Department of Physics and Astronomy
Heidelberg University

Bachelor thesis in Physics
submitted by

Mathieu Kaltschmidt

from Kappel-Grafenhausen

2019

The Asymptotic Safety approach to Quantum Gravity

This bachelor thesis has been carried out by

Mathieu Kaltschmidt

at the

Institute for Theoretical Physics

at

Heidelberg University

under the supervision of

Prof. Dr. Jan M. Pawlowski

The Asymptotic Safety approach to Quantum Gravity

Mathieu Kaltschmidt

Abstract

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Zusammenfassung

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Contents

1. Introduction **1**

2. Functional methods in Quantum Field Theory **3**
2.1. Generating Functionals and Correlation Functions 3
2.2. The Functional Renormalization Group 4
2.3. Renormalization Group Consistency 5
2.4. Flow Equations for Generating Functionals 6

3. Fundamentals of General Relativity **9**
3.1. The Einstein Equations 9
3.2. Perturbative Non-Renormalizability of Gravity 11

4. Quantum Gravity in the Einstein-Hilbert Truncation **13**
4.1. RG approach to Quantum Gravity 13
4.2. Einstein-Hilbert truncation 13

5. Conclusions and Outlook **15**

A. Mathematical Appendix **17**
A.1. Heat Kernel techniques 17
A.2. York decomposition 17

References **I**

List of Figures **III**

Chapter 1.

Introduction

Throughout this thesis we use units such that $\hbar = c = G \equiv 1$.

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

This is the second paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

And after the second paragraph follows the third paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

After this fourth paragraph, we start a new paragraph sequence. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Chapter 2.

Functional methods in Quantum Field Theory

This chapter introduces the the treatment of Quantum Field Theory (QFT) using functional methods. The main goal is to derive the flow equation for the effective action functional, the generating functional for the one-particle irreducible (1PI) correlation functions. The flow equation was first derived by Christof Wetterich in 1993 [10]. It is the foundation for our treatment of Quantum Gravity in the Asymptotic Safety approach, discussed in more detail later on.

2.1. Generating Functionals and Correlation Functions

We consider a theory setting of N scalar fields $\varphi_a(x), a \in \{1, \dots, N\}$ in d-dimensional Euclidean space. The corresponding partition sum in presence of sources $J_a(x)$ reads

$$Z[J] = \int \mathcal{D}\varphi \, \mathrm{e}^{-\mathcal{S}[\varphi]+J\cdot\varphi} \,. \tag{2.1}$$

The information content of the partition sum results mainly from the classical action functional $\mathcal{S}[\varphi]$, which determines the classical field equations

$$\frac{\delta \mathcal{S}}{\delta \varphi(x)} = 0. \tag{2.2}$$

Notation: The scalar product sums over field components and integrates over all space …

$$J \cdot \varphi = \int_x J_a(x) \, \varphi_a(x) = \int_p \tilde{J}_a(p) \, \tilde{\varphi}_a(p) \tag{2.3}$$

with

$$\int_x = \int_{\mathbb{R}^d} \mathrm{d}^d x \qquad \text{and} \qquad \int_p = \int_{\mathbb{R}^d} \frac{\mathrm{d}^d p}{(2\pi)^d} \tag{2.4}$$

Mean field description:

$$\phi := \langle \varphi \rangle = \frac{1}{Z} \frac{\delta Z}{\delta J}\bigg|_{J=0} = \int \mathcal{D}\varphi \; \varphi \; \mathrm{e}^{-\mathcal{S}[\varphi]+J\cdot\varphi} \tag{2.5}$$

Higher correlations:

$$\langle \varphi_1 \cdots \varphi_n \rangle := \langle \varphi^n \rangle = \frac{1}{Z} \frac{\delta^n Z}{\delta^n J} = \int \mathcal{D}\varphi \, \overbrace{\varphi_1 \cdots \varphi_n}^{:= \varphi^n} \, \mathrm{e}^{-\mathcal{S}[\varphi] + J \cdot \varphi} \tag{2.6}$$

The Schwinger functional W is then defined as

$$Z[J] = \mathrm{e}^{W[J]} \tag{2.7}$$

For the special case of $n = 2$ the correlation function yields the connected 2-point function which is also known as the propagator $G_{ab}(x, y) = G_{\alpha\beta}$ correlating the field φ_a at spacetime point x with the field φ_b at y.

$$\begin{aligned} G_{\alpha\beta} &= \frac{\delta^2 W[J]}{\delta J_\alpha \delta J_\beta} = \frac{\delta}{\delta J_\alpha} \left(\frac{1}{Z} \frac{\delta Z}{\delta J_\beta} \right) \\ &= \frac{1}{Z} \left(\frac{\delta^2 Z}{\delta J_\alpha \delta J_\beta} \right) - \frac{1}{Z^2} \left(\frac{\delta Z}{\delta J_\alpha} \right) \left(\frac{\delta Z}{\delta J_\beta} \right) \\ &= \langle \varphi_\alpha \varphi_\beta \rangle - \phi_\alpha \phi_\beta = \langle \varphi_\alpha \varphi_\beta \rangle_{\mathrm{c}} \end{aligned} \tag{2.8}$$

The Effective Action:

The effective action can be obtained by performing a Legendre transform of the Schwinger funtional, i. e.:

$$\Gamma[\phi] = \sup_J \left\{ \int_x J(x)\phi(x) - \mathcal{W}[J] \right\} = \int_x J_{\text{sub}}(x)\phi(x) - \mathcal{W}[J_{\text{sub}}] \tag{2.9}$$

Quantum equation of motion:

$$\frac{\delta \Gamma[\phi]}{\delta \phi(x)} = J(x) \tag{2.10}$$

Dyson-Schwinger equation:

$$\frac{\delta \Gamma[\phi]}{\delta \phi(x)} = \frac{\delta \mathcal{S}}{\delta \varphi(x)} \left[\varphi = G \cdot \frac{\delta}{\delta \phi} + \phi \right] \tag{2.11}$$

2.2. The Functional Renormalization Group

- Kadanoff Block-Spin model

- maybe visualization of Ising model + phase transitions

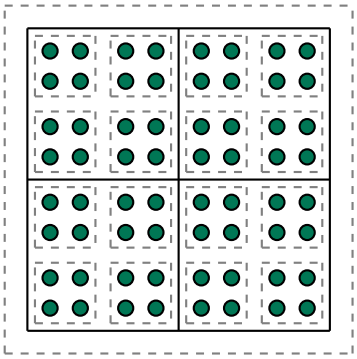


Figure 2.1.: Visualization of the Kadanoff Block-Spin model.[1]

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

2.3. Renormalization Group Consistency

This section is mainly based on [1].

Cutoff independence of the full quantum effective action:

$$\Lambda \frac{\mathrm{d}\Gamma}{\mathrm{d}\Lambda} = 0 \tag{2.12}$$

Full effective action in a generic representation:

$$\Gamma[\phi] = \mathcal{D}_\Lambda[\phi] + \Gamma_\Lambda[\phi] \tag{2.13}$$

Formal discussion:

$$\Gamma_k[\phi] = \Gamma_\Lambda[\phi] + \int_\Lambda^k \frac{\mathrm{d}k'}{k'} \mathcal{F}_{k'}[\phi] \tag{2.14}$$

1. This visualization is inspired by an image provided in the PhD thesis of J.R. Laguna.

2.4. Flow Equations for Generating Functionals

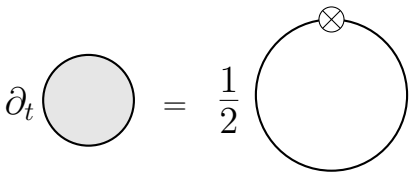
We introduce the RG time scale t:

$$\partial_t = \frac{\partial}{\partial \ln(k/\Lambda)} = \frac{k}{\Lambda}\frac{\partial}{\partial(k/\Lambda)} = k\partial_k \tag{2.15}$$

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

$$\begin{aligned}\partial_t \Gamma_k[\phi] &= \frac{1}{2}\,\mathrm{Tr}\left[\frac{1}{\Gamma_k^{(2)}[\phi] + R_k}\partial_t R_k\right] \\ &= \frac{1}{2}\int\limits_p \frac{1}{\Gamma_k^{(2)}[\phi] + R_k}(p,-p)\;\partial_t R_k(p^2)\end{aligned} \tag{2.16}$$

This translates directly into the following diagrammic representation:



(2.17)

where $\otimes = \partial_t R_k$ represents the insertion of the respective regulator.

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

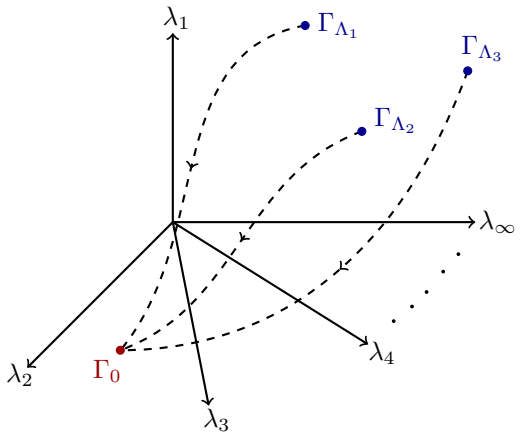


Figure 2.2.: Flow of Γ_k through infinite-dimensional theory space for different regulators.

Chapter 3.

Fundamentals of General Relativity

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

3.1. The Einstein Equations

This is the second paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

And after the second paragraph follows the third paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

After this fourth paragraph, we start a new paragraph sequence. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

This is the second paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

The Einstein-Hilbert action:

$$\mathcal{S}_{\text{EH}}[g_{\mu\nu}] = \frac{1}{16\pi G}\int\limits_x \sqrt{-\det g_{\mu\nu}}(\mathcal{R} - 2\Lambda) \tag{3.1}$$

Varying this action as usual yields the Einstein equations in absence of matter:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \tag{3.2}$$

where we used $G_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R}$.

Diffeomorphism invariance, Lie derivatives:

$$\mathcal{L}_\omega \phi = \omega^\mu \partial^\mu \phi = \omega^\mu \nabla^\mu \phi \tag{3.3}$$

Now we include matter.

Energy-Momentum Tensor:

$$T_{\mu\nu} = \frac{-2}{\sqrt{-\det g_{\mu\nu}}}\frac{\delta \mathcal{S}_{\text{matter}}}{\delta g^{\mu\nu}} \tag{3.4}$$

Matter part of the action for a minimally coupled scalar field ϕ:

$$\mathcal{S}_{\text{matter}}[g_{\mu\nu}, \phi] = -\frac{1}{2} \int_x \sqrt{-\det g_{\mu\nu}} \left(g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} V(\phi) \right) \tag{3.5}$$

From this, we get the Einstein equations including matter by demanding the variation $\sqrt{-\det g_{\mu\nu}} \frac{\delta \mathcal{S}}{\delta g^{\mu\nu}}$ to vanish. This yields:

$$\frac{1}{8\pi G} \left[\mathcal{R}_{\mu\nu} - \frac{1}{2} (\mathcal{R} - 2\Lambda) g_{\mu\nu} \right] = T_{\mu\nu} \tag{3.6}$$

3.2. Perturbative Non-Renormalizability of Gravity

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Chapter 4.

Quantum Gravity in the Einstein-Hilbert Truncation

4.1. RG approach to Quantum Gravity

Flow equation for QG:

$$\partial_t \Gamma_k[\overline{g}, \Phi] = \frac{1}{2} \operatorname{Tr} \; G_{\mathrm{hh}}[\Phi] \; \partial_t R_k - \operatorname{Tr} \; G_{\mathrm{c\bar{c}}}[\Phi] \; \partial_t R_k \tag{4.1}$$

4.2. Einstein-Hilbert truncation

We want to solve the Flow equation (4.2) approximately. All terms that are invariant under the imposed symmetry, i. e. invariant under diffeomorphism transformations need to be taken into account.

Easiest truncation takes only the scalar curvature $\mathcal{R}$ and the cosmological constant Λ into account (No higher order terms …) and was performed by Martin Reuter in 1993 [8].

This truncation reads

$$\Gamma_k = 2\kappa^2 Z_k \int_x \sqrt{g} \left[-\mathcal{R} + 2\Lambda_k\right] + \mathcal{S}_{\mathrm{gf}} + \mathcal{S}_{\mathrm{gh}} \tag{4.2}$$

with

$$\kappa^2 = \frac{1}{32\pi G}, \qquad\qquad G_k = G Z_k^{-1} \tag{4.3}$$

anomalous dimension:

$$\eta_g = -\frac{\partial_t Z_k}{Z_k} = -\partial_t \ln Z_k$$

dimensionless renormalized cosmological constant:

$$\lambda_k = \Lambda_k k^{-2}$$

dimensionless renormalized cosmological constant:

$$g_k = G_k k^{d-2} = \frac{G k^{d-2}}{Z_k}$$

corresponding beta function:

$$\beta_g = \partial_t g_k = \left(d - 2 + \eta_g\right) g_k \tag{4.4}$$

maximally symmetric space:

$$\overline{\mathcal{R}}_{\mu\nu} = \frac{1}{d}\,\overline{g}_{\mu\nu}\overline{\mathcal{R}} \tag{4.5}$$

$$\overline{\mathcal{R}}_{\mu\nu\rho\sigma} = \frac{1}{d(d-1)}\,(\overline{g}_{\mu\rho}\overline{g}_{\nu\sigma} - \overline{g}_{\mu\sigma}\overline{g}_{\nu\rho})\overline{\mathcal{R}} \tag{4.6}$$

suitable tensor basis:

$$h_{\mu\nu} = h^{\mathrm{TT}}_{\mu\nu} + \overline{\nabla}_\mu \xi_\nu + \left(\overline{\nabla}_\mu \overline{\nabla}_\nu - \frac{1}{d}\,\overline{g}_{\mu\nu}\overline{\Delta}\right)\sigma + \frac{1}{d}\,\overline{g}_{\mu\nu} h \tag{4.7}$$

Chapter 5.

Conclusions and Outlook

And after the second paragraph follows the third paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

After this fourth paragraph, we start a new paragraph sequence. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

This is the second paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

And after the second paragraph follows the third paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there

a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Appendix A.

Mathematical Appendix

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

A.1. Heat Kernel techniques

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

A.2. York decomposition

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

References

[1] Jens Braun, Marc Leonhardt, and Jan M. Pawlowski. "Renormalization group consistency and low-energy effective theories". In: (2018). arXiv: 1806.04432 [hep-ph].

[2] Nicolai Christiansen et al. "Asymptotic safety of gravity with matter". In: *Phys. Rev.* D97.10 (2018), p. 106012. arXiv: 1710.04669 [hep-th].

[3] Pietro Donà, Astrid Eichhorn, and Roberto Percacci. "Matter matters in asymptotically safe quantum gravity". In: *Phys. Rev.* D89.8 (2014), p. 084035. arXiv: 1311.2898 [hep-th].

[4] Stefan Floerchinger and Christof Wetterich. *Lectures on Quantum Field Theory*. Lecture Notes (Access currently restricted to students). Heidelberg University. 2019.

[5] Jan Meibohm, Jan M. Pawlowski, and Manuel Reichert. "Asymptotic safety of gravity-matter systems". In: *Phys. Rev.* D93.8 (2016), p. 084035. arXiv: 1510.07018 [hep-th].

[6] Jan. M. Pawlowski et al. *The Functional Renormalization Group & applications to gauge theories and gravity*. Lecture Notes (Access currently restricted to students). Heidelberg University. 2019.

[7] Martin Reuter and Frank Saueressig. *Quantum Gravity and the Functional Renormalization Group: The Road towards Asymptotic Safety*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2019.

[8] Martin Reuter and Frank Saueressig. "Renormalization group flow of quantum gravity in the Einstein-Hilbert truncation". In: *Phys. Rev.* D65 (2002), p. 065016. arXiv: hep-th/0110054 [hep-th].

[9] Timo Weigand. *Quantum Field Theory I+II*. Lecture Notes (Link). Heidelberg University. 2014.

[10] Christof Wetterich. "Exact evolution equation for the effective potential". In: *Phys. Lett.* B301 (1993), pp. 90–94. arXiv: 1710.05815 [hep-th].

List of Figures

2.1. Visualization of the Kadanoff Block-Spin model. 5
2.2. Flow of Γ_k through infinite-dimensional theory space for different regulators. 7

Acknowledgements

First and foremost i would like to thank my supervisor Prof. Jan Pawlowski for giving me the opportunity to work on such interesting topic and for his excellent guidance throughout the last months. I learned a lot about theoretical physics …

I would like the thank the whole Quantum Gravity group at ITP for the nice atmosphere and many interesting and helpful discussions.

Group, proofreaders, Heidelberger dudes..

Not to forget, i have to thank all my friends from home, especially Bastian, Chiara, Helena, Jakob, Jana and Lea for all the amazing time we spent together during the last years.

Lastly, i want to thank my parents Marie-Paule and Bernd Kaltschmidt and my sister Céline for their constant support and love and for always allowing me to pursue my dreams.

Declaration of Authorship

I hereby certify that this thesis has been composed by me and is based on my own work, unless stated otherwise.

Heidelberg, _____ _____