The ideas of gravitational effective field theory

John F. Donoghue

Department of Physics and Astronomy
University of Amherst, MA 01003

Abstract

I give a very brief introduction to the use of effective field theory techniques in quantum calculations of general relativity. The gravitational interaction is naturally organized as a quantum effective field theory and a certain class of quantum corrections can be calculated. [Talk presented at the XXVIII International Conference on High Energy Physics, ICHEP94, Glasgow, Aug. 1994, to be published in the proceedings.]

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We expect that there will be new interactions and new degrees of freedom at the Planck scale, if not sooner. Many discussions of quantum mechanics and gravity involve speculations about Planck scale physics. This talk describes a more conservative approach as we will use quantum mechanics and general relativity at ordinary energies, where we expect that they should both be valid. The goal is to argue that general relativity forms a fine quantum theory at ordinary energies, and to identify a class of "leading quantum corrections" which are the dominant quantum effects at long distance and which are reliably calculable. The apparent obstacle to such a program is the fact that the quantum corrections involve integration over all energy scales, including extreme high energies. The solution to this is the use of effective field theory. Because of the briefness of this report, the discussion here is necessarily superficial, but I will concentrate on the basic ideas of such an approach [1].

Effective field theory is a technique that allows one to separate the effects of high energy scales from low energy ones. In many cases, such as the theory of gravity, one does not know the correct high energy theory. However as a consequence of the uncertainty principle we do know that, when viewed at low energy, the high energy degrees of freedom do not propagate far. They can be integrated out of the theory leaving a local Lagrangian, although this Lagrangian will in general contain nonrenormalizable interactions. In contrast, the low energy degrees of freedom propagate long distances and cannot be summarized by a local interaction. They must be included explicitly. From an unknown high energy theory, we are then led to write the most general Lagrangian containing the low energy particles which is consistent with the symmetries and vacuum structure of the theory. In the case of gravity interacting with a massive matter field we impose general covariance and find that

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\begin{aligned}
\mathcal{L} &= \mathcal{L}_{gr} + \mathcal{L}_{matter} \\
\mathcal{L}_{gr} &= \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots \right\} \\
\mathcal{L}_{matter} &= \sqrt{-g} \left\{ \frac{1}{2} \left(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - m^2 \phi^2 \right) \right. \\
&+ d_1 R_{\mu\nu} \partial^\mu \phi \partial^\nu \phi \\
&+ \left. R \left(d_2 \partial_\lambda \phi \partial^\lambda \phi - d_3 m^2 \phi^2 \right) + \ldots \right\}
\end{aligned}
\tag{1}
$$

where $\Lambda \approx 0$ is related to the cosmological constant (we will set this equal to zero), $\kappa^2 = 32\pi G$, and c_i, d_i are unknown constants. The second key ingredient to effective field theory is the energy expansion, in which the many terms in the effective Lagrangian are ordered in powers of the low energy scale over the high energy scale. In gravity, since R involves two derivatives (which will become two factors of momentum q in matrix elements), the R^2 terms will be of order q^4 and hence much smaller than the R term at low enough energies. It is for this reason that we have essentially no phenomenological constraint on the R^2 terms ($i.e. c_1, c_2 < 10^{74}$) [2].

While it may seem relatively obvious that a classical Lagrangian can be ordered in an energy expansion, it is perhaps less obvious that quantum effects of the low energy particles can also be so ordered[3]. However, in loop diagrams the high momentum portions of the integration and all the ultraviolet divergences are also equivalent to local counterterms in a Lagrangian. For example the effect of gravitons at one-loop order has the high energy behavior (in dimensional regularization) of [4]

$$\mathcal{L}_{div} = \sqrt{-g}\frac{1}{8\pi^2(4-d)}\left(\frac{1}{120}R^2 + \frac{7}{20}R_{\mu\nu}R^{\mu\nu}\right). \tag{2}$$

This is probably not an accurate description of the full high energy behavior, but this does not matter because such quantum effects are not themselves observable and can be absorbed into renormalized values of the unknown constants c_i. However within the same Feynman diagrams there are also low energy quantum effects which correspond to the long range propagation of gravitons. These are reliable because they are independent of the unknown high energy theory, depending only on the massless degrees of freedom (gravitons) and their couplings at the lowest energies (which follow from the Einstein action). In calculations, the distinguishing characteristic is the analytic structure of the amplitudes. Effects which are able to be expanded in a power series in the momentum are thereby in a form that has the same structure as operators which arise in a local Lagrangian. These are then in most cases indistinguishable from possible effects from a high energy theory, which we argued above would be contained in the unknown coefficients of a local Lagrangian. However, non-analytic effects in the matrix element cannot come from a local Lagrangian, and only arise from long range propagation of light particles. One can use this distinction to separate out the low energy quantum effects. In most cases, the nonanalytic terms are larger numeri-

cally when one works at extremely low energies, so they are the leading long distance corrections. Effective field theory is a procedure which carries out these ideas in a straightforward way.

These ideas can perhaps be best explained by an example. The usual gravitational interactions between two masses can be obtained from the one graviton exchange potential

$$\frac{\kappa^2 m_1 m_2}{8q^2} \rightarrow -\frac{Gm_1 m_2}{r} \tag{3}$$

The effects of the R^2 terms in the effective Lagrangian appear at one higher power of q^2. Loop diagrams also give contribution at this order but the nonlocal effects of low energy are represented by nonanalytic terms in momentum space, i.e., schematically

$$V(q) \sim \kappa^2 m_1 m_2 \left[\frac{1}{q^2} + \left(c_i + \kappa^2 \ell_i \right) \right.$$
$$\left. + \kappa^2 \left(a \sqrt{\frac{m^2}{-q^2}} + b ln(-q^2) \right) + \ldots \right] \tag{4}$$

where a, b and ℓ_i arise form the calculation of a set of one-loop diagrams. (ℓ_i is divergent and is absorbed into the renormalized value of the parameter c_i.) The coefficient of the nonanalytic terms (a, b) are finite and are a consequence of the low energy part of the theory. While most work in the field has focussed on the divergent portion, it is these latter finite terms which are the most predictive part of the diagrams. Note that at low enough momentum, the non-analytic terms are larger than the constant terms. In addition, they are distinguished by a different spatial dependence. When one forms a nonrelativistic potential one finds

$$V(r) \quad \sim \quad -Gm_1 m_2 \left[\frac{1}{r} + 4\pi \left(c_i + \kappa^2 \ell_i \right) \delta^3(x) \right.$$
$$\left. + \quad \frac{2}{\pi} \frac{\kappa^2 am}{r^2} - \frac{2}{\pi} \frac{\kappa^2 b}{r^3} + \ldots \right] \tag{5}$$

The nonanalytic terms give power-law corrections while the local Lagrangian and high energy loop effects give a delta function. Thus the long distance

quantum correction to the Newtonian potential is calculable. An explicit calculation [1] yields

$$V(r) = -\frac{Gm_1m_2}{r}\left[1 - \frac{G(m_1 + m_2)}{rc^2}\right.$$
$$\left. -\frac{127G\hbar}{30\pi^2r^2c^3} + \cdots\right] \tag{6}$$

The idea of a gravitational effective field theory extends well beyond this calculation. In general, effective field theory techniques will organize any given matrix element into the calculable effects of low energy and the unknown effects of the full high energy theory. Most commonly, the nonanalytic terms are the leading contributions at large distance. This division has not been commonly applied to the gravitational interactions and much of the standard wisdom of general relativity needs to be scrutinized through the eyes of effective field theory. The quantum corrections are numerically small in macroscopic phenomena, and I know of no such effects that can influence present day experimental relativity. However, these ideas may be useful in elucidating some of the theoretical issues of general relativity, and perhaps can be compared to the work being done in lattice simulations of quantum gravity.

As far as quantum mechanics are concerned, effective field theories are as natural as the more restrictive class of renormalizable field theories (and are perhaps even more natural). From this point of view, the quantum theory of gravity does not seem more problematic at ordinary energies than the rest of the Standard Model.

References

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