The Minimal Supersymmetric Standard Model

Part I: Theory

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Supersymmetry Seminar
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Outline

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Standard Model — The Basics

The gauge group of the Standard Model (SM) is

$$\mathcal{G}_{\mathrm{SM}} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_T \times \mathrm{U}(1)_Y \,.$$

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$$\begin{aligned}
s = 0: &\quad \underset{\ni \phi}{(\mathbf{1},\mathbf{2})_{\frac{1}{2}}} \\
s = \frac{1}{2}: &\quad \underset{\ni q_L}{(\mathbf{3},\mathbf{2})_{\frac{1}{6}}} \oplus \underset{\ni (d_R)^c}{(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}} \oplus \underset{\ni (u_R)^c}{(\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}} \oplus \underset{\ni \ell_L}{(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}} \oplus \underset{\ni (e_R)^c}{(\mathbf{1},\mathbf{1})_{1}} \\
s = 1: &\quad \underset{\ni A^a_\mu}{(\mathbf{8},\mathbf{1})_0} \oplus \underset{\ni W^k_\mu}{(\mathbf{1},\mathbf{3})_0} \oplus \underset{\ni B_\mu}{(\mathbf{1},\mathbf{1})_0}.
\end{aligned}$$

Standard Model — The Basics

Beware! We only use lefthanded fields $\psi_L = \begin{pmatrix} \psi_\alpha \\ 0 \end{pmatrix}$, righthanded fields are included via charge conjugation $(\psi_R)^c = \begin{pmatrix} 0 \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}^c = \begin{pmatrix} \psi_\alpha \\ 0 \end{pmatrix}$.

The $\mathrm{SU}(2)$-doublets are written as:

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \qquad q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad \ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}.$$

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\end{aligned}$$

Standard Model — The Basics (Well, more or less...)

Georgi-Glashow model: Using the breaking pattern $\mathrm{SU}(5) \to \mathcal{G}_{\mathrm{SM}}$ the previous fields fit nicely into representations of $\mathrm{SU}(5)$:

$$\begin{aligned}
\bar{\mathbf{5}} &= (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} \oplus (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}} \\
\mathbf{10} &= (\mathbf{3}, \mathbf{2})_{\frac{1}{6}} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}} \oplus (\mathbf{1}, \mathbf{1})_{1}
\end{aligned}$$

$$\mathbf{24} = (\mathbf{8}, \mathbf{1})_0 \oplus (\mathbf{1}, \mathbf{3})_0 \oplus (\mathbf{1}, \mathbf{1})_0 \oplus (\mathbf{3}, \mathbf{2})_{-\frac{5}{3}} \oplus (\bar{\mathbf{3}}, \mathbf{2})_{\frac{5}{3}} \, .$$

Standard Model — The Lagrangian

The SM Lagrangian contains a variety of terms which roughly fall into three categories:

gauge terms | kinetic terms | Higgs sector

The gauge part is straightforward, albeit there being additional gauge configurations:

$$\begin{aligned}\mathcal{L}_{\text{gauge}} &\supset \frac{1}{2g_i^2}\,\mathrm{tr}\left[F^{(i)}_{\mu\nu}F^{(i)\mu\nu}\right]\\ &\supset \frac{\theta_{\mathrm{QCD}}}{16\pi^2 g_{\mathrm{s}}^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}\left[F^{(3)}_{\mu\nu}F^{(3)}_{\rho\sigma}\right].\end{aligned}$$

Kinetic terms for the fermions are constructed with the covariant derivative

$$\begin{aligned}\mathcal{L}_{\text{kin}} &\supset \bar{\psi}_i \mathrm{i}\not{D}\psi_i\\ D_\mu &= \partial_\mu - \mathrm{i}qA^k_\mu\mathcal{R}(T_k).\end{aligned}$$

Standard Model — Higgs sector

The Higgs part is made of a scalar kinetic term, the quartic potential and the Yukawa couplings:

$$\mathcal{L}_{\mathrm{Higgs}} \supset -\left(D^{\mu}\phi\right)^{\dagger}\left(D_{\mu}\phi\right) + \mu\phi^{\dagger}\phi - \lambda\left(\phi^{\dagger}\phi\right)^{2} - \left(\lambda^{\psi}\left[\bar{\psi}\phi\psi\right]_{\mathbf{1}} + \mathrm{h.c.}\right).$$

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The fermions gain masses $m_\psi = v\lambda^\psi$ when the Higgs field acquires its vacuum expectation value (VEV):

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \text{(shorthand: } \langle \phi_0 \rangle = v\text{)},$$

while the Higgs mass becomes $m_h = 2\sqrt{\lambda}v$.

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For our three types of massive fermions (electron, up-quark, down-quark) the corresponding singlets look like:[1]

$$\lambda^{u}\left[\bar{q}_L\tilde{\phi}u_R\right]_{\mathbf{1}}, \qquad \lambda^{d}\left[\bar{q}_L\phi d_R\right]_{\mathbf{1}}, \qquad \lambda^{e}\left[\bar{\ell}_L\phi e_R\right]_{\mathbf{1}}.$$

[1] Careful, since $\tilde{\phi} = \epsilon\phi^{*}$ to account for the u-type quarks.

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Furthermore, the SM fermion fields consists of three generations, thus promoting the λ^ψ to complex 3x3 matrices λ^e_{mn}, λ^d_{mn} and λ^u_{mn}. These λ^f_{mn} can be diagonalised via bi-unitary transformations:

$$V_f^\dagger\lambda^f U_f \propto \operatorname{diag}\left(m_f^{(1)}, m_f^{(2)}, m_f^{(3)}\right)/v.$$

[1]Careful, since $\tilde{\phi} = \epsilon\phi^*$ to account for the u-type quarks.

Standard Model — Parameter count

Count parameters and gauge redundancies:

$+$

- 3 couplings g, g' and g_{s}, one vacuum angle θ_{QCD} (4 parameters)
- Higgs parameters v, λ (2 parameters)
- 3 (complex) mass matrices λ^f (3x18 parameters)

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- Quark flavour symmetry $\mathrm{U}(3)_{q_L} \times \mathrm{U}(3)_{u_R} \times \mathrm{U}(3)_{d_R}/\mathrm{U}(1)_B$ (3x9-1 parameters)
- Lepton flavour symmetry $\mathrm{U}(3)_{\ell_L} \times \mathrm{U}(3)_{e_R}/\mathrm{U}(1)_{L_e} \times \mathrm{U}(1)_{L_\mu} \times \mathrm{U}(1)_{L_\tau}$ (2x9-3 parameters)

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The Standard Model of Particle Physics has 19 free parameters with v being the only one carrying a physical dimension.[2]

[2] 14 real parameters, 3 mixing angles, 2 CP-violating phase.

Outline

MSSM — The fields

First of all we promote all our previous fields to real (chiral) superfields resulting in our renewed table:

super field	bosonic field	fermionic field	representation
$\hat{V}_8$	g	$\tilde{g}$	$(\mathbf{8},\mathbf{1})_0$
$\hat{V}$	W^0, $W^\pm$	$\tilde{W}^0$, $\tilde{W}^\pm$	$(\mathbf{1},\mathbf{3})_0$
$\hat{V}'$	B	$\tilde{B}$	$(\mathbf{1},\mathbf{1})_0$
$\hat{L}$	$(\tilde{\nu}_L, \tilde{e}_L)$	(ν_L, e_L)	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$
$\hat{E}^c$	$\tilde{e}_R^c$	e_R^c	$(\mathbf{1},\mathbf{1})_1$
$\hat{Q}$	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$(\mathbf{3},\mathbf{1})_{\frac{1}{6}}$
$\hat{U}^c$	$\tilde{u}_R^c$	u_R^c	$(\mathbf{3},\mathbf{1})_{-\frac{2}{3}}$
$\hat{D}^c$	$\tilde{d}_R^c$	d_R^c	$(\mathbf{3},\mathbf{1})_{\frac{1}{3}}$
$\hat{H}_u$	(H_u^+, H_u^0)	$\left(\tilde{H}_u^+, \tilde{H}_u^0\right)$	$(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$
$\hat{H}_d$	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$

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Beware! We need a second Higgs doublet to cancel the gauge anomaly introduced by the Higgsinos!

MSSM — SUSY terms

For the gauge part the ususal field strength super fields

$$\mathcal{W}_{i,\alpha} = -\frac{1}{4}\bar{D}^2 e^{-\hat{V}} D_\alpha e^{\hat{V}},$$

are constructed and included in the Lagrangian:

$$\mathcal{L}^{\mathrm{MSSM}}_{\mathrm{gauge}} \supset \frac{1}{2g_i^2}\,\mathrm{tr}\left[\int \mathrm{d}^2\theta\,(\mathcal{W}_i)^\alpha\,(\mathcal{W}_i)_\alpha + \mathrm{h.c.}\right].$$

The kinetic terms for the fields read:

$$\begin{aligned}\mathcal{L}^{\mathrm{MSSM}}_{K} \supset& \int \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}\left[\hat{\Phi}_i^\dagger e^{2V_i}\hat{\Phi}_i\right]_{\mathbf{1}}\\ V_i =& \hat{V}_8^a \mathcal{R}_i(T_a) + \hat{V}^k \mathcal{R}_i(T_k) + Y_i \hat{V}'.\end{aligned}$$

MSSM — Superpotential

The superpotential term is simply:

$$\mathcal{L}_W^{\mathrm{MSSM}} = \int \mathrm{d}^2\theta W + \mathrm{h.c.}$$

What terms are contained in W?

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In general, the superpotential contains a great variety of different terms, under them the Yukawa couplings:

$$\begin{aligned} W =& \lambda_d \left[\hat{H}_d \hat{Q} \hat{D}\right]_{\mathbf{1}} + \lambda_e \left[\hat{H}_d \hat{L} \hat{E}\right]_{\mathbf{1}} - \lambda_u \left[\hat{H}_u \hat{Q} \hat{U}\right]_{\mathbf{1}} + \mu \left[\hat{H}_u \hat{H}_d\right]_{\mathbf{1}} \\ &+ a \left[\hat{L} \hat{H}_u\right]_{\mathbf{1}} + b \left[\hat{Q} \hat{L} \hat{D}\right]_{\mathbf{1}} + c \left[\hat{U} \hat{U} \hat{D}\right]_{\mathbf{1}} + d \left[\hat{L} \hat{L} \hat{E}\right]_{\mathbf{1}} . \end{aligned}$$

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The terms in the last line introduce B-number violation via proton decay as well as lepton number violation, but by imposing R-parity

$$R = (-1)^{3(B-L)+2s},$$

we can get rid of them. Beware! This is not obligatory!

R-conservation implies the existance of a lightest supersymmetric particle (LSP) thus providing us with a dark matter candidate.

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We could use matter parity

$$P_{\mathrm{m}} = (-1)^{3(B-L)},$$

instead and see directly how the lower line gets thrown out.

MSSM — Soft SUSY breaking terms

Introduce explicitly SUSY breaking terms to generate masses and additional interactions

$$
\begin{aligned}
-\mathcal{L}_{\text{soft}}^{\text{MSSM}} \supset & \frac{1}{2} M_i \tilde{\bar{\lambda}}_i \tilde{\lambda}_i + M_{\tilde{F}}^2 \tilde{f}^\dagger \tilde{f} \\
& + m_1^2 H_d^\dagger H_d + m_2^2 H_u^\dagger H_u + m_{12}^2 \left(H_u \cdot H_d + \text{h.c.} \right) \\
& + T_U H_u \tilde{Q} \tilde{U} + T_D H_d \tilde{Q} \tilde{D} + T_E H_d \tilde{L} \tilde{E} + \text{h.c.}
\end{aligned}
$$

Often, a parametrisation $m_{12}^2 = \mu B$ (and $T_F = \lambda_f A_F$) is chosen. Therefore, the corresponding terms are called A and B-terms.

MSSM — Note on the Higgs sector

Repeat the SM steps:
In the MSSM the quartic coupling is generated by the D-terms of the Kähler potential, and the SUSY breaking terms, leading to an effective Higgs potential.

$$
\begin{aligned}
V_{\text{Higgs}} = {} & \left(m_1^2 + |\mu|^2\right) H_d^\dagger H_d + \left(m_2^2 + |\mu|^2\right) H_u^\dagger H_u + m_{12}^2 \left(H_u \cdot H_d + \text{h.c.}\right) \\
& + \frac{g^2 + {g'}^2}{8} \left(H_d^\dagger H_d - H_u^\dagger H_u\right) + \frac{1}{2} g^2 \left|H_d^\dagger H_u\right|^2 ,
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The two doublets acquire separate VEVs

$$\left\langle H_f^0 \right\rangle = v_f ,$$

related to the previous v via

$$\sqrt{v_u^2 + v_d^2} = v,$$

by convention, the angle β is defined as

$$\tan\beta = \frac{v_u}{v_d}.$$

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At tree level this implies an upper bound on the mass of the lightest Higgs:

$$m_h^2 \leq m_Z^2 \cos^2 2\beta.$$

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At tree level this implies an upper bound on the mass of the lightest Higgs:

$$
m_h^2 \leq m_Z^2 \cos^2 2\beta + \cdots .
$$

MSSM — Mixing caveats

The SM particle spectrum looks like:

$$
\begin{array}{ccccc}
u & c & t & B & W^0 \\
d & s & b & g & W^\pm \\
 & & & & \\
e & \mu & \tau & h^0 & \\
\nu_e & \nu_\mu & \nu_\tau & &
\end{array}
$$

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The SM particle spectrum looks like:

$$\begin{array}{ccccc} u & c & t & B & W^0 \\ d & s & b & g & W^\pm \\ \\ e & \mu & \tau & H_d^- & H_d^0 \\ \nu_e & \nu_\mu & \nu_\tau & H_u^0 & H_u^+ \end{array}$$

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$$\begin{array}{ccccc} u & c & t & \gamma & Z^0 \\ d & s & b & g & W^\pm \\ & & & & \\ e & \mu & \tau & h^0 & H^0 \\ \nu_e & \nu_\mu & \nu_\tau & A^0 & H^\pm \end{array}$$

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\end{array}
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$$
\begin{array}{cccccccc}
\tilde{u}_L & \tilde{u}_R & \tilde{c}_L & \tilde{c}_R & \tilde{t}_L & \tilde{t}_R & \tilde{B} & \tilde{W}^0 \\
\tilde{d}_L & \tilde{d}_R & \tilde{s}_L & \tilde{s}_R & \tilde{b}_L & \tilde{b}_R & \tilde{g} & \tilde{W}^\pm \\
 & & & & & & & \\
\tilde{e}_L & \tilde{e}_R & \tilde{\mu}_L & \tilde{\mu}_R & \tilde{\tau}_L & \tilde{\tau}_R & \tilde{H}_d^- & \tilde{H}_d^0 \\
\tilde{\nu}_e & & \tilde{\nu}_\mu & & \tilde{\nu}_\tau & & \tilde{H}_u^0 & \tilde{H}_u^+
\end{array}
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$$\begin{array}{ccccc} u & c & t & \gamma & Z^0 \\ d & s & b & g & W^\pm \\ \\ e & \mu & \tau & h^0 & H^0 \\ \nu_e & \nu_\mu & \nu_\tau & A^0 & H^\pm \end{array}$$

The MSSM particle spectrum[3] looks like:

$$\begin{array}{cccccccc} \tilde{u}_1 & \tilde{u}_2 & \tilde{u}_3 & \tilde{u}_4 & \tilde{u}_5 & \tilde{u}_6 & \tilde{\chi}_1^\pm & \tilde{\chi}_2^\pm \\ \tilde{d}_1 & \tilde{d}_2 & \tilde{d}_3 & \tilde{d}_4 & \tilde{d}_5 & \tilde{d}_6 & \tilde{g} & \\ \\ \tilde{\ell}_1 & \tilde{\ell}_2 & \tilde{\ell}_3 & \tilde{\ell}_4 & \tilde{\ell}_5 & \tilde{\ell}_6 & \tilde{\chi}_1^0 & \tilde{\chi}_2^0 \\ \tilde{\nu}_1 & & \tilde{\nu}_2 & & \tilde{\nu}_3 & & \tilde{\chi}_3^0 & \tilde{\chi}_4^0 \end{array}$$

[3] Worst case scenario.

MSSM — Mixing caveats

Summary of mixed states:

- The Higgs bosons (H_d^-, H_d^0, H_u^0, H_u^+) form: a charged scalar pair $H^\pm$, two neutral scalars h^0, H^0, and a neutral pseudoscalar A^0.
- The charged bosinos ($\tilde{W}^\pm$, $\tilde{H}_u^+$, $\tilde{H}_d^-$) form the charginos $\tilde{\chi}_i^\pm$.
- The neutral bosinos ($\tilde{B}$, $\tilde{W}^0$, $\tilde{H}_u^0$, $\tilde{H}_d^0$) form the neutralinos $\tilde{\chi}_i^0$.
- The squarks ($\tilde{q}_{i,L}$, $\tilde{q}_{i,R}$) form mass eigenstates labeled $\tilde{q}_i$.
- The charged sleptons ($\tilde{e}_{i,L}$, $\tilde{e}_{i,R}$) form eigenstates $\tilde{\ell}_i$.
- The sneutrinos $\tilde{\nu}_i$ form eigenstates $\tilde{\nu}_i$.

Only for certain ranges of the parameters the particle spectrum will resemble a 'double-SM'.

MSSM — Parameter count

$+$

- 3 couplings g_i, one vacuum angle θ_{QCD} (4 parameters)
- 3 (complex) gaugino masses M_i (6 parameters)
- 2 Higgs mass parameters v, β (2 parameters)
- 2 (complex) Higgs/ino mass parameters μ, B (4 parameters)
- 5 hermitian scalar mass matrices $M^2_{\tilde{F}}$ (5x9 parameters)
- 3 mass matrices λ^f (3x18 parameters)
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The full Minimal Supersymmetric Standard Model has 124 free parameters[3] (MSSM-124).

[3] Consisting of 3 couplings, 37 real masses, 39 mixing angles and 45 CP-violating phases.

The Minimal Supersymmetric Standard Model

Part II: Phenomenology

Mathieu Kaltschmidt & **Jonah Cedric Strauß**

ITP Heidelberg

Supersymmetry Seminar
supervised by
Prof. Jörg Jäckel

Heidelberg, February 15th 2021

Outline

Outline

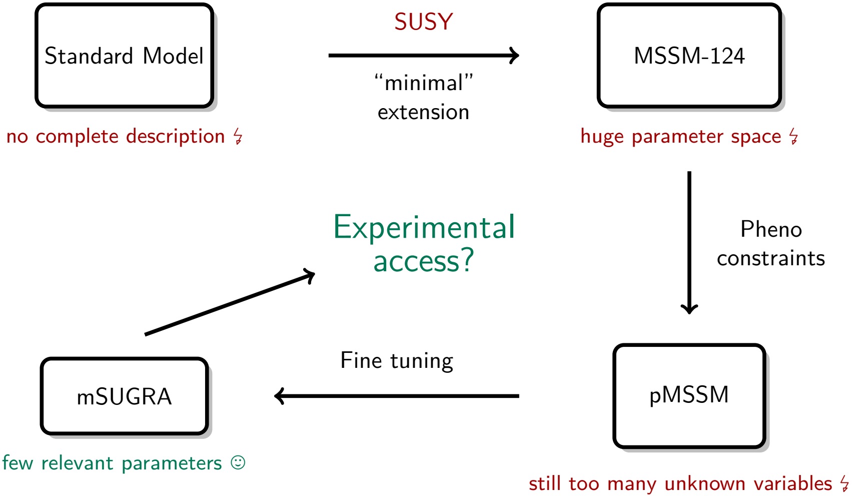
What have we learned so far and what needs to be discussed?

- **State of the Art:** We promoted the SM to the MSSM-124 and found a variety of new degrees of freedom that need to be taken into account in further investigations.
- Now we want to understand how to make sense of a theoretical model with $\mathcal{O}(100)$ parameters.
- **Central Question:** Is it possible to reduce this huge number of parameters to a reasonable number that may be tested in future experiments?[1]
- **Phenomenology of the MSSM**: Focus on the "pMSSM" and Gauge Coupling Unification.
- Which problems are still present? Why might the "minimal" SSM not be enough?

[1] Experimental details $\rightarrow$ talk next week!

Outline

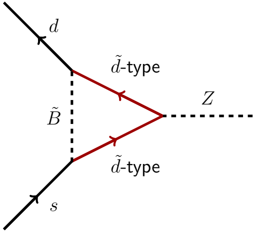
How to test the MSSM?



Some Observations

This huge additional parameter space in general comes with some severe problems :

- The respective lepton numbers L_e, L_μ and L_τ are not conserved ϟ
- Flavor-changing neutral currents (FCNCs) are unsuppressed ϟ
- New sources of CP-violation are present ϟ



For all of the above mentioned problems we have rather strict experimental bounds, for example:

Want to avoid contributions to electric dipole moments $\Longrightarrow$ complex phases of the gaugino mass parameters, A-parameters and $|\mu| \lesssim 10^{-2} - 10^{-3}$ for TeV-ish sfermion and gaugino masses [12].

General Requirements

We use these phenomenological constraints to define the phenomenological MSSM (pMSSM) [11]:

- No additional sources of CP-violation $\implies$ all phases in the soft-SUSY-breaking potential zero
- No FCNCs $\implies$ simple, diagonal structure of the sfermion and trilinear coupling matrices
- First and second generation universality $\implies$ soft-SUSY-breaking scalar masses + trilinear couplings[2] A^u, A^d and A^ℓ are the same for the first two generations

In addition, for different regions of the parameter space, one can use even more severe constraints[3]!

[2] Usually they are just set to zero for the first two generations

[3] In actual experiments only very simplified models with two or three parameters are tested.

Which Parameters do we need?

This already reduces the number of relevant parameters by a large amount, a summary is given below:

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- $m_{\tilde{Q}}$, $m_{\tilde{t}_R}$, $m_{\tilde{b}_R}$, $m_{\tilde{L}}$ and $m_{\tilde{\tau}_R}$, the third generation sfermion masses
- A^t, A^b and A^τ, the third generation trilinear couplings

In total we are left with 19 additional parameters in the pMSSM.

What about SUSY-breaking?

- The MSSM-124 fails to explain the fundamental origin of the SUSY-breaking parameters.
- **Last talks:** Gauge- and gravity-mediated SUSY breaking may provide a way around!
- The phenomenon of gauge coupling unification hints at a simpler structure of the MSSM at high energies.

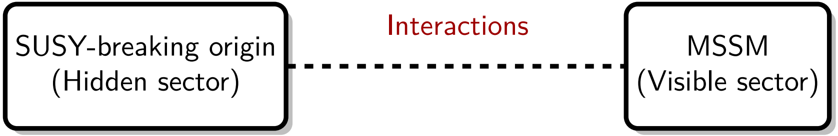
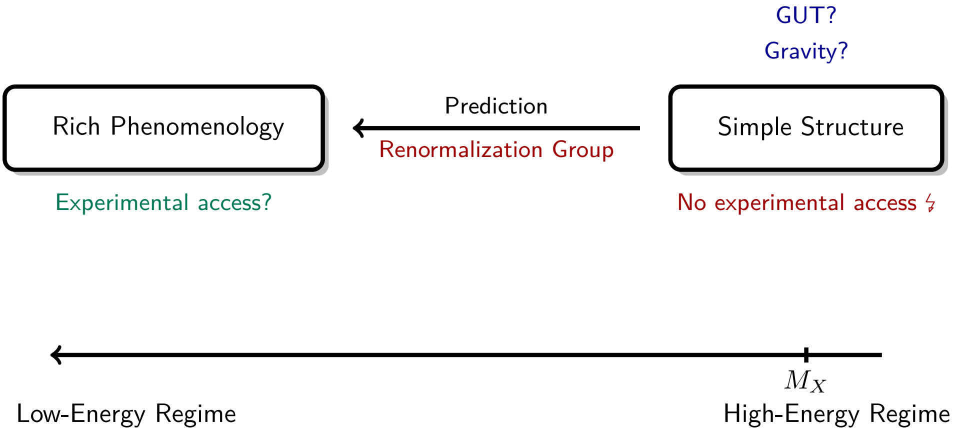


Figure: Gravity-mediated SUSY-breaking in the hidden sector[4].

[4]Figure inspired by the visualization in: https://www.thphys.uni-heidelberg.de/~plehn/includes/bad_honnef_12/kribs_2.pdf (13.02.15)

Outline

The mSUGRA Framework I



The mSUGRA framework II

- Want to naturally include gravity, or more precisely local SUGRA.
- SUGRA is broken at some high energy scale, usually at around $M_{\text{Planck}} \sim 10^{19}$ GeV.
- Assume gravity-mediated SUSY breaking, i. e. $m_{\text{soft}} \sim \frac{\langle F \rangle}{M_{\text{Planck}}}$

The general idea is now to assume a "minimal" normalization of the kinetic terms in the SUGRA Lagrangian, i. e.

$$\mathcal{L}_{\text{SUGRA}} \supset -\frac{1}{M_{\text{Planck}}} F \left(\frac{1}{2}\, \alpha\, \lambda\lambda + \frac{1}{6}\, \beta\, \phi\phi\phi + \frac{1}{2}\, \gamma\, \phi\phi \right) + \text{h.c.} - \frac{1}{M_{\text{Planck}}^2} F F^*\, \delta\, \phi\phi^*$$

Comparison with initial $\mathcal{L}_{\text{soft}} \implies$ Simple relations for the input parameters!

Finding the correct initial Conditions

The relatively simple form of the kinetic terms allows to simplify the structure of the relevant couplings at the initial scale as follows:

- scalar squared-masses are flavor-diagonal and universal:

$$m^2_{\tilde{q}}(M_X) = m^2_{\tilde{u}}(M_X) = m^2_{\tilde{d}}(M_X) = m^2_0 \mathbb{1}$$

$$m^2_{\tilde{\ell}}(M_X) = m^2_{\tilde{e}}(M_X) = m^2_0 \mathbb{1}$$

$$m^2_1(M_X) = m^2_2(M_X) = m^2_0$$

- the same is true for the A-parameters:

$$A^u(M_X) = A^d(M_X) = A^e(M_X) = A^2_0 \mathbb{1}$$

- Additionally we assume unification of the (tree level) gaugino masses:

$$M_1(M_X) = M_2(M_X) = M_3(M_X) = m_{1/2}$$

Renormalization Group Equations

This allows us to compute the low-energy MSSM parameters with the help of standard renormalization group techniques!

Reminder: The beta function $\beta(g) = \frac{\partial g}{\partial \ln M}$ is the rate of change of the renormalized coupling at the scale M, where the bare coupling is fixed [17].

For example, we get the following result for the low-energy gaugino mass parameters:

$$M_i = \frac{\alpha_i\left(M_Z\right)}{\alpha_{\mathrm{GUT}}} m_{1/2} \longrightarrow M_3\left(M_Z\right) = \frac{\alpha_3\left(M_Z\right)}{\alpha_2\left(M_Z\right)} M_2\left(M_Z\right) = \frac{\alpha_3\left(M_Z\right)}{\alpha_1\left(M_Z\right)} M_1\left(M_Z\right)$$

This results in the well known relation:

$$M_1 = \frac{5}{3} \tan^2 \theta_W M_2 \sim \frac{1}{2} M_2$$

Solution of the RGEs for mSUGRA

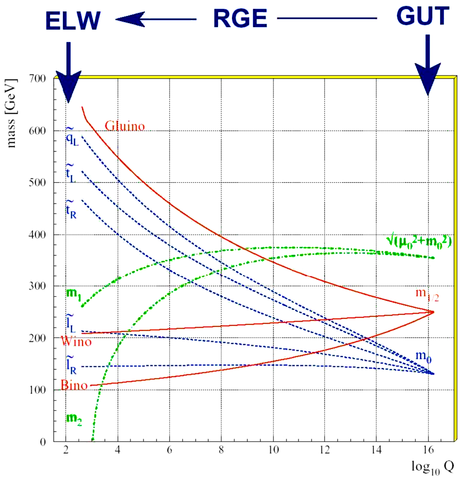


Figure: Running of the mSUGRA parameters[5].

[5]Figure taken from: https://www.physi.uni-heidelberg.de/~uwer/lectures/ParticlePhysics/Vorlesung/Lect-10b.pdf (10.02.21)

Parameter Count in the mSUGRA Framework

This means in total we are left with only the following four continuous and one discrete free parameters in the mSUGRA model:

- $\tan\beta$, the ratio of the VEVs in the two-Higgs doublet model
- $m_{1/2}$, the universal gaugino mass
- m_0, the universal scalar (sfermion/Higgs) mass
- A_0, the universal trilinear coupling
- $\mathrm{sign}(\mu)$, the sign of the Higgs-higgsino mass parameter

The relations for $\tan\beta$ and $|\mu|$ come from the two minimum conditions for the Higgs potential.

Remark: Additional requirements such as the unification of the top, bottom and tau Yukawa couplings at the GUT scale further restricts the possible values of $\tan\beta$ and A_0.

Map of the mSUGRA Parameter Space

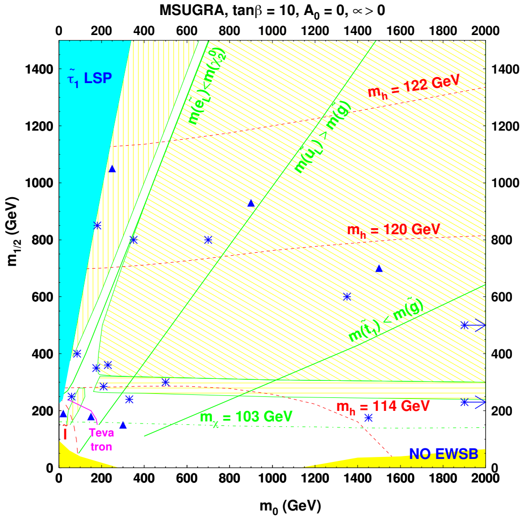


Figure: Map of the mSUGRA parameter space for different values of the universal mass parameters, slightly adapted from [10].

Outline

Gauge Couplings in the Standard Model

The Standard Model is described as the most general renormalizable field theory with gauge group

$$\mathcal{G}_{\mathrm{SM}} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1),$$

with associated gauge couplings α_3, α_2 and α_1, three generations of fermions and a scalar [4].

- The couplings are larger for the larger component of the gauge group, i. e.

$$\alpha_3(m_Z) > \alpha_2(m_Z) > \alpha_1(m_Z)$$

- Interesting observation: Values of the running couplings come close together at some high energy scale $\Lambda_{\mathrm{GUT}} \sim 10^{16}\ \mathrm{GeV}$ (cf. next slide).
- Georgi-Glashow [13]: Embed $\mathcal{G}_{\mathrm{SM}}$ in larger gauge group, i. e. $\mathrm{SU}(5) \implies \mathrm{GUT?}$

One Loop Running of the SM Gauge Couplings I

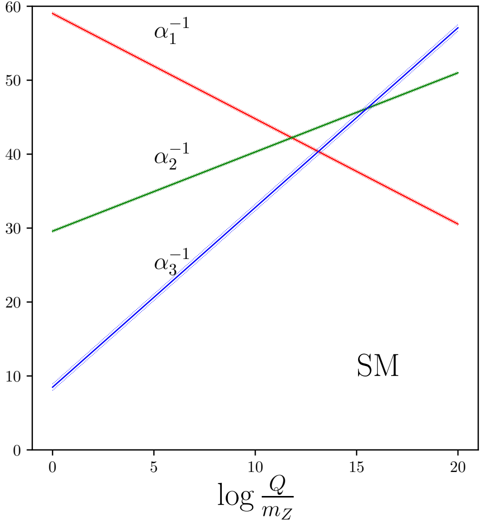
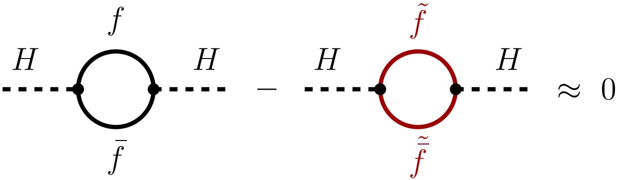


Figure: Running of the (inverse) SM gauge couplings, plot inspired by [15].

One Loop Running of the SM Gauge Couplings II

- In the Georgi-Glashow model, we need $-\mu^2 \sim -(100\ \mathrm{GeV})^2$ to reproduce the correct W and Z masses $\Longrightarrow$ Gauge hierarchy problem! [17]

- SUSY provides way out: If SUSY breaking works such that the mass differences between the superpartners are large enough, one can reproduce the correct Higgs mass $\Longrightarrow$ superpartners influence the running of the (MS)SM gauge couplings (cf. next slide)



- In the end it remains a complicated fine tuning task!

Comparison: SM vs. MSSM

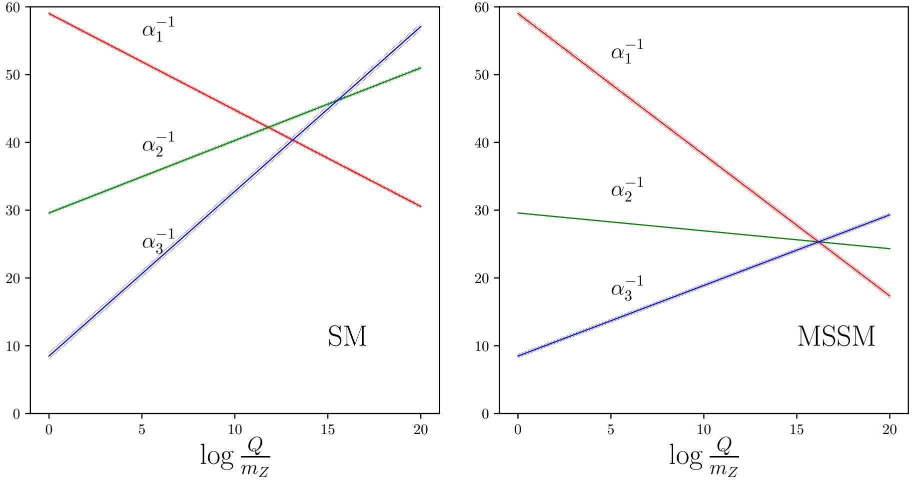


Figure: Running of the (inverse) gauge couplings in the SM and the MSSM, plots inspired by [15].

Outline

Are all Problems solved now?

The MSSM as introduced in the first part of our talk still fails to answer some of the key questions:

- How do we explain the value of the μ parameter in the MSSM?
- What about the “total” particle content?
- Is there a “natural” way to implement SUSY-breaking?
- What is $\mathcal{G}_{\mathrm{MSSM}}$?
- and (many) more ...

To conclude our talk we want to have a look at some ideas concerning the first mentioned problem, the value of the μ parameter.

Different Approaches to explain the Value of the μ Parameter

- **General Problem:** μ is a SUSY-preserving parameter, but from phenomenology we know that it must be of the order of the SUSY-breaking scale.

- "Natural" solution: Symmetry enforcing $\mu = 0$, and a small SUSY-breaking parameter that generates a value of μ which is not parametrically larger than the SUSY-breaking scale.

In proposed extensions of the MSSM, some other approaches have been presented, for example:

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4. Possible connection to the strong CP problem? $\Longrightarrow$ PQ symmetry $\Longrightarrow$ Axion Physics?
5. Higher-dimensional Higgs multiplets?

Outline

What have we learned today?

- The SM can be promoted to the MSSM using the concepts and methods introduced in the scope of this seminar.
- In addition to the 19 parameters of the SM we get $\mathcal{O}(100)$ new ones, which complicates the experimental access to the MSSM a lot!
- Phenomenological models of the MSSM allow for a drastic reduction of the number of independent parameters that have to be measured experimentally.
- SUSY may lead to a better understanding of the connection between Gravity and the SM. Additionally SUGRA helps us to further reduce the number of dof's in the MSSM.
- In the MSSM the unification of the SM gauge couplings at some high-energy scale M_X can be realized due to the effect of loop-corrections arising from the additional superpartners.
- There are still plenty of problems where the MSSM fails to explain the experimental results.

$\Longrightarrow$ The MSSM is not the "ultimate" model to describe our Universe.

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