THE MINIMALLY SUPERSYMMETRIC STANDARD MODEL

Mathieu Kaltschmidt & Jonah Cedric Strauß

ITP Heidelberg

Supersymmetry Seminar supervised by Prof. Jörg Jäckel

Heidelberg, February 15th 2021

- 1. Introduction
- 2. The Standard Model
- 3. The MSSM

- Introduction
- Parameter Count
 2 The Standard Model
 Basics
 The Lagrangian
 Parameter Count
- 3 The MSSN

The Lagrangian — SUSY conserving The Lagrangian — SUSY breaking Mixing Caveats Parameter Count

- Introduction
- The Standard Model Basics
 The Lagrangian
 Parameter Count
- 3 The MSSM

The Lagrangian — SUSY conserving
The Lagrangian — SUSY breaking
Mixing Caveats
Parameter Count

The gauge group of the Standard Model (SM) is

$$\mathcal{G}_{SM} = SU(3)_C \times SU(2)_T \times U(1)_Y.$$
(1)

The gauge group of the Standard Model (SM) is

$$\mathcal{G}_{SM} = SU(3)_C \times SU(2)_T \times U(1)_Y. \tag{1}$$

The SM fields transform in representations of this group:

$$s = 0: \quad (\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$$

$$s = \frac{1}{2}: \quad (\mathbf{3}, \mathbf{2})_{\frac{1}{6}} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}} \oplus (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}} \oplus (\mathbf{1}, \mathbf{1})_{1}$$

$$g_{L} \quad g_{L} \quad g_$$

Beware! We only use lefthanded fields
$$\psi_L = \begin{pmatrix} \psi_\alpha \\ 0 \end{pmatrix}$$
, righthanded fields are included via charge conjugation $(\psi_R)^c = \begin{pmatrix} 0 \\ \overline{\psi}^{\dot{\alpha}} \end{pmatrix}^c = \begin{pmatrix} \psi_\alpha \\ 0 \end{pmatrix}$.

The SU(2)-doublets are written as:

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \qquad q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad \ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}.$$

The gauge group of the Standard Model (SM) is

$$\mathcal{G}_{SM} = SU(3)_C \times SU(2)_T \times U(1)_Y. \tag{1}$$

The SM fields transform in representations of this group:

$$s = 0: \quad (\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$$

$$s = \frac{1}{2}: \quad (\mathbf{3}, \mathbf{2})_{\frac{1}{6}} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}} \oplus (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}} \oplus (\mathbf{1}, \mathbf{1})_{1}$$

$$g_{L} \quad g_{L} \quad g_$$

Standard Model — The Lagrangian

The SM Lagrangian contains a variety of terms which roughly fall into three categories:

gauge terms kinetic terms Higgs sector

The gauge part is straightforward albeit there being additional gauge configurations:

$$\mathcal{L}_{\mathsf{gauge}} \supset \frac{1}{2g_i^2} \operatorname{tr} \left[F_{\mu\nu}^{(i)} F^{(i)\mu\nu} \right] \tag{5}$$

$$\supset \frac{\theta_{\text{QCD}}}{16\pi^2 g_s^2} \epsilon^{\mu\nu\rho\sigma} \operatorname{tr} \left[F_{\mu\nu}^{(3)} F_{\rho\sigma}^{(3)} \right]. \tag{6}$$

Kinetic terms for the fermions are constructed with the covariant derivative

$$\mathcal{L}_{\rm kin} \supset \overline{\psi}_i i \not\!\!\!D \psi_i \tag{7}$$

$$D_{\mu} = \partial_{\mu} - iq A_{\mu}^{k} \mathcal{R}(T_{k}) \tag{8}$$

The Higgs part is made of a scalar kinetic term, the quartic potential and the Yukawa couplings:

$$\mathcal{L}_{\text{Higgs}} \supset -\left(D^{\mu}\phi\right)^{\dagger} \left(D_{\mu}\phi\right) + \mu\phi^{\dagger}\phi - \lambda\left(\phi^{\dagger}\phi\right)^{2} - \lambda^{\psi}\left[\overline{\psi}\phi\psi\right]_{1}. \tag{9}$$

The Higgs part is made of a scalar kinetic term, the quartic potential and the Yukawa couplings:

$$\mathcal{L}_{\text{Higgs}} \supset -\left(D^{\mu}\phi\right)^{\dagger} \left(D_{\mu}\phi\right) + \mu\phi^{\dagger}\phi - \lambda\left(\phi^{\dagger}\phi\right)^{2} - \lambda^{\psi} \left[\overline{\psi}\phi\psi\right]_{1}. \tag{9}$$

The fermions gain masses $m_{\psi}=v\lambda^{\psi}$ when the Higgs field acquires its vacuum expectation value (VEV):

$$\langle \phi \rangle = \left(\begin{array}{c} 0 \\ v \end{array} \right)$$
 (shorthand: $\langle \phi_0 \rangle = v$).

The Higgs part is made of a scalar kinetic term, the quartic potential and the Yukawa couplings:

$$\mathcal{L}_{\text{Higgs}} \supset -\left(D^{\mu}\phi\right)^{\dagger} \left(D_{\mu}\phi\right) + \mu\phi^{\dagger}\phi - \lambda\left(\phi^{\dagger}\phi\right)^{2} - \lambda^{\psi} \left[\overline{\psi}\phi\psi\right]_{1}. \tag{9}$$

For our three types of massive fermions (electron, up-quark, down-quark) the corresponding singlets look like:¹

$$\lambda^{u} \left[\overline{q}_{L} \tilde{\phi} u_{R} \right]_{1} \qquad \lambda^{d} \left[\overline{q}_{L} \phi d_{R} \right]_{1} \qquad \lambda^{e} \left[\overline{\ell}_{L} \phi e_{R} \right]_{1}. \tag{10}$$

¹Careful, since $\tilde{\phi} = \epsilon \phi^*$ to account for the u-type quarks.

The Higgs part is made of a scalar kinetic term, the quartic potential and the Yukawa couplings:

$$\mathcal{L}_{\text{Higgs}} \supset -\left(D^{\mu}\phi\right)^{\dagger} \left(D_{\mu}\phi\right) + \mu\phi^{\dagger}\phi - \lambda\left(\phi^{\dagger}\phi\right)^{2} - \lambda^{\psi} \left[\overline{\psi}\phi\psi\right]_{1}. \tag{9}$$

For our three types of massive fermions (electron, up-quark, down-quark) the corresponding singlets look like: 1

$$\lambda^{u} \left[\overline{q}_{L} \tilde{\phi} u_{R} \right]_{1} \qquad \lambda^{d} \left[\overline{q}_{L} \phi d_{R} \right]_{1} \qquad \lambda^{e} \left[\overline{\ell}_{L} \phi e_{R} \right]_{1}. \tag{10}$$

Furthermore, the SM consists of three generations of fermions thus promoting the λ^{ψ} to complex 3x3 matrices λ^{e}_{mn} , λ^{d}_{mn} and λ^{u}_{mn} . The λ^{f}_{mn} can be diagonalised via bi-unitary transformations:

$$V_f^{\dagger} \lambda^f U_f \propto \operatorname{diag}\left(m_f^{(1)}, m_f^{(2)}, m_f^{(3)}\right). \tag{11}$$

¹Careful, since $\tilde{\phi} = \epsilon \phi^*$ to account for the u-type quarks.

Standard Model — Parameter count

Count parameters and gauge redundancies:

+

- ullet 3 couplings g, g' and $g_{
 m s}$, one vacuum angle $heta_{
 m QCD}$ (4 parameters)
- Higgs parameters v, λ (2 parameters)
- 3 (complex) mass matrices λ^f (3x18 parameters)

Standard Model — Parameter count

Count parameters and gauge redundancies:

```
\exists
```

- 3 couplings g, g' and $g_{
 m s}$, one vacuum angle $heta_{
 m QCD}$ (4 parameters)
- Higgs parameters v, λ (2 parameters)
- 3 (complex) mass matrices λ^f (3x18 parameters)
- -
- Quark flavour symmetry $\mathrm{U}(3)_{q_L} \times \mathrm{U}(3)_{u_R} \times \mathrm{U}(3)_{d_R}/\mathrm{U}(1)_B$ (3x9-1 parameters)
- Lepton flavour symmetry $U(3)_{\ell_L} \times U(3)_{e_R}/U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$ (2x9-3 parameters)

Standard Model — Parameter count

Count parameters and gauge redundancies:

+

- ullet 3 couplings g, g' and $g_{
 m s}$, one vacuum angle $heta_{
 m QCD}$ (4 parameters)
- Higgs parameters v, λ (2 parameters)
- 3 (complex) mass matrices λ^f (3x18 parameters)

-

- Quark flavour symmetry $\mathrm{U}(3)_{q_L} imes \mathrm{U}(3)_{u_R} imes \mathrm{U}(3)_{d_R}/\mathrm{U}(1)_B$ (3x9-1 parameters)
- Lepton flavour symmetry $U(3)_{\ell_L} \times U(3)_{e_R}/U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$ (2x9-3 parameters)

The Standard Model of Particle Physics has 19 free parameters with \boldsymbol{v} being the only one carrying a physical dimension.²

²15 real parameters, 3 mixing angles, 1 CP-violating phase.

- Introduction
- 2 The Standard Model
 Basics
 The Lagrangian
 Parameter Count

3 The MSSM

The Lagrangian — SUSY conserving The Lagrangian — SUSY breaking Mixing Caveats
Parameter Count

MSSM — The fields

First of all we promote all our previous fields to real (chiral) superfields resulting in our renewed table:

super field	bosonic field	fermionic field	representation
\hat{V}_8	g	$ ilde{g}$	$(8,1)_0$
\hat{V}	W^0 , W^\pm	$ ilde{W}^0$, $ ilde{W}^\pm$	$(1,3)_0$
\hat{V}'	B	$ ilde{B}$	$({f 1},{f 1})_0$
\hat{L}	$(ilde{ u}_e, ilde{e})$	(ν_L,e_L)	$({f 1},{f 2})_{-rac{1}{2}}$
\hat{E}^c	$ ilde{e}_R^c$	e_R^c	$\left(1,1 ight)_1$
\hat{Q}	$(ilde{u}_L, ilde{d}_L)$	(u_L,d_L)	$({f 3},{f 1})_{rac{1}{6}}$
\hat{U}^c	$ ilde{u}_R^c$	u_R^c	$({f 3},{f 1})_{-rac{2}{3}}$
\hat{D}^c	$ ilde{d}_R^c$	d_R^c	$({f 3},{f 1})_{rac{1}{3}}$
\hat{H}_u	$\left(H_u^+,H_u^0\right)$	$\left(ilde{H}_{u}^{+}, ilde{H}_{u}^{0} ight)$	$({f 1},{f 2})_{rac{1}{2}}$
\hat{H}_d	(H_d^0,H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	$({f 1},{f 2})_{-rac{1}{2}}^{-rac{1}{2}}$

MSSM — SUSY terms

For the gauge part the ususal field strength super fields $W_{i,\alpha}$ are constructed and included in the Lagrangian:

$$\mathcal{L}_{\text{gauge}}^{\text{MSSM}} \supset \frac{1}{2g_i^2} \operatorname{tr} \left[\int d^2 \theta \left(\mathcal{W}_i \right)^{\alpha} \left(\mathcal{W}_i \right)_{\alpha} + \text{h.c.} \right]. \tag{12}$$

The kinetic terms for the fields read:

$$\mathcal{L}_K^{\text{MSSM}} \supset \int d^2\theta d^2\bar{\theta} \left[\hat{\Phi}_i^{\dagger} e^{2V_i} \hat{\Phi}_i \right]_1 \tag{13}$$

$$V_i = \hat{V}_8^a \mathcal{R}_i(T_a) + \hat{V}^k \mathcal{R}_i(T_k) + Y_i \hat{V}'. \tag{14}$$

The superpotential term is simply:

$$\mathcal{L}_W^{\text{MSSM}} = \int d^2 \theta W + \text{h.c.}$$
 (15)

What terms are contained in W?

The superpotential term is simply:

$$\mathcal{L}_W^{\text{MSSM}} = \int d^2 \theta W + \text{h.c.}$$
 (15)

In general, the superpotential contains a great variety of different terms, under them the Yukawa couplings:

$$W = \lambda_d [H_d Q U]_1 + \lambda_d [H_d L E]_1 - \lambda_u [H_u Q U]_1 + \mu [H_u H_d]_1 + a [\hat{L} \hat{H}_u]_1 + b [\hat{Q} \hat{L} \hat{D}]_1 + c [\hat{U} \hat{U} \hat{D}]_1 + d [\hat{L} \hat{L} \hat{E}]_1.$$
(16)

The superpotential term is simply:

$$\mathcal{L}_W^{\text{MSSM}} = \int d^2 \theta W + \text{h.c.}$$
 (15)

In general, the superpotential contains a great variety of different terms, under them the Yukawa couplings:

$$W = \lambda_d [H_d Q U]_1 + \lambda_d [H_d L E]_1 - \lambda_u [H_u Q U]_1 + \mu [H_u H_d]_1 + a [\hat{L} \hat{H}_u]_1 + b [\hat{Q} \hat{L} \hat{D}]_1 + c [\hat{U} \hat{U} \hat{D}]_1 + d [\hat{L} \hat{L} \hat{E}]_1.$$
(16)

The terms in the last line introduce B-number violation via proton decay as well as lepton number violation, but by imposing R-parity

$$R = (-1)^{3(B-L)+2s}, (17)$$

we can get rid of them. Beware! This is not obligatory!

R-conservation implies the existance of a lightest supersymmetric particle (LSP) thus providing us with a dark matter candidate.

The superpotential term is simply:

$$\mathcal{L}_W^{\text{MSSM}} = \int d^2 \theta W + \text{h.c.}$$
 (15)

In general, the superpotential contains a great variety of different terms, under them the Yukawa couplings:

$$W = \lambda_d [H_d Q U]_1 + \lambda_d [H_d L E]_1 - \lambda_u [H_u Q U]_1 + \mu [H_u H_d]_1 + a [\hat{L} \hat{H}_u]_1 + b [\hat{Q} \hat{L} \hat{D}]_1 + c [\hat{U} \hat{U} \hat{D}]_1 + d [\hat{L} \hat{L} \hat{E}]_1.$$
(16)

We could use matter parity

$$P_{\rm m} = (-1)^{3(B-L)},\tag{17}$$

instead and see directly how the lower line gets thrown out.

MSSM — Soft SUSY breaking terms

Introduce explicitly SUSY breaking terms to generate masses and additional interactions

$$-\mathcal{L}_{\text{soft}}^{\text{MSSM}} \supset \frac{1}{2} M_{i} \tilde{\lambda}_{i} \tilde{\lambda}_{i} + M_{\tilde{F}}^{2} \tilde{f}^{\dagger} \tilde{f}$$

$$+ m_{1}^{2} H_{d}^{\dagger} H_{d} + m_{2}^{2} H_{u}^{\dagger} H_{u} + m_{12}^{2} (H_{u} H_{d} + \text{h.c.})$$

$$+ T_{U} H_{u} Q U + T_{D} H_{d} Q D + T_{E} H_{d} L E + \text{h.c.}$$
(18)

Often, a parametrisation $m_{12}^2 = \mu B$ (and $T_F = \lambda_f A_F$) is chosen. Therefore, the corresponding terms are called A and B-terms.

MSSM — Note on the Higgs sector

Repeat the SM steps: In the MSSM the quartic coupling is generated by the kinetic and soft SUSY breaking terms leading to an effective Higgs potential.

$$V_{\text{Higgs}} = (m_1^2 + |\mu|^2) H_d^{\dagger} H_d + (m_2^2 + |\mu|^2) H_u^{\dagger} H_u + m_{12}^2 (H_u \cdot H_d + \text{h.c.})$$

$$+ \frac{g^2 + {g'}^2}{8} (H_d^{\dagger} H_d - H_u^{\dagger} H_u) + \frac{1}{2} g^2 |H_d^{\dagger} H_u|^2,$$
(19)

MSSM — Note on the Higgs sector

Repeat the SM steps: In the MSSM the quartic coupling is generated by the kinetic and soft SUSY breaking terms leading to an effective Higgs potential.

$$V_{\text{Higgs}} = \left(m_1^2 + |\mu|^2\right) H_d^{\dagger} H_d + \left(m_2^2 + |\mu|^2\right) H_u^{\dagger} H_u + m_{12}^2 \left(H_u \cdot H_d + \text{h.c.}\right) + \frac{g^2 + {g'}^2}{8} \left(H_d^{\dagger} H_d - H_u^{\dagger} H_u\right) + \frac{1}{2} g^2 \left|H_d^{\dagger} H_u\right|^2,$$
(19)

The two doublets acquire separate VEVs:

$$\langle H_f^0 \rangle = v_f, \tag{20}$$

related to the previous v via:

$$\sqrt{v_u^2 + v_d^2} = v, (21)$$

by convention the angle β is defined as

$$\tan \beta = \frac{v_u}{v_d}.\tag{22}$$

The SM particle spectrum looks like:

The SM particle spectrum looks like:

The SM particle spectrum looks like:

The SM particle spectrum looks like:

The MSSM particle spectrum looks like:

The SM particle spectrum looks like:

The MSSM particle spectrum³ looks like:

³Worst case scenario.

Summary of mixed states:

- The charged gauginos $(\tilde{W}^{\pm}, \tilde{H}_u^+, \tilde{H}_d^-)$ form the charginos $\tilde{\chi}_i^{\pm}$.
- The neutral gauginos $(\tilde{B},\,\tilde{W}^0,\,\tilde{H}^0_u,\,\tilde{H}^0_d)$ form the neutralinos $\tilde{\chi}^0_i.$
- The squarks $(\tilde{q}_{i,L}, \tilde{q}_{i,R})$ form mass eigenstates labeled \tilde{q}_i .
- The sleptons $(\tilde{e}_{i,L}, \tilde{e}_{i,R})$ form eigenstates $\tilde{\ell}_i$.
- The Higgs bosons $(H_d^-, H_d^0, H_u^0, H_u^+)$ form: a charged pair H^\pm , two CP-even neutral scalars h^0, H^0 and a CP-odd A^0 .

Only for certain ranges of the parameters the particle spectrum will resemble a 'double-SM'.

+

- 3 couplings g_i , one vacuum angle $\theta_{\rm QCD}$ (4 parameters)
- 3 (complex) gaugino masses M_i (6 parameters)
- 2 Higgs mass parameters v, β (2 parameters)
- 2 (complex) Higgs/ino mass parameters μ , B (4 parameters)
- ullet 5 hermitian scalar mass matrices $M_{ ilde{F}}^2$ (5x9 parameters)
- 3 mass matrices λ^f (3x18 parameters)
- 3 trilinear couplings T_F (3x18 parameters)

```
+
```

- 3 couplings g_i , one vacuum angle $\theta_{\rm QCD}$ (4 parameters)
- 3 (complex) gaugino masses M_i (6 parameters)
- 2 Higgs mass parameters v, β (2 parameters)
- 2 (complex) Higgs/ino mass parameters μ , B (4 parameters)
- 5 hermitian scalar mass matrices $M_{ ilde{E}}^2$ (5x9 parameters)
- 3 mass matrices λ^f (3x18 parameters)
- 3 trilinear couplings T_F (3x18 parameters)
- Flavour symmetry $U(3)^5/U(1)^2$ (5x9-2 parameters)

```
• 3 couplings q_i, one vacuum angle \theta_{QCD} (4 parameters)
• 3 (complex) gaugino masses M_i (6 parameters)
• 2 Higgs mass parameters v, \beta (2 parameters)
• 2 (complex) Higgs/ino mass parameters \mu, B (4 parameters)
• 5 hermitian scalar mass matrices M_{\tilde{E}}^2 (5x9 parameters)
• 3 mass matrices \lambda^f (3x18 parameters)
• 3 trilinear couplings T_F (3x18 parameters)
• Flavour symmetry U(3)^5/U(1)^2 (5x9-2 parameters)
• R- and Peccei-Quinn symmetry U(1)_R \times U(1)_{PQ} (2 parameters)
```

+

- 3 couplings g_i , one vacuum angle $\theta_{\rm QCD}$ (4 parameters)
- 3 (complex) gaugino masses M_i (6 parameters)
- 2 Higgs mass parameters v, β (2 parameters)
- 2 (complex) Higgs/ino mass parameters μ , B (4 parameters)
- ullet 5 hermitian scalar mass matrices $M_{ ilde{E}}^2$ (5x9 parameters)
- 3 mass matrices λ^f (3x18 parameters)
- 3 trilinear couplings T_F (3x18 parameters)

• Flavour symmetry $U(3)^5/U(1)^2$ (5x9-2 parameters)

+

• R- and Peccei-Quinn symmetry $U(1)_R \times U(1)_{PQ}$ (2 parameters)

The full Minimal Supersymmetric Standard Model has 124 free parameters³ (MSSM-124).

 $^{^3}$ Consisting of 3 couplings, 37 real masses, 39 mixing angles and 45 CP-violating phases.

References

- DJH Chung, LL Everett, GL Kane, SF King, J Lykken, and Lian-Tao Wang. The soft supersymmetry-breaking Lagrangian: Theory and applications. *Physics Reports* 407.1-3 (2005), pp. 1–203.
 - Patrick Draper and Heidi Rzehak.

 A review of Higgs mass calculations in supersymmetric models.

 Physics Reports 619 (2016), pp. 1–24.
- Howard E Haber.
 The status of the minimal supersymmetric standard model and beyond
- Nuclear Physics B-Proceedings Supplements 62.1-3 (1998), pp. 469–484.

 Arthur Hebecker
- Lectures on Beyond the Standard Model and the String Theory Landscape.
 Heidelberg University, 2020.
 - Luis E. Ibanez and Angel M. Uranga.

 String Theory and Particle Physics: An Introduction to String Phenomenology.

 Cambridge University Press, 2012.
 - Joseph D. Lykken.
 - Introduction to Supersymmetry.
 Theoretical Advanced Study Institute in Elementary Particle Physics (TASI 96): Fields, Strings, and Duality. 1996. pp. 88–154.
- 7] Stephen P. Martin. A Supersymmetry Primer.
- Advanced Series on Directions in High Energy Physics (1998), pp. 1–98.
- Hans Peter Nilles
- Supersymmetry, Supergravity and Particle Physics. Phys. Rept. 110 (1984), pp. 1–162.
- Michael E. Peskin.
- Supersymmetry in Elementary Particle Physics.
 - Theoretical Advanced Study Institute in Elementary Particle Physics: Exploring New Frontiers Using Colliders and Neutrinos. 2008. pp. 609–704.
- [10] Adrian Signer.
 ABC of SUSY.
 - Journal of Physics G: Nuclear and Particle Physics 36.7 (2009), p. 073002.
- [11] Julius Wess and Jonathan A. Bagger.

 Supersymmetry and Supergravity.

 Princeton University Press, 1992