

# Zeeman effect

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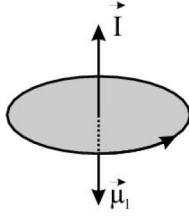
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## Introduction

The most important experimental technique triggering the development of the modern atomic theory is spectroscopy, the observation of the characteristic wavelengths of light that atoms can absorb and emit. A so-called wavelength dispersive element such as a water drop, a prism or a grating, spatially separates different wavelengths of light emitted from one source. If the light source emits white light, a band of all colours will be found after the light passes the wavelength dispersive element (e.g. rainbow). Observing the light from a lamp that contains only atoms of a certain element, one finds distinct lines instead of the complete band on the detector. The positioning of the lines on the detector is called a spectrum, and the corresponding wavelengths are hence called "spectral lines" of the element. The additional information needed to be able to draw conclusions about the atomic structure from spectral lines was provided by the experimental observation of Rutherford that in an atom, electrons surround a heavy, positively charged nucleus in the center, and by Planck's explanation of the blackbody spectrum by quanta of light, photons, whose energy  $E_{ph}$  is connected with the wavelength  $\lambda$  by

$$E_{ph} = h\nu = \frac{hc}{\lambda} \quad (1)$$

where  $\nu$  is the frequency,  $h$  is the Planck constant and  $c$  is the speed of light. With this knowledge it became clear that electrons in an atom can occupy only certain energy levels  $E_i$ , and that the transition of an electron from a higher energy level  $E_2$  to a lower energy level  $E_1$  leads to the emission of a photon:  $E_2 - E_1 = E_{ph}$ . The energy needed to



**Figure 1:** Classical model of an electron with angular momentum  $\mathbf{l}$  inducing an angular magnetic moment  $\vec{\mu}_l$ .

lift the electron to the excited energy level can be provided by absorption of a photon of the very same energy or by collisions.

The increasing experimental precision reached with spectroscopy inspired scientists to propose different atomic models, leading to the development of quantum mechanics. With refinements to the quantum mechanical description of the atom, relativistic quantum mechanics (necessary to explain the splitting of energy levels of same main quantum number  $n$ ) and quantum electrodynamics (QED, necessary to explain the splitting of levels with same quantum number  $j$  but different quantum number  $l$ ), theory is by now able to make highly accurate predictions about the atomic structure.

## Effects of an external magnetic field on the atomic structure

If the atom is placed in an external magnetic field, the atomic energy levels split and the observed spectrum changes. This was first observed by the dutch physicist Pieter Zeeman and is hence called the Zeeman effect. In an intuitive approach to explain the Zeeman effect, consider an atom with an electron with charge  $e$  moving with the speed  $v$  around the nucleus on a Bohr orbit with radius  $r$ . This orbiting electron can be described by a current

$$I = -e \cdot \frac{v}{2\pi r} \quad (2)$$

inducing an orbital magnetic moment

$$\vec{\mu}_l = I \cdot \mathbf{A} = I \cdot \pi r^2 \mathbf{n} = \frac{evr}{2} \mathbf{n} \quad (3)$$

where  $\mathbf{A} = \pi r^2 \mathbf{n}$  is the area vector perpendicular to the orbit area  $\pi r^2$ . The angular momentum of the electron with mass  $m_e$  is

$$\mathbf{l} = \mathbf{r} \times \mathbf{p} = m_e \cdot r \cdot v \cdot \mathbf{n}. \quad (4)$$

An external magnetic Field  $\mathbf{B}$  interacts with the angular magnetic moment and, hence,

changes the potential energy of electron:

$$\Delta E_{pot} = -\vec{\mu}_l \cdot \mathbf{B} = \frac{e}{2m_e} \cdot \mathbf{l} \cdot \mathbf{B}. \quad (5)$$

Using the direction of the magnetic Field vector  $\mathbf{B}$  as the quantization axis z and the quantized angular momentum of the electron

$$|\mathbf{l}| = \sqrt{l(l+1)\hbar} \quad \text{with} \quad l = 0, 1, \dots, n-1 \quad \text{and} \quad (6)$$

$$l_z = m_l \cdot \hbar \quad \text{with} \quad -l \leq m_l \leq l, \quad (7)$$

Eqn. 5 can be simplified to

$$\Delta E_{pot} = \frac{e \cdot \hbar}{2m_e} \cdot m_l \cdot B = \mu_B \cdot m_l \cdot B, \quad (8)$$

where  $\mu_B$  is called the Bohr magneton. The energy shift of the original energy level of the electron by  $\Delta E_{pot}$  due to the coupling of the external magnetic field to the angular magnetic moment of the electron leads to a splitting of the original level into the previously degenerated  $2l + 1$  sublevels of same angular momentum  $l$ , but different magnetic quantum number  $m_l$ .

The quantum mechanical way for deriving this energy splitting, where also the electron spin is taken into account, starts off by treating the external magnetic field as a perturbation  $\hat{H}_B$  of the unperturbed Hamilton Operator  $\hat{H}_0$ . In general, an atom with more than one electron has to be considered. In the case of an external magnetic field being small compared to the magnetic coupling energies between the  $i$ th orbital and spin momenta of the electron  $\mathbf{l}_i$  and  $\mathbf{s}_i$ , respectively, one may use the so-called LS-coupling approximation. This approximation is appropriate when the coupling energy between the different orbital momenta of the electrons is large compared to the coupling of a single orbital momentum of one electron with the same electron spin momentum. Hence, all orbital and spin momenta are added separately to total orbital

and total spin momenta  $\mathbf{L}$  and  $\mathbf{S}$  before determining the total angular momentum  $\mathbf{J}$ :

$$\mathbf{L} = \sum_i \mathbf{l}_i \quad \text{with} \quad |\mathbf{L}| = \sqrt{L(L+1)}\hbar, \quad (9)$$

$$\mathbf{S} = \sum_i \mathbf{s}_i \quad \text{with} \quad |\mathbf{S}| = \sqrt{S(S+1)}\hbar, \quad (10)$$

$$\mathbf{J} = \mathbf{L} + \mathbf{S} \quad \text{with} \quad |\mathbf{J}| = \sqrt{J(J+1)}\hbar, \quad (11)$$

$$J_z = M_J \cdot \hbar \quad \text{with} \quad -J \leq M_J \leq J. \quad (12)$$

With these momenta introduced, the Hamiltonian of an atom in an external magnetic field is

$$\hat{H}_0 + \hat{H}_B = \hat{H}_0 - \frac{\mu_B}{\hbar} (\mathbf{L} + \mathbf{S}) \cdot \mathbf{B} \quad (13)$$

and the solution for the shift of the level energy becomes

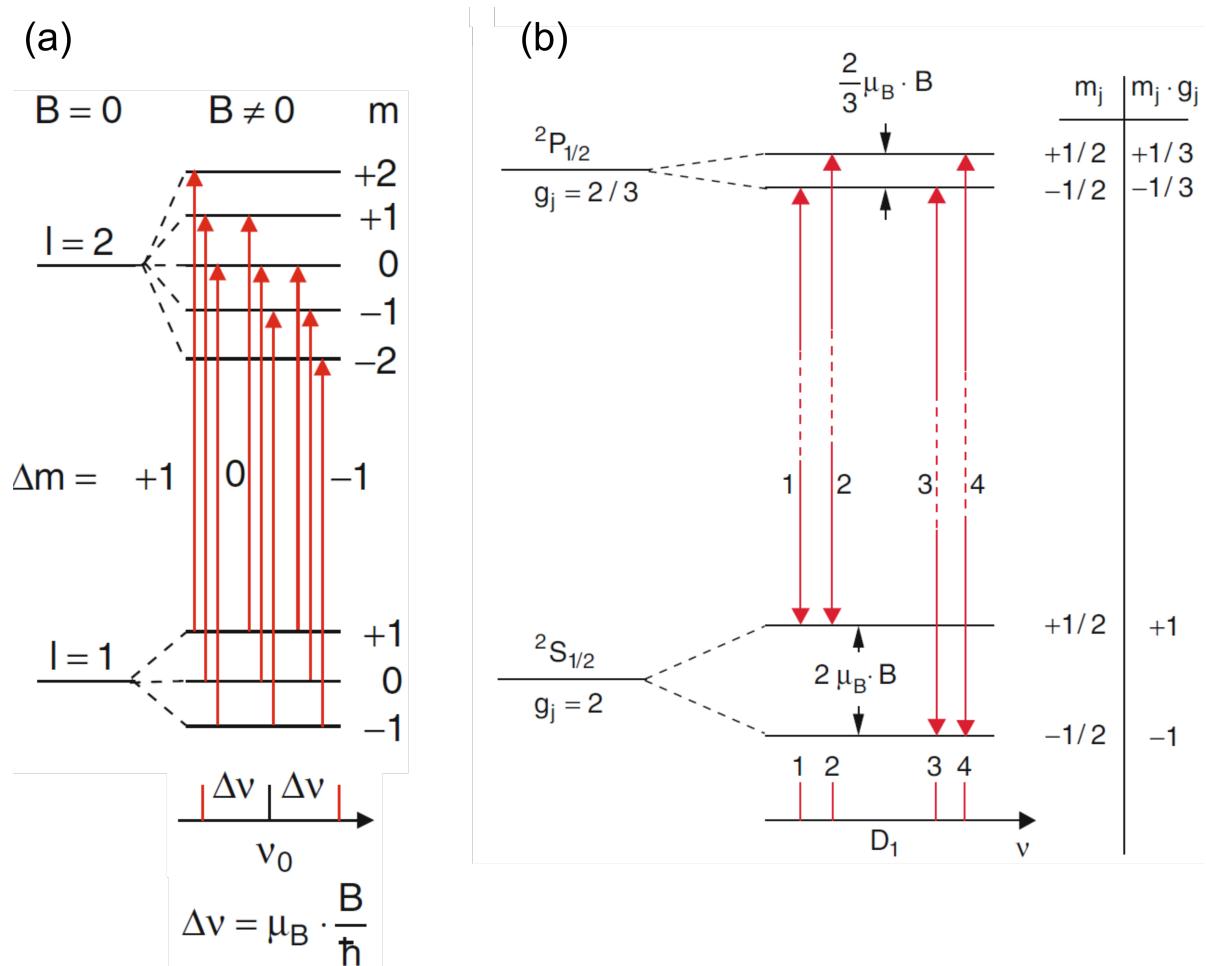
$$\Delta E_{pot} = \mu_B \cdot B \cdot M_J \cdot g_J \quad (14)$$

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}, \quad (15)$$

with the Landé factor  $g_J$ .

If all electron spins in the atom cancel each other, ( $S = 0$ ), Eqn. 5 becomes identical to Eqn. 14 because  $g_J = 1$ .

For historical reasons, the splitting in absence of spin is called the *normal Zeeman effect*, because the electron spin was not yet discovered at that time. The more general case with spin ( $S \neq 0$ ) is named the *anomalous Zeeman effect*. In the normal Zeeman effect, the size of the energy splitting from the unperturbed level energy depends only on the magnetic quantum number  $m_l$ . Therefore, it is of the same size for all levels in the atom. In the case of the anomalous Zeeman effect, the energy splitting depends on the quantum numbers  $J$ ,  $L$  and  $S$ , which is reflected by more complicated structures observed in the corresponding spectra (Fig. 2). In a strong external field, the  $LS$ -coupling approximation is no longer appropriate. The field-induced precessions are so rapid that the total orbital momentum  $\mathbf{L}$  and spin  $\mathbf{S}$  have to be considered separately, because they individually precess around  $\mathbf{B}$ . This means that  $\mathbf{L}$  and  $\mathbf{S}$  are decoupled and  $\mathbf{J}$  is rendered meaningless. This is called the Paschen-Back effect.



**Figure 2:** Left: Normal Zeeman effect for a  $p \rightarrow d$  transition. The field splits the degenerate  $M_J$  levels by  $\Delta E = \mu_B \cdot B \cdot M_J$ . Right: Anomalous Zeeman effect. Here, the levels split by  $\Delta E = \mu_B \cdot B \cdot M_J \cdot g_J$ . In both cases, transitions can only occur according to the selection rules. (Source: Experimentalphysik 3, W. Demtröder, 2015)

## Selection rules and polarization of emitted light

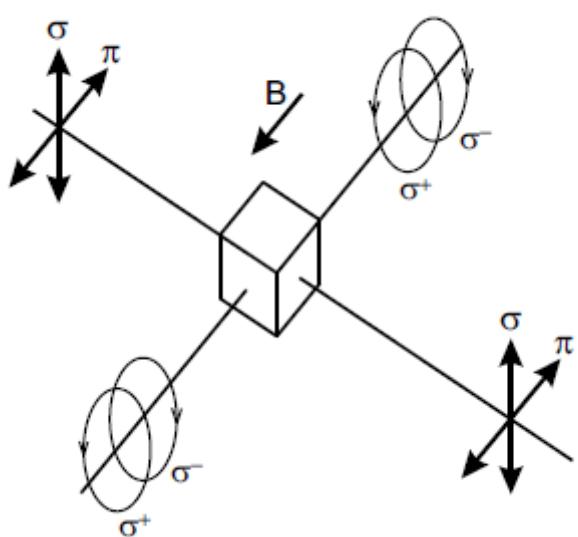
In transitions from an excited level  $i$  to a lower level  $k$ , a photon with the energy  $E_{ph} = \Delta E = E_i - E_k$  is emitted. Still, as indicated in Fig. 2, not all energy differences between existing levels are actually found in the spectra. The reason is that not only the energy, but also momentum conservation, angular momentum conservation and symmetry rules play a role on whether or not a transition can occur. Only those transitions are allowed where the transition dipole matrix element

$$\mathbf{M}_{ik} = e \int \Psi_i^* \mathbf{r} \Psi_k dV \quad (16)$$

has at least one non-zero component

$$(M_{ik})_q = e \int \Psi_i^* q \Psi_k dq, \quad \text{with } q = x, y, z. \quad (17)$$

Evaluating these three integrals yields that only those transitions have non-zero components where the change of the magnetic quantum number fulfills  $\Delta M_J = M_{J,i} - M_{J,k} = 0, \pm 1$ . Furthermore, electric dipole transitions require that the change of the total orbital momentum is  $\Delta L = L_i - L_k = \pm 1$  and that the spin does not change, i.e.  $\Delta S = 0$ . If the quantisation axis (i.e. the direction of the magnetic field vector) is the z-axis, the matrix element component  $(M_{ik})_z$  is the only non-zero one for  $\Delta M_J = 0$  (so-called ' $\pi$ '-transitions). Considering a dipole and its characteristics of emission, this means the dipole is oscillating along the axis of the magnetic field vector and, hence, does not emit any radiation in that direction. Also, since the electric field vector is oscillating in only one direction, also the field vector of the emitted light wave oscillates in this direction, meaning that the light emitted by  $\Delta M_J = 0$  transitions is linearly polarised (see Fig. 3). In the case of  $\Delta M_J = \pm 1$  (' $\sigma$ '-transitions),  $(M_{ik})_z$  is zero and both  $(M_{ik})_x$  and  $(M_{ik})_y$  are non-zero and equal in magnitude, but phase-shifted by  $\pi/2$ . This phase shift between the two superimposed dipoles leads to the observation of circularly polarised light when viewing in z-direction ('longitudinal'). Viewing the emitted light in x- or y-direction ('transversal'), only the projection of the superimposed dipoles is seen, and thus the light from  $\Delta M_J = \pm 1$  transitions appears linearly polarised when observed transversal to the magnetic field vector.



**Figure 3:** Isometric depiction of the polarisation of the different Zeeman components.

# The Experiment

The experiment consists of two parts. In the first part, the Zeeman effect in transverse and longitudinal directions is investigated, and the polarisation of the observed lines are examined. The second part is dedicated to precision spectroscopy with a grating spectrometer. In both parts of the experiment the light emitted from a Cadmium lamp (Cd) will be used. The electronic configuration of Cadmium is  $[\text{Kr}]4\text{d}^{10}5\text{s}^2$ . In this configuration all spins of the shell electrons compensate for each other, such that  $S = 0$ . In visible light, the Cd spectrum consists of four lines, given in Tab. 1. All measurements will be made on the red line,  $\lambda_0 = 643.8 \text{ nm}$  ( $\nu_0 = 465.7 \text{ THz}$ ), which arises from the transition between the two excited levels  ${}^1\text{D}_2 \rightarrow {}^1\text{P}_1$  in the fifth shell of Cd.

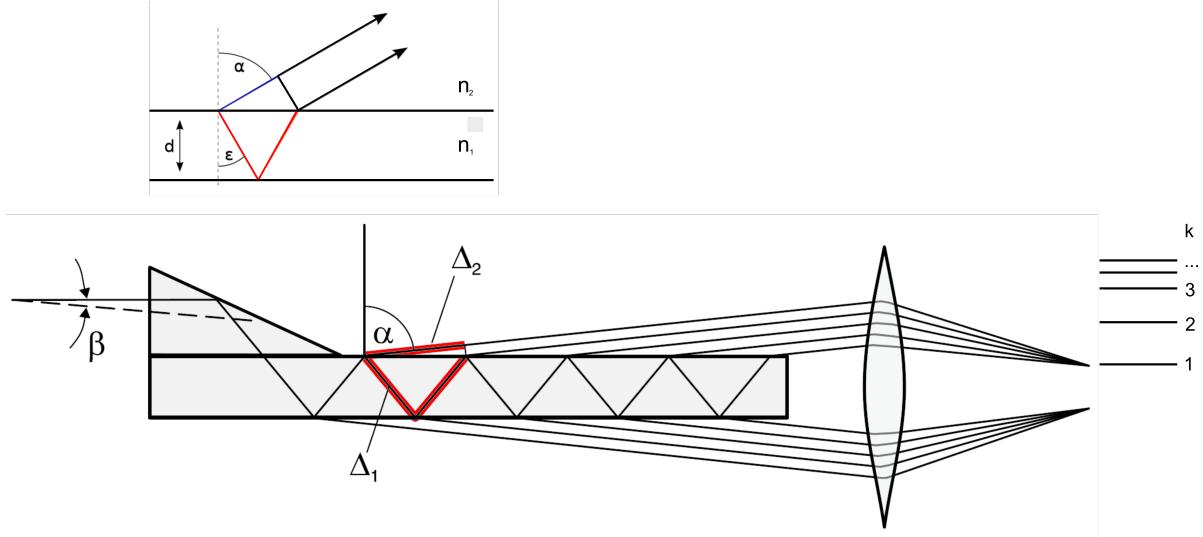
**Table 1:** Spectral lines of Cadmium in the visible range.

Colour	violet	blue	green	red
Wavelength (nm)	467.9	480.1	508.7	643.8

## Part 1: Spectroscopy of the Zeeman Effect

As the expected relative shift in wavelength due to the external magnetic field is smaller than 0.1 nm, a very high resolution is necessary for the effect to be observable. In this experiment, a so-called Lummer-Gehrcke Plate will be used (Fig. 4). It is a quartz glass plate with extremely plane parallel surfaces. The light enters into the plate through the use of a prism, and is then reflected to and from between the inner surfaces of the plate under an angle near to total internal reflection. In this way, only a small part of the light escapes from the plate at every point of reflection. The escaping light can be observed with a telescope fixed behind the plate, whose focal point is at infinity. As the thickness of the plate is a lot smaller than its length and has a higher coefficient of reflection, many light rays can interfere with each other. This leads to a very high resolution. The optical path length difference between two light rays, one of which is delayed by the glass plate, is given by

$$\Delta = 2n_2 \cdot \frac{d}{\cos(\epsilon)} - 2n_1 \cdot d \cdot \tan(\epsilon) \cdot \sin(\alpha), \quad (18)$$



**Figure 4:** Schematic of the Lummer-Gehrcke-Plate

with the thickness of the plate  $d$ , the indices of refraction  $n_1$  and  $n_2$  inside and outside the plate, respectively, and the angles  $\alpha$  and  $\epsilon$  of the light rays to the normal of the plate (outside and inside). With  $\sin(\alpha) = \frac{n_2}{n_1} \cdot \sin(\epsilon)$  and  $\sin^2(\epsilon) + \cos^2(\epsilon) = 1$  Eqn. 18 becomes:

$$\Delta = 2n_2 \cdot d \cdot \cos(\epsilon) = 2 \cdot d \cdot \sqrt{n_2^2 - n_1^2 \sin^2(\alpha)}. \quad (19)$$

For a plate in air ( $n_2 \approx n$ ), ( $n_1 \approx 1$ ) and  $\alpha \approx 90^\circ$  Eqn. 19 reduces to

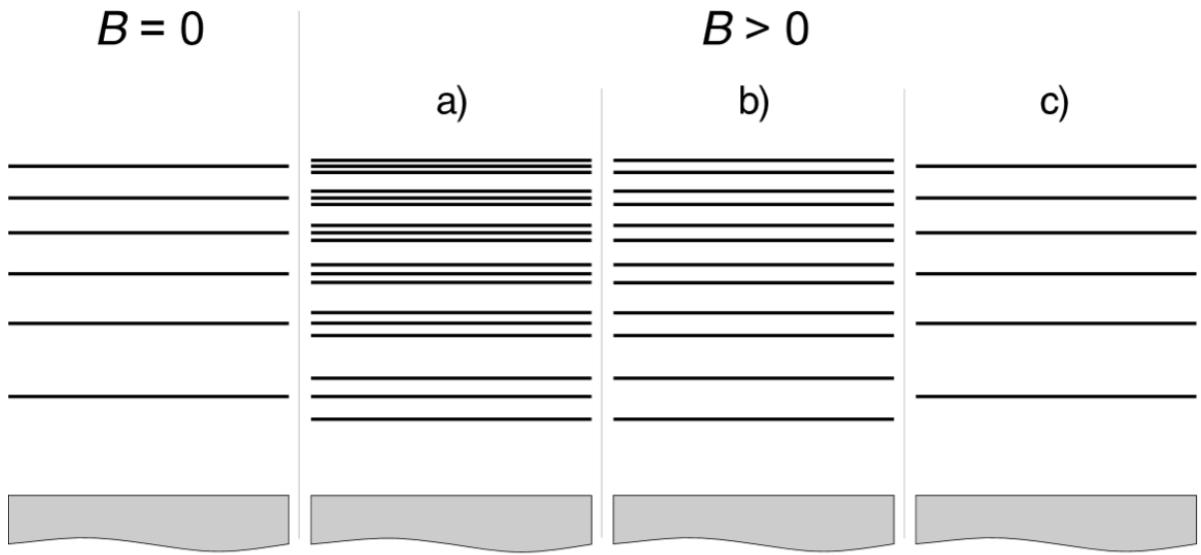
$$\Delta = \Delta_1 - \Delta_2 = 2d \cdot \sqrt{n^2 - 1} \quad (20)$$

where the  $\Delta_i$  are marked in the right hand part of Fig. 4.

For constructive interference the path difference must satisfy  $\Delta = k \cdot \lambda$ , for any whole number  $k$  and wavelength  $\lambda$ . Whether constructive or destructive interference occurs when the light behind the Lummer-Gehrcke Plate is observed with the telescope, depends on the wavelength  $\lambda$ , the escape angle  $\alpha$  and hence also on the angle  $\beta$  under which the light enters the plate. For a given wavelength  $\lambda$ , many orders  $k$  for different entering angles  $\beta$ , at which the escaping light rays interfere constructively, are recovered (compare Fig. 5).

The order  $k$  of wavelength  $\lambda + \Delta\lambda$  and order  $k + 1$  of wavelength  $\lambda$  overlap spatially when

$$k \cdot (\lambda + \Delta\lambda) = (k + 1) \cdot \lambda \rightarrow \Delta\lambda = \frac{\lambda}{k}. \quad (21)$$



**Figure 5:** Interference pattern for the Zeeman effect, as would be observed in the transverse direction for the case a) without polarisation filter, b) with polarisation direction of the filter perpendicular to the magnetic field and c) with polarisation direction of the filter parallel to the magnetic field.

With Eqn. 20 the free spectral range of the Lummer-Gehrcke plate is given by

$$\Delta\lambda = \frac{\lambda^2}{2d \cdot \sqrt{n^2 - 1}}. \quad (22)$$

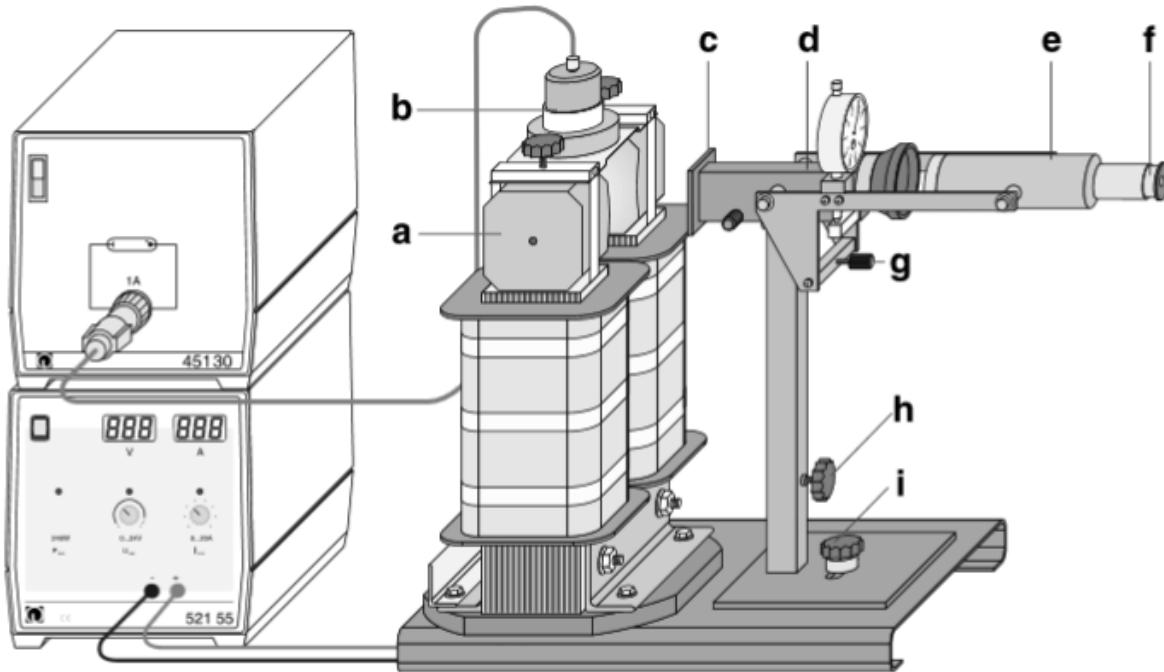
To a first approximation, a small change in wavelength  $\delta\lambda \ll \Delta\lambda$  leads to a shift of

$$\delta\lambda = \frac{\delta a}{\Delta a} \cdot \Delta\lambda, \quad (23)$$

with the distance  $\delta a$  between the line with wavelength  $\lambda + \delta\lambda$  and the position of the line with wavelength  $\lambda$ , and the distance  $\Delta a$  between two orders  $k$  and  $k + 1$  of wavelength  $\lambda$ . Therefore, for small shifts in wavelength as it is the case for the Zeeman effect, one obtains the following:

$$\delta\lambda = \frac{\delta a}{\Delta a} \cdot \frac{\lambda^2}{2d \cdot \sqrt{n^2 - 1}}. \quad (24)$$

The experimental setup is sketched in Fig. 6. The Cd-lamp is placed between the pole pieces of the magnet. Light emitted from this lamp enters the Lummer-Gehrcke



**Figure 6:** Drawing of the experimental set-up for investigating the normal Zeeman effect (Taken from Leybold, *Physics Leaflets*). a) Pole pieces of the magnets, b) Cd lamp, c) Red light filter, d) Lummer-Gehrcke Plate, e) Telescope, f) Eyepiece, g) Height adjustment for the telescope, h) Locking screw for the telescope holder, i) Locking screw for the base.

plate after passing through the red filter. This filter only lets the red part of the Cd spectrum through. The magnet can be turned by  $90^\circ$ , so that the lamp can be observed in transverse and longitudinal directions. The longitudinal direction may be observed through the hole in the pole piece. The interference pattern produced by the Lummer-Gehrcke plate can be observed through the telescope. In order to observe different polarisations, a polarisation filter and a  $\lambda/4$  waveplate may be fixed between the plate and the telescope. With the height adjustment of the telescope, a crosshair in the eyepiece can be moved over the interference pattern.

## Part 1: Execution

- Use the teslameter (Hall probe) to determine the strength of the magnetic field at the position of the Cd-lamp. The magnetic field strength should be measured as a function of the current. Carry out the measurements once with increasing and once with decreasing current (Hysteresis effect). Take three measurements

for each current value and average. Make sure to use the right power supply settings (constant voltage vs. constant current).

- Carefully set the Cd-lamp into the holder with the magnetic field turned off (without using force!) and observe the Zeeman effect in transverse and longitudinal directions with the telescope and with the camera. → Let turning the magnets be demonstrated by the supervisor first!
- Use the polarisation filter and the  $\lambda/4$ -waveplate (how can the two be distinguished?) in order to verify the polarisations of the different Zeeman components with different observation directions (transverse and longitudinal).
- Using the **ThorCam** program, record an image of the  $\sigma$ - and  $\pi$ -lines with  $\approx\sim 9$  orders of interference  $k$  for four different currents of the magnet between  $I = 8 \text{ A}$  and  $I = 13 \text{ A}$ .
- **ThorCam program** (see Appendix):
  - Open program
  - Select camera
  - Settings: Optimize
    - \* Gain: 'Image'-register
    - \* Exposure time: 'Camera'-register
  - Start capture
  - Save image (*Desktop/F44/your\_names*)
- **ImageJ program** (see Appendix):
  - Open image
  - Mark lines on the image (zoom and center the end points of the line)
  - PlugIns → Spectroscopy → Shift (produced from a projection of the image)
  - To save your data: *list* → *save as ..txt*'

## Security Advice:

- The Lummer-Gehrcke plate and its cover may not be taken away. The plate is very expensive!
- The current for the magnetic field should always be turned off when the Hall probe or the Cd-lamp is being placed in or taken out of the holder.
- Never touch the Cd-lamp with bare fingers. Moisture and grease on the glass reduce the lifetime of the lamp and it is also very hot. Take care when inserting and removing the lamp from the magnet.
- Only turn the magnet, when the current is switched off.

## Part 1: Evaluation

- Create a graph with the magnetic field strength as a function of the current. Assess the strength of the hysteresis effect. Use a linear fit to obtain the function  $B(I)$  and the magnetic field strength at the currents measured by you. Note: If the fit doesn't correspond to your datapoints you might directly use the measured values and think about reasons, why a linear fit seems wrong.
- Note qualitatively what you see in observations (transversal and longitudinal) of the Zeeman effect. (For long/short evaluations: save images in the **ThorCam** program for the long protocol.)
- When using the polarisation filter and the  $\lambda/4$ -waveplate, note which lines are visible, when and why.
- In the transverse direction: Determine from the graphs, that you have produced with the program **ImageJ**, the positions  $a$  of all  $\sigma$ - and  $\pi$ -lines for all orders of interference. This should be done for each of the three measurements at different current for the magnetic field. Fit the line positions (always fit the main and two neighbouring maxima at once, by choosing a suitable fitting model). Hence, you obtain the line positions in pixel. What lineshape did you use for your fit? Justify your choice. Is the error, given by the fit, reasonable? If this is not the case, estimate a new error based on the region of the fitted curve.

- For each of the three magnetic fields, plot the order of interference  $k$  against the position  $a$  of the  $\pi$ -lines. Find a polynomial function  $k = f(a)$  (which order is sufficient?) to fit the trend of the points. The distance of the different orders on the y-axis is  $f(\Delta a) = f(a_2) - f(a_1) = 1$ . Determine the relation  $\delta a / \Delta a$  based on the fitted curve. Calculate  $\delta\lambda$  using Eqn. 24 (parameters of the Lummer-Gehrcke plate:  $n = 1.4567$ ; thickness  $d = 4.04$  mm). The error for  $\delta\lambda$  is determined from the standard deviation of the order. For  $\lambda$ , use the value that you determined in part II of the experiment.
- Calculate  $\mu_B$  for the three magnetic field strengths. Form an average and compare this with the literature value of  $\mu_B = 9.27400949 \cdot 10^{-24}$  J/T.
- Long / short evaluation: Plot  $\Delta E$  against  $B$ , fit a function and determine  $\mu_B$  based on this plot, in order to compare the value with the literature value.

## Part II: Precision Spectroscopy

The Lummer-Gehrcke plate, which served to determine the small differences between wavelengths in the first part of the experiment, can only be used to determine the relative wavelengths. The absolute wavelength will be precisely determined in the second part with a grating spectrometer.

### Czerny-Turner Spectrometer

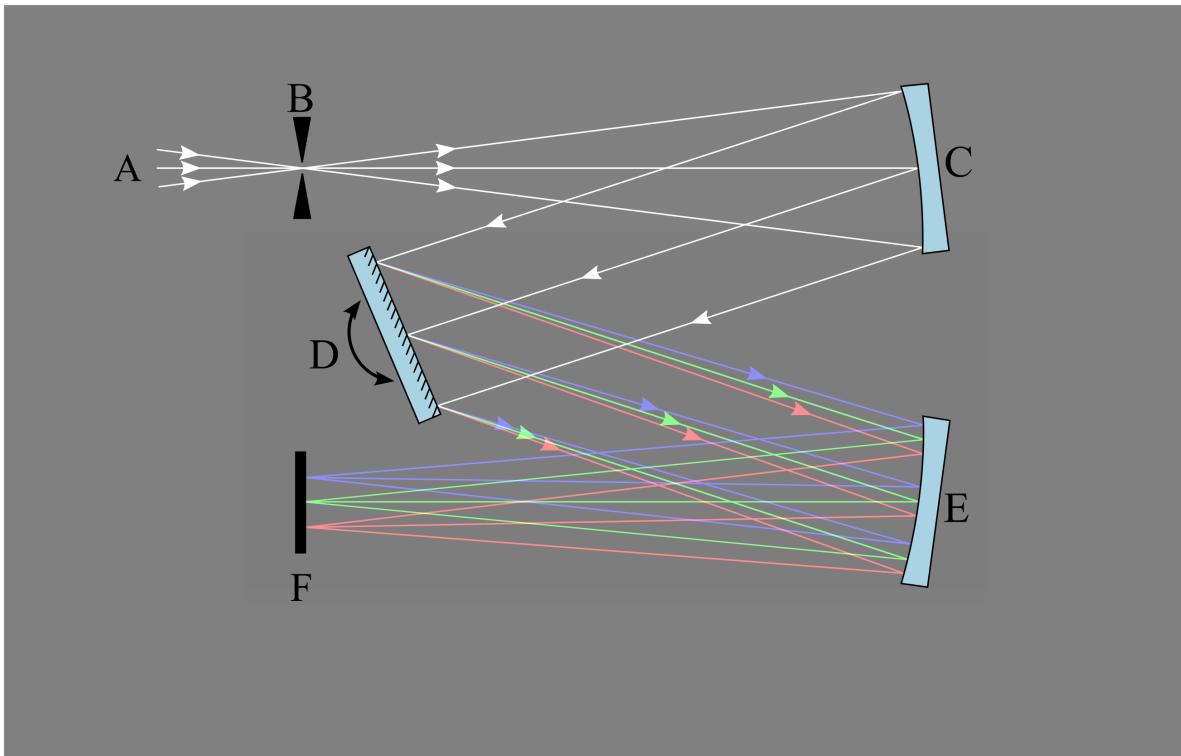
The Czerny-Turner (CT) Spectrometer splits up light into discrete wavelengths by a diffraction grating. The spectrum appears in the focal plane of the exit and is recorded there by a CCD camera. As shown in Fig. 7, a curved mirror (C) is used to focus the light rays, which reach the spectrometer through the entrance slit. After the light is diffracted at the grating, it is focused by another mirror onto the CCD camera.

#### Grating Characteristics

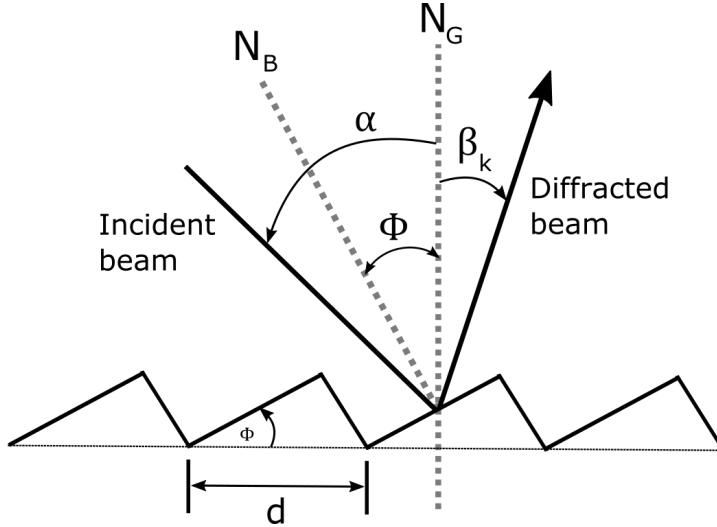
At a grating, the angles of the incoming and outgoing light rays with wavelength  $\lambda$  satisfy the following relationship:

$$\sin \alpha + \sin \beta_k = kn\lambda \quad (25)$$

with the angle of incidence  $\alpha$ , the diffraction angle  $\beta_k$  relative to the normal ( $N_G$ ) of the grating, the braking index  $k$  and the line density  $n$  (corresponding to the reciprocal of the grating constant  $1/d$ ). When  $k = 0$ , then Eqn. 25 reduces to  $\alpha = \beta_0$ , the law of reflection. If the diffraction angle is assigned a fixed value, then the difference between  $\alpha$  and  $\beta_k$ , the so called deflection angle ( $D_V$ ) is constant (Fig. 8). The grating can concentrate a large portion of the diffracted radiation into one spectral order. Hence the intensity of the other orders decreases. This redistribution of the intensity between the orders depends on the angle between the reflected elements and the grating surface, the so called *blaze angle*  $\Phi$  (Fig. 8). In the visible spectrum the red region of the spectrum is diverted the least and the violet region the most.



**Figure 7:** A diagram of a Czerny-Turner monochromator. Light (A) is focused onto an entrance slit (B) and is collimated by a concave mirror (C) with focal length  $f$ . The collimated beam is diffracted from a rotatable grating (D) and the dispersed beam re-focussed by a second mirror (E) with the same focal length  $f$  at the camera (F). Each wavelength of light is focused at a different position at the camera. (Adapted from wikipedia.org)



**Figure 8:** Schematic diagram of the reflection of a photon ray after interacting with a grating.  $\Phi$  defines the *blaze angle*. The distance between consecutive lines is given by  $d$ .

### Dispersion

The derivation of the diffraction angle after the wavelength is also referred to as angular dispersion. It is a measure of the angular distance between light beams with adjacent wavelengths. Through the derivation of Eqn. 25 for a fixed angle  $\alpha$ , the following expression for the angular dispersion is obtained:

$$\frac{\partial \beta_k}{\partial \lambda} = \frac{kn}{\cos \beta_k}. \quad (26)$$

A high dispersion can be achieved if either a grating with a high line density ( $n$ ) or a coarse grating in high diffraction orders is used. The linear wavelength dispersion in the exit plane of a spectroscopic element is referred to as *reciprocal linear dispersion* and given in [nm/mm]. For measurement apparatus with focal length  $f$ , the reciprocal linear dispersion follows:

$$D(\lambda) = \frac{\partial \lambda}{\partial p} = \frac{kn \cos \beta_k}{f} \quad (27)$$

with the distance  $p$  in mm. As the size of the measuring apparatus depends on the focal length of the optical system, it can be designed more compactly by using a grating with a higher line density. As shown in Eqn. 27, the relation between the pixel position  $p$  on the CCD camera and the real wavelength  $\lambda$  is given by the dispersion function  $D(\lambda)$ . This function depends on the wavelength. At least one reference point  $p_0$  in the

spectrum (at a known wavelength  $\lambda_0$ ) is necessary in order to calibrate the wavelength scale, if  $D(\lambda)$  is known. Then the value  $\lambda$  of every other line in the spectrum can be obtained through

$$\lambda = \lambda_0 + \int_{p_0}^p D(\lambda) dp. \quad (28)$$

This is often numerically solvable. Though if the dispersion function is independent of  $\lambda$ , then Eqn. 28 can be expressed as following

$$\lambda = \lambda_0 + D(p - p_0), \quad (29)$$

and the desired wavelength  $\lambda$  can be determined. In this experiment reference lines should be used in order to find the dispersion function of the spectrometer. With the positions of the reference lines in the detector  $p_i$  and their wavelengths  $\lambda_i$ , the dispersion can be approximated by a polynomial function  $\lambda(p)$ .

$$\lambda(p) = A + B \cdot p + C \cdot p^2 + \dots, \quad (30)$$

with the free parameters  $A, B, C$ , etc.

### Resolution

The *resolution* or *chromatic resolution* of a grating describes its ability to distinguish neighbouring spectral lines. The resolution is defined as  $R = \frac{\lambda}{\Delta\lambda}$ , with the wavelength difference  $\Delta\lambda$  between two spectral lines of the same intensity, that can be distinguished. The limit of resolution of a grating is  $R = kN$ , with the number  $N$  of the illuminated lines of the grating. An appropriate expression for the resolution is obtained through the use of Eqn. 25:

$$R = kN = \frac{Nd(\sin \alpha + \sin \beta_k)}{\lambda} \quad (31)$$

### CCD Camera

A CCD camera is an arrangement of silicon solid state detectors, which are sensitive in the range of 400 to 1100 nm. They convert the incoming light into electrons through the internal photoelectric effect. These free electrons are saved in a rectangular arrangement of sensors, called pixels. The CCD camera provides information about the intensity and position at the same time. The size of the CCD-chips used

is  $4.61 \times 3.69 \text{ mm}^2$  and consists of  $1280 \times 1024$  pixels. Each pixel has an area of  $3.6 \times 3.6 \mu\text{m}^2$ .

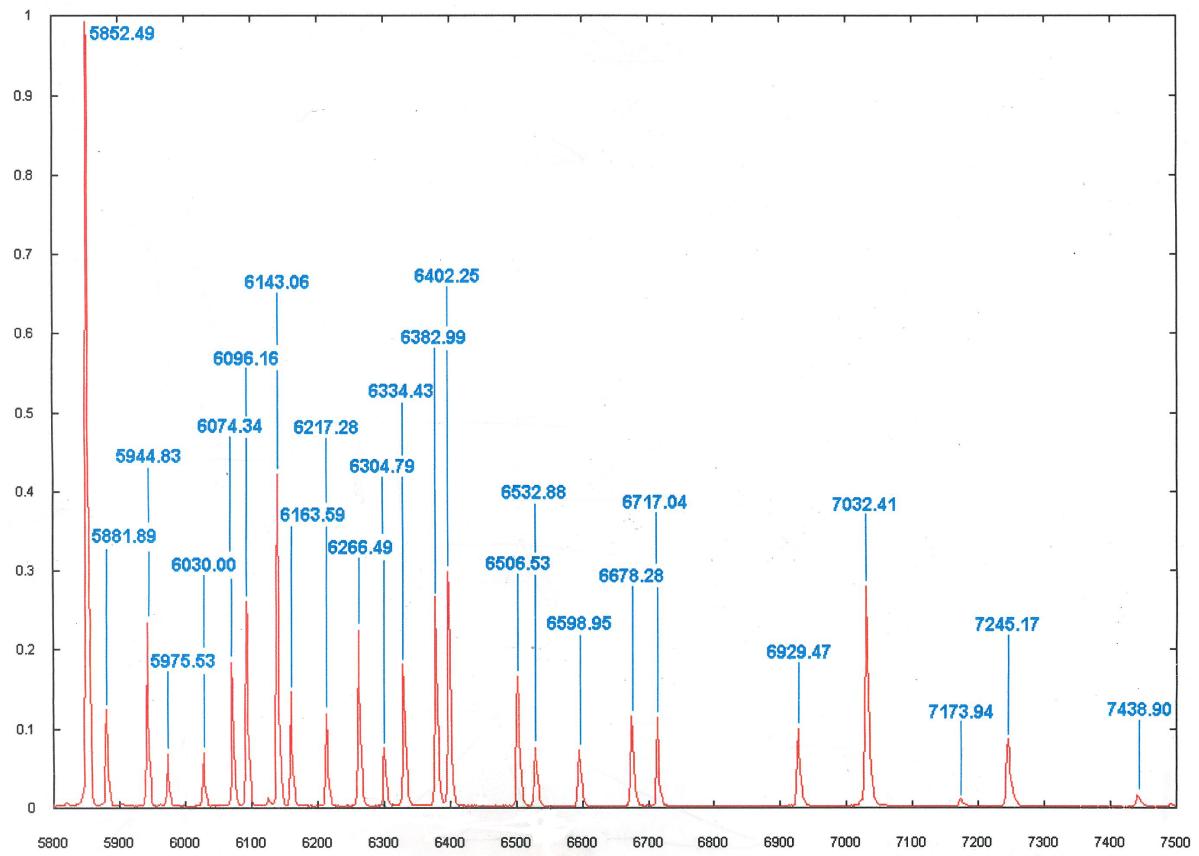
## Calibration

### Calibration lamp

In order to calibrate the spectrometer, for example to find the dispersion, reference lines will be necessary. For this reason, a neon lamp will be used for calibration. The necessary reference lines are listed in Tab. 2 (data from the NIST, National Institute for Standards and Technology, spectral line database, <http://www.nist.gov>) and are displayed in the form of an overview spectrum of the experiment.

**Table 2:** Neon reference lines for the calibration (source: <http://www.nist.gov>)

Species	$\lambda$ (nm)	Species	$\lambda$ (nm)	Species	$\lambda$ (nm)
Ne I	585.24878	Ne I	621.72812	Ne I	667.82766
Ne I	588.1895	Ne I	626.64952	Ne I	671.7043
Ne I	594.4834	Ne I	630.47893	Ne I	692.94672
Ne I	597.55343	Ne I	633.44276	Ne I	702.405
Ne I	602.99968	Ne I	638.29914	Ne I	703.24128
Ne I	607.43376	Ne I	640.2248	Ne I	717.3938
Ne I	609.6163	Ne I	650.65277	Ne I	724.51665
Ne I	614.30627	Ne I	653.28824	Ne I	743.88981
Ne I	616.35937	Ne I	659.89528		



**Figure 9:** Normalized spectrum of the Ne-Calibration lamp. The spectral lines correspond to the ones in Tab 2. The wavelength is given in Angstroem.

## Part II: Execution

The aim of this experiment is to measure the wavelength of the red spectral line of Cadmium (Cd). Furthermore, another very weak line will be emitted from the Cd lamp with a wavelength of  $\approx 652$  nm. This is a spectral line of Xenon. Why can you observe this line? Which line do you expect? (To identify the line go to the NIST Atomic Spectra Database ([http://physics.nist.gov/PhysRefData/ASD/lines\\_form.html](http://physics.nist.gov/PhysRefData/ASD/lines_form.html)))

- Questions to be prepared: The bandwidth, which will be observed by the detector, (CCD) is  $\approx 15$  nm. Therefore, the grating must be moved to map various regions of the Ne- and Cd-spectra. The grating converts the wavelength/energy into a geometrical value, namely the dispersion of the CCD detector. Hence, you must proceed carefully with the alignment of the various components along the light path of the experimental set-up. Are there sources of error in the set-up? How can these be prevented? What line shape do the spectral lines have? Give reasons.
- Consider a strategy for measuring the Cd-line that will minimise the systematic errors (integration time of the camera, calibration procedure etc.).
- Discuss this strategy with the supervisor before you start taking measurements.
- Use the provided optical fiber to guide the light to the spectrometer.
- Optimise the optical set-up in front of the spectrometer.
- Use the program **ThorCam** to take a spectrum of the neon-calibration lamp and compare the images to the overview spectrum, which is displayed on the experiment. Note: Make sure the camera is mounted the right way up.  
Now attempt to find the characteristic region of the neon-spectrum (around the expected wavelength for the Cadmium line) and make separate recordings of this region (4 lines).
- Now take an image of the Cd-line. The integration time/gain must be adapted to the intensity of the Cd-lamp.
- Long / short evaluation: Create an image, in which light from the neon- and the Cd- lamps enter the spectrometer at the same time.

- Long / short evaluation: Create an image of the whole neon spectrum by scanning the grating angle and taking overlapping images. Make sure that all images contain at least one line in the overlapping region in order to stitch all images together during evaluation. Note: The scanning range is limited by the distance between the peaks. The limit is reached, if the distance between lines is too large to create overlapping images.

## Part II: Evaluation

- Fit the position of the 4 neon lines. Fit a calibration function, such that pixels can be converted into wavelengths.
- Determine the wavelength of the Cd-line.
- Long / short evaluation: Create a graph from an image in which both neon lines and Cd lines can be seen. Consider how to present the graph, such that the strong and weak intensity lines are portrayed equally.
- Long / short evaluation: Create a graph of the whole neon spectrum, by stitching the images together.

Compare yours with a graph plotted from the neon values of the NIST database (Tab. 2).

# Preperation

Topics to be prepared are:

- Zeeman effect
- Hysteresis
- Hall effect
- Polarisation (filter, waveplate)
- Lummer-Gehrke plate
- Grating spectrometer (Czerny-Turner Monochromator)
- Charged coupled Device (CCD)
- Internal photoelectric effect
- Line width and line broadening
- **Don't forget to bring an USB stick!**

# Appendix

## ThorCam

- Open the **ThorCam** program and klick on the camera symbol to select the camera (Fig. 10).

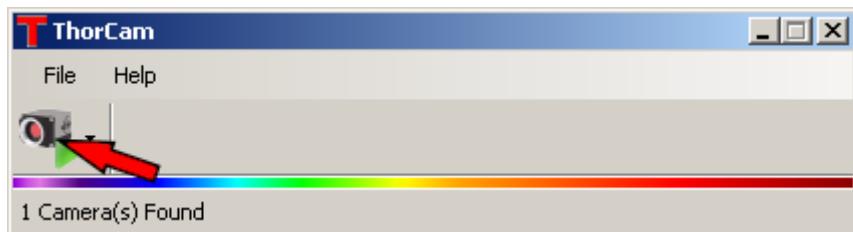


Figure 10: Starting screen of ThorCam

- Now the program opens. When you klick on **start capture**, the camera image is shown on the screen.

To optimize your image (e.g. so you can see faint lines and no lines are saturated), you can change the **options**. After you acquired an image to your satisfaction, you can **save** the image (Fig. 11).

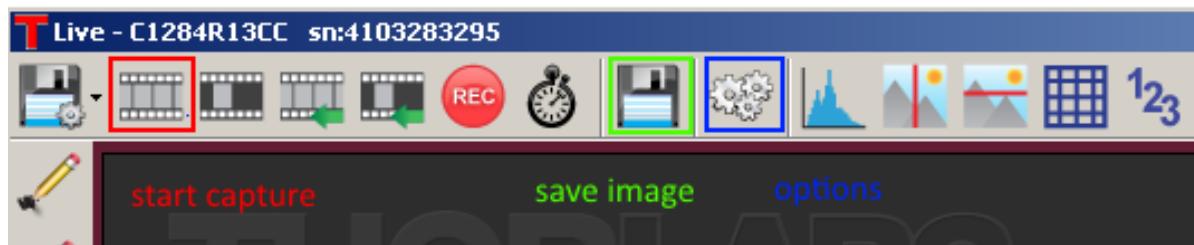


Figure 11: ThorCam toolbar

- In the **Options** section you can change the gain ('Image'-register) and the exposure time ('Camera'-register) to improve your image (Fig. 12). You can also reduce the color gain of blue and green and increase the red one.

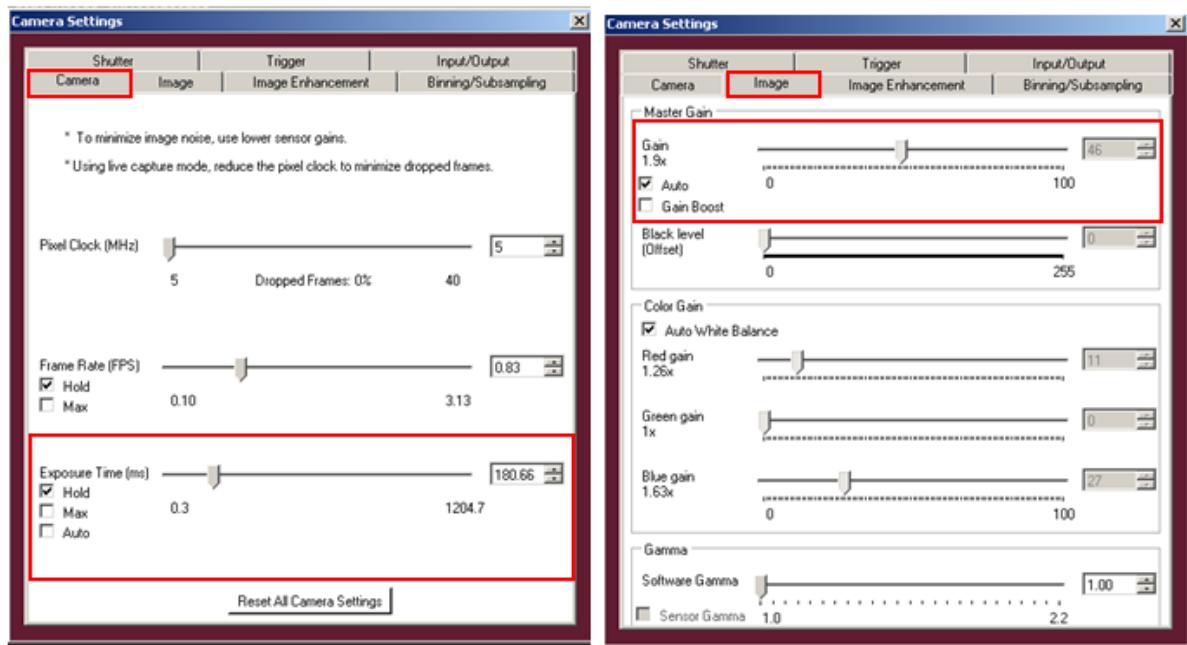


Figure 12: ThorCam option settings

## ImageJ

- Open the program ImageJ which you can find on the desktop.  
Open your picture taken with ThorCam: *File → open*

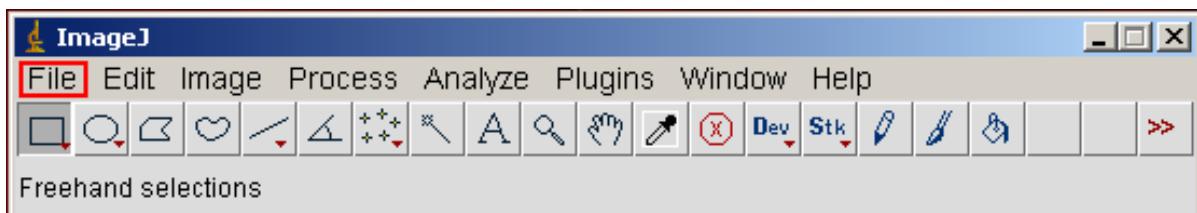
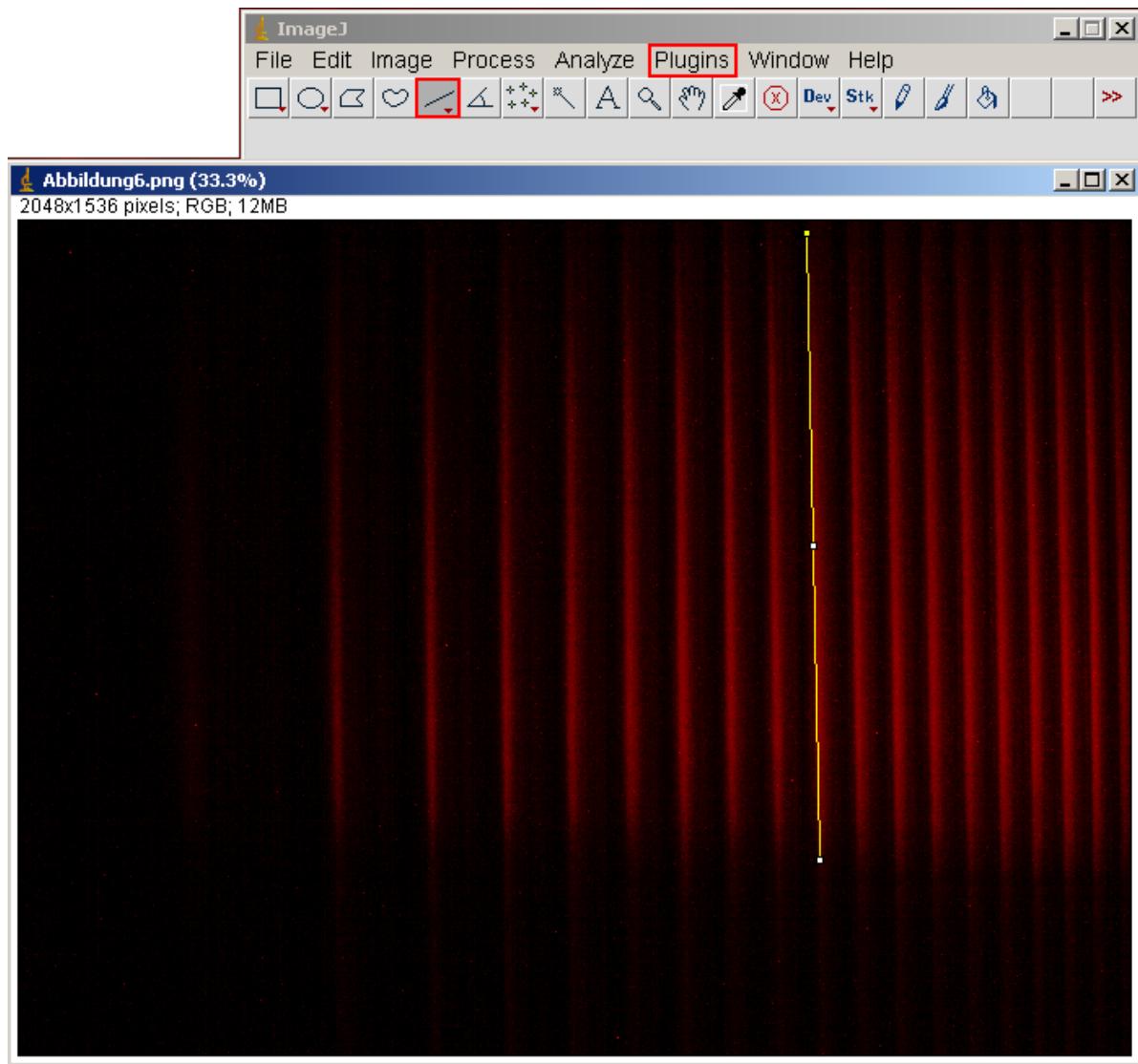


Figure 13: ImageJ toolbar

- In the next step click on the line symbol in the toolbar (Fig. 14) and mark one spectral line in the image.  
If you can not recognize the lines, increase the contrast: *Process → enhance contrast*



**Figure 14:** Projection creation with ImageJ

- To get a projection of your image, click on *Plugins* → *Spectroscopy* → *Shift*
- To save your data: *list* → *save as .txt*