The Minimally Supersymmetric Standard Model

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Supersymmetry Seminar
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Outline

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Standard Model — The Basics

The gauge group of the Standard Model (SM) is

$$\mathcal{G}_{\mathrm{SM}} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_T \times \mathrm{U}(1)_Y\,. \tag{1}$$

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The SM fields transform in representations of this group:

$$s = 0 : \quad \underset{\ni \phi}{(\mathbf{1}, \mathbf{2})_{\frac{1}{2}}} \tag{2}$$

$$s = \frac{1}{2} : \quad \underset{\ni q_L}{(\mathbf{3}, \mathbf{2})_{\frac{1}{6}}} \oplus \underset{\ni (d_R)^c}{(\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}} \oplus \underset{\ni (u_R)^c}{(\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}}} \oplus \underset{\ni \ell_L}{(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}} \oplus \underset{\ni (e_R)^c}{(\mathbf{1}, \mathbf{1})_{1}} \tag{3}$$

$$s = 1 : \quad \underset{\ni A_\mu^a}{(\mathbf{8}, \mathbf{1})_0} \oplus \underset{\ni W_\mu^k}{(\mathbf{1}, \mathbf{3})_0} \oplus \underset{\ni B_\mu}{(\mathbf{1}, \mathbf{1})_0} . \tag{4}$$

Standard Model — The Basics

Beware! We only use lefthanded fields $\psi_L = \begin{pmatrix} \psi_\alpha \\ 0 \end{pmatrix}$, righthanded fields are included via charge conjugation $(\psi_R)^c = \begin{pmatrix} 0 \\ \overline{\psi}^{\dot{\alpha}} \end{pmatrix}^c = \begin{pmatrix} \psi_\alpha \\ 0 \end{pmatrix}$.

The $\mathrm{SU}(2)$-doublets are written as:

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \qquad q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad \ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}.$$

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Standard Model — The Lagrangian

The SM Lagrangian contains a variety of terms which roughly fall into three categories:

gauge terms | kinetic terms | Higgs sector

The gauge part is straightforward albeit there being additional gauge configurations:

$$\mathcal{L}_{\text{gauge}} \supset \frac{1}{2g_i^2}\,\mathrm{tr}\left[F^{(i)}_{\mu\nu}F^{(i)\mu\nu}\right] \tag{5}$$

$$\supset \frac{\theta_{\mathrm{QCD}}}{16\pi^2 g_{\mathrm{s}}^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}\left[F^{(3)}_{\mu\nu}F^{(3)}_{\rho\sigma}\right]. \tag{6}$$

Kinetic terms for the fermions are constructed with the covariant derivative

$$\mathcal{L}_{\mathrm{kin}} \supset \overline{\psi}_i \mathrm{i}\not{D}\psi_i \tag{7}$$

$$D_\mu = \partial_\mu - \mathrm{i}qA^k_\mu\mathcal{R}(T_k) \tag{8}$$

Standard Model — Higgs sector

The Higgs part is made of a scalar kinetic term, the quartic potential and the Yukawa couplings:

$$\mathcal{L}_{\text{Higgs}} \supset -\left(D^{\mu}\phi\right)^{\dagger}\left(D_{\mu}\phi\right) + \mu\phi^{\dagger}\phi - \lambda\left(\phi^{\dagger}\phi\right)^{2} - \lambda^{\psi}\left[\overline{\psi}\phi\psi\right]_{\mathbf{1}}. \tag{9}$$

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The fermions gain masses $m_\psi = v \lambda^\psi$ when the Higgs field acquires its vacuum expectation value (VEV):

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \text{(shorthand: } \langle \phi_0 \rangle = v\text{)} .$$

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For our three types of massive fermions (electron, up-quark, down-quark) the corresponding singlets look like:[1]

$$\lambda^{u}\left[\overline{q}_L\tilde{\phi}u_R\right]_{\mathbf{1}} \qquad \lambda^{d}\left[\overline{q}_L\phi d_R\right]_{\mathbf{1}} \qquad \lambda^{e}\left[\overline{\ell}_L\phi e_R\right]_{\mathbf{1}}. \tag{10}$$

[1] Careful, since $\tilde{\phi} = \epsilon\phi^{*}$ to account for the u-type quarks.

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Furthermore, the SM consists of three generations of fermions thus promoting the λ^{ψ} to complex 3x3 matrices λ^{e}_{mn}, λ^{d}_{mn} and λ^{u}_{mn}. The λ^{f}_{mn} can be diagonalised via bi-unitary transformations:

$$V_f^{\dagger}\lambda^{f}U_f \propto \operatorname{diag}\left(m_f^{(1)}, m_f^{(2)}, m_f^{(3)}\right)/v. \tag{11}$$

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Standard Model — Parameter count

Count parameters and gauge redundancies:

$+$

- 3 couplings g, g' and g_{s}, one vacuum angle θ_{QCD} (4 parameters)
- Higgs parameters v, λ (2 parameters)
- 3 (complex) mass matrices λ^f (3x18 parameters)

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- Quark flavour symmetry $\mathrm{U}(3)_{q_L} \times \mathrm{U}(3)_{u_R} \times \mathrm{U}(3)_{d_R}/\mathrm{U}(1)_B$ (3x9-1 parameters)
- Lepton flavour symmetry $\mathrm{U}(3)_{\ell_L} \times \mathrm{U}(3)_{e_R}/\mathrm{U}(1)_{L_e} \times \mathrm{U}(1)_{L_\mu} \times \mathrm{U}(1)_{L_\tau}$ (2x9-3 parameters)

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The Standard Model of Particle Physics has 19 free parameters with v being the only one carrying a physical dimension.[2]

[2] 14 real parameters, 3 mixing angles, 2 CP-violating phase.

Outline

MSSM — The fields

First of all we promote all our previous fields to real (chiral) superfields resulting in our renewed table:

super field	bosonic field	fermionic field	representation
$\hat{V}_8$	g	$\tilde{g}$	$(\mathbf{8},\mathbf{1})_0$
$\hat{V}$	W^0, $W^\pm$	$\tilde{W}^0$, $\tilde{W}^\pm$	$(\mathbf{1},\mathbf{3})_0$
$\hat{V}'$	B	$\tilde{B}$	$(\mathbf{1},\mathbf{1})_0$
$\hat{L}$	$(\tilde{\nu}_e, \tilde{e})$	(ν_L, e_L)	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$
$\hat{E}^c$	$\tilde{e}_R^c$	e_R^c	$(\mathbf{1},\mathbf{1})_1$
$\hat{Q}$	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$(\mathbf{3},\mathbf{1})_{\frac{1}{6}}$
$\hat{U}^c$	$\tilde{u}_R^c$	u_R^c	$(\mathbf{3},\mathbf{1})_{-\frac{2}{3}}$
$\hat{D}^c$	$\tilde{d}_R^c$	d_R^c	$(\mathbf{3},\mathbf{1})_{\frac{1}{3}}$
$\hat{H}_u$	(H_u^+, H_u^0)	$\left(\tilde{H}_u^+, \tilde{H}_u^0\right)$	$(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$
$\hat{H}_d$	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$

MSSM — SUSY terms

For the gauge part the ususal field strength super fields $\mathcal{W}_{i,\alpha}$ are constructed and included in the Lagrangian:

$$\mathcal{L}^{\mathrm{MSSM}}_{\mathrm{gauge}} \supset \frac{1}{2g_i^2}\,\mathrm{tr}\left[\int \mathrm{d}^2\theta\,(\mathcal{W}_i)^\alpha\,(\mathcal{W}_i)_\alpha + \mathrm{h.c.}\right]. \tag{12}$$

The kinetic terms for the fields read:

$$\mathcal{L}^{\mathrm{MSSM}}_{K} \supset \int \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}\left[\hat{\Phi}_i^\dagger e^{2V_i}\hat{\Phi}_i\right]_{\mathbf{1}} \tag{13}$$

$$V_i = \hat{V}_8^a \mathcal{R}_i(T_a) + \hat{V}^k \mathcal{R}_i(T_k) + Y_i \hat{V}'. \tag{14}$$

MSSM — Superpotential

The superpotential term is simply:

$$\mathcal{L}_W^{\mathrm{MSSM}} = \int \mathrm{d}^2\theta W + \mathrm{h.c.} \tag{15}$$

What terms are contained in W?

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In general, the superpotential contains a great variety of different terms, under them the Yukawa couplings:

$$\begin{aligned} W =& \lambda_d \left[\hat{H}_d \hat{Q} \hat{D}\right]_{\mathbf{1}} + \lambda_e \left[\hat{H}_d \hat{L} \hat{E}\right]_{\mathbf{1}} - \lambda_u \left[\hat{H}_u \hat{Q} \hat{U}\right]_{\mathbf{1}} + \mu \left[\hat{H}_u \hat{H}_d\right]_{\mathbf{1}} \\ &+ a \left[\hat{L} \hat{H}_u\right]_{\mathbf{1}} + b \left[\hat{Q} \hat{L} \hat{D}\right]_{\mathbf{1}} + c \left[\hat{U} \hat{U} \hat{D}\right]_{\mathbf{1}} + d \left[\hat{L} \hat{L} \hat{E}\right]_{\mathbf{1}} . \end{aligned} \tag{16}$$

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The terms in the last line introduce B-number violation via proton decay as well as lepton number violation, but by imposing R-parity

$$R = (-1)^{3(B-L)+2s}, \tag{17}$$

we can get rid of them. Beware! This is not obligatory!

R-conservation implies the existance of a lightest supersymmetric particle (LSP) thus providing us with a dark matter candidate.

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We could use matter parity

$$P_{\mathrm{m}} = (-1)^{3(B-L)}, \tag{17}$$

instead and see directly how the lower line gets thrown out.

MSSM — Soft SUSY breaking terms

Introduce explicitly SUSY breaking terms to generate masses and additional interactions

$$
\begin{aligned}
-\mathcal{L}_{\text{soft}}^{\text{MSSM}} \supset & \frac{1}{2} M_i \bar{\tilde{\lambda}}_i \tilde{\lambda}_i + M_{\tilde{F}}^2 \tilde{f}^\dagger \tilde{f} \\
& + m_1^2 H_d^\dagger H_d + m_2^2 H_u^\dagger H_u + m_{12}^2 \left(H_u \cdot H_d + \text{h.c.} \right) \\
& + T_U H_u \tilde{Q} \tilde{U} + T_D H_d \tilde{Q} \tilde{D} + T_E H_d \tilde{L} \tilde{E} + \text{h.c.}
\end{aligned}
\tag{18}
$$

Often, a parametrisation $m_{12}^2 = \mu B$ (and $T_F = \lambda_f A_F$) is chosen. Therefore, the corresponding terms are called A and B-terms.

MSSM — Note on the Higgs sector

Repeat the SM steps: In the MSSM the quartic coupling is generated by the kinetic and soft SUSY breaking terms leading to an effective Higgs potential.

$$
\begin{aligned}
V_{\text{Higgs}} = {} & \left(m_1^2 + |\mu|^2\right) H_d^\dagger H_d + \left(m_2^2 + |\mu|^2\right) H_u^\dagger H_u + m_{12}^2 \left(H_u \cdot H_d + \text{h.c.}\right) \\
& + \frac{g^2 + {g'}^2}{8} \left(H_d^\dagger H_d - H_u^\dagger H_u\right) + \frac{1}{2} g^2 \left| H_d^\dagger H_u \right|^2 ,
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The two doublets acquire separate VEVs:

$$\left\langle H_f^0 \right\rangle = v_f , \tag{20}$$

related to the previous v via:

$$\sqrt{v_u^2 + v_d^2} = v, \tag{21}$$

by convention the angle β is defined as

$$\tan\beta = \frac{v_u}{v_d}. \tag{22}$$

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At tree level this implies an upper bound on the mass of the lightest Higgs:

$$m_h \leq m_Z \left|\cos 2\beta\right| . \tag{20}$$

MSSM — Mixing caveats

The SM particle spectrum looks like:

$$\begin{array}{ccccc} u & c & t & & B & W^0 \\ d & s & b & & g & W^\pm \\ \\ e & \mu & \tau & & h^0 & \\ \nu_e & \nu_\mu & \nu_\tau & & & \end{array}$$

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$$\begin{array}{cccccccc} \tilde{u}_L & \tilde{u}_R & \tilde{c}_L & \tilde{c}_R & \tilde{t}_L & \tilde{t}_R & \tilde{B} & \tilde{W}^0 \\ \tilde{d}_L & \tilde{d}_R & \tilde{s}_L & \tilde{s}_R & \tilde{b}_L & \tilde{b}_R & \tilde{g} & \tilde{W}^\pm \\ \\ \tilde{e}_L & \tilde{e}_R & \tilde{\mu}_L & \tilde{\mu}_R & \tilde{\tau}_L & \tilde{\tau}_R & \tilde{H}_d^- & \tilde{H}_d^0 \\ \tilde{\nu}_e & & \tilde{\nu}_\mu & & \tilde{\nu}_\tau & & \tilde{H}_u^0 & \tilde{H}_u^+ \end{array}$$

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The MSSM particle spectrum[3] looks like:

$$\begin{array}{cccccccc} \tilde{u}_1 & \tilde{u}_2 & \tilde{u}_3 & \tilde{u}_4 & \tilde{u}_5 & \tilde{u}_6 & \tilde{\chi}_1^\pm & \tilde{\chi}_2^\pm \\ \tilde{d}_1 & \tilde{d}_2 & \tilde{d}_3 & \tilde{d}_4 & \tilde{d}_5 & \tilde{d}_6 & \tilde{g} & \\ \\ \tilde{\ell}_1 & \tilde{\ell}_2 & \tilde{\ell}_3 & \tilde{\ell}_4 & \tilde{\ell}_5 & \tilde{\ell}_6 & \tilde{\chi}_1^0 & \tilde{\chi}_2^0 \\ \tilde{\nu}_1 & & \tilde{\nu}_2 & & \tilde{\nu}_3 & & \tilde{\chi}_3^0 & \tilde{\chi}_4^0 \end{array}$$

[3] Worst case scenario.

MSSM — Mixing caveats

Summary of mixed states:

- The Higgs bosons (H_d^-, H_d^0, H_u^0, H_u^+) form: a charged pair $H^\pm$, two CP-even neutral scalars h^0, H^0 and a CP-odd A^0.
- The sneutrinos $\tilde{\nu}_i$ form eigenstates $\tilde{\nu}_i$.
- The charged bosinos ($\tilde{W}^\pm$, $\tilde{H}_u^+$, $\tilde{H}_d^-$) form the charginos $\tilde{\chi}_i^\pm$.
- The neutral bosinos ($\tilde{B}$, $\tilde{W}^0$, $\tilde{H}_u^0$, $\tilde{H}_d^0$) form the neutralinos $\tilde{\chi}_i^0$.
- The squarks ($\tilde{q}_{i,L}$, $\tilde{q}_{i,R}$) form mass eigenstates labeled $\tilde{q}_i$.
- The sleptons ($\tilde{e}_{i,L}$, $\tilde{e}_{i,R}$) form eigenstates $\tilde{\ell}_i$.

Only for certain ranges of the parameters the particle spectrum will resemble a 'double-SM'.

MSSM — Parameter count

$+$

- 3 couplings g_i, one vacuum angle θ_{QCD} (4 parameters)
- 3 (complex) gaugino masses M_i (6 parameters)
- 2 Higgs mass parameters v, β (2 parameters)
- 2 (complex) Higgs/ino mass parameters μ, B (4 parameters)
- 5 hermitian scalar mass matrices $M^2_{\tilde{F}}$ (5x9 parameters)
- 3 mass matrices λ^f (3x18 parameters)
- 3 trilinear couplings T_F (3x18 parameters)

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The full Minimal Supersymmetric Standard Model has 124 free parameters[3] (MSSM-124).

[3] Consisting of 3 couplings, 37 real masses, 39 mixing angles and 45 CP-violating phases.

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