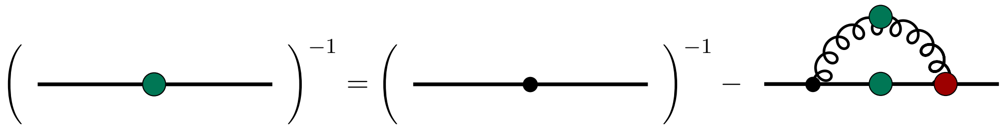
# The Quark Propagator DSE

The Dyson-Schwinger equation for the quark propagator, a.k.a. the quark gap equation, relates the inverse full quark propagator $\Gamma^{(2)}_{q\bar{q}}(p)$ to its classical counterpart $S^{(2)}_{q\bar{q}}(p)$, the respective quark and gluon propagators and the classical and full quark-gluon vertices $\Gamma^{(3)}_{q\bar{q}A}(p)$. It can be formulated in terms of diagrams as follows:



(0.1)

Here, green (red) blobs denote full propagators (vertices), black blobs denote classical vertices. Schematically this translates to:

$$\Gamma^{(2)}_{q\bar{q}}(p) = S^{(2)}_{q\bar{q}}(p) + Z_1^f g_s \int_q G_{AA}(q)(-i\gamma)G_{q\bar{q}}(p+q)\Gamma^{(3)}_{q\bar{q}A}(p+q,-p). \tag{0.2}$$

We suppressed all Lorenz and color indices for convenience. Since in our work, we will use a classical vertex approximation, $\Gamma^{(3)}_{q\bar{q}A}(p) \equiv S^{(3)}_{q\bar{q}A}(p)$, we won't go into details about the wave function renormalization of the quark-gluon-vertex $Z_1^f$. The two relevant renormalization constants, the wave function renormalization $Z_2$ and the mass renormalization $Z_{m_q}$ of the quark enter the gap equation via $S^{(2)}_{q\bar{q}}(p)$, i. e.

$$S^{(2)}_{q\bar{q}}(p) = iZ_2 \not{p} + Z_{m_q} m_q, \tag{0.3}$$

where $m_q$ denotes the bare current quark mass.
Random question: Where does the $i$ come from??
The full gluon and quark propagators in the Landau gauge, $\xi = 0$, are given by

$$G^{ab}_{AA,\mu\nu}(p) = \delta^{ab}\Pi^{\mu\nu}_{\perp} G_A(p) \tag{0.4}$$
$$G^{ab}_{q\bar{q}}(p) = \delta^{ab} G_q(p) \tag{0.5}$$

with

$$G_A(p) = Z_A^{-1}(p)\frac{1}{p^2} \tag{0.6}$$
$$G_q(p) = Z_q^{-1}(p)\frac{1}{i\not{p} + M_q(p)}. \tag{0.7}$$

In our calculation we need to replace these definitions with the spectral representations of the respective propagators.

With all these definitions in mind we can formulate the "standard" form of the gap equation as follows:

$$Z_q(p)\left[i\not{p}+M_q(p)\right]=Z_2 i\not{p}+Z_{m_q}m_q+\Sigma(p) \tag{0.8}$$

The next step is to project the gap equation onto its Dirac vector and scalar parts by multiplying it with either $\mathbb{1}$ or $\not{p}$ and performing the corresponding traces. This leaves us with a set of coupled DSEs for $Z_q(p)$ and $M_q(p)$:

$$Z_q(p)p^2 = Z_2 p^2 - Z_1^f \,\mathrm{Tr}\left[i\not{p}\Sigma(p)\right] \tag{0.9}$$

$$M_q(p) = Z_q^{-1}(p)\left(Z_{m_q}m_q + Z_1^f \,\mathrm{Tr}\left[\Sigma(p)\right]\right) \tag{0.10}$$