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Bachelor thesis in Physics
submitted by

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Functional Renormalization and Quantum Gravity

This bachelor thesis has been carried out by

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at the

Institute for Theoretical Physics

at

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under the supervision of

Prof. Dr. Jan M. Pawlowski

Functional Renormalization and Quantum Gravity

Mathieu Kaltschmidt

Abstract

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Zusammenfassung

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Contents

Chapter 1.

Introduction

Throughout this thesis we use units such that $\hbar = c = G \equiv 1$.

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Chapter 2.

Functional methods in Quantum Field Theory

This chapter introduces the the treatment of Quantum Field Theory (QFT) using functional methods. The main goal is to derive the flow equation for the effective action functional, the generating functional for the one-particle irreducible (1PI) correlation functions. The flow equation was first derived by Christof Wetterich in 1993 [9]. It is the foundation for our treatment of Quantum Gravity in the Asymptotic Safety approach, discussed in more detail later on.

2.1. Generating Functionals and Correlation Functions

We consider a theory setting of N scalar fields $\varphi_a(x), a \in \{1, \dots, N\}$ in d-dimensional Euclidean space. The corresponding partition sum in presence of sources $J_a(x)$ reads

$$Z[J] = \int \mathcal{D}\varphi \, \mathrm{e}^{-\mathcal{S}[\varphi] + J\cdot\varphi} \,. \tag{2.1}$$

The information content of the partition sum results mainly from the classical action functional $\mathcal{S}[\varphi]$, which determines the classical field equations

$$\frac{\delta \mathcal{S}}{\delta \varphi(x)} = 0. \tag{2.2}$$

Notation: The scalar product sums over field components and integrates over all space …

$$J \cdot \varphi = \int_x J_a(x) \, \varphi_a(x) = \int_p \tilde{J}_a(p) \, \tilde{\varphi}_a(p) \tag{2.3}$$

with

$$\int_x = \int_{\mathbb{R}^d} \mathrm{d}^d x \qquad \text{and} \qquad \int_p = \int_{\mathbb{R}^d} \frac{\mathrm{d}^d p}{(2\pi)^d} \tag{2.4}$$

Mean field description:

$$\phi := \langle \varphi \rangle = \frac{1}{Z} \frac{\delta Z}{\delta J}\bigg|_{J=0} = \int \mathcal{D}\varphi \; \varphi \; \mathrm{e}^{-\mathcal{S}[\varphi] + J\cdot\varphi} \tag{2.5}$$

Higher correlations:

$$\langle\varphi_1\cdots\varphi_n\rangle := \langle\varphi^n\rangle = \frac{1}{Z}\frac{\delta^n Z}{\delta^n J} = \int \mathcal{D}\varphi\ \overbrace{\varphi_1\cdots\varphi_n}^{:=\varphi^n}\ \mathrm{e}^{-\mathcal{S}[\varphi]+J\cdot\varphi} \tag{2.6}$$

We obtain the Schwinger functional by taking the logarithm:

$$W[J] = \ln Z[J] \tag{2.7}$$

For the special case of $n = 2$ the correlation function yields the connected 2-point function which is also known as the propagator $G_{ab}(x,y) = G_{\alpha\beta}$ correlating the field φ_a at spacetime point x with the field φ_b at y.

$$\begin{aligned} G_{\alpha\beta} &= \frac{\delta^2 W[J]}{\delta J_\alpha \delta J_\beta} = \frac{\delta}{\delta J_\alpha}\left(\frac{1}{Z}\frac{\delta Z}{\delta J_\beta}\right) \\ &= \frac{1}{Z}\left(\frac{\delta^2 Z}{\delta J_\alpha \delta J_\beta}\right) - \frac{1}{Z^2}\left(\frac{\delta Z}{\delta J_\alpha}\right)\left(\frac{\delta Z}{\delta J_\beta}\right) \\ &= \langle\varphi_\alpha\varphi_\beta\rangle - \phi_\alpha\phi_\beta = \langle\varphi_\alpha\varphi_\beta\rangle_{\mathrm{c}} \end{aligned} \tag{2.8}$$

The Effective Action:

The effective action can be obtained by performing a Legendre transform of the Schwinger funtional, i. e.:

$$\Gamma[\phi] = \sup_J \left\{ \int_x J(x)\phi(x) - \mathcal{W}[J] \right\} = \int_x J_{\mathrm{sub}}(x)\phi(x) - \mathcal{W}[J_{\mathrm{sub}}] \tag{2.9}$$

Quantum equation of motion:

$$\frac{\delta\Gamma[\phi]}{\delta\phi(x)} = J(x) \tag{2.10}$$

Dyson-Schwinger equation:

$$\frac{\delta\Gamma[\phi]}{\delta\phi(x)} = \frac{\delta\mathcal{S}}{\delta\varphi(x)}\left[\varphi = G\cdot\frac{\delta}{\delta\phi} + \phi\right] \tag{2.11}$$

2.2. The Functional Renormalization Group

- Kadanoff Block-Spin model

- maybe visualization of Ising model + phase transitions

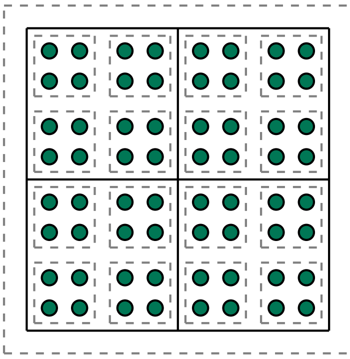

Figure 2.1.: Visualization of the Kadanoff Block-Spin model.[1]

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2.3. Renormalization Group Consistency

This section is mainly based on [1].

Cutoff independence of the full quantum effective action:

$$\Lambda \frac{\mathrm{d}\Gamma}{\mathrm{d}\Lambda} = 0 \tag{2.12}$$

Full effective action in a generic representation:

$$\Gamma[\phi] = \mathcal{D}_\Lambda[\phi] + \Gamma_\Lambda[\phi] \tag{2.13}$$

Formal discussion:

$$\Gamma_k[\phi] = \Gamma_\Lambda[\phi] + \int_\Lambda^k \frac{\mathrm{d}k'}{k'} \mathcal{F}_{k'}[\phi] \tag{2.14}$$

1. This visualization is inspired by an image provided in the PhD thesis of J.R. Laguna.

2.4. Flow Equations for Generating Functionals

We introduce the RG time scale t:

$$\partial_t = \frac{\partial}{\partial \ln(k/\Lambda)} = \frac{k}{\Lambda}\frac{\partial}{\partial(k/\Lambda)} = k\partial_k \tag{2.15}$$

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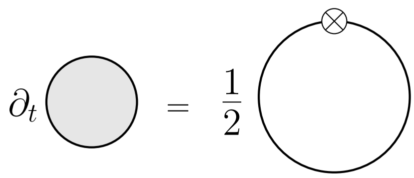
$$\begin{aligned} \partial_t \Gamma_k[\phi] &= \frac{1}{2}\,\mathrm{Tr}\left[\frac{1}{\Gamma_k^{(2)}[\phi] + R_k}\partial_t R_k\right] \\ &= \frac{1}{2}\int_p \frac{1}{\Gamma_k^{(2)}[\phi] + R_k}(p,-p)\;\partial_t R_k(p^2) \end{aligned} \tag{2.16}$$

This translates directly into the following diagrammic representation:



(2.17)

where $\otimes = \partial_t R_k$ represents the insertion of the respective regulator.

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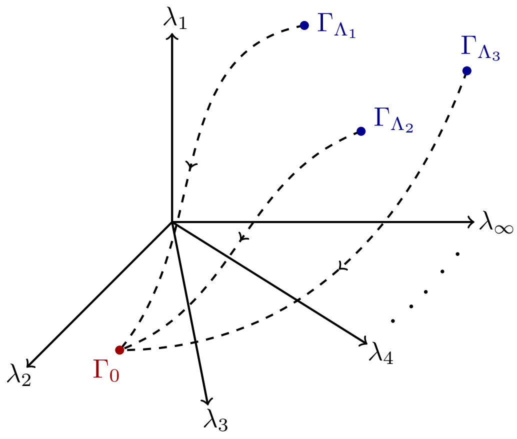

Figure 2.2.: Flow of Γ_k through infinite-dimensional theory space for different regulators.

Chapter 3.

Fundamentals of General Relativity

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3.1. The Einstein Equations

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The Einstein-Hilbert action:

$$\mathcal{S}_{\mathrm{EH}}[g_{\mu\nu}] = \frac{1}{16\pi G} \int\limits_x \sqrt{-\det g_{\mu\nu}} (\mathcal{R} - 2\Lambda) \tag{3.1}$$

Varying this action as usual yields the Einstein equations in absence of matter:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \tag{3.2}$$

where we used $G_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R}$.

Diffeomorphism invariance, Lie derivatives:

$$\mathcal{L}_\omega \phi = \omega^\mu \partial^\mu \phi = \omega^\mu \nabla^\mu \phi \tag{3.3}$$

Now we include matter.

Energy-Momentum Tensor:

$$T_{\mu\nu} = \frac{-2}{\sqrt{-\det g_{\mu\nu}}} \frac{\delta \mathcal{S}_{\text{matter}}}{\delta g^{\mu\nu}} \tag{3.4}$$

Matter part of the action for a minimally coupled scalar field ϕ:

$$\mathcal{S}_{\text{matter}}[g_{\mu\nu}, \phi] = -\frac{1}{2} \int\limits_x \sqrt{-\det g_{\mu\nu}} \left(g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} V(\phi) \right) \tag{3.5}$$

From this, we get the Einstein equations including matter by demanding the variation $\sqrt{-\det g_{\mu\nu}} \frac{\delta \mathcal{S}}{\delta g^{\mu\nu}}$ to vanish. This yields:

$$\frac{1}{8\pi G} \left[\mathcal{R}_{\mu\nu} - \frac{1}{2} (\mathcal{R} - 2\Lambda) g_{\mu\nu} \right] = T_{\mu\nu} \tag{3.6}$$

3.2. Perturbative Non-Renormalizability of Gravity

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Chapter 4.

Quantum Gravity in the Einstein-Hilbert Truncation

4.1. RG approach to Quantum Gravity

Flow equation for QG:

$$\partial_t \Gamma_k[\bar{g}, \Phi] = \frac{1}{2} \operatorname{Tr} \, G_{\mathrm{hh}}[\Phi] \, \partial_t R_k - \operatorname{Tr} \, G_{\mathrm{c\bar{c}}}[\Phi] \, \partial_t R_k \tag{4.1}$$

4.2. Einstein-Hilbert truncation

We want to solve the Flow equation (4.2) approximately. All terms that are invariant under the imposed symmetry, i. e. invariant under diffeomorphism transformations need to be taken into account.

Easiest truncation takes only the scalar curvature $\mathcal{R}$ and the cosmological constant Λ into account (No higher order terms ...) and was performed by Martin Reuter in 1993 [7].

This truncation reads

$$\Gamma_k = 2\kappa^2 Z_k \int_x \sqrt{g} \left[-\mathcal{R} + 2\Lambda_k\right] + \mathcal{S}_{\mathrm{gf}} + \mathcal{S}_{\mathrm{gh}} \tag{4.2}$$

with

$$\kappa^2 = \frac{1}{32\pi G}, \qquad G_k = G Z_k^{-1} \tag{4.3}$$

anomalous dimension:

$$\eta_g = -\frac{\partial_t Z_k}{Z_k} = -\partial_t \ln Z_k$$

dimensionless renormalized cosmological constant:

$$\lambda_k = \Lambda_k k^{-2}$$

dimensionless renormalized cosmological constant:

$$g_k = G_k k^{d-2} = \frac{G k^{d-2}}{Z_k}$$

corresponding beta function:

$$\beta_g = \partial_t g_k = \left(d - 2 + \eta_g\right) g_k \tag{4.4}$$

maximally symmetric space:

$$\overline{\mathcal{R}}_{\mu\nu} = \frac{1}{d}\,\overline{g}_{\mu\nu}\overline{\mathcal{R}} \tag{4.5}$$

$$\overline{\mathcal{R}}_{\mu\nu\rho\sigma} = \frac{1}{d(d-1)}\,(\overline{g}_{\mu\rho}\overline{g}_{\nu\sigma} - \overline{g}_{\mu\sigma}\overline{g}_{\nu\rho})\overline{\mathcal{R}} \tag{4.6}$$

suitable tensor basis:

$$h_{\mu\nu} = h^{\mathrm{TT}}_{\mu\nu} + \overline{\nabla}_\mu \xi_\nu + \left(\overline{\nabla}_\mu \overline{\nabla}_\nu - \frac{1}{d}\,\overline{g}_{\mu\nu}\overline{\Delta}\right)\sigma + \frac{1}{d}\,\overline{g}_{\mu\nu} h \tag{4.7}$$

Chapter 5.

Conclusions and Outlook

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Appendix A.

Mathematical Appendix

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A.1. Heat Kernel techniques

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A.2. York decomposition

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Appendix B.

Numerical Implementation

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B.1. Determination of the Fixed Points

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List of Figures

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Group, proofreaders, Heidelberger dudes..

Not to forget, i have to thank all my friends from home, especially Bastian, Chiara, Helena, Jakob, Jana and Lea for all the amazing time we spent together during the last years.

Lastly, i want to thank my parents Marie-Paule and Bernd Kaltschmidt and my sister Céline for their constant support and love and for always allowing me to pursue my dreams.

Declaration of Authorship

I hereby certify that this thesis has been composed by me and is based on my own work, unless stated otherwise.

Heidelberg, ________________ ____________________________