The Minimally Supersymmetric Standard Model

Mathieu Kaltschmidt & Jonah Cedric Strauß

ITP Heidelberg

Supersymmetry Seminar
supervised by
Prof. Jörg Jäckel

Heidelberg, February 15th 2021

Outline

Outline

Standard Model — The Basics

The gauge group of the Standard Model (SM) is

$$\mathcal{G}_{\mathrm{SM}} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_T \times \mathrm{U}(1)_Y. \tag{1}$$

Standard Model — The Basics

The gauge group of the Standard Model (SM) is

$$\mathcal{G}_{\mathrm{SM}} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_T \times \mathrm{U}(1)_Y. \tag{1}$$

The SM fields transform in representations of this group:

$$s = 0: \quad \underset{\ni\,\phi}{(\mathbf{1},\mathbf{2})_{\frac{1}{2}}} \tag{2}$$

$$s = \frac{1}{2}: \quad \underset{\ni\,q_L}{(\mathbf{3},\mathbf{2})_{\frac{1}{6}}} \oplus \underset{\ni\,(d_R)^c}{(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}} \oplus \underset{\ni\,(u_R)^c}{(\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}} \oplus \underset{\ni\,\ell_L}{(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}} \oplus \underset{\ni\,(e_R)^c}{(\mathbf{1},\mathbf{1})_{1}} \tag{3}$$

$$s = 1: \quad \underset{\ni\,A_\mu^a}{(\mathbf{8},\mathbf{1})_0} \oplus \underset{\ni\,W_\mu^k}{(\mathbf{1},\mathbf{3})_0} \oplus \underset{\ni\,B_\mu}{(\mathbf{1},\mathbf{1})_0}. \tag{4}$$

Standard Model — The Basics

Beware! We only use lefthanded fields $\psi_L = \begin{pmatrix} \psi_\alpha \\ 0 \end{pmatrix}$, righthanded fields are included via charge conjugation $(\psi_R)^c = \begin{pmatrix} 0 \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}^c = \begin{pmatrix} \psi_\alpha \\ 0 \end{pmatrix}$.

The $SU(2)$-doublets are written as:

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \qquad q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad \ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}.$$

Standard Model — The Basics

The gauge group of the Standard Model (SM) is

$$\mathcal{G}_{\mathrm{SM}} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_T \times \mathrm{U}(1)_Y. \tag{1}$$

The SM fields transform in representations of this group:

$$s = 0 : \quad \underset{\ni \phi}{(\mathbf{1}, \mathbf{2})_{\frac{1}{2}}} \tag{2}$$

$$s = \frac{1}{2} : \quad \underset{\ni q_L}{(\mathbf{3}, \mathbf{2})_{\frac{1}{6}}} \oplus \underset{\ni (d_R)^c}{(\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}} \oplus \underset{\ni (u_R)^c}{(\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}}} \oplus \underset{\ni \ell_L}{(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}} \oplus \underset{\ni (e_R)^c}{(\mathbf{1}, \mathbf{1})_1} \tag{3}$$

$$s = 1 : \quad \underset{\ni A_\mu^a}{(\mathbf{8}, \mathbf{1})_0} \oplus \underset{\ni W_\mu^k}{(\mathbf{1}, \mathbf{3})_0} \oplus \underset{\ni B_\mu}{(\mathbf{1}, \mathbf{1})_0}. \tag{4}$$

Standard Model — The Lagrangian

The SM Lagrangian contains a variety of terms which roughly fall into three categories:

<div align="center">gauge terms   kinetic terms   Higgs sector</div>

The gauge part is straightforward albeit there being additional gauge configurations:

$$\mathcal{L}_{\text{gauge}} \supset \frac{1}{2g_i^2} \operatorname{tr}\left[F_{\mu\nu}^{(i)} F^{(i)\mu\nu} \right] \tag{5}$$

$$\supset \frac{\theta_{\text{QCD}}}{16\pi^2 g_{\text{s}}^2} \epsilon^{\mu\nu\rho\sigma} \operatorname{tr}\left[F_{\mu\nu}^{(3)} F_{\rho\sigma}^{(3)} \right]. \tag{6}$$

Kinetic terms for the fermions are constructed with the covariant derivative

$$\mathcal{L}_{\text{kin}} \supset \overline{\psi}_i \mathrm{i} \slashed{D} \psi_i \tag{7}$$

$$D_\mu = \partial_\mu - \mathrm{i} q A_\mu^k \mathcal{R}(T_k) \tag{8}$$

Standard Model — Higgs sector

The Higgs part is made of a scalar kinetic term, the quartic potential and the Yukawa couplings:

$$\mathcal{L}_{\text{Higgs}} \supset -\left(D^\mu \phi\right)^\dagger \left(D_\mu \phi\right) + \mu \phi^\dagger \phi - \lambda \left(\phi^\dagger \phi\right)^2 - \lambda^\psi \left[\overline{\psi} \phi \psi\right]_{\mathbf{1}}. \tag{9}$$

Standard Model — Higgs sector

The Higgs part is made of a scalar kinetic term, the quartic potential and the Yukawa couplings:

$$\mathcal{L}_{\text{Higgs}} \supset -\left(D^\mu \phi\right)^\dagger \left(D_\mu \phi\right) + \mu \phi^\dagger \phi - \lambda \left(\phi^\dagger \phi\right)^2 - \lambda^\psi \left[\overline{\psi} \phi \psi\right]_{\mathbf{1}}. \tag{9}$$

The fermions gain masses $m_\psi = v \lambda^\psi$ when the Higgs field acquires its vacuum expectation value (VEV):

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad (\text{shorthand:} \ \langle \phi_0 \rangle = v).$$

Standard Model — Higgs sector

The Higgs part is made of a scalar kinetic term, the quartic potential and the Yukawa couplings:

$$\mathcal{L}_{\text{Higgs}} \supset - \left(D^\mu \phi\right)^\dagger \left(D_\mu \phi\right) + \mu \phi^\dagger \phi - \lambda \left(\phi^\dagger \phi\right)^2 - \lambda^\psi \left[\overline{\psi} \phi \psi\right]_{\mathbf{1}}. \tag{9}$$

For our three types of massive fermions (electron, up-quark, down-quark) the corresponding singlets look like:[1]

$$\lambda^u \left[\overline{q}_L \tilde{\phi} u_R\right]_{\mathbf{1}} \qquad \lambda^d \left[\overline{q}_L \phi d_R\right]_{\mathbf{1}} \qquad \lambda^e \left[\overline{\ell}_L \phi e_R\right]_{\mathbf{1}}. \tag{10}$$

---

[1] Careful, since $\tilde{\phi} = \epsilon \phi^*$ to account for the u-type quarks.

Standard Model — Higgs sector

The Higgs part is made of a scalar kinetic term, the quartic potential and the Yukawa couplings:

$$\mathcal{L}_{\text{Higgs}} \supset - \left(D^\mu \phi\right)^\dagger \left(D_\mu \phi\right) + \mu \phi^\dagger \phi - \lambda \left(\phi^\dagger \phi\right)^2 - \lambda^\psi \left[\overline{\psi} \phi \psi\right]_{\mathbf{1}}. \tag{9}$$

For our three types of massive fermions (electron, up-quark, down-quark) the corresponding singlets look like:[1]

$$\lambda^u \left[\overline{q}_L \tilde{\phi} u_R\right]_{\mathbf{1}} \qquad \lambda^d \left[\overline{q}_L \phi d_R\right]_{\mathbf{1}} \qquad \lambda^e \left[\overline{\ell}_L \phi e_R\right]_{\mathbf{1}}. \tag{10}$$

Furthermore, the SM consists of three generations of fermions thus promoting the λ^ψ to complex 3x3 matrices λ^e_{mn}, λ^d_{mn} and λ^u_{mn}. The λ^f_{mn} can be diagonalised via bi-unitary transformations:

$$V_f^\dagger \lambda^f U_f \propto \text{diag}\left(m_f^{(1)}, m_f^{(2)}, m_f^{(3)}\right). \tag{11}$$

---

[1]Careful, since $\tilde{\phi} = \epsilon \phi^*$ to account for the u-type quarks.

Standard Model — Parameter count

Count parameters and gauge redundancies:

$+$
- 3 couplings g, g' and g_{s}, one vacuum angle θ_{QCD} (4 parameters)
- Higgs parameters v, λ (2 parameters)
- 3 (complex) mass matrices λ^f (3x18 parameters)

Standard Model — Parameter count

Count parameters and gauge redundancies:

$+$

- 3 couplings g, g' and g_{s}, one vacuum angle θ_{QCD} (4 parameters)
- Higgs parameters v, λ (2 parameters)
- 3 (complex) mass matrices λ^f (3x18 parameters)

$-$

- Quark flavour symmetry $\mathrm{U}(3)_{q_L} \times \mathrm{U}(3)_{u_R} \times \mathrm{U}(3)_{d_R}/\mathrm{U}(1)_B$ (3x9-1 parameters)
- Lepton flavour symmetry $\mathrm{U}(3)_{\ell_L} \times \mathrm{U}(3)_{e_R}/\mathrm{U}(1)_{L_e} \times \mathrm{U}(1)_{L_\mu} \times \mathrm{U}(1)_{L_\tau}$ (2x9-3 parameters)

Standard Model — Parameter count

Count parameters and gauge redundancies:

$+$

- 3 couplings g, g' and g_s, one vacuum angle θ_{QCD} (4 parameters)
- Higgs parameters v, λ (2 parameters)
- 3 (complex) mass matrices λ^f (3x18 parameters)

$-$

- Quark flavour symmetry $\mathrm{U}(3)_{q_L} \times \mathrm{U}(3)_{u_R} \times \mathrm{U}(3)_{d_R}/\mathrm{U}(1)_B$ (3x9-1 parameters)
- Lepton flavour symmetry $\mathrm{U}(3)_{\ell_L} \times \mathrm{U}(3)_{e_R}/\mathrm{U}(1)_{L_e} \times \mathrm{U}(1)_{L_\mu} \times \mathrm{U}(1)_{L_\tau}$ (2x9-3 parameters)

The Standard Model of Particle Physics has 19 free parameters with v being the only one carrying a physical dimension.[2]

---

[2]15 real parameters, 3 mixing angles, 1 CP-violating phase.

Outline

MSSM — The fields

First of all we promote all our previous fields to real (chiral) superfields resulting in our renewed table:

super field	bosonic field	fermionic field	representation
$\hat{V}_8$	g	$\tilde{g}$	$(\mathbf{8}, \mathbf{1})_0$
$\hat{V}$	W^0, $W^\pm$	$\tilde{W}^0$, $\tilde{W}^\pm$	$(\mathbf{1}, \mathbf{3})_0$
$\hat{V}'$	B	$\tilde{B}$	$(\mathbf{1}, \mathbf{1})_0$
$\hat{L}$	$(\tilde{\nu}_e, \tilde{e})$	(ν_L, e_L)	$(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$
$\hat{E}^c$	$\tilde{e}_R^c$	e_R^c	$(\mathbf{1}, \mathbf{1})_1$
$\hat{Q}$	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$(\mathbf{3}, \mathbf{1})_{\frac{1}{6}}$
$\hat{U}^c$	$\tilde{u}_R^c$	u_R^c	$(\mathbf{3}, \mathbf{1})_{-\frac{2}{3}}$
$\hat{D}^c$	$\tilde{d}_R^c$	d_R^c	$(\mathbf{3}, \mathbf{1})_{\frac{1}{3}}$
$\hat{H}_u$	(H_u^+, H_u^0)	$(\tilde{H}_u^+, \tilde{H}_u^0)$	$(\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$
$\hat{H}_d$	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	$(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$

MSSM — SUSY terms

For the gauge part the ususal field strength super fields $\mathcal{W}_{i,\alpha}$ are constructed and included in the Lagrangian:

$$\mathcal{L}_{\text{gauge}}^{\text{MSSM}} \supset \frac{1}{2g_i^2} \operatorname{tr} \left[\int \mathrm{d}^2\theta \left(\mathcal{W}_i \right)^{\alpha} \left(\mathcal{W}_i \right)_{\alpha} + \text{h.c.} \right]. \tag{12}$$

The kinetic terms for the fields read:

$$\mathcal{L}_K^{\text{MSSM}} \supset \int \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} \left[\hat{\Phi}_i^{\dagger} e^{2V_i} \hat{\Phi}_i \right]_{\mathbf{1}} \tag{13}$$

$$V_i = \hat{V}_8^a \mathcal{R}_i(T_a) + \hat{V}^k \mathcal{R}_i(T_k) + Y_i \hat{V}'. \tag{14}$$

MSSM — Superpotential

The superpotential term is simply:

$$\mathcal{L}_W^{\mathrm{MSSM}} = \int \mathrm{d}^2\theta\, W + \mathrm{h.c.} \tag{15}$$

What terms are contained in W?

MSSM — Superpotential

The superpotential term is simply:

$$\mathcal{L}_W^{\text{MSSM}} = \int \mathrm{d}^2\theta W + \text{h.c.} \tag{15}$$

In general, the superpotential contains a great variety of different terms, under them the Yukawa couplings:

$$\begin{aligned}
W =& \lambda_d \left[H_d Q U\right]_{\mathbf{1}} + \lambda_d \left[H_d L E\right]_{\mathbf{1}} - \lambda_u \left[H_u Q U\right]_{\mathbf{1}} + \mu \left[H_u H_d\right]_{\mathbf{1}} \\
&+ a \left[\hat{L}\hat{H}_u\right]_{\mathbf{1}} + b \left[\hat{Q}\hat{L}\hat{D}\right]_{\mathbf{1}} + c \left[\hat{U}\hat{U}\hat{D}\right]_{\mathbf{1}} + d \left[\hat{L}\hat{L}\hat{E}\right]_{\mathbf{1}}.
\end{aligned} \tag{16}$$

MSSM — Superpotential

The superpotential term is simply:

$$\mathcal{L}_W^{\mathrm{MSSM}} = \int \mathrm{d}^2\theta W + \mathrm{h.c.} \tag{15}$$

In general, the superpotential contains a great variety of different terms, under them the Yukawa couplings:

$$\begin{aligned}
W = &\lambda_d \left[H_d Q U\right]_{\mathbf{1}} + \lambda_d \left[H_d L E\right]_{\mathbf{1}} - \lambda_u \left[H_u Q U\right]_{\mathbf{1}} + \mu \left[H_u H_d\right]_{\mathbf{1}} \\
&+ a \left[\hat{L}\hat{H}_u\right]_{\mathbf{1}} + b \left[\hat{Q}\hat{L}\hat{D}\right]_{\mathbf{1}} + c \left[\hat{U}\hat{U}\hat{D}\right]_{\mathbf{1}} + d \left[\hat{L}\hat{L}\hat{E}\right]_{\mathbf{1}}.
\end{aligned} \tag{16}$$

The terms in the last line introduce B-number violation via proton decay as well as lepton number violation, but by imposing R-parity

$$R = (-1)^{3(B-L)+2s}, \tag{17}$$

we can get rid of them. Beware! This is not obligatory!
R-conservation implies the existance of a lightest supersymmetric particle (LSP) thus providing us with a dark matter candidate.

MSSM — Superpotential

The superpotential term is simply:

$$\mathcal{L}_W^{\mathrm{MSSM}} = \int \mathrm{d}^2\theta W + \mathrm{h.c.} \tag{15}$$

In general, the superpotential contains a great variety of different terms, under them the Yukawa couplings:

$$\begin{aligned}
W =& \lambda_d \left[H_d QU\right]_{\mathbf{1}} + \lambda_d \left[H_d LE\right]_{\mathbf{1}} - \lambda_u \left[H_u QU\right]_{\mathbf{1}} + \mu \left[H_u H_d\right]_{\mathbf{1}} \\
&+ a \left[\hat{L}\hat{H}_u\right]_{\mathbf{1}} + b \left[\hat{Q}\hat{L}\hat{D}\right]_{\mathbf{1}} + c \left[\hat{U}\hat{U}\hat{D}\right]_{\mathbf{1}} + d \left[\hat{L}\hat{L}\hat{E}\right]_{\mathbf{1}}.
\end{aligned} \tag{16}$$

We could use matter parity

$$P_{\mathrm{m}} = (-1)^{3(B-L)}, \tag{17}$$

instead and see directly how the lower line gets thrown out.

MSSM — Soft SUSY breaking terms

Introduce explicitly SUSY breaking terms to generate masses and additional interactions

$$
\begin{aligned}
-\mathcal{L}_{\text{soft}}^{\text{MSSM}} \supset \ & \frac{1}{2} M_i \tilde{\lambda}_i \tilde{\lambda}_i + M_{\tilde{F}}^2 \tilde{f}^\dagger \tilde{f} \\
& + m_1^2 H_d^\dagger H_d + m_2^2 H_u^\dagger H_u + m_{12}^2 \left(H_u H_d + \text{h.c.} \right) \\
& + T_U H_u Q U + T_D H_d Q D + T_E H_d L E + \text{h.c.}
\end{aligned}
\tag{18}
$$

Often, a parametrisation $m_{12}^2 = \mu B$ (and $T_F = \lambda_f A_F$) is chosen. Therefore, the corresponding terms are called A and B-terms.

MSSM — Note on the Higgs sector

Repeat the SM steps: In the MSSM the quartic coupling is generated by the kinetic and soft SUSY breaking terms leading to an effective Higgs potential.

$$V_{\text{Higgs}} = \left(m_1^2 + |\mu|^2\right) H_d^\dagger H_d + \left(m_2^2 + |\mu|^2\right) H_u^\dagger H_u + m_{12}^2 \left(H_u \cdot H_d + \text{h.c.}\right)$$
$$+ \frac{g^2 + g'^2}{8} \left(H_d^\dagger H_d - H_u^\dagger H_u\right) + \frac{1}{2} g^2 \left|H_d^\dagger H_u\right|^2, \tag{19}$$

MSSM — Note on the Higgs sector

Repeat the SM steps: In the MSSM the quartic coupling is generated by the kinetic and soft SUSY breaking terms leading to an effective Higgs potential.

$$V_{\text{Higgs}} = \left(m_1^2 + |\mu|^2\right) H_d^\dagger H_d + \left(m_2^2 + |\mu|^2\right) H_u^\dagger H_u + m_{12}^2 \left(H_u \cdot H_d + \text{h.c.}\right)$$
$$+ \frac{g^2 + g'^2}{8} \left(H_d^\dagger H_d - H_u^\dagger H_u\right) + \frac{1}{2} g^2 \left|H_d^\dagger H_u\right|^2, \tag{19}$$

The two doublets acquire separate VEVs:

$$\left\langle H_f^0 \right\rangle = v_f, \tag{20}$$

related to the previous v via:

$$\sqrt{v_u^2 + v_d^2} = v, \tag{21}$$

by convention the angle β is defined as

$$\tan \beta = \frac{v_u}{v_d}. \tag{22}$$

MSSM — Mixing in the particle spectrum

The SM particle spectrum looks like:

$$
\begin{array}{ccccc}
u & c & t & B & W^0 \\
d & s & b & g & W^\pm
\end{array}
$$

$$
\begin{array}{cccc}
e & \mu & \tau & h^0 \\
\nu_e & \nu_\mu & \nu_\tau &
\end{array}
$$

MSSM — Mixing in the particle spectrum

The SM particle spectrum looks like:

$$
\begin{array}{cccccc}
u & c & t & & B & W^0 \\
d & s & b & & g & W^\pm \\
\\
e & \mu & \tau & & H_d^- & H_d^0 \\
\nu_e & \nu_\mu & \nu_\tau & & H_u^0 & H_u^+
\end{array}
$$

MSSM — Mixing in the particle spectrum

The SM particle spectrum looks like:

$$
\begin{array}{ccccc}
u & c & t & \gamma & Z^0 \\
d & s & b & g & W^{\pm} \\
\\
e & \mu & \tau & h^0 & H^0 \\
\nu_e & \nu_\mu & \nu_\tau & A^0 & H^{\pm}
\end{array}
$$

MSSM — Mixing in the particle spectrum

The SM particle spectrum looks like:

$$
\begin{array}{cccccc}
u & c & t & \gamma & Z^0 \\
d & s & b & g & W^{\pm}
\end{array}
$$

$$
\begin{array}{cccccc}
e & \mu & \tau & h^0 & H^0 \\
\nu_e & \nu_\mu & \nu_\tau & A^0 & H^{\pm}
\end{array}
$$

The MSSM particle spectrum looks like:

$$
\begin{array}{cccccccc}
\tilde{u}_L & \tilde{u}_R & \tilde{c}_L & \tilde{c}_R & \tilde{t}_L & \tilde{t}_R & \tilde{B} & \tilde{W}^0 \\
\tilde{d}_L & \tilde{d}_R & \tilde{s}_L & \tilde{s}_R & \tilde{b}_L & \tilde{b}_R & \tilde{g} & \tilde{W}^{\pm}
\end{array}
$$

$$
\begin{array}{cccccccc}
\tilde{e}_L & \tilde{e}_R & \tilde{\mu}_L & \tilde{\mu}_R & \tilde{\tau}_L & \tilde{\tau}_R & \tilde{H}_d^- & \tilde{H}_d^0 \\
\tilde{\nu}_e & & \tilde{\nu}_\mu & & \tilde{\nu}_\tau & & \tilde{H}_u^0 & \tilde{H}_u^+
\end{array}
$$

MSSM — Mixing in the particle spectrum

The SM particle spectrum looks like:

$$u \quad c \quad t \qquad \gamma \quad Z^0$$
$$d \quad s \quad b \qquad g \quad W^\pm$$

$$e \quad \mu \quad \tau \qquad h^0 \quad H^0$$
$$\nu_e \quad \nu_\mu \quad \nu_\tau \qquad A^0 \quad H^\pm$$

The MSSM particle spectrum[3] looks like:

$$\tilde{u}_1 \quad \tilde{u}_2 \quad \tilde{u}_3 \quad \tilde{u}_4 \quad \tilde{u}_5 \quad \tilde{u}_6 \qquad \tilde{\chi}_1^\pm \quad \tilde{\chi}_2^\pm$$
$$\tilde{d}_1 \quad \tilde{d}_2 \quad \tilde{d}_3 \quad \tilde{d}_4 \quad \tilde{d}_5 \quad \tilde{d}_6 \qquad \tilde{g}$$

$$\tilde{\ell}_1 \quad \tilde{\ell}_2 \quad \tilde{\ell}_3 \quad \tilde{\ell}_4 \quad \tilde{\ell}_5 \quad \tilde{\ell}_6 \qquad \tilde{\chi}_1^0 \quad \tilde{\chi}_2^0$$
$$\tilde{\nu}_1 \qquad \tilde{\nu}_2 \qquad \tilde{\nu}_3 \qquad \tilde{\chi}_3^0 \quad \tilde{\chi}_4^0$$

[3]Worst case scenario.

MSSM — Mixing in the particle spectrum

Summary of mixed states:

- The charged gauginos ($\tilde{W}^{\pm}$, $\tilde{H}_u^+$, $\tilde{H}_d^-$) form the charginos $\tilde{\chi}_i^{\pm}$.
- The neutral gauginos ($\tilde{B}$, $\tilde{W}^0$, $\tilde{H}_u^0$, $\tilde{H}_d^0$) form the neutralinos $\tilde{\chi}_i^0$.
- The squarks ($\tilde{q}_{i,L}$, $\tilde{q}_{i,R}$) form mass eigenstates labeled $\tilde{q}_i$.
- The sleptons ($\tilde{e}_{i,L}$, $\tilde{e}_{i,R}$) form eigenstates $\tilde{\ell}_i$.
- The Higgs bosons (H_d^-, H_d^0, H_u^0, H_u^+) form: a charged pair $H^{\pm}$, two CP-even neutral scalars h^0, H^0 and a CP-odd A^0.

Only for certain ranges of the parameters the particle spectrum will resemble a 'double-SM'.

MSSM — Parameter count

$+$

- 3 couplings g_i, one vacuum angle θ_{QCD} (4 parameters)
- 3 (complex) gaugino masses M_i (6 parameters)
- 2 Higgs mass parameters v, β (2 parameters)
- 2 (complex) Higgs/ino mass parameters μ, B (4 parameters)
- 5 hermitian scalar mass matrices $M_{\tilde{F}}^2$ (5x9 parameters)
- 3 mass matrices λ^f (3x18 parameters)
- 3 trilinear couplings T_F (3x18 parameters)

MSSM — Parameter count

$+$

- 3 couplings g_i, one vacuum angle θ_{QCD} (4 parameters)
- 3 (complex) gaugino masses M_i (6 parameters)
- 2 Higgs mass parameters v, β (2 parameters)
- 2 (complex) Higgs/ino mass parameters μ, B (4 parameters)
- 5 hermitian scalar mass matrices $M_{\tilde{F}}^2$ (5x9 parameters)
- 3 mass matrices λ^f (3x18 parameters)
- 3 trilinear couplings T_F (3x18 parameters)

$-$

- Flavour symmetry $\mathrm{U}(3)^5/\mathrm{U}(1)^2$ (5x9-2 parameters)

MSSM — Parameter count

$+$

- 3 couplings g_i, one vacuum angle θ_{QCD} (4 parameters)
- 3 (complex) gaugino masses M_i (6 parameters)
- 2 Higgs mass parameters v, β (2 parameters)
- 2 (complex) Higgs/ino mass parameters μ, B (4 parameters)
- 5 hermitian scalar mass matrices $M_{\tilde{F}}^2$ (5x9 parameters)
- 3 mass matrices λ^f (3x18 parameters)
- 3 trilinear couplings T_F (3x18 parameters)

$-$

- Flavour symmetry $\mathrm{U}(3)^5/\mathrm{U}(1)^2$ (5x9-2 parameters)

$+$

- R- and Peccei-Quinn symmetry $\mathrm{U}(1)_R \times \mathrm{U}(1)_{\mathrm{PQ}}$ (2 parameters)

MSSM — Parameter count

$+$

- 3 couplings g_i, one vacuum angle θ_{QCD} (4 parameters)
- 3 (complex) gaugino masses M_i (6 parameters)
- 2 Higgs mass parameters v, β (2 parameters)
- 2 (complex) Higgs/ino mass parameters μ, B (4 parameters)
- 5 hermitian scalar mass matrices $M_{\tilde{F}}^2$ (5x9 parameters)
- 3 mass matrices λ^f (3x18 parameters)
- 3 trilinear couplings T_F (3x18 parameters)

$-$

- Flavour symmetry $\text{U}(3)^5/\text{U}(1)^2$ (5x9-2 parameters)

$+$

- R- and Peccei-Quinn symmetry $\text{U}(1)_R \times \text{U}(1)_{\text{PQ}}$ (2 parameters)

The full Minimal Supersymmetric Standard Model has 124 free parameters[3] (MSSM-124).

---

[3]Consisting of 3 couplings, 37 real masses, 39 mixing angles and 45 CP-violating phases.

References

[1] DJH Chung, LL Everett, GL Kane, SF King, J Lykken, and Lian-Tao Wang.
The soft supersymmetry-breaking Lagrangian: Theory and applications.
Physics Reports 407.1-3 (2005), pp. 1–203.

[2] Patrick Draper and Heidi Rzehak.
A review of Higgs mass calculations in supersymmetric models.
Physics Reports 619 (2016), pp. 1–24.

[3] Howard E Haber.
The status of the minimal supersymmetric standard model and beyond.
Nuclear Physics B-Proceedings Supplements 62.1-3 (1998), pp. 469–484.

[4] Arthur Hebecker.
Lectures on Beyond the Standard Model and the String Theory Landscape.
Heidelberg University, 2020.

[5] Luis E. Ibanez and Angel M. Uranga.
String Theory and Particle Physics: An Introduction to String Phenomenology.
Cambridge University Press, 2012.

[6] Joseph D. Lykken.
Introduction to Supersymmetry.
Theoretical Advanced Study Institute in Elementary Particle Physics (TASI 96): Fields, Strings, and Duality. 1996, pp. 85–153.

[7] Stephen P. Martin.
A Supersymmetry Primer.
Advanced Series on Directions in High Energy Physics (1998), pp. 1–98.

[8] Hans - Peter Nilles.
Supersymmetry, Supergravity and Particle Physics.
Phys. Rept. 110 (1984), pp. 1–162.

[9] Michael E. Peskin.
Supersymmetry in Elementary Particle Physics.
Theoretical Advanced Study Institute in Elementary Particle Physics: Exploring New Frontiers Using Colliders and Neutrinos. 2008, pp. 609–704.

[10] Adrian Signer.
ABC of SUSY.
Journal of Physics G: Nuclear and Particle Physics 36.7 (2009), p. 073002.

[11] Julius Wess and Jonathan A. Bagger.
Supersymmetry and Supergravity.
Princeton University Press, 1992.