The Status of the Minimal Supersymmetric Standard Model and Beyond

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Abstract

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The Status of the Minimal Supersymmetric Standard Model and Beyond

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1. UNIFICATION: PAST AND PRESENT

During the 1980s, a series of meetings were held called the nth Workshop on Grand Unification (nWOGU). By the end of the 1970s, the electroweak model of Glashow, Weinberg and Salam had been confirmed by the experimental observations of the $SU(2) \times U(1)$ structure of the weak neutral current. Models of grand unified theories (GUTs) of the strong and electroweak forces were being studied intensely, with some indications for the minimal $SU(5)$ unification model. In particular, the measurement of the weak mixing angle, $\sin^2 \theta_W$, seemed to in agreement with the $SU(5)$ prediction. At the 4th WOGU held in 1983 at the University of Pennsylvania, Marciano [1] reported the $SU(5)$ prediction of:

$$\sin^2 \theta_W(m_W) = 0.214^{+0.004}_{-0.003} \quad [\text{SU(5) GUT}], \quad (1)$$

which was in excellent agreement with the experimental results based on deep-inelastic νN scattering and polarized electron deuteron (ed) scattering asymmetry measurements (including $\mathcal{O}(\alpha)$ radiative corrections):

$$\sin^2 \theta_W(m_W) = 0.215 \pm 0.014 \quad [\nu N \text{ data}],$$
$$\sin^2 \theta_W(m_W) = 0.216 \pm 0.020 \quad [ed \text{ data}]. \quad (2)$$

The apparent success of the $SU(5)$ model encouraged theorists to take seriously its predictions for proton decay. There were already a couple of candidates from existing proton decay experiments which had generated some interest. New proton decay experiments with increased sensitivity and better resolution were beginning to take data. There were high expectations that proton decay would soon be observed in the new experiments. The organizers of WOGU optimistically anticipated the announcement of the discovery of proton decay during the early 1980s. More detailed measurements of proton decay branching ratios were expected later in the decade and would provide the necessary evidence to either confirm the simplest $SU(5)$ grand unified model or to distinguish among more complicated GUT scenarios.

In 1983, although the success of the gauge coupling unification prediction apparently provided strong evidence for the GUT approach, no evidence for proton decay was forthcoming. The IMB Collaboration reported $\Gamma^{-1}(p \to e^+ \pi^0) > 6 \times 10^{31}$ years, and Marciano concluded [1] that this "bound appears to rule out minimal $SU(5)$ with a great desert" unless the assumptions underlying the theoretical computation of the proton decay rate were significantly in error.

By 1987, proton decay bounds had increased, but a more significant change had taken place. At the 8th WOGU, Marciano reported [2] an experimental weak mixing angle determined from a global fit of all relevant experimental data[1] [3]:

$$\sin^2 \theta_W(m_W) = 0.228 \pm 0.0044 \quad [\text{global fit}], \quad (3)$$

[1]The global fit assumed that $m_t = 45$ GeV. For $m_t = 175$ GeV, the central value of $\sin^2 \theta_W(m_W)$ should be *increased* by 0.004 [3].

which is clearly in disagreement with the SU(5) prediction [eq. 1]. Equivalently, the strong and electroweak coupling constants do *not* unify in the minimal SU(5) model. Marciano explained [2] that the new experimental result [eq. 3] differed from the old result quoted in eq. 2 "primarily because of more precise deep-inelastic ν_μ scattering data and refinements in the $W^\pm$ and Z mass determinations." The simplest GUT models with the grand desert between the electroweak and GUT-scales were now decisively ruled out by experiment. Of course, one could always add an intermediate scale and make the GUT model sufficiently complex to reproduce the observed $\sin^2\theta_W(m_W)$ and to suppress the proton decay rate below its experimental bound. However, in the same paper [2], Marciano also notes that the supersymmetric SU(5) model predicts:

$$\sin^2\theta_W(m_W) = 0.237^{+0.003}_{-0.004} - \frac{4\alpha}{15\pi}\ln\frac{M_{\text{SUSY}}}{m_W}, \quad (4)$$

where M_{SUSY} characterizes the scale of low-energy supersymmetry breaking, which "is in good accord with experiment" (for $M_{\text{SUSY}} \sim$ 1 TeV). Of course, with more precise electroweak data obtained at LEP, SLC, and the Tevatron during the past eight years, the latter observation has become much sharper (see Section 3.4 for further discussions).

In 1989 the tenth and last WOGU was held. The demise of nWOGU was not a consequence of theorists giving up on unification. In fact, during the early 1990s, the suggestion that unification of couplings was a strong hint for low-energy supersymmetry received a significant boost from the new precision data from LEP [4]. Moreover, low-energy supersymmetry has the (possibly unique) potential for providing a theoretical understanding of the large hierarchy between the electroweak and Planck scales (the latter only a few orders of magnitude above the GUT-scale), while preserving the perturbativity of couplings [5]. Thus, starting in 1993, a new series of meetings, the nth International Conference on Supersymmetries in Physics (SUSY-yy)[2] was established. Since supersymmetry is believed to be an essential ingre-

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[2]Here, yy= $92 + n$ (mod 100), where $n = 1, 2, \ldots$.

dient in the unification of fundamental interactions and particles, SUSY-yy is a worthy successor to nWOGU. Like their predecessors, the organizers of SUSY-yy hold high expectations for future meetings. Low-energy supersymmetry implies the existence of supersymmetric phenomena associated with the electroweak scale. New experimental facilities at the Tevatron and LEP-2 have increased their sensitivities, and could provide the first hints for supersymmetric particles. Future experiments at the LHC (and other supercolliders now under development) have the potential to explore in detail the properties of supersymmetric particles and their interactions. These data could then provide crucial clues to the nature of Planck-scale physics.

Once again, we have come to Philadelphia to explore the consequences of unification physics. One hopes that nature will be kinder this second time around.

2. DEFINING THE MSSM

2.1. The MSSM particle spectrum

The minimal supersymmetric extension of the Standard Model (MSSM) consists of taking the Standard Model and adding the corresponding supersymmetric partners [6]. In addition, the MSSM contains two hypercharge $Y = \pm 1$ Higgs doublets, which is the minimal structure for the Higgs sector of an anomaly-free supersymmetric extension of the Standard Model. The supersymmetric structure of the theory also requires (at least) two Higgs doublets to generate mass for both "up"-type and "down"-type quarks (and charged leptons) [7,8]. All renormalizable supersymmetric interactions consistent with (global) B−L conservation (B=baryon number and L=lepton number) are included. The latter condition is achieved by employing the following R-parity conserving superpotential:

$$W = \epsilon_{ij}\left[(h_L)_{mn}\widehat{H}_1^i\widehat{L}_m^j\widehat{E}_n + (h_D)_{mn}\widehat{H}_1^i\widehat{Q}_m^j\widehat{D}_n\right.$$
$$\left. -(h_U)_{mn}\widehat{H}_2^i\widehat{Q}_m^j\widehat{U}_n - \mu\widehat{H}_1^i\widehat{H}_2^j\right], \quad (5)$$

where $\epsilon_{ij} = -\epsilon_{ji}$ (with $\epsilon_{12} = 1$) contracts SU(2) doublet fields. The parameters introduced above are the 3×3 Yukawa coupling matrices h_L, h_D and h_U (with corresponding generation labels m

and n) and the Higgs superfield mass parameter, μ. The gauge multiplets couple to matter multiplets in a manner consistent with supersymmetry and the SU(3)$\times$SU(2)$\times$U(1) gauge symmetry. The spectrum of the MSSM is exhibited in Table 1 (generation labels are suppressed). Note that Table 1 lists the interaction eigenstates. Particles with the same SU(3)$\times$U(1)$_{\rm EM}$ quantum numbers can mix. For example, the charginos $[\widetilde{\chi}_j^{\pm} \ (j = 1, 2)]$ are linear combinations of the charged winos and higgsinos, while the neutralinos $[\widetilde{\chi}_k^0 \ (k = 1, \ldots, 4)]$ are linear combinations of the neutral wino, bino and neutral higgsinos.

Table 1
The MSSM Particle Spectrum

Superfield	Boson Fields	Fermionic Partners
Gauge Multiplets		
$\widehat{G}$	g	$\widetilde{g}$
$\widehat{V}^a$	W^a	$\widetilde{W}^a$
$\widehat{V}'$	B	$\widetilde{B}$
Matter Multiplets		
$\widehat{L}$ leptons $\begin{cases} \widetilde{L}^j = (\widetilde{\nu}, \widetilde{e}^-)_L & (\nu, e^-)_L \\ \widetilde{E} = \widetilde{e}_R^+ & e_L^c \end{cases}$		
$\widehat{E}$		
$\widehat{Q}$ quarks $\begin{cases} \widetilde{Q}^j = (\widetilde{u}_L, \widetilde{d}_L) & (u, d)_L \\ \widetilde{U} = \widetilde{u}_R^* & u_L^c \\ \widetilde{D} = \widetilde{d}_R^* & d_L^c \end{cases}$		
$\widehat{U}$		
$\widehat{D}$		
$\widehat{H}_1$ Higgs $\begin{cases} H_1^i & (\widetilde{H}_1^0, \widetilde{H}_1^-)_L \\ H_2^i & (\widetilde{H}_2^+, \widetilde{H}_2^0)_L \end{cases}$		
$\widehat{H}_2$		

The fundamental origin of supersymmetry breaking is unknown. This ignorance can be parameterized by adding the most general soft-supersymmetry-breaking terms [9] consistent with gauge invariance and R-parity conservation. It is here where most of the new supersymmetric model parameters reside. These include three supersymmetry-breaking Higgs mass parameters,

five hermitian 3×3 scalar squared-mass matrices, three complex 3×3 matrix A-parameters and three (complex) Majorana gaugino masses:

$$
\begin{aligned}
V_{\rm soft} &= m_1^2|H_1|^2 + m_2^2|H_2|^2 - m_{12}^2(\epsilon_{ij}H_1^iH_2^j + {\rm h.c.}) \\
&+ (M_{\widetilde{Q}}^2)_{mn}\widetilde{Q}_m^{i*}\widetilde{Q}_n^i + (M_{\widetilde{U}}^2)_{mn}\widetilde{U}_m^*\widetilde{U}_n \\
&+ (M_{\widetilde{D}}^2)_{mn}\widetilde{D}_m^*\widetilde{D}_n \\
&+ (M_{\widetilde{L}}^2)_{mn}\widetilde{L}_m^{i*}\widetilde{L}_n^i + (M_{\widetilde{E}}^2)_{mn}\widetilde{E}_m^*\widetilde{E}_n \\
&+ \epsilon_{ij}\big[(h_LA_L)_{mn}\widetilde{H}_1^i\widetilde{L}_m^j\widetilde{E}_n + (h_DA_D)_{mn}\widetilde{H}_1^i\widetilde{Q}_m^j\widetilde{D}_n \\
&\quad - (h_UA_U)_{mn}\widetilde{H}_2^i\widetilde{Q}_m^j\widetilde{U}_n + {\rm h.c.}\big] \\
&+ \tfrac{1}{2}\Big[M_3\,\widetilde{g}\,\widetilde{g} + M_2\widetilde{W}^a\widetilde{W}^a + M_1\widetilde{B}\widetilde{B} + {\rm h.c.}\Big] \, . \quad (6)
\end{aligned}
$$

2.2. MSSM-124

It is instructive to count the number of independent parameters of the MSSM. To see how the counting works, consider the Standard Model with one complex hypercharge-one Higgs doublet. The gauge sector consists of three real gauge couplings (g_3, g_2 and g_1) and the QCD vacuum angle (θ_{QCD}). The Higgs sector consists of one Higgs squared-mass parameter and one Higgs self-coupling (m^2 and λ). Traditionally, one trades in the latter two real parameters for the vacuum expectation value ($v = 246$ GeV) and the physical Higgs mass. The fermion sector consists of three Higgs-Yukawa coupling matrices h_L, h_U, and h_D. Initially, h_L, h_U, and h_D are arbitrary complex 3×3 matrices, which in total depend on 27 real and 27 imaginary degrees of freedom.

But, most of these degrees of freedom are unphysical. In particular, in the limit where $h_L = h_U = h_D = 0$, the Standard Model possesses a global U(3)^5 symmetry corresponding to three generations of the five SU(3)$\times$SU(2)$\times$U(1) multiplets: $(\nu_m, \ e_m^-)_L$, $(e_m^c)_L$, $(u_m, d_m)_L$, $(u_m^c)_L$, $(d_m^c)_L$, where m is the generation label. Thus, one can make global U(3)^5 rotations on the fermion fields of the Standard Model to absorb the unphysical degrees of freedom of h_L, h_U, and h_D. A U(3) matrix can be parameterized by three real angles and six phases, so that with the most general U(3)^5 rotation, we can apparently remove 15 real angles and 30 phases from h_L, h_U, and h_D. However, the U(3)^5 rotations include four exact U(1) global symmetries of the Standard Model,

namely B and the three separate lepton numbers L_e, L_μ and L_τ. Thus, one can only remove 26 phases from h_L, h_U, and h_D. This leaves 12 real parameters (corresponding to six quark masses, three lepton masses,[3] and three CKM mixing angles) and one imaginary degree of freedom (the phase of the CKM matrix). Adding up to get the final result, one finds that the Standard Model possesses 19 independent parameters (of which 13 are associated with the flavor sector).

We now repeat the analysis for the MSSM [10]. The gauge sector consists of four Standard Model real parameters (g_3, g_2, g_1 and θ_{QCD}), and three complex gaugino mass parameters (M_3, M_2, and M_1). The Higgs sector adds two real squared-mass parameters (m_1^2 and m_2^2) and two complex mass parameters (m_{12}^2 and μ). In fact, two of the imaginary degrees of freedom can be removed. Consider the limit where $\mu = m_{12}^2 = 0$, all Majorana gaugino mass parameters are zero and all matrix A-parameters are zero. The theory in this limit possesses two flavor-conserving global U(1) symmetries [11]: a continuous R symmetry [U(1)$_{\rm R}$] and a Peccei-Quinn symmetry [U(1)$_{\rm PQ}$]. Thus, one can make global U(1)$_{\rm R}$ and U(1)$_{\rm PQ}$ rotations on the MSSM fields to remove two unphysical degrees of freedom from among μ, m_{12}^2 and the three complex gaugino Majorana mass parameters (unphysical degrees of freedom in the matrix A parameters will be addressed below). It is convenient to perform a U(1)$_{\rm R}$ rotation in order to make the gluino mass real and positive (*i.e.*, $M_3 > 0$), followed by a $U(1)_{\rm PQ}$ rotation to remove a complex phase from m_{12}^2. Since the tree-level Higgs potential depends on the Higgs mass parameters $m_i^2 + |\mu|^2$ ($i = 1, 2$) and m_{12}^2, it follows that the tree-level Higgs potential is CP-conserving [8]. Thus, three Higgs sector mass parameters can be traded in (at tree-level) for two real vacuum expectation values v_1 and v_2 [or equivalently, $v^2 \equiv v_1^2 + v_2^2 = (246 \text{ GeV})^2$ and $\tan\beta \equiv v_2/v_1$] and one Higgs mass [usually taken to be the mass of the CP-odd Higgs scalar (A^0)]. The parameters $\tan\beta$ and m_{A^0} can then be used to predict the masses of the other MSSM Higgs

bosons (the CP-even Higgs states h^0 and H^0 and a charged Higgs pair $H^\pm$) and their couplings [8]. Thus, among the gaugino and Higgs/higgsino mass parameters, there are seven real degrees of freedom (v, m_{A^0}, $\tan\beta$, M_3, $|M_2|$, $|M_1|$, and $|\mu|$) and three phases (arg M_2, arg M_1, and arg μ).

Finally, we must examine the flavor sector of the MSSM. In addition to h_L, h_U, and h_D of the Standard Model, we have three arbitrary complex 3×3 matrix A-parameters, A_L, A_U, and A_D which adds an additional 27 real and 27 imaginary parameters. Furthermore, the five scalar hermitian 3×3 squared-mass matrices $M_{\tilde{Q}}^2$, $M_{\tilde{U}}^2$, $M_{\tilde{D}}^2$, $M_{\tilde{L}}^2$, $M_{\tilde{E}}^2$ contribute a total of 30 real and 15 imaginary degrees of freedom. To remove the unphysical degrees of freedom, we employ global U(3)^5 rotations on the *superfields* of the model (thereby preserving the form of the interactions of the gauginos with matter). This analysis differs from the Standard Model analysis in that the MSSM possesses only one global lepton number L. (In particular, L$_e$, L$_\mu$, and L$_\tau$ are no longer separately conserved in general, for the case of arbitrary sneutrino masses). Thus, global U(3)^5 rotations can remove 15 real parameters and 28 phases. Hence, the flavor sector contains 69 real parameters and 41 phases. Of these, there are nine quark and lepton masses, three real CKM angles, and 21 squark and slepton masses. This leaves 36 new real mixing angles to describe the squark and slepton mass eigenstates and 40 new CP-violating phases that can appear in squark and slepton interactions!

The final count gives 124 independent parameters for the MSSM of which 110 are associated with the flavor sector. Of these 124 parameters, 18 correspond to Standard Model parameters, one corresponds to a Higgs sector parameter (the analogue of the Standard Model Higgs mass), and 105 are genuinely new parameters of the model. Thus, an appropriate name for the minimal supersymmetric extension of the Standard Model as described above is MSSM-124.

Even in the absence of a fundamental theory of supersymmetry breaking, one is hard-pressed to regard MSSM-124 as a fundamental theory. In particular, the "minimal" in MSSM refers to the

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[3]The neutrinos in the Standard Model are automatically massless and are not counted as independent degrees of freedom in the parameter count.

minimal particle content and not a minimal parameter count. Moreover, once low-energy supersymmetry is discovered, one of the main tasks of future experiments will be to measure as many of the 124 parameters as possible. Nevertheless, MSSM-124 is not a phenomenologically viable theory over most of its parameter space. Among the phenomenologically bad features of this model are: (i) no separate conservation of L_e, L_μ, and L_τ; (ii) unsuppressed flavor changing neutral currents (FCNC's); and (iii) electric dipole moments of the electron and neutron that are inconsistent with the experimental bounds. As a result, almost the entire MSSM-124 parameter space is ruled out! This theory is viable only at very special "exceptional" points of the full parameter space. MSSM-124 is also theoretically deficient since it provides no explanation for the origin of the flavor-sector parameters (and in particular, why these parameters conform to the exceptional points of the parameter space mentioned above). In addition, no fundamental explanation is provided for the origin of electroweak symmetry breaking.

There are two general approaches for treating MSSM-124. In the low-energy approach, an attempt is made to elucidate the nature of the exceptional points in the MSSM-124 parameter space that are phenomenologically viable. Consider the following two possible choices. First, one can arbitrarily assert that $M_{\tilde{Q}}^2$, $M_{\tilde{U}}^2$, $M_{\tilde{D}}^2$, $M_{\tilde{L}}^2$, $M_{\tilde{E}}^2$ and the matrix A-parameters are generation-independent (horizontal universality [10,12]). Alternatively, one can simply require that all the aforementioned matrices are flavor diagonal in a basis where the quark and lepton mass matrices are diagonal (flavor alignment [13]). In either case, L_e, L_μ and L_τ are separately conserved, while tree-level FCNC's are automatically absent. Of course, the number of free parameters characterizing the MSSM in either of these two cases is substantially less than 124. Both scenarios are phenomenologically viable. However, such approaches are almost certainly too restrictive. First, the phenomenologically viable region of MSSM-124 parameter space is surely larger than that of these two scenarios. Second, it is

likely that there is no fundamental theory of supersymmetry breaking that precisely produces either scenario above. Nevertheless, one could reasonably hope that one these two models might serve as useful first approximations to the correct theory. Of course, the deviations of the correct theory from the above approximations would contain critical clues for the origin of the flavor structure of the MSSM.

In the high-energy approach, one treats the parameters of the MSSM as running parameters and imposes a particular structure on the soft supersymmetry breaking terms at a common high energy scale [such as the Planck scale (M_P) or GUT scale (M_X)]. Using the renormalization group equations (RGEs), one can then derive the low-energy MSSM parameters. This approach is usually characterized by the mechanism in which supersymmetry breaking is communicated to the effective low energy theory. Two theoretical scenarios have been examined in detail: gravity-mediated and gauge-mediated supersymmetry breaking. One bonus of such approaches is that one of the diagonal Higgs squared-mass parameters is typically driven negative by renormalization group evolution. Thus, electroweak symmetry breaking is generated radiatively, and the resulting electroweak symmetry breaking scale is intimately tied to the scale of low-energy supersymmetry breaking.

A truly Minimal SSM does not (yet) exist. The MSSM particle content must be supplemented by assumptions about the origin of supersymmetry-breaking that lie outside the low-energy domain of the model. Moreover, a comprehensive map of the phenomenologically acceptable region of MSSM-124 parameter space does not yet exist. This presents a formidable challenge to supersymmetric particle searches that must impose some parameter constraints while trying to ensure that the search is as complete as possible.

2.3. The minimal-SUGRA-inspired MSSM

Consider a supergravity (SUGRA) theory consisting of two sectors: a "hidden" sector,[4] in

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[4]A hidden sector consists of fields that carry no SU(3)×SU(2)×U(1) quantum numbers and do not have any renormalizable interactions with the MSSM fields.

which supersymmetry is spontaneously broken and a "visible" sector consisting of the MSSM fields. Because of the gravitational interactions that necessarily couple the two sectors, the effects of the hidden sector supersymmetry breaking will be transmitted to the MSSM. One finds that the resulting low-energy effective theory below the Planck scale consists of the unbroken MSSM plus all possible soft supersymmetry breaking terms [14]. In a *minimal* SUGRA framework [15], the soft-supersymmetry breaking parameters at the Planck scale take a particularly simple form in which the scalar squared masses and the *A*-parameters are flavor diagonal and universal [16]:

$$M_Q^2(M_{\mathrm{P}}) = M_U^2(M_{\mathrm{P}}) = M_D^2(M_{\mathrm{P}}) = m_0^2 \mathbf{1} \,,$$
$$M_L^2(M_{\mathrm{P}}) = M_E^2(M_{\mathrm{P}}) = m_0^2 \mathbf{1} \,,$$
$$m_1^2(M_{\mathrm{P}}) = m_2^2(M_{\mathrm{P}}) = m_0^2 \,,$$
$$A_U(M_{\mathrm{P}}) = A_D(M_{\mathrm{P}}) = A_L(M_{\mathrm{P}}) = A_0 \mathbf{1} \,. \qquad (7)$$

In addition, the gauge couplings and gaugino mass parameters are assumed to unify at the grand unification scale, M_X. Note that this implies that:

$$c_1 g_1(M_X) = g_2(M_X) = g_3(M_X) = g_U \,,$$
$$M_1(M_X) = M_2(M_X) = M_3(M_X) = m_{1/2} \,. \qquad (8)$$

where $c_1 \equiv \sqrt{5/3}$ ensures proper normalization of the $U(1)_Y$ coupling constant. Eq. (8) implies that the low-energy gaugino mass parameters satisfy:

$$M_3 = \frac{g_3^2}{g_2^2} M_2 \,, \qquad M_1 = \frac{5}{3} \tan^2 \theta_W M_2 \,. \qquad (9)$$

Finally, μ, and m_{12}^2 and the gaugino mass parameters are assumed (rather arbitrarily) to be initially real at the high scale. The minimal SUGRA-inspired MSSM has been sometimes called the constrained MSSM (or CMSSM [17]); I will adopt this nomenclature in this paper. It is easy to count the number of free parameters of the CMSSM. The low-energy values of the MSSM parameters are determined by the MSSM RGEs and the above initial conditions [eqs. (7) and (8)]. Gauge coupling unification yields a prediction for one of the gauge couplings in terms of the other two. In this case, it is convenient to take g_1 and

g_2 as input, and $\alpha_s \equiv g_3^2/(4\pi)$ as a prediction. To the extent that gauge coupling unification is successful, the number of parameters of the Standard Model is reduced by one (since M_{X} is also *a priori* a free parameter). To be conservative, it is useful to introduce an additional parameter that reflects possible non-trivial thresholds at the GUT scale, which could lead to slight changes in the prediction for $\alpha_s(m_Z)$ [18]. In this case, the Standard Model parameter count would remain unchanged.

Thus, the number of parameters of the CMSSM are: 18 Standard Model parameters (excluding the Higgs mass), m_0, $m_{1/2}$, A_0, $\tan\beta$, and $\mathrm{sgn}(\mu)$ for a total of 23 parameters. Note that m_{12}^2 and μ^2 were traded in for v^2 (which is counted as one of the 18 Standard Model parameters) and $\tan\beta$. In this procedure the sign of μ is not fixed, and so it remains an independent degree of freedom. It is tempting to rename this theory MSSM-23.

Clearly, MSSM-23 is much more predictive than MSSM-124. In particular, one has only four genuinely new parameters beyond the Standard Model (plus a two-fold ambiguity corresponding to the sign of μ), from which one can predict the entire MSSM spectrum and its interactions.[5] The disadvantage of the CMSSM is that the theoretical motivation underlying the initial conditions given in eq. (7) is rather weak. Although these initial conditions correspond to a minimal SUGRA framework (specifically, the kinetic energy terms for the gauge and matter fields are assumed to take a minimal canonical form), there is no theoretical principle that enforces such a minimal structure. In fact, it is now generally believed that supergravity-based (or superstring-based) supersymmetry breaking theories generically predict non-universal scalar masses [20].

A number of attempts have been made to perturb the CMSSM initial conditions [21,22] in a phenomenologically viable manner (*e.g*, without generating dangerous FCNC's [23]). For example, one can introduce separate mass scales for the Higgs and squark/slepton soft-supersymmetry-breaking masses. One can also introduce non-

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[5]In some regions of CMSSM parameter space, infrared fixed point behavior reduces the number of new parameters even further [19].

universal scalar masses, but restrict the size of the non-universal terms to be consistent with phenomenology. String-inspired models have provided an interesting parameterization of the deviation from universality of the soft-supersymmetry-breaking parameters [20]. Both "bottom-up" and "top-down" approaches have been useful in studying the possible form for supersymmetry breaking at the low-energy and high-energy scales.

Finally, although gaugino mass unification [see eq. (8)] is an integral part of the CMSSM initial conditions, it is *not* required by phenomenological constraints. Thus, it is also of interest to consider the phenomenological consequences of non-universal gaugino masses. For example, the phenomenology of $M_1 \simeq M_2$ [in contrast to $M_1 \simeq 0.5M_2$ predicted by eq. (9)] has recently been advocated [24] in order to provide a possible explanation for the famous CDF $ee\gamma\gamma$ event.

2.4. Models of gauge-mediated supersymmetry breaking

In an alternative to the SUGRA approach, the theory of gauge-mediated supersymmetry breaking posits that supersymmetry breaking is transmitted to the MSSM via gauge forces. The canonical structure of such models involves a hidden sector where supersymmetry is broken, a "messenger" sector consisting of messenger fields with SU(3)×SU(2)×U(1) quantum numbers, and a sector containing the fields of the MSSM [25,26]. The direct coupling of the messengers to the hidden sector generates a supersymmetry-breaking spectrum in the messenger sector. Finally, supersymmetry breaking is transmitted to the MSSM via the virtual exchange of the messengers.

In models of gauge-mediated supersymmetry breaking, scalar squared-masses are expected to be flavor independent since gauge forces are flavor-blind. In the simplest models, there is one effective mass scale, Λ, that determines all low-energy scalar and gaugino mass parameters through loop-effects (while no A-parameters are generated). In order that the resulting superpartner masses be of order 1 TeV or less, one must have $\Lambda \sim 100$ TeV. The generation of μ and m_{12}^2 lies outside the ansatz of gauge-mediated su-

persymmetry breaking. The initial conditions for the soft-supersymmetry-breaking running parameters are fixed at the messenger scale M, which characterizes the average mass of messenger particles. In principle, M can lie anywhere between (roughly) Λ and 10^{16} GeV (in models with larger values of M, supergravity-mediated effects would dominate the gauge-mediated effects). Thus, the minimal gauge mediated model (MGM) [27] contains 18 Standard Model parameters, Λ (which determines the supersymmetric scalar and gaugino masses), and $\tan\beta$ and $\arg(\mu)$ [after trading in m_{12}^2 and $|\mu|^2$ for v and $\tan\beta$]. There is also a weak logarithmic dependence on M, which enters through RGE running. We thus end up with 22 free parameters, which implies that the MGM is even more predictive than the CMSSM. However, the MGM is not a fully realized model. The sector of supersymmetry-breaking dynamics can be very complex, and it is fair to say that no simple compelling model of gauge-mediated supersymmetry yet exists. Nevertheless, this is an area of intense theoretical activity, and it will be interesting to see the variety of MSSM's that emerge from this approach over the next few years.

3. PHENOMENOLOGICAL ISSUES

3.1. Can low-energy supersymmetry be excluded?

Supersymmetric particles have not yet been discovered.[6] Thus, direct searches for supersymmetric particles at colliders have so far provided lower bounds for supersymmetric particle masses. These results are summarized by the Particle Data Group [29].

Can the MSSM be ruled out if no supersymmetric particles are discovered at future colliders? It is generally believed that low-energy supersymmetric theories require that supersymmetric particle masses should be less than $\mathcal{O}(1$ TeV). The argument follows from the naturalness require-

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[6]A few intriguing experimental anomalies have encouraged various supersymmetric interpretations [24,28]. In most cases, data now being collected at LEP-2 will either provide substance to such claims or rule them out. Data from Run-II of the Tevatron (which is scheduled to begin in 1999) can also provide the important corroborating evidence.

ment that is invoked to explain the existence of the large hierarchy between the electroweak scale and the Planck scale. This hierarchy is unnatural in the Standard Model, since there is no natural mechanism for keeping scalar masses light. Supersymmetry can naturally incorporate light scalars by relating them to light fermions which can have small masses due to weakly broken chiral symmetries.

If the scale of supersymmetry breaking is of order 1 TeV or less, then a Higgs mass of order the electroweak scale is still natural. However, as the supersymmetry-breaking scale increases, the condition for the fine-tuning of parameters (in order to keep the Higgs mass light) becomes more severe. Theorists have attempted to quantify the "degree of naturalness", and thereby deduce upper bounds for supersymmetric particle masses (see, *e.g.*, Refs. [30,31]). The naturalness conditions obtained are somewhat arbitrary, as are the corresponding conclusions. Although some interesting observations can be made, it is not possible to obtain rigorous upper limits on supersymmetric particle masses. Personally speaking, I am willing to concede that if supersymmetric particles are not discovered at the LHC, then supersymmetry is not relevant for explaining the origin of the electroweak scale.

3.2. The MSSM Higgs Sector

There is one case in the MSSM where a particle mass upper limit can be rigorously obtained. The mass of the light CP-even neutral Higgs boson, h^0, in the MSSM can be calculated to arbitrary accuracy in terms of two parameters of the Higgs sector, m_{A^0} and $\tan\beta$, and other MSSM soft-supersymmetry-breaking parameters that affect the Higgs mass through virtual loops [32]. If the scale of supersymmetry breaking is much larger than m_Z, then large logarithmic terms arise in the perturbation expansion. These large logarithms can be resummed using renormalization group (RG) methods. The logarithmic sensitivity to the supersymmetry breaking scale implies that the Higgs mass upper bound depends only weakly on the choice of an upper bound for supersymmetric particle masses.

At tree level, the MSSM predicts that $m_{h^0} \leq$

$m_Z|\cos 2\beta| \leq m_Z$. If this prediction were accurate, it would imply that the Higgs boson must be discovered at the LEP-2 collider (running at a center-of-mass energy of 192 GeV, with an integrated luminosity of 300 pb^{-1}). Absence of a Higgs boson lighter than m_Z would naively rule out the MSSM. When radiative corrections are included, the light Higgs mass upper bound is increased significantly. In the one-loop leading logarithmic approximation [32],

$$m_{h^0}^2 \lesssim m_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{8\pi^2 m_W^2} \ln\left(\frac{M_{\tilde{t}}^2}{m_t^2}\right), \quad (10)$$

where $M_{\tilde{t}}$ is the (approximately) common mass of the top-squarks. Observe that the Higgs mass upper bound is very sensitive to the top quark mass and is logarithmically sensitive to the top-squark masses. Although eq. (10) provides a rough guide to the Higgs mass upper bound, it is certainly insufficient for Higgs searches at LEP-2, where the Higgs mass reach depends delicately on the MSSM parameters. Moreover, in order to compare precision Higgs measurements with theory, a more precise expression for the Higgs mass is needed. The formula for the full one-loop radiative corrected Higgs mass has been obtained in the literature, although it is very complicated since it depends in detail on the virtual contributions of the MSSM spectrum [33]. If the supersymmetry breaking scale is larger than a few hundred GeV, then RG methods are essential for summing up the effects of the leading logarithms and obtaining an accurate prediction.

The computation of the RG-improved one-loop corrections requires numerical integration of a coupled set of RGEs [34]. (The dominant two-loop next-to-leading logarithmic results are also known [35].) Although this program has been carried out in the literature, the procedure is unwieldy and not easily amenable to large-scale Monte-Carlo analyses. Recently, two groups have presented simple analytic procedures for accurately approximating m_{h^0}. These methods can be easily implemented, and incorporate both the leading one-loop and two-loop effects and the RG-improvement. Also included are the leading effects at one loop of supersymmetric thresholds

(the most important effects of this type are squark mixing effects in the third generation). Details of the techniques can be found in Refs. [36] and [37], along with other references to the original literature. Here, I shall quote two specific bounds, assuming $m_t = 175$ GeV and $M_{\tilde{t}} \lesssim 1$ TeV: $m_{h^0} \lesssim 112$ GeV if top-squark mixing is negligible, while $m_{h^0} \lesssim 125$ GeV if top-squark mixing is "maximal". Maximal mixing corresponds to an off-diagonal squark squared-mass that produces the largest value of m_{h^0}. This mixing leads to an extremely large splitting of top-squark mass eigenstates. A more realistic choice of top-squark parameters leads to a Higgs mass upper bound of about 120 GeV.

3.3. Implications of precision electroweak data

Virtual supersymmetric particle exchange can influence many experimental processes. After eight years of precision electroweak data from LEP, SLC, and the Tevatron, the Standard Model predictions for many observables have been confirmed with remarkable accuracy. The LEP Electroweak Working Group (LEPEWWG) continues to update its fits of precision electroweak data. In its most recent work [38], a Standard Model global fit to 21 electroweak observables is presented. Only two observables exhibit a pull of two standard deviations or greater, while the χ^2/d.o.f. for the fit is 17/15. Thus, the precision electroweak data shows no significant deviation from Standard Model expectations. There is some sensitivity to the Standard Model Higgs mass via its virtual effects. The result obtained from the global fit is $m_{h^0} = 115^{+116}_{-66}$ GeV, or $m_{h^0} < 420$ GeV at 95% CL. These results are relevant for the MSSM in the following sense. The MSSM is a decoupling theory. If all supersymmetric particle masses are large compared to m_Z, then the virtual effects of supersymmetric particle exchange decouple from electroweak observables measured at an energy scale of order m_Z or below. Moreover, if $m_{A^0} \gg m_Z$, then the effects of the non-minimal Higgs bosons H^0, A^0 and $H^\pm$ also decouple, while h^0 remains light ($m_{h^0} \lesssim 125$ GeV). In the decoupling limit, the light CP-even Higgs boson h^0 has Standard Model coupling strengths to the Standard Model fermions and gauge bosons, and in this sense is indistinguishable from the Higgs boson of the Standard Model [39]. Thus, as long as all supersymmetric particles (and the non-minimal Higgs bosons) are sufficiently heavy (in practice, masses above 200 GeV are sufficiently decoupled), then the MSSM[7] provides an equally good description of the precision electroweak data *as long as the Higgs boson mass obtained in the Standard Model global fit is consistent with the MSSM light Higgs mass upper bound.* This latter condition is indeed satisfied by the LEPEWWG global fit.

If some supersymmetric particles are light (say, below 200 GeV but above present experimental bounds), then it is possible that the MSSM could either improve or destroy the LEPEWWG global fit. A few years ago, when the rate for $Z \to b\bar{b}$ was four standard deviations above the Standard Model prediction, the possibility that the MSSM could improve the LEPEWWG fit was taken quite seriously. However, it is hard to imagine that the MSSM could substantively improve the present LEPEWWG fit (given the goodness of the Standard Model fit in comparison to an MSSM fit, which necessarily involves more degrees of freedom). On the other hand, the MSSM could significantly decrease the goodness of the fit. This possibility has been explored recently in Ref. [41]. It was shown that there exists a range of parameters of the CMSSM and the MGM in which all supersymmetric particle masses are above their direct search bounds, but the global fit of electroweak data is significantly worse than the corresponding Standard Model fit. Thus, the allowable CMSSM and MGM parameter spaces are slightly smaller than the regions ruled out by the direct supersymmetric particle searches.

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[7]We still must make the additional assumption that the appropriate region of MSSM-124 parameter space has been selected to avoid the flavor problems discussed in Section 2.2. In principle, the flavor problem could be solved by the decoupling properties of the MSSM. However, the strongest constraints that exist for FCNC processes would require first and second generation squark and slepton masses to be above about 50 TeV in the absence of any other FCNC suppression mechanism [40].

3.4. Supersymmetric unification revisited

Electroweak observables are also sensitive to the strong coupling constant through the QCD radiative corrections. The LEPEWWG global fit extracts a value of $\alpha_s(m_Z) = 0.120 \pm 0.003$, which is in good agreement with the world average of $\alpha_s(m_Z) = 0.118 \pm 0.003$ quoted by the Particle Data Group [29]. Thus, previous claims that $\alpha(m_Z) \simeq 0.11$ [42] now seem somewhat disfavored. This result has important implications for the viability of supersymmetric unification. In Section 1, I briefly reviewed the history of unification models by focusing on the prediction of $\sin^2\theta_W$. Given the $\sin^2\theta_W$ is so well measured at LEP and SLC, it makes more sense to use this as input data (along with the fine-structure constant). This data can be used to obtain accurate values for $g_1(m_Z)$ and $g_2(m_Z)$ [in either the $\overline{\text{MS}}$ or $\overline{\text{DR}}$ schemes]. Extrapolating to high energies using either Standard Model or MSSM RGEs (with appropriate treatment of the low-energy supersymmetric thresholds), one then defines the mass scale at which $c_1 g_1$ [$c_1 = \sqrt{5/3}$] and g_2 meet as the unification scale, M_X. If the strong coupling constant g_3 also coincides with $c_1 g_1$ and g_2 at M_X, then by extrapolating back down to m_Z, one obtains a prediction for $\alpha_s(m_Z)$. The result of this exercise for the CMSSM is $\alpha_s(m_Z) > 0.126$ [18],[8] assuming that all squark masses are below 1 TeV (similar results have been obtained in Ref. [44]).

Thus, naive unification of gauge couplings in the MSSM does not quite work. However, the results of the analysis quoted above do not include the effects of possible high energy thresholds generated from the spectrum of masses of superheavy GUT particles. Taking such effects into account could either improve the situation or make it worse, depending on the details of the model. Two examples taken from the recent literature exhibit some of the possibilities. Consider GUT models employing the missing doublet mechanism to solve the doublet–triplet splitting problem (*i.e.*, why are the Higgs weak doublets so much lighter than the superheavy Higgs triplets

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[8]The corresponding result for the Standard Model extrapolation, $\alpha_s(m_Z) \simeq 0.073 \pm 0.002$ [43], is of course many standard deviations away from the experimentally observed result.

that typically occur in GUT models). It has been shown [18,45] that there is a range of the model parameter space where the heavy threshold corrections are sufficient to lower $\alpha_s(m_Z)$ to a value in agreement with experimental observation. In Ref. [46], some SO(10) grand unified models are studied which also exhibit an acceptable prediction for $\alpha_s(m_Z)$ as a result of high energy threshold corrections. In addition, interesting correlations are found between the predicted value of $\alpha_s(m_Z)$ and the proton lifetime.

3.5. Search for supersymmetry at future colliders

The search for supersymmetry at future colliders presents some important challenges. Perhaps the first order of business is to discover the light CP-even Higgs boson. If no Higgs boson with mass below about 125 GeV is discovered, then MSSM-124 is ruled out. LEP-2 will eventually be sensitive to Higgs masses up to about 100 GeV. To close the gap completely, one must first look to the hadron colliders. It has recently been pointed out that an upgraded Tevatron (with luminosity a factor of ten larger than the Main Injector) has the potential to detect Higgs bosons with masses up to about 130 GeV [47]. Whether all the machine and detector requirements can be met to reach this goal remains to be seen. For the LHC, the designs of the ATLAS and CMS detectors are being optimized for a discovery of a Higgs boson with a mass between 90 and 130 GeV via its rare $\gamma\gamma$ decay mode (the expected branching ratio is about 10^{-3} in the Standard Model) [48]. To achieve success in the Higgs search via the $\gamma\gamma$ mode at the LHC will require high electromagnetic calorimeter resolution (at about the 1% level) and maximal luminosity. At present, the LHC coverage of the MSSM Higgs sector parameter space is nearly complete, although small gaps in the MSSM parameter space may still exist. Because the search techniques in some cases depend on the observation of small signals above significant Standard Model backgrounds, it may be difficult to definitively rule out the MSSM if no Higgs signal is observed at the LHC.

If a very high-energy (next) linear e^+e^- collider (NLC) is built, then it will be able to ex-

tend the LEP-2 Higgs search up to a Standard Model Higgs mass of about 350 (800) GeV for $\sqrt{s} = 500$ (1000) GeV [49]. With $\sqrt{s} \gtrsim 300$ GeV and a total integrated luminosity of $\gtrsim 1$ fb^{-1}, the NLC would either discover h^0 or rule out the MSSM. Note that although the discovery of h^0 is essential for the MSSM to remain viable, the discovery does not serve to confirm the MSSM. For example, if the other non-minimal Higgs bosons are significantly heavier than the Z, then the properties of the h^0 will be indistinguishable from the Standard Model Higgs boson.

Thus, the ultimate confirmation of low-energy supersymmetry requires the direct discovery of supersymmetric particles. A comprehensive experimental program to study low-energy supersymmetric phenomena must accomplish four goals: (i) initial discovery of the supersymmetric particles; (ii) verification of the supersymmetric structure of their interactions; (iii) identification of the low-energy supersymmetric spectrum and its symmetries [e.g., MSSM or beyond, R-parity conservation or violation]; and (iv) measurement of the low-energy supersymmetry model parameters. For example, in the case of the MSSM, step (iv) would consist of measuring as many of the MSSM-124 parameters as possible. One would then make use of these experimental measurements to determine whether these parameters approximately matched the expectations of particular models such as the CMSSM or the MGM.

This program is highly non-trivial. In developing strategies for supersymmetric particle searches, one is tempted to look for shortcuts. For example, devising a strategy for discovery and exploration of the CMSSM (a.k.a. MSSM-23) is clearly a simpler task than the corresponding strategy for MSSM-124. Even well established phenomenology, such as the missing-energy signal generated by the escaping lightest supersymmetric particle (LSP), can be altered by a change of assumptions.[9] Another example of a well known fact of canonical supersymmetric phenomenology—most supersymmetric decays do not involve photons—does not necessarily ap-

ply to models of gauge-mediated supersymmetry breaking [27,51].[10] In these models the gravitino ($\widetilde{g}_{3/2}$) is the LSP. The decays of a supersymmetric particle will eventually produce the next-to-lightest supersymmetric particle (NLSP), which is typically the lightest neutralino ($\widetilde{\chi}_1^0$). Over a significant fraction of the model parameter space, $\widetilde{\chi}_1^0$ decays inside the detector to $\gamma + \widetilde{g}_{3/2}$. In this example, *all* supersymmetric particle decay chains would contain a photon.

Perhaps a non-minimal model of low-energy supersymmetry is the correct theory of TeV-scale physics. The phenomenology of such models could be considerably different from the canonical supersymmetric phenomenology usually analyzed. For example, in R-parity violating models, the LSP can decay into visible matter. Here is an example where the missing energy signal could be irrelevant for low-energy supersymmetry searches. The challenge for supersymmetry searches at future colliders is to allow for all possible phenomenological scenarios.

4. BEYOND MSSM-124

In this paper, I have attempted to restrict the definition of the MSSM to the properties of the effective low-energy theory below the 1 TeV energy scale. I argued that the most general approach then leads to MSSM-124 which possesses a viable phenomenology only at exceptional points in its parameter space. More precisely, the phenomenology is viable at these exceptional points, and in the regions of parameter space defined by small perturbations around these exceptional points. Presumably these perturbations are generated by new physics at higher energy scales. Such perturbations can generate rare processes that lie outside the Standard Model (as well as outside the typical MSSM). Possible consequences include: proton decay, L_e, L_μ and/or L_τ violation, L violation (leading, *e.g.*, to neutrino masses), and new sources of CP-violation.

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[9]For example, consider the model recently proposed in Ref. [50], in which the gluino is the LSP.

[10]Ref. [24] reminds us that photons can be also produced with large branching ratio in neutralino decay, $\widetilde{\chi}_2^0 \to \widetilde{\chi}_1^0 \gamma$ [52], in models where the electroweak gaugino mass parameters are approximately equal ($M_2 \simeq M_1$). This is neither the CMSSM nor the MGM, but another interesting point in MSSM-124 parameter space.

For example, in Ref. [53] it was argued that supersymmetric grand unification models should typically generate $\mu \to e\gamma$, at a level that may be observable at future high precision experiments.

4.1. Supersymmetric models with non-zero neutrino masses

There is some evidence for very small but non-zero neutrino masses. Thus, it is of interest to consider a supersymmetric generalization of an extended Standard Model that contains nonzero neutrino masses. In the supersymmetric extension of the see-saw model of neutrino masses [54,55], one introduces a right-handed neutrino superfield $\widehat{N}$, with the following new terms in the superpotential:

$$\delta W = -\epsilon_{ij} h_N \widehat{H}_2^i \widehat{L}^j \widehat{N} - \tfrac{1}{2} M \widehat{N}\widehat{N}\,, \qquad (11)$$

where generation labels have been suppressed. Here, M is the scale of the right-handed neutrino. In addition, one adds new soft supersymmetry breaking terms:

$$\delta V_{\text{soft}} = m_N^2 \widetilde{N}^* \widetilde{N} + (m_{NN} \widetilde{N}\widetilde{N} + \text{h.c.})$$
$$-\epsilon_{ij}\left[h_N A_N \widehat{H}_2^i \widetilde{L}^j \widetilde{N} + \text{h.c.}\right]\,. \qquad (12)$$

Note that the supersymmetric see-saw model conserves R-parity since lepton number is violated by two units.

For simplicity, consider the one-generation case. Let $m_D \equiv h_N v_2/\sqrt{2}$. If $m_D \ll M$, then the fermion spectrum contains a very heavy neutrino with mass of order M and a very light neutrino with mass of order m_D^2/M. This is the see-saw; an appropriately large choice for M can naturally lead to neutrino masses in the eV range and below. In the supersymmetric model, the $\Delta L = 2$ interaction responsible for neutrino mass will also generate sneutrino-antisneutrino mixing. If CP is conserved, the $\widetilde{\nu}$–$\overline{\widetilde{\nu}}$ mixing angle is 45°, producing a CP-even and CP-odd scalar mass eigenstate. The mass splitting of these two states is of order the light neutrino mass (although enhancements of 10^3 are possible as shown in Ref. [55]). In favorable model circumstances, this small mass difference could be measured in sneutrino pair production at e^+e^- colliders by detecting like-sign di-leptons from sneutrino decays (in analogy with the B^0–$\overline{B^0}$ system).

4.2. New gauge and matter multiplets at the TeV-scale

Models of neutrino masses necessarily add new structure beyond the Standard Model. In the see-saw example, this new structure lives at a high energy scale. However, when the physics associated with the high scale is integrated out, there is a non-trivial remnant in the effective low-energy theory. A second possible approach is to add new structure beyond the Standard Model at the 1 TeV scale. The supersymmetric extension of such a theory would be a non-minimal extension of the MSSM. Possible new structures include [56]: (i) an enlarged electroweak gauge group beyond SU(2)×U(1); (ii) the addition of new, possibly exotic, matter multiplets [e.g., a vector-like color triplet with electric charge $(1/3)e$; such states sometimes occur as low-energy remnants in E_6 GUT models]; or (iii) the addition of low-energy SU(3)×SU(2)×U(1) singlets. A possible theoretical motivation for such new structure arises from the study of phenomenologically viable string theory ground states [57].

4.3. The next-to-minimal supersymmetric model

The next-to-minimal supersymmetric extension of the Standard Model (NMSSM) consists of adding one complex singlet Higgs superfield to the MSSM [58]. This example provides an instructive case study of a non-minimal supersymmetric Higgs sector. In particular, it was noted in Section 3.2 that the experimental absence of a light CP-even Higgs boson with $m_{h^0} \lesssim 125$ GeV would rule out the MSSM. In the NMSSM, the Higgs mass bound depends on an extra assumption beyond the physics of the low-energy effective theory. Specifically, the addition of the Higgs singlet adds a new Higgs self-coupling parameter λ to the theory.[11] The mass of the lightest neutral Higgs boson can be raised arbitrarily by increasing the value of λ (analogous to the behavior of the Higgs mass in the Standard Model). However, if λ is taken to be too large, then perturbation theory becomes unreliable. If one imposes the

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[11]In contrast, all Higgs self-couplings of the MSSM are related by supersymmetry to gauge couplings. This is the origin of the MSSM bound: $m_{h^0} \lesssim \mathcal{O}(m_Z)$.

condition that all couplings remain perturbative up to the Planck scale, one finds that at least one Higgs boson of the model must be lighter than about 150 GeV.

Could the Higgs bosons of the NMSSM escape detection at future colliders? Even though a light Higgs state must exist (under the perturbativity assumption introduced above), it may be very weakly coupled to quarks, leptons and gauge bosons if it is primarily composed of the singlet component. Thus, a detailed analysis is required to see whether the Higgs search at the NLC is sensitive to all regions of the NMSSM Higgs sector parameter space. The analysis of Ref. [59] demonstrates that even for $\sqrt{s} = 300$ GeV, the NLC search would easily detect at least one Higgs state of the NMSSM.

A similar question can be posed in the case of the LHC Higgs search. As mentioned in Section 3.5, the LHC search has nearly complete coverage of the MSSM Higgs sector parameter space. It is likely that further development of search techniques (and improvements of detector technology such as efficient b-tagging) will be able to close any final loopholes. Nevertheless, the LHC search is operating "at the edge" of its capabilities. By relaxing some of the MSSM constraints to Higgs sector parameters, we expect some holes to develop in the region of supersymmetric parameter space accessible to the LHC Higgs search. Ref. [60] examined this question in detail for the case of the NMSSM, and concluded that although the region of inaccessibility is not large, it is possible to find regions of NMSSM Higgs parameter space in which no Higgs boson state could be discovered at the LHC. This analysis does suggest the possibility that future improvements in search strategies and detector capabilities may be able to close these loopholes as well.

4.4. R-parity violating supersymmetry

So far, all non-minimal supersymmetric models considered here have retained R-parity as a discrete symmetry. R-parity is a $\mathbf{Z}_2$ symmetry that distinguishes between Higgs and matter superfields. For a particle of spin S, the R-parity quantum number is $R = (-1)^{3(B-L)+2S}$, so R-parity implies the conservation of B−L. Con-

sider the MSSM, but now relax the constraint of R-parity conservation [61]. In the absence of R-parity conservation, new terms can be added to the superpotential:

$$W_{\rm NR} = \epsilon_{ij}\big[(\lambda_L)_{pmn}\widehat{L}^i_p\widehat{L}^j_m\widehat{E}_n + (\lambda'_L)_{pmn}\widehat{L}^i_p\widehat{Q}^j_m\widehat{D}_n$$
$$-\mu'_p\widehat{L}^i_p\widehat{H}^j_2\big] + (\lambda_B)_{pmn}\widehat{U}_p\widehat{D}_m\widehat{D}_n . \quad (13)$$

New R-parity violating soft supersymmetry breaking terms can also be obtained by the usual procedure of replacing the superfields of the superpotential with their corresponding scalar partners, and introducing a new matrix coefficient for each term. Note that the term in eq. (13) proportional to λ_B violates B, while the other three terms (which have been obtained from eq. (5) by replacing $\widehat{H}_1$ with $\widehat{L}$) violate L. Phenomenological constraints on various low-energy B and L violating processes yield limits on each of the individual coefficients in eq. (13) taken one at a time [61]. If more than one coefficient is simultaneously active, the limits are in general more complex. All four terms in eq. (13) cannot be simultaneously present and unsuppressed; otherwise the proton decay rate would be many orders of magnitude larger than the present experimental bound. One way to avoid proton decay is to impose either B or L separately. For example, if B is conserved but L is not, then $\lambda_B = 0$ (while the other three terms in eq. (13) can be present). Such a model violates R-parity but preserves a $\mathbf{Z}_3$ baryon parity.

If R-parity is not conserved, supersymmetric phenomenology exhibits features that are quite distinct from that of the MSSM. The LSP is no longer stable, which implies that not all supersymmetric decay chains must yield missing energy events at colliders. Both $\Delta L = 1$ and $\Delta L = 2$ phenomena are allowed (assuming B is conserved), leading to neutrino masses and mixing [62], neutrinoless double beta decay [63], sneutrino-antisneutrino mixing [55,64], and s-channel resonant production of the sneutrino in e^+e^- collisions [65]. Since the distinction between the Higgs and matter multiplets is lost, R-parity violation permits the mixing of sleptons and Higgs bosons, the mixing of neutrinos and neutralinos, and the mixing of charged leptons and charginos, leading to more complicated mass

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matrices and mass eigenstates.

Squarks can be regarded as leptoquarks since the term in eq. (13) proportional to λ'_L permits processes such as:

$$e^+ \overline{u}_m \to \overline{\widetilde{d}}_n \to e^+ \overline{u}_m , \, \overline{\nu} \overline{d}_m$$
$$e^+ d_m \to \widetilde{u}_n \to e^+ d_m . \tag{14}$$

These processes have received much attention during the past year as a possible explanation for the HERA high Q^2 anomaly [28]. Note that the same term responsible for the processes displayed above could also generate purely hadronic decays for sleptons: $e.g.$, $\widetilde{\ell}_p^- \to \overline{u}_m d_n$ and $\widetilde{\nu}_p \to \overline{q}_m q_n$ ($q = u$ or d). If such decays were dominant, then the pair production of sleptons in $e^+ e^-$ events would lead to hadronic four-jet events [66], a signature quite different from the missing energy signals expected in the MSSM.

5. THE STATUS OF LOW-ENERGY SUPERSYMMETRY

The organizers of SUSY-97 asked me to summarize the status of low-energy supersymmetry.

• *Theory.* The origin of the soft supersymmetry breaking terms and the details of their structure remain a mystery. The interplay of supersymmetry and the origin of flavor needs elucidation. There are many ideas but as yet no compelling models. We do not understand the mechanism that requires nature to lie near one of the exceptional points in MSSM-124 parameter space.

• *Experiment.* Supersymmetric particles have not yet been discovered. Indirect hints such as gauge coupling unification, the existence of dark matter (for which the LSP is a natural candidate), and a few interesting collider "zoo" events are intriguing but not yet compelling.

• *Phenomenology.* Canonical supersymmetric signatures at future colliders are well analyzed and understood. Much of the recent efforts have been directed at trying to develop strategies for precision measurements to prove the underlying supersymmetric structure of the interactions and to distinguish among models [67]. However, we are far from understanding all possible facets of MSSM-124 parameter space (even restricted

to those regions that are phenomenologically viable). Moreover, the phenomenology of alternative low-energy supersymmetric models (such as models with R-parity violation) and its consequences for collider physics have only recently begun to attract significant attention. The variety of possible non-minimal models of low-energy supersymmetry presents an additional challenge to experimenters who plan on searching for supersymmetry at future colliders.

Low-energy supersymmetry remains the most elegant solution to the naturalness and hierarchy problems, while providing a possible link to Planck scale physics and the unification of particle physics and gravity. If nature has chosen this path, then the future of the SUSY-yy workshops is indeed bright.

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