the font color.

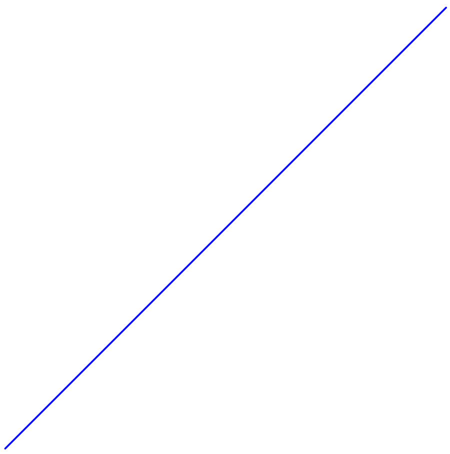
Graphics

`Graphics[`*primitives*`, `*options*`]`
: represents a graphic.

Options include:

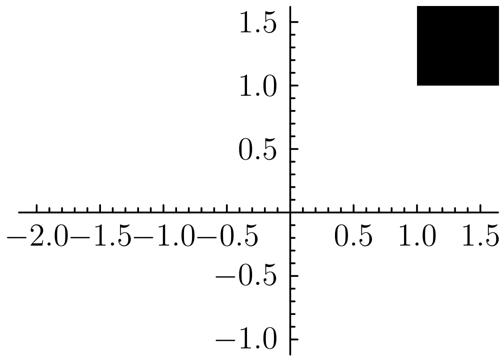
- Axes
- TicksStyle
- AxesStyle
- LabelStyle
- AspectRatio
- PlotRange
- PlotRangePadding
- ImageSize
- Background

```
>> Graphics[{Blue, Line[{{0,0},
   {1,1}}]}]
```



`Graphics` supports `PlotRange`:

```
>> Graphics[{Rectangle[{1, 1}]},
   Axes -> True, PlotRange -> {{-2,
    1.5}, {-1, 1.5}}]
```




```
>> Graphics[{Rectangle[],Red,Disk
   [{1,0}]},PlotRange
   ->{{0,1},{0,1}}]
```

`Graphics` produces `GraphicsBox` boxes:

```
>> Graphics[Rectangle[]] // ToBoxes
    // Head
```

GraphicsBox

In `TeXForm`, `Graphics` produces Asymptote figures:

```
>> Graphics[Circle[]] // TeXForm
```

\begin{asy}
usepackage("amsmath");
size(5.8556cm, 5.8333cm);
draw(ellipse((175,175),175,175),
rgb(0, 0, 0)+linewidth(0.66667));
clip(box((-0.33333,0.33333),
(350.33,349.67)));
\end{asy}

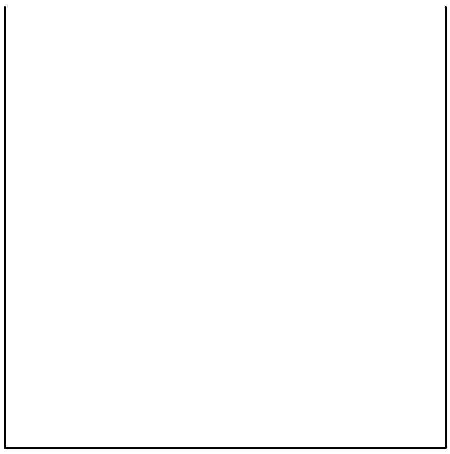
Large

`ImageSize -> Large`
produces a large image.

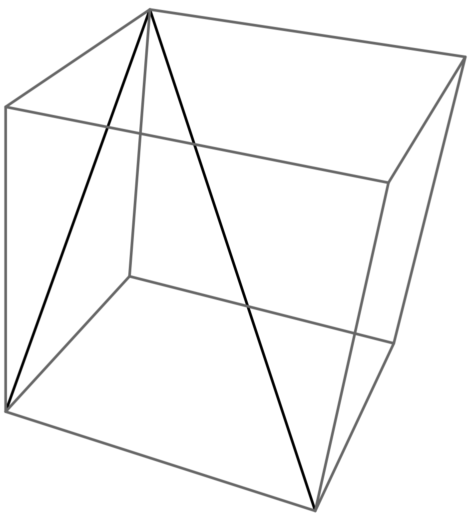
Line

`Line[{`*point_1*`, `*point_2*` ...}]`
represents the line primitive.
`Line[{{`*p_11*`, `*p_12*`, ...}, {`*p_21*`, `*p_22*`, ...}, ...}]`
represents a number of line primitives.

```
>> Graphics[Line
   [{{0,1},{0,0},{1,0},{1,1}}]]
```



```
>> Graphics3D[Line
   [{{0,0,0},{0,1,1},{1,0,0}}]]
```



Medium

`ImageSize -> Medium`
produces a medium-sized image.

Point

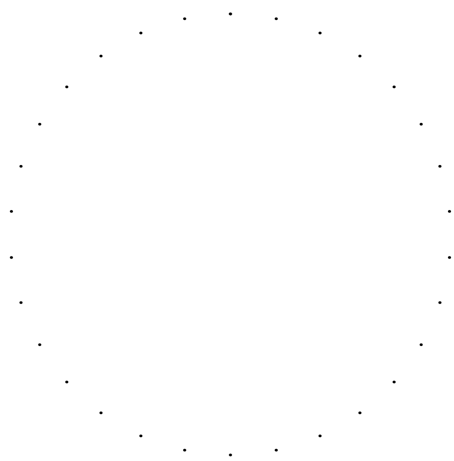
`Point[{`*point_1*`, `*point_2*` ...}]`
represents the point primitive.
`Point[{{`*p_11*`, `*p_12*`, ...}, {`*p_21*`, `*p_22*`, ...}, ...}]`
represents a number of point primitives.

Points are rendered if possible as circular regions. Their diameters can be specified using `PointSize`.
Points can be specified as $\{x, y\}$:

```
>> Graphics[Point[{0, 0}]]
```

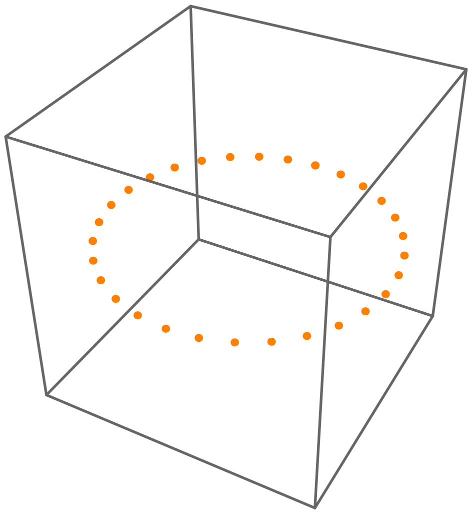


```
>> Graphics[Point[Table[{Sin[t],
   Cos[t]}, {t, 0, 2. Pi, Pi /
   15.}]]]
```



or as $\{x, y, z\}$:

```
>> Graphics3D[{Orange, PointSize[0.05], Point[Table[{Sin[t], Cos[t], 0}, {t, 0, 2 Pi, Pi / 15.}]]}]
```

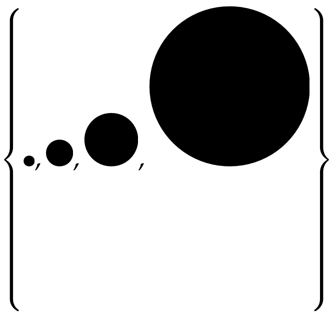


PointSize

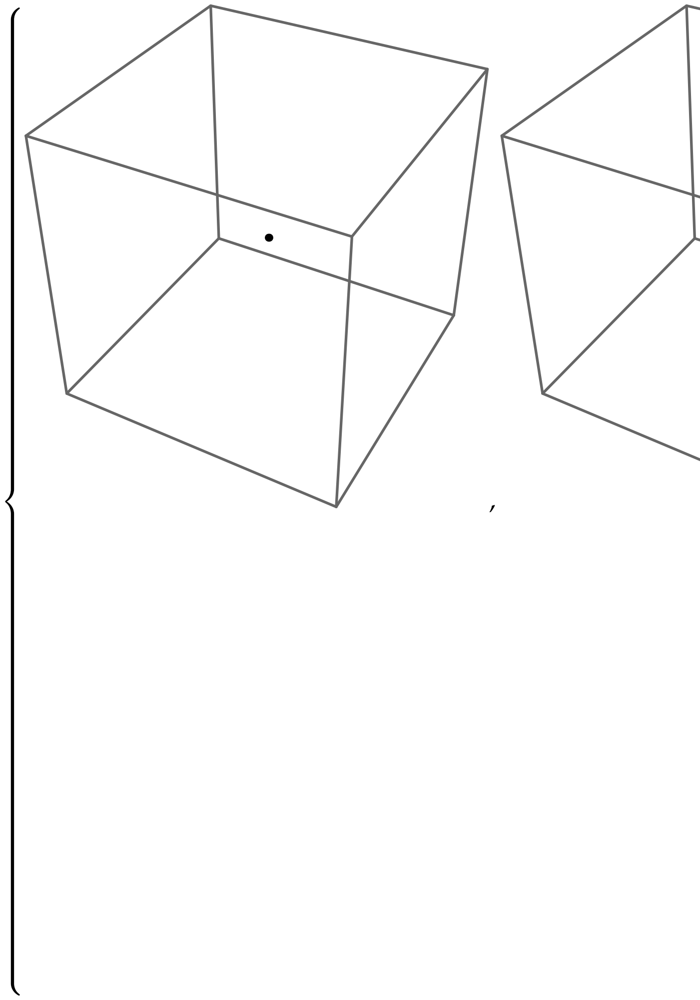
`PointSize[`*t*`]`
: sets the diameter of points to *t*, which is relative to the overall width.

`PointSize` can be used for both two- and three-dimensional graphics. The initial default pointsize is 0.008 for two-dimensional graphics and 0.01 for three-dimensional graphics.

```
>> Table[Graphics[{PointSize[r], Point[{0, 0}]}], {r, {0.02, 0.05, 0.1, 0.3}}]
```



```
>> Table[Graphics3D[{PointSize[r], Point[{0, 0, 0}]}], {r, {0.05, 0.1, 0.8}}]
```



Polygon

`Polygon[{`*point_1*`, `*point_2*` ...}]`
: represents the filled polygon primitive.

`Polygon[{{`*p_11*`, `*p_12*`, ...}, {`*p_21*`, `*p_22*`, ...}, ...}]`
: represents a number of filled polygon primitives.

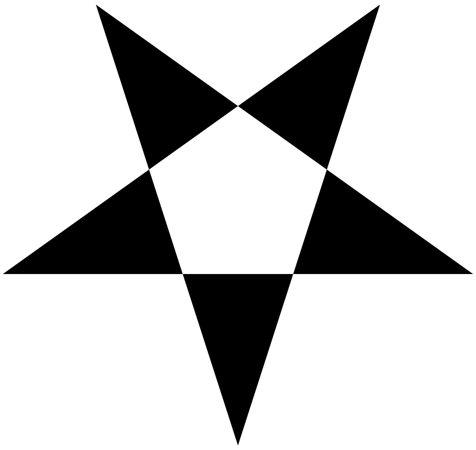
A Right Triangle:

```
>> Graphics[Polygon
   [{{1,0},{0,0},{0,1}}]]
```

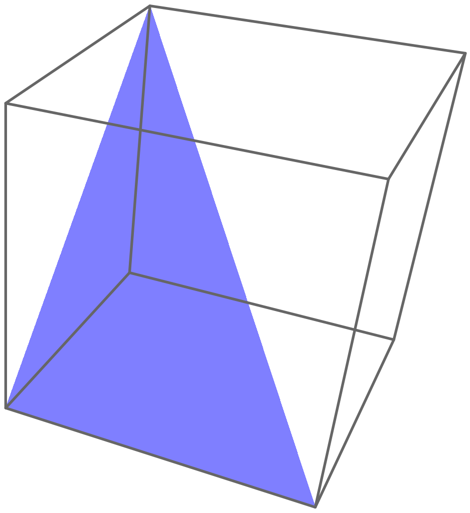
Notice that there is a line connecting from the last point to the first one.

A point is an element of the polygon if a ray from the point in any direction in the plane crosses the boundary line segments an odd number of times.

```
>> Graphics[Polygon
   [{{150,0},{121,90},{198,35},{102,35},{179,
```



```
>> Graphics3D[Polygon
   [{{0,0,0},{0,1,1},{1,0,0}}]]
```

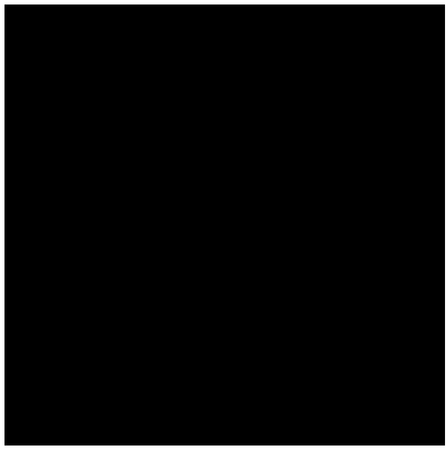


Rectangle

`Rectangle[{`*xmin*`, `*ymin*`}]`
: represents a unit square with bottom-left corner at {*xmin*, *ymin*}.

'Rectangle[{*xmin*, *ymin*}, {*xmax*, *ymax*}]
: is a rectange extending from {*xmin*, *ymin*} to {*xmax*, *ymax*}.

```
>> Graphics[Rectangle[]]
```



```
>> Graphics[{Blue, Rectangle[{0.5, 0}], Orange, Rectangle[{0, 0.5}]}]
```

RegularPolygon

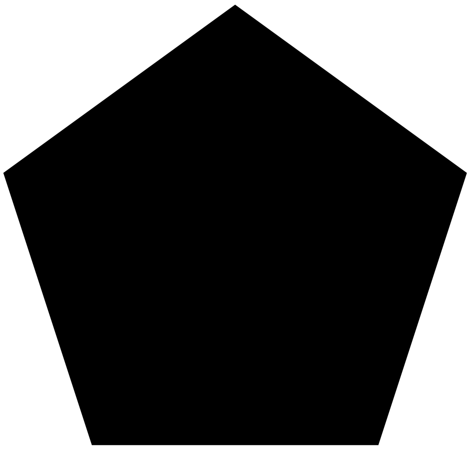
`RegularPolygon[`*n*`]`
: gives the regular polygon with *n* edges.

`RegularPolygon[`*r*`, `*n*`]`
: gives the regular polygon with *n* edges and radius *r*.

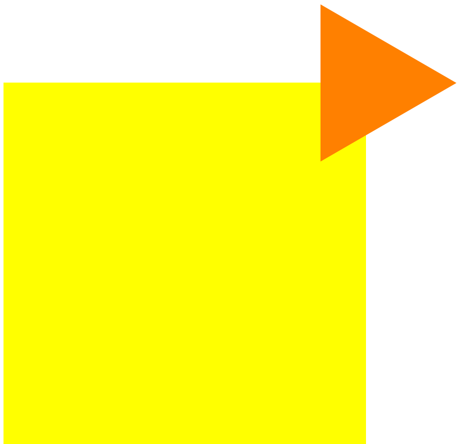
`RegularPolygon[{`*r*`, `*phi*`}, `*n*`]`
: gives the regular polygon with radius *r* with one vertex drawn at angle *phi*.

`RegularPolygon[{$x, $y}, `*r*`, `*n*`]`
: gives the regular polygon centered at the position {$x, $y}.

```
>> Graphics[RegularPolygon[5]]
```



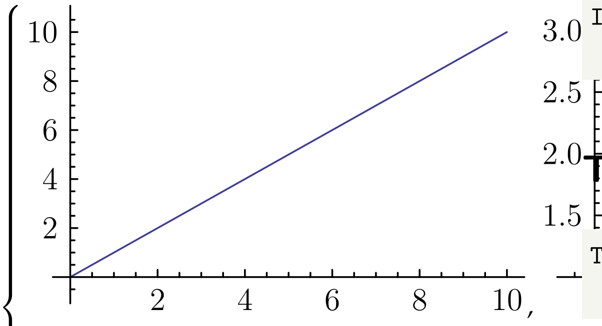
```
>> Graphics[{Yellow, Rectangle[], Orange, RegularPolygon[{1, 1}, {0.25, 0}, 3]}]
```



Show

`Show[`*graphics*`, `*options*`]`
: shows a list of graphics with the specified options added.

```
>> Show[{Plot[x, {x, 0, 10}],
   ListPlot[{1,2,3}]}]
```



− > Ignore, AspectRatio
− > Automatic, Axes
− > False, AxesStyle
− > {} , Background
− > Automatic, ImageSize
− > Automatic, LabelStyle
− > {} , PlotRange
− > Automatic, PlotRangePadding

− > Automatic, TicksStyle− > {} }

ıSyntax

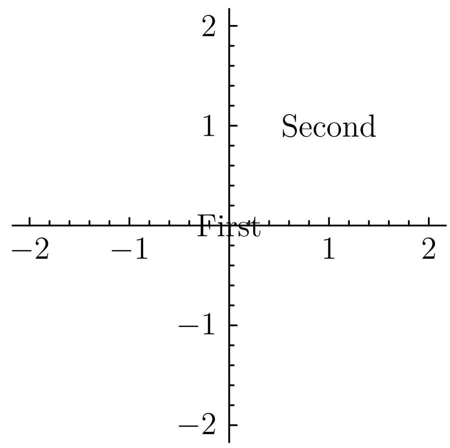
Small

`ImageSize -> Small`
  produces a small image.

Text

`Text["`*text*`", {`x`, `y`}]`
  draws *text* centered on position {x, y}.

```
>> Graphics[{Text["First", {0, 0}],
    Text["Second", {1, 1}]}, Axes->
   True, PlotRange->{{-2, 2}, {-2,
   2}}]
```



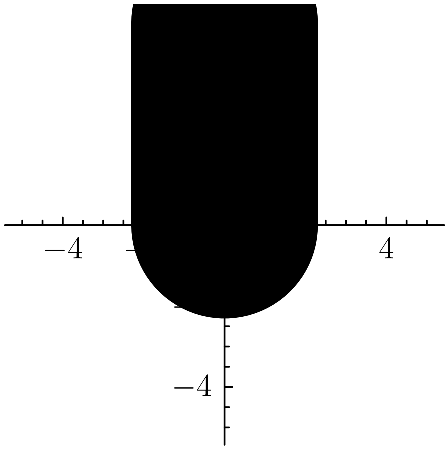
Thick

`Thick`
  sets the line width for subsequent graphics primitives to 2pt.

Thickness

`Thickness[`t`]`
  sets the line thickness for subsequent graphics primitives to t times the size of the plot area.

```
>> Graphics[{Thickness[0.2], Line
   [{{0, 0}, {0, 5}}]}, Axes->True,
    PlotRange->{{-5, 5}, {-5, 5}}]
```



Thin

`Thin`
: sets the line width for subsequent graphics primitives to 0.5pt.

Tiny

`ImageSize -> Tiny`
: produces a tiny image.

16. Strings and Characters - Miscellaneous

Contents

Alphabet

`Alphabet[]`
: gives the list of lowercase letters a-z in the English alphabet .

`Alphabet[`*type*`]`
: gives the alphabet for the language or class *type*.

```
>> Alphabet[]
```
{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z}

```
>> Alphabet["German"]
```
{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z}

$CharacterEncoding

`CharacterEncoding`
: specifies the default character encoding to use if no other encoding is specified.

HexidecimalCharacter

`HexidecimalCharacter`
: represents the characters 0-9, a-f and A-F.

```
>> StringMatchQ[#, HexidecimalCharacter] & /@ {"a", "1", "A", "x", "H", " ", "."}
```
{True, True, True, False, False, False, False}

LetterNumber

`LetterNumber[`*c*`]`
: returns the position of the character *c* in the English alphabet.

`LetterNumber[‘‘string’]’`
: returns a list of the positions of characters in string.

`LetterNumber[‘‘string’,’ `*alpha*`]`
: returns a list of the positions of characters in string, regarding the alphabet *alpha*.

```
>> LetterNumber["b"]
```
2

LetterNumber also works with uppercase characters

```
>> LetterNumber["B"]
```
2

```
>> LetterNumber["ss2!"]
```
{19, 19, 0, 0}

Get positions of each of the letters in a string:

```
>> LetterNumber[Characters["Peccary"]]
```
{16, 5, 3, 3, 1, 18, 25}

```
>> LetterNumber[{"P", "Pe", "P1", "
   eck"}]
```

{16, {16,5}, {16,0}, {5,3,11}}

```
>> LetterNumber["\[Beta]", "Greek"]
```

2

NumberString

`NumberString`
: represents the characters in a number.

```
>> StringMatchQ["1234",
   NumberString]
```

True

```
>> StringMatchQ["1234.5",
   NumberString]
```

True

```
>> StringMatchQ["1.2‘20",
   NumberString]
```

False

RemoveDiacritics

`RemoveDiacritics[`*s*`]`
: returns a version of *s* with all diacritics removed.

```
>> RemoveDiacritics["en prononçant
   pêcher et pécher"]
```

en prononcant pecher et pecher

```
>> RemoveDiacritics["piñata"]
```

pinata

StringContainsQ

`StringContainsQ["`*string*`", `*patt*`]`
: returns True if any part of *string* matches *patt*, and returns False otherwise.

`StringContainsQ[{‘‘s1’, "s2", ...}, patt]’`
: returns the list of results for each element of string list.

`StringContainsQ[patt]`
: represents an operator form of StringContainsQ that can be applied to an expression.

```
>> StringContainsQ["mathics", "m" ~
   ~__ ~~"s"]
```

True

```
>> StringContainsQ["mathics", "a" ~
   ~__ ~~"m"]
```

False

```
>> StringContainsQ["Mathics", "MA"
   , IgnoreCase -> True]
```

True

```
>> StringContainsQ[{"g", "a", "laxy
   ", "universe", "sun"}, "u"]
```

{False, False, False, True, True}

```
>> StringContainsQ["e" ~~___ ~~"u"]
    /@ {"The Sun", "Mercury", "
   Venus", "Earth", "Mars", "
   Jupiter", "Saturn", "Uranus", "
   Neptune"}
```

{True, True, True, False, False,
 False, False, False, True}

StringQ

`StringQ[`*expr*`]`
: returns True if *expr* is a `String`, or False otherwise.

```
>> StringQ["abc"]
```

True

```
>> StringQ[1.5]
```

False

```
>> Select[{"12", 1, 3, 5, "yz", x,
   y}, StringQ]
```
{12, yz}

StringRepeat

`StringRepeat["string", n]`
gives *string* repeated *n* times.
`StringRepeat["string", n, max]`
gives *string* repeated *n* times, but not more than *max* characters.

```
>> StringRepeat["abc", 3]
```
abcabcabc

```
>> StringRepeat["abc", 10, 7]
```
abcabca

String

`String`
is the head of strings.

```
>> Head["abc"]
```
String

```
>> "abc"
```
abc

Use `InputForm` to display quotes around strings:

```
>> InputForm["abc"]
```
"abc"

`FullForm` also displays quotes:

```
>> FullForm["abc" + 2]
```
Plus [2, "abc"]

$SystemCharacterEncoding

$SystemCharacterEncoding

ToExpression

`ToExpression[input]`
intepets a given string as Mathics input.
`ToExpression[input, form]`
reads the given input in the specified *form*.
`ToExpression[input, form, h]`
applies the head *h* to the expression before evaluating it.

```
>> ToExpression["1 + 2"]
```
3

```
>> ToExpression["{2, 3, 1}",
   InputForm, Max]
```
3

```
>> ToExpression["2 3", InputForm]
```
6

Note that newlines are like semicolons, not blanks. So so the return value is the second-line value.

```
>> ToExpression["2\[NewLine]3"]
```
3

ToString

`ToString[expr]`
returns a string representation of *expr*.
`ToString[expr, form]`
returns a string representation of *expr* in the form *form*.

```
>> ToString[2]
```
2

```
>> ToString[2] // InputForm
```
"2"

```
>> ToString[a+b]
```
a + b

```
>> "U" <> 2
```
Stringexpected.
U<>2

```
>> "U" <> ToString[2]
```
U2

```
>> ToString[Integrate[f[x],x],
   TeXForm]
```

\int f\left[x\right] \, dx

Transliterate

`Transliterate[s]`
: transliterates a text in some script into an ASCII string.

The following examples were taken from # https://en.wikipedia.org/wiki/Iliad, # https://en.wikipedia.org/wiki/Russian_language, and # https://en.wikipedia.org/wiki/Hiragana

Whitespace

`Whitespace`
: represents a sequence of whitespace characters.

```
>> StringMatchQ["\r \n", Whitespace
   ]
```

True

```
>> StringSplit["a \n b \r\n c d",
   Whitespace]
```

$\{a, b, c, d\}$

```
>> StringReplace[" this has leading
    and trailing whitespace \n ", (
   StartOfString ~~Whitespace)| (
   Whitespace ~~EndOfString)-> ""]
   <> " removed" // FullForm
```

"this has leading and trailing
whitespace removed"

17. Mathematical Optimization

Mathematical optimization is the selection of a best element, with regard to some criterion, from some set of available alternatives.
Optimization problems of sorts arise in all quantitative disciplines from computer science and engineering to operations research and economics, and the development of solution methods has been of interest in mathematics for centuries.
We intend to provide local and global optimization techniques, both numeric and symbolic.

Contents

Maximize

`Maximize[`f`, `x`]`
compute the maximum of f respect x that change between a and b

>> **`Maximize[-2 x^2 - 3 x + 5, x]`**

$$\left\{\left\{\frac{49}{8},\left\{x->-\frac{3}{4}\right\}\right\}\right\}$$

#» Maximize[1 - (x y - 3)^2, {x, y}] = {{1, {x -> 3, y -> 1}}}
#» Maximize[{x - 2 y, x^2 + y^2 <= 1}, {x, y}] = {{Sqrt[5], {x -> Sqrt[5] / 5, y -> -2 Sqrt[5] / 5}}}

Minimize

`Minimize[`f`, `x`]`
compute the minimum of f respect x that change between a and b

>> **`Minimize[2 x^2 - 3 x + 5, x]`**

$$\left\{\left\{\frac{31}{8},\left\{x->\frac{3}{4}\right\}\right\}\right\}$$

#» Minimize[(x y - 3)^2 + 1, {x, y}] = {{1, {x -> 3, y -> 1}}}
#» Minimize[{x - 2 y, x^2 + y^2 <= 1}, {x, y}] = {{-Sqrt[5], {x -> -Sqrt[5] / 5, y -> 2 Sqrt[5] / 5}}}

18. Drawing Options and Option Values

The various common Plot and Graphics options, along with the meaning of specific option values are described here.

Contents

Automatic

`Automatic`
is used to specify an automatically computed option value.

`Automatic` is the default for `PlotRange`, `ImageSize`, and other graphical options:

```
>> Cases[Options[Plot], HoldPattern
   [_ :> Automatic]]
```

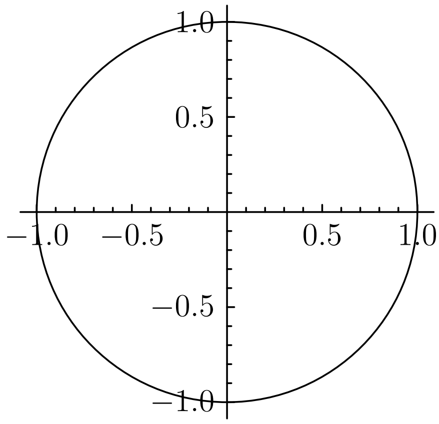
{Background:>Automatic, Exclusions:>Automatic, ImageSize:>Automatic, MaxRecursion:>Automatic, PlotRange:>Automatic, PlotRangePadding:>Automatic}

Axes

`Axes`
is an option for charting and graphics functions that specifies whether axes should be drawn.

- `Axes->True` draws all axes.
- `Axes->False` draws no axes.
- `Axes->{False,True}` draws an axis y but no x axis in two dimensions.

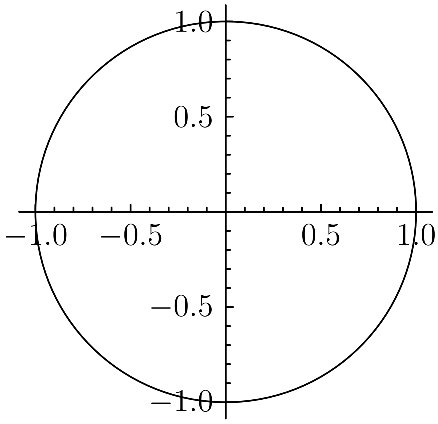
```
>> Graphics[Circle[], Axes -> True]
```



Axis

`Axis`
is a possible value for the `Filling` option.

```
>>  ListLinePlot[Table[Sin[x], {x,
    -5, 5, 0.5}], Filling->Axis]
```



Bottom

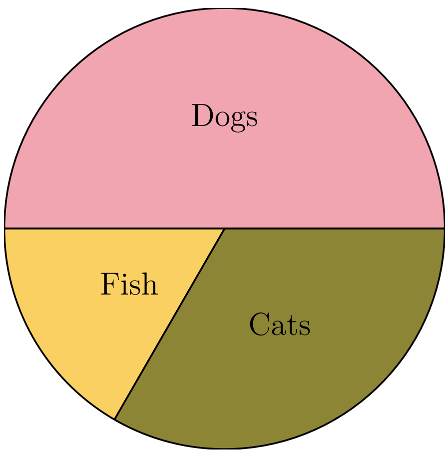
`Bottom`
is a possible value for the `Filling` option.

```
>>  ListLinePlot[Table[Sin[x], {x,
    -5, 5, 0.5}], Filling->Bottom]
```

ChartLabels

`ChartLabels`
is a charting option that specifies what labels should be used for chart elements.

```
>>  PieChart[{30, 20, 10},
    ChartLabels -> {Dogs, Cats, Fish
    }]
```



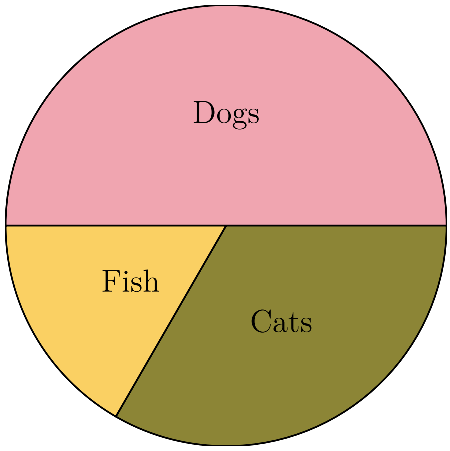
ChartLegends

`ChartLegends`
is a charting option.

Filling

`Filling Top |Bottom|Axis`
is a an option to Plot to specify what filling to add under point, curves, and surfaces

```
>>  ListLinePlot[Table[Sin[x], {x,
    -5, 5, 0.5}], Filling->Axis]
```



Full

`Full`
is a possible value for the `Mesh` and `PlotRange` options.

ImageSize

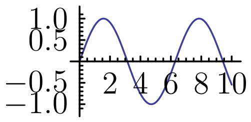
`ImageSize`
is an option that specifies the overall size of an image to display.

Specifications for both width and height can be any of the following:

Automatic
: determined by location or other dimension (default)

Tiny, Small, Medium, Large
: pre defined absolute sizes

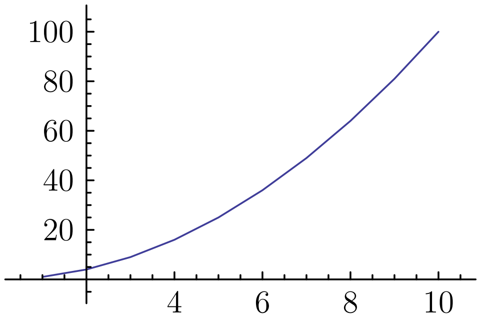
```
>> Plot[Sin[x], {x, 0, 10},
   ImageSize -> Small]
```



Joined

`Joined` *boolean*
: is an option for `Plot` that gives whether to join points to make lines.

```
>> ListPlot[Table[n ^ 2, {n, 10}],
   Joined->True]
```



MaxRecursion

`MaxRecursion`
: is an option for functions like NIntegrate and Plot that specifies how many recursive subdivisions can be made.

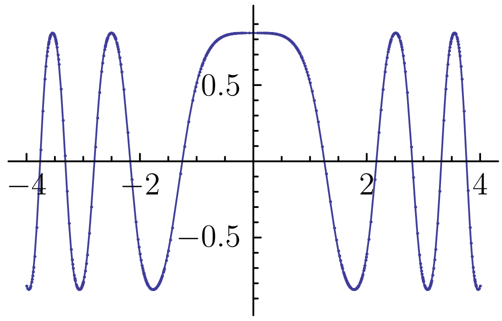
```
>> NIntegrate[Exp[-10^8 x^2], {x,
   -1, 1}, MaxRecursion -> 10]
```

1.97519×10^{-207}

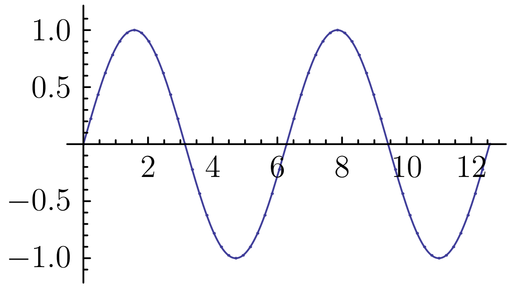
Mesh

`Mesh`
: is a charting option, such as for `Plot`, `BarChart`, `PieChart`, etc. that specifies the mesh to be drawn. The default is `Mesh->None`.

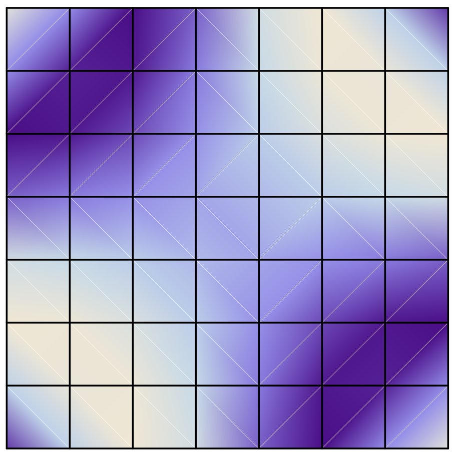
```
>> Plot[Sin[Cos[x^2]],{x,-4,4},Mesh
   ->All]
```



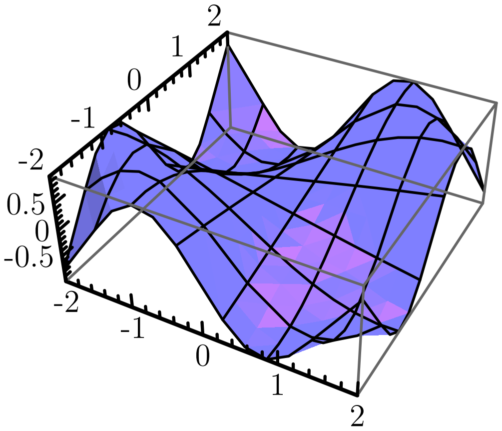

```
>> Plot[Sin[x], {x,0,4 Pi}, Mesh->
   Full]
```




```
>> DensityPlot[Sin[x y], {x, -2,
   2}, {y, -2, 2}, Mesh->Full]
```



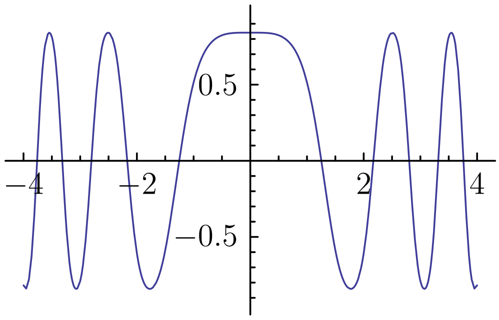
```
>> Plot3D[Sin[x y], {x, -2, 2}, {y,
   -2, 2}, Mesh->Full]
```



PlotPoints

PlotPoints *n*
A number specifies how many initial sample points to use.

```
>> Plot[Sin[Cos[x^2]],{x,-4,4},
   PlotPoints->22]
```

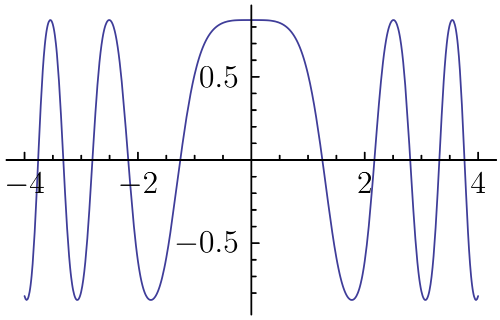


PlotRange

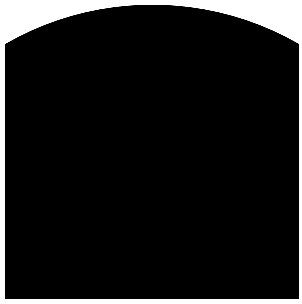
PlotRange
is a charting option, such as for `Plot`, `BarChart`, `PieChart`, etc. that gives the range of coordinates to include in a plot.

- All all points are included.
- Automatic - outlying points are dropped.
- *max* - explicit limit for each function.
- {*min*, *max*} - explicit limits for *y* (2D), *z* (3D), or array values.
- {{*x_min*, *x_max*}, {{$y_min}, {$y_max}} - explit limits for *x* and *y*.

```
>> Plot[Sin[Cos[x^2]],{x,-4,4},
   PlotRange -> All]
```




```
>> Graphics[Disk[], PlotRange ->
   {{-.5, .5}, {0, 1.5}}]
```

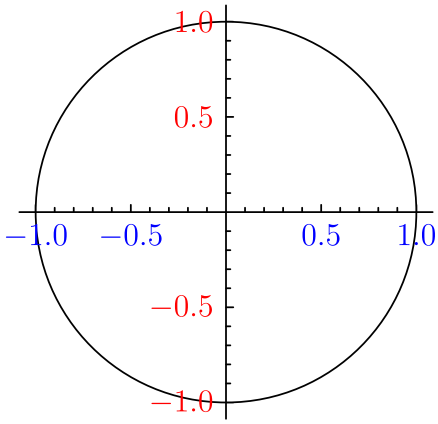


TicksStyle

TicksStyle
is an option for graphics functions which specifies how ticks should be rendered.

- TicksStyle gives styles for both tick marks and tick labels.
- TicksStyle can be used in both two and three-dimensional graphics.
- TicksStyle->*list* specifies the colors of each of the axes.

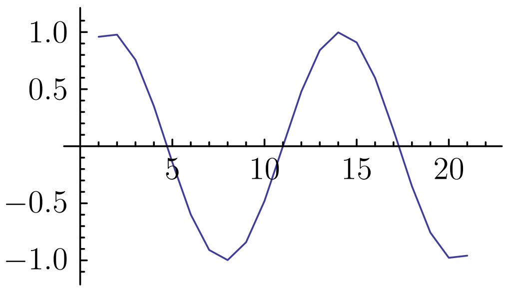
```
>> Graphics[Circle[], Axes-> True,
   TicksStyle -> {Blue, Red}]
```



Top

`Top`
is a possible value for the `Filling` option.

```
>> ListLinePlot[Table[Sin[x], {x,
   -5, 5, 0.5}], Filling->Axis|Top|
   Bottom]
```



19. Physical and Chemical data

Contents

ElementData

`ElementData["`*name*‘‘`,"`*property*`"]`
: gives the value of the *property* for the chemical specified by *name*.

`ElementData[`*n*`, "`*property*`"]`
: gives the value of the *property* for the *n*th chemical element.

```
>> ElementData[74]
```

Tungsten

```
>> ElementData["He", "
   AbsoluteBoilingPoint"]
```

4.22

```
>> ElementData["Carbon", "
   IonizationEnergies"]
```

{1 086.5, 2 352.6, 4 620.5
, 6 222.7, 37 831, 47 277.}

```
>> ElementData[16, "
   ElectronConfigurationString"]
```

[Ne] 3s2 3p4

```
>> ElementData[73, "
   ElectronConfiguration"]
```

{{2} , {2, 6} , {2, 6, 10} , {2,
6, 10, 14} , {2, 6, 3} , {2}}

The number of known elements:

```
>> Length[ElementData[All]]
```

118

Some properties are not appropriate for certain elements:

```
>> ElementData["He", "
   ElectroNegativity"]
```

Missing [NotApplicable]

Some data is missing:

```
>> ElementData["Tc", "SpecificHeat
   "]
```

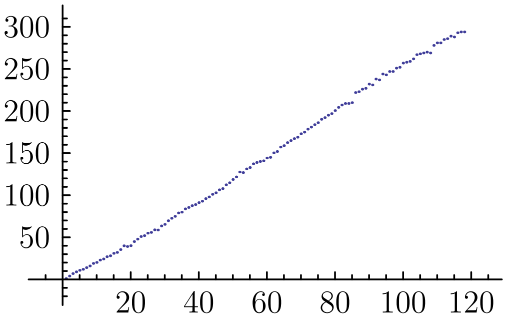
Missing [NotAvailable]

All the known properties:

```
>> ElementData["Properties"]
```

{Abbreviation,
AbsoluteBoilingPoint,
AbsoluteMeltingPoint,
AtomicNumber, AtomicRadius,
AtomicWeight, Block, BoilingPoint,
BrinellHardness, BulkModulus,
CovalentRadius, CrustAbundance,
Density, DiscoveryYear,
ElectroNegativity, ElectronAffinity,
ElectronConfiguration,
ElectronConfigurationString,
ElectronShellConfiguration,
FusionHeat, Group,
IonizationEnergies, LiquidDensity,
MeltingPoint, MohsHardness,
Name, Period, PoissonRatio,
Series, ShearModulus,
SpecificHeat, StandardName,
ThermalConductivity,
VanDerWaalsRadius,
VaporizationHeat,
VickersHardness, YoungModulus}

```
>> ListPlot[Table[ElementData[z, "
   AtomicWeight"], {z, 118}]]
```



20. List Functions - Miscellaneous

Contents

All

`All`
is a possible option value for `Span`, `Quiet`, `Part` and related functions. `All` specifies all parts at a particular level.

Association

`Association[`*key1* `->` *val1*`,` *key2* `->` *val2*`, ...]`
`<|`*key1* `->` *val1*`,` *key2* `->` *val2*`, ...|>`
represents an association between keys and values.

`Association` is the head of associations:

```
>> Head[<|a -> x, b -> y, c -> z|>]
```
Association

```
>> <|a -> x, b -> y|>
```
<|a− > x, b− > y|>

```
>> Association[{a -> x, b -> y}]
```
<|a− > x, b− > y|>

Associations can be nested:

```
>> <|a -> x, b -> y, <|a -> z, d ->
    t|>|>
```
<|a− > z, b− > y, d− > t|>

AssociationQ

`AssociationQ[`*expr*`]`
: return True if *expr* is a valid Association object, and False otherwise.

```
>> AssociationQ[<|a -> 1, b :> 2|>]
```
True

```
>> AssociationQ[<|a, b|>]
```
False

ByteArray

`ByteArray[{`*b_1*`, `*b_2*`, ...}]`
: Represents a sequence of Bytes *b_1*, *b_2*, ...

`ByteArray[''string']'`
: Constructs a byte array where bytes comes from decode a b64 encoded String

```
>> A=ByteArray[{1, 25, 3}]
```
ByteArray ["ARkD"]

```
>> A[[2]]
```
25

```
>> Normal[A]
```
$\{1,25,3\}$

```
>> ToString[A]
```
ByteArray["ARkD"]

```
>> ByteArray["ARkD"]
```
ByteArray ["ARkD"]

```
>> B=ByteArray["asy"]
```
The first argument in Bytearray[asy] should be a B64 encoded string or a vector of integers.

$Failed

Catenate

`Catenate[{`*l1*`, `*l2*`, ...}]`
: concatenates the lists *l1*, *l2*, ...

```
>> Catenate[{{1, 2, 3}, {4, 5}}]
```
$\{1,2,3,4,5\}$

CentralMoment

`CentralMoment[`*list*`, `*r*`]`
: gives the the *r*th central moment (i.e. the *r*th moment about the mean) of *list*.

```
>> CentralMoment[{1.1, 1.2, 1.4, 2.1, 2.4}, 4]
```
0.100845

ClusteringComponents

`ClusteringComponents[`*list*`]`
: forms clusters from *list* and returns a list of cluster indices, in which each element shows the index of the cluster in which the corresponding element in *list* ended up.

`ClusteringComponents[`*list*`, `*k*`]`
: forms *k* clusters from *list* and returns a list of cluster indices, in which each element shows the index of the cluster in which the corresponding element in *list* ended up.

For more detailed documentation regarding options and behavior, see FindClusters[].

```
>> ClusteringComponents[{1, 2, 3, 1, 2, 10, 100}]
```
$\{1,1,1,1,1,1,2\}$

```
>> ClusteringComponents[{10, 100, 20}, Method -> "KMeans"]
```
$\{1,0,1\}$

Complement

`Complement[`*all*`, `*e1*`, `*e2*`, ...]`
: returns an expression containing the elements in the set *all* that are not in any of *e1*, *e2*, etc.

`Complement[`*all*`, `*e1*`, `*e2*`, ..., SameTest->`*test*`]`
: applies *test* to the elements in *all* and each of the *ei* to determine equality.

The sets *all*, *e1*, etc can have any head, which must all match. The returned expression has the same head as the input expressions. The expression will be sorted and each element will only occur once.

```
>> Complement[{a, b, c}, {a, c}]
```

$\{b\}$

```
>> Complement[{a, b, c}, {a, c}, {b
   }]
```

$\{\}$

```
>> Complement[f[z, y, x, w], f[x],
   f[x, z]]
```

$f[w, y]$

```
>> Complement[{c, b, a}]
```

$\{a, b, c\}$

ContainsOnly

`ContainsOnly[`*list1*`, `*list2*`]`
: yields True if *list1* contains only elements that appear in *list2*.

```
>> ContainsOnly[{b, a, a}, {a, b, c
   }]
```

True

The first list contains elements not present in the second list:

```
>> ContainsOnly[{b, a, d}, {a, b, c
   }]
```

False

```
>> ContainsOnly[{}, {a, b, c}]
```

True

Use Equal as the comparison function to have numerical tolerance:

```
>> ContainsOnly[{a, 1.0}, {1, a, b
   }, {SameTest -> Equal}]
```

True

Correlation

`Correlation[`*a*`, `*b*`]`
: computes Pearson's correlation of two equal-sized vectors *a* and *b*.

An example from Wikipedia:

```
>> Correlation[{10, 8, 13, 9, 11,
   14, 6, 4, 12, 7, 5}, {8.04,
   6.95, 7.58, 8.81, 8.33, 9.96,
   7.24, 4.26, 10.84, 4.82, 5.68}]
```

0.816421

Covariance

`Covariance[`*a*`, `*b*`]`
: computes the covariance between the equal-sized vectors *a* and *b*.

```
>> Covariance[{0.2, 0.3, 0.1},
   {0.3, 0.3, -0.2}]
```

0.025

Delete

`Delete[`*expr*`, `*i*`]`
: deletes the element at position *i* in *expr*. The position is counted from the end if *i* is negative.

`Delete[`*expr*`, {`*m*`, `*n*`, ...}]`
: deletes the element at position $\{m, n, ...\}$.

`Delete[`*expr*`, {{`*m1*`, `*n1*`, ...}, {`*m2*`, `*n2*`, ...}, ...}]`
: deletes the elements at several positions.

Delete the element at position 3:

```
>> Delete[{a, b, c, d}, 3]
```

$\{a, b, d\}$

Delete at position 2 from the end:

```
>> Delete[{a, b, c, d}, -2]
```

$\{a, b, d\}$

Delete at positions 1 and 3:

```
>> Delete[{a, b, c, d}, {{1}, {3}}]
```

$\{b, d\}$

Delete in a 2D array:

```
>> Delete[{{a, b}, {c, d}}, {2, 1}]
```

$\{\{a, b\}, \{d\}\}$

Deleting the head of a whole expression gives a Sequence object:

```
>> Delete[{a, b, c}, 0]
```

Sequence $[a, b, c]$

Delete in an expression with any head:

```
>> Delete[f[a, b, c, d], 3]
```

$f[a,b,d]$

Delete a head to splice in its arguments:

```
>> Delete[f[a, b, u + v, c], {3, 0}]
```

$f[a,b,u,v,c]$

```
>> Delete[{a, b, c}, 0]
```

Sequence $[a,b,c]$

Delete without the position:

```
>> Delete[{a, b, c, d}]
```

Deletecalledwith1argument;2argumentsareexpecte

Delete $[\{a,b,c,d\}]$

Delete with many arguments:

```
>> Delete[{a, b, c, d}, 1, 2]
```

Deletecalledwith3arguments;2argumentsareexpected.

Delete $[\{a,b,c,d\},1,2]$

Delete the element out of range:

```
>> Delete[{a, b, c, d}, 5]
```

Part{5}of{a,b,c,d}doesnotexist.

Delete $[\{a,b,c,d\},5]$

Delete the position not integer:

```
>> Delete[{a, b, c, d}, {1, n}]
```

Positionspecificationnin{a, b,c,d}isnotamachine −sizedintegeroralistofmachine −sizedintegers.

Delete $[\{a,b,c,d\},\{1,n\}]$

DeleteDuplicates

`DeleteDuplicates[`*list*`]`
: deletes duplicates from *list*.

`DeleteDuplicates[`*list*`, `*test*`]`
: deletes elements from *list* based on whether the function *test* yields `True` on pairs of elements. DeleteDuplicates does not change the order of the remaining elements.

```
>> DeleteDuplicates[{1, 7, 8, 4, 3, 4, 1, 9, 9, 2, 1}]
```

$\{1,7,8,4,3,9,2\}$

```
>> DeleteDuplicates[{3,2,1,2,3,4}, Less]
```

$\{3,2,1\}$

DisjointQ

`DisjointQ[`*a*`, `*b*`]`
: gives True if $a and $b are disjoint, or False if $a and $b have any common elements.

Failure

Failure[*tag*, *assoc*]
: represents a failure of a type indicated by *tag*, with details given by the association *assoc*.

FindClusters

`FindClusters[`*list*`]`
: returns a list of clusters formed from the elements of *list*. The number of cluster is determined automatically.

`FindClusters[`*list*`, `*k*`]`
: returns a list of *k* clusters formed from the elements of *list*.

```
>> FindClusters[{1, 2, 20, 10, 11, 40, 19, 42}]
```

$\{\{1,2,20,10,11,19\},\{40,42\}\}$

```
>> FindClusters[{25, 100, 17, 20}]
```

$\{\{25,17,20\},\{100\}\}$

```
>> FindClusters[{3, 6, 1, 100, 20, 5, 25, 17, -10, 2}]
```

$\{\{3,6,1,5,-10,2\},\{100\},\{20,25,17\}\}$

```
>> FindClusters[{1, 2, 10, 11, 20, 21}]
```
$\{\{1,2\},\{10,11\},\{20,21\}\}$

```
>> FindClusters[{1, 2, 10, 11, 20, 21}, 2]
```
$\{\{1,2,10,11\},\{20,21\}\}$

```
>> FindClusters[{1 -> a, 2 -> b, 10 -> c}]
```
$\{\{a,b\},\{c\}\}$

```
>> FindClusters[{1, 2, 5} -> {a, b, c}]
```
$\{\{a,b\},\{c\}\}$

```
>> FindClusters[{1, 2, 3, 1, 2, 10, 100}, Method -> "Agglomerate"]
```
$\{\{1,2,3,1,2,10\},\{100\}\}$

```
>> FindClusters[{1, 2, 3, 10, 17, 18}, Method -> "Agglomerate"]
```
$\{\{1,2,3\},\{10\},\{17,18\}\}$

```
>> FindClusters[{{1}, {5, 6}, {7}, {2, 4}}, DistanceFunction -> (Abs[Length[#1] - Length[#2]]&)]
```
$\{\{\{1\},\{7\}\},\{\{5,6\},\{2,4\}\}\}$

```
>> FindClusters[{"meep", "heap", "deep", "weep", "sheep", "leap", "keep"}, 3]
```
{{meep, deep, weep, keep}, {heap, leap}, {sheep}}

FindClusters' automatic distance function detection supports scalars, numeric tensors, boolean vectors and strings.

The Method option must be either "Agglomerate" or "Optimize". If not specified, it defaults to "Optimize". Note that the Agglomerate and Optimize methods usually produce different clusterings.

The runtime of the Agglomerate method is quadratic in the number of clustered points n, builds the clustering from the bottom up, and is exact (no element of randomness). The Optimize method's runtime is linear in n, Optimize builds the clustering from top down, and uses random sampling.

Fold

`Fold[`*f*`, `*x*`, `*list*`]`
: returns the result of iteratively applying the binary operator *f* to each element of *list*, starting with *x*.

`Fold[`*f*`, `*list*`]`
: is equivalent to `Fold[`*f*`, First[`*list*`], Rest[`*list*`]]`.

```
>> Fold[Plus, 5, {1, 1, 1}]
```
8

```
>> Fold[f, 5, {1, 2, 3}]
```
$f[f[f[5,1],2],3]$

FoldList

`FoldList[`*f*`, `*x*`, `*list*`]`
: returns a list starting with *x*, where each element is the result of applying the binary operator *f* to the previous result and the next element of *list*.

`FoldList[`*f*`, `*list*`]`
: is equivalent to `FoldList[`*f*`, First[`*list*`], Rest[`*list*`]]`.

```
>> FoldList[f, x, {1, 2, 3}]
```
$\{x,f[x,1],f[f[x,1],2],f[f[f[x,1],2],3]\}$

```
>> FoldList[Times, {1, 2, 3}]
```
$\{1,2,6\}$

Gather

`Gather[`*list*`, `*test*`]`
: gathers leaves of *list* into sub lists of items that are the same according to *test*.

`Gather[`*list*`]`
: gathers leaves of *list* into sub lists of items that are the same.

The order of the items inside the sub lists is the same as in the original list.

```
>> Gather[{1, 7, 3, 7, 2, 3, 9}]
```
$\{\{1\},\{7,7\},\{3,3\},\{2\},\{9\}\}$

```
>> Gather[{1/3, 2/6, 1/9}]
```

$\left\{\left\{\frac{1}{3},\frac{1}{3}\right\},\left\{\frac{1}{9}\right\}\right\}$

GatherBy

`GatherBy[`*list*`, `*f*`]`
: gathers leaves of *list* into sub lists of items whose image under $f identical.

`GatherBy[`*list*`, {`*f*`, `*g*`, ...}]`
: gathers leaves of *list* into sub lists of items whose image under $f identical. Then, gathers these sub lists again into sub sub lists, that are identical under $g.

```
>> GatherBy[{{1, 3}, {2, 2}, {1, 1}}, Total]
```

$\{\{\{1,3\},\{2,2\}\},\{\{1,1\}\}\}$

```
>> GatherBy[{"xy", "abc", "ab"}, StringLength]
```

$\{\{\mathrm{xy},\mathrm{ab}\},\{\mathrm{abc}\}\}$

```
>> GatherBy[{{2, 0}, {1, 5}, {1, 0}}, Last]
```

$\{\{\{2,0\},\{1,0\}\},\{\{1,5\}\}\}$

```
>> GatherBy[{{1, 2}, {2, 1}, {3, 5}, {5, 1}, {2, 2, 2}}, {Total, Length}]
```

$\{\{\{\{1,2\},\{2,1\}\}\},\{\{\{3,5\}\}\},\{\{\{5,1\}\},\{\{2,2,2\}\}\}\}$

Insert

`Insert[`*list*`, `*elem*`, `*n*`]`
: inserts *elem* at position *n* in *list*. When *n* is negative, the position is counted from the end.

```
>> Insert[{a,b,c,d,e}, x, 3]
```

$\{a,b,x,c,d,e\}$

```
>> Insert[{a,b,c,d,e}, x, -2]
```

$\{a,b,c,d,x,e\}$

IntersectingQ

`IntersectingQ[`*a*`, `*b*`]`
: gives True if there are any common elements in $a and $b, or False if $a and $b are disjoint.

Intersection

`Intersection[`*a*`, `*b*`, ...]`
: gives the intersection of the sets. The resulting list will be sorted and each element will only occur once.

```
>> Intersection[{1000, 100, 10, 1}, {1, 5, 10, 15}]
```

$\{1,10\}$

```
>> Intersection[{{a, b}, {x, y}}, {{x, x}, {x, y}, {x, z}}]
```

$\{\{x,y\}\}$

```
>> Intersection[{c, b, a}]
```

$\{a,b,c\}$

```
>> Intersection[{1, 2, 3}, {2, 3, 4}, SameTest->Less]
```

$\{3\}$

Join

`Join[`*l1*`, `*l2*`]`
: concatenates the lists *l1* and *l2*.

`Join` concatenates lists:

```
>> Join[{a, b}, {c, d, e}]
```

$\{a,b,c,d,e\}$

```
>> Join[{{a, b}, {c, d}}, {{1, 2}, {3, 4}}]
```

$\{\{a,b\},\{c,d\},\{1,2\},\{3,4\}\}$

The concatenated expressions may have any head:

```
>> Join[a + b, c + d, e + f]
```

$a+b+c+d+e+f$

However, it must be the same for all expressions:

```
>> Join[a + b, c * d]
```

HeadsPlusandTimesareexpectedtobethesame.

Join $[a + b, cd]$

Key

`Key[key]`
: represents a key used to access a value in an association.

`Key[key][assoc]`

Keys

`Keys[<|key1 -> val1, key2 -> val2, ...|>]`
: return a list of the keys *keyi* in an association.

`Keys[{key1 -> val1, key2 -> val2, ...}]`
: return a list of the *keyi* in a list of rules.

```
>> Keys[<|a -> x, b -> y|>]
```

$\{a, b\}$

```
>> Keys[{a -> x, b -> y}]
```

$\{a, b\}$

Keys automatically threads over lists:

```
>> Keys[{<|a -> x, b -> y|>, {w -> z, {}}}]
```

$\{\{a, b\}, \{w, \{\}\}\}$

Keys are listed in the order of their appearance:

```
>> Keys[{c -> z, b -> y, a -> x}]
```

$\{c, b, a\}$

Kurtosis

`Kurtosis[list]`
: gives the Pearson measure of kurtosis for *list* (a measure of existing outliers).

```
>> Kurtosis[{1.1, 1.2, 1.4, 2.1, 2.4}]
```

1.42098

LeafCount

`LeafCount[expr]`
: returns the total number of indivisible subexpressions in *expr*.

```
>> LeafCount[1 + x + y^a]
```

6

```
>> LeafCount[f[x, y]]
```

3

```
>> LeafCount[{1 / 3, 1 + I}]
```

7

```
>> LeafCount[Sqrt[2]]
```

5

```
>> LeafCount[100!]
```

1

Level

`Level[expr, levelspec]`
: gives a list of all subexpressions of *expr* at the level(s) specified by *levelspec*.

Level uses standard level specifications:

n
: levels 1 through n

`Infinity`
: all levels from level 1

`{n}`
: level n only

`{m, n}`
: levels m through n

Level 0 corresponds to the whole expression.
A negative level $-n$ consists of parts with depth n.
Level -1 is the set of atoms in an expression:

```
>> Level[a + b ^ 3 * f[2 x ^ 2], {-1}]
```

$\{a, b, 3, 2, x, 2\}$

```
>> Level[{{{{a}}}}, 3]
```

$\{\{a\}, \{\{a\}\}, \{\{\{a\}\}\}\}$

```
>> Level[{{{{a}}}}, -4]
```

$\{\{\{\{a\}\}\}\}$

```
>> Level[{{{{a}}}}, -5]
```

$\{\}$

```
>> Level[h0[h1[h2[h3[a]]]], {0,
   -1}]
```

$\{a, \text{h3}[a], \text{h2}[\text{h3}[a]], \text{h1}[\text{h2}[\text{h3}[a]]], \text{h0}[\text{h1}[\text{h2}[\text{h3}[a]]]]\}$

Use the option `Heads -> True` to include heads:

```
>> Level[{{{{a}}}}, 3, Heads ->
   True]
```

$\{\text{List}, \text{List}, \text{List}, \{a\}, \{\{a\}\}, \{\{\{a\}\}\}\}$

```
>> Level[x^2 + y^3, 3, Heads ->
   True]
```

$\left\{\text{Plus}, \text{Power}, x, 2, x^2, \text{Power}, y, 3, y^3\right\}$

```
>> Level[a ^ 2 + 2 * b, {-1}, Heads
    -> True]
```

$\{\text{Plus}, \text{Power}, a, 2, \text{Times}, 2, b\}$

```
>> Level[f[g[h]][x], {-1}, Heads ->
    True]
```

$\{f, g, h, x\}$

```
>> Level[f[g[h]][x], {-2, -1},
   Heads -> True]
```

$\{f, g, h, g[h], x, f[g[h]][x]\}$

LevelQ

`LevelQ[`*expr*`]`
: tests whether *expr* is a valid level specification.

```
>> LevelQ[2]
```

True

```
>> LevelQ[{2, 4}]
```

True

```
>> LevelQ[Infinity]
```

True

```
>> LevelQ[a + b]
```

False

List

`List[`*e1*`, `*e2*`, ..., `*ei*`]`
`{`*e1*`, `*e2*`, ..., `*ei*`}`
: represents a list containing the elements *e1*...*ei*.

`List` is the head of lists:

```
>> Head[{1, 2, 3}]
```

List

Lists can be nested:

```
>> {{a, b, {c, d}}}
```

$\{\{a, b, \{c, d\}\}\}$

ListQ

`ListQ[`*expr*`]`
: tests whether *expr* is a `List`.

```
>> ListQ[{1, 2, 3}]
```

True

```
>> ListQ[{{1, 2}, {3, 4}}]
```

True

```
>> ListQ[x]
```

False

Lookup

Lookup[*assoc*, *key*]
: looks up the value associated with *key* in the association *assoc*, or Missing[*KeyAbsent*].

Median

`Median[`*list*`]`
: returns the median of *list*.

```
>> Median[{26, 64, 36}]
```

36

For lists with an even number of elements, Median returns the mean of the two middle values:

```
>> Median[{-11, 38, 501, 1183}]
```

$\frac{539}{2}$

Passing a matrix returns the medians of the respective columns:

```
>> Median[{{100, 1, 10, 50}, {-1,
   1, -2, 2}}]
```

$\left\{\frac{99}{2}, 1, 4, 26\right\}$

Nearest

`Nearest[`*list*`, `*x*`]`
: returns the one item in *list* that is nearest to *x*.

`Nearest[`*list*`, `*x*`, `*n*`]`
: returns the *n* nearest items.

`Nearest[`*list*`, `*x*`, {`*n*`, `*r*`}]`
: returns up to *n* nearest items that are not farther from *x* than *r*.

`Nearest[{`*p1*` -> `*q1*`, `*p2*` -> `*q2*`, ...}, `*x*`]`
: returns *q1*, *q2*, ... but measures the distances using *p1*, *p2*, ...

`Nearest[{`*p1*`, `*p2*`, ...} -> {`*q1*`, `*q2*`, ...}, `*x*`]`
: returns *q1*, *q2*, ... but measures the distances using *p1*, *p2*, ...

```
>> Nearest[{5, 2.5, 10, 11, 15,
   8.5, 14}, 12]
```

$\{11\}$

Return all items within a distance of 5:

```
>> Nearest[{5, 2.5, 10, 11, 15,
   8.5, 14}, 12, {All, 5}]
```

$\{11, 10, 14\}$

```
>> Nearest[{Blue -> "blue", White
   -> "white", Red -> "red", Green
   -> "green"}, {Orange, Gray}]
```

{{red}, {white}}

```
>> Nearest[{{0, 1}, {1, 2}, {2, 3}}
    -> {a, b, c}, {1.1, 2}]
```

$\{b\}$

None

`None`
: is a possible value for `Span` and `Quiet`.

NotListQ

`NotListQ[`*expr*`]`
: returns true if *expr* is not a list.

PadLeft

`PadLeft[`*list*`, `*n*`]`
: pads *list* to length *n* by adding 0 on the left.

`PadLeft[`*list*`, `*n*`, `*x*`]`
: pads *list* to length *n* by adding *x* on the left.

`PadLeft[`*list*`, {`*n1*`, $n2, ...}, `*x*`]`
: pads *list* to lengths *n1*, *n2* at levels 1, 2, ... respectively by adding *x* on the left.

`PadLeft[`*list*`, `*n*`, `*x*`, `*m*`]`
: pads *list* to length *n* by adding *x* on the left and adding a margin of *m* on the right.

`PadLeft[`*list*`, `*n*`, `*x*`, {`*m1*`, `*m2*`, ...}]`
: pads *list* to length *n* by adding *x* on the left and adding margins of *m1*, *m2*, ... on levels 1, 2, ... on the right.

`PadLeft[`*list*`]`
: turns the ragged list *list* into a regular list by adding 0 on the left.

```
>> PadLeft[{1, 2, 3}, 5]
```

$\{0, 0, 1, 2, 3\}$

```
>> PadLeft[x[a, b, c], 5]
```

$x[0, 0, a, b, c]$

```
>> PadLeft[{1, 2, 3}, 2]
```

$\{2, 3\}$

```
>> PadLeft[{{}, {1, 2}, {1, 2, 3}}]
```

$\{\{0, 0, 0\}, \{0, 1, 2\}, \{1, 2, 3\}\}$

```
>> PadLeft[{1, 2, 3}, 10, {a, b, c
   }, 2]
```

$\{b, c, a, b, c, 1, 2, 3, a, b\}$

```
>> PadLeft[{{1, 2, 3}}, {5, 2}, x,
1]
```

$\{\{x, x\}, \{x, x\}, \{x, x\}, \{3, x\}, \{x, x\}\}$

PadRight

`PadRight[`*list*`, ` *n*`]`
: pads *list* to length *n* by adding 0 on the right.

`PadRight[`*list*`, ` *n*`, ` *x*`]`
: pads *list* to length *n* by adding *x* on the right.

`PadRight[`*list*`, {`*n1*`, $n2, ...}, ` *x*`]`
: pads *list* to lengths *n1*, *n2* at levels 1, 2, ... respectively by adding *x* on the right.

`PadRight[`*list*`, ` *n*`, ` *x*`, ` *m*`]`
: pads *list* to length *n* by adding *x* on the left and adding a margin of *m* on the left.

`PadRight[`*list*`, ` *n*`, ` *x*`, {`*m1*`, ` *m2*`, ...}]`
: pads *list* to length *n* by adding *x* on the right and adding margins of *m1*, *m2*, ... on levels 1, 2, ... on the left.

`PadRight[`*list*`]`
: turns the ragged list *list* into a regular list by adding 0 on the right.

```
>> PadRight[{1, 2, 3}, 5]
```

$\{1, 2, 3, 0, 0\}$

```
>> PadRight[x[a, b, c], 5]
```

$x[a, b, c, 0, 0]$

```
>> PadRight[{1, 2, 3}, 2]
```

$\{1, 2\}$

```
>> PadRight[{{}, {1, 2}, {1, 2,
3}}]
```

$\{\{0, 0, 0\}, \{1, 2, 0\}, \{1, 2, 3\}\}$

```
>> PadRight[{1, 2, 3}, 10, {a, b, c
}, 2]
```

$\{b, c, 1, 2, 3, a, b, c, a, b\}$

```
>> PadRight[{{1, 2, 3}}, {5, 2}, x,
 1]
```

$\{\{x, x\}, \{x, 1\}, \{x, x\}, \{x, x\}, \{x, x\}\}$

Partition

`Partition[`*list*`, ` *n*`]`
: partitions *list* into sublists of length *n*.

`Parition[`*list*`, ` *n*`, ` *d*`]`
: partitions *list* into sublists of length *n* which overlap *d* indicies.

```
>> Partition[{a, b, c, d, e, f}, 2]
```

$\{\{a, b\}, \{c, d\}, \{e, f\}\}$

```
>> Partition[{a, b, c, d, e, f}, 3,
 1]
```

$\{\{a, b, c\}, \{b, c, d\}, \{c, d, e\}, \{d, e, f\}\}$

Position

`Position[`*expr*`, ` *patt*`]`
: returns the list of positions for which *expr* matches *patt*.

`Position[`*expr*`, ` *patt*`, ` *ls*`]`
: returns the positions on levels specified by levelspec *ls*.

```
>> Position[{1, 2, 2, 1, 2, 3, 2},
2]
```

$\{\{2\}, \{3\}, \{5\}, \{7\}\}$

Find positions upto 3 levels deep

```
>> Position[{1 + Sin[x], x, (Tan[x]
 - y)^2}, x, 3]
```

$\{\{1, 2, 1\}, \{2\}\}$

Find all powers of x

```
>> Position[{1 + x^2, x y ^ 2, 4 y,
 x ^ z}, x^_]
```

$\{\{1, 2\}, \{4\}\}$

Use Position as an operator

```
>> Position[_Integer][{1.5, 2,
2.5}]
```

$\{\{2\}\}$

Quantile

`Quantile[`*list*`, ` *q*`]`
: returns the *q*th quantile of *list*.

```
>> Quantile[Range[11], 1/3]
```
4

```
>> Quantile[Range[16], 1/4]
```
5

Quartiles

`Quartiles[`*list*`]`
: returns the 1/4, 1/2, and 3/4 quantiles of *list*.

```
>> Quartiles[Range[25]]
```
$\left\{\frac{27}{4}, 13, \frac{77}{4}\right\}$

RankedMax

`RankedMax[`*list*`, `*n*`]`
: returns the *n*th largest element of *list* (with $n = 1$ yielding the largest element, $n = 2$ yielding the second largest element, and so on).

```
>> RankedMax[{482, 17, 181, -12},
2]
```
181

RankedMin

`RankedMin[`*list*`, `*n*`]`
: returns the *n*th smallest element of *list* (with $n = 1$ yielding the smallest element, $n = 2$ yielding the second smallest element, and so on).

```
>> RankedMin[{482, 17, 181, -12},
2]
```
17

Reverse

`Reverse[`*expr*`]`
: reverses the order of *expr*'s items (on the top level)

`Reverse[`*expr*`, `*n*`]`
: reverses the order of items in *expr* on level *n*

`Reverse[`*expr*`, {`*n1*`, `*n2*`, ...}]`
: reverses the order of items in *expr* on levels *n1*, *n2*, ...

```
>> Reverse[{1, 2, 3}]
```
$\{3, 2, 1\}$

```
>> Reverse[x[a, b, c]]
```
$x[c, b, a]$

```
>> Reverse[{{1, 2}, {3, 4}}, 1]
```
$\{\{3, 4\}, \{1, 2\}\}$

```
>> Reverse[{{1, 2}, {3, 4}}, 2]
```
$\{\{2, 1\}, \{4, 3\}\}$

```
>> Reverse[{{1, 2}, {3, 4}}, {1,
2}]
```
$\{\{4, 3\}, \{2, 1\}\}$

Riffle

`Riffle[`*list*`, `*x*`]`
: inserts a copy of *x* between each element of *list*.

`Riffle[{`*a1*`, `*a2*`, ...}, {`*b1*`, `*b2*`, ...}]`
: interleaves the elements of both lists, returning `{`*a1*`, `*b1*`, `*a2*`, `*b2*`, ...}`.

```
>> Riffle[{a, b, c}, x]
```
$\{a, x, b, x, c\}$

```
>> Riffle[{a, b, c}, {x, y, z}]
```
$\{a, x, b, y, c, z\}$

```
>> Riffle[{a, b, c, d, e, f}, {x, y
, z}]
```
$\{a, x, b, y, c, z, d, x, e, y, f\}$

RotateLeft

`RotateLeft[`*expr*`]`
: rotates the items of *expr'* by one item to the left.

`RotateLeft[`*expr*`, ` *n*`]`
: rotates the items of *expr'* by *n* items to the left.

`RotateLeft[`*expr*`, {`*n1*`, ` *n2*`, ...}]`
: rotates the items of *expr'* by *n1* items to the left at the first level, by *n2* items to the left at the second level, and so on.

```
>> RotateLeft[{1, 2, 3}]
```
$\{2,3,1\}$

```
>> RotateLeft[Range[10], 3]
```
$\{4,5,6,7,8,9,10,1,2,3\}$

```
>> RotateLeft[x[a, b, c], 2]
```
$x[c,a,b]$

```
>> RotateLeft[{{a, b, c}, {d, e, f
}, {g, h, i}}, {1, 2}]
```
$\{\{f,d,e\},\{i,g,h\},\{c,a,b\}\}$

RotateRight

`RotateRight[`*expr*`]`
: rotates the items of *expr'* by one item to the right.

`RotateRight[`*expr*`, ` *n*`]`
: rotates the items of *expr'* by *n* items to the right.

`RotateRight[`*expr*`, {`*n1*`, ` *n2*`, ...}]`
: rotates the items of *expr'* by *n1* items to the right at the first level, by *n2* items to the right at the second level, and so on.

```
>> RotateRight[{1, 2, 3}]
```
$\{3,1,2\}$

```
>> RotateRight[Range[10], 3]
```
$\{8,9,10,1,2,3,4,5,6,7\}$

```
>> RotateRight[x[a, b, c], 2]
```
$x[b,c,a]$

```
>> RotateRight[{{a, b, c}, {d, e, f
}, {g, h, i}}, {1, 2}]
```
$\{\{h,i,g\},\{b,c,a\},\{e,f,d\}\}$

Skewness

`Skewness[`*list*`]`
: gives Pearson's moment coefficient of skewness for *list* (a measure for estimating the symmetry of a distribution).

```
>> Skewness[{1.1, 1.2, 1.4, 2.1,
2.4}]
```
0.407041

Split

`Split[`*list*`]`
: splits *list* into collections of consecutive identical elements.

`Split[`*list*`, ` *test*`]`
: splits *list* based on whether the function *test* yields `True` on consecutive elements.

```
>> Split[{x, x, x, y, x, y, y, z}]
```
$\{\{x,x,x\},\{y\},\{x\},\{y,y\},\{z\}\}$

Split into increasing or decreasing runs of elements

```
>> Split[{1, 5, 6, 3, 6, 1, 6, 3,
4, 5, 4}, Less]
```
$\{\{1,5,6\},\{3,6\},\{1,6\},\{3,4,5\},\{4\}\}$

```
>> Split[{1, 5, 6, 3, 6, 1, 6, 3,
4, 5, 4}, Greater]
```
$\{\{1\},\{5\},\{6,3\},\{6,1\},\{6,3\},\{4\},\{5,4\}\}$

Split based on first element

```
>> Split[{x -> a, x -> y, 2 -> a, z
-> c, z -> a}, First[#1] ===
First[#2] &]
```
$\{\{x->a,x->y\},\{2->a\},\{z->c,z->a\}\}$

SplitBy

`SplitBy[`*list*`, `*f*`]`
: splits *list* into collections of consecutive elements that give the same result when *f* is applied.

```
>> SplitBy[Range[1, 3, 1/3], Round]
```
$\left\{\left\{1,\frac{4}{3}\right\},\left\{\frac{5}{3},2,\frac{7}{3}\right\},\left\{\frac{8}{3},3\right\}\right\}$

```
>> SplitBy[{1, 2, 1, 1.2}, {Round, Identity}]
```
$\{\{\{1\}\},\{\{2\}\},\{\{1\},\{1.2\}\}\}$

StandardDeviation

`StandardDeviation[`*list*`]`
: computes the standard deviation of $list. *list* may consist of numerical values or symbols. Numerical values may be real or complex.

StandardDeviation[{{*a1*, *a2*, ...}, {*b1*, *b2*, ...}, ...}] will yield {StandardDeviation[{*a1*, *b1*, ...}, StandardDeviation[{*a2*, *b2*, ...}], ...}.

```
>> StandardDeviation[{1, 2, 3}]
```
1

```
>> StandardDeviation[{7, -5, 101, 100}]
```
$\frac{\sqrt{13297}}{2}$

```
>> StandardDeviation[{a, a}]
```
0

```
>> StandardDeviation[{{1, 10}, {-1, 20}}]
```
$\left\{\sqrt{2},5\sqrt{2}\right\}$

SubsetQ

`SubsetQ[`*list1*`, `*list2*`]`
: returns True if *list2* is a subset of *list1*, and False otherwise.

```
>> SubsetQ[{1, 2, 3}, {3, 1}]
```
True

The empty list is a subset of every list:

```
>> SubsetQ[{}, {}]
```
True

```
>> SubsetQ[{1, 2, 3}, {}]
```
True

Every list is a subset of itself:

```
>> SubsetQ[{1, 2, 3}, {1, 2, 3}]
```
True

TakeLargest

`TakeLargest[`*list*`, `*f*`, `*n*`]`
: returns the a sorted list of the *n* largest items in *list*.

```
>> TakeLargest[{100, -1, 50, 10}, 2]
```
$\{100,50\}$

None, Null, Indeterminate and expressions with head Missing are ignored by default:

```
>> TakeLargest[{-8, 150, Missing[abc]}, 2]
```
$\{150,-8\}$

You may specify which items are ignored using the option ExcludedForms:

```
>> TakeLargest[{-8, 150, Missing[abc]}, 2, ExcludedForms -> {}]
```
{Missing[abc], 150}

TakeLargestBy

`TakeLargestBy[`*list*`, `*f*`, `*n*`]`
: returns the a sorted list of the *n* largest items in *list* using *f* to retrieve the items' keys to compare them.

For details on how to use the ExcludedForms option, see TakeLargest[].

```
>> TakeLargestBy[{{1, -1}, {10, 100}, {23, 7, 8}, {5, 1}}, Total, 2]
```
$\{\{10,100\},\{23,7,8\}\}$

>> `TakeLargestBy[{"abc", "ab", "x"}, StringLength, 1]`

{abc}

TakeSmallest

`TakeSmallest[`*list*`, `*f*`, `*n*`]`
: returns the a sorted list of the n smallest items in *list*.

For details on how to use the ExcludedForms option, see TakeLargest[].

>> `TakeSmallest[{100, -1, 50, 10}, 2]`

$\{-1, 10\}$

TakeSmallestBy

`TakeSmallestBy[`*list*`, `*f*`, `*n*`]`
: returns the a sorted list of the n smallest items in *list* using f to retrieve the items' keys to compare them.

For details on how to use the ExcludedForms option, see TakeLargest[].

>> `TakeSmallestBy[{{1, -1}, {10, 100}, {23, 7, 8}, {5, 1}}, Total, 2]`

$\{\{1, -1\}, \{5, 1\}\}$

>> `TakeSmallestBy[{"abc", "ab", "x"}, StringLength, 1]`

{x}

Tally

`Tally[`*list*`]`
: counts and returns the number of occurences of objects and returns the result as a list of pairs {object, count}.

`Tally[`*list*`, `*test*`]`
: counts the number of occurences of objects and uses $test to determine if two objects should be counted in the same bin.

>> `Tally[{a, b, c, b, a}]`

$\{\{a, 2\}, \{b, 2\}, \{c, 1\}\}$

Tally always returns items in the order as they first appear in *list*:

>> `Tally[{b, b, a, a, a, d, d, d, d, c}]`

$\{\{b, 2\}, \{a, 3\}, \{d, 4\}, \{c, 1\}\}$

Union

`Union[`*a*`, `*b*`, ...]`
: gives the union of the given set or sets. The resulting list will be sorted and each element will only occur once.

>> `Union[{5, 1, 3, 7, 1, 8, 3}]`

$\{1, 3, 5, 7, 8\}$

>> `Union[{a, b, c}, {c, d, e}]`

$\{a, b, c, d, e\}$

>> `Union[{c, b, a}]`

$\{a, b, c\}$

>> `Union[{{a, 1}, {b, 2}}, {{c, 1}, {d, 3}}, SameTest->(SameQ[Last[#1],Last[#2]]&)]`

$\{\{b, 2\}, \{c, 1\}, \{d, 3\}\}$

>> `Union[{1, 2, 3}, {2, 3, 4}, SameTest->Less]`

$\{1, 2, 2, 3, 4\}$

UnitVector

`UnitVector[`*n*`, `*k*`]`
: returns the n-dimensional unit vector with a 1 in position k.

`UnitVector[`*k*`]`
: is equivalent to `UnitVector[2, `*k*`]`.

>> `UnitVector[2]`

$\{0, 1\}$

>> `UnitVector[4, 3]`

$\{0, 0, 1, 0\}$

Values

`Values[<|`*key1* `->` *val1*`,` *key2* `->` *val2*`, ...|>]`
: return a list of the values *vali* in an association.

`Values[{`*key1* `->` *val1*`,` *key2* `->` *val2*`, ...}]`
: return a list of the *vali* in a list of rules.

```
>> Values[<|a -> x, b -> y|>]
```
$\{x, y\}$

```
>> Values[{a -> x, b -> y}]
```
$\{x, y\}$

Values automatically threads over lists:
```
>> Values[{<|a -> x, b -> y|>, {c -> z, {}}}]
```
$\{\{x, y\}, \{z, \{\}\}\}$

Values are listed in the order of their appearance:
```
>> Values[{c -> z, b -> y, a -> x}]
```
$\{z, y, x\}$

Variance

`Variance[`*list*`]`
: computes the variance of $list. *list* may consist of numerical values or symbols. Numerical values may be real or complex.

Variance[{{*a1*, *a2*, ...}, {*b1*, *b2*, ...}, ...}] will yield {Variance[{*a1*, *b1*, ...}, Variance[{*a2*, *b2*, ...}], ...}.

```
>> Variance[{1, 2, 3}]
```
1

```
>> Variance[{7, -5, 101, 3}]
```
$\frac{7475}{3}$

```
>> Variance[{1 + 2I, 3 - 10I}]
```
74

```
>> Variance[{a, a}]
```
0

```
>> Variance[{{1, 3, 5}, {4, 10, 100}}]
```
$\left\{\frac{9}{2}, \frac{49}{2}, \frac{9025}{2}\right\}$

21. Numeric Evaluation and Precision

Support for numeric evaluation with arbitrary precision is just a proof-of-concept.
Precision is not “guarded” through the evaluation process. Only integer precision is supported. However, things like `N[Pi, 100]` should work as expected.

Contents

Chop

`Chop[`*expr*`]`
: replaces floating point numbers close to 0 by 0.

`Chop[`*expr*`, `*delta*`]`
: uses a tolerance of *delta*. The default tolerance is `10^-10`.

```
>> Chop[10.0 ^ -16]
```
0

```
>> Chop[10.0 ^ -9]
```
$1. \times 10^{-9}$

```
>> Chop[10 ^ -11 I]
```
$\frac{I}{100\,000\,000\,000}$

```
>> Chop[0. + 10 ^ -11 I]
```
0

Hash

`Hash[`*expr*`]`
: returns an integer hash for the given *expr*.

`Hash[`*expr*`, `*type*`]`
: returns an integer hash of the specified *type* for the given *expr*.
The types supported are “MD5”, “Adler32”, “CRC32”, “SHA”, “SHA224”, “SHA256”, “SHA384”, and “SHA512”.

`Hash[`*expr*`, `*type*`, `*format*`]`
: Returns the hash in the specified format.

> Hash[“The Adventures of Huckleberry Finn”] = 213425047836523694663619736686226550816
> Hash[“The Adventures of Huckleberry Finn”, “SHA256”] = 950926495945903842880571834086092549189343518
> Hash[1/3] = 56073172797010645108327809727054836008
> Hash[{a, b, {c, {d, e, f}}}] = 135682164776235407777080772547528
> Hash[SomeHead[3.1415]] = 5804231647347187731544201546970

```
>> Hash[{a, b, c}, "xyzstr"]
```
Hash [{a, b, c} , xyzstr, Integer]

IntegerDigits

`IntegerDigits[`*n*`]`
: returns a list of the base-10 digits in the integer n.

`IntegerDigits[`*n*`, `*base*`]`
: returns a list of the base-*base* digits in n.

`IntegerDigits[`*n*`, `*base*`, `*length*`]`
: returns a list of length *length*, truncating or padding with zeroes on the left as necessary.

```
>> IntegerDigits[76543]
```
$\{7, 6, 5, 4, 3\}$

The sign of n is discarded:

```
>> IntegerDigits[-76543]
```
$\{7, 6, 5, 4, 3\}$

```
>> IntegerDigits[15, 16]
```
$\{15\}$

```
>> IntegerDigits[1234, 16]
```
$\{4, 13, 2\}$

```
>> IntegerDigits[1234, 10, 5]
```
$\{0, 1, 2, 3, 4\}$

$MachineEpsilon

`$MachineEpsilon`
: is the distance between `1.0` and the next nearest representable machine-precision number.

```
>> $MachineEpsilon
```
2.22045×10^{-16}

```
>> x = 1.0 + {0.4, 0.5, 0.6} $MachineEpsilon;
```

```
>> x - 1
```
$\left\{0., 0., 2.22045 \times 10^{-16}\right\}$

MachinePrecision

`MachinePrecision`
: represents the precision of machine precision numbers.

```
>> N[MachinePrecision]
```
15.9546

```
>> N[MachinePrecision, 30]
```
15.9545897701910033463281614204

$MachinePrecision

`$MachinePrecision`
: is the number of decimal digits of precision for machine-precision numbers.

```
>> $MachinePrecision
```
15.9546

$MaxPrecision

`$MaxPrecision`
: represents the maximum number of digits of precision permitted in abitrary-precision numbers.

```
>> $MaxPrecision
```
∞

```
>> $MaxPrecision = 10;
```

```
>> N[Pi, 11]
```
Requestedprecision11islargerthan$MaxPrecision.Usingcurre

= Infinityspecifiesthatanyprecisionshouldbeallowed.

3.141592654

$MinPrecision

`$MinPrecision`
: represents the minimum number of digits of precision permitted in abitrary-precision numbers.

```
>> $MinPrecision
```
0

```
>> $MinPrecision = 10;
```

```
>> N[Pi, 9]
```
Requestedprecision9issmallerthan$MinPrecision.Usingcurrent$MinPrecisionof10.instead.

3.141592654

N

`N[expr, prec]`
evaluates *expr* numerically with a precision of *prec* digits.

```
>> N[Pi, 50]
```
3.14159265358979323846264338327950288419716939937510

```
>> N[1/7]
```
0.142857

```
>> N[1/7, 5]
```
0.14286

You can manually assign numerical values to symbols. When you do not specify a precision, `MachinePrecision` is taken.

```
>> N[a] = 10.9
```
10.9

```
>> a
```
a

`N` automatically threads over expressions, except when a symbol has attributes `NHoldAll`, `NHoldFirst`, or `NHoldRest`.

```
>> N[a + b]
```
10.9 + *b*

```
>> N[a, 20]
```
a

```
>> N[a, 20] = 11;
```

```
>> N[a + b, 20]
```
11.000000000000000000 + *b*

```
>> N[f[a, b]]
```
f [10.9, *b*]

```
>> SetAttributes[f, NHoldAll]
```

```
>> N[f[a, b]]
```
f [*a*, *b*]

The precision can be a pattern:

```
>> N[c, p_?(#>10&)] := p
```

```
>> N[c, 3]
```
c

```
>> N[c, 11]
```
11.000000000

You can also use `UpSet` or `TagSet` to specify values for `N`:

```
>> N[d] ^= 5;
```

However, the value will not be stored in `UpValues`, but in `NValues` (as for `Set`):

```
>> UpValues[d]
```
{}

```
>> NValues[d]
```
{HoldPattern [*N* [*d*, MachinePrecision]] :>5}

```
>> e /: N[e] = 6;
```

```
>> N[e]
```
6.

Values for `N[`*expr*`]` must be associated with the head of *expr*:

```
>> f /: N[e[f]] = 7;
```
Tagfnotfoundortoodeepforanassignedrule.

You can use `Condition`:

```
>> N[g[x_, y_], p_] := x + y * Pi
   /; x + y > 3
```

```
>> SetAttributes[g, NHoldRest]
```

```
>> N[g[1, 1]]
```
g [1., 1]

```
>> N[g[2, 2]] // InputForm
```
8.283185307179586

The precision of the result is no higher than the precision of the input

```
>> N[Exp[0.1], 100]
```
1.10517

```
>> % // Precision
```
MachinePrecision

```
>> N[Exp[1/10], 100]
```
1.105170918075647624811707~
~826490246668224547194737~
~518718792863289440967966~
~747654302989143318970748654

```
>> % // Precision
```
100.

```
>> N[Exp[1.0`20], 100]
```
2.7182818284590452354

```
>> % // Precision
```
20.

NIntegrate

`NIntegrate[`*expr*`, `*interval*`]`
: returns a numeric approximation to the definite integral of *expr* with limits *interval* and with a precision of *prec* digits.

`NIntegrate[`*expr*`, `*interval1*`, `*interval2*`, ...]`
: returns a numeric approximation to the multiple integral of *expr* with limits *interval1*, *interval2* and with a precision of *prec* digits.

```
>> NIntegrate[Exp[-x],{x,0,Infinity},Tolerance->1*^-6]
```
1.

```
>> NIntegrate[Exp[x],{x,-Infinity,0},Tolerance->1*^-6]
```
1.

```
>> NIntegrate[Exp[-x^2/2.],{x,-Infinity, Infinity},Tolerance->1*^-6]
```
2.50663

```
>> Table[1./NIntegrate[x^k,{x,0,1}],Tolerance->1*^-6], {k,0,6}]
```
Thespecifiedmethodfailedtoreturnanumber.Falling

{1., 2., 3., 4., 5., 6., 7.}

```
>> NIntegrate[1 / z, {z, -1 - I, 1 - I, 1 + I, -1 + I, -1 - I}, Tolerance->1.*^-4]
```
Integrationoveracomplexdomainisnotimplementedyet

$$\text{NIntegrate}\left[\frac{1}{z}, \{z, -1-I, 1-I, 1+I, -1+I, -1-I\}, \text{Tolerance}->0.0001\right]$$

Integrate singularities with weak divergences:

```
>> Table[ NIntegrate[x^(1./k-1.), {x,0,1.}, Tolerance->1*^-6], {k,1,7.} ]
```
{1., 2., 3., 4., 5., 6., 7.}

Mutiple Integrals :

```
>> NIntegrate[x * y,{x, 0, 1}, {y, 0, 1}]
```
0.25

NumericQ

`NumericQ[`*expr*`]`
: tests whether *expr* represents a numeric quantity.

```
>> NumericQ[2]
```
True

```
>> NumericQ[Sqrt[Pi]]
```
True

```
>> NumberQ[Sqrt[Pi]]
```
False

Precision

`Precision[`*expr*`]`
: examines the number of significant digits of *expr*.

This is rather a proof-of-concept than a full implementation. Precision of compound expression is not supported yet.

```
>> Precision[1]
```
∞

```
>> Precision[1/2]
```
∞

```
>> Precision[0.5]
```
MachinePrecision

Rationalize

`Rationalize[x]`
: converts a real number x to a nearby rational number.

`Rationalize[x, dx]`
: finds the rational number within dx of x with the smallest denominator.

```
>> Rationalize[2.2]
```
$$\frac{11}{5}$$

Not all numbers can be well approximated.

```
>> Rationalize[N[Pi]]
```
3.14159

Find the exact rational representation of `N[Pi]`

```
>> Rationalize[N[Pi], 0]
```
$$\frac{245\,850\,922}{78\,256\,779}$$

RealDigits

`RealDigits[n]`
: returns the decimal representation of the real number n as list of digits, together with the number of digits that are to the left of the decimal point.

`RealDigits[n, b]`
: returns a list of base_b representation of the real number n.

`RealDigits[n, b, len]`
: returns a list of len digits.

`RealDigits[n, b, len, p]`
: return len digits starting with the coefficient of b^p

Return the list of digits and exponent:

```
>> RealDigits[123.55555]
```
{{1, 2, 3, 5, 5, 5, 5, 5, 0, 0, 0, 0, 0, 0, 0, 0}, 3}

Return an explicit recurring decimal form:

```
>> RealDigits[19 / 7]
```
{{2, {7, 1, 4, 2, 8, 5}}, 1}

The 10000th digit of is an 8:

```
>> RealDigits[Pi, 10, 1, -10000]
```
{{8}, −9 999}

20 digits starting with the coefficient of 10^-5:

```
>> RealDigits[Pi, 10, 20, -5]
```
{{9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3, 2, 3, 8, 4, 6, 2, 6, 4, 3}, −4}

RealDigits gives Indeterminate if more digits than the precision are requested:

```
>> RealDigits[123.45, 10, 18]
```
{{1, 2, 3, 4, 5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, Indeterminate, Indeterminate}, 3}

Return 25 digits of in base 10:

```
>> RealDigits[Pi, 10, 25]
```
{{3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3, 2, 3, 8, 4, 6, 2, 6, 4, 3}, 1}

Round

`Round[expr]`
: rounds $expr$ to the nearest integer.

`Round[expr, k]`
: rounds $expr$ to the closest multiple of k.

```
>> Round[10.6]
```
11

```
>> Round[0.06, 0.1]
```
0.1

```
>> Round[0.04, 0.1]
```
0.

Constants can be rounded too

```
>> Round[Pi, .5]
```
3.

```
>> Round[Pi^2]
```
10

Round to exact value

```
>> Round[2.6, 1/3]
```
$$\frac{8}{3}$$

>> `Round[10, Pi]`

3Pi

Round complex numbers

>> `Round[6/(2 + 3 I)]`

$1 - I$

>> `Round[1 + 2 I, 2 I]`

$2I$

Round Negative numbers too

>> `Round[-1.4]`

-1

Expressions other than numbers remain unevaluated:

>> `Round[x]`

Round $[x]$

>> `Round[1.5, k]`

Round $[1.5, k]$

22. Arithmetic Functions

Arithmetic Functions are functions that work on individual numbers, lists, and arrays: in either symbolic or algebraic forms.

23. Basic Arithmetic

The functions here are the basic arithmetic operations that you might find on a calculator.

Contents

CubeRoot

`CubeRoot[`*n*`]`
: finds the real-valued cube root of the given *n*.

```
>> CubeRoot[16]
```
$2 2^{\frac{1}{3}}$

Divide (/)

`Divide[`*a*, *b*`]`
a / *b*
: represents the division of *a* by *b*.

```
>> 30 / 5
```
6

```
>> 1 / 8
```
$\frac{1}{8}$

```
>> Pi / 4
```
$\frac{\text{Pi}}{4}$

Use `N` or a decimal point to force numeric evaluation:

```
>> Pi / 4.0
```
0.785398

```
>> 1 / 8
```
$\frac{1}{8}$

```
>> N[%]
```
0.125

Nested divisions:

```
>> a / b / c
```
$\frac{a}{bc}$

```
>> a / (b / c)
```
$\frac{ac}{b}$

```
>> a / b / (c / (d / e))
```
$\frac{ad}{bce}$

```
>> a / (b ^ 2 * c ^ 3 / e)
```
$\frac{ae}{b^2c^3}$

Minus (-)

`Minus[`*expr*`]`
: is the negation of *expr*.

```
>> -a //FullForm
```
Times [− 1, *a*]

`Minus` automatically distributes:

```
>> -(x - 2/3)
```
$\frac{2}{3} - x$

`Minus` threads over lists:

```
>> -Range[10]
```
$\{-1, -2, -3, -4, -5,$
$-6, -7, -8, -9, -10\}$

Plus (+)

`Plus[`a`, `b`, ...]`
$a + b + \ldots$
    represents the sum of the terms a, b, ...

```
>> 1 + 2
```
3

`Plus` performs basic simplification of terms:

```
>> a + b + a
```
$2a + b$

```
>> a + a + 3 * a
```
$5a$

```
>> a + b + 4.5 + a + b + a + 2 +
   1.5 b
```
$6.5 + 3a + 3.5b$

Apply `Plus` on a list to sum up its elements:

```
>> Plus @@ {2, 4, 6}
```
12

The sum of the first 1000 integers:

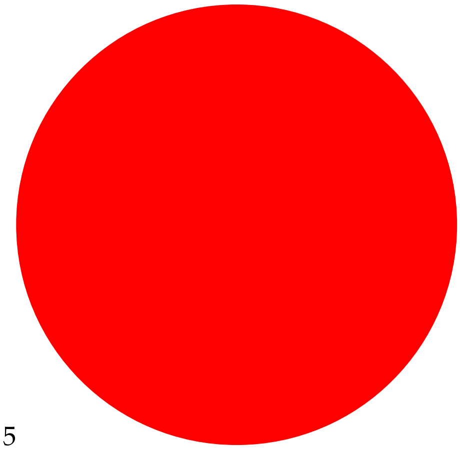
```
>> Plus @@ Range[1000]
```
500 500

`Plus` has default value 0:

```
>> DefaultValues[Plus]
```
{HoldPattern [Default [Plus]] :>0}

```
>> a /. n_. + x_ :> {n, x}
```
$\{0, a\}$

The sum of 2 red circles and 3 red circles is...

```
>> 2 Graphics[{Red,Disk[]}] + 3
   Graphics[{Red,Disk[]}]
```
5 

Power (^)

`Power[`a`, `b`]`
a `^` b
    represents a raised to the power of b.

```
>> 4 ^ (1/2)
```
2

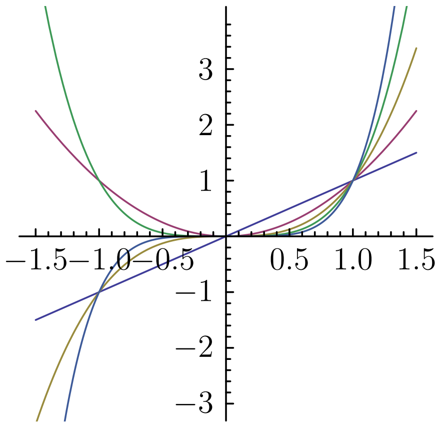
```
>> 4 ^ (1/3)
```
$2^{\frac{2}{3}}$

```
>> 3^123
```
48 519 278 097 689 642 681 ~
~155 855 396 759 336 072 ~
~749 841 943 521 979 872 827

```
>> (y ^ 2)^ (1/2)
```
$\sqrt{y^2}$

```
>> (y ^ 2)^ 3
```
y^6

```
>> Plot[Evaluate[Table[x^y, {y, 1,
   5}]], {x, -1.5, 1.5},
   AspectRatio -> 1]
```



Use a decimal point to force numeric evaluation:

```
>> 4.0 ^ (1/3)
```

1.5874

`Power` has default value 1 for its second argument:

```
>> DefaultValues[Power]
```

{HoldPattern [Default [Power, 2]] :>1}

```
>> a /. x_ ^ n_. :> {x, n}
```

$\{a, 1\}$

`Power` can be used with complex numbers:

```
>> (1.5 + 1.0 I)^ 3.5
```

$-3.68294 + 6.95139I$

```
>> (1.5 + 1.0 I)^ (3.5 + 1.5 I)
```

$-3.19182 + 0.645659I$

Sqrt

`Sqrt[`*expr*`]`
    returns the square root of *expr*.

```
>> Sqrt[4]
```

2

```
>> Sqrt[5]
```

$\sqrt{5}$

```
>> Sqrt[5] // N
```

2.23607

```
>> Sqrt[a]^2
```

a

Complex numbers:

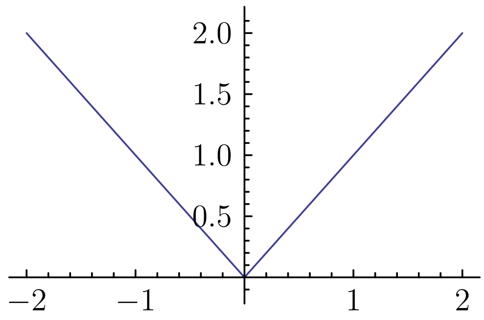
```
>> Sqrt[-4]
```

$2I$

```
>> I == Sqrt[-1]
```

True

```
>> Plot[Sqrt[a^2], {a, -2, 2}]
```



Subtract (-)

`Subtract[`*a*`, `*b*`]`
a `-` *b*
    represents the subtraction of *b* from *a*.

```
>> 5 - 3
```

2

```
>> a - b // FullForm
```

Plus [a, Times [$-1, b$]]

```
>> a - b - c
```

$a - b - c$

```
>> a - (b - c)
```

$a - b + c$

Times (*)

`Times[`*a*`, `*b*`, ...]`
a `*` *b* `* ...`
a *b* `...`
    represents the product of the terms *a*, *b*, ...

```
>> 10 * 2
```

20

```
>> 10 2
```

20

```
>> a * a
```
a^2

```
>> x ^ 10 * x ^ -2
```
x^8

```
>> {1, 2, 3} * 4
```
$\{4, 8, 12\}$

```
>> Times @@ {1, 2, 3, 4}
```
24

```
>> IntegerLength[Times@@Range
   [5000]]
```
16 326

`Times` has default value 1:

```
>> DefaultValues[Times]
```
{HoldPattern [Default [Times]] :>1}

```
>> a /. n_. * x_ :> {n, x}
```
$\{1, a\}$

24. Sums, Simple Statistics

These functions perform a simple arithmetic computation over a list.

Contents

Accumulate

`Accumulate[`*list*`]`
: accumulates the values of *list*, returning a new list.

```
>> Accumulate[{1, 2, 3}]
```
$\{1, 3, 6\}$

Mean

`Mean[`*list*`]`
: returns the statistical mean of *list*.

```
>> Mean[{26, 64, 36}]
```
42

```
>> Mean[{1, 1, 2, 3, 5, 8}]
```
$\frac{10}{3}$

```
>> Mean[{a, b}]
```
$\frac{a+b}{2}$

Total

`Total[`*list*`]`
: adds all values in *list*.

`Total[`*list*`, ` *n*`]`
: adds all values up to level *n*.

`Total[`*list*`, {`*n*`}]`
: totals only the values at level $\{n\}$.

`Total[`*list*`, {`*n_1*`, ` *n_2*`}]`
: totals at levels $\{n\_1, n\_2\}$.

```
>> Total[{1, 2, 3}]
```
6

```
>> Total[{{1, 2, 3}, {4, 5, 6}, {7,
   8 ,9}}]
```
$\{12, 15, 18\}$

Total over rows and columns

```
>> Total[{{1, 2, 3}, {4, 5, 6}, {7,
   8 ,9}}, 2]
```
45

Total over rows instead of columns

```
>> Total[{{1, 2, 3}, {4, 5, 6}, {7,
   8 ,9}}, {2}]
```
$\{6, 15, 24\}$

25. Colors

Programmatic support for symbolic colors.

26. Color Directives

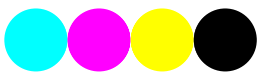
There are many different way to specify color; we support all of the color formats below and will convert between the different color formats.

Contents

CMYKColor

`CMYKColor[`*c*`,` *m*`,` *y*`,` *k*`]`
: represents a color with the specified cyan, magenta, yellow and black components.

```
>> Graphics[MapIndexed[{CMYKColor
   @@ #1, Disk[2*#2 ~Join~{0}]} &,
   IdentityMatrix[4]], ImageSize->
   Small]
```



ColorDistance

`ColorDistance[`*c1*`,` *c2*`]`
: returns a measure of color distance between the colors *c1* and *c2*.

`ColorDistance[`*list*`,` *c2*`]`
: returns a list of color distances between the colors in *list* and *c2*.

The option DistanceFunction specifies the method used to measure the color distance. Available options are:

- CIE76: Euclidean distance in the LABColor space
- CIE94: Euclidean distance in the LCHColor space
- CIE2000 or CIEDE2000: CIE94 distance with corrections
- CMC: Color Measurement Committee metric (1984)
- DeltaL: difference in the L component of LCHColor
- DeltaC: difference in the C component of LCHColor
- DeltaH: difference in the H component of LCHColor

It is also possible to specify a custom distance.

```
>> ColorDistance[Magenta, Green]
```

2.2507

```
>> ColorDistance[{Red, Blue}, {
   Green, Yellow}, DistanceFunction
   -> {"CMC", "Perceptibility"}]
```

$\{1.0495, 1.27455\}$

GrayLevel

`GrayLevel[`*g*`]`
: represents a shade of gray specified by *g*, ranging from 0 (black) to 1 (white).

`GrayLevel[`*g*`,` *a*`]`
: represents a shade of gray specified by *g* with opacity *a*.

Hue

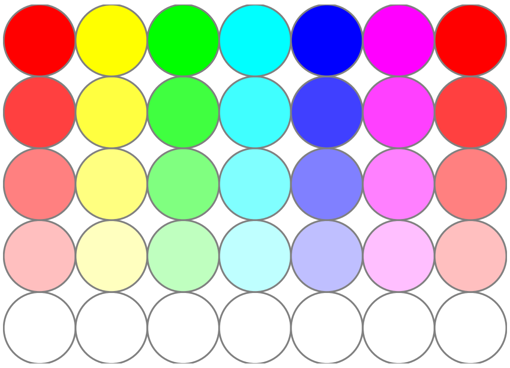
`Hue[`*h*`, `*s*`, `*l*`, `*a*`]`
: represents the color with hue *h*, saturation *s*, lightness *l* and opacity *a*.

`Hue[`*h*`, `*s*`, `*l*`]`
: is equivalent to `Hue[`*h*`, `*s*`, `*l*`, 1]`.

`Hue[`*h*`, `*s*`]`
: is equivalent to `Hue[`*h*`, `*s*`, 1, 1]`.

`Hue[`*h*`]`
: is equivalent to `Hue[`*h*`, 1, 1, 1]`.

```
>> Graphics[Table[{EdgeForm[Gray],
   Hue[h, s], Disk[{12h, 8s}]}, {h,
    0, 1, 1/6}, {s, 0, 1, 1/4}]]
```



```
>> Graphics[Table[{EdgeForm[{
   GrayLevel[0, 0.5]}], Hue[(-11+q
   +10r)/72, 1, 1, 0.6], Disk[(8-r)
   {Cos[2Pi q/12], Sin[2Pi q/12]},
   (8-r)/3]}, {r, 6}, {q, 12}]]
```

LABColor

`LABColor[`*l*`, `*a*`, `*b*`]`
: represents a color with the specified lightness, red/green and yellow/blue components in the CIE 1976 L*a*b* (CIELAB) color space.

LCHColor

`LCHColor[`*l*`, `*c*`, `*h*`]`
: represents a color with the specified lightness, chroma and hue components in the CIELCh CIELab cube color space.

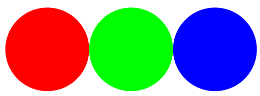
LUVColor

`LCHColor[`*l*`, `*u*`, `*v*`]`
: represents a color with the specified components in the CIE 1976 L*u*v* (CIELUV) color space.

RGBColor

`RGBColor[`*r*`, `*g*`, `*b*`]`
: represents a color with the specified red, green and blue components.

```
>> Graphics[MapIndexed[{RGBColor @@
    #1, Disk[2*#2 ~Join~{0}]} &,
   IdentityMatrix[3]], ImageSize->
   Small]
```



```
>> RGBColor[0, 1, 0]
```

>> `RGBColor[0, 1, 0] // ToBoxes`

StyleBox [GraphicsBox [
{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [0, 1, 0] ,
RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

XYZColor

`XYZColor[`x`, `y`, `z`]`
: represents a color with the specified components in the CIE 1931 XYZ color space.

27. Color Operations

Functions for manipulating colors and color images.

Contents

Blend

`Blend[{`*c1*`, `*c2*`}]`
: represents the color between *c1* and *c2*.

`Blend[{`*c1*`, `*c2*`}, `*x*`]`
: represents the color formed by blending *c1* and *c2* with factors 1 - *x* and *x* respectively.

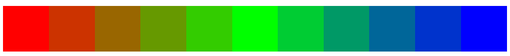
`Blend[{`*c1*`, `*c2*`, ..., `*cn*`}, `*x*`]`
: blends between the colors *c1* to *cn* according to the factor *x*.

```
>> Blend[{Red, Blue}]
```

```
>> Blend[{Red, Blue}, 0.3]
```

```
>> Blend[{Red, Blue, Green}, 0.75]
```

```
>> Graphics[Table[{Blend[{Red,
   Green, Blue}, x], Rectangle[{10
   x, 0}]}, {x, 0, 1, 1/10}]]
```



```
>> Graphics[Table[{Blend[{RGBColor
   [1, 0.5, 0, 0.5], RGBColor[0, 0,
    1, 0.5]}, x], Disk[{5x, 0}]}, {
   x, 0, 1, 1/10}]]
```

ColorConvert

`ColorConvert[`*c*`, `*colspace*`]`
: returns the representation of *c* in the color space *colspace*. *c* may be a color or an image.

Valid values for *colspace* are:
CMYK: convert to CMYKColor Grayscale: convert to GrayLevel HSB: convert to Hue LAB: concert to LABColor LCH: convert to LCHColor LUV: convert to LUVColor RGB: convert to RGBColor XYZ: convert to XYZColor

ColorNegate

<dl> <dt>`ColorNegate[`*image*`]` <dd>returns the negative of *image* in which colors have been negated.
<dt>`ColorNegate[`*color*`]` <dd>returns the negative of a color.
Yellow is RGBColor[1.0, 1.0, 0.0]

```
>> ColorNegate[Yellow]
```

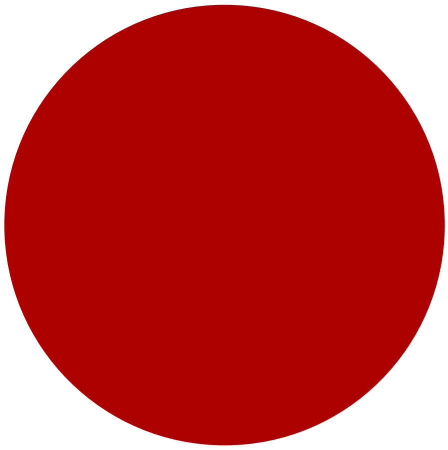
</dl>

Darker

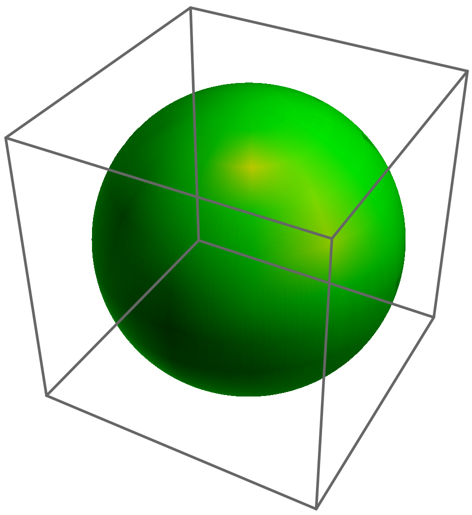
`Darker[`*c*`, `*f*`]`
: is equivalent to `Blend[{`*c*`, Black}, `*f*`]`.

`Darker[`*c*`]`
: is equivalent to `Darker[`*c*`, 1/3]`.

```
>> Graphics[{Darker[Red], Disk[]}]
```



```
>> Graphics3D[{Darker[Green],
   Sphere[]}]
```



```
>> Graphics[Table[{Darker[Yellow, x
   ], Disk[{12x, 0}]}, {x, 0, 1,
   1/6}]]
```

DominantColors

`DominantColors[`*image*`]`
: gives a list of colors which are dominant in the given image.

`DominantColors[`*image*`, ` *n*`]`
: returns at most *n* colors.

`DominantColors[`*image*`, ` *n*`, ` *prop*`]`
: returns the given property *prop*, which may be "Color" (return RGB colors), "LABColor" (return LAB colors), "Count" (return the number of pixels a dominant color covers), "Coverage" (return the fraction of the image a dominant color covers), or "CoverageImage" (return a black and white image indicating with white the parts that are covered by a dominant color).

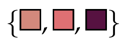
The option "ColorCoverage" specifies the minimum amount of coverage needed to include a dominant color in the result.
The option "MinColorDistance" specifies the distance (in LAB color space) up to which colors are merged and thus regarded as belonging to the same dominant color.

```
>> img = Import["ExampleData/lena.
   tif"]
```

−Image−

```
>> DominantColors[img]
```

```
>> DominantColors[img, 3]
```



```
>> DominantColors[img, 3, "Coverage
   "]
```

$$\left\{\frac{28\,579}{131\,072}, \frac{751}{4\,096}, \frac{23\,841}{131\,072}\right\}$$

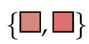
```
>> DominantColors[img, 3, "
   CoverageImage"]
```

{*−Image−*, *−Image−*, *−Image−*}

```
>> DominantColors[img, 3, "Count"]
```

{57 158, 48 064, 47 682}

```
>> DominantColors[img, 2, "LABColor
   "]
```



```
>> DominantColors[img,
   MinColorDistance -> 0.5]
```

{}

```
>> DominantColors[img,
   ColorCoverage -> 0.15]
```

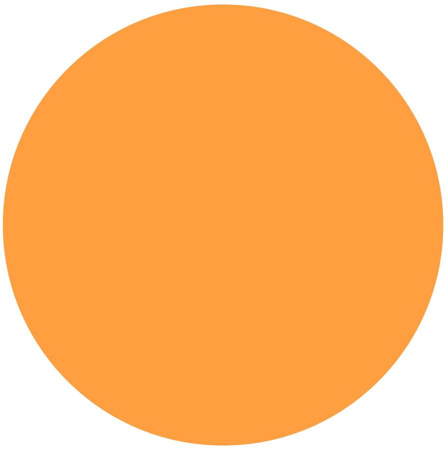
{}

Lighter

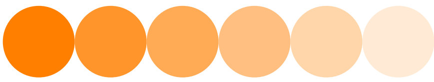
`Lighter[`*c*`, `*f*`]`
    is equivalent to `Blend[{`*c*`, White}, `*f*`]`.
`Lighter[`*c*`]`
    is equivalent to `Lighter[`*c*`, 1/3]`.

```
>> Lighter[Orange, 1/4]
```

```
>> Graphics[{Lighter[Orange, 1/4],
   Disk[]}]
```



```
>> Graphics[Table[{Lighter[Orange,
   x], Disk[{12x, 0}]}, {x, 0, 1,
   1/6}]]
```



28. Named Colors

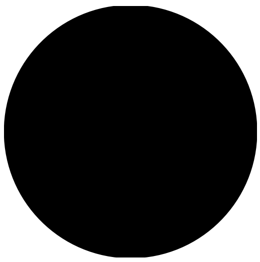
Mathics has definitions for the most common color names which can be used in a graphics or style specification.

Contents

Black

`Black`
: represents the color black in graphics.

```
>> Graphics[{EdgeForm[Black], Black, Disk[]}, ImageSize->Small]
```



```
>> Black // ToBoxes
```

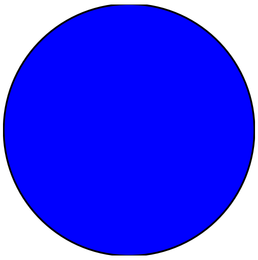
StyleBox [GraphicsBox [{EdgeForm [RGBColor [0, 0, 0]] , RGBColor [0, 0, 0] , RectangleBox [{0, 0}] } , $OptionSyntax− > Ignore, AspectRatio− > Automatic, Axes− > False, AxesStyle− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle− > {}] , ImageSizeMultipliers− > {1, 1}]

```
>> Black
```

Blue

`Blue`
: represents the color blue in graphics.

```
>> Graphics[{EdgeForm[Black], Blue,
   Disk[]}, ImageSize->Small]
```



```
>> Blue // ToBoxes
```

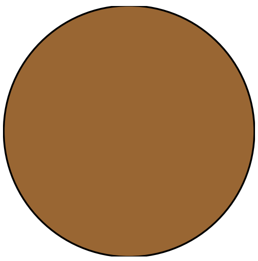
StyleBox [GraphicsBox [
{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [0, 0, 1] ,
RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

```
>> Blue
```

Brown

`Brown`
represents the color brown in graphics.

```
>> Graphics[{EdgeForm[Black], Brown
   , Disk[]}, ImageSize->Small]
```



```
>> Brown // ToBoxes
```

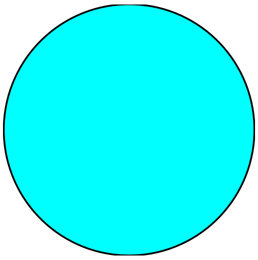
StyleBox [GraphicsBox [
{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [0.6, 0.4,
0.2] , RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

```
>> Brown
```

Cyan

`Cyan`
represents the color cyan in graphics.

```
>> Graphics[{EdgeForm[Black], Cyan,
   Disk[]}, ImageSize->Small]
```



```
>> Cyan // ToBoxes
```

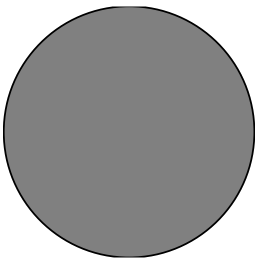
StyleBox [GraphicsBox [
{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [0, 1, 1] ,
RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

```
>> Cyan
```

Gray

`Gray`
: represents the color gray in graphics.

```
>> Graphics[{EdgeForm[Black], Gray,
    Disk[]}, ImageSize->Small]
```



```
>> Gray // ToBoxes
```

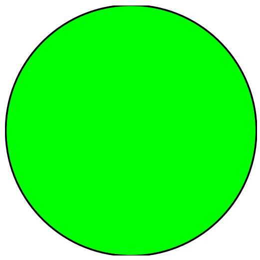
StyleBox [GraphicsBox [
{EdgeForm [RGBColor [0, 0, 0]] ,
GrayLevel [0.5] , RectangleBox [{0,
0}] } , $OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

```
>> Gray
```

Green

`Green`
: represents the color green in graphics.

```
>> Graphics[{EdgeForm[Black], Green
   , Disk[]}, ImageSize->Small]
```



>> `Green // ToBoxes`

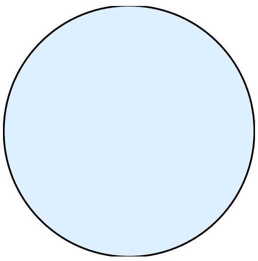
StyleBox [GraphicsBox [
{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [0, 1, 0] ,
RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

>> `Green`

LightBlue

`LightBlue`
represents the color light blue in graphics.

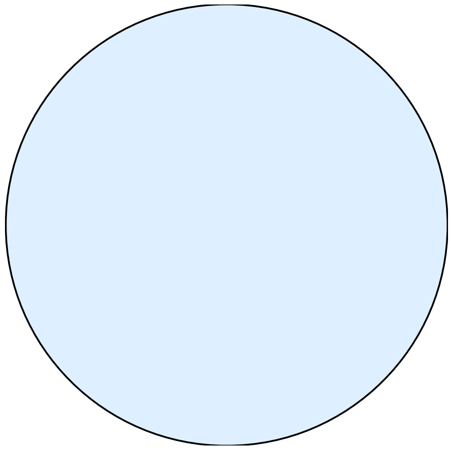
>> `Graphics[{EdgeForm[Black], LightBlue, Disk[]}, ImageSize->Small]`



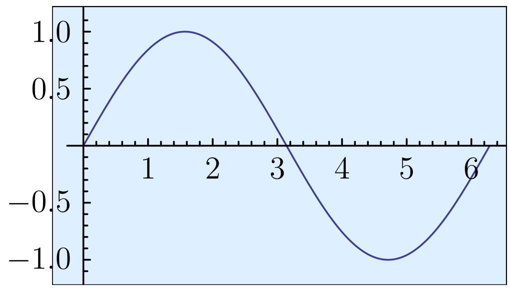
>> `LightBlue // ToBoxes`

StyleBox [GraphicsBox [
{EdgeForm [RGBColor [0,
0, 0]] , RGBColor [0.87, 0.94,
1] , RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

>> `Graphics[{LightBlue, EdgeForm[Black], Disk[]}]`



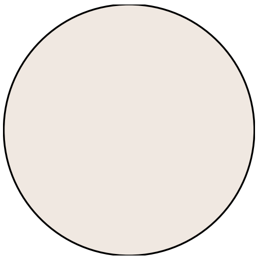
>> `Plot[Sin[x], {x, 0, 2 Pi}, Background -> LightBlue]`



LightBrown

`LightBrown`
: represents the color light brown in graphics.

```
>> Graphics[{EdgeForm[Black], LightBrown, Disk[]}, ImageSize->Small]
```



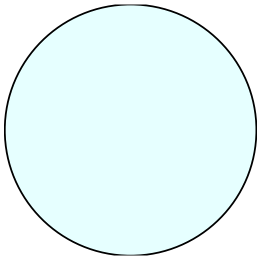
```
>> LightBrown // ToBoxes
```

StyleBox [GraphicsBox [{EdgeForm [RGBColor [0, 0, 0]] , RGBColor [0.94, 0.91, 0.88] , RectangleBox [{0, 0}] } , $OptionSyntax− > Ignore, AspectRatio− > Automatic, Axes− > False, AxesStyle− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle− > {}] , ImageSizeMultipliers− > {1, 1}]

LightCyan

`LightCyan`
: represents the color light cyan in graphics.

```
>> Graphics[{EdgeForm[Black], LightCyan, Disk[]}, ImageSize->Small]
```



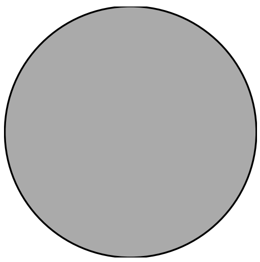
```
>> LightCyan // ToBoxes
```

StyleBox [GraphicsBox [{EdgeForm [RGBColor [0, 0, 0]] , RGBColor [0.9, 1., 1.] , RectangleBox [{0, 0}] } , $OptionSyntax− > Ignore, AspectRatio− > Automatic, Axes− > False, AxesStyle− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle− > {}] , ImageSizeMultipliers− > {1, 1}]

LightGray

`LightGray`
: represents the color light gray in graphics.

```
>> Graphics[{EdgeForm[Black], LightGray, Disk[]}, ImageSize->Small]
```



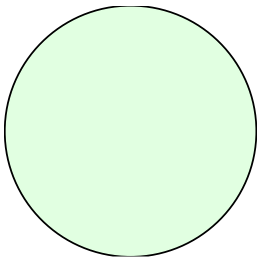
>> **LightGray // ToBoxes**

StyleBox [GraphicsBox [
{EdgeForm [RGBColor [0,
0, 0]] , GrayLevel [0.666667,
1.] , RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

LightGreen

`LightGreen`
represents the color light green in graphics.

>> **Graphics[{EdgeForm[Black], LightGreen, Disk[]}, ImageSize->Small]**



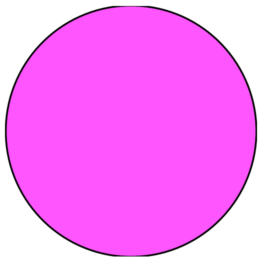
>> **LightGreen // ToBoxes**

StyleBox [GraphicsBox [
{EdgeForm [RGBColor [0, 0,
0]] , RGBColor [0.88, 1., 0.88
] , RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

LightMagenta

`LightMagenta`
represents the color light magenta in graphics.

>> **Graphics[{EdgeForm[Black], LightMagenta, Disk[]}, ImageSize->Small]**



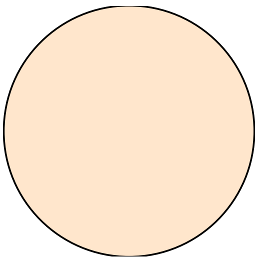
>> `LightMagenta // ToBoxes`

StyleBox [GraphicsBox [{EdgeForm [RGBColor [0, 0, 0]] , RGBColor [1., 0.333333, 1.] , RectangleBox [{0, 0}] } , $OptionSyntax− > Ignore, AspectRatio− > Automatic, Axes− > False, AxesStyle− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle− > {}] , ImageSizeMultipliers− > {1, 1}]

LightOrange

`LightOrange`
: represents the color light orange in graphics.

>> `Graphics[{EdgeForm[Black], LightOrange, Disk[]}, ImageSize->Small]`



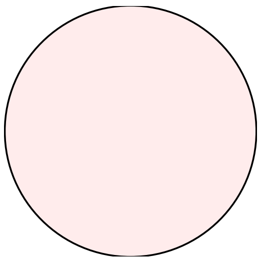
>> `LightOrange // ToBoxes`

StyleBox [GraphicsBox [{EdgeForm [RGBColor [0, 0, 0]] , RGBColor [1, 0.9, 0.8] , RectangleBox [{0, 0}] } , $OptionSyntax− > Ignore, AspectRatio− > Automatic, Axes− > False, AxesStyle− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle− > {}] , ImageSizeMultipliers− > {1, 1}]

LightPink

`LightPink`
: represents the color light pink in graphics.

>> `Graphics[{EdgeForm[Black], LightPink, Disk[]}, ImageSize-> Small]`



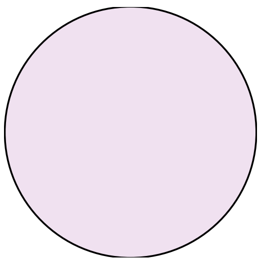
```
>> LightPink // ToBoxes
```

StyleBox [GraphicsBox [
{EdgeForm [RGBColor [0, 0,
0]] , RGBColor [1., 0.925, 0.925
] , RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

LightPurple

`LightPurple`
represents the color light purple in graphics.

```
>> Graphics[{EdgeForm[Black],
LightPurple, Disk[]}, ImageSize
->Small]
```



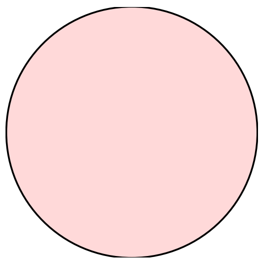
```
>> LightPurple // ToBoxes
```

StyleBox [GraphicsBox [
{EdgeForm [RGBColor [0, 0,
0]] , RGBColor [0.94, 0.88, 0.94
] , RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

LightRed

`LightRed`
represents the color light red in graphics.

```
>> Graphics[{EdgeForm[Black],
LightRed, Disk[]}, ImageSize->
Small]
```



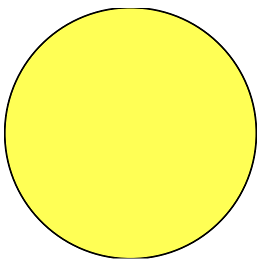
```
>> LightRed // ToBoxes
```

StyleBox [GraphicsBox [
{EdgeForm [RGBColor [0, 0,
0]] , RGBColor [1., 0.85, 0.85
] , RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

LightYellow

`LightYellow`
represents the color light yellow in graphics.

```
>> Graphics[{EdgeForm[Black],
   LightYellow, Disk[]}, ImageSize
   ->Small]
```



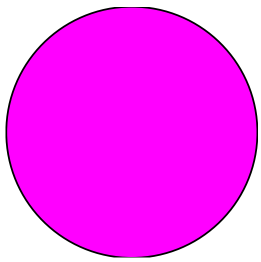
```
>> LightYellow // ToBoxes
```

StyleBox [GraphicsBox [
{EdgeForm [RGBColor [0, 0,
0]] , RGBColor [1., 1., 0.333333
] , RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

Magenta

`Magenta`
represents the color magenta in graphics.

```
>> Graphics[{EdgeForm[Black],
   Magenta, Disk[]}, ImageSize->
   Small]
```



>> `Magenta // ToBoxes`

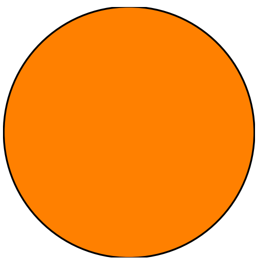
StyleBox [GraphicsBox [
{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [1, 0, 1] ,
RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

>> `Magenta`

Orange

`Orange`
  represents the color orange in graphics.

>> `Graphics[{EdgeForm[Black], Orange, Disk[]}, ImageSize->Small]`



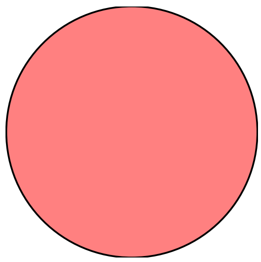
>> `Orange // ToBoxes`

StyleBox [GraphicsBox [
{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [1, 0.5,
0] , RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

Pink

`Pink`
  represents the color pink in graphics.

>> `Graphics[{EdgeForm[Black], Pink, Disk[]}, ImageSize->Small]`



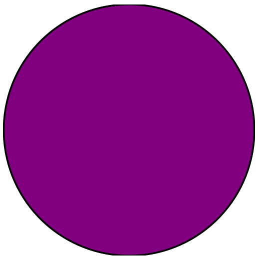
>> `Pink // ToBoxes`

StyleBox [GraphicsBox [
{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [1., 0.5,
0.5] , RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

Purple

`Purple`
represents the color purple in graphics.

```
>> Graphics[{EdgeForm[Black], Purple, Disk[]}, ImageSize->Small]
```



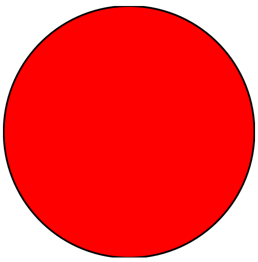
```
>> Purple // ToBoxes
```

StyleBox [GraphicsBox [
{EdgeForm [RGBColor [0, 0, 0]] , RGBColor [0.5, 0, 0.5] , RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

Red

`Red`
represents the color red in graphics.

```
>> Graphics[{EdgeForm[Black], Red, Disk[]}, ImageSize->Small]
```



```
>> Red // ToBoxes
```

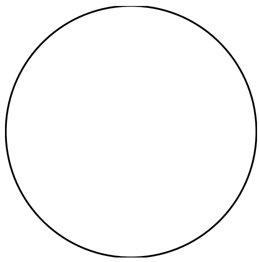
StyleBox [GraphicsBox [
{EdgeForm [RGBColor [0, 0, 0]] , RGBColor [1, 0, 0] , RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

```
>> Red
```

White

`White`
represents the color white in graphics.

```
>> Graphics[{EdgeForm[Black], White, Disk[]}, ImageSize->Small]
```



```
>> White // ToBoxes
```

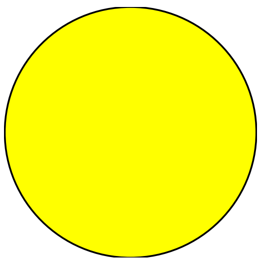
StyleBox [GraphicsBox [
{EdgeForm [RGBColor [0, 0, 0]] ,
GrayLevel [1] , RectangleBox [{0,
0}] } , $OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

```
>> White
```

Yellow

`Yellow`
represents the color yellow in graphics.

```
>> Graphics[{EdgeForm[Black],
   Yellow, Disk[]}, ImageSize->
   Small]
```



```
>> Yellow // ToBoxes
```

StyleBox [GraphicsBox [
{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [1, 1, 0] ,
RectangleBox [{0, 0}] } ,
$OptionSyntax− > Ignore,
AspectRatio− > Automatic,
Axes− > False, AxesStyle− > {} ,
Background− > Automatic,
ImageSize− > 16,
LabelStyle− > {} ,
PlotRange− > Automatic,
PlotRangePadding− > Automatic,
TicksStyle− > {}] ,
ImageSizeMultipliers− > {1, 1}]

```
>> Yellow
```

29. Distance and Similarity Measures

Different measures of distance or similarity for different types of analysis.

30. String Distances and Similarity Measures

Contents

DamerauLevenshteinDistance

`DamerauLevenshteinDistance[`*a*`, `*b*`]`
returns the Damerau-Levenshtein distance of *a* and *b*, which is defined as the minimum number of transpositions, insertions, deletions and substitutions needed to transform one into the other. In contrast to EditDistance, DamerauLevenshteinDistance counts transposition of adjacent items (e.g. “ab” into “ba”) as one operation of change.

```
>> DamerauLevenshteinDistance["
   kitten", "kitchen"]
   2
>> DamerauLevenshteinDistance["abc
   ", "ac"]
   1
>> DamerauLevenshteinDistance["abc
   ", "acb"]
   1
>> DamerauLevenshteinDistance["azbc
   ", "abxyc"]
   3
```

The IgnoreCase option makes DamerauLevenshteinDistance ignore the case of letters:

```
>> DamerauLevenshteinDistance["time
   ", "Thyme"]
   3
>> DamerauLevenshteinDistance["time
   ", "Thyme", IgnoreCase -> True]
   2
```

DamerauLevenshteinDistance also works on lists:

```
>> DamerauLevenshteinDistance[{1, E
   , 2, Pi}, {1, E, Pi, 2}]
   1
```

EditDistance

`EditDistance[`*a*`, `*b*`]`
returns the Levenshtein distance of *a* and *b*, which is defined as the minimum number of insertions, deletions and substitutions on the constituents of *a* and *b* needed to transform one into the other.

```
>> EditDistance["kitten", "kitchen
   "]
   2
>> EditDistance["abc", "ac"]
   1
>> EditDistance["abc", "acb"]
   2
>> EditDistance["azbc", "abxyc"]
   3
```

The IgnoreCase option makes EditDistance ignore the case of letters:

```
>> EditDistance["time", "Thyme"]
   3
>> EditDistance["time", "Thyme",
   IgnoreCase -> True]
   2
```

EditDistance also works on lists:

>> `EditDistance[{1, E, 2, Pi}, {1, E, Pi, 2}]`

2

HammingDistance

`HammingDistance[`*u*`, `*v*`]`
: returns the Hamming distance between *u* and *v*, i.e. the number of different elements. *u* and *v* may be lists or strings.

>> `HammingDistance[{1, 0, 1, 0}, {1, 0, 0, 1}]`

2

>> `HammingDistance["time", "dime"]`

1

>> `HammingDistance["TIME", "dime", IgnoreCase -> True]`

1

31. Graphics, Drawing, and Images

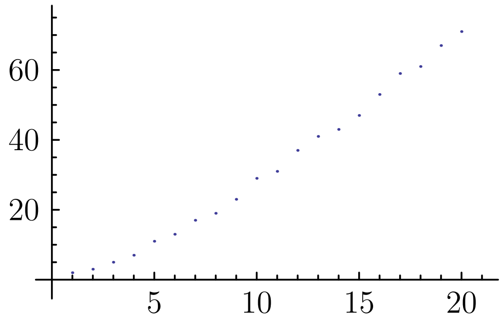
Functions like `Plot` and `ListPlot` can be used to draw graphs of functions and data.

Graphics is implemented as a collection of *graphics primitives*. Primatives are objects like `Point`, `Line`, and `Polygon` and become elements of a *graphics object*.

A graphics object can have directives as well such as `RGBColor`, and `Thickness`.

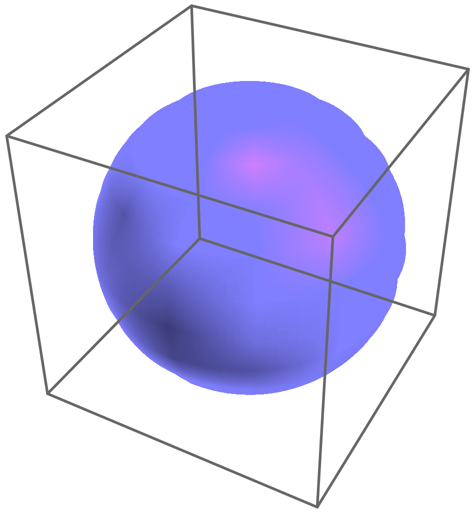
There are several kinds of graphics objects; each kind has a head which identifies its type.

```
>> ListPlot[ Table[Prime[n], {n,
   20} ]]
```




```
>> Head[%]
```
Graphics

```
>> Graphics3D[Sphere[]]
```



```
>> Head[%]
```
Graphics3D

»

32. Three-Dimensional Graphics

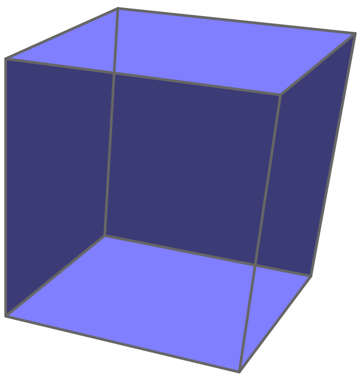
Contents

Cuboid

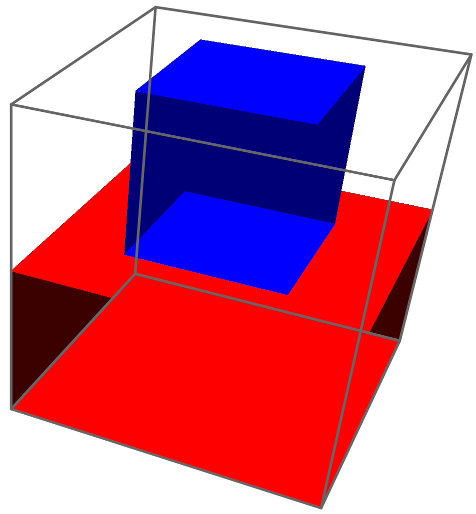
`Cuboid[{`*xmin*`, `*ymin*`, `*zmin*`}]`
  is a unit cube.

`Cuboid[{`*xmin*`, `*ymin*`, `*zmin*`}, {`*xmax*`, `*ymax*`, `*zmax*`}]`
  represents a cuboid extending from {*xmin*, *ymin*, *zmin*} to {*xmax*, *ymax*, *zmax*}.

```
>> Graphics3D[Cuboid[{0, 0, 1}]]
```



```
>> Graphics3D[{Red, Cuboid[{0, 0, 0}, {1, 1, 0.5}], Blue, Cuboid[{0.25, 0.25, 0.5}, {0.75, 0.75, 1}]}]
```



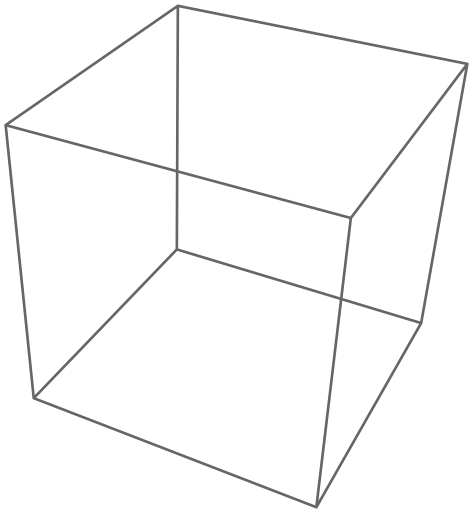
Cylinder

`Cylinder[{{`*x1*`, `*y1*`, `*z1*`}, {`*x2*`, `*y2*`, `*z2*`}}]`
  represents a cylinder of radius 1.

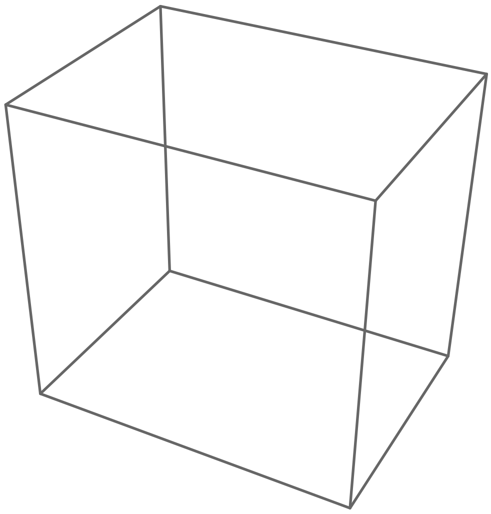
`Cylinder[{{`*x1*`, `*y1*`, `*z1*`}, {`*x2*`, `*y2*`, `*z2*`}}, `*r*`]`
  is a cylinder of radius *r* starting at (*x1*, *y1*, *z1*) and ending at (*x2*, *y2*, *z2*).

`Cylinder[{{`*x1*`, `*y1*`, `*z1*`}, {`*x2*`, `*y2*`, `*z2*`}, ... }, `*r*`]`
  is a collection cylinders of radius *r*

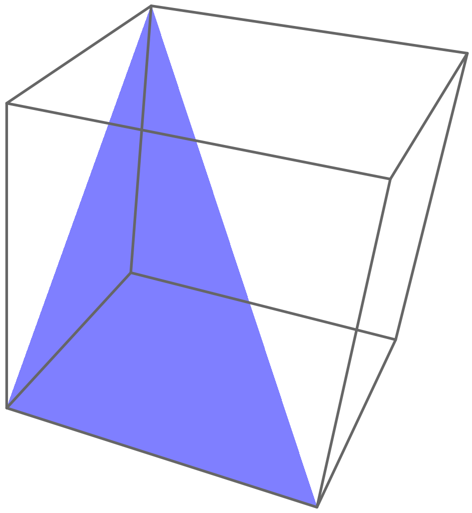
>> `Graphics3D[Cylinder[{{0, 0, 0}, {1, 1, 1}}, 1]]`



>> `Graphics3D[{Yellow, Cylinder [{{-1, 0, 0}, {1, 0, 0}, {0, 0, Sqrt[3]}, {1, 1, Sqrt[3]}}, 1]}]`



>> `Graphics3D[Polygon[{{0,0,0}, {0,1,1}, {1,0,0}}]]`



In `TeXForm`, `Graphics3D` creates Asymptote figures:

Graphics3D

`Graphics3D[`*primitives*`, `*options*`]`
    represents a three-dimensional graphic.
See also the Section "Plotting" for a list of Plot options.

```
>> Graphics3D[Sphere[]] // TeXForm
```

\begin{asy}
import three;
import solids;
size(6.6667cm, 6.6667cm);
currentprojection=perspective(2.6,-4.8,4.0);
currentlight=light(rgb(0.5,0.5,1),
specular=red, (2,0,2), (2,2,2), (0,2,2));
draw(surface(sphere((0, 0, 0), 1)),
rgb(1,1,1));
draw(((-1,-1,-1)–(1,-1,-1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((-1,1,-1)–(1,1,-1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((-1,-1,1)–(1,-1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((-1,1,1)–(1,1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((-1,-1,-1)–(-1,1,-1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((1,-1,-1)–(1,1,-1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((-1,-1,1)–(-1,1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((1,-1,1)–(1,1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((-1,-1,-1)–(-1,-1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((1,-1,-1)–(1,-1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((-1,1,-1)–(-1,1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
draw(((1,1,-1)–(1,1,1)), rgb(0.4, 0.4,
0.4)+linewidth(1));
\end{asy}

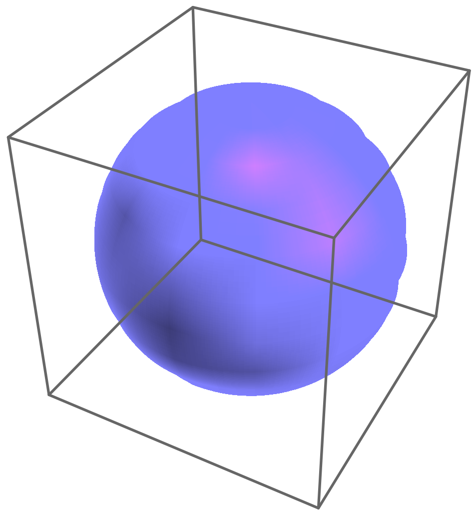
Sphere

`Sphere[{`*x*`, `*y*`, `*z*`}]`
: is a sphere of radius 1 centered at the point $\{x, y, z\}$.

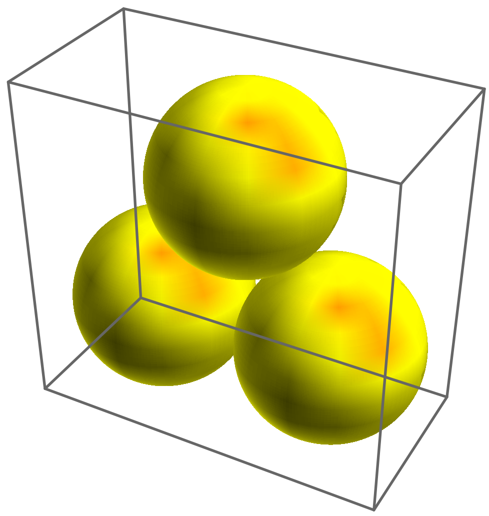
`Sphere[{`*x*`, `*y*`, `*z*`}, `*r*`]`
: is a sphere of radius r centered at the point $\{x, y, z\}$.

`Sphere[{{`*x1*`, `*y1*`, `*z1*`}, {`*x2*`, `*y2*`, `*z2*`}, ... }, `*r*`]`
: is a collection spheres of radius r centered at the points $\{x1, y2, z2\}$, $\{x2, y2, z2\}$, ...

```
>> Graphics3D[Sphere[{0, 0, 0}, 1]]
```



```
>> Graphics3D[{Yellow, Sphere[{{-1,
    0, 0}, {1, 0, 0}, {0, 0, Sqrt
   [3.]}}, 1]}]
```



33. Image[] and image related functions.

Note that you (currently) need scikit-image installed in order for this module to work.

Contents

Binarize

`Binarize[`*image*`]`
: gives a binarized version of *image*, in which each pixel is either 0 or 1.

`Binarize[`*image*`, `*t*`]`
: map values $x > t$ to 1, and values $x <= t$ to 0.

`Binarize[`*image*`, {`*t1*`, `*t2*`}]`
: map $t1 < x < t2$ to 1, and all other values to 0.

```
>> img = Import["ExampleData/lena.
   tif"];
```

```
>> Binarize[img]
```
–Image–

```
>> Binarize[img, 0.7]
```
–Image–

```
>> Binarize[img, {0.2, 0.6}]
```
–Image–

BinaryImageQ

`BinaryImageQ[$image]`
: returns True if the pixels of $image are binary bit values, and False otherwise.

```
>> img = Import["ExampleData/lena.
   tif"];
```

```
>> BinaryImageQ[img]
```
False

```
>> BinaryImageQ[Binarize[img]]
```
True

Blur

`Blur[`*image*`]`
: gives a blurred version of *image*.

`Blur[`*image*`, `*r*`]`
: blurs *image* with a kernel of size *r*.

```
>> lena = Import["ExampleData/lena.
tif"];
```

```
>> Blur[lena]
```
—Image—

```
>> Blur[lena, 5]
```
—Image—

BoxMatrix

`BoxMatrix[$s]`
: Gives a box shaped kernel of size 2 s + 1.

```
>> BoxMatrix[3]
```
{{1,1,1,1,1,1,1}, {1,1,1,1,1, 1,1}, {1,1,1,1,1,1,1}, {1,1,1, 1,1,1,1}, {1,1,1,1,1,1,1}, {1, 1,1,1,1,1,1}, {1,1,1,1,1,1,1}}

Closing

`Closing[`*image*`,` *ker*`]`
: Gives the morphological closing of *image* with respect to structuring element *ker*.

```
>> ein = Import["ExampleData/
Einstein.jpg"];
```

```
>> Closing[ein, 2.5]
```
—Image—

ColorCombine

`ColorCombine[`*channels*`,` *colorspace*`]`
: Gives an image with *colorspace* and the respective components described by the given channels.

```
>> ColorCombine[{{{1, 0}, {0,
0.75}}, {{0, 1}, {0, 0.25}},
{{0, 0}, {1, 0.5}}}, "RGB"]
```
—Image—

ColorQuantize

`ColorQuantize[`*image*`,` *n*`]`
: gives a version of *image* using only *n* colors.

```
>> img = Import["ExampleData/lena.
tif"];
```

```
>> ColorQuantize[img, 6]
```
—Image—

ColorSeparate

`ColorSeparate[`*image*`]`
: Gives each channel of *image* as a separate grayscale image.

Colorize

`Colorize[`*values*`]`
: returns an image where each number in the rectangular matrix *values* is a pixel and each occurence of the same number is displayed in the same unique color, which is different from the colors of all non-identical numbers.

`Colorize[`*image*`]`
: gives a colorized version of *image*.

```
>> Colorize[{{1.3, 2.1, 1.5}, {1.3,
 1.3, 2.1}, {1.3, 2.1, 1.5}}]
```
—Image—

```
>> Colorize[{{1, 2}, {2, 2}, {2,
3}}, ColorFunction -> (Blend[{
White, Blue}, #]&)]
```
—Image—

DiamondMatrix

`DiamondMatrix[$s]`
: Gives a diamond shaped kernel of size 2 s + 1.

```
>> DiamondMatrix[3]
```

{{0,0,0,1,0,0,0}, {0,0,1,1,1,0,0}, {0,1,1,1,1,1,0}, {1,1,1,1,1,1,1}, {0,1,1,1,1,1,0}, {0,0,1,1,1,0,0}, {0,0,0,1,0,0,0}}

Dilation

`Dilation[`*image*`, `*ker*`]`
: Gives the morphological dilation of *image* with respect to structuring element *ker*.

```
>> ein = Import["ExampleData/Einstein.jpg"];
```

```
>> Dilation[ein, 2.5]
```

–Image–

DiskMatrix

`DiskMatrix[$s]`
: Gives a disk shaped kernel of size $2s+1$.

```
>> DiskMatrix[3]
```

{{0,0,1,1,1,0,0}, {0,1,1,1,1,1,0}, {1,1,1,1,1,1,1}, {1,1,1,1,1,1,1}, {1,1,1,1,1,1,1}, {0,1,1,1,1,1,0}, {0,0,1,1,1,0,0}}

EdgeDetect

`EdgeDetect[`*image*`]`
: returns an image showing the edges in *image*.

```
>> lena = Import["ExampleData/lena.tif"];
```

```
>> EdgeDetect[lena]
```

–Image–

```
>> EdgeDetect[lena, 5]
```

–Image–

```
>> EdgeDetect[lena, 4, 0.5]
```

–Image–

Erosion

`Erosion[`*image*`, `*ker*`]`
: Gives the morphological erosion of *image* with respect to structuring element *ker*.

```
>> ein = Import["ExampleData/Einstein.jpg"];
```

```
>> Erosion[ein, 2.5]
```

–Image–

GaussianFilter

`GaussianFilter[`*image*`, `*r*`]`
: blurs *image* using a Gaussian blur filter of radius *r*.

```
>> lena = Import["ExampleData/lena.tif"];
```

```
>> GaussianFilter[lena, 2.5]
```

–Image–

ImageAdd

`ImageAdd[`*image*`, `*expr_1*`, `*expr_2*`, ...]`
: adds all *expr_i* to *image* where each *expr_i* must be an image or a real number.

```
>> i = Image[{{0, 0.5, 0.2, 0.1, 0.9}, {1.0, 0.1, 0.3, 0.8, 0.6}}];
```

```
>> ImageAdd[i, 0.5]
```

–Image–

```
>> ImageAdd[i, i]
```

–Image–

```
>> ein = Import["ExampleData/Einstein.jpg"];
```

```
>> noise = RandomImage[{-0.1, 0.1}, ImageDimensions[ein]];
```

```
>> ImageAdd[noise, ein]
```

–Image–

```
>> lena = Import["ExampleData/lena.tif"];
```

```
>> noise = RandomImage[{-0.2, 0.2}, ImageDimensions[lena], ColorSpace -> "RGB"];
```

```
>> ImageAdd[noise, lena]
```

–Image–

ImageAdjust

`ImageAdjust[`*image*`]`
: adjusts the levels in *image*.

`ImageAdjust[`*image*`, `*c*`]`
: adjusts the contrast in *image* by *c*.

`ImageAdjust[`*image*`, {`*c*`, `*b*`}]`
: adjusts the contrast *c*, and brightness *b* in *image*.

`ImageAdjust[`*image*`, {`*c*`, `*b*`, `*g*`}]`
: adjusts the contrast *c*, brightness *b*, and gamma *g* in *image*.

```
>> lena = Import["ExampleData/lena.tif"];
```

```
>> ImageAdjust[lena]
```

–Image–

ImageAspectRatio

`ImageAspectRatio[`*image*`]`
: gives the aspect ratio of *image*.

```
>> img = Import["ExampleData/lena.tif"];
```

```
>> ImageAspectRatio[img]
```

1

```
>> ImageAspectRatio[Image[{{0, 1}, {1, 0}, {1, 1}}]]
```

$\frac{3}{2}$

Image

ImageChannels

`ImageChannels[`*image*`]`
: gives the number of channels in *image*.

```
>> ImageChannels[Image[{{0, 1}, {1, 0}}]]
```

1

```
>> img = Import["ExampleData/lena.tif"];
```

```
>> ImageChannels[img]
```

3

ImageColorSpace

`ImageColorSpace[`*image*`]`
: gives *image*'s color space, e.g. "RGB" or "CMYK".

```
>> img = Import["ExampleData/lena.tif"];
```

```
>> ImageColorSpace[img]
```

RGB

ImageConvolve

`ImageConvolve[`*image*`, `*kernel*`]`
: Computes the convolution of *image* using *kernel*.

```
>> img = Import["ExampleData/lena.tif"];
```

```
>> ImageConvolve[img, DiamondMatrix[5] / 61]
```

–Image–

```
>> ImageConvolve[img, DiskMatrix[5] / 97]
```

–Image–

```
>> ImageConvolve[img, BoxMatrix[5]
   / 121]
```
−Image−

ImageData

`ImageData[`*image*`]`
: gives a list of all color values of *image* as a matrix.

`ImageData[`*image*`, `*stype*`]`
: gives a list of color values in type *stype*.

```
>> img = Image[{{0.2, 0.4}, {0.9,
   0.6}, {0.5, 0.8}}];
```

```
>> ImageData[img]
```
$\{\{0.2, 0.4\}, \{0.9, 0.6\}, \{0.5, 0.8\}\}$

```
>> ImageData[img, "Byte"]
```
$\{\{51, 102\}, \{229, 153\}, \{127, 204\}\}$

```
>> ImageData[Image[{{0, 1}, {1, 0},
    {1, 1}}], "Bit"]
```
$\{\{0, 1\}, \{1, 0\}, \{1, 1\}\}$

ImageDimensions

`ImageDimensions[`*image*`]`
: Returns the dimensions of *image* in pixels.

```
>> lena = Import["ExampleData/lena.
   tif"];
```

```
>> ImageDimensions[lena]
```
$\{512, 512\}$

```
>> ImageDimensions[RandomImage[1,
   {50, 70}]]
```
$\{50, 70\}$

ImageImport

```
>> Import["ExampleData/Einstein.jpg
   "]
```
−Image−

```
>> Import["ExampleData/MadTeaParty.
   gif"]
```
−Image−

```
>> Import["ExampleData/moon.tif"]
```
−Image−

ImageMultiply

`ImageMultiply[`*image*`, `*expr_1*`, `*expr_2*`, ...]`
: multiplies all *expr_i* with *image* where each *expr_i* must be an image or a real number.

```
>> i = Image[{{0, 0.5, 0.2, 0.1,
   0.9}, {1.0, 0.1, 0.3, 0.8,
   0.6}}];
```

```
>> ImageMultiply[i, 0.2]
```
−Image−

```
>> ImageMultiply[i, i]
```
−Image−

```
>> ein = Import["ExampleData/
   Einstein.jpg"];
```

```
>> noise = RandomImage[{0.7, 1.3},
   ImageDimensions[ein]];
```

```
>> ImageMultiply[noise, ein]
```
−Image−

ImagePartition

`ImagePartition[`*image*`, `*s*`]`
: Partitions an image into an array of *s* x *s* pixel subimages.

`ImagePartition[`*image*`, {`*w*`, `*h*`}]`
: Partitions an image into an array of *w* x *h* pixel subimages.

```
>> lena = Import["ExampleData/lena.
   tif"];
```

```
>> ImageDimensions[lena]
```
$\{512, 512\}$

```
>> ImagePartition[lena, 256]
```
{{−*Image*−, − *Image*−},
{−*Image*−, − *Image*−}}

```
>> ImagePartition[lena, {512, 128}]
```
{{−*Image*−}, {−*Image*−},
{−*Image*−}, {−*Image*−}}

ImageQ

`ImageQ[Image[$pixels]]`
returns True if $pixels has dimensions from which an Image can be constructed, and False otherwise.

```
>> ImageQ[Image[{{0, 1}, {1, 0}}]]
```
True

```
>> ImageQ[Image[{{{0, 0, 0}, {0, 1,
   0}}, {{0, 1, 0}, {0, 1, 1}}}]]
```
True

```
>> ImageQ[Image[{{{0, 0, 0}, {0,
  1}}, {{0, 1, 0}, {0, 1, 1}}}]]
```
False

```
>> ImageQ[Image[{1, 0, 1}]]
```
False

```
>> ImageQ["abc"]
```
False

ImageReflect

`ImageReflect[`*image*`]`
Flips *image* top to bottom.
`ImageReflect[`*image*`, `*side*`]`
Flips *image* so that *side* is interchanged with its opposite.
`ImageReflect[`*image*`, `*side_1* `->` *side_2*`]`
Flips *image* so that *side_1* is interchanged with *side_2*.

```
>> ein = Import["ExampleData/
  Einstein.jpg"];
```

```
>> ImageReflect[ein]
```
−*Image*−

```
>> ImageReflect[ein, Left]
```
−*Image*−

```
>> ImageReflect[ein, Left -> Top]
```
−*Image*−

ImageResize

`ImageResize[`*image*`, `*width*`]`
`ImageResize[`*image*`, {`*width*`, `*height*`}]`

```
>> ein = Import["ExampleData/
  Einstein.jpg"];
```

```
>> ImageDimensions[ein]
```
{615, 768}

```
>> ImageResize[ein, {400, 600}]
```
−*Image*−

```
>> ImageResize[ein, 256]
```
−*Image*−

```
>> ImageDimensions[%]
```
{256, 320}

The default sampling method is Bicubic

```
>> ImageResize[ein, 256, Resampling
    -> "Bicubic"]
```
−*Image*−

```
>> ImageResize[ein, 256, Resampling
    -> "Nearest"]
```
−*Image*−

```
>> ImageResize[ein, 256, Resampling
    -> "Gaussian"]
```
−*Image*−

ImageRotate

`ImageRotate[`*image*`]`
Rotates *image* 90 degrees counterclockwise.
`ImageRotate[`*image*`, `*theta*`]`
Rotates *image* by a given angle *theta*

```
>> ein = Import["ExampleData/Einstein.jpg"];
```

```
>> ImageRotate[ein]
```
—Image—

```
>> ImageRotate[ein, 45 Degree]
```
—Image—

```
>> ImageRotate[ein, Pi / 2]
```
—Image—

ImageSubtract

`ImageSubtract[`*image*`, `*expr_1*`, `*expr_2*`, ...]`
: subtracts all *expr_i* from *image* where each *expr_i* must be an image or a real number.

```
>> i = Image[{{0, 0.5, 0.2, 0.1, 0.9}, {1.0, 0.1, 0.3, 0.8, 0.6}}];
```

```
>> ImageSubtract[i, 0.2]
```
—Image—

```
>> ImageSubtract[i, i]
```
—Image—

ImageTake

`ImageTake[`*image*`, `*n*`]`
: gives the first *n* rows of *image*.

`ImageTake[`*image*`, -`*n*`]`
: gives the last *n* rows of *image*.

`ImageTake[`*image*`, {`*r1*`, `*r2*`}]`
: gives rows *r1*, ..., *r2* of *image*.

`ImageTake[`*image*`, {`*r1*`, `*r2*`}, {`*c1*`, `*c2*`}]`
: gives a cropped version of *image*.

ImageType

`ImageType[`*image*`]`
: gives the interval storage type of *image*, e.g. “Real”, “Bit32”, or “Bit”.

```
>> img = Import["ExampleData/lena.tif"];
```

```
>> ImageType[img]
```
Byte

```
>> ImageType[Image[{{0, 1}, {1, 0}}]]
```
Real

```
>> ImageType[Binarize[img]]
```
Bit

MaxFilter

`MaxFilter[`*image*`, `*r*`]`
: gives *image* with a maximum filter of radius *r* applied on it. This always picks the largest value in the filter's area.

```
>> lena = Import["ExampleData/lena.tif"];
```

```
>> MaxFilter[lena, 5]
```
—Image—

MedianFilter

`MedianFilter[`*image*`, `*r*`]`
: gives *image* with a median filter of radius *r* applied on it. This always picks the median value in the filter's area.

```
>> lena = Import["ExampleData/lena.tif"];
```

```
>> MedianFilter[lena, 5]
```
—Image—

MinFilter

`MinFilter[`*image*`, `*r*`]`
: gives *image* with a minimum filter of radius *r* applied on it. This always picks the smallest value in the filter's area.

```
>> lena = Import["ExampleData/lena.tif"];
```

```
>> MinFilter[lena, 5]
```
−Image−

Opening

`Opening[`*image*`, `*ker*`]`
: Gives the morphological opening of *image* with respect to structuring element *ker*.

```
>> ein = Import["ExampleData/Einstein.jpg"];
```

```
>> Opening[ein, 2.5]
```
−Image−

PixelValue

`PixelValue[`*image*`, {`*x*`, `*y*`}]`
: gives the value of the pixel at position {*x*, *y*} in *image*.

```
>> lena = Import["ExampleData/lena.tif"];
```

```
>> PixelValue[lena, {1, 1}]
```
$\{0.321569, 0.0862745, 0.223529\}$

PixelValuePositions

`PixelValuePositions[`*image*`, `*val*`]`
: gives the positions of all pixels in *image* that have value *val*.

```
>> PixelValuePositions[Image[{{0, 1}, {1, 0}, {1, 1}}], 1]
```
$\{\{1,1\}, \{1,2\}, \{2,1\}, \{2,3\}\}$

```
>> PixelValuePositions[Image[{{0.2, 0.4}, {0.9, 0.6}, {0.3, 0.8}}], 0.5, 0.15]
```
$\{\{2,2\}, \{2,3\}\}$

```
>> img = Import["ExampleData/lena.tif"];
```

```
>> PixelValuePositions[img, 3 / 255, 0.5 / 255]
```
$\{\{180,192,2\}, \{181,192,2\}, \{181,193,2\}, \{188,204,2\}, \{265,314,2\}, \{364,77,2\}, \{365,72,2\}, \{365,73,2\}, \{365,77,2\}, \{366,70,2\}, \{367,65,2\}\}$

```
>> PixelValue[img, {180, 192}]
```
$\{0.25098, 0.0117647, 0.215686\}$

RandomImage

`RandomImage[`*max*`]`
: creates an image of random pixels with values 0 to *max*.

`RandomImage[{`*min*`, `*max*`}]`
: creates an image of random pixels with values *min* to *max*.

`RandomImage[..., `*size*`]`
: creates an image of the given *size*.

```
>> RandomImage[1, {100, 100}]
```
−Image−

Sharpen

`Sharpen[`*image*`]`
: gives a sharpened version of *image*.

`Sharpen[`*image*`, `*r*`]`
: sharpens *image* with a kernel of size *r*.

```
>> lena = Import["ExampleData/lena.tif"];
```

```
>> Sharpen[lena]
```
−Image−

```
>> Sharpen[lena, 5]
```
−Image−

TextRecognize

`TextRecognize[{`*image*`}]`
: Recognizes text in *image* and returns it as string.

Threshold

`Threshold[`*image*`]`
: gives a value suitable for binarizing *image*.

The option "Method" may be "Cluster" (use Otsu's threshold), "Median", or "Mean".

```
>> img = Import["ExampleData/lena.tif"];
```

```
>> Threshold[img]
```
0.456739

```
>> Binarize[img, %]
```
—Image—

```
>> Threshold[img, Method -> "Mean"]
```
0.486458

```
>> Threshold[img, Method -> "Median"]
```
0.504726

WordCloud

`WordCloud[{`*word1*`, `*word2*`, ...}]`
: Gives a word cloud with the given list of words.

`WordCloud[{`*weight1*` -> `*word1*`, `*weight2*` -> `*word2*`, ...}]`
: Gives a word cloud with the words weighted using the given weights.

`WordCloud[{`*weight1*`, `*weight2*`, ...} -> {`*word1*`, `*word2*`, ...}]`
: Also gives a word cloud with the words weighted using the given weights.

`WordCloud[{{`*word1*`, `*weight1*`}, {`*word2*`, `*weight2*`}, ...}]`
: Gives a word cloud with the words weighted using the given weights.

```
>> WordCloud[StringSplit[Import["ExampleData/EinsteinSzilLetter.txt"]]]
```
—Image—

```
>> WordCloud[Range[50] -> ToString /@ Range[50]]
```
—Image—

34. Plotting

Contents

BarChart

`BarChart[{b1, b2 ...}]`
    makes a bar chart with lengths *b1*, *b2*, ...

Drawing options include - Charting:

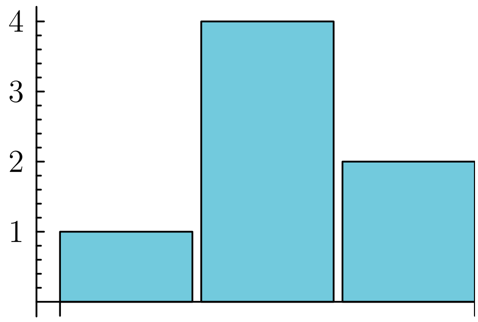
- Mesh
- PlotRange
- ChartLabels
- ChartLegends
- ChartStyle

BarChart specific:

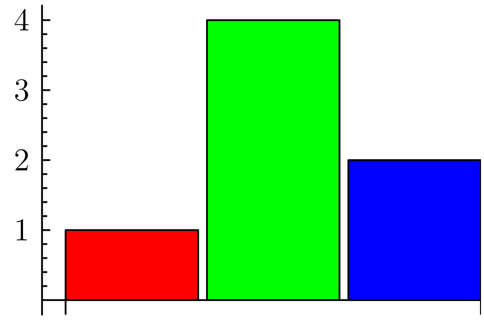
- Axes (default {False, True})
- AspectRatio: (default 1 / GoldenRatio)

A bar chart of a list of heights:

```
>> BarChart[{1, 4, 2}]
```




```
>> BarChart[{1, 4, 2}, ChartStyle
   -> {Red, Green, Blue}]
```




```
>> BarChart[{{1, 2, 3}, {2, 3, 4}}]
```

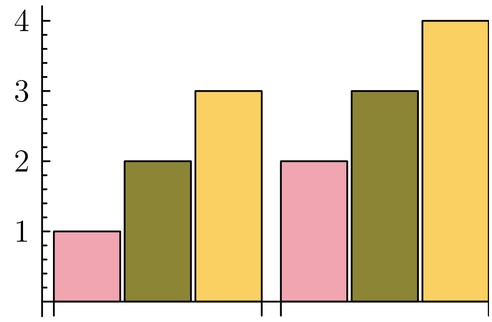
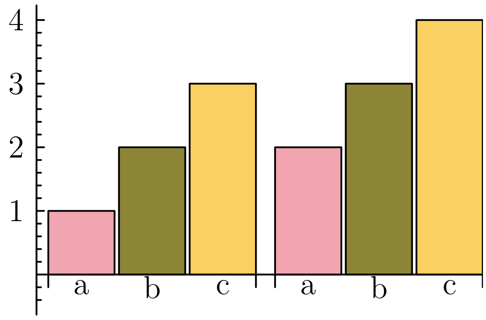
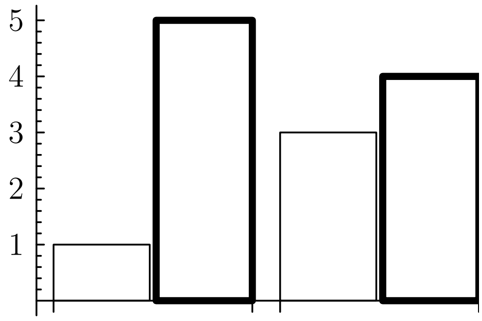


Chart several datasets with categorical labels:

```
>> BarChart[{{1, 2, 3}, {2, 3, 4}},
    ChartLabels -> {"a", "b", "c"}]
```




```
>> BarChart[{{1, 5}, {3, 4}},
   ChartStyle -> {{EdgeForm[Thin],
   White}, {EdgeForm[Thick], White
   }}]
```



ColorData

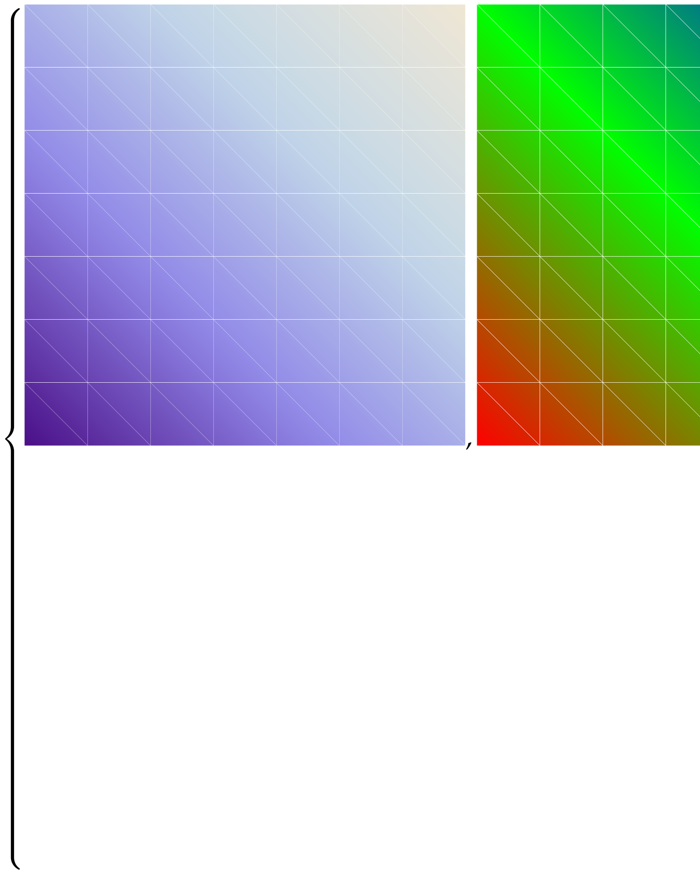
`ColorData["`*name*`"]`
: returns a color function with the given *name*.

Define a user-defined color function:

```
>> Unprotect[ColorData]; ColorData
   ["test"] := ColorDataFunction["
   test", "Gradients", {0, 1},
   Blend[{Red, Green, Blue}, #1]
   &]; Protect[ColorData]
```

Compare it to the default color function, `LakeColors`:

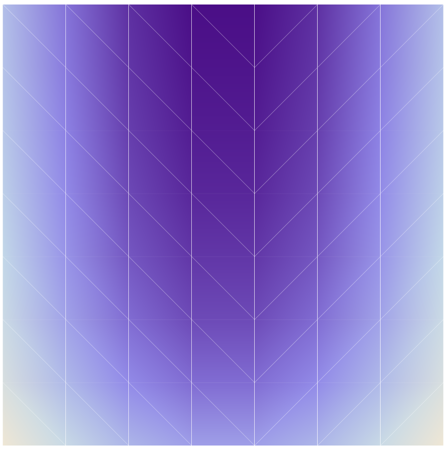
```
>> {DensityPlot[x + y, {x, -1, 1},
   {y, -1, 1}], DensityPlot[x + y,
   {x, -1, 1}, {y, -1, 1},
   ColorFunction->"test"]}
```



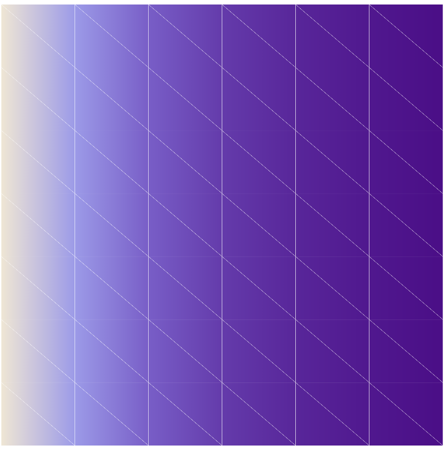
DensityPlot

`DensityPlot[`*f*`, {`*x*`, `*xmin*`, `*xmax*`}, {`*y*`, `*ymin*`, `*ymax*`}]`
: plots a density plot of *f* with *x* ranging from *xmin* to *xmax* and *y* ranging from *ymin* to *ymax*.

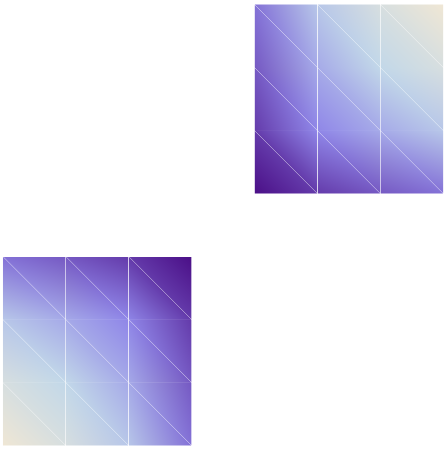
```
>> DensityPlot[x ^ 2 + 1 / y, {x,
   -1, 1}, {y, 1, 4}]
```



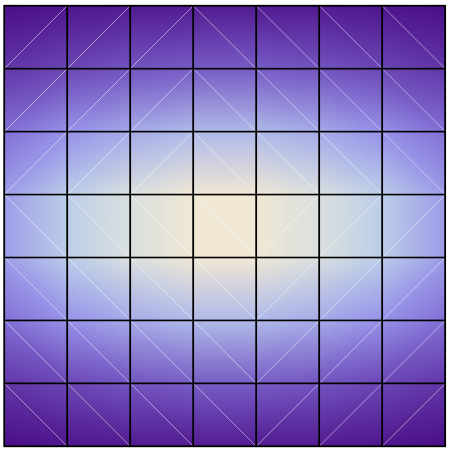
```
>> DensityPlot[1 / x, {x, 0, 1}, {y
   , 0, 1}]
```



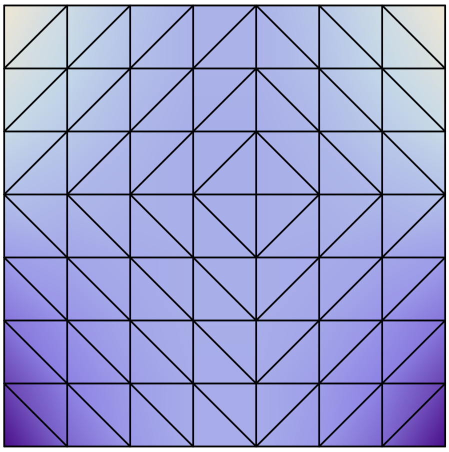
```
>> DensityPlot[Sqrt[x * y], {x, -1,
    1}, {y, -1, 1}]
```



```
>> DensityPlot[1/(x^2 + y^2 + 1), {
   x, -1, 1}, {y, -2,2}, Mesh->Full
   ]
```



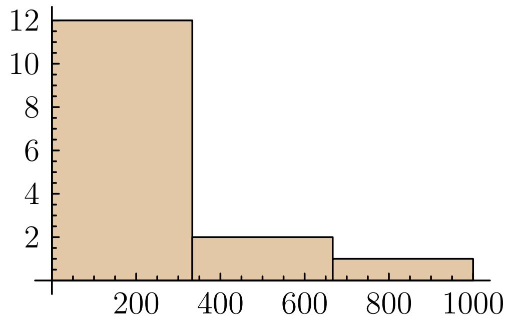
```
>> DensityPlot[x^2 y, {x, -1, 1}, {
   y, -1, 1}, Mesh->All]
```



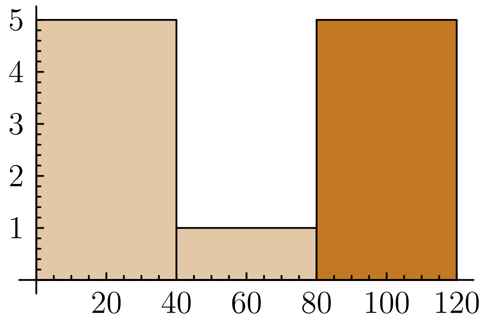
Histogram

`Histogram[{`*x1*`, `*x2*` ...}]`
: plots a histogram using the values *x1*, *x2*, ...

```
>> Histogram[{3, 8, 10, 100, 1000,
500, 300, 200, 10, 20, 200, 100,
 200, 300, 500}]
```




```
>> Histogram[{{1, 2, 10, 5, 50,
20}, {90, 100, 101, 120, 80}}]
```



ListLinePlot

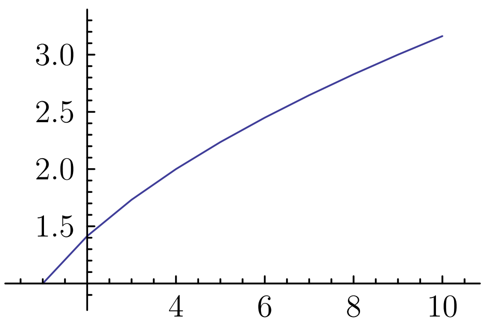
`ListLinePlot[{`*y_1*`, `*y_2*`, ...}]`
plots a line through a list of y-values, assuming integer x-values 1, 2, 3, ...

`ListLinePlot[{{`*x_1*`, `*y_1*`}, {`*x_2*`, `*y_2*`}, ...}]`
plots a line through a list of x, y pairs.

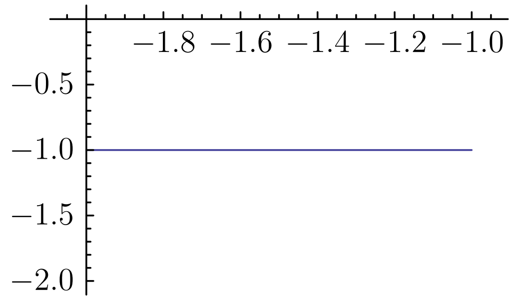
`ListLinePlot[{`*list_1*`, `*list_2*`, ...}]`
plots several lines.

ListPlot accepts a superset of the Graphics options.

```
>> ListLinePlot[Table[{n, n ^ 0.5},
 {n, 10}]]
```




```
>> ListLinePlot[{{-2, -1}, {-1,
-1}}]
```



ListPlot

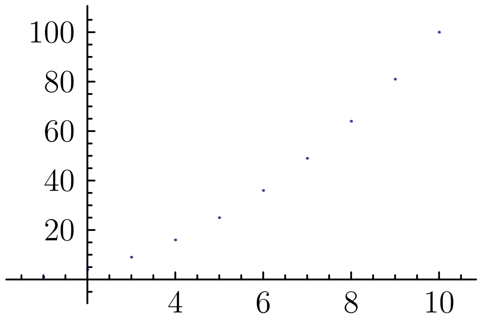
`ListPlot[{`*y_1*`, `*y_2*`, ...}]`
plots a list of y-values, assuming integer x-values 1, 2, 3, ...

`ListPlot[{{`*x_1*`, `*y_1*`}, {`*x_2*`, `*y_2*`}, ...}]`
plots a list of x, y pairs.

`ListPlot[{`*list_1*`, `*list_2*`, ...}]`
plots several lists of points.

ListPlot accepts a superset of the Graphics options.

```
>> ListPlot[Table[n ^ 2, {n, 10}]]
```



ParametricPlot

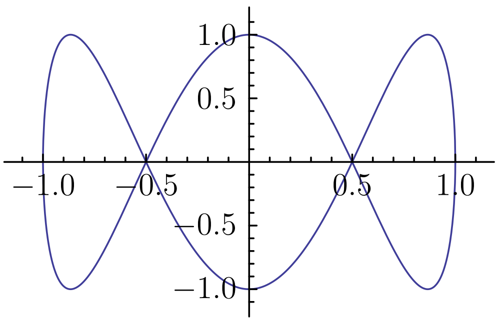
`ParametricPlot[{`*f_x*`, `*f_y*`}, {`*u*`, `*umin*`, `*umax*`}]`
: plots a parametric function *f* with the parameter *u* ranging from *umin* to *umax*.

`ParametricPlot[{{`*f_x*`, `*f_y*`}, {`*g_x*`, `*g_y*`}, ...}, {`*u*`, `*umin*`, `*umax*`}]`
: plots several parametric functions *f*, *g*, ...

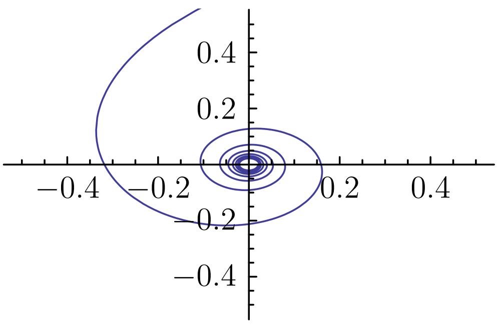
`ParametricPlot[{`*f_x*`, `*f_y*`}, {`*u*`, `*umin*`, `*umax*`}, {`*v*`, `*vmin*`, `*vmax*`}]`
: plots a parametric area.

`ParametricPlot[{{`*f_x*`, `*f_y*`}, {`*g_x*`, `*g_y*`}, ...}, {`*u*`, `*umin*`, `*umax*`}, {`*v*`, `*vmin*`, `*vmax*`}]`
: plots several parametric areas.

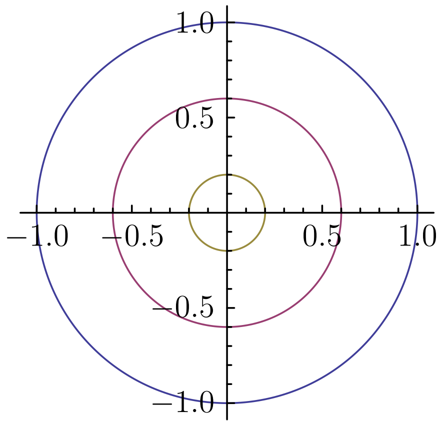
```
>> ParametricPlot[{Sin[u], Cos[3 u
   ]}, {u, 0, 2 Pi}]
```




```
>> ParametricPlot[{Cos[u] / u, Sin[
   u] / u}, {u, 0, 50}, PlotRange
   ->0.5]
```




```
>> ParametricPlot[{{Sin[u], Cos[u
   ]},{0.6 Sin[u], 0.6 Cos[u]},
   {0.2 Sin[u], 0.2 Cos[u]}}, {u,
   0, 2 Pi}, PlotRange->1,
   AspectRatio->1]
```



PieChart

`PieChart[{`*a1*`, `*a2*` ...}]`
: draws a pie chart with sector angles proportional to *a1*, *a2*, ...

Drawing options include - Charting:

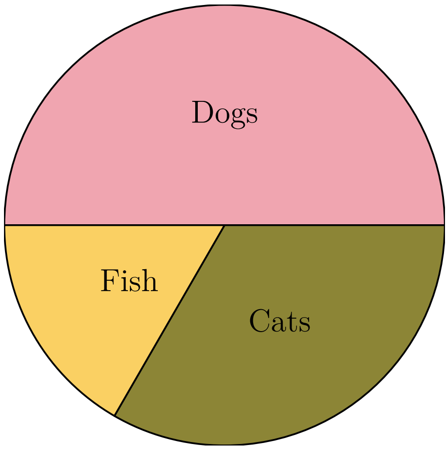
- Mesh
- PlotRange
- ChartLabels
- ChartLegends
- ChartStyle

PieChart specific:

- Axes (default: False, False)
- AspectRatio (default 1)
- SectorOrigin: (default {Automatic, 0})
- SectorSpacing" (default Automatic)

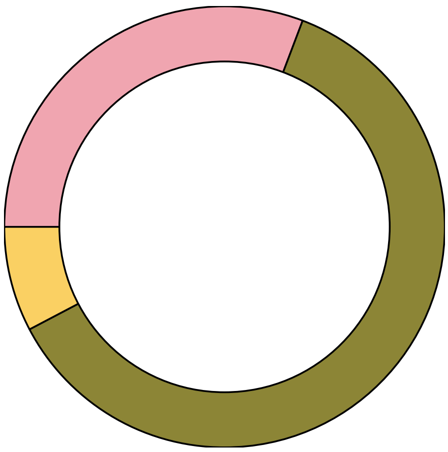
A hypothetical comparsion between types of pets owned:

```
>> PieChart[{30, 20, 10},
   ChartLabels -> {Dogs, Cats, Fish
   }]
```



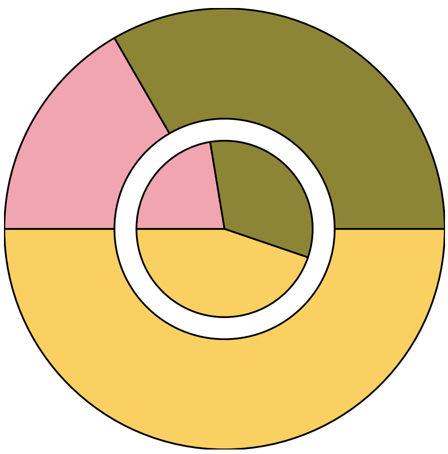
A doughnut chart for a list of values:

```
>> PieChart[{8, 16, 2},
   SectorOrigin -> {Automatic,
   1.5}]
```



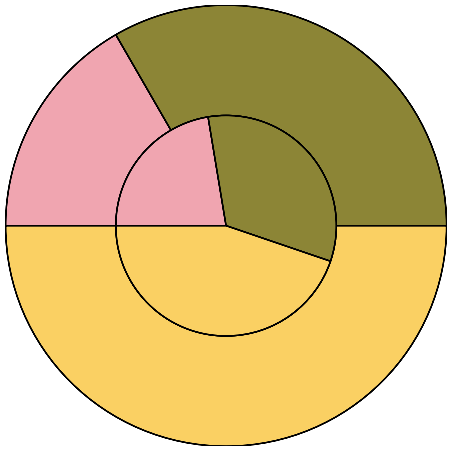
A Pie chart with multple datasets:

```
>> PieChart[{{10, 20, 30}, {15, 22,
    30}}]
```



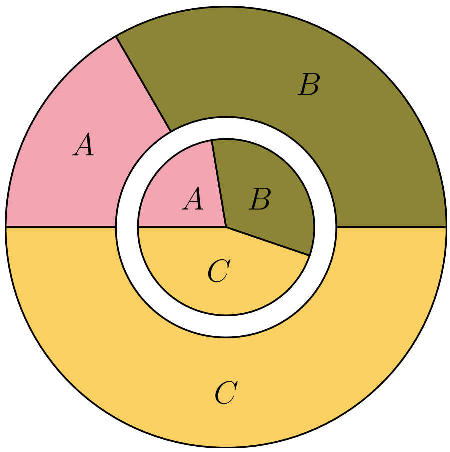
Same as the above, but without gaps between the groups of data:

```
>> PieChart[{{10, 20, 30}, {15, 22,
    30}}, SectorSpacing -> None]
```



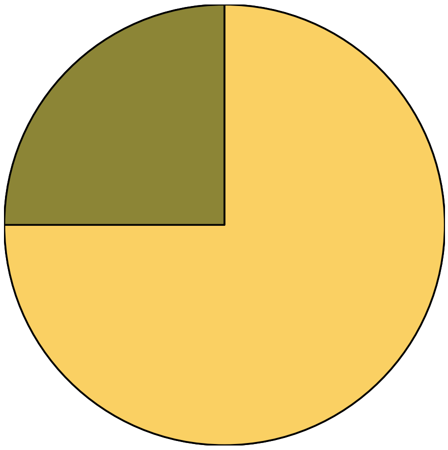
The doughnut chart above with labels on each of the 3 pieces:

```
>> PieChart[{{10, 20, 30}, {15, 22,
    30}}, ChartLabels -> {A, B, C}]
```



Negative values are removed, the data below is the same as {1, 3}:

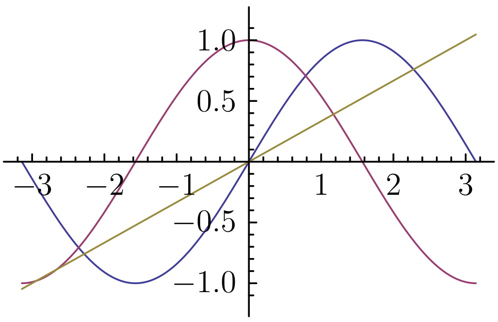
>>
```
PieChart[{1, -1, 3}]
```



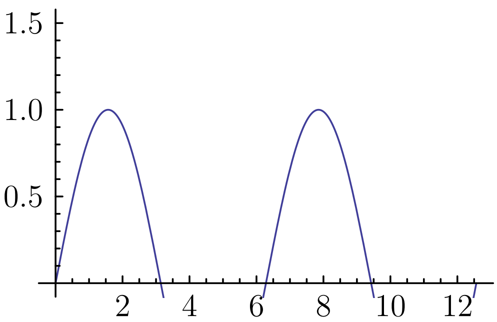
Plot

`Plot[`*f*`, {`*x*`, `*xmin*`, `*xmax*`}]`
plots *f* with *x* ranging from *xmin* to *xmax*.
`Plot[{`*f1*`, `*f2*`, ...}, {`*x*`, `*xmin*`, `*xmax*`}]`
plots several functions *f1*, *f2*, ...

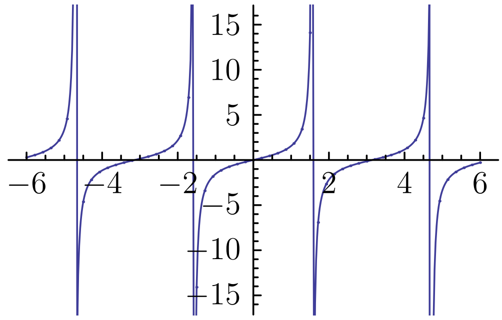
>>
```
Plot[{Sin[x], Cos[x], x / 3}, {x
, -Pi, Pi}]
```



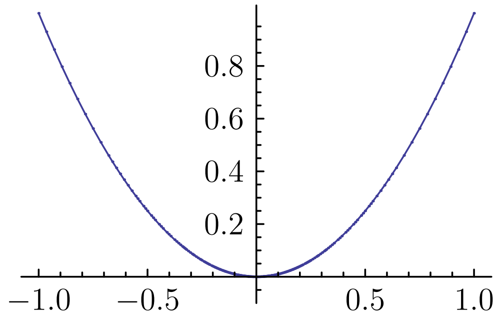
>>
```
Plot[Sin[x], {x, 0, 4 Pi},
PlotRange->{{0, 4 Pi}, {0,
1.5}}]
```



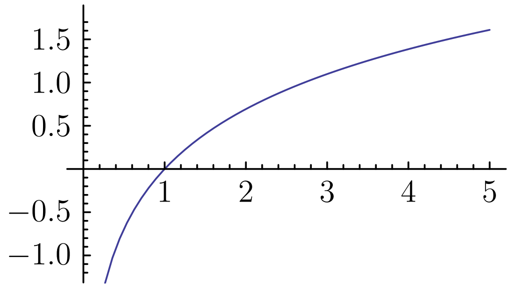
>>
```
Plot[Tan[x], {x, -6, 6}, Mesh->
Full]
```



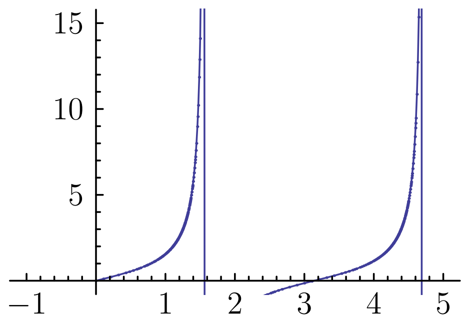
>>
```
Plot[x^2, {x, -1, 1},
MaxRecursion->5, Mesh->All]
```



>>
```
Plot[Log[x], {x, 0, 5},
MaxRecursion->0]
```

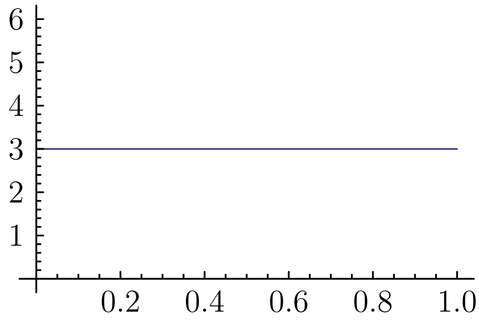


>>
```
Plot[Tan[x], {x, 0, 6}, Mesh->
All, PlotRange->{{-1, 5}, {0,
15}}, MaxRecursion->10]
```



A constant function:

```
>> Plot[3, {x, 0, 1}]
```



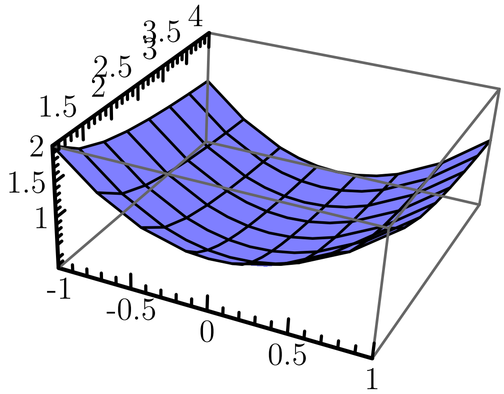
Plot3D

`Plot3D[`*f*`, {`*x*`, `*xmin*`, `*xmax*`}, {`*y*`, `*ymin*`, `*ymax*`}]`
: creates a three-dimensional plot of *f* with *x* ranging from *xmin* to *xmax* and *y* ranging from *ymin* to *ymax*.

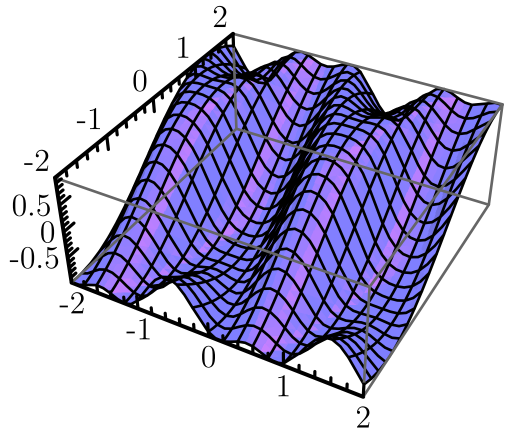
Plot3D has the same options as Graphics3D, in particular:

- Mesh
- PlotPoints
- MaxRecursion

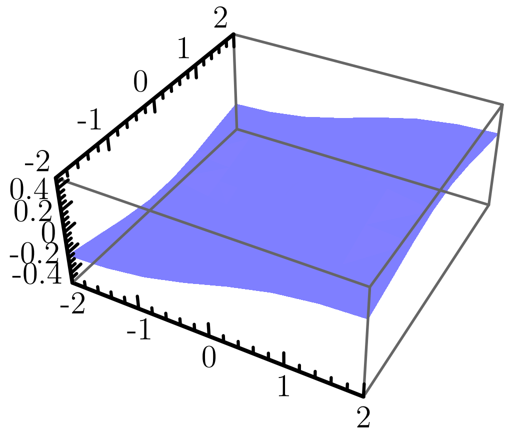
```
>> Plot3D[x ^ 2 + 1 / y, {x, -1,
   1}, {y, 1, 4}]
```



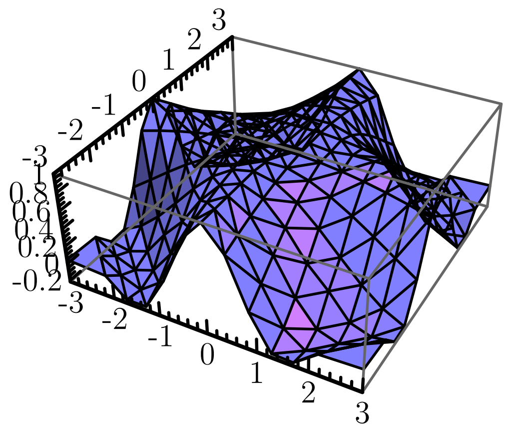

```
>> Plot3D[Sin[y + Sin[3 x]], {x,
   -2, 2}, {y, -2, 2}, PlotPoints
   ->20]
```



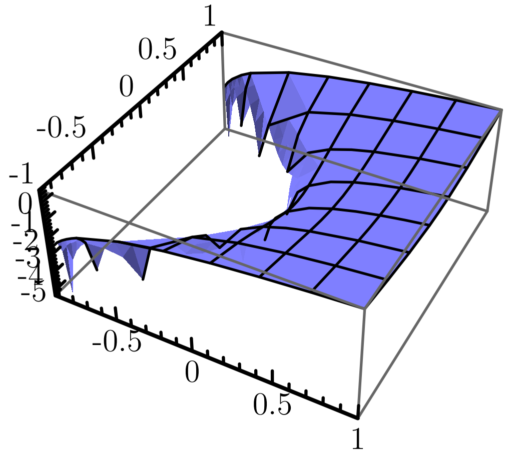

```
>> Plot3D[x / (x ^ 2 + y ^ 2 + 1),
   {x, -2, 2}, {y, -2, 2}, Mesh->
   None]
```



```
>> Plot3D[Sin[x y] /(x y), {x, -3,
   3}, {y, -3, 3}, Mesh->All]
```



```
>> Plot3D[Log[x + y^2], {x, -1, 1},
    {y, -1, 1}]
```



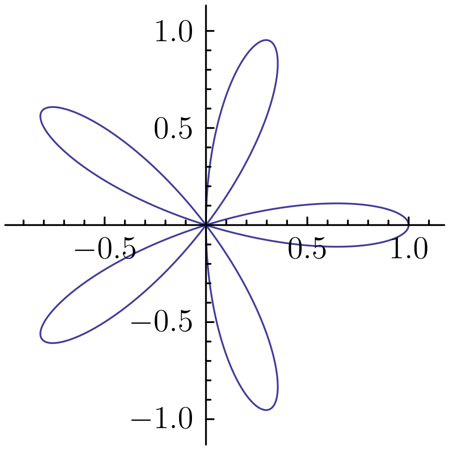
PolarPlot

PolarPlot[*r*, {*t*, *t_min*, *t_max*}]
: creates a polar plot of curve with radius *r* as a function of angle *t* ranging from *t_min* to *t_max*.

In a Polar Plot, a polar coordinate system is used. A polar coordinate system is a two-dimensional coordinate system in which each point on a plane is determined by a distance from a reference point and an angle from a reference direction.

Here is a 5-blade propeller, or maybe a flower, using `PolarPlot`:

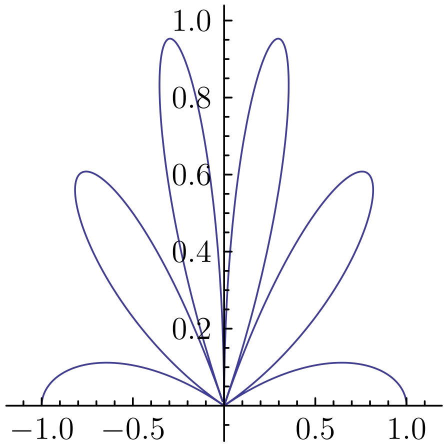
```
>> PolarPlot[Cos[5t], {t, 0, Pi}]
```



The number of blades and be change by adjusting the *t* multiplier.

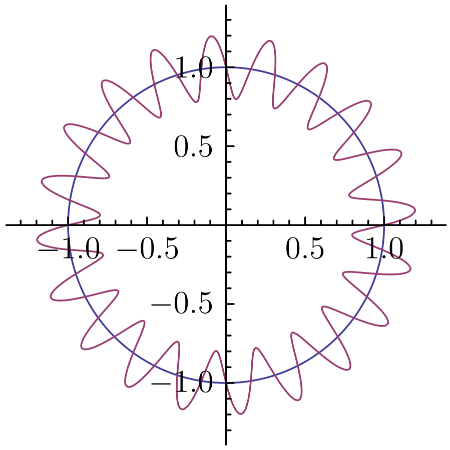
A slight change adding `Abs` turns this a clump of grass:

```
>> PolarPlot[Abs[Cos[5t]], {t, 0,
   Pi}]
```



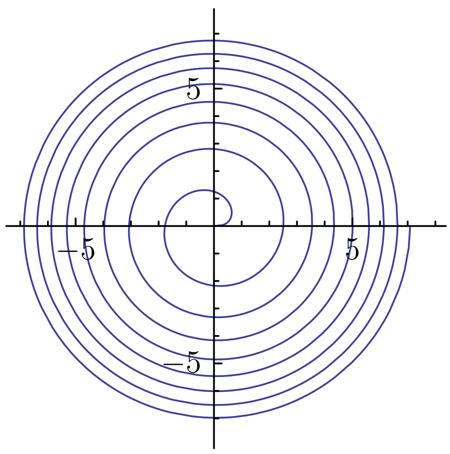
Coils around a ring:

```
>> PolarPlot[{1, 1 + Sin[20 t] /
   5}, {t, 0, 2 Pi}]
```



A spring having 16 turns:

```
>> PolarPlot[Sqrt[t], {t, 0, 16 Pi
   }]
```



35. Splines

Splines are used both in graphics and computations.

Contents

BernsteinBasis

`BernsteinBasis[`*d*`,`*n*`,`*x*`]`
returns the *n*th Bernstein basis of degree *d* at *x*.

A Bernstein polynomial is a polynomial that is a linear combination of Bernstein basis polynomials.
With the advent of computer graphics, Bernstein polynomials, restricted to the interval [0, 1], became important in the form of Bézier curves.
`BernsteinBasis[d,n,x]` equals `Binomial[d, n] x^n (1-x)^(d-n)` in the interval [0, 1] and zero elsewhere.

```
>> BernsteinBasis[4, 3, 0.5]
```
0.25

BezierCurve

`BezierCurve[{`*pt_1*`, `*pt_2*` ...}]`
represents a Bézier curve with control points *p_i*.

Option:

- `SplineDegree->`*d* specifies that the underlying polynomial basis should have maximal degree d.

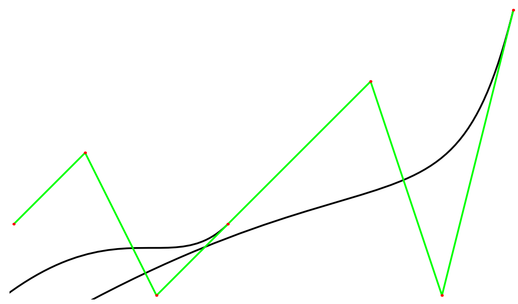
Set up some points...

```
>> pts = {{0, 0}, {1, 1}, {2, -1}, {3, 0}, {5, 2}, {6, -1}, {7, 3}};
```

=

A composite Bézier curve and its control points:

```
>> Graphics[{BezierCurve[pts], Green, Line[pts], Red, Point[pts]}]
```



=

BezierFunction

`BezierFunction[{`*pt_1*`, `*pt_2*`, ...}]`
returns a Bézier function for the curve defined by points *pt_i*. The embedding dimension for the curve represented by `BezierFunction[{`*pt_1*`,`*pt_2*`,...}]` is given by the length of the lists *pt_i*.

```
>> f = BezierFunction[{{0, 0}, {1, 1}, {2, 0}, {3, 2}}];
```

=

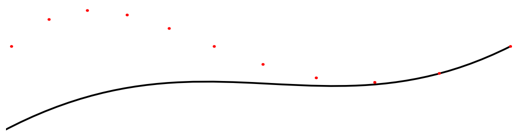
```
>> f[.5]
```
{1.5, 0.625}

=

Plotting the Bézier Function accoss a Bézier curve:

>>

```
Module[{p={{0, 0},{1, 1},{2,
-1},{4, 0}}}, Graphics[{
BezierCurve[p], Red, Point[Table
[BezierFunction[p][x], {x, 0, 1,
 0.1}]]}]]
```



36. Input/Output, Files, and Filesystem

37. File and Stream Operations

Contents

BinaryRead

`BinaryRead[`*stream*`]`
: reads one byte from the stream as an integer from 0 to 255.

`BinaryRead[`*stream*`, `*type*`]`
: reads one object of specified type from the stream.

`BinaryRead[`*stream*`, {`*type1*`, `*type2*`, ...}]`
: reads a sequence of objects of specified types.

```
>> strm = OpenWrite[BinaryFormat -> True]
```

OutputStream [/tmp/tmpncbldyy2, 153]

```
>> BinaryWrite[strm, {97, 98, 99}]
```

OutputStream [/tmp/tmpncbldyy2, 153]

```
>> Close[strm]
```

/tmp/tmpncbldyy2

```
>> strm = OpenRead[%, BinaryFormat -> True]
```

InputStream [/tmp/tmpncbldyy2, 154]

```
>> BinaryRead[strm, {"Character8", "Character8", "Character8"}]
```

{a, b, c}

```
>> Close[strm];
```

BinaryWrite

`BinaryWrite[`*channel*`, `*b*`]`
: writes a single byte given as an integer from 0 to 255.

`BinaryWrite[`*channel*`, {b1, b2, ...}]`
: writes a sequence of byte.

`BinaryWrite[`*channel*`, ‘‘string’]’`
: writes the raw characters in a string.

`BinaryWrite[`*channel*`, `*x*`, `*type*`]`
: writes *x* as the specified type.

`BinaryWrite[`*channel*`, {`*x1*`, `*x2*`, ...}, `*type*`]`
: writes a sequence of objects as the specified type.

`BinaryWrite[`*channel*`, {`*x1*`, `*x2*`, ...}, {`*type1*`, `*type2*`, ...}]`
: writes a sequence of objects using a sequence of specified types.

```
>> strm = OpenWrite[BinaryFormat -> True]
```
OutputStream [/tmp/tmphg8qm476, 273]

```
>> BinaryWrite[strm, {39, 4, 122}]
```
OutputStream [/tmp/tmphg8qm476, 273]

```
>> Close[strm]
```
/tmp/tmphg8qm476

```
>> strm = OpenRead[%, BinaryFormat -> True]
```
InputStream [/tmp/tmphg8qm476, 274]

```
>> BinaryRead[strm]
```
39

```
>> BinaryRead[strm, "Byte"]
```
4

```
>> BinaryRead[strm, "Character8"]
```
z

```
>> Close[strm];
```

Write a String

```
>> strm = OpenWrite[BinaryFormat -> True]
```
OutputStream [/tmp/tmp5vfjbqkb, 275]

```
>> BinaryWrite[strm, "abc123"]
```
OutputStream [/tmp/tmp5vfjbqkb, 275]

```
>> Close[%]
```
/tmp/tmp5vfjbqkb

Read as Bytes

```
>> strm = OpenRead[%, BinaryFormat -> True]
```
InputStream [/tmp/tmp5vfjbqkb, 276]

```
>> BinaryRead[strm, {"Character8", "Character8", "Character8", "Character8", "Character8", "Character8", "Character8"}]
```
{a, b, c, 1, 2, 3, EndOfFile}

```
>> Close[strm]
```
/tmp/tmp5vfjbqkb

Read as Characters

```
>> strm = OpenRead[%, BinaryFormat -> True]
```
InputStream [/tmp/tmp5vfjbqkb, 277]

```
>> BinaryRead[strm, {"Byte", "Byte", "Byte", "Byte", "Byte", "Byte", "Byte"}]
```
{97, 98, 99, 49, 50, 51, EndOfFile}

```
>> Close[strm]
```
/tmp/tmp5vfjbqkb

Write Type

```
>> strm = OpenWrite[BinaryFormat -> True]
```
OutputStream [/tmp/tmpmgc5s9as, 278]

```
>> BinaryWrite[strm, 97, "Byte"]
```
OutputStream [/tmp/tmpmgc5s9as, 278]

```
>> BinaryWrite[strm, {97, 98, 99}, {"Byte", "Byte", "Byte"}]
```
OutputStream [/tmp/tmpmgc5s9as, 278]

```
>> Close[%]
```
/tmp/tmpmgc5s9as

Byte

`Byte`
is a data type for `Read`.

Character

`Character`
is a data type for `Read`.

Close

`Close[`*stream*`]`
: closes an input or output stream.

```
>> Close[StringToStream["123abc"]]
```
String

```
>> Close[OpenWrite[]]
```
/tmp/tmpj1y_j14q

EndOfFile

`EndOfFile`
: is returned by `Read` when the end of an input stream is reached.

Expression

`Expression`
: is a data type for `Read`.

FilePrint

`FilePrint[`*file*`]`
: prints the raw contents of *file*.

Find

`Find[`*stream*`, ` *text*`]`
: find the first line in *stream* that contains *text*.

```
>> stream = OpenRead["ExampleData/
EinsteinSzilLetter.txt"];
```

```
>> Find[stream, "uranium"]
```
in manuscript, leads me
  to expect that the element
  uranium may be turned into

```
>> Find[stream, "uranium"]
```
become possible to set up
  a nuclear chain reaction in
  a large mass of uranium,

```
>> Close[stream]
```
ExampleData/EinsteinSzilLetter.txt

```
>> stream = OpenRead["ExampleData/
EinsteinSzilLetter.txt"];
```

```
>> Find[stream, {"energy", "power"}
]
```
a new and important source
  of energy in the immediate
  future. Certain aspects

```
>> Find[stream, {"energy", "power"}
]
```
by which vast amounts of
  power and large quantities
  of new radium-like

```
>> Close[stream]
```
ExampleData/EinsteinSzilLetter.txt

Get (<<)

`<<`*name*
: reads a file and evaluates each expression, returning only the last one.

`Get[`*name*`, Trace->True]`
: Runs Get tracing each line before it is evaluated.

```
>> filename = $TemporaryDirectory
<> "/example_file";
```

```
>> Put[x + y, filename]
```

```
>> Get[filename]
```
"x"cannotbefollowedby"
text{+}y"(line1of"/tmp/example_file").

```
>> filename = $TemporaryDirectory
<> "/example_file";
```

```
>> Put[x + y, 2x^2 + 4z!, Cos[x] +
I Sin[x], filename]
```

```
>> Get[filename]
```
"x"cannotbefollowedby" text{+}y"(line1of"/tmp/example_file").

```
>> DeleteFile[filename]
```

$Input

`$Input`
: is the name of the stream from which input is currently being read.

```
>> $Input
```

$InputFileName

`$InputFileName`
: is the name of the file from which input is currently being read.

While in interactive mode, `$InputFileName` is "".

```
>> $InputFileName
```

InputStream

`InputStream[name, n]`
: represents an input stream.

```
>> stream = StringToStream["Mathics
    is cool!"]
```
InputStream [String, 294]

```
>> Close[stream]
```
String

Number

`Number`
: is a data type for `Read`.

OpenAppend

`OpenAppend[''file']'`
: opens a file and returns an OutputStream to which writes are appended.

```
>> OpenAppend[]
```
OutputStream [
/tmp/tmp5n8nnl9n, 297]

OpenRead

`OpenRead[''file']'`
: opens a file and returns an InputStream.

```
>> OpenRead["ExampleData/
   EinsteinSzilLetter.txt"]
```
InputStream [
ExampleData/EinsteinSzilLetter.txt,
301]

```
>> OpenRead["https://raw.
   githubusercontent.com/mathics/
   Mathics/master/README.rst"]
```
InputStream [
https://raw.githubusercontent.com/mathics/Mathics/mas
302]

```
>> Close[%];
```

OpenWrite

`OpenWrite[''file']'`
: opens a file and returns an OutputStream.

```
>> OpenWrite[]
```
OutputStream [
/tmp/tmp9wbwt_kr, 305]

OutputStream

`OutputStream[name, n]`
represents an output stream.

```
>> OpenWrite[]
```

OutputStream [
/tmp/tmp0vk_2azf, 307]

```
>> Close[%]
```

/tmp/tmp0vk_2azf

Put (>>)

expr >> *filename*
write *expr* to a file.
`Put[expr1, expr2, ..., filename]`
write a sequence of expressions to a file.

```
>> Put[40!, fortyfactorial]
```

fortyfactorialisnotstring,
InputStream[], orOutputStream[]

815 915 283 247 897 734 345 ~
~611 269 596 115 894 272 ~
~000 000 000»fortyfactorial

```
>> filename = $TemporaryDirectory
<> "/fortyfactorial";
```

```
>> Put[40!, filename]
```

```
>> FilePrint[filename]
```

815 915 283 247 897 734 345 611 ~
~269 596 115 894 272 000 000 000

```
>> Get[filename]
```

815 915 283 247 897 734 345 611 ~
~269 596 115 894 272 000 000 000

```
>> DeleteFile[filename]
```

```
>> filename = $TemporaryDirectory
<> "/fiftyfactorial";
```

```
>> Put[10!, 20!, 30!, filename]
```

```
>> FilePrint[filename]
```

3 628 800
2 432 902 008 176 640 000
265 252 859 812 191 ~
~058 636 308 480 000 000

```
>> DeleteFile[filename]
```

=

```
>> filename = $TemporaryDirectory
<> "/example_file";
```

```
>> Put[x + y, 2x^2 + 4z!, Cos[x] +
I Sin[x], filename]
```

```
>> FilePrint[filename]
```

x
text{+}*y*
2 ∗ *x*{}{
wedge}2
text{+}4 ∗ *z*!
text{*Cos*}
left[*x*
right]
text{+}*I* ∗
text{*Sin*}
left[*x*
right]

```
>> DeleteFile[filename]
```

PutAppend (>>>)

expr >>> *filename*
append *expr* to a file.
`PutAppend[expr1, expr2, ..., $‘‘filename’$]’`
write a sequence of expressions to a file.

```
>> Put[50!, "factorials"]
```

```
>> FilePrint["factorials"]
```

30 414 093 201 713 378 043 612 ~
~608 166 064 768 844 377 641 ~
~568 960 512 000 000 000 000

```
>> PutAppend[10!, 20!, 30!, "
factorials"]
```

```
>> FilePrint["factorials"]
```
30 414 093 201 713 378 043 612~
~608 166 064 768 844 377 641~
~568 960 512 000 000 000 000
3 628 800
2 432 902 008 176 640 000
265 252 859 812 191~
~058 636 308 480 000 000

```
>> 60! >>> "factorials"
```

```
>> FilePrint["factorials"]
```
30 414 093 201 713 378 043 612~
~608 166 064 768 844 377 641~
~568 960 512 000 000 000 000
3 628 800
2 432 902 008 176 640 000
265 252 859 812 191~
~058 636 308 480 000 000
8 320 987 112 741 390 144~
~276 341 183 223 364 380 754~
~172 606 361 245 952 449 277~
~696 409 600 000 000 000 000

```
>> "string" >>> factorials
```

```
>> FilePrint["factorials"]
```
30 414 093 201 713 378 043 612~
~608 166 064 768 844 377 641~
~568 960 512 000 000 000 000
3 628 800
2 432 902 008 176 640 000
265 252 859 812 191~
~058 636 308 480 000 000
8 320 987 112 741 390 144~
~276 341 183 223 364 380 754~
~172 606 361 245 952 449 277~
~696 409 600 000 000 000 000
"string"

Read

`Read[`*stream*`]`
: reads the input stream and returns one expression.

`Read[`*stream*`, `*type*`]`
: reads the input stream and returns an object of the given type.

`Read[`*stream*`, `*type*`]`
: reads the input stream and returns an object of the given type.

`Read[`*stream*`, Hold[Expression]]`
: reads the input stream for an Expression and puts it inside `Hold`.

type is one of:

- Byte
- Character
- Expression
- HoldExpression
- Number
- Real
- Record
- String
- Word

```
>> stream = StringToStream["abc123
   "];
```

```
>> Read[stream, String]
```
abc123

```
>> stream = StringToStream["abc
   123"];
```

```
>> Read[stream, Word]
```
abc

```
>> Read[stream, Word]
```
123

```
>> stream = StringToStream["123,
   4"];
```

```
>> Read[stream, Number]
```
123

```
>> Read[stream, Number]
```
4

```
>> stream = StringToStream["2+2\n2
   +3"];
```

`Read` with a `Hold[Expression]` returns the expression it reads unevaluated so it can be later inspected and evaluated:

```
>> Read[stream, Hold[Expression]]
   Hold[2 + 2]

>> Read[stream, Expression]
   5

>> Close[stream];
```

Reading a comment however will return the empy list:

```
>> stream = StringToStream["(* ::
   Package:: *)"];

>> Read[stream, Hold[Expression]]
   {}

>> Close[stream];

>> stream = StringToStream["123 abc
   "];

>> Read[stream, {Number, Word}]
   {123, abc}
```

Multiple lines:

```
>> stream = StringToStream["\"Tengo
    una\nvaca lechera.\""]; Read[
   stream]
   Tengo una
     vaca lechera.
```

ReadList

`ReadList["file"]`
: Reads all the expressions until the end of file.

`ReadList["file", type]`
: Reads objects of a specified type until the end of file.

`ReadList["file", {type1, type2, ...}]`
: Reads a sequence of specified types until the end of file.

```
>> ReadList[StringToStream["a 1 b
   2"], {Word, Number}]
   {{a, 1}, {b, 2}}

>> stream = StringToStream["\"
   abc123\""];

>> ReadList[stream]
   {abc123}

>> InputForm[%]
   {"abc123"}
```

Record

`Record`
: is a data type for `Read`.

SetStreamPosition

`SetStreamPosition[stream, n]`
: sets the current position in a stream.

```
>> stream = StringToStream["Mathics
    is cool!"]
   InputStream[String, 352]

>> SetStreamPosition[stream, 8]
   8

>> Read[stream, Word]
   is

>> SetStreamPosition[stream,
   Infinity]
   16
```

Skip

`Skip[stream, type]`
: skips ahead in an input steream by one object of the specified *type*.

`Skip[stream, type, n]`
: skips ahead in an input steream by *n* objects of the specified *type*.

```
>> stream = StringToStream["a b c d
   "];

>> Read[stream, Word]
   a

>> Skip[stream, Word]
```

```
>> Read[stream, Word]
c
```

```
>> stream = StringToStream["a b c d
"];
```

```
>> Read[stream, Word]
a
```

```
>> Skip[stream, Word, 2]
```

```
>> Read[stream, Word]
d
```

StreamPosition

`StreamPosition[stream]`
: returns the current position in a stream as an integer.

```
>> stream = StringToStream["Mathics
 is cool!"]
InputStream [String, 358]
```

```
>> Read[stream, Word]
Mathics
```

```
>> StreamPosition[stream]
7
```

Streams

`Streams[]`
: returns a list of all open streams.

```
>> Streams[]
{InputStream [stdin, 0] ,
OutputStream [stdout, 1] ,
OutputStream [stderr, 2] ,
OutputStream [/tmp/tmp5n8nnl9n,
297] , InputStream [
/src/external-vcs/github/mathics/Mathics/mathics/data/
301] , OutputStream [
/tmp/tmp9wbwt_kr,
305] , InputStream [
String, 326] , InputStream [
String, 338] , InputStream [
String, 339] , InputStream [
String, 340] , InputStream [
String, 341] , InputStream [
String, 344] , InputStream [
String, 345] , InputStream [
String, 346] , InputStream [
String, 348] , InputStream [
String, 349] , InputStream [
String, 350] , InputStream [
String, 351] , InputStream [
String, 352] , InputStream [
String, 355] , InputStream [
String, 356] , InputStream [
String, 357] , InputStream [
String, 358] , OutputStream [
/tmp/tmp3ci5nb6a, 359] }
```

```
>> Streams["stdout"]
{OutputStream [stdout, 1]}
```

StringToStream

`StringToStream[string]`
: converts a *string* to an open input stream.

```
>> strm = StringToStream["abc 123"]
InputStream [String, 361]
```

Word

`Word`
: is a data type for `Read`.

Write

`Write[`*channel*`, `*expr1*`, `*expr2*`, ...]`
writes the expressions to the output channel followed by a newline.

```
>> stream = OpenWrite[]
```
OutputStream [/tmp/tmpwb5ie82h, 364]

```
>> Write[stream, 10 x + 15 y ^ 2]
```

```
>> Write[stream, 3 Sin[z]]
```

```
>> Close[stream]
```
/tmp/tmpwb5ie82h

```
>> stream = OpenRead[%];
```

```
>> ReadList[stream]
```
$\{10x + 15y^2, 3\text{Sin}[z]\}$

WriteString

`WriteString[`*stream*`, $str1, `*str2*`, ... ]`
writes the strings to the output stream.

```
>> stream = OpenWrite[];
```

```
>> WriteString[stream, "This is a
test 1"]
```

```
>> WriteString[stream, "This is
also a test 2"]
```

```
>> Close[stream]
```
/tmp/tmpdqv9ujjn

```
>> FilePrint[%]
```
Thisisatest1Thisisalsoatest2

```
>> stream = OpenWrite[];
```

```
>> WriteString[stream, "This is a
test 1", "This is also a test
2"]
```

```
>> Close[stream]
```
/tmp/tmpnyw8xmb8

```
>> FilePrint[%]
```
Thisisatest1Thisisalsoatest2

38. Filesystem Operations

Contents

AbsoluteFileName

`AbsoluteFileName["`*name*`"]`
returns the absolute version of the given filename.

```
>> AbsoluteFileName["ExampleData/
   sunflowers.jpg"]
```

/src/external-vcs/github/mathics/Mathics/mathics/data/ExampleData/sunflowers.jpg

$BaseDirectory

`$UserBaseDirectory`
returns the folder where user configurations are stored.

```
>> $RootDirectory
```

/

CopyDirectory

`CopyDirectory["`*dir1*`‘‘,"`*dir2*`"]`
copies directory *dir1* to *dir2*.

CopyFile

`CopyFile["`*file1*`‘‘,"`*file2*`"]`
copies *file1* to *file2*.

```
>> CopyFile["ExampleData/sunflowers
   .jpg", "MathicsSunflowers.jpg"]
```

MathicsSunflowers.jpg

```
>> DeleteFile["MathicsSunflowers.
   jpg"]
```

CreateDirectory

`CreateDirectory["`*dir*`"]`
: creates a directory called *dir*.

`CreateDirectory[]`
: creates a temporary directory.

```
>> dir = CreateDirectory[]
```
/tmp/mub92ii5k

CreateFile

`CreateFile[‘‘filename’]’`
: Creates a file named “filename” temporary file, but do not open it.

`CreateFile[]`
: Creates a temporary file, but do not open it.

CreateTemporary

`CreateTemporary[]`
: Creates a temporary file, but do not open it.

DeleteDirectory

`DeleteDirectory["`*dir*`"]`
: deletes a directory called *dir*.

```
>> dir = CreateDirectory[]
```
/tmp/mf1ofp352

```
>> DeleteDirectory[dir]
```

```
>> DirectoryQ[dir]
```
False

DeleteFile

`Delete["`*file*`"]`
: deletes *file*.

`Delete[{"`*file1*`‘‘,"`*file2*`", ...}]`
: deletes a list of files.

```
>> CopyFile["ExampleData/sunflowers
.jpg", "MathicsSunflowers.jpg"];
```

```
>> DeleteFile["MathicsSunflowers.
jpg"]
```

```
>> CopyFile["ExampleData/sunflowers
.jpg", "MathicsSunflowers1.jpg
"];
```

```
>> CopyFile["ExampleData/sunflowers
.jpg", "MathicsSunflowers2.jpg
"];
```

```
>> DeleteFile[{"MathicsSunflowers1.
jpg", "MathicsSunflowers2.jpg"}]
```

Directory

`Directory[]`
: returns the current working directory.

```
>> Directory[]
```
/src/external-vcs/github/mathics/Mathics

DirectoryName

`DirectoryName["`*name*`"]`
: extracts the directory name from a filename.

```
>> DirectoryName["a/b/c"]
```
a/b

```
>> DirectoryName["a/b/c", 2]
```
a

DirectoryQ

`DirectoryQ["`*name*`"]`
: returns `True` if the directory called *name* exists and `False` otherwise.

```
>> DirectoryQ["ExampleData/"]
```
True

>> `DirectoryQ["ExampleData/MythicalSubdir/"]`

False

DirectoryStack

`DirectoryStack[]`
returns the directory stack.

>> `DirectoryStack[]`

{/src/external-vcs/github/mathics/Mathics}

ExpandFileName

`ExpandFileName["`*name*`"]`
expands *name* to an absolute filename for your system.

>> `ExpandFileName["ExampleData/sunflowers.jpg"]`

/src/external-vcs/github/mathics/Mathics/ExampleData/sunflowers.jpg

FileBaseName

`FileBaseName["`*file*`"]`
gives the base name for the specified file name.

>> `FileBaseName["file.txt"]`

file

>> `FileBaseName["file.tar.gz"]`

file.tar

FileByteCount

`FileByteCount[`*file*`]`
returns the number of bytes in *file*.

>> `FileByteCount["ExampleData/sunflowers.jpg"]`

142 286

FileDate

`FileDate[`*file*`, `*types*`]`
returns the time and date at which the file was last modified.

>> `FileDate["ExampleData/sunflowers.jpg"]`

{2 120, 9, 7, 7, 16, 33.2822}

>> `FileDate["ExampleData/sunflowers.jpg", "Access"]`

{2 121, 7, 3, 17, 18, 41.2356}

>> `FileDate["ExampleData/sunflowers.jpg", "Creation"]`

Missing [NotApplicable]

>> `FileDate["ExampleData/sunflowers.jpg", "Change"]`

{2 120, 9, 7, 7, 16, 33.2822}

>> `FileDate["ExampleData/sunflowers.jpg", "Modification"]`

{2 120, 9, 7, 7, 16, 33.2822}

>> `FileDate["ExampleData/sunflowers.jpg", "Rules"]`

{Access− > {2 121, 7, 3, 17, 18, 41.2356} , Creation− > Missing [NotApplicable] , Change− > {2 120, 9, 7, 7, 16, 33.282˜˜2} , Modification− > {2 120, 9, 7, 7, 16, 33.2822}}

FileExistsQ

`FileExistsQ["`*file*`"]`
returns `True` if *file* exists and `False` otherwise.

>> `FileExistsQ["ExampleData/sunflowers.jpg"]`

True

>> `FileExistsQ["ExampleData/sunflowers.png"]`

False

FileExtension

`FileExtension["`*file*`"]`
: gives the extension for the specified file name.

```
>> FileExtension["file.txt"]
```
txt

```
>> FileExtension["file.tar.gz"]
```
gz

FileHash

`FileHash[`*file*`]`
: returns an integer hash for the given *file*.

`FileHash[`*file*`, `*type*`]`
: returns an integer hash of the specified *type* for the given *file*.
The types supported are "MD5", "Adler32", "CRC32", "SHA", "SHA224", "SHA256", "SHA384", and "SHA512".

`FileHash[`*file*`, `*type*`, `*format*`]`
: gives a hash code in the specified format.

```
>> FileHash["ExampleData/sunflowers.jpg"]
```
109 937 059 621 979 839~
~952 736 809 235 486 742 106

```
>> FileHash["ExampleData/sunflowers.jpg", "MD5"]
```
109 937 059 621 979 839~
~952 736 809 235 486 742 106

```
>> FileHash["ExampleData/sunflowers.jpg", "Adler32"]
```
1 607 049 478

```
>> FileHash["ExampleData/sunflowers.jpg", "SHA256"]
```
111 619 807 552 579 450 300 684 600~
~241 129 773 909 359 865 098 672~
~286 468 229 443 390 003 894 913 065

FileInformation

`FileInformation["`*file*`"]`
: returns information about *file*.

This function is totally undocumented in MMA!

```
>> FileInformation["ExampleData/sunflowers.jpg"]
```
{File
$-$ > /src/external-vcs/github/mathics/Mathics/ExampleD
FileType$-$ > File, ByteCount$-$ >
142 286, Date$-$ > 6.96413×10^{9}}

FileNameDepth

`FileNameDepth["`*name*`"]`
: gives the number of path parts in the given filename.

```
>> FileNameDepth["a/b/c"]
```
3

```
>> FileNameDepth["a/b/c/"]
```
3

FileNameJoin

`FileNameJoin[{"`*dir_1*`‘‘,"`*dir_2*`", ...}]`
: joins the *dir_i* together into one path.

`FileNameJoin[..., OperatingSystem->‘‘os’]’`
: yields a file name in the format for the specified operating system. Possible choices are "Windows", "MacOSX", and "Unix".

```
>> FileNameJoin[{"dir1", "dir2", "dir3"}]
```
dir1/dir2/dir3

```
>> FileNameJoin[{"dir1", "dir2", "dir3"}, OperatingSystem -> "Unix"]
```
dir1/dir2/dir3

```
>> FileNameJoin[{"dir1", "dir2", "dir3"}, OperatingSystem -> "Windows"]
```
dir1\dir2\dir3

FileNameSplit

`FileNameSplit["`*filenams*`"]`
: splits a *filename* into a list of parts.

```
>> FileNameSplit["example/path/file
   .txt"]
```

{example, path, file.txt}

FileNameTake

`FileNameTake["`*file*`"]`
: returns the last path element in the file name *name*.

`FileNameTake["`*file*`", `*n*`]`
: returns the first *n* path elements in the file name *name*.

`FileNameTake["`*file*`", $-n$]`
: returns the last *n* path elements in the file name *name*.

FileNames

`FileNames[]`
: Returns a list with the filenames in the current working folder.

`FileNames[`*form*`]`
: Returns a list with the filenames in the current working folder that matches with *form*.

`FileNames[{`*form_1*`, `*form_2*`, ...}]`
: Returns a list with the filenames in the current working folder that matches with one of *form_1*, *form_2*, ...

`FileNames[{`*form_1*`, `*form_2*`, ...},{`*dir_1*`, `*dir_2*`, ...}]`
: Looks into the directories *dir_1*, *dir_2*, ...

`FileNames[{`*form_1*`, `*form_2*`, ...},{`*dir_1*`, `*dir_2*`, ...}]`
: Looks into the directories *dir_1*, *dir_2*, ...

`FileNames[{`*forms*`, `*dirs*`, `*n*`]`
: Look for files up to the level *n*.

```
>> SetDirectory[
   $InstallationDirectory <> "/
   autoload"];
```

```
>> FileNames["*.m", "formats"]//
   Length
```

0

```
>> FileNames["*.m", "formats", 3]//
   Length
```

14

```
>> FileNames["*.m", "formats",
   Infinity]//Length
```

14

FileType

`FileType["`*file*`"]`
: gives the type of a file, a string. This is typically `File`, `Directory` or `None`.

```
>> FileType["ExampleData/sunflowers
   .jpg"]
```

File

```
>> FileType["ExampleData"]
```

Directory

```
>> FileType["ExampleData/
   nonexistant"]
```

None

FindFile

`FindFile[`*name*`]`
: searches `$Path` for the given filename.

```
>> FindFile["ExampleData/sunflowers
   .jpg"]
```

/src/external-vcs/github/mathics/Mathics/mathics/data/

```
>> FindFile["VectorAnalysis`"]
```

/src/external-vcs/github/mathics/Mathics/mathics/packa

```
>> FindFile["VectorAnalysis`
   VectorAnalysis`"]
```

/src/external-vcs/github/mathics/Mathics/mathics/packa

FindList

`FindList[`*file*`, `*text*`]`
: returns a list of all lines in *file* that contain *text*.

`FindList[`*file*`, {`*text1*`, `*text2*`, ...}]`
: returns a list of all lines in *file* that contain any of the specified string.

`FindList[{`*file1*`, `*file2*`, ...}, ...]`
: returns a list of all lines in any of the *filei* that contain the specified strings.

```
>> stream = FindList["ExampleData/EinsteinSzilLetter.txt", "uranium"];
```

```
>> FindList["ExampleData/EinsteinSzilLetter.txt", "uranium", 1]
```

{in manuscript, leads me
to expect that the element
uranium may be turned into}

$HomeDirectory

`$HomeDirectory`
: returns the users HOME directory.

```
>> $HomeDirectory
```

/home/rocky

$InitialDirectory

`$InitialDirectory`
: returns the directory from which *Mathics* was started.

```
>> $InitialDirectory
```

/src/external-vcs/github/mathics/Mathics

$InstallationDirectory

`$InstallationDirectory`
: returns the top-level directory in which *Mathics* was installed.

```
>> $InstallationDirectory
```

/src/external-vcs/github/mathics/Mathics/mathics

Needs

`Needs["context`"]`
: loads the specified context if not already in `$Packages`.

```
>> Needs["VectorAnalysis`"]
```

$OperatingSystem

`$OperatingSystem`
: gives the type of operating system running Mathics.

```
>> $OperatingSystem
```

Unix

ParentDirectory

`ParentDirectory[]`
: returns the parent of the current working directory.

`ParentDirectory["`*dir*`"]`
: returns the parent *dir*.

```
>> ParentDirectory[]
```

/src/external-vcs/github/mathics/Mathics/mathics

$Path

`$Path`
: returns the list of directories to search when looking for a file.

```
>> $Path
```

{., /home/rocky,
/src/external-vcs/github/mathics/Mathics/mathics/data,
/src/external-vcs/github/mathics/Mathics/mathics/pack

$PathnameSeparator

`$PathnameSeparator`
: returns a string for the seperator in paths.

```
>> $PathnameSeparator
```
/

RenameDirectory

`RenameDirectory["`*dir1*`‘‘,"`*dir2*`"]`
: renames directory *dir1* to *dir2*.

RenameFile

`RenameFile["`*file1*`‘‘,"`*file2*`"]`
: renames *file1* to *file2*.

```
>> CopyFile["ExampleData/sunflowers
.jpg", "MathicsSunflowers.jpg"]
```
MathicsSunflowers.jpg

```
>> RenameFile["MathicsSunflowers.
jpg", "MathicsSunnyFlowers.jpg"]
```
MathicsSunnyFlowers.jpg

```
>> DeleteFile["MathicsSunnyFlowers.
jpg"]
```

ResetDirectory

`ResetDirectory[]`
: pops a directory from the directory stack and returns it.

```
>> ResetDirectory[]
```
/src/external-vcs/github/mathics/Mathics/matl

$RootDirectory

`$RootDirectory`
: returns the system root directory.

```
>> $RootDirectory
```
/

SetDirectory

`SetDirectory[`*dir*`]`
: sets the current working directory to *dir*.

```
>> SetDirectory[]
```
/home/rocky

SetFileDate

`SetFileDate["`*file*`"]`
: set the file access and modification dates of *file* to the current date.

`SetFileDate["`*file*`", `*date*`]`
: set the file access and modification dates of *file* to the specified date list.

`SetFileDate["`*file*`", `*date*`, "`*type*`"]`
: set the file date of *file* to the specified date list. The "*type*“ can be one of ”*Access*“, ”*Creation*“, ”*Modification*", or `All`.

Create a temporary file (for example purposes)

```
>> tmpfilename =
$TemporaryDirectory <> "/tmp0";
```

```
>> Close[OpenWrite[tmpfilename]];
```

```
>> SetFileDate[tmpfilename, {2002,
1, 1, 0, 0, 0.}, "Access"];
```

```
>> FileDate[tmpfilename, "Access"]
```
$\{2002, 1, 1, 0, 0, 0.\}$

$TemporaryDirectory

`$TemporaryDirectory`
: returns the directory used for temporary files.

```
>> $TemporaryDirectory
```
/tmp

ToFileName

`ToFileName[{"`*dir_1*`‘‘,"`*dir_2*`", ...}]`
: joins the *dir_i* together into one path.

`ToFileName` has been superseded by `FileNameJoin`.

>> **`ToFileName[{"dir1", "dir2"}, "file"]`**

dir1/dir2/file

>> **`ToFileName["dir1", "file"]`**

dir1/file

>> **`ToFileName[{"dir1", "dir2", "dir3"}]`**

dir1/dir2/dir3

URLSave

`URLSave[‘‘url’]’`
: Save “url” in a temporary file.

`URLSave[‘‘url’,’` *filename*`]`
: Save “url” in *filename*.

$UserBaseDirectory

`$UserBaseDirectory`
: returns the folder where user configurations are stored.

>> **`$RootDirectory`**

/

39. Importing and Exporting

Contents

System`Convert`B64Dump`B64Decode

System`Convert`B64Dump`B64Decode[*string*]
: Decode *string* in Base64 coding to an expression.

```
>> System`Convert`B64Dump`B64Decode["R!="]
```

String"R!
= "isnotavalidb64encodedstring.

$Failed

System`Convert`B64Dump`B64Encode

System`Convert`B64Dump`B64Encode[*expr*]
: Encodes *expr* in Base64 coding

```
>> System`Convert`B64Dump`B64Encode["Hello world"]
```

SGVsbG8gd29ybGQ=

```
>> System`Convert`B64Dump`B64Decode[%]
```

Hello world

```
>> System`Convert`B64Dump`B64Encode[Integrate[f[x],{x,0,2}]]
```

SW50ZWdyYXRlW2ZbeF0sIHt4LCAwLCAyfV0=

```
>> System`Convert`B64Dump`B64Decode[%]
```

Integrate[f[x], {x, 0, 2}]

System`Convert`CommonDump`RemoveLin

System`Convert`CommonDump`RemoveLinearSyntax[*something*]
: Keine anung... Undocumented in wma

System`ConvertersDump`$extensionMappin

$extensionMappings
: Returns a list of associations between file extensions and file types.

System`ConvertersDump`$formatMappings

$formatMappings
: Returns a list of associations between file extensions and file types.

Export

`Export["`*file.ext*`", `*expr*`]`
: exports *expr* to a file, using the extension *ext* to determine the format.

`Export["`*file*`", `*expr*`, "`*format*`"]`
: exports *expr* to a file in the specified format.

`Export["`*file*`", `*exprs*`, `*elems*`]`
: exports *exprs* to a file as elements specified by *elems*.

$ExportFormats

`$ExportFormats`
: returns a list of file formats supported by Export.

```
>> $ExportFormats
```

{BMP, Base64, CSV, GIF, JPEG, JPEG2 000, PBM, PCX, PGM, PNG, PPM, SVG, TIFF, Text, asy}

ExportString

`ExportString[`*expr*`, `*form*`]`
: exports *expr* to a string, in the format *form*.

`Export["`*file*`", `*exprs*`, `*elems*`]`
: exports *exprs* to a string as elements specified by *elems*.

```
>> ExportString
   [{{1,2,3,4},{3},{2},{4}}, "CSV"]
```

1,2,3,4
3,
2,
4,

```
>> ExportString[{1,2,3,4}, "CSV"]
```

1,
2,
3,
4,

```
>> ExportString[Integrate[f[x],{x
   ,0,2}], "SVG"]//Head
```

String

FileFormat

`FileFormat["`*name*`"]`
: attempts to determine what format `Import` should use to import specified file.

```
>> FileFormat["ExampleData/
   sunflowers.jpg"]
```

JPEG

```
>> FileFormat["ExampleData/
   EinsteinSzilLetter.txt"]
```

Text

```
>> FileFormat["ExampleData/lena.tif
   "]
```

TIFF

Import

`Import["`*file*`"]`
: imports data from a file.

`Import["`*file*`", `*elements*`]`
: imports the specified elements from a file.

`Import["http://`*url*`", ...]` and `Import["ftp://`*url*`", ...]`
: imports from a URL.

```
>> Import["ExampleData/ExampleData.
   txt", "Elements"]
```

{Data, Lines, Plaintext, String, Words}

```
>> Import["ExampleData/ExampleData.
   txt", "Lines"]
```

{Example File Format, Created by Angus, 0.629452 0.586355, 0.711009 0.687453, 0.246540 0.433973, 0.926871 0.887255, 0.825141 0.940900, 0.847035 0.127464, 0.054348 0.296494, 0.838545 0.247025, 0.838697 0.436220, 0.309496 0.833591}

```
>> Import["ExampleData/colors.json"]
```

{colorsArray
 -> {{colorName-> black,
 rgbValue-> (0, 0, 0),
 hexValue-> #000 000} ,
 {colorName-> red,
 rgbValue-> (255, 0, 0),
 hexValue-> #FF0 000} ,
 {colorName-> green,
 rgbValue-> (0, 255, 0),
 hexValue-> #00FF00} ,
 {colorName-> blue,
 rgbValue-> (0, 0, 255),
 hexValue-> #0 000FF} ,
 {colorName-> yellow,
 rgbValue-> (255, 255, 0),
 hexValue-> #FFFF00} ,
 {colorName-> cyan,
 rgbValue-> (0, 255, 255),
 hexValue-> #00FFFF} ,
 {colorName-> magenta,
 rgbValue-> (255, 0, 255),
 hexValue-> #FF00FF} ,
 {colorName-> white,
 rgbValue-> (255, 255, 255),
 hexValue-> #FFFFFF}}}

$ImportFormats

`$ImportFormats`
: returns a list of file formats supported by Import.

```
>> $ImportFormats
```

{BMP, Base64, CSV, GIF, HTML, ICO, JPEG, JPEG2 000, JSON, PBM, PCX, PGM, PNG, PPM, Package, TGA, TIFF, Text, XML}

ImportString

`ImportString["`*data*`‘‘, "`*format*`"]`
: imports data in the specified format from a string.

`ImportString["`*file*`", `*elements*`]`
: imports the specified elements from a string.

`ImportString["`*data*`"]`
: attempts to determine the format of the string from its content.

```
>> str = "Hello!\n This is a testing text\n";
```

```
>> ImportString[str, "Elements"]
```

{Data, Lines, Plaintext, String, Words}

```
>> ImportString[str, "Lines"]
```

{Hello!, This is a testing text}

ImportExport‘RegisterExport

`RegisterExport["`*format*`", `*func*`]`
: register *func* as the default function used when exporting from a file of type "*format*".

Simple text exporter

```
>> ExampleExporter1[filename_, data_, opts___] := Module[{strm = OpenWrite[filename], char = data}, WriteString[strm, char]; Close[strm]]
```

```
>> ImportExport‘RegisterExport["ExampleFormat1", ExampleExporter1]
```

```
>> Export["sample.txt", "Encode this string!", "ExampleFormat1"];
```

```
>> FilePrint["sample.txt"]
```

Encodethisstring!

Very basic encrypted text exporter

```
>> ExampleExporter2[filename_,
   data_, opts___] := Module[{strm
   = OpenWrite[filename], char}, (*
    TODO: Check data *)char =
   FromCharacterCode[Mod[
   ToCharacterCode[data] - 84, 26]
   + 97]; WriteString[strm, char];
   Close[strm]]
```

```
>> ImportExport`RegisterExport["
   ExampleFormat2",
   ExampleExporter2]
```

```
>> Export["sample.txt", "
   encodethisstring", "
   ExampleFormat2"];
```

```
>> FilePrint["sample.txt"]
```

rapbqrguvffgevat

ImportExport`RegisterImport

`RegisterImport["`*format*`", `*defaultFunction*`]`
: register *defaultFunction* as the default function used when importing from a file of type "*format*".

`RegisterImport["`*format*`", {"`*elem1*`" :> `*conditionalFunction1*`, "`*elem2*`" :> `*conditionalFunction2*`, ..., `*defaultFunction*`}]`
: registers multiple elements (*elem1*, ...) and their corresponding converter functions (*conditionalFunction1*, ...) in addition to the *defaultFunction*.

`RegisterImport["`*format*`", {"`*conditionalFunctions*`, `*defaultFunction*`, "`*elem3*`" :> `*postFunction3*`, "`*elem4*`" :> `*postFunction4*`, ...}]`
: also registers additional elements (*elem3*, ...) whose converters (*postFunction3*, ...) act on output from the low-level funcions.

First, define the default function used to import the data.

```
>> ExampleFormat1Import[
   filename_String] := Module[{
   stream, head, data}, stream =
   OpenRead[filename]; head =
   ReadList[stream, String, 2];
   data = Partition[ReadList[stream
   , Number], 2]; Close[stream]; {"
   Header" -> head, "Data" -> data
   }]
```

`RegisterImport` is then used to register the above function to a new data format.

```
>> ImportExport`RegisterImport["
   ExampleFormat1",
   ExampleFormat1Import]
```

```
>> FilePrint["ExampleData/
   ExampleData.txt"]
```

ExampleFileFormat

CreatedbyAngus

0.6294520.586355

0.7110090.687453

0.2465400.433973

0.9268710.887255

0.8251410.940900

0.8470350.127464

0.0543480.296494

0.8385450.247025

0.8386970.436220

0.3094960.833591

```
>> Import["ExampleData/ExampleData.
   txt", {"ExampleFormat1", "
   Elements"}]
```

{Data, Header}

```
>> Import["ExampleData/ExampleData.
   txt", {"ExampleFormat1", "Header
   "}]
```

{Example File Format,
 Created by Angus}

Conditional Importer:

```
>> ExampleFormat2DefaultImport[
   filename_String] := Module[{
   stream, head}, stream = OpenRead
   [filename]; head = ReadList[
   stream, String, 2]; Close[stream
   ]; {"Header" -> head}]
```

```
>> ExampleFormat2DataImport[
   filename_String] := Module[{
   stream, data}, stream = OpenRead
   [filename]; Skip[stream, String,
    2]; data = Partition[ReadList[
   stream, Number], 2]; Close[
   stream]; {"Data" -> data}]
```

```
>> ImportExport`RegisterImport["
   ExampleFormat2", {"Data" :>
   ExampleFormat2DataImport,
   ExampleFormat2DefaultImport}]
```

```
>> Import["ExampleData/ExampleData.
   txt", {"ExampleFormat2", "
   Elements"}]
```

{Data, Header}

```
>> Import["ExampleData/ExampleData.
   txt", {"ExampleFormat2", "Header
   "}]
```

{Example File Format,
Created by Angus}

```
>> Import["ExampleData/ExampleData.
   txt", {"ExampleFormat2", "Data
   "}] // Grid
```

0.629452	0.586355
0.711009	0.687453
0.24654	0.433973
0.926871	0.887255
0.825141	0.9409
0.847035	0.127464
0.054348	0.296494
0.838545	0.247025
0.838697	0.43622
0.309496	0.833591

URLFetch

`URLFetch[`*URL*`]`
Returns the content of *URL* as a string.

= ...

40. The Main Loop

An interactive session operates a loop, called the "main loop" in this way:

- read input
- process input
- format and print results
- repeat

As part of this loop, various global objects in this section are consulted.
There are a variety of "hooks" that allow you to insert functions to be applied to the expresssions at various stages in the main loop.
If you assign a function to the global variable `$PreRead` it will be applied with the input that is read in the first step listed above.
Similarly, if you assign a function to global variable `$Pre`, it will be applied with the input before processing the input, the second step listed above.

Contents

$HistoryLength

`$HistoryLength`
specifies the maximum number of `In` and `Out` entries.

```
>> $HistoryLength
```
100

```
>> $HistoryLength = 1;
```

```
>> 42
```
42

```
>> %
```
42

```
>> %%
```
%3

```
>> $HistoryLength = 0;
```

```
>> 42
```
42

```
>> %
```
%7

$Post

$Post
is a global variable whose value, if set, is applied to every output expression.

$Pre

$Pre
is a global variable whose value, if set, is applied to every input expression.

Set *Timing* as the $Pre function, stores the enlapsed time in a variable, stores just the result in Out[$Line] and print a formated version showing the enlapsed time

```
>> $Pre := (Print["[Processing
   input...]"];#1)&
```

```
>> $Post := (Print["[Storing result
   ...]"]; #1)&
```
[Processinginput...]

[Storingresult...]

```
>> $PrePrint := (Print["The result is:"]; {TimeUsed[], #1})&
```

[Processinginput...]

[Storingresult...]

```
>> 2 + 2
```

[Processinginput...]

[Storingresult...]

Theresultis :

$\{341.557, 4\}$

```
>> $Pre = .; $Post = .; $PrePrint = .; $EnlapsedTime = .;
```

[Processinginput...]

```
>> 2 + 2
```

4

$PrePrint

$PrePrint
is a global variable whose value, if set, is applied to every output expression before it is printed.

$PreRead

$PreRead
is a global variable whose value, if set, is applied to the text or box form of every input expression before it is fed to the parser.
(Not implemented yet)

$SyntaxHandler

$SyntaxHandler
is a global variable whose value, if set, is applied to any input string that is found to contain a syntax error.
(Not implemented yet)

In

`In[k]`
gives the kth line of input.

```
>> x = 1
```

1

```
>> x = x + 1
```

2

```
>> Do[In[2], {3}]
```

```
>> x
```

5

```
>> In[-1]
```

5

```
>> Definition[In]
```

$$\begin{aligned} \text{Attributes}[\text{In}] &= \{\text{Listable}, \text{Protected}\} \\ \text{In}[6] &= \text{Definition}[\text{In}] \\ \text{In}[5] &= \text{In}[-1] \\ \text{In}[4] &= x \\ \text{In}[3] &= \text{Do}\left[\text{In}[2], \{3\}\right] \\ \text{In}[2] &= x = x + 1 \\ \text{In}[1] &= x = 1 \end{aligned}$$

$Line

`$Line`
holds the current input line number.

```
>> $Line
```

1

```
>> $Line
```

2

```
>> $Line = 12;
```

```
>> 2 * 5
```

10

```
>> Out[13]
```

10

```
>> $Line = -1;
```

Non − negativeintegerexpected.

Out

`Out[`k`]`
`%`k
: gives the result of the kth input line.

`%`, `%%`, etc.
: gives the result of the previous input line, of the line before the previous input line, etc.

```
>> 42
```
42

```
>> %
```
42

```
>> 43;
```

```
>> %
```
43

```
>> 44
```
44

```
>> %1
```
42

```
>> %%
```
44

```
>> Hold[Out[-1]]
```
Hold [%]

```
>> Hold[%4]
```
Hold [%4]

```
>> Out[0]
```
Out [0]

41. Integer Functions

Integer Functions can work on integers of any size.

42. Combinatorial Functions

Combinatorics is an area of mathematics primarily concerned with counting, both as a means and an end in obtaining results, and certain properties of finite structures.

It is closely related to many other areas of mathematics and has many applications ranging from logic to statistical physics, from evolutionary biology to computer science, etc.

Contents

Binomial

`Binomial[`*n*`, `*k*`]`
: gives the binomial coefficient *n* choose *k*.

```
>> Binomial[5, 3]
   10
```

`Binomial` supports inexact numbers:

```
>> Binomial[10.5,3.2]
   165.286
```

Some special cases:

```
>> Binomial[10, -2]
   0
```

```
>> Binomial[-10.5, -3.5]
   0.
```

DiceDissimilarity

`DiceDissimilarity[`*u*`, `*v*`]`
: returns the Dice dissimilarity between the two boolean 1-D lists *u* and *v*, which is defined as (c_tf + c_ft) / (2 * c_tt + c_ft + c_tf), where n is len(*u*) and c_ij is the number of occurrences of *u*[k]=i and *v*[k]=j for k<n.

```
>> DiceDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
```

$\frac{1}{2}$

JaccardDissimilarity

`JaccardDissimilarity[`*u*`, `*v*`]`
: returns the Jaccard-Needham dissimilarity between the two boolean 1-D lists *u* and *v*, which is defined as (c_tf + c_ft) / (c_tt + c_ft + c_tf), where n is len(*u*) and c_ij is the number of occurrences of *u*[k]=i and *v*[k]=j for k<n.

```
>> JaccardDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
```

$\frac{2}{3}$

MatchingDissimilarity

`MatchingDissimilarity[`*u*`, `*v*`]`
: returns the Matching dissimilarity between the two boolean 1-D lists *u* and *v*, which is defined as (c_tf + c_ft) / n, where n is len(*u*) and c_ij is the number of occurrences of *u*[k]=i and *v*[k]=j for k<n.

```
>> MatchingDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
```
$\frac{4}{7}$

Multinomial

`Multinomial[`*n1*`, `*n2*`, ...]`
: gives the multinomial coefficient `(`*n1*`+`*n2*`+...)!/(`*n1*`!`*n2*`!...)`.

```
>> Multinomial[2, 3, 4, 5]
```
2 522 520

```
>> Multinomial[]
```
1

Multinomial is expressed in terms of `Binomial`:

```
>> Multinomial[a, b, c]
```
Binomial $[a, a]$ Binomial $[a + b, b]$ Binomial $[a + b + c, c]$

`Multinomial[`*n*`-`*k*`, `*k*`]` is equivalent to `Binomial[`*n*`, `*k*`]`.

```
>> Multinomial[2, 3]
```
10

RogersTanimotoDissimilarity

`RogersTanimotoDissimilarity[`*u*`, `*v*`]`
: returns the Rogers-Tanimoto dissimilarity between the two boolean 1-D lists *u* and *v*, which is defined as R / (c_tt + c_ff + R) where n is len(*u*), c_ij is the number of occurrences of *u*[k]=i and *v*[k]=j for k<n, and R = 2 * (c_tf + c_ft).

```
>> RogersTanimotoDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
```
$\frac{8}{11}$

RussellRaoDissimilarity

`RussellRaoDissimilarity[`*u*`, `*v*`]`
: returns the Russell-Rao dissimilarity between the two boolean 1-D lists *u* and *v*, which is defined as (n - c_tt) / c_tt where n is len(*u*) and c_ij is the number of occurrences of *u*[k]=i and *v*[k]=j for k<n.

```
>> RussellRaoDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
```
$\frac{5}{7}$

SokalSneathDissimilarity

`SokalSneathDissimilarity[`*u*`, `*v*`]`
: returns the Sokal-Sneath dissimilarity between the two boolean 1-D lists *u* and *v*, which is defined as R / (c_tt + R) where n is len(*u*), c_ij is the number of occurrences of *u*[k]=i and *v*[k]=j for k<n, and R = 2 * (c_tf + c_ft).

```
>> SokalSneathDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
```
$\frac{4}{5}$

Subsets

`Subsets[`*list*`]`
: finds a list of all possible subsets of *list*.

`Subsets[`*list*`, `*n*`]`
: finds a list of all possible subsets containing at most *n* elements.

`Subsets[`*list*`, {`*n*`}]`
: finds a list of all possible subsets containing exactly *n* elements.

`Subsets[`*list*`, {`*min*`, `*max*`}]`
: finds a list of all possible subsets containing between *min* and *max* elements.

`Subsets[`*list*`, `*spec*`, `*n*`]`
: finds a list of the first *n* possible subsets.

`Subsets[`*list*`, `*spec*`, {`*n*`}]`
: finds the *n*th possible subset.

All possible subsets (power set):

>> **`Subsets[{a, b, c}]`**

$\{\{\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

All possible subsets containing up to 2 elements:

>> **`Subsets[{a, b, c, d}, 2]`**

$\{\{\}, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$

Subsets containing exactly 2 elements:

>> **`Subsets[{a, b, c, d}, {2}]`**

$\{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$

The first 5 subsets containing 3 elements:

>> **`Subsets[{a, b, c, d, e}, {3}, 5]`**

$\{\{a, b, c\}, \{a, b, d\}, \{a, b, e\}, \{a, c, d\}, \{a, c, e\}\}$

All subsets with even length:

>> **`Subsets[{a, b, c, d, e}, {0, 5, 2}]`**

$\{\{\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, e\}, \{b, c\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}, \{a, b, c, d\}, \{a, b, c, e\}, \{a, b, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}\}$

The 25th subset:

>> **`Subsets[Range[5], All, {25}]`**

$\{\{2, 4, 5\}\}$

The odd-numbered subsets of {a,b,c,d} in reverse order:

>> **`Subsets[{a, b, c, d}, All, {15, 1, -2}]`**

$\{\{b, c, d\}, \{a, b, d\}, \{c, d\}, \{b, c\}, \{a, c\}, \{d\}, \{b\}, \{\}\}$

YuleDissimilarity

`YuleDissimilarity[`*u*`, `*v*`]`
: returns the Yule dissimilarity between the two boolean 1-D lists *u* and *v*, which is defined as R / (c_tt * c_ff + R / 2) where n is len(*u*), c_ij is the number of occurrences of *u*[k]=i and *v*[k]=j for k<n, and R = 2 * c_tf * c_ft.

>> **`YuleDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]`**

$\frac{6}{5}$

43. Division-Related Functions

Contents

CoprimeQ

`CoprimeQ[`*x*`, `*y*`]`
: tests whether *x* and *y* are coprime by computing their greatest common divisor.

```
>> CoprimeQ[7, 9]
   True
>> CoprimeQ[-4, 9]
   True
>> CoprimeQ[12, 15]
   False
```

CoprimeQ also works for complex numbers

```
>> CoprimeQ[1+2I, 1-I]
   True
>> CoprimeQ[4+2I, 6+3I]
   True
>> CoprimeQ[2, 3, 5]
   True
>> CoprimeQ[2, 4, 5]
   False
```

EvenQ

`EvenQ[`*x*`]`
: returns `True` if *x* is even, and `False` otherwise.

```
>> EvenQ[4]
   True
>> EvenQ[-3]
   False
>> EvenQ[n]
   False
```

GCD

`GCD[`*n1*`, `*n2*`, ...]`
: computes the greatest common divisor of the given integers.

```
>> GCD[20, 30]
   10
>> GCD[10, y]
```
$\text{GCD}[10, y]$

`GCD` is `Listable`:

```
>> GCD[4, {10, 11, 12, 13, 14}]
```
$\{2, 1, 4, 1, 2\}$

`GCD` does not work for rational numbers and Gaussian integers yet.

LCM

`LCM[`*n1*`, `*n2*`, ...]`
: computes the least common multiple of the given integers.

```
>> LCM[15, 20]
60
```

```
>> LCM[20, 30, 40, 50]
600
```

Mod

`Mod[x, m]`
returns x modulo m.

```
>> Mod[14, 6]
2
```

```
>> Mod[-3, 4]
1
```

```
>> Mod[-3, -4]
-3
```

```
>> Mod[5, 0]
Theargument0shouldbenonzero.
Mod[5, 0]
```

OddQ

`OddQ[x]`
returns `True` if x is odd, and `False` otherwise.

```
>> OddQ[-3]
True
```

```
>> OddQ[0]
False
```

PowerMod

`PowerMod[x, y, m]`
computes x^y modulo m.

```
>> PowerMod[2, 10000000, 3]
1
```

```
>> PowerMod[3, -2, 10]
9
```

```
>> PowerMod[0, -1, 2]
0isnotinvertiblemodulo2.
PowerMod[0, -1, 2]
```

```
>> PowerMod[5, 2, 0]
Theargument0shouldbenonzero.
PowerMod[5, 2, 0]
```

`PowerMod` does not support rational coefficients (roots) yet.

PrimeQ

`PrimeQ[n]`
returns `True` if n is a prime number.

For very large numbers, `PrimeQ` uses probabilistic prime testing, so it might be wrong sometimes (a number might be composite even though `PrimeQ` says it is prime). The algorithm might be changed in the future.

```
>> PrimeQ[2]
True
```

```
>> PrimeQ[-3]
True
```

```
>> PrimeQ[137]
True
```

```
>> PrimeQ[2 ^ 127 - 1]
True
```

All prime numbers between 1 and 100:

```
>> Select[Range[100], PrimeQ]
{2, 3, 5, 7, 11, 13, 17, 19, 23,
  29, 31, 37, 41, 43, 47, 53, 59,
  61, 67, 71, 73, 79, 83, 89, 97}
```

`PrimeQ` has attribute `Listable`:

```
>> PrimeQ[Range[20]]
{False, True, True, False, True,
  False, True, False, False, False,
  True, False, True, False, False,
  False, True, False, True, False}
```

Quotient

`Quotient[m, n]`
: computes the integer quotient of *m* and *n*.

```
>> Quotient[23, 7]
```
3

QuotientRemainder

`QuotientRemainder[m, n]`
: computes a list of the quotient and remainder from division of *m* by *n*.

```
>> QuotientRemainder[23, 7]
```
$\{3, 2\}$

44. Recurrence and Sum Functions

A recurrence relation is an equation that recursively defines a sequence or multidimensional array of values, once one or more initial terms are given; each further term of the sequence or array is defined as a function of the preceding terms.

Contents

Fibonacci

`Fibonacci[n]`
: computes the nth Fibonacci number.

```
>> Fibonacci[0]
0
>> Fibonacci[1]
1
>> Fibonacci[10]
55
>> Fibonacci[200]
280 571 172 992 510 140 037~
~611 932 413 038 677 189 525
```

HarmonicNumber

`HarmonicNumber[n]`
: returns the nth harmonic number.

```
>> Table[HarmonicNumber[n], {n, 8}]
```

$$\left\{1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, \frac{137}{60}, \frac{49}{20}, \frac{363}{140}, \frac{761}{280}\right\}$$

```
>> HarmonicNumber[3.8]
2.03806
```

StirlingS1

`StirlingS1[n, m]`
: gives the Stirling number of the first kind $ _n^m$.

Integer mathematical function, suitable for both symbolic and numerical manipulation. gives the number of permutations of n elements that contain exactly m cycles.

```
>> StirlingS1[50, 1]
-608 281 864 034 267 560 872~
~252 163 321 295 376 887 552~
~831 379 210 240 000 000 000
```

StirlingS2

`StirlingS2[n, m]`
: gives the Stirling number of the second kind _n^m.

returns the number of ways of partitioning a set of n elements into m non empty subsets.

```
>> Table[StirlingS2[10, m], {m,
10}]
{1, 511, 9 330, 34 105, 42 525
, 22 827, 5 880, 750, 45, 1}
```

45. List Functions

S-Expressions make up a core part of Mathics. The parsed and internal representation of an an S-Expression is nothing more than a list with possibliy nested elements.
As a result, there about a hundred list functions.

46. Constructing Lists

Functions for constructing lists of various sizes and structure.

Contents

Array

`Array[`*f*`, `*n*`]`
: returns the *n*-element list `{`*f*`[1], ..., `*f*`[`*n*`]}`.

`Array[`*f*`, `*n*`, `*a*`]`
: returns the *n*-element list `{`*f*`[`*a*`], ..., `*f*`[`*a* `+` *n*`]}`.

`Array[`*f*`, {`*n*`, `*m*`}, {`*a*`, `*b*`}]`
: returns an *n*-by-*m* matrix created by applying *f* to indices ranging from `(`*a*`, `*b*`)` to `(`*a* `+` *n*`, `*b* `+` *m*`)`.

`Array[`*f*`, `*dims*`, `*origins*`, `*h*`]`
: returns an expression with the specified dimensions and index origins, with head *h* (instead of `List`).

```
>> Array[f, 4]
```
$\{f[1], f[2], f[3], f[4]\}$

```
>> Array[f, {2, 3}]
```
$\{\{f[1,1], f[1,2], f[1,3]\}, \{f[2,1], f[2,2], f[2,3]\}\}$

```
>> Array[f, {2, 3}, 3]
```
$\{\{f[3,3], f[3,4], f[3,5]\}, \{f[4,3], f[4,4], f[4,5]\}\}$

```
>> Array[f, {2, 3}, {4, 6}]
```
$\{\{f[4,6], f[4,7], f[4,8]\}, \{f[5,6], f[5,7], f[5,8]\}\}$

```
>> Array[f, {2, 3}, 1, Plus]
```
$f[1,1] + f[1,2] + f[1,3] + f[2,1] + f[2,2] + f[2,3]$

ConstantArray

`ConstantArray[`*expr*`, `*n*`]`
: returns a list of *n* copies of *expr*.

```
>> ConstantArray[a, 3]
```
$\{a, a, a\}$

```
>> ConstantArray[a, {2, 3}]
```
$\{\{a, a, a\}, \{a, a, a\}\}$

Normal

`Normal[expr_]`
: Brings especial expressions to a normal expression from different especial forms.

Permutations

`Permutations[`*list*`]`
: gives all possible orderings of the items in *list*.

`Permutations[`*list*`, `*n*`]`
: gives permutations up to length *n*.

`Permutations[`*list*`, {`*n*`}]`
: gives permutations of length *n*.

```
>> Permutations[{y, 1, x}]
```
$\{\{y, 1, x\}, \{y, x, 1\}, \{1, y, x\}, \{1, x, y\}, \{x, y, 1\}, \{x, 1, y\}\}$

Elements are differentiated by their position in *list*, not their value.

```
>> Permutations[{a, b, b}]
```
$\{\{a,b,b\},\{a,b,b\},\{b,a,b\},\{b,b,a\},\{b,a,b\},\{b,b,a\}\}$

```
>> Permutations[{1, 2, 3}, 2]
```
$\{\{\},\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,1\},\{2,3\},\{3,1\},\{3,2\}\}$

```
>> Permutations[{1, 2, 3}, {2}]
```
$\{\{1,2\},\{1,3\},\{2,1\},\{2,3\},\{3,1\},\{3,2\}\}$

Range

`Range[`*n*`]`
: returns a list of integers from 1 to *n*.

`Range[`*a*`, `*b*`]`
: returns a list of integers from *a* to *b*.

```
>> Range[5]
```
$\{1,2,3,4,5\}$

```
>> Range[-3, 2]
```
$\{-3,-2,-1,0,1,2\}$

```
>> Range[0, 2, 1/3]
```
$\left\{0,\frac{1}{3},\frac{2}{3},1,\frac{4}{3},\frac{5}{3},2\right\}$

Reap

`Reap[`*expr*`]`
: gives the result of evaluating *expr*, together with all values sown during this evaluation. Values sown with different tags are given in different lists.

`Reap[`*expr*`, `*pattern*`]`
: only yields values sown with a tag matching *pattern*. `Reap[`*expr*`]` is equivalent to `Reap[`*expr*`, _]`.

`Reap[`*expr*`, {`*pattern1*`, `*pattern2*`, ...}]`
: uses multiple patterns.

`Reap[`*expr*`, `*pattern*`, `*f*`]`
: applies *f* on each tag and the corresponding values sown in the form *f*`[tag, {e1, e2, ...}]`.

```
>> Reap[Sow[3]; Sow[1]]
```
$\{1,\{\{3,1\}\}\}$

```
>> Reap[Sow[2, {x, x, x}]; Sow[3, x]; Sow[4, y]; Sow[4, 1], {_Symbol, _Integer, x}, f]
```
$\{4,\{\{f[x,\{2,2,2,3\}],f[y,\{4\}]\},\{f[1,\{4\}]\},\{f[x,\{2,2,2,3\}]\}\}\}$

Find the unique elements of a list, keeping their order:

```
>> Reap[Sow[Null, {a, a, b, d, c, a}], _, # &][[2]]
```
$\{a,b,d,c\}$

Sown values are reaped by the innermost matching `Reap`:

```
>> Reap[Reap[Sow[a, x]; Sow[b, 1], _Symbol, Print["Inner: ", #1]&];, _, f]
```
Inner : x

$\{\mathrm{Null},\{f[1,\{b\}]\}\}$

When no value is sown, an empty list is returned:

```
>> Reap[x]
```
$\{x,\{\}\}$

Sow

`Sow[`*e*`]`
: sends the value *e* to the innermost `Reap`.

`Sow[`*e*`, `*tag*`]`
: sows *e* using *tag*. `Sow[`*e*`]` is equivalent to `Sow[`*e*`, Null]`.

`Sow[`*e*`, {`*tag1*`, `*tag2*`, ...}]`
: uses multiple tags.

Table

`Table[`*expr*`, `*n*`]`
: generates a list of *n* copies of *expr*.

`Table[`*expr*`, {`*i*`, `*n*`}]`
: generates a list of the values of expr when *i* runs from 1 to *n*.

`Table[`*expr*`, {`*i*`, `*start*`, `*stop*`, `*step*`}]`
: evaluates *expr* with *i* ranging from *start* to *stop*, incrementing by *step*.

`Table[`*expr*`, {`*i*`, {`*e1*`, `*e2*`, ..., `*ei*`}}]`
: evaluates *expr* with *i* taking on the values *e1*, *e2*, ..., *ei*.

```
>> Table[x, 3]
```
$\{x, x, x\}$

```
>> n = 0; Table[n = n + 1, {5}]
```
$\{1, 2, 3, 4, 5\}$

```
>> Table[i, {i, 4}]
```
$\{1, 2, 3, 4\}$

```
>> Table[i, {i, 2, 5}]
```
$\{2, 3, 4, 5\}$

```
>> Table[i, {i, 2, 6, 2}]
```
$\{2, 4, 6\}$

```
>> Table[i, {i, Pi, 2 Pi, Pi / 2}]
```
$$\left\{\mathrm{Pi}, \frac{3\mathrm{Pi}}{2}, 2\mathrm{Pi}\right\}$$

```
>> Table[x^2, {x, {a, b, c}}]
```
$$\left\{a^2, b^2, c^2\right\}$$

`Table` supports multi-dimensional tables:

```
>> Table[{i, j}, {i, {a, b}}, {j,
   1, 2}]
```
$\{\{\{a, 1\}, \{a, 2\}\}, \{\{b, 1\}, \{b, 2\}\}\}$

Tuples

`Tuples[`*list*`, `*n*`]`
: returns a list of all *n*-tuples of elements in *list*.

`Tuples[{`*list1*`, `*list2*`, ...}]`
: returns a list of tuples with elements from the given lists.

```
>> Tuples[{a, b, c}, 2]
```
$\{\{a, a\}, \{a, b\}, \{a, c\}, \{b, a\}, \{b, b\}, \{b, c\}, \{c, a\}, \{c, b\}, \{c, c\}\}$

```
>> Tuples[{}, 2]
```
$\{\}$

```
>> Tuples[{a, b, c}, 0]
```
$\{\{\}\}$

```
>> Tuples[{{a, b}, {1, 2, 3}}]
```
$\{\{a, 1\}, \{a, 2\}, \{a, 3\}, \{b, 1\}, \{b, 2\}, \{b, 3\}\}$

The head of *list* need not be `List`:

```
>> Tuples[f[a, b, c], 2]
```
$\{f[a, a], f[a, b], f[a, c], f[b, a], f[b, b], f[b, c], f[c, a], f[c, b], f[c, c]\}$

However, when specifying multiple expressions, `List` is always used:

```
>> Tuples[{f[a, b], g[c, d]}]
```
$\{\{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\}$

47. Elements of Lists

Functions for accessing elements of lists using either indices, positions, or patterns of criteria.

Contents

Append

`Append[`*expr*`, `*elem*`]`
: returns *expr* with *elem* appended.

```
>> Append[{1, 2, 3}, 4]
```
$\{1,2,3,4\}$

`Append` works on expressions with heads other than `List`:

```
>> Append[f[a, b], c]
```
$f[a,b,c]$

Unlike `Join`, `Append` does not flatten lists in *item*:

```
>> Append[{a, b}, {c, d}]
```
$\{a,b,\{c,d\}\}$

AppendTo

`AppendTo[`*s*`, `*item*`]`
: append *item* to value of *s* and sets *s* to the result.

```
>> s = {};
```

```
>> AppendTo[s, 1]
```
$\{1\}$

```
>> s
```
$\{1\}$

`Append` works on expressions with heads other than `List`:

```
>> y = f[];
```

```
>> AppendTo[y, x]
```
$f[x]$

```
>> y
```
$f[x]$

Cases

`Cases[`*list*`, `*pattern*`]`
: returns the elements of *list* that match *pattern*.

`Cases[`*list*`, `*pattern*`, `*ls*`]`
: returns the elements matching at levelspec *ls*.

`Cases[`*list*`, `*pattern*`, Heads->`*bool*`]`
: Match including the head of the expression in the search.

```
>> Cases[{a, 1, 2.5, "string"},
   _Integer|_Real]
```
$\{1,2.5\}$

```
>> Cases[_Complex][{1, 2I, 3, 4-I,
   5}]
```

$\{2I, 4 - I\}$

Find symbols among the elements of an expression:

```
>> Cases[{b, 6, \[Pi]}, _Symbol]
```

$\{b, \mathrm{Pi}\}$

Also include the head of the expression in the previous search:

```
>> Cases[{b, 6, \[Pi]}, _Symbol,
   Heads -> True]
```

$\{\mathrm{List}, b, \mathrm{Pi}\}$

Count

`Count[`*list*`, `*pattern*`]`
: returns the number of times *pattern* appears in *list*.

`Count[`*list*`, `*pattern*`, `*ls*`]`
: counts the elements matching at levelspec *ls*.

```
>> Count[{3, 7, 10, 7, 5, 3, 7,
   10}, 3]
```

2

```
>> Count[{{a, a}, {a, a, a}, a}, a,
    {2}]
```

5

DeleteCases

`DeleteCases[`*list*`, `*pattern*`]`
: returns the elements of *list* that do not match *pattern*.

`DeleteCases[`*list*`, `*pattern*`, `*levelspec*`]`
: removes all parts of $list on levels specified by *levelspec* that match pattern (not fully implemented).

`DeleteCases[`*list*`, `*pattern*`, `*levelspec*`, `*n*`]`
: removes the first *n* parts of *list* that match *pattern*.

```
>> DeleteCases[{a, 1, 2.5, "string
   "}, _Integer|_Real]
```

$\{a, \mathrm{string}\}$

```
>> DeleteCases[{a, b, 1, c, 2, 3},
   _Symbol]
```

$\{1, 2, 3\}$

Drop

`Drop[`*expr*`, `*n*`]`
: returns *expr* with the first *n* leaves removed.

```
>> Drop[{a, b, c, d}, 3]
```

$\{d\}$

```
>> Drop[{a, b, c, d}, -2]
```

$\{a, b\}$

```
>> Drop[{a, b, c, d, e}, {2, -2}]
```

$\{a, e\}$

Drop a submatrix:

```
>> A = Table[i*10 + j, {i, 4}, {j,
   4}]
```

$\{\{11, 12, 13, 14\}, \{21, 22, 23, 24\}, \{31, 32, 33, 34\}, \{41, 42, 43, 44\}\}$

```
>> Drop[A, {2, 3}, {2, 3}]
```

$\{\{11, 14\}, \{41, 44\}\}$

Extract

`Extract[`*expr*`, `*list*`]`
: extracts parts of *expr* specified by *list*.

`Extract[`*expr*`, {`*list1*`, `*list2*`, ...}]`
: extracts a list of parts.

`Extract[`*expr*`, `*i*`, `*j*`, ...]` is equivalent to `Part[`*expr*`, {`*i*`, `*j*`, ...}]`.

```
>> Extract[a + b + c, {2}]
```

b

```
>> Extract[{{a, b}, {c, d}}, {{1},
   {2, 2}}]
```

$\{\{a, b\}, d\}$

First

`First`[*expr*]
: returns the first element in *expr*.

`First`[*expr*] is equivalent to *expr*`[[1]]`.

```
>> First[{a, b, c}]
```
a

```
>> First[a + b + c]
```
a

```
>> First[x]
```
Nonatomicexpressionexpected.

First[*x*]

FirstCase

FirstCase[{*e1*, *e2*, ...}, *pattern*]
: gives the first *ei* to match *pattern*, or $Missing["NotFound"]$ if none matching pattern is found.

FirstCase[{*e1*,*e2*, ...}, *pattern* -> *rhs*]
: gives the value of *rhs* corresponding to the first *ei* to match pattern.

FirstCase[*expr*, *pattern*, *default*]
: gives *default* if no element matching *pattern* is found.

FirstCase[*expr*, *pattern*, *default*, *levelspec*]
: finds only objects that appear on levels specified by *levelspec*.

FirstCase[*pattern*]
: represents an operator form of FirstCase that can be applied to an expression.

FirstPosition

`FirstPosition`[*expr*, *pattern*]
: gives the position of the first element in *expr* that matches *pattern*, or Missing["NotFound"] if no such element is found.

`FirstPosition`[*expr*, *pattern*, *default*]
: gives default if no element matching *pattern* is found.

`FirstPosition`[*expr*, *pattern*, *default*, *levelspec*]
: finds only objects that appear on levels specified by *levelspec*.

```
>> FirstPosition[{a, b, a, a, b, c,
    b}, b]
```
$\{2\}$

```
>> FirstPosition[{{a, a, b}, {b, a,
    a}, {a, b, a}}, b]
```
$\{1,3\}$

```
>> FirstPosition[{x, y, z}, b]
```
Missing[NotFound]

Find the first position at which x^2 to appears:

```
>> FirstPosition[{1 + x^2, 5, x^4,
   a + (1 + x^2)^2}, x^2]
```
$\{1,2\}$

Last

`Last`[*expr*]
: returns the last element in *expr*.

`Last`[*expr*] is equivalent to *expr*`[[-1]]`.

```
>> Last[{a, b, c}]
```
c

```
>> Last[x]
```
Nonatomicexpressionexpected.

Last[*x*]

Length

`Length`[*expr*]
: returns the number of leaves in *expr*.

Length of a list:

```
>> Length[{1, 2, 3}]
```
3

`Length` operates on the `FullForm` of expressions:

```
>> Length[Exp[x]]
```
2

```
>> FullForm[Exp[x]]
```
Power[*E*, *x*]

The length of atoms is 0:

```
>> Length[a]
```
0

Note that rational and complex numbers are atoms, although their `FullForm` might suggest the opposite:

```
>> Length[1/3]
   0

>> FullForm[1/3]
   Rational[1,3]
```

MemberQ

`MemberQ[list, pattern]`
: returns `True` if *pattern* matches any element of *list*, or `False` otherwise.

```
>> MemberQ[{a, b, c}, b]
   True

>> MemberQ[{a, b, c}, d]
   False

>> MemberQ[{"a", b, f[x]}, _?
   NumericQ]
   False

>> MemberQ[_List][{{}}]
   True
```

Most

`Most[expr]`
: returns *expr* with the last element removed.

`Most[expr]` is equivalent to `expr[[;;-2]]`.

```
>> Most[{a, b, c}]
   {a,b}

>> Most[a + b + c]
   a + b

>> Most[x]
   Nonatomicexpressionexpected.
   Most[x]
```

Part

`Part[expr, i]`
: returns part *i* of *expr*.

Extract an element from a list:

```
>> A = {a, b, c, d};

>> A[[3]]
   c
```

Negative indices count from the end:

```
>> {a, b, c}[[-2]]
   b
```

`Part` can be applied on any expression, not necessarily lists.

```
>> (a + b + c)[[2]]
   b
```

`expr[[0]]` gives the head of *expr*:

```
>> (a + b + c)[[0]]
   Plus
```

Parts of nested lists:

```
>> M = {{a, b}, {c, d}};

>> M[[1, 2]]
   b
```

You can use `Span` to specify a range of parts:

```
>> {1, 2, 3, 4}[[2;;4]]
   {2,3,4}

>> {1, 2, 3, 4}[[2;;-1]]
   {2,3,4}
```

A list of parts extracts elements at certain indices:

```
>> {a, b, c, d}[[{1, 3, 3}]]
   {a,c,c}
```

Get a certain column of a matrix:

```
>> B = {{a, b, c}, {d, e, f}, {g, h
   , i}};

>> B[[;;, 2]]
   {b,e,h}
```

Extract a submatrix of 1st and 3rd row and the two last columns:

```
>> B = {{1, 2, 3}, {4, 5, 6}, {7,
   8, 9}};
```

```
>> B[[{1, 3}, -2;;-1]]
```
$\{\{2,3\},\{8,9\}\}$

The 3d column of a matrix:

```
>> {{a, b, c}, {d, e, f}, {g, h, i
   }}[[All, 3]]
```
$\{c,f,i\}$

Further examples:

```
>> (a+b+c+d)[[-1;;-2]]
```
0

```
>> x[[2]]
```
Partspecificationislongerthandepthofobject.

$x[[2]]$

Assignments to parts are possible:

```
>> B[[;;, 2]] = {10, 11, 12}
```
$\{10,11,12\}$

```
>> B
```
$\{\{1,10,3\},\{4,11,6\},\{7,12,9\}\}$

```
>> B[[;;, 3]] = 13
```
13

```
>> B
```
$\{\{1,10,13\},\{4,11,13\},\{7,12,13\}\}$

```
>> B[[1;;-2]] = t;
```

```
>> B
```
$\{t,t,\{7,12,13\}\}$

```
>> F = Table[i*j*k, {i, 1, 3}, {j,
   1, 3}, {k, 1, 3}];
```

```
>> F[[;; All, 2 ;; 3, 2]] = t;
```

```
>> F
```
$\{\{\{1,2,3\},\{2,t,6\},\{3,t,9\}\},\{\{2,4,6\},\{4,t,12\},\{6,t,18\}\},\{\{3,6,9\},\{6,t,18\},\{9,t,27\}\}\}$

```
>> F[[;; All, 1 ;; 2, 3 ;; 3]] = k;
```

```
>> F
```
$\{\{\{1,2,k\},\{2,t,k\},\{3,t,9\}\},\{\{2,4,k\},\{4,t,k\},\{6,t,18\}\},\{\{3,6,k\},\{6,t,k\},\{9,t,27\}\}\}$

Of course, part specifications have precedence over most arithmetic operations:

```
>> A[[1]] + B[[2]] + C[[3]] // Hold
   // FullForm
```
Hold [Plus [Part [A, 1], Part [B, 2], Part [C, 3]]]

Pick

`Pick[`*list*, *sel*`]`
: returns those items in *list* that are True in *sel*.

`Pick[`*list*, *sel*, *patt*`]`
: returns those items in *list* that match *patt* in *sel*.

```
>> Pick[{a, b, c}, {False, True,
   False}]
```
$\{b\}$

```
>> Pick[f[g[1, 2], h[3, 4]], {{True
   , False}, {False, True}}]
```
$f[g[1],h[4]]$

```
>> Pick[{a, b, c, d, e}, {1, 2,
   3.5, 4, 5.5}, _Integer]
```
$\{a,b,d\}$

Prepend

`Prepend[`*expr*, *item*`]`
: returns *expr* with *item* prepended to its leaves.

`Prepend[`*expr*`]`
: `Prepend[`*elem*`][`*expr*`]` is equivalent to `Prepend[`*expr*,*elem*`]`.

`Prepend` is similar to `Append`, but adds *item* to the beginning of *expr*:

```
>> Prepend[{2, 3, 4}, 1]
```
$\{1,2,3,4\}$

`Prepend` works on expressions with heads other than `List`:

```
>> Prepend[f[b, c], a]
```
$f[a,b,c]$

Unlike `Join`, `Prepend` does not flatten lists in *item*:

```
>> Prepend[{c, d}, {a, b}]
```
$\{\{a,b\},c,d\}$

PrependTo

`PrependTo[`*s*`, `*item*`]`
prepends *item* to value of *s* and sets *s* to the result.

Assign s to a list
```
>> s = {1, 2, 4, 9}
```
$\{1,2,4,9\}$

Add a new value at the beginning of the list:
```
>> PrependTo[s, 0]
```
$\{0,1,2,4,9\}$

The value assigned to s has changed:
```
>> s
```
$\{0,1,2,4,9\}$

`PrependTo` works with a head other than `List`:
```
>> y = f[a, b, c];
```

```
>> PrependTo[y, x]
```
$f[x,a,b,c]$

```
>> y
```
$f[x,a,b,c]$

ReplacePart

`ReplacePart[`*expr*`, `*i*` -> `*new*`]`
replaces part *i* in *expr* with *new*.
`ReplacePart[`*expr*`, {{`*i*`, `*j*`} -> `*e1*`, {`*k*`, `*l*`} -> `*e2*`}]`
replaces parts *i* and *j* with *e1*, and parts *k* and *l* with *e2*.

```
>> ReplacePart[{a, b, c}, 1 -> t]
```
$\{t,b,c\}$

```
>> ReplacePart[{{a, b}, {c, d}}, {2, 1} -> t]
```
$\{\{a,b\},\{t,d\}\}$

```
>> ReplacePart[{{a, b}, {c, d}}, {{2, 1} -> t, {1, 1} -> t}]
```
$\{\{t,b\},\{t,d\}\}$

```
>> ReplacePart[{a, b, c}, {{1}, {2}} -> t]
```
$\{t,t,c\}$

Delayed rules are evaluated once for each replacement:
```
>> n = 1;
```

```
>> ReplacePart[{a, b, c, d}, {{1}, {3}} :> n++]
```
$\{1,b,2,d\}$

Non-existing parts are simply ignored:
```
>> ReplacePart[{a, b, c}, 4 -> t]
```
$\{a,b,c\}$

You can replace heads by replacing part 0:
```
>> ReplacePart[{a, b, c}, 0 -> Times]
```
abc

(This is equivalent to `Apply`.)
Negative part numbers count from the end:
```
>> ReplacePart[{a, b, c}, -1 -> t]
```
$\{a,b,t\}$

Rest

`Rest[`*expr*`]`
returns *expr* with the first element removed.

`Rest[`*expr*`]` is equivalent to *expr*`[[2;;]]`.
```
>> Rest[{a, b, c}]
```
$\{b,c\}$

```
>> Rest[a + b + c]
```
$b+c$

```
>> Rest[x]
```
Nonatomicexpressionexpected.

Rest[x]

Select

`Select[{e1, e2, ...}, f]`
: returns a list of the elements *ei* for which *f*[*ei*] returns `True`.

Find numbers greater than zero:

```
>> Select[{-3, 0, 1, 3, a}, #>0&]
   {1,3}
```

`Select` works on an expression with any head:

```
>> Select[f[a, 2, 3], NumberQ]
   f[2,3]
```

```
>> Select[a, True]
   Nonatomicexpressionexpected.
   Select[a, True]
```

Span (;;)

`Span`
: is the head of span ranges like `1;;3`.

```
>> ;; // FullForm
   Span[1, All]
```

```
>> 1;;4;;2 // FullForm
   Span[1, 4, 2]
```

```
>> 2;;-2 // FullForm
   Span[2, -2]
```

```
>> ;;3 // FullForm
   Span[1, 3]
```

Take

`Take[expr, n]`
: returns *expr* with all but the first *n* leaves removed.

```
>> Take[{a, b, c, d}, 3]
   {a,b,c}
```

```
>> Take[{a, b, c, d}, -2]
   {c,d}
```

```
>> Take[{a, b, c, d, e}, {2, -2}]
   {b,c,d}
```

Take a submatrix:

```
>> A = {{a, b, c}, {d, e, f}};
```

```
>> Take[A, 2, 2]
   {{a,b}, {d,e}}
```

Take a single column:

```
>> Take[A, All, {2}]
   {{b}, {e}}
```

48. Integer and Number-Theoretical Functions

49. Algebraic Manipulation

Contents

Apart

`Apart[`*expr*`]`
: writes *expr* as a sum of individual fractions.

`Apart[`*expr*`, `*var*`]`
: treats *var* as the main variable.

```
>> Apart[1 / (x^2 + 5x + 6)]
```

$$\frac{1}{2+x}-\frac{1}{3+x}$$

When several variables are involved, the results can be different depending on the main variable:

```
>> Apart[1 / (x^2 - y^2), x]
```

$$-\frac{1}{2y\left(x+y\right)}+\frac{1}{2y\left(x-y\right)}$$

```
>> Apart[1 / (x^2 - y^2), y]
```

$$\frac{1}{2x\left(x+y\right)}+\frac{1}{2x\left(x-y\right)}$$

`Apart` is `Listable`:

```
>> Apart[{1 / (x^2 + 5x + 6)}]
```

$$\left\{\frac{1}{2+x}-\frac{1}{3+x}\right\}$$

But it does not touch other expressions:

```
>> Sin[1 / (x ^ 2 - y ^ 2)] //
   Apart
```

$$\text{Sin}\left[\frac{1}{x^2-y^2}\right]$$

Cancel

`Cancel[`*expr*`]`
: cancels out common factors in numerators and denominators.

```
>> Cancel[x / x ^ 2]
```

$$\frac{1}{x}$$

`Cancel` threads over sums:

```
>> Cancel[x / x ^ 2 + y / y ^ 2]
```

$$\frac{1}{x}+\frac{1}{y}$$

```
>> Cancel[f[x] / x + x * f[x] / x ^
   2]
```

$$\frac{2f\left[x\right]}{x}$$

Coefficient

`Coefficient[expr, form]`
: returns the coefficient of *form* in the polynomial *expr*.

`Coefficient[expr, form, n]`
: return the coefficient of *form*^*n* in *expr*.

```
>> Coefficient[(x + y)^4, (x^2)* (y
^2)]
```
6

```
>> Coefficient[a x^2 + b y^3 + c x
+ d y + 5, x]
```
c

```
>> Coefficient[(x + 3 y)^5, x]
```
$405y^4$

```
>> Coefficient[(x + 3 y)^5, x * y
^4]
```
405

```
>> Coefficient[(x + 2)/(y - 3)+ (x
+ 3)/(y - 2), x]
```
$$\frac{1}{-3+y}+\frac{1}{-2+y}$$

```
>> Coefficient[x*Cos[x + 3] + 6*y,
x]
```
$\mathrm{Cos}[3+x]$

```
>> Coefficient[(x + 1)^3, x, 2]
```
3

```
>> Coefficient[a x^2 + b y^3 + c x
+ d y + 5, y, 3]
```
b

Find the free term in a polynomial:

```
>> Coefficient[(x + 2)^3 + (x + 3)
^2, x, 0]
```
17

```
>> Coefficient[(x + 2)^3 + (x + 3)
^2, y, 0]
```
$(2+x)^3+(3+x)^2$

```
>> Coefficient[a x^2 + b y^3 + c x
+ d y + 5, x, 0]
```
$5+by^3+dy$

CoefficientArrays

`CoefficientArrays[`*polys*`, `*vars*`]`
returns a list of arrays of coefficients of the variables *vars* in the polynomial *poly*.

```
>> CoefficientArrays[1 + x^3, x]
```
{1, {0}, {{0}}, {{{1}}}}

```
>> CoefficientArrays[1 + x y+ x^3,
{x, y}]
```
{1, {0,0}, {{0,1}, {0,0}}, {{{1,
0}, {0,0}}, {{0,0}, {0,0}}}}

```
>> CoefficientArrays[{1 + x^2, x y
}, {x, y}]
```
{{1,0}, {{0,0}, {0,0}}, {{{1,
0}, {0,0}}, {{0,1}, {0,0}}}}

```
>> CoefficientArrays[(x+y+Sin[z])
^3, {x,y}]
```
$\{\mathrm{Sin}[z]^3, \{3\mathrm{Sin}[z]^2, 3\mathrm{Sin}[z]^2\}, \{\{3\mathrm{Sin}[z], 6\mathrm{Sin}[z]\}, \{0, 3\mathrm{Sin}[z]\}\}, \{\{\{1,3\}, \{0,3\}\}, \{\{0,0\}, \{0,1\}\}\}\}$

```
>> CoefficientArrays[(x + y + Sin[z
])^3, {x, z}]
```
$(x+y+Sin[z])^3 isnotapolynomialin\{x,z\}$

$\mathrm{CoefficientArrays}\left[(x+y+\mathrm{Sin}[z])^3, \{x,z\}\right]$

CoefficientList

`CoefficientList[poly, var]`
returns a list of coefficients of powers of *var* in *poly*, starting with power 0.

`CoefficientList[poly, {var1, var2, ...}]`
returns an array of coefficients of the *vari*.

```
>> CoefficientList[(x + 3)^5, x]
```
{243, 405, 270, 90, 15, 1}

```
>> CoefficientList[(x + y)^4, x]
```
$\{y^4, 4y^3, 6y^2, 4y, 1\}$

```
>> CoefficientList[a x^2 + b y^3 +
c x + d y + 5, x]
```
$\{5+by^3+dy, c, a\}$

```
>> CoefficientList[(x + 2)/(y - 3)+
   x/(y - 2), x]
```

$$\left\{\frac{2}{-3+y}, \frac{1}{-3+y}+\frac{1}{-2+y}\right\}$$

```
>> CoefficientList[(x + y)^3, z]
```

$$\left\{(x+y)^3\right\}$$

```
>> CoefficientList[a x^2 + b y^3 +
   c x + d y + 5, {x, y}]
```

$\{\{5, d, 0, b\}, \{c, 0, 0, 0\}, \{a, 0, 0, 0\}\}$

```
>> CoefficientList[(x - 2 y + 3 z)
   ^3, {x, y, z}]
```

$\{\{\{0,0,0,27\}, \{0,0,-54,0\}, \{0,36,0,0\}, \{-8,0,0,0\}\}, \{\{0,0,27,0\}, \{0,-36,0,0\}, \{12,0,0,0\}, \{0,0,0,0\}\}, \{\{0,9,0,0\}, \{-6,0,0,0\}, \{0,0,0,0\}, \{0,0,0,0\}\}, \{\{1,0,0,0\}, \{0,0,0,0\}, \{0,0,0,0\}, \{0,0,0,0\}\}\}$

Collect

`Collect[`*expr*`, `*x*`]`
: Expands *expr* and collect together terms having the same power of *x*.

`Collect[`*expr*`, {`*x_1*`, `*x_2*`, ...}]`
: Expands *expr* and collect together terms having the same powers of *x_1*, *x_2*, ...

`Collect[`*expr*`, {`*x_1*`, `*x_2*`, ...}, `*filter*`]`
: After collect the terms, applies *filter* to each coefficient.

```
>> Collect[(x+y)^3, y]
```

$x^3 + 3x^2y + 3xy^2 + y^3$

```
>> Collect[2 Sin[x z] (x+2 y^2 +
   Sin[y] x), y]
```

$2x\mathrm{Sin}[xz] + 2x\mathrm{Sin}[xz]\,\mathrm{Sin}[y] + 4y^2\mathrm{Sin}[xz]$

```
>> Collect[3 x y+2 Sin[x z] (x+2 y
   ^2 + x)+ (x+y)^3, y]
```

$4x\mathrm{Sin}[xz] + x^3 + y\left(3x + 3x^2\right) + y^2\left(3x + 4\mathrm{Sin}[xz]\right) + y^3$

```
>> Collect[3 x y+2 Sin[x z] (x+2 y
   ^2 + x)+ (x+y)^3, {x,y}]
```

$4x\mathrm{Sin}[xz] + x^3 + 3xy + 3x^2y + 4y^2\mathrm{Sin}[xz] + 3xy^2 + y^3$

```
>> Collect[3 x y+2 Sin[x z] (x+2 y
   ^2 + x)+ (x+y)^3, {x,y}, h]
```

$xh[4\mathrm{Sin}[xz]] + x^3h[1] + xyh[3] + x^2yh[3] + y^2h[4\mathrm{Sin}[xz]] + xy^2h[3] + y^3h[1]$

Denominator

`Denominator[`*expr*`]`
: gives the denominator in *expr*.

```
>> Denominator[a / b]
```

b

```
>> Denominator[2 / 3]
```

3

```
>> Denominator[a + b]
```

1

Expand

`Expand[`*expr*`]`
: expands out positive integer powers and products of sums in *expr*, as well as trigonometric identities.

Expand[*expr*, *target*]
: just expands those parts involving *target*.

```
>> Expand[(x + y)^ 3]
```

$x^3 + 3x^2y + 3xy^2 + y^3$

```
>> Expand[(a + b)(a + c + d)]
```

$a^2 + ab + ac + ad + bc + bd$

```
>> Expand[(a + b)(a + c + d)(e + f)
   + e a a]
```

$2a^2e + a^2f + abe + abf + ace + acf + ade + adf + bce + bcf + bde + bdf$

```
>> Expand[(a + b)^ 2 * (c + d)]
```

$a^2c + a^2d + 2abc + 2abd + b^2c + b^2d$

```
>> Expand[(x + y)^ 2 + x y]
```
$x^2 + 3xy + y^2$

```
>> Expand[((a + b)(c + d))^ 2 + b
   (1 + a)]
```
$a^2c^2 + 2a^2cd + a^2d^2 + b + ab + 2abc^2 + 4abcd + 2abd^2 + b^2c^2 + 2b^2cd + b^2d^2$

Expand expands items in lists and rules:

```
>> Expand[{4 (x + y), 2 (x + y)-> 4
    (x + y)}]
```
$\{4x + 4y, 2x + 2y- > 4x + 4y\}$

Expand expands trigonometric identities

```
>> Expand[Sin[x + y], Trig -> True]
```
$\mathrm{Cos}\,[x]\,\mathrm{Sin}\,[y] + \mathrm{Cos}\,[y]\,\mathrm{Sin}\,[x]$

```
>> Expand[Tanh[x + y], Trig -> True
   ]
```
$$\frac{\mathrm{Cosh}\,[x]\,\mathrm{Sinh}\,[y]}{\mathrm{Cosh}\,[x]\,\mathrm{Cosh}\,[y] + \mathrm{Sinh}\,[x]\,\mathrm{Sinh}\,[y]} + \frac{\mathrm{Cosh}\,[y]\,\mathrm{Sinh}\,[x]}{\mathrm{Cosh}\,[x]\,\mathrm{Cosh}\,[y] + \mathrm{Sinh}\,[x]\,\mathrm{Sinh}\,[y]}$$

Expand does not change any other expression.

```
>> Expand[Sin[x (1 + y)]]
```
$\mathrm{Sin}\,[x\,(1 + y)]$

Using the second argument, the expression only expands those subexpressions containing *pat*:

```
>> Expand[(x+a)^2+(y+a)^2+(x+y)(x+a
   ), y]
```
$a^2 + 2ay + x\,(a + x) + y\,(a + x) + y^2 + (a + x)^2$

Expand also works in Galois fields

```
>> Expand[(1 + a)^12, Modulus -> 3]
```
$1 + a^3 + a^9 + a^{12}$

```
>> Expand[(1 + a)^12, Modulus -> 4]
```
$1 + 2a^2 + 3a^4 + 3a^8 + 2a^{10} + a^{12}$

ExpandAll

`ExpandAll[`*expr*`]`
: expands out negative integer powers and products of sums in *expr*.

`ExpandAll[`*expr*`, `*target*`]`
: just expands those parts involving *target*.

```
>> ExpandAll[(a + b)^ 2 / (c + d)
   ^2]
```
$$\frac{a^2}{c^2 + 2cd + d^2} + \frac{2ab}{c^2 + 2cd + d^2} + \frac{b^2}{c^2 + 2cd + d^2}$$

ExpandAll descends into sub expressions

```
>> ExpandAll[(a + Sin[x (1 + y)])
   ^2]
```
$2a\mathrm{Sin}\,[x + xy] + a^2 + \mathrm{Sin}\,[x + xy]^2$

```
>> ExpandAll[Sin[(x+y)^2]]
```
$\mathrm{Sin}\,[x^2 + 2xy + y^2]$

```
>> ExpandAll[Sin[(x+y)^2], Trig->
   True]
```
$-\mathrm{Sin}\,[x^2]\,\mathrm{Sin}\,[2xy]\,\mathrm{Sin}\,[y^2] + \mathrm{Cos}\,[x^2]\,\mathrm{Cos}\,[2xy]\,\mathrm{Sin}\,[y^2] + \mathrm{Cos}\,[x^2]\,\mathrm{Cos}\,[y^2]\,\mathrm{Sin}\,[2xy] + \mathrm{Cos}\,[2xy]\,\mathrm{Cos}\,[y^2]\,\mathrm{Sin}\,[x^2]$

ExpandAll also expands heads

```
>> ExpandAll[((1 + x)(1 + y))[x]]
```
$(1 + x + y + xy)\,[x]$

ExpandAll can also work in finite fields

```
>> ExpandAll[(1 + a)^ 6 / (x + y)
   ^3, Modulus -> 3]
```
$$\frac{1 + 2a^3 + a^6}{x^3 + y^3}$$

ExpandDenominator

`ExpandDenominator[`*expr*`]`
: expands out negative integer powers and products of sums in *expr*.

```
>> ExpandDenominator[(a + b)^ 2 /
   ((c + d)^2 (e + f))]
```
$$\frac{(a + b)^2}{c^2e + c^2f + 2cde + 2cdf + d^2e + d^2f}$$

Exponent

`Exponent[expr, form]`
: returns the maximum power with which *form* appears in the expanded form of *expr*.

`Exponent[expr, form, h]`
: applies *h* to the set of exponents with which *form* appears in *expr*.

```
>> Exponent[5 x^2 - 3 x + 7, x]
```
2

```
>> Exponent[(x^3 + 1)^2 + 1, x]
```
6

```
>> Exponent[x^(n + 1)+ Sqrt[x] + 1,
    x]
```
$\mathrm{Max}\left[\frac{1}{2}, 1+n\right]$

```
>> Exponent[x / y, y]
```
-1

```
>> Exponent[(x^2 + 1)^3 - 1, x, Min
   ]
```
2

```
>> Exponent[0, x]
```
$-\infty$

```
>> Exponent[1, x]
```
0

Factor

`Factor[`*expr*`]`
: factors the polynomial expression *expr*.

```
>> Factor[x ^ 2 + 2 x + 1]
```
$(1+x)^2$

```
>> Factor[1 / (x^2+2x+1)+ 1 / (x
   ^4+2x^2+1)]
```
$$\frac{2+2x+3x^2+x^4}{(1+x)^2\left(1+x^2\right)^2}$$

FactorTermsList

`FactorTermsList[poly]`
: returns a list of 2 elements. The first element is the numerical factor in *poly*. The second one is the remaining of the polynomial with numerical factor removed

`FactorTermsList[poly, {x1, x2, ...}]`
: returns a list of factors in *poly*. The first element is the numerical factor in *poly*. The next ones are factors that are independent of variables lists which are created by removing each variable *xi* from right to left. The last one is the remaining of polynomial after dividing *poly* to all previous factors

```
>> FactorTermsList[2 x^2 - 2]
```
$\left\{2, -1+x^2\right\}$

```
>> FactorTermsList[x^2 - 2 x + 1]
```
$\left\{1, 1-2x+x^2\right\}$

```
>> f = 3 (-1 + 2 x)(-1 + y)(1 - a)
```
$3(-1+2x)\left(-1+y\right)(1-a)$

```
>> FactorTermsList[f]
```
$\{-3, -1+a-2ax-ay$
$+2x+y-2xy+2axy\}$

```
>> FactorTermsList[f, x]
```
$\{-3, 1-a-y+ay, -1+2x\}$

FullSimplify

`FullSimplify[`*expr*`]`
: simplifies *expr* using an extended set of simplification rules.

`FullSimplify[`*expr*`, `*assump*`]`
: simplifies *expr* assuming *assump* instead of *Assumptions*.

TODO: implement the extension. By now, this

does the same than Simplify...

```
>> FullSimplify[2*Sin[x]^2 + 2*Cos[
   x]^2]
```
2

MinimalPolynomial

`MinimalPolynomial[s, x]`
: gives the minimal polynomial in x for which the algebraic number s is a root.

```
>> MinimalPolynomial[7, x]
```
$-7 + x$

```
>> MinimalPolynomial[Sqrt[2] + Sqrt
   [3], x]
```
$1 - 10x^2 + x^4$

```
>> MinimalPolynomial[Sqrt[1 + Sqrt
   [3]], x]
```
$-2 - 2x^2 + x^4$

```
>> MinimalPolynomial[Sqrt[I + Sqrt
   [6]], x]
```
$49 - 10x^4 + x^8$

Numerator

`Numerator[`*expr*`]`
: gives the numerator in *expr*.

```
>> Numerator[a / b]
```
a

```
>> Numerator[2 / 3]
```
2

```
>> Numerator[a + b]
```
$a + b$

PolynomialQ

`PolynomialQ[expr, var]`
: returns True if *expr* is a polynomial in *var*, and returns False otherwise.

`PolynomialQ[expr, {var1, ...}]`
: tests whether *expr* is a polynomial in the *vari*.

```
>> PolynomialQ[x^3 - 2 x/y + 3xz, x
   ]
```
True

```
>> PolynomialQ[x^3 - 2 x/y + 3xz, y
   ]
```
False

```
>> PolynomialQ[f[a] + f[a]^2, f[a]]
```
True

```
>> PolynomialQ[x^2 + axy^2 - bSin[c
   ], {x, y}]
```
True

```
>> PolynomialQ[x^2 + axy^2 - bSin[c
   ], {a, b, c}]
```
False

PowerExpand

`PowerExpand[`*expr*`]`
: expands out powers of the form `(x^y)^z` and `(x*y)^z` in *expr*.

```
>> PowerExpand[(a ^ b)^ c]
```
a^{bc}

```
>> PowerExpand[(a * b)^ c]
```
$a^c b^c$

`PowerExpand` is not correct without certain assumptions:

```
>> PowerExpand[(x ^ 2)^ (1/2)]
```
x

Simplify

`Simplify[`*expr*`]`
: simplifies *expr*.

`Simplify[`*expr*`, `*assump*`]`
: simplifies *expr* assuming *assump* instead of *Assumptions*.

```
>> Simplify[2*Sin[x]^2 + 2*Cos[x
   ]^2]
```
2

```
>> Simplify[x]
```
x

```
>> Simplify[f[x]]
```
$f[x]$

Simplify over conditional expressions uses $Assumptions, or *assump* to evaluate the condition: # TODO: enable this once the logic for conditional expression # be restaured. # » $Assumptions={a <= 0}; # » Simplify[ConditionalExpression[1, a > 0]] # = Undefined # » Simplify[ConditionalExpression[1, a > 0], { a > 0 }] # = 1

Together

`Together[`*expr*`]`
: writes sums of fractions in *expr* together.

```
>> Together[a / c + b / c]
```

$$\frac{a+b}{c}$$

`Together` operates on lists:

```
>> Together[{x / (y+1)+ x / (y+1)
^2}]
```

$$\left\{\frac{x\left(2+y\right)}{\left(1+y\right)^2}\right\}$$

But it does not touch other functions:

```
>> Together[f[a / c + b / c]]
```

$$f\left[\frac{a}{c}+\frac{b}{c}\right]$$

Variables

`Variables[`*expr*`]`
: gives a list of the variables that appear in the polynomial *expr*.

```
>> Variables[a x^2 + b x + c]
```

$\{a,b,c,x\}$

```
>> Variables[{a + b x, c y^2 + x
/2}]
```

$\{a,b,c,x,y\}$

```
>> Variables[x + Sin[y]]
```

$\{x,\mathrm{Sin}\left[y\right]\}$

50. Calculus

Contents

Complexes

`Complexes`
    is the set of complex numbers.

D

`D[`*f*`, `*x*`]`
    gives the partial derivative of f with respect to x.
`D[`*f*`, `*x*`, `*y*`, ...]`
    differentiates successively with respect to x, y, etc.
`D[`*f*`, {`*x*`, `*n*`}]`
    gives the multiple derivative of order n.
`D[`*f*`, {{`*x1*`, `*x2*`, ...}}]`
    gives the vector derivative of f with respect to $x1$, $x2$, etc.

First-order derivative of a polynomial:

```
>> D[x^3 + x^2, x]
```

$2x + 3x^2$

Second-order derivative:

```
>> D[x^3 + x^2, {x, 2}]
```

$2 + 6x$

Trigonometric derivatives:

```
>> D[Sin[Cos[x]], x]
```

$-\mathrm{Cos}\,[\mathrm{Cos}\,[x]]\,\mathrm{Sin}\,[x]$

```
>> D[Sin[x], {x, 2}]
```

$-\mathrm{Sin}\,[x]$

```
>> D[Cos[t], {t, 2}]
```

$-\mathrm{Cos}\,[t]$

Unknown variables are treated as constant:

```
>> D[y, x]
```

0

```
>> D[x, x]
```

1

```
>> D[x + y, x]
```

1

Derivatives of unknown functions are represented using `Derivative`:

```
>> D[f[x], x]
```

$f'\,[x]$

```
>> D[f[x, x], x]
```

$f^{(0,1)}\,[x, x] + f^{(1,0)}\,[x, x]$

```
>> D[f[x, x], x] // InputForm
```

$\mathrm{Derivative}\,[0, 1]\,\left[f\right]\,[x, x] + \mathrm{Derivative}\,[1, 0]\,\left[f\right]\,[x, x]$

Chain rule:

```
>> D[f[2x+1, 2y, x+y], x]
```

$2f^{(1,0,0)}\left[1 + 2x, 2y, x + y\right] + f^{(0,0,1)}\left[1 + 2x, 2y, x + y\right]$

```
>> D[f[x^2, x, 2y], {x,2}, y] //
Expand
```

$8xf^{(1,1,1)}\left[x^2,x,2y\right]+8x^2f^{(2,0,1)}\left[x^2,x,2y\right]+2f^{(0,2,1)}\left[x^2,x,2y\right]+4f^{(1,0,1)}\left[x^2,x,2y\right]$

Compute the gradient vector of a function:

```
>> D[x ^ 3 * Cos[y], {{x, y}}]
```

$\left\{3x^2\mathrm{Cos}\left[y\right], -x^3\mathrm{Sin}\left[y\right]\right\}$

Hesse matrix:

```
>> D[Sin[x] * Cos[y], {{x,y}, 2}]
```

$\{\{-\mathrm{Cos}\left[y\right]\mathrm{Sin}\left[x\right], -\mathrm{Cos}\left[x\right]\mathrm{Sin}\left[y\right]\}, \{-\mathrm{Cos}\left[x\right]\mathrm{Sin}\left[y\right], -\mathrm{Cos}\left[y\right]\mathrm{Sin}\left[x\right]\}\}$

Derivative (')

`Derivative[`*n*`][`*f*`]`
: represents the *n*th derivative of the function *f*.

`Derivative[`*n1*`, `*n2*`, ...][`*f*`]`
: represents a multivariate derivative.

```
>> Derivative[1][Sin]
```

$\mathrm{Cos}[\#1]\&$

```
>> Derivative[3][Sin]
```

$-\mathrm{Cos}[\#1]\&$

```
>> Derivative[2][# ^ 3&]
```

$6\#1\&$

`Derivative` can be entered using `'`:

```
>> Sin'[x]
```

$\mathrm{Cos}[x]$

```
>> (# ^ 4&)''
```

$12\#1^2\&$

```
>> f'[x] // InputForm
```

$\mathrm{Derivative}[1]\left[f\right][x]$

```
>> Derivative[1][#2 Sin[#1]+Cos
[#2]&]
```

$\mathrm{Cos}[\#1]\#2\&$

```
>> Derivative[1,2][#2^3 Sin[#1]+Cos
[#2]&]
```

$6\mathrm{Cos}[\#1]\#2\&$

Deriving with respect to an unknown parameter yields 0:

```
>> Derivative[1,2,1][#2^3 Sin[#1]+
Cos[#2]&]
```

$0\&$

The 0th derivative of any expression is the expression itself:

```
>> Derivative[0,0,0][a+b+c]
```

$a+b+c$

You can calculate the derivative of custom functions:

```
>> f[x_] := x ^ 2
```

```
>> f'[x]
```

$2x$

Unknown derivatives:

```
>> Derivative[2, 1][h]
```

$h^{(2,1)}$

```
>> Derivative[2, 0, 1, 0][h[g]]
```

$h\left[g\right]^{(2,0,1,0)}$

DiscreteLimit

`DiscreteLimit[`*f*`, `*k*`->Infinity]`
: gives the limit of the sequence *f* as *k* tends to infinity.

```
>> DiscreteLimit[n/(n + 1), n ->
Infinity]
```

1

```
>> DiscreteLimit[f[n], n ->
Infinity]
```

$f[\infty]$

FindRoot

`FindRoot[`f`, {`x`, `$x0$`}]`
: searches for a numerical root of f, starting from x=$x0$.

`FindRoot[`lhs `==` rhs`, {`x`, `$x0$`}]`
: tries to solve the equation lhs `==` rhs.

`FindRoot` by default uses Newton's method, so the function of interest should have a first derivative.

```
>> FindRoot[Cos[x], {x, 1}]
```

$\{x->1.5708\}$

```
>> FindRoot[Sin[x] + Exp[x],{x, 0}]
```

$\{x->-0.588533\}$

```
>> FindRoot[Sin[x] + Exp[x] == Pi,{
   x, 0}]
```

$\{x->0.866815\}$

`FindRoot` has attribute `HoldAll` and effectively uses `Block` to localize x. However, in the result x will eventually still be replaced by its value.

```
>> x = "I am the result!";
```

```
>> FindRoot[Tan[x] + Sin[x] == Pi,
   {x, 1}]
```

{I am the result!$->1.14911$}

```
>> Clear[x]
```

`FindRoot` stops after 100 iterations:

```
>> FindRoot[x^2 + x + 1, {x, 1}]
```

Themaximumnumberofiterationswasexceeded.Theresultmightbeinaccurate.

$\{x->-1.\}$

Find complex roots:

```
>> FindRoot[x ^ 2 + x + 1, {x, -I}]
```

$\{x->-0.5-0.866025I\}$

The function has to return numerical values:

```
>> FindRoot[f[x] == 0, {x, 0}]
```

Thefunctionvalueisnotanumberatx = 0..

$\text{FindRoot}\left[f[x]-0,\ \{x,0\}\right]$

The derivative must not be 0:

```
>> FindRoot[Sin[x] == x, {x, 0}]
```

Encounteredasingularderivativeatthepointx = 0..

$\text{FindRoot}\left[\text{Sin}[x]-x,\ \{x,0\}\right]$

```
>> FindRoot[x^2 - 2, {x, 1,3},
   Method->"Secant"]
```

$\{x->1.41421\}$

Integers

`Integers`
: is the set of integer numbers.

Limit a solution to integer numbers:

```
>> Solve[-4 - 4 x + x^4 + x^5 == 0,
    x, Integers]
```

$\{\{x->-1\}\}$

```
>> Solve[x^4 == 4, x, Integers]
```

$\{\}$

Integrate

`Integrate[`f`, `x`]`
: integrates f with respect to x. The result does not contain the additive integration constant.

`Integrate[`f`, {`x`, `a`, `b`}]`
: computes the definite integral of f with respect to x from a to b.

Integrate a polynomial:

```
>> Integrate[6 x ^ 2 + 3 x ^ 2 - 4
   x + 10, x]
```

$x\left(10-2x+3x^2\right)$

Integrate trigonometric functions:

```
>> Integrate[Sin[x] ^ 5, x]
```

$$-\text{Cos}[x]-\frac{\text{Cos}[x]^5}{5}+\frac{2\text{Cos}[x]^3}{3}$$

Definite integrals:

```
>> Integrate[x ^ 2 + x, {x, 1, 3}]
```

$\frac{38}{3}$

```
>> Integrate[Sin[x], {x, 0, Pi/2}]
```

1

Some other integrals:

```
>> Integrate[1 / (1 - 4 x + x^2), x
]
```

$$\frac{\sqrt{3}\left(\text{Log}\left[-2-\sqrt{3}+x\right]-\text{Log}\left[-2+\sqrt{3}+x\right]\right)}{6}$$

```
>> Integrate[4 Sin[x] Cos[x], x]
```

$2\text{Sin}[x]^2$

Integration in TeX:

```
>> Integrate[f[x], {x, a, b}] //
TeXForm
```

\int_a^b f\left[x\right] \, dx

Sometimes there is a loss of precision during integration. You can check the precision of your result with the following sequence of commands.

```
>> Integrate[Abs[Sin[phi]], {phi,
0, 2Pi}] // N
```

4.

```
>> % // Precision
```

MachinePrecision

```
>> Integrate[ArcSin[x / 3], x]
```

$x\text{ArcSin}\left[\frac{x}{3}\right]+\sqrt{9-x^2}$

```
>> Integrate[f'[x], {x, a, b}]
```

$f[b]-f[a]$

Limit

`Limit[`*expr*`, `*x*`->`*x0*`]`
: gives the limit of *expr* as *x* approaches *x0*.

`Limit[`*expr*`, `*x*`->`*x0*`, Direction->1]`
: approaches *x0* from smaller values.

`Limit[`*expr*`, `*x*`->`*x0*`, Direction->-1]`
: approaches *x0* from larger values.

```
>> Limit[x, x->2]
```

2

```
>> Limit[Sin[x] / x, x->0]
```

1

```
>> Limit[1/x, x->0, Direction->-1]
```

∞

```
>> Limit[1/x, x->0, Direction->1]
```

$-\infty$

O

`O[`*x*`]^n`
: Represents a term of order x^n. O[x]^n is generated to represent omitted higher order terms in power series.

```
>> Series[1/(1-x),{x,0,2}]
```

$1+x+x^2+O[x]^3$

Reals

`Reals`
: is the set of real numbers.

Limit a solution to real numbers:

```
>> Solve[x^3 == 1, x, Reals]
```

$\{\{x->1\}\}$

Root

`Root[`*f*`, `*i*`]`
: represents the i-th complex root of the polynomial *f*

```
>> Root[#1 ^ 2 - 1&, 1]
```

-1

```
>> Root[#1 ^ 2 - 1&, 2]
```

1

Roots that can't be represented by radicals:

```
>> Root[#1 ^ 5 + 2 #1 + 1&, 2]
```

$\text{Root}\left[\#1^5+2\#1+1\&,2\right]$

Series

`Series[`*f*`, {`*x*`, `*x0*`, `*n*`}]`
: Represents the series expansion around *x*=*x0* up to order *n*.

```
>> Series[Exp[x],{x,0,2}]
```

$1+x+\frac{1}{2}x^2+O[x]^3$

```
>> Series[Exp[x^2],{x,0,2}]
```
$1 + x^2 + O[x]^3$

SeriesData

`SeriesData[...]`
: Represents a series expansion

TODO: - Implement sum, product and composition of series

Solve

`Solve[`*equation*`, `*vars*`]`
: attempts to solve *equation* for the variables *vars*.

`Solve[`*equation*`, `*vars*`, `*domain*`]`
: restricts variables to *domain*, which can be `Complexes` or `Reals` or `Integers`.

```
>> Solve[x ^ 2 - 3 x == 4, x]
```
$\{\{x- > -1\}, \{x- > 4\}\}$

```
>> Solve[4 y - 8 == 0, y]
```
$\{\{y- > 2\}\}$

Apply the solution:

```
>> sol = Solve[2 x^2 - 10 x - 12 ==
   0, x]
```
$\{\{x- > -1\}, \{x- > 6\}\}$

```
>> x /. sol
```
$\{-1, 6\}$

Contradiction:

```
>> Solve[x + 1 == x, x]
```
$\{\}$

Tautology:

```
>> Solve[x ^ 2 == x ^ 2, x]
```
$\{\{\}\}$

Rational equations:

```
>> Solve[x / (x ^ 2 + 1)== 1, x]
```
$$\left\{\left\{x- > \frac{1}{2} - \frac{I}{2}\sqrt{3}\right\}, \left\{x- > \frac{1}{2} + \frac{I}{2}\sqrt{3}\right\}\right\}$$

```
>> Solve[(x^2 + 3 x + 2)/(4 x - 2)
   == 0, x]
```
$\{\{x- > -2\}, \{x- > -1\}\}$

Transcendental equations:

```
>> Solve[Cos[x] == 0, x]
```
$$\left\{\left\{x- > \frac{\text{Pi}}{2}\right\}, \left\{x- > \frac{3\text{Pi}}{2}\right\}\right\}$$

Solve can only solve equations with respect to symbols or functions:

```
>> Solve[f[x + y] == 3, f[x + y]]
```
$\{\{f[x + y] - > 3\}\}$

```
>> Solve[a + b == 2, a + b]
```
a + bisnotavalidvariable.

Solve $[a + b{==}2, a + b]$

This happens when solving with respect to an assigned symbol:

```
>> x = 3;
```

```
>> Solve[x == 2, x]
```
3isnotavalidvariable.

Solve [False, 3]

```
>> Clear[x]
```

```
>> Solve[a < b, a]
```
$a < bisnotawell - formedequation.$

Solve $[a < b, a]$

Solve a system of equations:

```
>> eqs = {3 x ^ 2 - 3 y == 0, 3 y ^
   2 - 3 x == 0};
```

```
>> sol = Solve[eqs, {x, y}] //
   Simplify
```

$$\left\{\{x->0,y->0\},\{x->1,y->1\},\left\{x->-\frac{1}{2}+\frac{I}{2}\sqrt{3},y->-\frac{1}{2}-\frac{I}{2}\sqrt{3}\right\},\left\{x->\frac{\left(1-I\sqrt{3}\right)^2}{4},y->-\frac{1}{2}+\frac{I}{2}\sqrt{3}\right\}\right\}$$

```
>> eqs /. sol // Simplify
```

{{True, True}, {True, True}, {True, True}, {True, True}}

An underdetermined system:

```
>> Solve[x^2 == 1 && z^2 == -1, {x,
    y, z}]
```

Equationsmaynotgivesolutionsforall"solve"variables.

$\{\{x->-1,z->-I\},\{x->-1,z->I\},\{x->1,z->-I\},\{x->1,z->I\}\}$

Domain specification:

```
>> Solve[x^2 == -1, x, Reals]
```

{}

```
>> Solve[x^2 == 1, x, Reals]
```

$\{\{x->-1\},\{x->1\}\}$

```
>> Solve[x^2 == -1, x, Complexes]
```

$\{\{x->-I\},\{x->I\}\}$

```
>> Solve[4 - 4 * x^2 - x^4 + x^6 ==
    0, x, Integers]
```

$\{\{x->-1\},\{x->1\}\}$

51. Mathematical Constants

Numeric, Arithmetic, or Symbolic constants like Pi, E, or Infinity.

Contents

Catalan

`Catalan`
is Catalan's constant with numerical value $\simeq 0.915966$.

```
>> Catalan // N
```
0.915965594177219

```
>> N[Catalan, 20]
```
0.91596559417721901505

ComplexInfinity

`ComplexInfinity`
represents an infinite complex quantity of undetermined direction.

```
>> 1 / ComplexInfinity
```
0

```
>> ComplexInfinity * Infinity
```
ComplexInfinity

```
>> FullForm[ComplexInfinity]
```
DirectedInfinity []

Degree

`Degree`
is the number of radians in one degree. It has a numerical value of π / 180.

```
>> Cos[60 Degree]
```
$\frac{1}{2}$

Degree has the value of Pi / 180

```
>> Degree == Pi / 180
```
True

```
>> N[\[Degree]] == N[Degree]
```
True

E

`E`
is the constant e with numerical value $\simeq$ 2.71828.

```
>> N[E]
```
2.71828

```
>> N[E, 50]
```
2.7182818284590452353602874713526624977572470937000

EulerGamma

`EulerGamma`
is Euler's constant γ with numerial value $\simeq 0.577216$.

```
>> EulerGamma // N
```
0.577216

```
>> N[EulerGamma, 40]
```
0.577215664901532860~
~6065120900824024310422

Glaisher

`Glaisher`
is Glaisher's constant, with numerical value ≃ 1.28243.

```
>> N[Glaisher]
```
1.28242712910062

```
>> N[Glaisher, 50]
```
1.2824271291006226368753425~
~688697917277676889273250

1.2824271291006219541941391071304678916931152343750

GoldenRatio

`GoldenRatio`
is the golden ratio, = (1+Sqrt[5])/2.

```
>> GoldenRatio // N
```
1.61803398874989

```
>> N[GoldenRatio, 40]
```
1.618033988749894848~
~204586834365638117720

Indeterminate

`Indeterminate`
represents an indeterminate result.

```
>> 0^0
```
Indeterminateexpression0^0*encountered.*

Indeterminate

```
>> Tan[Indeterminate]
```
Indeterminate

Infinity

`Infinity`
represents an infinite real quantity.

```
>> 1 / Infinity
```
0

```
>> Infinity + 100
```
∞

Use `Infinity` in sum and limit calculations:

```
>> Sum[1/x^2, {x, 1, Infinity}]
```
$\frac{\text{Pi}^2}{6}$

Khinchin

`Khinchin`
is Khinchin's constant, with numerical value ≃ 2.68545.

```
>> N[Khinchin]
```
2.68545200106531

```
>> N[Khinchin, 50]
```
2.685452001065306445309714~
~8354817956938203822939945

= 2.6854520010653075701156922150403261184692382812500

Pi

`Pi`
is the constant π.

```
>> N[Pi]
```
3.14159

Pi to a numeric precision of 20 digits:

```
>> N[Pi, 20]
```
3.1415926535897932385

Note that the above is not the same thing as the number of digits *after* the decimal point. This may differ from similar concepts from other mathematical libraries, including those which Mathics uses!

Use numpy to compute Pi to 20 digits:

```
>> N[Pi, 20, Method->"numpy"]
```
3.1415926535897930000

"sympy" is the default method.

```
>> Attributes[Pi]
```
{Constant, Protected, ReadProtected}

52. Differential Equations

Contents

C

`C[`n`]`
: represents the nth constant in a solution to a differential equation.

DSolve

`DSolve[`eq`, `$y[x]$`, `x`]`
: solves a differential equation for the function $y[x]$.

```
>> DSolve[y''[x] == 0, y[x], x]
```

$\{\{y[x]->xC[2]+C[1]\}\}$

```
>> DSolve[y''[x] == y[x], y[x], x]
```

$\{\{y[x]->C[1]E^{-x}+C[2]E^{x}\}\}$

```
>> DSolve[y''[x] == y[x], y, x]
```

$\{\{y->(\text{Function}[\{x\}, C[1]E^{-x}+C[2]E^{x}])\}\}$

DSolve can also solve basic PDE

```
>> DSolve[D[f[x, y], x] / f[x, y] +
    3 D[f[x, y], y] / f[x, y] == 2,
    f, {x, y}]
```

$\left\{\left\{f->\left(\text{Function}\left[\{x,y\}, E^{\frac{x}{5}+\frac{3y}{5}}C[1][3x-y]\right]\right)\right\}\right\}$

```
>> DSolve[D[f[x, y], x] x + D[f[x,
   y], y] y == 2, f[x, y], {x, y}]
```

$\left\{\left\{f[x,y]->2\text{Log}[x]+C[1]\left[\frac{y}{x}\right]\right\}\right\}$

```
>> DSolve[D[y[x, t], t] + 2 D[y[x,
   t], x] == 0, y[x, t], {x, t}]
```

$\{\{y[x,t]->C[1][x-2t]\}\}$

53. Exponential, Trigonometric and Hyperbolic Functions

Mathics basically supports all important trigonometric and hyperbolic functions.
Numerical values and derivatives can be computed; however, most special exact values and simplification rules are not implemented yet.

Contents

AnglePath

`AnglePath[{`*phi1*`, `*phi2*`, ...}]`
: returns the points formed by a turtle starting at {0, 0} and angled at 0 degrees going through the turns given by angles *phi1*, *phi2*, ... and using distance 1 for each step.

`AnglePath[{{`*r1*`, `*phi1*`}, {`*r2*`, `*phi2*`}, ...}]`
: instead of using 1 as distance, use *r1*, *r2*, ... as distances for the respective steps.

`AngleVector[`*phi0*`, {`*phi1*`, `*phi2*`, ...}]`
: returns the points on a path formed by a turtle starting with direction *phi0* instead of 0.

`AngleVector[{`*x*`, `*y*`}, {`*phi1*`, `*phi2*`, ...}]`
: returns the points on a path formed by a turtle starting at {$x, $y} instead of {0, 0}.

`AngleVector[{{`*x*`, `*y*`}, `*phi0*`}, {`*phi1*`, `*phi2*`, ...}]`
: specifies initial position {*x*, *y*} and initial direction *phi0*.

`AngleVector[{{`*x*`, `*y*`}, {`*dx*`, `*dy*`}}, {`*phi1*`, `*phi2*`, ...}]`
: specifies initial position {*x*, *y*} and a slope {*dx*, *dy*} that is understood to be the initial direction of the turtle.

>> `AnglePath[{90 Degree, 90 Degree, 90 Degree, 90 Degree}]`

$\{\{0,0\},\{0,1\},\{-1,1\},\{-1,0\},\{0,0\}\}$

>> `AnglePath[{{1, 1}, 90 Degree}, {{1, 90 Degree}, {2, 90 Degree}, {1, 90 Degree}, {2, 90 Degree}}]`

$\{\{1,1\},\{0,1\},\{0,-1\},\{1,-1\},\{1,1\}\}$

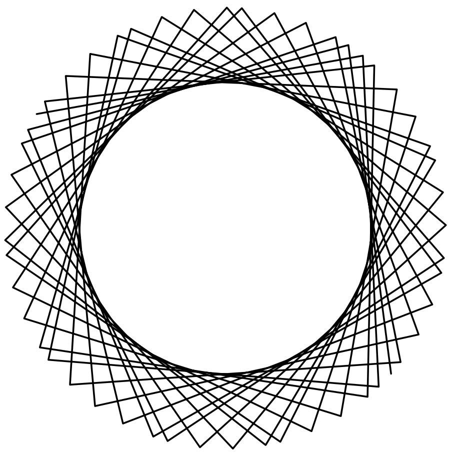
>> `AnglePath[{a, b}]`

$\{\{0,0\},\{\mathrm{Cos}[a],\mathrm{Sin}[a]\},\{\mathrm{Cos}[a]+\mathrm{Cos}[a+b],\mathrm{Sin}[a]+\mathrm{Sin}[a+b]\}\}$

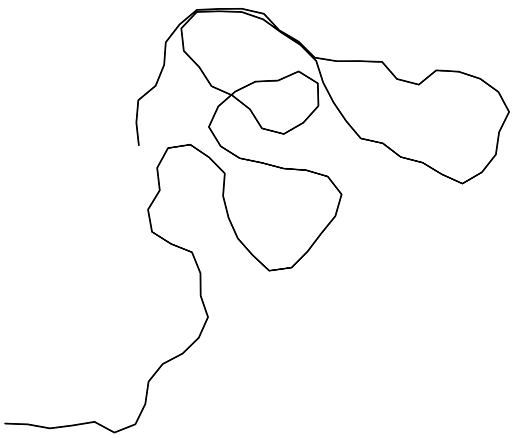
>> `Precision[Part[AnglePath[{N[1/3, 100], N[2/3, 100]}], 2, 1]]`

100.

>> `Graphics[Line[AnglePath[Table[1.7, {50}]]]]`



>> `Graphics[Line[AnglePath[RandomReal[{-1, 1}, {100}]]]]`



AngleVector

`AngleVector[`*phi*`]`
: returns the point at angle *phi* on the unit circle.

`AngleVector[{`*r*`, `*phi*`}]`
: returns the point at angle *phi* on a circle of radius *r*.

`AngleVector[{`*x*`, `*y*`}, `*phi*`]`
: returns the point at angle *phi* on a circle of radius 1 centered at {*x*, *y*}.

`AngleVector[{`*x*`, `*y*`}, {`*r*`, `*phi*`}]`
: returns point at angle *phi* on a circle of radius *r* centered at {*x*, *y*}.

>> `AngleVector[90 Degree]`

$\{0,1\}$

>> `AngleVector[{1, 10}, a]`

$\{1+\mathrm{Cos}[a],10+\mathrm{Sin}[a]\}$

ArcCos

`ArcCos[`*z*`]`
: returns the inverse cosine of *z*.

>> `ArcCos[1]`

0

>> `ArcCos[0]`

$\frac{\mathrm{Pi}}{2}$

>> `Integrate[ArcCos[x], {x, -1, 1}]`

Pi

ArcCosh

`ArcCosh[`*z*`]`
: returns the inverse hyperbolic cosine of *z*.

>> `ArcCosh[0]`

$\frac{I}{2}\mathrm{Pi}$

>> `ArcCosh[0.]`

$0.+1.5708I$

>> `ArcCosh[0.00000000000000000000000000000000000000]`

1.570796326794896619~
~2313216916397514421I

ArcCot

`ArcCot[z]`
: returns the inverse cotangent of z.

>> `ArcCot[0]`

$\frac{\text{Pi}}{2}$

>> `ArcCot[1]`

$\frac{\text{Pi}}{4}$

ArcCoth

`ArcCoth[z]`
: returns the inverse hyperbolic cotangent of z.

>> `ArcCoth[0]`

$\frac{I}{2}\text{Pi}$

>> `ArcCoth[1]`

∞

>> `ArcCoth[0.0]`

$0. + 1.5708I$

>> `ArcCoth[0.5]`

$0.549306 - 1.5708I$

ArcCsc

`ArcCsc[z]`
: returns the inverse cosecant of z.

>> `ArcCsc[1]`

$\frac{\text{Pi}}{2}$

>> `ArcCsc[-1]`

$-\frac{\text{Pi}}{2}$

ArcCsch

`ArcCsch[z]`
: returns the inverse hyperbolic cosecant of z.

>> `ArcCsch[0]`

ComplexInfinity

>> `ArcCsch[1.0]`

0.881374

ArcSec

`ArcSec[z]`
: returns the inverse secant of z.

>> `ArcSec[1]`

0

>> `ArcSec[-1]`

Pi

ArcSech

`ArcSech[z]`
: returns the inverse hyperbolic secant of z.

>> `ArcSech[0]`

∞

>> `ArcSech[1]`

0

>> `ArcSech[0.5]`

1.31696

ArcSin

`ArcSin[z]`
: returns the inverse sine of z.

>> `ArcSin[0]`

0

>> `ArcSin[1]`
$\frac{\text{Pi}}{2}$

ArcSinh

`ArcSinh[`*z*`]`
returns the inverse hyperbolic sine of *z*.

>> `ArcSinh[0]`
0

>> `ArcSinh[0.]`
0.

>> `ArcSinh[1.0]`
0.881374

ArcTan

`ArcTan[`*z*`]`
returns the inverse tangent of *z*.

>> `ArcTan[1]`
$\frac{\text{Pi}}{4}$

>> `ArcTan[1.0]`
0.785398

>> `ArcTan[-1.0]`
-0.785398

>> `ArcTan[1, 1]`
$\frac{\text{Pi}}{4}$

ArcTanh

`ArcTanh[`*z*`]`
returns the inverse hyperbolic tangent of *z*.

>> `ArcTanh[0]`
0

>> `ArcTanh[1]`
∞

>> `ArcTanh[0]`
0

>> `ArcTanh[.5 + 2 I]`
$0.0964156 + 1.12656I$

>> `ArcTanh[2 + I]`
$\text{ArcTanh}\,[2 + I]$

Cos

`Cos[`*z*`]`
returns the cosine of *z*.

>> `Cos[3 Pi]`
-1

Cosh

`Cosh[`*z*`]`
returns the hyperbolic cosine of *z*.

>> `Cosh[0]`
1

Cot

`Cot[`*z*`]`
returns the cotangent of *z*.

>> `Cot[0]`
ComplexInfinity

>> `Cot[1.]`
0.642093

Coth

`Coth[`*z*`]`
returns the hyperbolic cotangent of *z*.

>> `Coth[0]`
ComplexInfinity

Csc

`Csc[z]`
returns the cosecant of z.

>> `Csc[0]`
ComplexInfinity

>> `Csc[1] (* Csc[1] in Mathematica *)`
$\frac{1}{\mathrm{Sin}[1]}$

>> `Csc[1.]`
1.1884

Csch

`Csch[z]`
returns the hyperbolic cosecant of z.

>> `Csch[0]`
ComplexInfinity

Exp

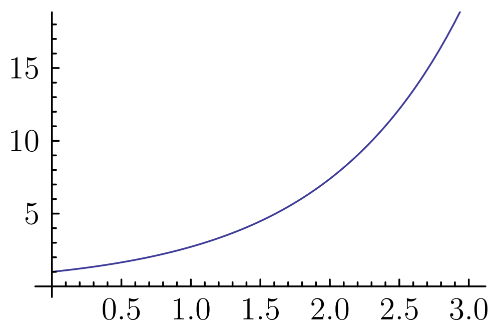
`Exp[z]`
returns the exponential function of z.

>> `Exp[1]`
E

>> `Exp[10.0]`
22 026.5

>> `Exp[x] //FullForm`
Power$[E, x]$

>> `Plot[Exp[x], {x, 0, 3}]`



Haversine

`Haversine[z]`
returns the haversine function of z.

>> `Haversine[1.5]`
0.464631

>> `Haversine[0.5 + 2I]`
$-1.15082 + 0.869405I$

InverseHaversine

`InverseHaversine[z]`
returns the inverse haversine function of z.

>> `InverseHaversine[0.5]`
1.5708

>> `InverseHaversine[1 + 2.5 I]`
$1.76459 + 2.33097I$

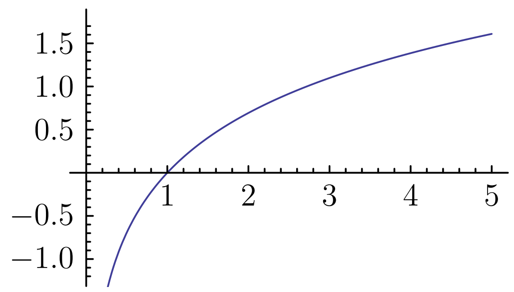
Log

`Log[z]`
returns the natural logarithm of z.

>> `Log[{0, 1, E, E * E, E ^ 3, E ^ x}]`
$\{-\infty, 0, 1, 2, 3, \mathrm{Log}[E^x]\}$

>> `Log[0.]`
Indeterminate

>> `Plot[Log[x], {x, 0, 5}]`



Log10

`Log10[`z`]`
    returns the base-10 logarithm of z.

```
>> Log10[1000]
```
3

```
>> Log10[{2., 5.}]
```
$\{0.30103, 0.69897\}$

```
>> Log10[E ^ 3]
```
$$\frac{3}{\text{Log}[10]}$$

Log2

`Log2[`z`]`
    returns the base-2 logarithm of z.

```
>> Log2[4 ^ 8]
```
16

```
>> Log2[5.6]
```
2.48543

```
>> Log2[E ^ 2]
```
$$\frac{2}{\text{Log}[2]}$$

LogisticSigmoid

`LogisticSigmoid[`z`]`
    returns the logistic sigmoid of z.

```
>> LogisticSigmoid[0.5]
```
0.622459

```
>> LogisticSigmoid[0.5 + 2.3 I]
```
$1.06475 + 0.808177I$

```
>> LogisticSigmoid[{-0.2, 0.1,
   0.3}]
```
$\{0.450166, 0.524979, 0.574443\}$

Sec

`Sec[`z`]`
    returns the secant of z.

```
>> Sec[0]
```
1

```
>> Sec[1] (* Sec[1] in Mathematica
   *)
```
$$\frac{1}{\text{Cos}[1]}$$

```
>> Sec[1.]
```
1.85082

Sech

`Sech[`z`]`
    returns the hyperbolic secant of z.

```
>> Sech[0]
```
1

Sin

`Sin[`z`]`
    returns the sine of z.

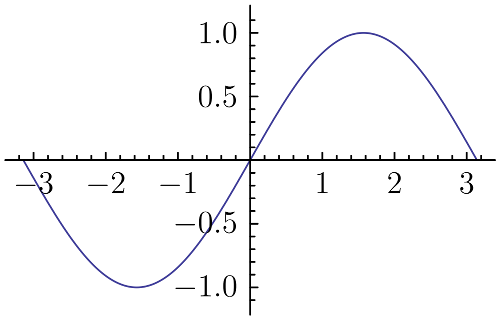
```
>> Sin[0]
```
0

```
>> Sin[0.5]
```
0.479426

```
>> Sin[3 Pi]
```
0

```
>> Sin[1.0 + I]
```
$1.29846 + 0.634964I$

>> `Plot[Sin[x], {x, -Pi, Pi}]`



Sinh

`Sinh[z]`
returns the hyperbolic sine of z.

>> `Sinh[0]`
0

Tan

`Tan[z]`
returns the tangent of z.

>> `Tan[0]`
0

>> `Tan[Pi / 2]`
ComplexInfinity

Tanh

`Tanh[z]`
returns the hyperbolic tangent of z.

>> `Tanh[0]`
0

54. Integer Functions

Contents

BitLength

`BitLength[`*x*`]`
: gives the number of bits needed to represent the integer x. x's sign is ignored.

>> `BitLength[1023]`
10

>> `BitLength[100]`
7

>> `BitLength[-5]`
3

>> `BitLength[0]`
0

Ceiling

`Ceiling[`*x*`]`
: gives the first integer greater than x.

>> `Ceiling[1.2]`
2

>> `Ceiling[3/2]`
2

For complex x, take the ceiling of real an imaginary parts.

>> `Ceiling[1.3 + 0.7 I]`
$2 + I$

DigitCount

`DigitCount[`*n*`, `*b*`, `*d*`]`
: returns the number of times digit d occurs in the base b representation of n.

`DigitCount[`*n*`, `*b*`]`
: returns a list indicating the number of times each digit occurs in the base b representation of n.

`DigitCount[`*n*`, `*b*`]`
: returns a list indicating the number of times each digit occurs in the decimal representation of n.

>> `DigitCount[1022]`
$\{1, 2, 0, 0, 0, 0, 0, 0, 0, 1\}$

>> `DigitCount[Floor[Pi * 10^100]]`
$\{8, 12, 12, 10, 8, 9, 8, 12, 14, 8\}$

>> `DigitCount[1022, 2]`
$\{9, 1\}$

>> `DigitCount[1022, 2, 1]`
9

Floor

`Floor[`*x*`]`
: gives the smallest integer less than or equal to x.

`Floor[`*x*`, `*a*`]`
: gives the smallest multiple of a less than or equal to x.

```
>> Floor[10.4]
   10

>> Floor[10/3]
   3

>> Floor[10]
   10

>> Floor[21, 2]
   20

>> Floor[2.6, 0.5]
   2.5

>> Floor[-10.4]
   -11
```

For complex x, take the floor of real an imaginary parts.

```
>> Floor[1.5 + 2.7 I]
   1 + 2I
```

For negative a, the smallest multiple of a greater than or equal to x is returned.

```
>> Floor[10.4, -1]
   11

>> Floor[-10.4, -1]
   -10
```

FromDigits

FromDigits[*l*]
: returns the integer corresponding to the decimal representation given by l. l can be a list of digits or a string.

FromDigits[*l*, *b*]
: returns the integer corresponding to the base b representation given by l. l can be a list of digits or a string.

```
>> FromDigits["123"]
   123

>> FromDigits[{1, 2, 3}]
   123

>> FromDigits[{1, 0, 1}, 1000]
   1 000 001
```

FromDigits can handle symbolic input:

```
>> FromDigits[{a, b, c}, 5]
   c + 5 (5a + b)
```

Note that FromDigits does not automatically detect if you are providing a non-decimal representation:

```
>> FromDigits["a0"]
   100

>> FromDigits["a0", 16]
   160
```

FromDigits on empty lists or strings returns 0:

```
>> FromDigits[{}]
   0

>> FromDigits[""]
   0
```

IntegerDigits

IntegerDigits[*n*]
: returns the decimal representation of integer x as list of digits. x's sign is ignored.

IntegerDigits[*n*, *b*]
: returns the base b representation of integer x as list of digits. x's sign is ignored.

IntegerDigits[*n*, *b*, *length*]
: returns a list of length *length*. If the number is too short, the list gets padded with 0 on the left. If the number is too long, the *length* least significant digits are returned.

```
>> IntegerDigits[12345]
   {1, 2, 3, 4, 5}

>> IntegerDigits[-500]
   {5, 0, 0}

>> IntegerDigits[12345, 10, 8]
   {0, 0, 0, 1, 2, 3, 4, 5}

>> IntegerDigits[12345, 10, 3]
   {3, 4, 5}

>> IntegerDigits[11, 2]
   {1, 0, 1, 1}

>> IntegerDigits[123, 8]
   {1, 7, 3}

>> IntegerDigits[98765, 20]
   {12, 6, 18, 5}
```

IntegerLength

`IntegerLength[`x`]`
: gives the number of digits in the base-10 representation of x.

`IntegerLength[`x`, `b`]`
: gives the number of base-b digits in x.

```
>> IntegerLength[123456]
```
6

```
>> IntegerLength[10^10000]
```
10 001

```
>> IntegerLength[-10^1000]
```
1 001

`IntegerLength` with base 2:

```
>> IntegerLength[8, 2]
```
4

Check that `IntegerLength` is correct for the first 100 powers of 10:

```
>> IntegerLength /@ (10 ^ Range
   [100])== Range[2, 101]
```
True

The base must be greater than 1:

```
>> IntegerLength[3, -2]
```
*Base − 2isnotanintegergreaterthan*1.

IntegerLength [3, − 2]

`0` is a special case:

```
>> IntegerLength[0]
```
0

IntegerReverse

`IntegerReverse[`n`]`
: returns the integer that has the reverse decimal representation of x without sign.

`IntegerReverse[`n`, `b`]`
: returns the integer that has the reverse base b represenation of x without sign.

```
>> IntegerReverse[1234]
```
4 321

```
>> IntegerReverse[1022, 2]
```
511

```
>> IntegerReverse[-123]
```
321

IntegerString

`IntegerString[`n`]`
: returns the decimal representation of integer x as string. x's sign is ignored.

`IntegerString[`n`, `b`]`
: returns the base b representation of integer x as string. x's sign is ignored.

`IntegerString[`n`, `b`, `*length*`]`
: returns a string of length *length*. If the number is too short, the string gets padded with 0 on the left. If the number is too long, the *length* least significant digits are returned.

For bases > 10, alphabetic characters a, b, ... are used to represent digits 11, 12, ... Note that base must be an integer in the range from 2 to 36.

```
>> IntegerString[12345]
```
12 345

```
>> IntegerString[-500]
```
500

```
>> IntegerString[12345, 10, 8]
```
00 012 345

```
>> IntegerString[12345, 10, 3]
```
345

```
>> IntegerString[11, 2]
```
1 011

```
>> IntegerString[123, 8]
```
173

```
>> IntegerString[32767, 16]
```
7fff

```
>> IntegerString[98765, 20]
```
c6i5

55. Linear algebra

Contents

BrayCurtisDistance

`BrayCurtisDistance[`*u*`, `*v*`]`
returns the Bray Curtis distance between *u* and *v*.

```
>> BrayCurtisDistance[-7, 5]
```
6

```
>> BrayCurtisDistance[{-1, -1}, {10, 10}]
```
$\frac{11}{9}$

CanberraDistance

`CanberraDistance[`*u*`, `*v*`]`
returns the canberra distance between *u* and *v*, which is a weighted version of the Manhattan distance.

```
>> CanberraDistance[-7, 5]
```
1

```
>> CanberraDistance[{-1, -1}, {1, 1}]
```
2

ChessboardDistance

`ChessboardDistance[`*u*`, `*v*`]`
returns the chessboard distance (also known as Chebyshev distance) between *u* and *v*, which is the number of moves a king on a chessboard needs to get from square *u* to square *v*.

```
>> ChessboardDistance[-7, 5]
```
12

```
>> ChessboardDistance[{-1, -1}, {1, 1}]
```
2

CosineDistance

`CosineDistance[`*u*`, `*v*`]`
returns the cosine distance between *u* and *v*.

```
>> N[CosineDistance[{7, 9}, {71, 89}]]
```
0.0000759646

```
>> CosineDistance[{a, b}, {c, d}]
```

$$1+\frac{-ac-bd}{\sqrt{\text{Abs}[a]^2+\text{Abs}[b]^2}\sqrt{\text{Abs}[c]^2+\text{Abs}[d]^2}}$$

Cross

`Cross[a, b]`
: computes the vector cross product of a and b.

```
>> Cross[{x1, y1, z1}, {x2, y2, z2}]
```

$\{y1z2-y2z1, -x1z2+x2z1, x1y2-x2y1\}$

```
>> Cross[{x, y}]
```

$\{-y, x\}$

```
>> Cross[{1, 2}, {3, 4, 5}]
```

The arguments are expected to be vectors of equal length, and the number of arguments is expected to be 1 less than their length.

Cross [{1,2} , {3,4,5}]

DesignMatrix

`DesignMatrix[m, f, x]`
: returns the design matrix.

```
>> DesignMatrix[{{2, 1}, {3, 4}, {5, 3}, {7, 6}}, x, x]
```

$\{\{1,2\},\{1,3\},\{1,5\},\{1,7\}\}$

```
>> DesignMatrix[{{2, 1}, {3, 4}, {5, 3}, {7, 6}}, f[x], x]
```

$\{\{1,f[2]\},\{1,f[3]\},\{1,f[5]\},\{1,f[7]\}\}$

Det

`Det[m]`
: computes the determinant of the matrix m.

```
>> Det[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}]
```

-2

Symbolic determinant:

```
>> Det[{{a, b, c}, {d, e, f}, {g, h, i}}]
```

$aei-afh-bdi+bfg+cdh-ceg$

Eigensystem

`Eigensystem[m]`
: returns the list `{Eigenvalues[m], Eigenvectors[m]}`.

```
>> Eigensystem[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}]
```

$\{\{2,-1,1\},\{\{1,1,1\},\{1,-2,1\},\{-1,0,1\}\}\}$

Eigenvalues

`Eigenvalues[m]`
: computes the eigenvalues of the matrix m. By default Sympy's routine is used. Sometimes this is slow and less good than the corresponding mpmath routine. Use option Method->"mpmath" if you want to use mpmath's routine instead.

Numeric eigenvalues are sorted in order of decreasing absolute value:

```
>> Eigenvalues[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}]
```

$\{2,-1,1\}$

Symbolic eigenvalues:

```
>> Eigenvalues[{{Cos[theta],Sin[theta],0},{-Sin[theta],Cos[theta],0},{0,0,1}}] // Sort
```

$$\left\{1, \text{Cos}[\text{theta}]+\sqrt{(-1+\text{Cos}[\text{theta}])(1+\text{Cos}[\text{theta}])}, \text{Cos}[\text{theta}]-\sqrt{(-1+\text{Cos}[\text{theta}])(1+\text{Cos}[\text{theta}])}\right\}$$

>> `Eigenvalues[{{7, 1}, {-4, 3}}]`

{5,5}

>> `Eigenvalues[{{7, 1}, {-4, 3}}]`

{5,5}

Eigenvectors

`Eigenvectors[`*m*`]`
: computes the eigenvectors of the matrix *m*.

>> `Eigenvectors[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}]`

{{1,1,1}, {1, −2,1}, {−1,0,1}}

>> `Eigenvectors[{{1, 0, 0}, {0, 1, 0}, {0, 0, 0}}]`

{{0,1,0}, {1,0,0}, {0,0,1}}

>> `Eigenvectors[{{2, 0, 0}, {0, -1, 0}, {0, 0, 0}}]`

{{1,0,0}, {0,1,0}, {0,0,1}}

>> `Eigenvectors[{{0.1, 0.2}, {0.8, 0.5}}]`

{{−0.355518216481267016676297~
~559501705929896062805897153~
~500209120909839738411406528~
~939551208168268203735351562~
~500000000000000000000000000~
~000000000000000000000000000~
~000000000000000000000000000~
~0000000000000, − 1.150481115~
~772866118834236549972506478~
~611688789589714534394707980~
~136107750013252370990812778~
~472900390625000000000000000~
~000000000000000000000000000~
~000000000000000000000000000~
~000000000000000000000000000},
{−0.628960169645094045731745~
~684302104224901929314653543~
~850901708147770746704097177~
~826042752712965011596679687~
~500000000000000000000000000~
~000000000000000000000000000~
~000000000000000000000000000~
~0000000000000, 0.777437524821~
~136041447958386087174831147~
~822934682708885214954348721~
~189125726027668861206620931~
~625366210937500000000000000~
~000000000000000000000000000~
~000000000000000000000000000~
~0000000000000000000000000}}

EuclideanDistance

`EuclideanDistance[`*u*`, `*v*`]`
: returns the euclidean distance between *u* and *v*.

>> `EuclideanDistance[-7, 5]`

12

>> `EuclideanDistance[{-1, -1}, {1, 1}]`

$2\sqrt{2}$

```
>> EuclideanDistance[{a, b}, {c, d}]
```

$$\sqrt{\text{Abs}[a-c]^2+\text{Abs}[b-d]^2}$$

Inverse

`Inverse[m]`
computes the inverse of the matrix m.

```
>> Inverse[{{1, 2, 0}, {2, 3, 0}, {3, 4, 1}}]
```

$\{\{-3,2,0\},\{2,-1,0\},\{1,-2,1\}\}$

```
>> Inverse[{{1, 0}, {0, 0}}]
```

Thematrix{{1,0},{0,0}}issingular.

Inverse [{{1,0}, {0,0}}]

```
>> Inverse[{{1, 0, 0}, {0, Sqrt[3]/2, 1/2}, {0,-1 / 2, Sqrt[3]/2}}]
```

$$\left\{\{1,0,0\},\left\{0,\frac{\sqrt{3}}{2},-\frac{1}{2}\right\},\left\{0,\frac{1}{2},\frac{\sqrt{3}}{2}\right\}\right\}$$

LeastSquares

`LeastSquares[m, b]`
computes the least squares solution to m $x = b$, finding an x that solves for b optimally.

```
>> LeastSquares[{{1, 2}, {2, 3}, {5, 6}}, {1, 5, 3}]
```

$$\left\{-\frac{28}{13},\frac{31}{13}\right\}$$

```
>> Simplify[LeastSquares[{{1, 2}, {2, 3}, {5, 6}}, {1, x, 3}]]
```

$$\left\{\frac{12}{13}-\frac{8x}{13},-\frac{4}{13}+\frac{7x}{13}\right\}$$

```
>> LeastSquares[{{1, 1, 1}, {1, 1, 2}}, {1, 3}]
```

Solvingforunderdeterminedsystemnotimplemented.

LeastSquares [{{1,1, 1}, {1,1,2}}, {1,3}]

LinearModelFit

`LinearModelFit[m, f, x]`
returns the design matrix.

```
>> m = LinearModelFit[{{2, 1}, {3, 4}, {5, 3}, {7, 6}}, x, x];
```

```
>> m["BasisFunctions"]
```

$\{1,x\}$

```
>> m["BestFit"]
```

$0.186441+0.779661x$

```
>> m["BestFitParameters"]
```

$\{0.186441,0.779661\}$

```
>> m["DesignMatrix"]
```

$\{\{1,2\},\{1,3\},\{1,5\},\{1,7\}\}$

```
>> m["Function"]
```

0.186441 + 0.779661#1&

```
>> m["Response"]
```

$\{1,4,3,6\}$

```
>> m["FitResiduals"]
```

$\{-0.745763,1.47458,-1.08475,0.355932\}$

```
>> m = LinearModelFit[{{2, 2, 1}, {3, 2, 4}, {5, 6, 3}, {7, 9, 6}}, {Sin[x], Cos[y]}, {x, y}];
```

```
>> m["BasisFunctions"]
```

{1, Sin [x], Cos [y]}

```
>> m["Function"]
```

3.33077 − 5.65221Cos [#2] − 5.01042Sin [#1] &

```
>> m = LinearModelFit[{{{1, 4}, {1, 5}, {1, 7}}, {1, 2, 3}}];
```

```
>> m["BasisFunctions"]
```

{#1, #2}

```
>> m["FitResiduals"]
```

$\{-0.142857, 0.214286, -0.0714286\}$

LinearSolve

`LinearSolve[`*matrix*`, `*right*`]`
: solves the linear equation system *matrix* . *x* = *right* and returns one corresponding solution *x*.

```
>> LinearSolve[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}, {1, 2, 3}]
```

$\{0, 1, 2\}$

Test the solution:

```
>> {{1, 1, 0}, {1, 0, 1}, {0, 1, 1}} . {0, 1, 2}
```

$\{1, 2, 3\}$

If there are several solutions, one arbitrary solution is returned:

```
>> LinearSolve[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}, {1, 1, 1}]
```

$\{-1, 1, 0\}$

Infeasible systems are reported:

```
>> LinearSolve[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}, {1, -2, 3}]
```

Linearequationencounteredthathasnosolution.

LinearSolve $[\{\{1,2,3\},\{4,5,6\},\{7,8,9\}\},\{1,-2,3\}]$

ManhattanDistance

`ManhattanDistance[`*u*`, `*v*`]`
: returns the Manhattan distance between *u* and *v*, which is the number of horizontal or vertical moves in the gridlike Manhattan city layout to get from *u* to *v*.

```
>> ManhattanDistance[-7, 5]
```

12

```
>> ManhattanDistance[{-1, -1}, {1, 1}]
```

4

MatrixExp

`MatrixExp[`*m*`]`
: computes the exponential of the matrix *m*.

```
>> MatrixExp[{{0, 2}, {0, 1}}]
```

$\{\{1, -2+2E\}, \{0, E\}\}$

```
>> MatrixExp[{{1.5, 0.5}, {0.5, 2.0}}]
```

$\{\{5.16266, 3.02952\}, \{3.02952, 8.19218\}\}$

MatrixPower

`MatrixPower[`*m*`, `*n*`]`
: computes the *n*th power of a matrix *m*.

```
>> MatrixPower[{{1, 2}, {1, 1}}, 10]
```

$\{\{3\,363, 4\,756\}, \{2\,378, 3\,363\}\}$

```
>> MatrixPower[{{1, 2}, {2, 5}}, -3]
```

$\{\{169, -70\}, \{-70, 29\}\}$

MatrixRank

`MatrixRank[`*matrix*`]`
: returns the rank of *matrix*.

```
>> MatrixRank[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}]
```

2

```
>> MatrixRank[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}]
```

3

```
>> MatrixRank[{{a, b}, {3 a, 3 b}}]
```

1

Norm

`Norm[`m`, `l`]`
: computes the l-norm of matrix m (currently only works for vectors!).

`Norm[`m`]`
: computes the 2-norm of matrix m (currently only works for vectors!).

```
>> Norm[{1, 2, 3, 4}, 2]
```
$\sqrt{30}$

```
>> Norm[{10, 100, 200}, 1]
```
310

```
>> Norm[{a, b, c}]
```
$\sqrt{\text{Abs}\,[a]^2 + \text{Abs}\,[b]^2 + \text{Abs}\,[c]^2}$

```
>> Norm[{-100, 2, 3, 4}, Infinity]
```
100

```
>> Norm[1 + I]
```
$\sqrt{2}$

Normalize

`Normalize[`v`]`
: calculates the normalized vector v.

`Normalize[`z`]`
: calculates the normalized complex number z.

```
>> Normalize[{1, 1, 1, 1}]
```
$\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}$

```
>> Normalize[1 + I]
```
$\left(\frac{1}{2} + \frac{I}{2}\right)\sqrt{2}$

NullSpace

`NullSpace[`$matrix$`]`
: returns a list of vectors that span the nullspace of $matrix$.

```
>> NullSpace[{{1, 2, 3}, {4, 5, 6},
    {7, 8, 9}}]
```
$\{\{1, -2, 1\}\}$

```
>> A = {{1, 1, 0}, {1, 0, 1}, {0,
   1, 1}};
```

```
>> NullSpace[A]
```
$\{\}$

```
>> MatrixRank[A]
```
3

PseudoInverse

`PseudoInverse[`m`]`
: computes the Moore-Penrose pseudoinverse of the matrix m. If m is invertible, the pseudoinverse equals the inverse.

```
>> PseudoInverse[{{1, 2}, {2, 3},
   {3, 4}}]
```
$\left\{\left\{-\frac{11}{6}, -\frac{1}{3}, \frac{7}{6}\right\}, \left\{\frac{4}{3}, \frac{1}{3}, -\frac{2}{3}\right\}\right\}$

```
>> PseudoInverse[{{1, 2, 0}, {2, 3,
    0}, {3, 4, 1}}]
```
$\{\{-3, 2, 0\}, \{2, -1, 0\}, \{1, -2, 1\}\}$

```
>> PseudoInverse[{{1.0, 2.5}, {2.5,
    1.0}}]
```
$\{\{-0.190476, 0.47619\},$
$\{0.47619, -0.190476\}\}$

QRDecomposition

`QRDecomposition[`m`]`
: computes the QR decomposition of the matrix m.

```
>> QRDecomposition[{{1, 2}, {3, 4},
    {5, 6}}]
```

$$\left\{\left\{\left\{\frac{\sqrt{35}}{35},\frac{3\sqrt{35}}{35},\frac{\sqrt{35}}{7}\right\},\left\{\frac{13\sqrt{210}}{210},\frac{2\sqrt{210}}{105},-\frac{\sqrt{210}}{42}\right\}\right\},\left\{\left\{\sqrt{35},\frac{44\sqrt{35}}{35}\right\},\left\{0,\frac{2\sqrt{210}}{35}\right\}\right\}\right\}$$

RowReduce

`RowReduce[`*matrix*`]`
returns the reduced row-echelon form of *matrix*.

```
>> RowReduce[{{1, 0, a}, {1, 1, b
   }}]
```

$\{\{1,0,a\},\{0,1,-a+b\}\}$

```
>> RowReduce[{{1, 2, 3}, {4, 5, 6},
    {7, 8, 9}}] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

SingularValueDecomposition

`SingularValueDecomposition[`*m*`]`
calculates the singular value decomposition for the matrix *m*.

`SingularValueDecomposition` returns u, s, w such that m=u s v, uu=1, vv=1, and s is diagonal.

```
>> SingularValueDecomposition
   [{{1.5, 2.0}, {2.5, 3.0}}]
```

$\{\{\{0.538954,0.842335\},\{0.842335,-0.538954\}\},\{\{4.63555,0.\},\{0.,0.107862\}\},\{\{0.628678,0.777666\},\{-0.777666,0.628678\}\}\}$

SquaredEuclideanDistance

`SquaredEuclideanDistance[`*u*`, `*v*`]`
returns squared the euclidean distance between *u* and *v*.

```
>> SquaredEuclideanDistance[-7, 5]
```

144

```
>> SquaredEuclideanDistance[{-1,
   -1}, {1, 1}]
```

8

Tr

`Tr[`*m*`]`
computes the trace of the matrix *m*.

```
>> Tr[{{1, 2, 3}, {4, 5, 6}, {7, 8,
    9}}]
```

15

Symbolic trace:

```
>> Tr[{{a, b, c}, {d, e, f}, {g, h,
    i}}]
```

$a+e+i$

VectorAngle

`VectorAngle[`*u*`, `*v*`]`
gives the angles between vectors *u* and *v*

```
>> VectorAngle[{1, 0}, {0, 1}]
```

$\frac{\text{Pi}}{2}$

```
>> VectorAngle[{1, 2}, {3, 1}]
```

$\frac{\text{Pi}}{4}$

```
>> VectorAngle[{1, 1, 0}, {1, 0,
   1}]
```

$\frac{\text{Pi}}{3}$

56. Number theoretic functions

Contents

ContinuedFraction

`ContinuedFraction[`*x*`, `*n*`]`
: generate the first *n* terms in the continued fraction reprentation of *x*.

`ContinuedFraction[`*x*`]`
: the complete continued fraction representation for a rational or quadradic irrational number.

>> **`ContinuedFraction[Pi, 10]`**

$\{3, 7, 15, 1, 292, 1, 1, 1, 2, 1\}$

>> **`ContinuedFraction[(1 + 2 Sqrt[3])/5]`**

$\{0, 1, \{8, 3, 34, 3\}\}$

>> **`ContinuedFraction[Sqrt[70]]`**

$\{8, \{2, 1, 2, 1, 2, 16\}\}$

Divisors

`Divisors[`*n*`]`
: returns a list of the integers that divide *n*.

>> **`Divisors[96]`**

$\{1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96\}$

>> **`Divisors[704]`**

$\{1, 2, 4, 8, 11, 16, 22, 32, 44, 64, 88, 176, 352, 704\}$

>> **`Divisors[{87, 106, 202, 305}]`**

$\{\{1, 3, 29, 87\}, \{1, 2, 53, 106\}, \{1, 2, 101, 202\}, \{1, 5, 61, 305\}\}$

FactorInteger

`FactorInteger[`*n*`]`
: returns the factorization of *n* as a list of factors and exponents.

>> **`factors = FactorInteger[2010]`**

$\{\{2, 1\}, \{3, 1\}, \{5, 1\}, \{67, 1\}\}$

To get back the original number:

>> **`Times @@ Power @@@ factors`**

2 010

`FactorInteger` factors rationals using negative exponents:

>> **`FactorInteger[2010 / 2011]`**

$\{\{2, 1\}, \{3, 1\}, \{5, 1\}, \{67, 1\}, \{2\,011, -1\}\}$

FractionalPart

`FractionalPart[`*n*`]`
: finds the fractional part of *n*.

>> **`FractionalPart[4.1]`**

0.1

>> `FractionalPart[-5.25]`

-0.25

FromContinuedFraction

`FromContinuedFraction[`*list*`]`
: reconstructs a number from the list of its continued fraction terms.

>> `FromContinuedFraction[{3, 7, 15, 1, 292, 1, 1, 1, 2, 1}]`

$\frac{1146408}{364913}$

>> `FromContinuedFraction[Range[5]]`

$\frac{225}{157}$

IntegerExponent

`IntegerExponent[`*n*`, `*b*`]`
: gives the highest exponent of b that divides n.

>> `IntegerExponent[16, 2]`

4

>> `IntegerExponent[-510000]`

4

>> `IntegerExponent[10, b]`

IntegerExponent $[10, b]$

MantissaExponent

`MantissaExponent[`*n*`]`
: finds a list containing the mantissa and exponent of a given number n.

`MantissaExponent[`*n*`, `*b*`]`
: finds the base b mantissa and exponent of n.

>> `MantissaExponent[2.5*10^20]`

$\{0.25, 21\}$

>> `MantissaExponent[125.24]`

$\{0.12524, 3\}$

>> `MantissaExponent[125., 2]`

$\{0.976563, 7\}$

>> `MantissaExponent[10, b]`

MantissaExponent $[10, b]$

NextPrime

`NextPrime[`*n*`]`
: gives the next prime after n.

`NextPrime[`*n*`,`*k*`]`
: gives the kth prime after n.

>> `NextPrime[10000]`

10 007

>> `NextPrime[100, -5]`

73

>> `NextPrime[10, -5]`

-2

>> `NextPrime[100, 5]`

113

>> `NextPrime[5.5, 100]`

563

>> `NextPrime[5, 10.5]`

NextPrime $[5, 10.5]$

PartitionsP

`PartitionsP[`*n*`]`
: return the number $p(n)$ of unrestricted partitions of the integer n.

>> `Table[PartitionsP[k], {k, -2, 12}]`

$\{0, 0, 1, 1, 2, 3, 5, 7, 11, 15, 22, 30, 42, 56, 77\}$

Prime

`Prime[n]`
`Prime[{n0, n1, ...}]`
: returns the *n*th prime number where *n* is an positive Integer. If given a list of integers, the return value is a list with `Prime` applied to each.

Note that the first prime is 2, not 1:

```
>> Prime[1]
```
2

```
>> Prime[167]
```
991

When given a list of integers, a list is returned:

```
>> Prime[{5, 10, 15}]
```
{11, 29, 47}

1.2 isn't an integer

```
>> Prime[1.2]
```
Prime [1.2]

Since 0 is less than 1, like 1.2 it is invalid.

```
>> Prime[{0, 1, 1.2, 3}]
```
{Prime [0] , 2, Prime [1.2] , 5}

PrimePi

`PrimePi[x]`
: gives the number of primes less than or equal to *x*.

PrimePi is the inverse of Prime:

```
>> PrimePi[2]
```
1

```
>> PrimePi[100]
```
25

```
>> PrimePi[-1]
```
0

```
>> PrimePi[3.5]
```
2

```
>> PrimePi[E]
```
1

PrimePowerQ

`PrimePowerQ[n]`
: returns `True` if *n* is a power of a prime number.

```
>> PrimePowerQ[9]
```
True

```
>> PrimePowerQ[52142]
```
False

```
>> PrimePowerQ[-8]
```
True

```
>> PrimePowerQ[371293]
```
True

RandomPrime

`RandomPrime[{imin, $imax}]`
: gives a random prime between *imin* and *imax*.

`RandomPrime[imax]`
: gives a random prime between 2 and *imax*.

`RandomPrime[range, n]`
: gives a list of *n* random primes in *range*.

```
>> RandomPrime[{14, 17}]
```
17

```
>> RandomPrime[{14, 16}, 1]
```
Therearenoprimesinthespecifiedinterval.

RandomPrime [{14, 16} , 1]

```
>> RandomPrime[{8,12}, 3]
```
{11, 11, 11}

```
>> RandomPrime[{10,30}, {2,5}]
```
{{19, 19, 19, 19, 19} , {19, 19, 19, 19, 19}}

57. Random number generation

Random numbers are generated using the Mersenne Twister.

Contents

Random

Legacy function. Superseded by RandomReal, RandomInteger and RandomComplex.

RandomChoice

`RandomChoice[`*items*`]`
: randomly picks one item from *items*.

`RandomChoice[`*items*`, `*n*`]`
: randomly picks *n* items from *items*. Each pick in the *n* picks happens from the given set of *items*, so each item can be picked any number of times.

`RandomChoice[`*items*`, {`*n1*`, `*n2*`, ...}]`
: randomly picks items from *items* and arranges the picked items in the nested list structure described by {*n1*, *n2*, ...}.

`RandomChoice[`*weights*` -> `*items*`, `*n*`]`
: randomly picks *n* items from *items* and uses the corresponding numeric values in *weights* to determine how probable it is for each item in *items* to get picked (in the long run, items with higher weights will get picked more often than ones with lower weight).

`RandomChoice[`*weights*` -> `*items*`]`
: randomly picks one items from *items* using weights *weights*.

`RandomChoice[`*weights*` -> `*items*`, {`*n1*`, `*n2*`, ...}]`
: randomly picks a structured list of items from *items* using weights *weights*.

```
>> SeedRandom[42]
```

```
>> RandomChoice[{a, b, c}]
```
$\{c\}$

```
>> SeedRandom[42]
```

```
>> RandomChoice[{a, b, c}, 20]
```
$\{c,a,c,c,a,a,c,b,c,c,c,c,a,c,b,a,b,b,b,b\}$

```
>> SeedRandom[42]
```

```
>> RandomChoice[{"a", {1, 2}, x, {}}, 10]
```
{x, {}, a, x, x, {}, a, a, x, {1,2}}

```
>> SeedRandom[42]
```

```
>> RandomChoice[{a, b, c}, {5, 2}]
```
$\{\{c,a\},\{c,c\},\{a,a\},\{c,b\},\{c,c\}\}$

```
>> SeedRandom[42]
```

```
>> RandomChoice[{1, 100, 5} -> {a, b, c}, 20]
```
$\{b,b,b,b,b,b,b,b,b,b,b,c,b,b,b,b,b,b,b,b\}$

RandomComplex

`RandomComplex[{`*z_min*`, `*z_max*`}]`
: yields a pseudorandom complex number in the rectangle with complex corners *z_min* and *z_max*.

`RandomComplex[`*z_max*`]`
: yields a pseudorandom complex number in the rectangle with corners at the origin and at *z_max*.

`RandomComplex[]`
: yields a pseudorandom complex number with real and imaginary parts from 0 to 1.

`RandomComplex[`*range*`, `*n*`]`
: gives a list of *n* pseudorandom complex numbers.

`RandomComplex[`*range*`, {`*n1*`, `*n2*`, ...}]`
: gives a nested list of pseudorandom complex numbers.

>> **`RandomComplex[]`**

$0.223828 + 0.430861I$

>> **`RandomComplex[{1+I, 5+5I}]`**

$2.15219 + 2.5964I$

>> **`RandomComplex[1+I, 5]`**

$\{0.651258 + 0.113927I, 0.480\tilde{}$
$\tilde{}653 + 0.190907I, 0.351188 +$
$0.0222305I, 0.850087 + 0.817\tilde{}$
$\tilde{}387I, 0.796483 + 0.351061I\}$

>> **`RandomComplex[{1+I, 2+2I}, {2, 2}]`**

$\{\{1.47823 + 1.16764I, 1.536\tilde{}$
$\tilde{}9 + 1.44804I\}, \{1.37622 +$
$1.3907I, 1.04106 + 1.25756I\}\}$

RandomInteger

`RandomInteger[{`*min*`, `*max*`}]`
: yields a pseudorandom integer in the range from *min* to *max* inclusive.

`RandomInteger[`*max*`]`
: yields a pseudorandom integer in the range from 0 to *max* inclusive.

`RandomInteger[]`
: gives 0 or 1.

`RandomInteger[`*range*`, `*n*`]`
: gives a list of *n* pseudorandom integers.

`RandomInteger[`*range*`, {`*n1*`, `*n2*`, ...}]`
: gives a nested list of pseudorandom integers.

>> **`RandomInteger[{1, 5}]`**

1

>> **`RandomInteger[100, {2, 3}] // TableForm`**

32	85	5
25	33	75

Calling `RandomInteger` changes `$RandomState`:

>> **`previousState = $RandomState;`**

>> **`RandomInteger[]`**

0

>> **`$RandomState != previousState`**

True

RandomReal

`RandomReal[{`*min*`, `*max*`}]`
: yields a pseudorandom real number in the range from *min* to *max*.

`RandomReal[`*max*`]`
: yields a pseudorandom real number in the range from 0 to *max*.

`RandomReal[]`
: yields a pseudorandom real number in the range from 0 to 1.

`RandomReal[`*range*`, `*n*`]`
: gives a list of *n* pseudorandom real numbers.

`RandomReal[`*range*`, {`*n1*`, `*n2*`, ...}]`
: gives a nested list of pseudorandom real numbers.

```
>> RandomReal[]
   0.133018

>> RandomReal[{1, 5}]
   4.29275
```

RandomSample

`RandomSample[`*items*`]`
: randomly picks one item from *items*.

`RandomSample[`*items*`, `*n*`]`
: randomly picks *n* items from *items*. Each pick in the *n* picks happens after the previous items picked have been removed from *items*, so each item can be picked at most once.

`RandomSample[`*items*`, {`*n1*`, `*n2*`, ...}]`
: randomly picks items from *items* and arranges the picked items in the nested list structure described by {*n1*, *n2*, ...}. Each item gets picked at most once.

`RandomSample[`*weights*` -> `*items*`, `*n*`]`
: randomly picks *n* items from *items* and uses the corresponding numeric values in *weights* to determine how probable it is for each item in *items* to get picked (in the long run, items with higher weights will get picked more often than ones with lower weight). Each item gets picked at most once.

`RandomSample[`*weights*` -> `*items*`]`
: randomly picks one items from *items* using weights *weights*. Each item gets picked at most once.

`RandomSample[`*weights*` -> `*items*`, {`*n1*`, `*n2*`, ...}]`
: randomly picks a structured list of items from *items* using weights *weights*. Each item gets picked at most once.

```
>> SeedRandom[42]

>> RandomSample[{a, b, c}]
   {a}

>> SeedRandom[42]

>> RandomSample[{a, b, c, d, e, f, g, h}, 7]
   {b, f, a, h, c, e, d}

>> SeedRandom[42]

>> RandomSample[{"a", {1, 2}, x, {}}, 3]
   {{1,2}, {}, a}

>> SeedRandom[42]

>> RandomSample[Range[100], {2, 3}]
   {{84, 54, 71}, {46, 45, 40}}

>> SeedRandom[42]

>> RandomSample[Range[100] -> Range[100], 5]
   {62, 98, 86, 78, 40}
```

$RandomState

`$RandomState`
: is a long number representing the internal state of the pseudorandom number generator.

```
>> Mod[$RandomState, 10^100]
   1 922 741 579 711 707 052 868 202~
   ~168 987 310 452 796 849 361 011~
   ~441 840 662 726 639 187 953 358~
   ~001 669 151 583 460 868 769 255 470

>> IntegerLength[$RandomState]
   6 466
```

So far, it is not possible to assign values to `$RandomState`.

```
>> $RandomState = 42
   Itisnotpossibletochangetherandomstate.
   42
```

Not even to its own value:

```
>> $RandomState = $RandomState;
   Itisnotpossibletochangetherandomstate.
```

SeedRandom

`SeedRandom[`*n*`]`
: resets the pseudorandom generator with seed *n*.

`SeedRandom[]`
: uses the current date and time as the seed.

`SeedRandom` can be used to get reproducible random numbers:

```
>> SeedRandom[42]

>> RandomInteger[100]
   51

>> RandomInteger[100]
   92

>> SeedRandom[42]

>> RandomInteger[100]
   51

>> RandomInteger[100]
   92
```

String seeds are supported as well:

```
>> SeedRandom["Mathics"]

>> RandomInteger[100]
   27
```

Calling `SeedRandom` without arguments will seed the random number generator to a random state:

```
>> SeedRandom[]

>> RandomInteger[100]
   86
```

58. Special Functions

There are a number of functions found in mathematical physics and found in standard handbooks.

One thing to note is that the technical literature often contains several conflicting definitions. So beware and check for conformance with the Mathics documentation.

A number of special functions can be evaluated for arbitrary complex values of their arguments. However defining relations may apply only for some special choices of arguments. Here, the full function corresponds to an extension or "analytic continuation" of the defining relation.

For example, integral representations of functions are only valid when the integral exists, but the functions can usually be defined b by analytic continuation.

59. Bessel and Related Functions

Contents

AiryAi

`AiryAi[`*x*`]`
returns the Airy function Ai(x).

Exact values:

```
>> AiryAi[0]
```

$$\frac{3^{\frac{1}{3}}}{3\text{Gamma}\left[\frac{2}{3}\right]}$$

`AiryAi` can be evaluated numerically:

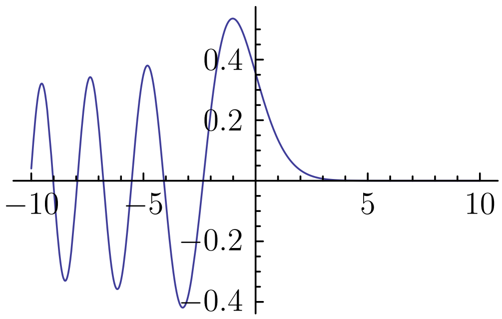
```
>> AiryAi[0.5]
```

0.231694

```
>> AiryAi[0.5 + I]
```

$0.157118 - 0.24104I$

```
>> Plot[AiryAi[x], {x, -10, 10}]
```



AiryAiPrime

`AiryAiPrime[`*x*`]`
returns the derivative of the Airy function `AiryAi[`*x*`]`.

Exact values:

```
>> AiryAiPrime[0]
```

$$-\frac{3^{\frac{2}{3}}}{3\text{Gamma}\left[\frac{1}{3}\right]}$$

Numeric evaluation:

```
>> AiryAiPrime[0.5]
```

-0.224911

AiryAiZero

`AiryAiZero[`*k*`]`
returns the kth zero of the Airy function Ai(z).

```
>> N[AiryAiZero[1]]
```

-2.33811

AiryBi

`AiryBi[`x`]`
: returns the Airy function of the second kind Bi(x).

Exact values:

```
>> AiryBi[0]
```

$$\frac{3^{\frac{5}{6}}}{3\text{Gamma}\left[\frac{2}{3}\right]}$$

Numeric evaluation:

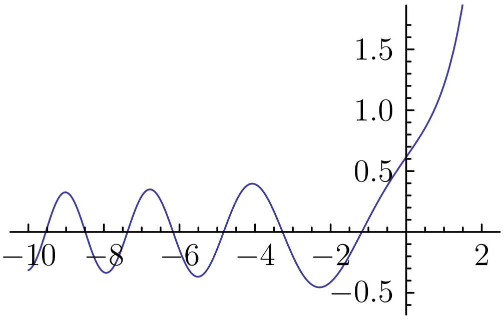
```
>> AiryBi[0.5]
```

0.854277

```
>> AiryBi[0.5 + I]
```

$0.688145 + 0.370815I$

```
>> Plot[AiryBi[x], {x, -10, 2}]
```



AiryBiPrime

`AiryBiPrime[`x`]`
: returns the derivative of the Airy function of the second kind `AiryBi[`x`]`.

Exact values:

```
>> AiryBiPrime[0]
```

$$\frac{3^{\frac{1}{6}}}{\text{Gamma}\left[\frac{1}{3}\right]}$$

Numeric evaluation:

```
>> AiryBiPrime[0.5]
```

0.544573

AiryBiZero

`AiryBiZero[`k`]`
: returns the kth zero of the Airy function Bi(z).

```
>> N[AiryBiZero[1]]
```

-1.17371

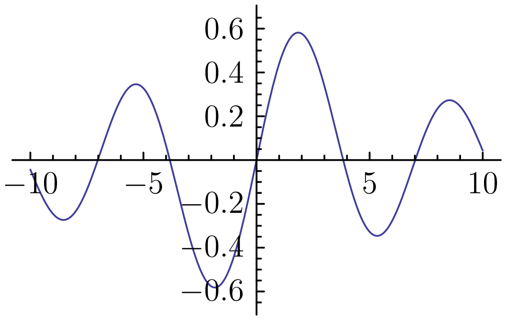
AngerJ

`AngerJ[`n`, `z`]`
: returns the Anger function J_n(z).

```
>> AngerJ[1.5, 3.5]
```

0.294479

```
>> Plot[AngerJ[1, x], {x, -10, 10}]
```



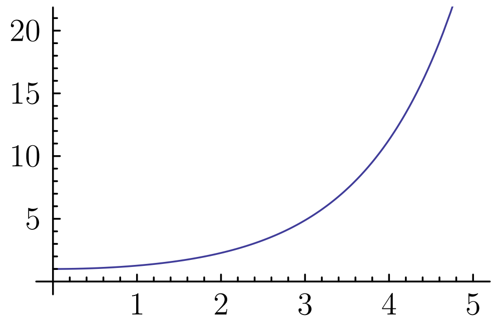
BesselI

`BesselI[`n`, `z`]`
: returns the modified Bessel function of the first kind I_n(z).

```
>> BesselI[1.5, 4]
```

8.17263

```
>> Plot[BesselI[0, x], {x, 0, 5}]
```

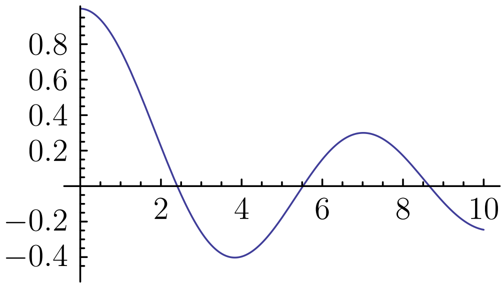


BesselJ

`BesselJ[`n`, `z`]`
: returns the Bessel function of the first kind J_n(z).

```
>> BesselJ[0, 5.2]
```
−0.11029

```
>> D[BesselJ[n, z], z]
```
$$-\frac{\text{BesselJ}[1+n,z]}{2}+\frac{\text{BesselJ}[-1+n,z]}{2}$$

```
>> Plot[BesselJ[0, x], {x, 0, 10}]
```


BesselJZero

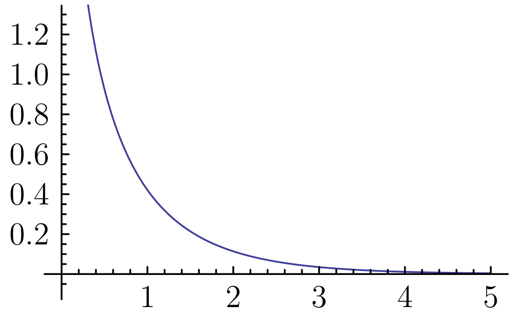
`BesselJZero[`n`, `k`]`
: returns the kth zero of the Bessel function of the first kind J_n(z).

```
>> N[BesselJZero[0, 1]]
```
2.40483

BesselK

`BesselK[`n`, `z`]`
: returns the modified Bessel function of the second kind K_n(z).

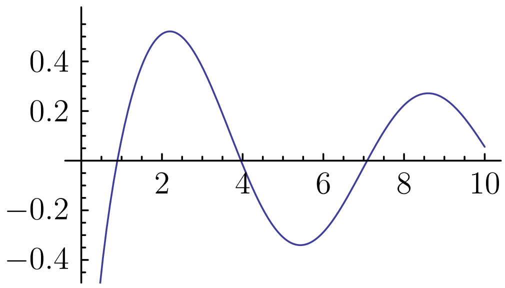
```
>> BesselK[1.5, 4]
```
0.014347

```
>> Plot[BesselK[0, x], {x, 0, 5}]
```


BesselY

`BesselY[`n`, `z`]`
: returns the Bessel function of the second kind Y_n(z).

```
>> BesselY[1.5, 4]
```
0.367112

```
>> Plot[BesselY[0, x], {x, 0, 10}]
```


BesselYZero

`BesselYZero[`n`, `k`]`
: returns the kth zero of the Bessel function of the second kind Y_n(z).

```
>> N[BesselYZero[0, 1]]
```
0.893577

HankelH1

`HankelH1[`n`, `z`]`
: returns the Hankel function of the first kind H_n^1 (z).

>> HankelH1[1.5, 4]
0.185286 + 0.367112I

HankelH2

HankelH2[n, z]
  returns the Hankel function of the second kind H_n^2 (z).

>> HankelH2[1.5, 4]
0.185286 − 0.367112I

KelvinBei

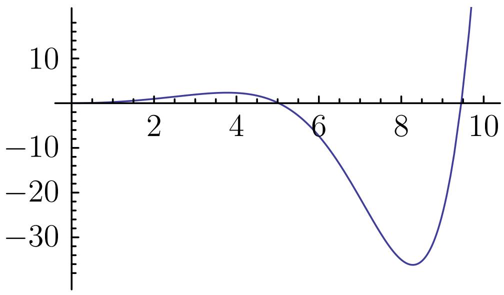
KelvinBei[z]
  returns the Kelvin function bei(z).
KelvinBei[n, z]
  returns the Kelvin function bei_n(z).

>> KelvinBei[0.5]
0.0624932

>> KelvinBei[1.5 + I]
0.326323 + 0.755606I

>> KelvinBei[0.5, 0.25]
0.370153

>> Plot[KelvinBei[x], {x, 0, 10}]



KelvinBer

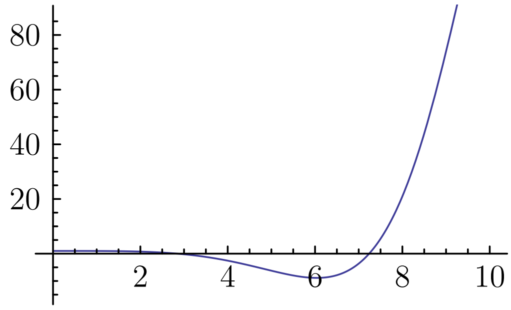
KelvinBer[z]
  returns the Kelvin function ber(z).
KelvinBer[n, z]
  returns the Kelvin function ber_n(z).

>> KelvinBer[0.5]
0.999023

>> KelvinBer[1.5 + I]
1.1162 − 0.117944I

>> KelvinBer[0.5, 0.25]
0.148824

>> Plot[KelvinBer[x], {x, 0, 10}]



KelvinKei

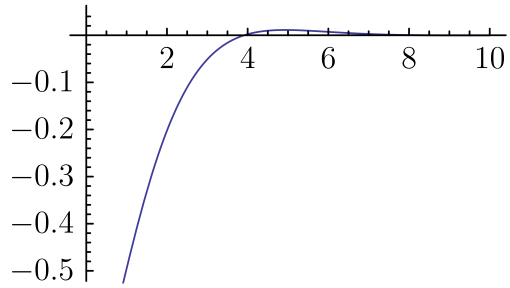
KelvinKei[z]
  returns the Kelvin function kei(z).
KelvinKei[n, z]
  returns the Kelvin function kei_n(z).

>> KelvinKei[0.5]
−0.671582

>> KelvinKei[1.5 + I]
−0.248994 + 0.303326I

>> KelvinKei[0.5, 0.25]
−2.0517

>> Plot[KelvinKei[x], {x, 0, 10}]



KelvinKer

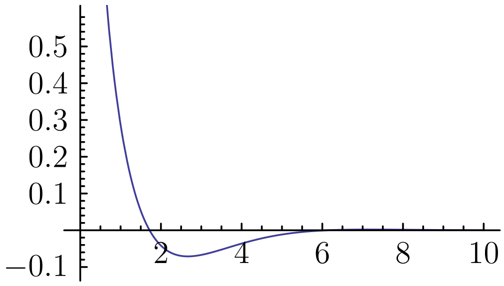
KelvinKer[z]
  returns the Kelvin function ker(z).
KelvinKer[n, z]
  returns the Kelvin function ker_n(z).

>> `KelvinKer[0.5]`
0.855906

>> `KelvinKer[1.5 + I]`
$-0.167162 - 0.184404I$

>> `KelvinKer[0.5, 0.25]`
0.450023

>> `Plot[KelvinKer[x], {x, 0, 10}]`

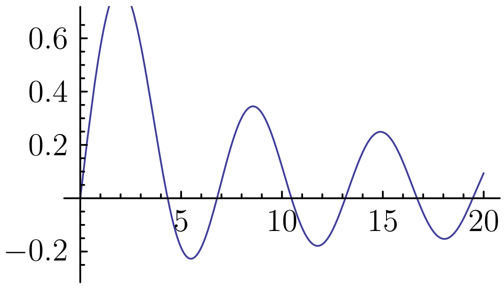


StruveH

`StruveH[n, z]`
returns the Struve function H_n(z).

>> `StruveH[1.5, 3.5]`
1.13192

>> `Plot[StruveH[0, x], {x, 0, 20}]`

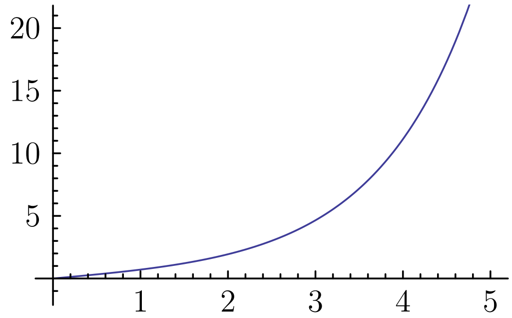


StruveL

`StruveL[n, z]`
returns the modified Struve function L_n(z).

>> `StruveL[1.5, 3.5]`
4.41126

>> `Plot[StruveL[0, x], {x, 0, 5}]`

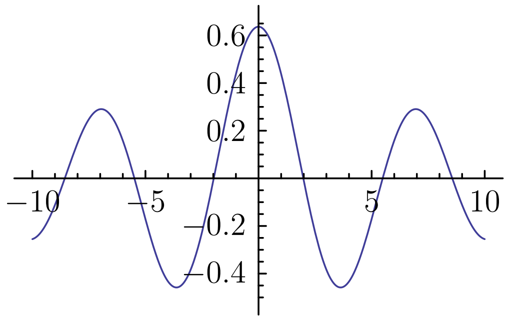


WeberE

`WeberE[n, z]`
returns the Weber function E_n(z).

>> `WeberE[1.5, 3.5]`
−0.397256

>> `Plot[WeberE[1, x], {x, -10, 10}]`



60. Error Function and Related Functions

Contents

Erf

`Erf[`*z*`]`
: returns the error function of *z*.

`Erf[`*z0*`, `*z1*`]`
: returns the result of `Erf[`*z1*`] - Erf[`*z0*`]`.

`Erf[`*x*`]` is an odd function:

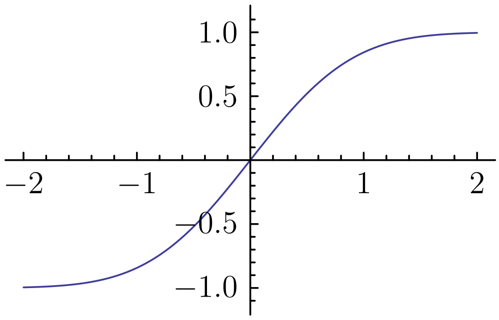
```
>> Erf[-x]
```
$-\text{Erf}[x]$

```
>> Erf[1.0]
```
0.842701

```
>> Erf[0]
```
0

```
>> {Erf[0, x], Erf[x, 0]}
```
$\{\text{Erf}[x], -\text{Erf}[x]\}$

```
>> Plot[Erf[x], {x, -2, 2}]
```



Erfc

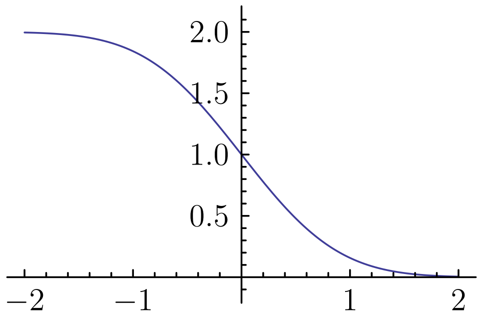
`Erfc[`*z*`]`
: returns the complementary error function of *z*.

```
>> Erfc[-x] / 2
```
$$\frac{2-\text{Erfc}[x]}{2}$$

```
>> Erfc[1.0]
```
0.157299

```
>> Erfc[0]
```
1

```
>> Plot[Erfc[x], {x, -2, 2}]
```



FresnelC

`FresnelC[`*z*`]`
: is the Fresnel C integral $C(z)$.

```
>> FresnelC[{0, Infinity}]
```
$$\left\{0, \frac{1}{2}\right\}$$

```
>> Integrate[Cos[x^2 Pi/2], {x, 0, z}]
```
FresnelC[z]

FresnelS

`FresnelS[`z`]`
: is the Fresnel S integral $S(z)$.

```
>> FresnelS[{0, Infinity}]
```

$\left\{0, \frac{1}{2}\right\}$

```
>> Integrate[Sin[x^2 Pi/2], {x, 0,
z}]
```

FresnelS [z]

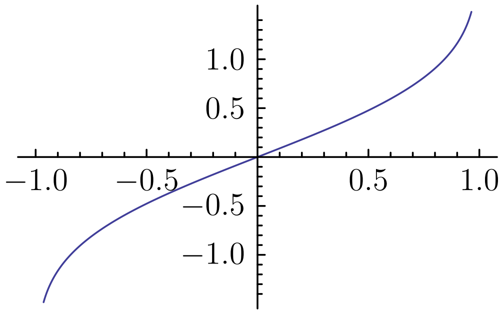
InverseErf

`InverseErf[`z`]`
: returns the inverse error function of z.

```
>> InverseErf /@ {-1, 0, 1}
```

$\{-\infty, 0, \infty\}$

```
>> Plot[InverseErf[x], {x, -1, 1}]
```



`InverseErf[`z`]` only returns numeric values for `-1 <= `z` <= 1`:

```
>> InverseErf /@ {0.9, 1.0, 1.1}
```

$\{1.16309, \infty,$ InverseErf [1.1]$\}$

InverseErfc

`InverseErfc[`z`]`
: returns the inverse complementary error function of z.

```
>> InverseErfc /@ {0, 1, 2}
```

$\{\infty, 0, -\infty\}$

61. Exponential Integral and Special Functions

Contents

ExpIntegralE

`ExpIntegralE[`n`, `z`]`
: returns the exponential integral function $E_n(z)$.

```
>> ExpIntegralE[2.0, 2.0]
   0.0375343
```

ExpIntegralEi

`ExpIntegralEi[`z`]`
: returns the exponential integral function $Ei(z)$.

```
>> ExpIntegralEi[2.0]
   4.95423
```

ProductLog

`ProductLog[`z`]`
: returns the value of the Lambert W function at z.

The defining equation:

```
>> z == ProductLog[z] * E ^
   ProductLog[z]
   True
```

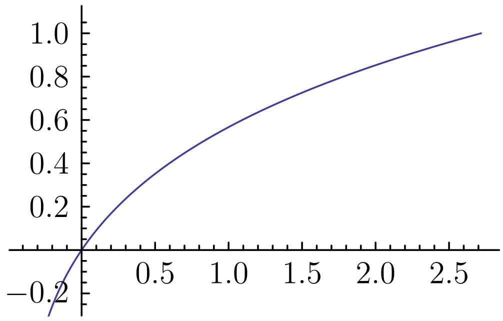
Some special values:

```
>> ProductLog[0]
   0
```

```
>> ProductLog[E]
   1
```

The graph of `ProductLog`:

```
>> Plot[ProductLog[x], {x, -1/E, E
   }]
```



62. Gamma and Related Functions

Contents

Gamma

In number theory the logarithm of the gamma function often appears. For positive real numbers, this can be evaluated as `Log[Gamma[z]]`.

`Gamma[`*z*`]`
: is the gamma function on the complex number *z*.

`Gamma[`*z*`, `*x*`]`
: is the upper incomplete gamma function.

`Gamma[`*z*`, `*x0*`, `*x1*`]`
: is equivalent to `Gamma[`*z*`, `*x0*`] - Gamma[`*z*`, `*x1*`]`.

`Gamma[`*z*`]` is equivalent to `(`*z*` - 1)!`:

```
>> Simplify[Gamma[z] - (z - 1)!]
```
0

Exact arguments:

```
>> Gamma[8]
```
5 040

```
>> Gamma[1/2]
```
$\sqrt{\text{Pi}}$

```
>> Gamma[1, x]
```
E^{-x}

```
>> Gamma[0, x]
```
ExpIntegralE $[1, x]$

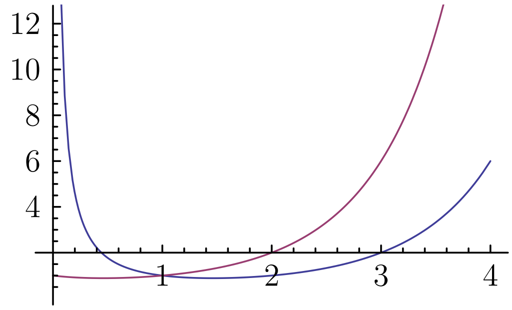
Numeric arguments:

```
>> Gamma[123.78]
```
4.21078×10^{204}

```
>> Gamma[1. + I]
```
$0.498016 - 0.154951 I$

Both `Gamma` and `Factorial` functions are continuous:

```
>> Plot[{Gamma[x], x!}, {x, 0, 4}]
```



Pochhammer

The Pochhammer symbol or rising factorial often appears in series expansions for hypergeometric functions. The Pochammer symbol has a definie value even when the gamma functions which appear in its definition are infinite.

`Pochhammer[`*a*`, `*n*`]`
: is the Pochhammer symbol (a)_n.

```
>> Pochhammer[4, 8]
```
6 652 800

63. Orthogonal Polynomials

Contents

ChebyshevT

`ChebyshevT[n, x]`
: returns the Chebyshev polynomial of the first kind T_n(x).

```
>> ChebyshevT[8, x]
```
$1 - 32x^2 + 160x^4 - 256x^6 + 128x^8$

```
>> ChebyshevT[1 - I, 0.5]
```
$0.800143 + 1.08198I$

ChebyshevU

`ChebyshevU[n, x]`
: returns the Chebyshev polynomial of the second kind U_n(x).

```
>> ChebyshevU[8, x]
```
$1 - 40x^2 + 240x^4 - 448x^6 + 256x^8$

```
>> ChebyshevU[1 - I, 0.5]
```
$1.60029 + 0.721322I$

GegenbauerC

`GegenbauerC[n, m, x]`
: returns the Gegenbauer polynomial C_n^(m)(x).

```
>> GegenbauerC[6, 1, x]
```
$-1 + 24x^2 - 80x^4 + 64x^6$

```
>> GegenbauerC[4 - I, 1 + 2 I, 0.7]
```
$-3.2621 - 24.9739I$

HermiteH

`HermiteH[n, x]`
: returns the Hermite polynomial H_n(x).

```
>> HermiteH[8, x]
```
$1\,680 - 13\,440x^2 + 13\,440x^4 - 3\,584x^6 + 256x^8$

```
>> HermiteH[3, 1 + I]
```
$-28 + 4I$

```
>> HermiteH[4.2, 2]
```
77.5291

JacobiP

`JacobiP[n, a, b, x]`
: returns the Jacobi polynomial P_n^(a,b)(x).

```
>> JacobiP[1, a, b, z]
```
$$\frac{a}{2} - \frac{b}{2} + z\left(1 + \frac{a}{2} + \frac{b}{2}\right)$$

```
>> JacobiP[3.5 + I, 3, 2, 4 - I]
```
$1\,410.02 + 5\,797.3I$

LaguerreL

`LaguerreL[`*n*`, `*x*`]`
: returns the Laguerre polynomial L_*n*(*x*).

`LaguerreL[`*n*`, `*a*`, `*x*`]`
: returns the generalised Laguerre polynomial L^*a*_*n*(*x*).

```
>> LaguerreL[8, x]
```

$$1 - 8x + 14x^2 - \frac{28x^3}{3} + \frac{35x^4}{12} - \frac{7x^5}{15} + \frac{7x^6}{180} - \frac{x^7}{630} + \frac{x^8}{40320}$$

```
>> LaguerreL[3/2, 1.7]
```

-0.947134

```
>> LaguerreL[5, 2, x]
```

$$21 - 35x + \frac{35x^2}{2} - \frac{7x^3}{2} + \frac{7x^4}{24} - \frac{x^5}{120}$$

LegendreP

`LegendreP[`*n*`, `*x*`]`
: returns the Legendre polynomial P_*n*(*x*).

`LegendreP[`*n*`, `*m*`, `*x*`]`
: returns the associated Legendre polynomial P^*m*_*n*(*x*).

```
>> LegendreP[4, x]
```

$$\frac{3}{8} - \frac{15x^2}{4} + \frac{35x^4}{8}$$

```
>> LegendreP[5/2, 1.5]
```

4.17762

```
>> LegendreP[1.75, 1.4, 0.53]
```

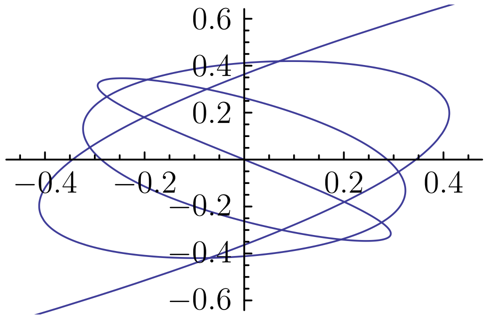
-1.32619

```
>> LegendreP[1.6, 3.1, 1.5]
```

$-0.303998 - 1.91937I$

`LegendreP` can be used to draw generalized Lissajous figures:

```
>> ParametricPlot[ {LegendreP[7, x
], LegendreP[5, x]}, {x, -1, 1}]
```



LegendreQ

`LegendreQ[`*n*`, `*x*`]`
: returns the Legendre function of the second kind Q_*n*(*x*).

`LegendreQ[`*n*`, `*m*`, `*x*`]`
: returns the associated Legendre function of the second Q^*m*_*n*(*x*).

```
>> LegendreQ[5/2, 1.5]
```

$0.036211 - 6.56219I$

```
>> LegendreQ[1.75, 1.4, 0.53]
```

2.05499

```
>> LegendreQ[1.6, 3.1, 1.5]
```

$-1.71931 - 7.70273I$

SphericalHarmonicY

`SphericalHarmonicY[`*l*`, `*m*`, `*theta*`, `*phi*`]`
: returns the spherical harmonic function Y_*l*^*m*(theta, phi).

```
>> SphericalHarmonicY[3/4, 0.5, Pi
/5, Pi/3]
```

$0.254247 + 0.14679I$

```
>> SphericalHarmonicY[3, 1, theta,
phi]
```

$$\frac{\sqrt{21}\left(1 - 5\text{Cos}[\text{theta}]^2\right) E^{I\text{phi}}\text{Sin}[\text{theta}]}{8\sqrt{\text{Pi}}}$$

64. Exponential Integral and Special Functions

Contents

LerchPhi

`LerchPhi[z,s,a]`
gives the Lerch transcendent (z,s,a).

```
>> LerchPhi[2, 3, -1.5]
```
$19.3893 - 2.1346 I$

```
>> LerchPhi[1, 2, 1/4]
```
17.1973

Zeta

`Zeta[z]`
returns the Riemann zeta function of z.

```
>> Zeta[2]
```
$$\frac{\mathrm{Pi}^2}{6}$$

```
>> Zeta[-2.5 + I]
```
$0.0235936 + 0.0014078 I$

65. Strings and Characters

66. Characters in Strings

Contents

CharacterRange

`CharacterRange["`*a*`‘‘,"`*b*`"]`
returns a list of the Unicode characters from *a* to *b* inclusive.

\>> `CharacterRange["a", "e"]`
{a, b, c, d, e}

\>> `CharacterRange["b", "a"]`
{}

Characters

`Characters["`*string*`"]`
returns a list of the characters in *string*.

\>> `Characters["abc"]`
{a, b, c}

DigitQ

`DigitQ[`*string*`]` yields `True` if all the characters in the *string* are digits, and yields `False` otherwise.

\>> `DigitQ["9"]`
True

\>> `DigitQ["a"]`
False

\>> `DigitQ ["01001101011000010111010001101000011010010110001`
True

\>> `DigitQ["-123456789"]`
False

LetterQ

`LetterQ[`*string*`]` yields `True` if all the characters in the *string* are letters, and yields `False` otherwise.

\>> `LetterQ["m"]`
True

\>> `LetterQ["9"]`
False

\>> `LetterQ["Mathics"]`
True

\>> `LetterQ["Welcome to Mathics"]`
False

LowerCaseQ

`LowerCaseQ[`*s*`]`
returns True if *s* consists wholly of lower case characters.

\>> `LowerCaseQ["abc"]`
True

An empty string returns True.

```
>> LowerCaseQ[""]
```
True

ToLowerCase

`ToLowerCase[s]`
: returns *s* in all lower case.

```
>> ToLowerCase["New York"]
```
new york

ToUpperCase

`ToUpperCase[s]`
: returns *s* in all upper case.

```
>> ToUpperCase["New York"]
```
NEW YORK

UpperCaseQ

`UpperCaseQ[s]`
: returns True if *s* consists wholly of upper case characters.

```
>> UpperCaseQ["ABC"]
```
True

An empty string returns True.

```
>> UpperCaseQ[""]
```
True

67. Character Codes

Contents

FromCharacterCode

`FromCharacterCode[`*n*`]`
: returns the character corresponding to Unicode codepoint *n*.

`FromCharacterCode[{`*n1*`, `*n2*`, ...}]`
: returns a string with characters corresponding to *n_i*.

`FromCharacterCode[{{`*n11*`, `*n12*`, ...}, {`*n21*`, `*n22*`, ...}, ...}]`
: returns a list of strings.

```
>> FromCharacterCode[100]
```
d

```
>> FromCharacterCode[228, "ISO8859
   -1"]
```
ä

```
>> FromCharacterCode[{100, 101,
   102}]
```
def

```
>> ToCharacterCode[%]
```
{100, 101, 102}

```
>> FromCharacterCode[{{97, 98, 99},
    {100, 101, 102}}]
```
{abc, def}

```
>> ToCharacterCode["abc 123"] //
   FromCharacterCode
```
abc 123

ToCharacterCode

`ToCharacterCode["`*string*`"]`
: converts the string to a list of character codes (Unicode codepoints).

`ToCharacterCode[{"`*string1*`‘‘,"`*string2*`", ...}]`
: converts a list of strings to character codes.

```
>> ToCharacterCode["abc"]
```
{97, 98, 99}

```
>> FromCharacterCode[%]
```
abc

```
>> ToCharacterCode["\[Alpha]\[Beta
   ]\[Gamma]"]
```
{945, 946, 947}

```
>> ToCharacterCode["ä", "UTF8"]
```
{195, 164}

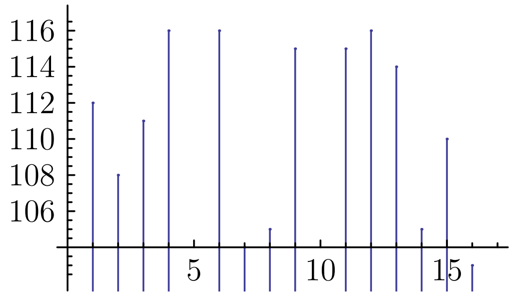
```
>> ToCharacterCode["ä", "ISO8859
   -1"]
```
{228}

```
>> ToCharacterCode[{"ab", "c"}]
```
{{97, 98}, {99}}

```
>> ToCharacterCode[{"ab", x}]
```
Stringorlistofstringsexpectedatposition1inToCharacterCode[{ab, x}].

ToCharacterCode [{ab, x}]

```
>> ListPlot[ToCharacterCode["plot
   this string"], Filling -> Axis]
```



68. Operations on Strings

Contents

StringDrop

`StringDrop["`*string*`", `*n*`]`
: gives *string* with the first *n* characters dropped.

`StringDrop["`*string*`", -`*n*`]`
: gives *string* with the last *n* characters dropped.

`StringDrop["`*string*`", {`*n*`}]`
: gives *string* with the *n*th character dropped.

`StringDrop["`*string*`", {`*m*`, `*n*`}]`
: gives *string* with the characters *m* through *n* dropped.

```
>> StringDrop["abcde", 2]
```
cde

```
>> StringDrop["abcde", -2]
```
abc

```
>> StringDrop["abcde", {2}]
```
acde

```
>> StringDrop["abcde", {2,3}]
```
ade

```
>> StringDrop["abcd",{3,2}]
```
abcd

```
>> StringDrop["abcd",0]
```
abcd

StringInsert

`StringInsert["`*string*`‘‘,"`*snew*`", `*n*`]`
: yields a string with *snew* inserted starting at position *n* in *string*.

`StringInsert["`*string*`‘‘,"`*snew*`", -`*n*`]`
: inserts a at position *n* from the end of "*string*".

`StringInsert["`*string*`‘‘,"`*snew*`", {`*n_1*`, `*n_2*`, ...}]`
: inserts a copy of *snew* at each position *n_i* in *string*; the *n_i* are taken before any insertion is done.

`StringInsert[{`*s_1*`, `*s_2*`, ...}, "`*snew*`", `*n*`]`
: gives the list of resutls for each of the *s_i*.

```
>> StringInsert["noting", "h", 4]
```
nothing

```
>> StringInsert["note", "d", -1]
```
noted

```
>> StringInsert["here", "t", -5]
```
there

```
>> StringInsert["adac", "he", {1,
   5}]
```
headache

```
>> StringInsert[{"something", "
   sometimes"}, " ", 5]
```
{some thing, some times}

```
>> StringInsert["1234567890123456",
    ".", Range[-16, -4, 3]]
```
1.234.567.890.123.456

StringJoin (<>)

`StringJoin["s1‘‘,"s2", ...]`
: returns the concatenation of the strings *s1, s2, .*

```
>> StringJoin["a", "b", "c"]
```
abc

```
>> "a" <> "b" <> "c" // InputForm
```
"abc"

`StringJoin` flattens lists out:

```
>> StringJoin[{"a", "b"}] // InputForm
```
"ab"

```
>> Print[StringJoin[{"Hello", " ", {"world"}}, "!"]]
```
Helloworld!

StringLength

`StringLength["string"]`
: gives the length of *string*.

```
>> StringLength["abc"]
```
3

`StringLength` is listable:

```
>> StringLength[{"a", "bc"}]
```
{1,2}

```
>> StringLength[x]
```
Stringexpected.

StringLength [*x*]

StringPosition

`StringPosition["string", patt]`
: gives a list of starting and ending positions where *patt* matches "*string*".

`StringPosition["string", patt, n]`
: returns the first *n* matches only.

`StringPosition["string", {patt1, patt2, ...}, n]`
: matches multiple patterns.

`StringPosition[{s1, s2, ...}, patt]`
: returns a list of matches for multiple strings.

```
>> StringPosition["123 ABCxyABCzzzABCABC", "ABC"]
```
{{4,6}, {9,11}, {15,17}, {18,20}}

```
>> StringPosition["123 ABCxyABCzzzABCABC", "ABC", 2]
```
{{4,6}, {9,11}}

`StringPosition` can be useful for searching through text.

```
>> data = Import["ExampleData/EinsteinSzilLetter.txt"];
```

```
>> StringPosition[data, "uranium"]
```
{{299,305}, {870,876}, {1 538,1~
~544}, {1 671,1 677}, {2 300,2 306
}, {2 784,2 790}, {3 093,3 099}}

StringReplace

`StringReplace["string‘‘,"a"->"b"]`
: replaces each occurrence of *old* with *new* in *string*.

`StringReplace["string", {"s1"->"sp1‘‘, "s2"->"sp2"}]`
: performs multiple replacements of each *si* by the corresponding *spi* in *string*.

`StringReplace["string", srules, n]`
: only performs the first *n* replacements.

`StringReplace[{"string1‘‘,"string2", ...}, srules]`
: performs the replacements specified by *srules* on a list of strings.

StringReplace replaces all occurrences of one substring with another:

```
>> StringReplace["xyxyxyyyxxxyyxy",
   "xy" -> "A"]
```

AAAyyxxAyA

Multiple replacements can be supplied:

```
>> StringReplace["xyzwxyzwxxyzxyzw
  ", {"xyz" -> "A", "w" -> "BCD"}]
```

ABCDABCDxAABCD

Only replace the first 2 occurences:

```
>> StringReplace["xyxyxyyyxxxyyxy",
   "xy" -> "A", 2]
```

AAxyyyxxxyyxy

Also works for multiple rules:

```
>> StringReplace["abba", {"a" -> "A
  ", "b" -> "B"}, 2]
```

ABba

StringReplace acts on lists of strings too:

```
>> StringReplace[{"xyxyxxy", "
  yxyxyxxxyyxy"}, "xy" -> "A"]
```

{AAxA, yAAxxAyA}

StringReplace also can be used as an operator:

```
>> StringReplace["y" -> "ies"]["
  city"]
```

cities

StringReverse

`StringReverse["`*string*`"]`
: reverses the order of the characters in "string".

```
>> StringReverse["live"]
```

evil

StringRiffle

`StringRiffle[{s1, s2, s3, ...}]`
: returns a new string by concatenating all the *si*, with spaces inserted between them.

`StringRiffle[list, sep]`
: inserts the separator *sep* between all elements in *list*.

`StringRiffle[list, {‘‘left’, "sep", "right"}]’`
: use *left* and *right* as delimiters after concatenation.

```
>> StringRiffle[{"a", "b", "c", "d
  ", "e"}]
```

a b c d e

```
>> StringRiffle[{"a", "b", "c", "d
  ", "e"}, ", "]
```

a, b, c, d, e

```
>> StringRiffle[{"a", "b", "c", "d
  ", "e"}, {"(", " ", ")"}]
```

(a b c d e)

StringSplit

`StringSplit["`*s*`"]`
: splits the string *s* at whitespace, discarding the whitespace and returning a list of strings.

`StringSplit["`*s*`‘‘,"`*d*`"]`
: splits *s* at the delimiter *d*.

`StringSplit[`*s*`, {"`*d1*`‘‘,"`*d2*`", ...}]`
: splits *s* using multiple delimiters.

`StringSplit[{`*s_1*`, $s_2, ...}, {"`*d1*`‘‘, "`*d2*`", ...}]`
: returns a list with the result of applying the function to each element.

```
>> StringSplit["abc,123", ","]
```

{abc, 123}

```
>> StringSplit["abc 123"]
```

{abc, 123}

```
>> StringSplit["abc,123.456", {",",
   "."}]
```

{abc, 123, 456}

```
>> StringSplit["a b c",
   RegularExpression[" +"]]
```

{a, b, c}

```
>> StringSplit[{"a b", "c d"},
   RegularExpression[" +"]]
```

{{a, b} , {c, d}}

StringTake

`StringTake["`*string*`", `*n*`]`
: gives the first *n* characters in *string*.

`StringTake["`*string*`", -`*n*`]`
: gives the last *n* characters in *string*.

`StringTake["`*string*`", {`*n*`}]`
: gives the *n*th character in *string*.

`StringTake["`*string*`", {`*m*`, `*n*`}]`
: gives characters *m* through *n* in *string*.

`StringTake["`*string*`", {`*m*`, `*n*`, `*s*`}]`
: gives characters *m* through *n* in steps of *s*.

`StringTake[{`*s1*`, `*s2*`, ...} `*spec*`}]`
: gives the list of results for each of the *si*.

```
>> StringTake["abcde", 2]
```

ab

```
>> StringTake["abcde", 0]
```

```
>> StringTake["abcde", -2]
```

de

```
>> StringTake["abcde", {2}]
```

b

```
>> StringTake["abcd", {2,3}]
```

bc

```
>> StringTake["abcdefgh", {1, 5,
   2}]
```

ace

Take the last 2 characters from several strings:

```
>> StringTake[{"abcdef", "stuv", "
   xyzw"}, -2]
```

{ef, uv, zw}

StringTake also supports standard sequence specifications

```
>> StringTake["abcdef", All]
```

abcdef

StringTrim

`StringTrim[`*s*`]`
: returns a version of *s* with whitespace removed from start and end.

```
>> StringJoin["a", StringTrim[" \tb
   \n "], "c"]
```

abc

```
>> StringTrim["ababaxababyaabab",
   RegularExpression["(ab)+"]]
```

axababya

69. String Patterns

Contents

DigitCharacter

`DigitCharacter`
  represents the digits 0-9.

```
>> StringMatchQ["1", DigitCharacter
   ]
```
True

```
>> StringMatchQ["a", DigitCharacter
   ]
```
False

```
>> StringMatchQ["12",
   DigitCharacter]
```
False

```
>> StringMatchQ["123245",
   DigitCharacter..]
```
True

EndOfLine

`EndOfString`
  represents the end of a line in a string.

```
>> StringReplace["aba\nbba\na\nab",
    "a" ~~EndOfLine -> "c"]
```
abc
  bbc
  c
  ab

```
>> StringSplit["abc\ndef\nhij",
   EndOfLine]
```
{abc,
  def,
  hij}

EndOfString

`EndOfString`
  represents the end of a string.

Test whether strings end with “e”:

```
>> StringMatchQ[#, __ ~~"e" ~~
   EndOfString] &/@ {"apple", "
   banana", "artichoke"}
```
{True, False, True}

```
>> StringReplace["aab\nabb", "b" ~~
   EndOfString -> "c"]
```
aab
  abc

LetterCharacter

`LetterCharacter`
  represents letters.

```
>> StringMatchQ[#, LetterCharacter]
    & /@ {"a", "1", "A", " ", "."}
```
{True, False, True, False, False}

LetterCharacter also matches unicode characters.

```
>> StringMatchQ["\[Lambda]",
   LetterCharacter]
```

True

StartOfLine

`StartOfString`
: represents the start of a line in a string.

```
>> StringReplace["aba\nbba\na\nab",
    StartOfLine ~~"a" -> "c"]
```

cba
 bba
 c
 cb

```
>> StringSplit["abc\ndef\nhij",
   StartOfLine]
```

{abc
 , def
 , hij}

StartOfString

`StartOfString`
: represents the start of a string.

Test whether strings start with "a":

```
>> StringMatchQ[#, StartOfString ~~
   "a" ~~__] &/@ {"apple", "banana
   ", "artichoke"}
```

{True, False, True}

```
>> StringReplace["aba\nabb",
   StartOfString ~~"a" -> "c"]
```

cba
 abb

StringCases

`StringCases["`*string*`", `*pattern*`]`
: gives all occurences of *pattern* in *string*.

`StringReplace["`*string*`", `*pattern* `->` *form*`]`
: gives all instances of *form* that stem from occurences of *pattern* in *string*.

`StringCases["`*string*`", {`*pattern1*`, `*pattern2*`, ...}]`
: gives all occurences of *pattern1*, *pattern2*, ...

`StringReplace["`*string*`", `*pattern*`, `*n*`]`
: gives only the first *n* occurences.

`StringReplace[{"`*string1*`‘‘,"`*string2*`", ...}, `*pattern*`]`
: gives occurences in *string1*, *string2*, ...

```
>> StringCases["axbaxxb", "a" ~~x_
   ~~"b"]
```

{axb}

```
>> StringCases["axbaxxb", "a" ~~x__
    ~~"b"]
```

{axbaxxb}

```
>> StringCases["axbaxxb", Shortest
   ["a" ~~x__ ~~"b"]]
```

{axb, axxb}

```
>> StringCases["-abc- def -uvw- xyz
   ", Shortest["-" ~~x__ ~~"-"] ->
   x]
```

{abc, uvw}

```
>> StringCases["-öhi- -abc- -.-",
   "-" ~~x : WordCharacter .. ~~"-"
    -> x]
```

{öhi, abc}

```
>> StringCases["abc-abc xyz-uvw",
   Shortest[x : WordCharacter .. ~~
   "-" ~~x_] -> x]
```

{abc}

```
>> StringCases["abba", {"a" -> 10,
   "b" -> 20}, 2]
```

{10, 20}

```
>> StringCases["a#ä_123",
   WordCharacter]
```

{a, ä, 1, 2, 3}

```
>> StringCases["a#ä_123", LetterCharacter]
```

{a, ä}

StringExpression (~~)

`StringExpression[s_1, s_2, ...]`
: represents a sequence of strings and symbolic string objects *s_i*.

```
>> "a" ~~ "b" // FullForm
```

"ab"

StringFreeQ

`StringFreeQ["string", patt]`
: returns True if no substring in *string* matches the string expression *patt*, and returns False otherwise.

`StringFreeQ[{‘‘s1’, "s2", ...}, patt]’`
: returns the list of results for each element of string list.

`StringFreeQ[‘‘string’, {p1, p2, ...}]’`
: returns True if no substring matches any of the *pi*.

`StringFreeQ[patt]`
: represents an operator form of StringFreeQ that can be applied to an expression.

```
>> StringFreeQ["mathics", "m" ~~__ ~~"s"]
```

False

```
>> StringFreeQ["mathics", "a" ~~__ ~~"m"]
```

True

```
>> StringFreeQ["Mathics", "MA" , IgnoreCase -> True]
```

False

```
>> StringFreeQ[{"g", "a", "laxy", "
universe", "sun"}, "u"]
```

{True, True, True, False, False}

```
>> StringFreeQ["e" ~~___ ~~"u"] /@
{"The Sun", "Mercury", "Venus",
"Earth", "Mars", "Jupiter", "
Saturn", "Uranus", "Neptune"}
```

{False, False, False, True,
True, True, True, True, False}

```
>> StringFreeQ[{"A", "Galaxy", "Far
", "Far", "Away"}, {"F" ~~__ ~~"
r", "aw" ~~___}, IgnoreCase ->
True]
```

{True, True, False, False, False}

StringMatchQ

```
>> StringMatchQ["abc", "abc"]
```

True

```
>> StringMatchQ["abc", "abd"]
```

False

```
>> StringMatchQ["15a94xcZ6", (
DigitCharacter | LetterCharacter
)..]
```

True

Use StringMatchQ as an operator

```
>> StringMatchQ[LetterCharacter]["a
"]
```

True

WhitespaceCharacter

`WhitespaceCharacter`
: represents a single whitespace character.

```
>> StringMatchQ["\n", WhitespaceCharacter]
```

True

```
>> StringSplit["a\nb\r\nc\rd", WhitespaceCharacter]
```

{a, b, c, d}

For sequences of whitespace characters use `Whitespace`:

```
>> StringMatchQ[" \n", WhitespaceCharacter]
```

False

```
>> StringMatchQ[" \n", Whitespace]
```
True

WordBoundary

`WordBoundary`
: represents the boundary between words.

```
>> StringReplace["apple banana
   orange artichoke", "e" ~~
   WordBoundary -> "E"]
```
applE banana orangE artichokE

WordCharacter

`WordCharacter`
: represents a single letter or digit character.

```
>> StringMatchQ[#, WordCharacter]
   &/@ {"1", "a", "A", ",", " "}
```
{True, True, True, False, False}

Test whether a string is alphanumeric:

```
>> StringMatchQ["abc123DEF",
   WordCharacter..]
```
True

```
>> StringMatchQ["$b;123",
   WordCharacter..]
```
False

70. Regular Expressions

Contents

RegularExpression

`RegularExpression[‘‘regex’]’`
: represents the regex specified by the string $“regex”$.

```
>> StringSplit["1.23, 4.56 7.89",
   RegularExpression["(\\s|,)+"]]
```

$\{1.23, 4.56, 7.89\}$

71. File Formats

Built-in Importers.

72. HTML

Basic implementation for a HTML importer

Contents

HTML'DataImport

```
>> Import["ExampleData/
PrimeMeridian.html", "Data"][[1,
 1, 2, 3]]
```

{Washington, D.C., 77ř0356.07 W (1 897) or 77ř0402.24 W (NAD 27) or 77ř0401.16 W (NAD 83), New Naval Observatory meridian}

HTML'Parser'HTMLGetString

HTML'HyperlinksImport

```
>> Import["ExampleData/
PrimeMeridian.html", "Hyperlinks
"][[1]]
```

/wiki/Prime_meridian_(Greenwich)

HTML'ImageLinksImport

```
>> Import["ExampleData/
PrimeMeridian.html", "ImageLinks
"][[6]]
```

//upload.wikimedia.org/wikipedia/commons/thumb/d/d5/Prime_meridian.jpg/180px-Prime_meridian.jpg

HTML'PlaintextImport

```
>> DeleteDuplicates[StringCases[
Import["ExampleData/
PrimeMeridian.html"],
RegularExpression["Wiki[a-z
]+"]]]
```

{Wikipedia, Wikidata, Wikibase, Wikimedia}

HTML'SourceImport

```
>> DeleteDuplicates[StringCases[
Import["ExampleData/
PrimeMeridian.html", "Source"],
RegularExpression["<t[a-z]+>"]]]
```

{<title>, <tr>, <th>, <td>}

HTML'TitleImport

```
>> Import["ExampleData/
PrimeMeridian.html", "Title"]
```

Prime meridian - Wikipedia

HTML'XMLObjectImport

```
>> Part[Import["ExampleData/
PrimeMeridian.html", "XMLObject
"], 2, 3, 1, 3, 2]
```

XMLElement [title, {}, {Prime meridian - Wikipedia}]

73. HTML

Basic implementation for a HTML importer

Contents

HTML‘DataImport

```
>> Import["ExampleData/
PrimeMeridian.html", "Data"][[1,
 1, 2, 3]]
```

{Washington, D.C., 77ř0356.07 W (1 897) or 77ř0402.24 W (NAD 27) or 77ř0401.16 W (NAD 83), New Naval Observatory meridian}

HTML‘Parser‘HTMLGetString

HTML‘HyperlinksImport

```
>> Import["ExampleData/
PrimeMeridian.html", "Hyperlinks
"][[1]]
```

/wiki/Prime_meridian_(Greenwich)

HTML‘ImageLinksImport

```
>> Import["ExampleData/
PrimeMeridian.html", "ImageLinks
"][[6]]
```

//upload.wikimedia.org/wikipedia/commons/thumb/d/d5/Prime_meridian.jpg/180px-Prime_meridian.jpg

HTML‘PlaintextImport

```
>> DeleteDuplicates[StringCases[
Import["ExampleData/
PrimeMeridian.html"],
RegularExpression["Wiki[a-z
]+"]]]
```

{Wikipedia, Wikidata, Wikibase, Wikimedia}

HTML‘SourceImport

```
>> DeleteDuplicates[StringCases[
Import["ExampleData/
PrimeMeridian.html", "Source"],
RegularExpression["<t[a-z]+>"]]]
```

{<title>, <tr>, <th>, <td>}

HTML‘TitleImport

```
>> Import["ExampleData/
PrimeMeridian.html", "Title"]
```

Prime meridian - Wikipedia

HTML‘XMLObjectImport

```
>> Part[Import["ExampleData/
PrimeMeridian.html", "XMLObject
"], 2, 3, 1, 3, 2]
```

XMLElement [title, {}, {Prime meridian - Wikipedia}]

74. XML

Contents

XML‘PlaintextImport

```
>> StringReplace[StringTake[Import
   ["ExampleData/InventionNo1.xml",
    "Plaintext"],31],
   FromCharacterCode[10]->"/"]
```

MuseScore 1.2/2 012-09-12/5.7/40

XML‘TagsImport

```
>> Take[Import["ExampleData/
   InventionNo1.xml", "Tags"], 10]
```

{accidental, alter, arpeggiate, articulations, attributes, backup, bar-style, barline, beam, beat-type}

XML‘Parser‘XMLGetString

```
>> Head[XML‘Parser‘XMLGetString["<a
   ></a>"]]
```

XMLObject [Document]

XML‘XMLObjectImport

```
>> Part[Import["ExampleData/
   InventionNo1.xml", "XMLObject"],
    2, 3, 1]
```

XMLElement [identification, {} , {XMLElement [encoding, {} , {XMLElement [software, {} , {MuseScore 1.2}] , XMLElement [encoding-date, {} , {2 012-09-12}] }] }]

```
>> Part[Import["ExampleData/
   Namespaces.xml"], 2]
```

XMLElement [book, {{http://www.w3.org/2 000/xmlns/, xmlns} − > urn:loc.gov:books} , {XMLElement [title, {} , {Cheaper by the Dozen}] , XMLElement [{urn:ISBN:0-395-36 341-6, number} , {} , {1 568 491 379}] , XMLElement [notes, {} , {XMLElement [p, {{http://www.w3.org/2 000/xmlns/, xmlns} − > http://www.w3.org/1 999/xhtml} , {This is a, XMLElement [i, {} , {funny, book!}] }] }] }]

Part III.

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scriptaculous

SymPy

Three.js

Index