Department of Physics and Astronomy
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Bachelor thesis in Physics
submitted by

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Functional Renormalization and Quantum Gravity

This bachelor thesis has been carried out by

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at the

Institute for Theoretical Physics

at

Heidelberg University

under the supervision of

Prof. Dr. Jan M. Pawlowski

Functional Renormalization and Quantum Gravity

Mathieu Kaltschmidt

Abstract

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Zusammenfassung

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Chapter 1

Introduction

"Einsetzen ist was für Loser!"
Oliver Isak, 2017

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Chapter 2

Functional methods in Quantum Field Theory

2.1 Generating Functionals and Correlation Functions

Path Integral formulation of the partition sum:

$$\mathcal{Z}[J] = \frac{1}{N}\int \mathcal{D}\varphi \exp\left\{-\mathcal{S}[\varphi] + \int_x J(x)\varphi(x)\right\} \tag{2.1}$$

2.1.1 The Schwinger Functional

We obtain the Schwinger functional by taking the logarithm of $\mathcal{Z}[J]$:

$$\mathcal{W}[J] = \ln \mathcal{Z}[J] \tag{2.2}$$

The connected two point functions, also known as the propagators, can be computed simply by differentiating the Schwinger functional:

$$\begin{aligned} G(x_1, x_2) &= \frac{\delta^2 \mathcal{W}[J]}{\delta J(x_1)\delta J(x_2)} \\ &= \langle\varphi(x_1)\varphi(x_2)\rangle - \langle\varphi(x_1)\rangle\langle\varphi(x_2)\rangle \\ &= \langle\varphi(x_1)\varphi(x_2)\rangle_{\mathrm{c}} \end{aligned} \tag{2.3}$$

2.1.2 The Effective Action

The effective action can be obtained by performing a Legendre transform of the Schwinger funtional, i. e.:

$$\Gamma[\phi] = \sup_J \left\{\int_x J(x)\phi(x) - \mathcal{W}[J]\right\} = \int_x J_{\mathrm{sub}}(x)\phi(x) - \mathcal{W}[J_{\mathrm{sub}}] \tag{2.4}$$

Quantum equation of motion:

$$\frac{\delta\Gamma[\phi]}{\delta\phi(x)} = J(x) \tag{2.5}$$

Dyson-Schwinger equation:

$$\frac{\delta\Gamma[\phi]}{\delta\phi(x)} = \frac{\delta\mathcal{S}}{\delta\varphi(x)}\left[\varphi = G\cdot\frac{\delta}{\delta\phi} + \phi\right] \tag{2.6}$$

2.2 The Functional Renormalization Group

- Kadanoff Block-Spin model
- maybe visualization of Ising model + phase transitions

2.2.1 Renormalization group consistency

Cutoff independence of the full quantum effective action:

$$\Lambda\frac{\mathrm{d}\Gamma}{\mathrm{d}\Lambda} = 0 \tag{2.7}$$

Full effective action in a generic representation:

$$\Gamma[\phi] = \mathcal{D}_\Lambda[\phi] + \Gamma_\Lambda[\phi] \tag{2.8}$$

Formal discussion:

$$\Gamma_k[\phi] = \Gamma_\Lambda[\phi] + \int\limits_\Lambda^k \frac{\mathrm{d}k'}{k'}\mathcal{F}_{k'}[\phi] \tag{2.9}$$

2.3 The Renormalisation Group Flow for the Effective Action

We introduce the RG time scale as $t = \ln\frac{k}{\Lambda}$

$$\partial_t\Gamma_k[\phi] = \frac{1}{2}\,\mathrm{Tr}\left[\frac{1}{\Gamma_k^{(2)}[\phi] + R_k}\partial_t R_k\right] \tag{2.10}$$

Chapter 3

Fundamentals of General Relativity

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3.1 The Einstein Equations

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The Einstein-Hilbert action:

$$\mathcal{S}_{\mathrm{EH}}[g_{\mu\nu}] = \frac{1}{16\pi G}\int_x \sqrt{-\det g_{\mu\nu}}(\mathcal{R} - 2\Lambda) \tag{3.1}$$

Varying this action as usual yields the Einstein equations in absence of matter:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \tag{3.2}$$

where we used $G_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R}$.

Diffeomorphism invariance, Lie derivatives:

$$\mathcal{L}_\omega \phi = \omega^\mu \partial^\mu \phi = \omega^\mu \nabla^\mu \phi \tag{3.3}$$

Now we include matter.

Energy-Momentum Tensor:

$$T_{\mu\nu} = \frac{-2}{\sqrt{-\det g_{\mu\nu}}}\frac{\delta \mathcal{S}_{\text{matter}}}{\delta g^{\mu\nu}} \tag{3.4}$$

Matter part of the action for a minimally coupled scalar field ϕ:

$$\mathcal{S}_{\text{matter}}[g_{\mu\nu}, \phi] = -\frac{1}{2} \int\limits_x \sqrt{-\det g_{\mu\nu}} \left(g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} V(\phi) \right) \tag{3.5}$$

From this, we get the Einstein equations including matter by demanding the variation $\sqrt{-\det g_{\mu\nu}} \frac{\delta \mathcal{S}}{\delta g^{\mu\nu}}$ to vanish. This yields:

$$\frac{1}{8\pi G} \left[\mathcal{R}_{\mu\nu} - \frac{1}{2} (\mathcal{R} - 2\Lambda) g_{\mu\nu} \right] = T_{\mu\nu} \tag{3.6}$$

3.2 Perturbative Non-Renormalizability of Gravity

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Chapter 4

Quantum Gravity in the Einstein-Hilbert Truncation

4.1 RG approach to Quantum Gravity

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4.2 Truncations of the theory space

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4.3 The effective action for Quantum Gravity

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4.4 Non-Gaussian Fixed Points

Chapter 5

Conclusion

5.1 Summary of the results

5.2 Outlook

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- Pavel + Gruppe
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- Freunde
- Familie

List of Figures

Declaration of Authorship

I hereby certify that this thesis has been composed by me and is based on my own work, unless stated otherwise. No other person's work has been used without due acknowledgement in this thesis. All references have been quoted and all sources of information, including graphs and data sets, have been specifically acknowledged.

Mathieu Kaltschmidt

Heidelberg, July 8^{th} 2019