The Minimal Supersymmetric Standard Model

Part I: Theory

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Outline

1. The Standard Model

2. The Minimal Supersymmetric Standard Model

Outline

Standard Model — The Basics

The gauge group of the Standard Model (SM) is

$$\mathcal{G}_{\mathrm{SM}} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_T \times \mathrm{U}(1)_Y\,.$$

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$$\begin{aligned}
s = 0 : &\quad \underset{\ni \phi}{(\mathbf{1},\mathbf{2})_{\frac{1}{2}}} \\
s = \frac{1}{2} : &\quad \underset{\ni q_L}{(\mathbf{3},\mathbf{2})_{\frac{1}{6}}} \oplus \underset{\ni (d_R)^c}{(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}} \oplus \underset{\ni (u_R)^c}{(\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}} \oplus \underset{\ni \ell_L}{(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}} \oplus \underset{\ni (e_R)^c}{(\mathbf{1},\mathbf{1})_{1}} \\
s = 1 : &\quad \underset{\ni A_\mu^a}{(\mathbf{8},\mathbf{1})_0} \oplus \underset{\ni W_\mu^k}{(\mathbf{1},\mathbf{3})_0} \oplus \underset{\ni B_\mu}{(\mathbf{1},\mathbf{1})_0} .
\end{aligned}$$

Standard Model — The Basics

Beware! We only use lefthanded fields $\psi_L = \begin{pmatrix} \psi_\alpha \\ 0 \end{pmatrix}$, righthanded fields are included via charge conjugation $(\psi_R)^c = \begin{pmatrix} 0 \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}^c = \begin{pmatrix} \psi_\alpha \\ 0 \end{pmatrix}$.

The $\mathrm{SU}(2)$-doublets are written as:

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \qquad q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad \ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}.$$

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\end{aligned}$$

Standard Model — The Basics (Well, more or less...)

Georgi-Glashow model: Using the breaking pattern $\mathrm{SU}(5) \to \mathcal{G}_{\mathrm{SM}}$ the previous fields fit nicely into representations of $\mathrm{SU}(5)$:

$$\begin{aligned}
\bar{\mathbf{5}} &= (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} \oplus (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}} \\
\mathbf{10} &= (\mathbf{3}, \mathbf{2})_{\frac{1}{6}} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}} \oplus (\mathbf{1}, \mathbf{1})_{1}
\end{aligned}$$

$$\mathbf{24} = (\mathbf{8}, \mathbf{1})_0 \oplus (\mathbf{1}, \mathbf{3})_0 \oplus (\mathbf{1}, \mathbf{1})_0 \oplus (\mathbf{3}, \mathbf{2})_{-\frac{5}{3}} \oplus (\bar{\mathbf{3}}, \mathbf{2})_{\frac{5}{3}} \,.$$

Standard Model — The Lagrangian

The SM Lagrangian contains a variety of terms which roughly fall into three categories:

gauge terms kinetic terms Higgs sector

The gauge part is straightforward albeit there being additional gauge configurations:

$$\mathcal{L}_{\text{gauge}} \supset \frac{1}{2g_i^2}\,\mathrm{tr}\left[F^{(i)}_{\mu\nu}F^{(i)\mu\nu}\right]$$
$$\supset \frac{\theta_{\mathrm{QCD}}}{16\pi^2 g_{\mathrm{s}}^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}\left[F^{(3)}_{\mu\nu}F^{(3)}_{\rho\sigma}\right].$$

Kinetic terms for the fermions are constructed with the covariant derivative

$$\mathcal{L}_{\text{kin}} \supset \bar{\psi}_i \mathrm{i}\not{D}\psi_i$$
$$D_\mu = \partial_\mu - \mathrm{i}qA^k_\mu\mathcal{R}(T_k).$$

Standard Model — Higgs sector

The Higgs part is made of a scalar kinetic term, the quartic potential and the Yukawa couplings:

$$\mathcal{L}_{\text{Higgs}} \supset -\left(D^{\mu}\phi\right)^{\dagger}\left(D_{\mu}\phi\right) + \mu\phi^{\dagger}\phi - \lambda\left(\phi^{\dagger}\phi\right)^{2} - \left(\lambda^{\psi}\left[\bar{\psi}\phi\psi\right]_{\mathbf{1}} + \text{h.c.}\right).$$

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The fermions gain masses $m_{\psi} = v\lambda^{\psi}$ when the Higgs field acquires its vacuum expectation value (VEV):

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \text{(shorthand: } \langle\phi_0\rangle = v\text{)},$$

while the Higgs mass becomes $m_h = 2\sqrt{\lambda}v$.

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For our three types of massive fermions (electron, up-quark, down-quark) the corresponding singlets look like:[1]

$$\lambda^{u}\left[\bar{q}_L\tilde{\phi}u_R\right]_{\mathbf{1}}, \qquad \lambda^{d}\left[\bar{q}_L\phi d_R\right]_{\mathbf{1}}, \qquad \lambda^{e}\left[\bar{\ell}_L\phi e_R\right]_{\mathbf{1}}.$$

[1] Careful, since $\tilde{\phi} = \epsilon\phi^{*}$ to account for the u-type quarks.

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Furthermore, the SM fermion fields consists of three generations, thus promoting the λ^ψ to complex 3x3 matrices λ^e_{mn}, λ^d_{mn} and λ^u_{mn}. These λ^f_{mn} can be diagonalised via bi-unitary transformations:

$$V_f^\dagger\lambda^f U_f \propto \text{diag}\left(m_f^{(1)}, m_f^{(2)}, m_f^{(3)}\right)/v.$$

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Standard Model — Parameter count

Count parameters and gauge redundancies:

$+$

- 3 couplings g, g' and g_s, one vacuum angle θ_QCD (4 parameters)
- Higgs parameters v, λ (2 parameters)
- 3 (complex) mass matrices λ^f (3x18 parameters)

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- Quark flavour symmetry $\mathrm{U}(3)_{q_L} \times \mathrm{U}(3)_{u_R} \times \mathrm{U}(3)_{d_R}/\mathrm{U}(1)_B$ (3x9-1 parameters)
- Lepton flavour symmetry $\mathrm{U}(3)_{\ell_L} \times \mathrm{U}(3)_{e_R}/\mathrm{U}(1)_{L_e} \times \mathrm{U}(1)_{L_\mu} \times \mathrm{U}(1)_{L_\tau}$ (2x9-3 parameters)

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The Standard Model of Particle Physics has 19 free parameters with v being the only one carrying a physical dimension.[2]

[2] 14 real parameters, 3 mixing angles, 2 CP-violating phase.

Outline

MSSM — The fields

First of all we promote all our previous fields to real (chiral) superfields resulting in our renewed table:

super field	bosonic field	fermionic field	representation
$\hat{V}_8$	g	$\tilde{g}$	$(\mathbf{8},\mathbf{1})_0$
$\hat{V}$	W^0, $W^\pm$	$\tilde{W}^0$, $\tilde{W}^\pm$	$(\mathbf{1},\mathbf{3})_0$
$\hat{V}'$	B	$\tilde{B}$	$(\mathbf{1},\mathbf{1})_0$
$\hat{L}$	$(\tilde{\nu}_L, \tilde{e}_L)$	(ν_L, e_L)	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$
$\hat{E}^c$	$\tilde{e}_R^c$	e_R^c	$(\mathbf{1},\mathbf{1})_1$
$\hat{Q}$	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$(\mathbf{3},\mathbf{1})_{\frac{1}{6}}$
$\hat{U}^c$	$\tilde{u}_R^c$	u_R^c	$(\mathbf{3},\mathbf{1})_{-\frac{2}{3}}$
$\hat{D}^c$	$\tilde{d}_R^c$	d_R^c	$(\mathbf{3},\mathbf{1})_{\frac{1}{3}}$
$\hat{H}_u$	(H_u^+, H_u^0)	$\left(\tilde{H}_u^+, \tilde{H}_u^0\right)$	$(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$
$\hat{H}_d$	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$

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Beware! We need a second Higgs doublet to cancel the gauge anomaly introduced by the Higgsinos!

MSSM — SUSY terms

For the gauge part the ususal field strength super fields

$$\mathcal{W}_{i,\alpha} = -\frac{1}{4}\bar{D}^2 e^{-\hat{V}} D_\alpha e^{\hat{V}},$$

are constructed and included in the Lagrangian:

$$\mathcal{L}^{\mathrm{MSSM}}_{\mathrm{gauge}} \supset \frac{1}{2g_i^2}\,\mathrm{tr}\left[\int \mathrm{d}^2\theta \, (\mathcal{W}_i)^\alpha \, (\mathcal{W}_i)_\alpha + \mathrm{h.c.}\right].$$

The kinetic terms for the fields read:

$$\begin{aligned}\mathcal{L}^{\mathrm{MSSM}}_{K} \supset & \int \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} \left[\hat{\Phi}_i^\dagger e^{2V_i} \hat{\Phi}_i\right]_{\mathbf{1}} \\ V_i = & \hat{V}_8^a \mathcal{R}_i(T_a) + \hat{V}^k \mathcal{R}_i(T_k) + Y_i \hat{V}'.\end{aligned}$$

MSSM — Superpotential

The superpotential term is simply:

$$\mathcal{L}_W^{\mathrm{MSSM}} = \int \mathrm{d}^2\theta W + \mathrm{h.c.}$$

What terms are contained in W?

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In general, the superpotential contains a great variety of different terms, under them the Yukawa couplings:

$$\begin{aligned} W = & \lambda_d \left[\hat{H}_d \hat{Q} \hat{D}\right]_{\mathbf{1}} + \lambda_e \left[\hat{H}_d \hat{L} \hat{E}\right]_{\mathbf{1}} - \lambda_u \left[\hat{H}_u \hat{Q} \hat{U}\right]_{\mathbf{1}} + \mu \left[\hat{H}_u \hat{H}_d\right]_{\mathbf{1}} \\ & + a \left[\hat{L} \hat{H}_u\right]_{\mathbf{1}} + b \left[\hat{Q} \hat{L} \hat{D}\right]_{\mathbf{1}} + c \left[\hat{U} \hat{U} \hat{D}\right]_{\mathbf{1}} + d \left[\hat{L} \hat{L} \hat{E}\right]_{\mathbf{1}} . \end{aligned}$$

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The terms in the last line introduce B-number violation via proton decay as well as lepton number violation, but by imposing R-parity

$$R = (-1)^{3(B-L)+2s},$$

we can get rid of them. Beware! This is not obligatory!
R-conservation implies the existance of a lightest supersymmetric particle (LSP) thus providing us with a dark matter candidate.

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We could use matter parity

$$P_{\mathrm{m}} = (-1)^{3(B-L)},$$

instead and see directly how the lower line gets thrown out.

MSSM — Soft SUSY breaking terms

Introduce explicitly SUSY breaking terms to generate masses and additional interactions

$$
\begin{aligned}
-\mathcal{L}_{\text{soft}}^{\text{MSSM}} \supset & \frac{1}{2} M_i \tilde{\bar{\lambda}}_i \tilde{\lambda}_i + M_{\tilde{F}}^2 \tilde{f}^\dagger \tilde{f} \\
& + m_1^2 H_d^\dagger H_d + m_2^2 H_u^\dagger H_u + m_{12}^2 \left(H_u \cdot H_d + \text{h.c.}\right) \\
& + T_U H_u \tilde{Q} \tilde{U} + T_D H_d \tilde{Q} \tilde{D} + T_E H_d \tilde{L} \tilde{E} + \text{h.c.}
\end{aligned}
$$

Often, a parametrisation $m_{12}^2 = \mu B$ (and $T_F = \lambda_f A_F$) is chosen. Therefore, the corresponding terms are called A and B-terms.

MSSM — Note on the Higgs sector

Repeat the SM steps:
In the MSSM the quartic coupling is generated by the D-terms of the Kähler potential, and the SUSY breaking terms, leading to an effective Higgs potential.

$$V_{\text{Higgs}} = \left(m_1^2 + |\mu|^2\right) H_d^\dagger H_d + \left(m_2^2 + |\mu|^2\right) H_u^\dagger H_u + m_{12}^2 \left(H_u \cdot H_d + \text{h.c.}\right)$$
$$+ \frac{g^2 + {g'}^2}{8} \left(H_d^\dagger H_d - H_u^\dagger H_u\right) + \frac{1}{2} g^2 \left|H_d^\dagger H_u\right|^2 ,$$

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The two doublets acquire separate VEVs

$$\left\langle H_f^0 \right\rangle = v_f ,$$

related to the previous v via

$$\sqrt{v_u^2 + v_d^2} = v,$$

by convention, the angle β is defined as

$$\tan\beta = \frac{v_u}{v_d}.$$

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At tree level this implies an upper bound on the mass of the lightest Higgs:

$$m_h^2 \leq m_Z^2 \cos^2 2\beta .$$

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MSSM — Mixing caveats

The SM particle spectrum looks like:

$$\begin{array}{ccccc} u & c & t & B & W^0 \\ d & s & b & g & W^\pm \\ \\ e & \mu & \tau & h^0 & \\ \nu_e & \nu_\mu & \nu_\tau & & \end{array}$$

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d & s & b & g & W^\pm \\
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e & \mu & \tau & H_d^- & H_d^0 \\
\nu_e & \nu_\mu & \nu_\tau & H_u^0 & H_u^+
\end{array}
$$

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\begin{array}{ccccc}
u & c & t & \gamma & Z^0 \\
d & s & b & g & W^\pm \\
 & & & & \\
e & \mu & \tau & h^0 & H^0 \\
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The MSSM particle spectrum looks like:

$$\begin{array}{cccccccc} \tilde{u}_L & \tilde{u}_R & \tilde{c}_L & \tilde{c}_R & \tilde{t}_L & \tilde{t}_R & \tilde{B} & \tilde{W}^0 \\ \tilde{d}_L & \tilde{d}_R & \tilde{s}_L & \tilde{s}_R & \tilde{b}_L & \tilde{b}_R & \tilde{g} & \tilde{W}^\pm \\ \\ \tilde{e}_L & \tilde{e}_R & \tilde{\mu}_L & \tilde{\mu}_R & \tilde{\tau}_L & \tilde{\tau}_R & \tilde{H}_d^- & \tilde{H}_d^0 \\ \tilde{\nu}_e & & \tilde{\nu}_\mu & & \tilde{\nu}_\tau & & \tilde{H}_u^0 & \tilde{H}_u^+ \end{array}$$

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The MSSM particle spectrum[3] looks like:

$$\begin{array}{cccccccc} \tilde{u}_1 & \tilde{u}_2 & \tilde{u}_3 & \tilde{u}_4 & \tilde{u}_5 & \tilde{u}_6 & \tilde{\chi}_1^\pm & \tilde{\chi}_2^\pm \\ \tilde{d}_1 & \tilde{d}_2 & \tilde{d}_3 & \tilde{d}_4 & \tilde{d}_5 & \tilde{d}_6 & \tilde{g} & \\ \\ \tilde{\ell}_1 & \tilde{\ell}_2 & \tilde{\ell}_3 & \tilde{\ell}_4 & \tilde{\ell}_5 & \tilde{\ell}_6 & \tilde{\chi}_1^0 & \tilde{\chi}_2^0 \\ \tilde{\nu}_1 & & \tilde{\nu}_2 & & \tilde{\nu}_3 & & \tilde{\chi}_3^0 & \tilde{\chi}_4^0 \end{array}$$

[3] Worst case scenario.

MSSM — Mixing caveats

Summary of mixed states:

- The Higgs bosons (H_d^-, H_d^0, H_u^0, H_u^+) form: a charged scalar pair $H^\pm$, two neutral scalars h^0, H^0, and a neutral pseudoscalar A^0.
- The charged bosinos ($\tilde{W}^\pm$, $\tilde{H}_u^+$, $\tilde{H}_d^-$) form the charginos $\tilde{\chi}_i^\pm$.
- The neutral bosinos ($\tilde{B}$, $\tilde{W}^0$, $\tilde{H}_u^0$, $\tilde{H}_d^0$) form the neutralinos $\tilde{\chi}_i^0$.
- The squarks ($\tilde{q}_{i,L}$, $\tilde{q}_{i,R}$) form mass eigenstates labeled $\tilde{q}_i$.
- The charged sleptons ($\tilde{e}_{i,L}$, $\tilde{e}_{i,R}$) form eigenstates $\tilde{\ell}_i$.
- The sneutrinos $\tilde{\nu}_i$ form eigenstates $\tilde{\nu}_i$.

Only for certain ranges of the parameters the particle spectrum will resemble a 'double-SM'.

MSSM — Parameter count

$+$

- 3 couplings g_i, one vacuum angle θ_{QCD} (4 parameters)
- 3 (complex) gaugino masses M_i (6 parameters)
- 2 Higgs mass parameters v, β (2 parameters)
- 2 (complex) Higgs/ino mass parameters μ, B (4 parameters)
- 5 hermitian scalar mass matrices $M^2_{\tilde{F}}$ (5x9 parameters)
- 3 mass matrices λ^f (3x18 parameters)
- 3 trilinear couplings T_F (3x18 parameters)

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$-$

- Flavour symmetry $\mathrm{U}(3)^5/\mathrm{U}(1)^2$ (5x9-2 parameters)

MSSM — Parameter count

$+$

- 3 couplings g_i, one vacuum angle $\theta_{\rm QCD}$ (4 parameters)
- 3 (complex) gaugino masses M_i (6 parameters)
- 2 Higgs mass parameters v, β (2 parameters)
- 2 (complex) Higgs/ino mass parameters μ, B (4 parameters)
- 5 hermitian scalar mass matrices $M^2_{\tilde{F}}$ (5x9 parameters)
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$-$

- Flavour symmetry $\mathrm{U}(3)^5/\mathrm{U}(1)^2$ (5x9-2 parameters)

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- R- and Peccei-Quinn symmetry $\mathrm{U}(1)_R \times \mathrm{U}(1)_{\rm PQ}$ (2 parameters)

MSSM — Parameter count

$+$

- 3 couplings g_i, one vacuum angle θ_{QCD} (4 parameters)
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The full Minimal Supersymmetric Standard Model has 124 free parameters[3] (MSSM-124).

[3] Consisting of 3 couplings, 37 real masses, 39 mixing angles and 45 CP-violating phases.

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