Course Scheduling

Stanford CS221 Fall 2015-2016

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Note: grader.py only provides basic tests. Passing grader.py does not by any means guarantee full points.

a. This icon means a written answer is expected in your writeup.pdf

b. This icon means a coded solution is expected in your submission.py

c. There will be a lot of reading in this assignment. Be patient. It's worth your time! :)

d. **Start early**. Ask questions. Have fun.

What courses should you take in a given quarter? Answering this question requires balancing your interests, satisfying prerequisite chains, graduation requirements, availability of courses; this can be a complex tedious process. In this assignment, you will write a program that does automatic course scheduling for you based on your preferences and constraints. The program will cast the course scheduling problem (CSP) as a constraint satisfaction problem (CSP) and then use backtracking search to solve that CSP to give you your optimal course schedule.

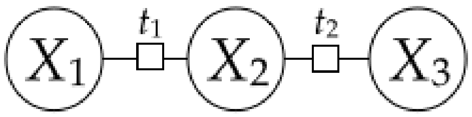
You will first get yourself familiar with CSP by doing warmup exercises in Problem 0. In Problem 1, you will implement two of the three heuristics you learned from the lectures that will make CSP solving much faster. In problem 2, you will add a helper function to reduce n-ary factors to unary and binary factors. Lastly, in Problem 3, you will create the course scheduling CSP and solve it using the code from previous parts.

Problem 0: Warmup

a. Let's create a CSP. Suppose you have n light bulbs, where each light bulb $i = 1, \ldots, n$ is initially off. You also have m buttons which control the lights. For each button $j = 1, \ldots, m$, we know the subset $T_j \subseteq \{1, \ldots, n\}$ of light bulbs that it controls. When button j is pressed, it toggles the state of each light bulb in T_j (For example, if $3 \in T_j$ and light bulb 3 is off, then after the button is pressed, light bulb 3 will be on, and vice versa). Note here each button can be pressed multiple times.

Your goal is to turn on all the light bulbs by pressing a subset of the buttons. Construct a CSP to solve this problem. Your CSP should have m variables and n constraints. *For this problem only*, you can use n-ary constraints. Describe your CSP precisely and concisely.

b. Let's consider a simple CSP with 3 variables and 2 binary factors:



where $X_1, X_2, X_3 \in \{0, 1\}$ and t_1, t_2 are XOR functions (that is $t_1(X) = x_1 \bigoplus x_2$ and $t_2(X) = x_2 \bigoplus x_3$).

i. How many consistent assignments are there for this CSP?

ii. To see why variable ordering is important, let's use backtracking search to solve the CSP *without* *using any heuristics (MCV, LCV, AC-3)*. How many times will `backtrack()` be called to get all consistent assignments if we use the fixed ordering X_1, X_3, X_2? Draw the call stack for `backtrack()`. (You should use the Backtrack algorithm from the slides. The initial arguments are $x = \emptyset$, $w = 1$, and the original Domain.)

In the code, this will be `BacktrackingSearch.numOperations`.

iii. To see why lookahead can be useful, let's do it again with the ordering X_1, X_3, X_2 and AC-3. How many times will Backtrack be called to get all consistent assignments? Draw the call stack for `backtrack()`.

c. Now let's consider a general case: given a factor graph with n variables $X_1, \dots, X_n$ and $n-1$ binary factors $t_1, \dots, t_{n-1}$ where $X_i \in \{0, 1\}$ and $t_i(X) = x_i \bigoplus x_{i+1}$. Note that the CSP has a chain structure. Implement `create_chain_csp()` by creating a generic chain CSP with XOR as factors.

Note: We've provided you with a CSP implementation in `util.py` which supports unary and binary factors. For now, you don't need to understand the implementation, but please read the comments and get yourself familiar with the CSP interface. For this problem, you'll need to use `CSP.add_variable()` and `CSP.add_binary_factor()`.

Problem 1: CSP solving

So far, we've only worked with unweighted CSPs, where $f_j(x) \in \{0, 1\}$. In this problem, we will work with weighted CSPs, which associates a weight for each assignment x based on the product of m factor functions $f_1, \dots, f_m$:

$$\text{Weight}(x) = \prod_{j=1}^{m} f_j(x)$$

where each factor $f_j(x) \geq 0$. Our goal is to find the assignment(s) x with the *highest* weight. As in problem 0, we will assume that each factor is either a unary factor (depends on exactly one variable) or a binary factor (depends on exactly two variables).

For weighted CSP construction, you can refer to the CSP examples we provided in `util.py` for guidance (`create_map_coloring_csp()` and `create_weighted_csp()`). You can try these examples out by running

```
python run_p1.py
```

Notice we are already able to solve the CSPs, because in `submission.py`, a basic backtracking search is already implemented. Recall that backtracking search operates over partial assignments and associates each partial assignment with a weight, which is the product of all the factors that depend only on the assigned variables. When we assign a value to a new variable X_i, we multiply in all the factors that depend only on X_i and the previously assigned variables. The function `get_delta_weight()` returns the contribution of these new factors based on the `unaryFactors` and `binaryFactors`. An important case is when `get_delta_weight()` returns 0. In this case, any full assignment that extends the new partial assignment will also be zero, so *there is no need to search further with that new partial assignment*.

Take a look at `BacktrackingSearch.reset_results()` to see the other fields which are set as a result of solving the weighted CSP. You should read `submission.BacktrackingSearch` carefully to make sure that you understand how the backtracking search is working on the CSP.

a. Let's create a CSP to solve the n-queens problem: Given an $n \times n$ board, we'd like to place n queens on this board such that no two queens are on the same row, column, or diagonal. Implement `create_nqueens_csp()` by **adding n variables** and some number of binary factors. Note that the solver collects some basic statistics on the performance of the algorithm. You should take advantage of these statistics for debugging and analysis. You should get 92 (optimal) assignments for $n = 8$ with exactly 2057 operations (number of calls to `backtrack()`).

Hint: If you get a larger number of operations, make sure your CSP is minimal. Try to define the variables such that the size of domain is O(n).

b. You might notice that our search algorithm explores quite a large number of states even for the 8×8 board. Let's see if we can do better. One heuristic we discussed in class is using most constrained variable (MCV): To choose an unassigned variable, pick the X_j that has the fewest number of values a which are consistent with the current partial assignment (a for which `get_delta_weight()` on $X_j = a$ returns a non-zero value). Implement this heuristic in `get_unassigned_variable()` under the condition `self.mcv = True`. It should take you exactly 1361 operations to find all optimal assignments for 8 queens CSP — that's 30% fewer!

Some useful fields:

- `csp.unaryFactors[var][val]` gives the unary factor value.
- `csp.binaryFactors[var1][var2][val1][val2]` gives the binary factor value. Here, `var1` and `var2` are variables and `val1` and `val2` are their corresponding values.
- In `BacktrackingSearch`, if `var` has been assigned a value, you can retrieve it using `assignment[var]`. Otherwise `var` is not in `assignment`.

c. The previous heuristics looked only at the local effects of a variable or value. Let's now implement arc consistency (AC-3) that we discussed in lecture. After we set variable X_j to value a, we remove the values b of all neighboring variables X_k that could cause arc-inconsistencies. If X_k's domain has changed, we use X_k's domain to remove values from the domains of its neighboring variables. This is repeated until no domains have changed. Note that this may significantly reduce your branching factor, although at some cost. In `backtrack()` we've implemented code which copies and restores domains for you. Your job is to fill in `arc_consistency_check()`.

You should make sure that your existing MCV implementation is compatible with your AC-3 algorithm as we will be using all three heuristics together during grading.

With AC-3 enabled, it should take you 769 operations only to find all optimal assignments to 8 queens CSP — That is almost 45% fewer even compared with MCV!

Take a deep breath! This part requires time and effort to implement — be patient.

Hint 1: documentation for `CSP.add_unary_factor()` and `CSP.add_binary_factor()` can be helpful.
Hint 2: although AC-3 works recursively, you may implement it iteratively. Using a queue might be a good idea. `li.pop(0)` removes and returns the first element for a python list `li`.

Problem 2: Handling n-ary factors

So far, our CSP solver only handles unary and binary factors, but for course scheduling (and really any non-trivial application), we would like to define factors that involve more than two variables. It would be nice if we could have a general way of reducing n-ary constraint to unary and binary constraints. In this problem, we will do exactly that for two types of n-ary constraints.

Suppose we have boolean variables X_1, X_2, X_3, where X_i represents whether the i-th course is taken. Suppose we want to enforce the constraint that $Y = X_1 \vee X_2 \vee X_3$, that is, Y is a boolean representing whether at least one course has been taken. For reference, in `util.py`, the function `get_or_variable()` does such a reduction. It takes in a list of variables and a target value, and returns a boolean variable with domain `[True, False]` whose value is constrained to the condition of having at least one of the variables assigned to the target value. For example, we would call `get_or_variable()` with arguments $(X_1, X_2, X_3, \mathrm{True})$, which would return a new (auxiliary) variable X_4 , and then add another constraint $[X_4 = \mathrm{True}]$.

The second type of n-ary factors is constraints on the sum over n variables. You are going to implement reduction of this type but let's first look at a simpler problem to get started:

a. Suppose we have a CSP with three variables X_1, X_2, X_3 with the same domain $\{0, 1, 2\}$ and a ternary constraint $[X_1 + X_2 + X_3 \leq K]$. How can we reduce this CSP to one with only unary and/or binary constraints? Explain what auxiliary variables we need to introduce, what their domains are, what unary/binary factors you'll add, and why your scheme works. Add a graph if you think that'll better explain your scheme.

Hint: draw inspiration from the example of enforcing $[X_i = 1 \text{ for exactly one } i]$ which Percy did in the lecture.

b. Now let's do the general case in code: implement `get_sum_variable()`, which takes in a sequence of non-negative integer-valued variables and returns a variable whose value is constrained to equal the sum of the variables. You will need to access the domains of the variables passed in, which you can assume contain only non-negative integers. The parameter `maxSum` is the maximum sum possible of all the variables. You can use this information to decide the proper domains for your auxiliary variables.

How can this function be useful? Suppose we wanted to enforce the constraint $[X_1 + X_2 + X_3 \leq K]$. We would call `get_sum_variable()` on (X_1, X_2, X_3) to get some auxiliary variable Y, and then add the constraint $[Y \leq K]$. Note: You don't have to implement the $\leq$ constraint for this part.

Problem 3: Course Scheduling

In this problem, we will apply your weighted CSP solver to the problem of course scheduling. We have scraped a subset of courses that are offered from Stanford's Bulletin. For each course in this dataset, we have information on which quarters it is offered, the prerequisites (which may not be fully accurate due to ambiguity in the listing), and the range of units allowed. You can take a look at all the courses in `courses.json`. Please refer to `util.Course` and `util.CourseBulletin` for more information.

To specify a desired course plan, you would need to provide a *profile* which specifies your constraints and preferences for courses. A profile is specified in a text file (see `profile*.txt` for examples). The profile file has four sections:

- The first section specifies a fixed minimum and maximum (inclusive) number of units you need to take for each quarter. For example:

  ```
  minUnits 0
  maxUnits 3
  ```

- In the second section, you `register` for the quarters that you want to take your courses in. For example,

  ```
  register Aut2015
  register Win2016
  register Spr2016
  ```

  would sign you up for this academic year. The quarters need not to be contiguous, but they must follow the exact format `XxxYYYY` where `Xxx` is one of `Aut, Win, Spr, Sum` and `YYYY` is the year.
- The third section specifies the list of courses that you've taken in the past and elsewhere using the `taken` keyword. For example, if you're in CS221, this is probably what you would put:

  ```
  taken CS103
  taken CS106B
  taken CS107
  taken CS109
  ```

- The last section is a list of courses that you would like to take during the registered quarters, specified using `request`. For example, two basic requests would look like this:

  ```
  request CS224N
  request CS229
  ```

  Not every request must be fulfilled, and indeed, due to the additional constraints described below, it is possible that not all of them can actually be fulfilled.

Constrained requests. To allow for more flexibility in your preferences, we allow some freedom to customize the requests:

- You can request to take exclusively one of several courses but not sure which one, then specify:

  ```
  request CS229 or CS229A or CS229T
  ```

  Note that these courses do not necessarily have to be offered in the same quarter. The final schedule can have at most one of these three courses. **Each course can only be requested at most once.**

- If you want to take a course in one of a specified set of quarters, use the `in` modifier. For example, if you want to take one of CS221 or CS229 in either Aut2015 **or** Sum2016, do:

  ```
  request CS221 or CS229 in Aut2015,Sum2016
  ```

- Another operator you can apply is `after`, which specifies that a course must be taken after another one. For example, if you want to choose one of CS221 or CS229 and take it after both CS109 **and** CS161, add:

  ```
  request CS221 or CS229 after CS109,CS161
  ```

Note that this implies that if you take CS221 or CS229, then you must take both CS109 and CS161. In this case, we say that CS109 and CS161 are `prereqs` of this request. (Note that there's **no space** after the comma.)

If you request course A and B (separately), and A is an official prerequisite of B based on the `CourseBulletin`, we will automatically add A as a prerequisite for B; that is, typing `request B` is equivalent to `request B after A`. Note that if B is a prerequisite of A, to request A, you must either request B or declare you've taken B before.

- Finally, the last operator you can add is `weight`, which adds non-negative weight to each request. All requests have a default weight value of 1. Requests with higher weight should be preferred by your CSP solver. Note that you can combine all of the aforementioned operators into one as follows (again, no space after comma):

```
request CS221 or CS229 in Win2016,Win2017 after CS131 weight 5
```

Each `request` line in your profile is represented in code as an instance of the `Request` class (see `util.py`). For example, the request above would have the following fields:

- `cids` (course IDs that you're choosing one of) with value `['CS221', 'CS229']`
- `quarters` (that you're allowed to take the courses) with value `['Win2016', 'Win2017']`
- `prereqs` (course IDs that you must take before) with value `['CS131']`
- `weight` (preference) with value `5.0`

It's important to note that a request does not have to be fulfilled, *but if it is*, the constraints specified by the various operators `after,in` must also be satisfied.

You shall not worry about parsing the profiles because we have done all the parsing of the bulletin and profile for you, so all you need to work with is the collection of `Request` objects in `Profile` and `CourseBulletin` to know when courses are offered and the number of units of courses.

Well, that's a lot of information! Let's open a python shell and see them in action:

```
import util
# load bulletin
bulletin = util.CourseBulletin('courses.json')
# retrieve information of CS221
cs221 = bulletin.courses['CS221']
print cs221
# look at various properties of the course
print cs221.cid
print cs221.minUnits
print cs221.maxUnits
print cs221.prereqs  # the prerequisites
print cs221.is_offered_in('Aut2016')
print cs221.is_offered_in('Win2017')

# load profile from profile_example.txt
profile = util.Profile(bulletin, 'profile_example.txt')
# see what it's about
profile.print_info()
# iterate over the requests and print out the properties
for req in profile.requests:
    print req.cids, req.quarters, req.prereqs, req.weight
```

Solving the CSP. Your task is to take a profile and bulletin and construct a CSP. We have started you off with code in `SchedulingCSPConstructor` that constructs the core variables of the CSP as well as some basic constraints. The variables are all pairs of requests and registered quarters `(request, quarter)`, and the value of such a variable is one of the course IDs in that Request or `None`, which indicates none of the courses should be taken in that quarter. We will add auxiliary variables later. We have also implemented some basic constraints: `add_bulletin_constraints()`, which enforces that a course can only be taken if it's offered in that quarter (according to the bulletin), and `add_norepeating_contstraints()`, which constrains that no course can be taken more than once.

You should take a look at `add_bulletin_constraints()` and `add_norepeating_contstraints()` to get a basic understanding how the CSP for scheduling is represented. Nevertheless, we'll highlight some important details to make it easier for you to implement:

- The existing variables are tuples of `(req, quarter)` where `req` is a `Request` object (like the one shown above) and `quarter` is a `str` representing a quarter (e.g. `'Aut2015'`). For detail please look at

`SchedulingCSPConstructor.add_variables()`.

- The domain for `req` is the course IDs of the request **plus** `None` (e.g. `['CS221', 'CS229', None]`). When `req` is `None`, this means no course is scheduled. **Always remember to check if `req` is `None`**.
- The domain for `quarter` is all possible quarters (`self.profile.quarters`, e.g. `['Win2016', 'Win2017']`).
- Given a course ID `cid`, you can get the corresponding `Course` object by `self.bulletin.courses[cid]`.

a. Implement the `add_quarter_constraints()`. This is when your profile specifies which quarter(s) you want your requested courses to be taken in. This does not saying that one of the courses must be taken, *but if it is*, then it must be taken in any one of the specified quarters. Also note that this constraint will apply to all courses in that request. We have written a `verify_schedule()` function in `grader.py` that determines if your schedule satisfies all of the given constraints. Note that since we are not dealing with units yet, it will print `None` for the number of units of each course. For profile3a.txt, you should find 3 optimal assignments with weight 1.0.

b. Let's now add the unit constraints in `add_unit_constraints()`. You must ensure that the sum of units per quarter for your schedule are within the min and max threshold inclusive. You should use `get_sum_variable()`. In order for our solution extractor to obtain the number of units, for every course, you must add a variable `(courseId, quarter)` to the CSP taking on a value equal to the number of units being taken for that course during that quarter. When the course is not taken during that quarter, the unit should be 0. **NOTE:** Each grader test only tests the function you are asked to implement. To test your CSP with multiple constraints you can use `run_p3.py` and changing the constraints that you want to add. For profile3b.txt, you should find 15 optimal assignments with weight 1.0.

Hint: If your code times out, your `maxSum` passed to `get_sum_variable()` might be too large.

c. Now try to use the course scheduler for the winter and spring (and next year if applicable). Create your own `profile.txt` and then run the course scheduler:

```
python run_p3.py profile.txt
```

You might want to turn on the appropriate heuristic flags to speed up the computation. Does it produce a reasonable course schedule? Please submit your `profile.txt`; we're curious how it worked out for you!

Extra Credit: weighted CSPs with notable patterns (3 points)

Want more challenges about CSP? Here we go. :D

Suppose we have a weighted CSP with variables $X_1, \dots, X_n$ with domains $\text{Domain}_i = \{1, \dots, K\}$. We have a set of basic factors which depend only on adjacent pairs of variables in the same way: there is some function g such that $f_i(x) = g(x_i, x_{i+1})$ for $i = 1, \dots, n-1$. In addition, we have a small set of *notable patterns* P, where each $p \in P$ is a sequence of elements from the domain.

Let n_p be the number of times that p occurs in an assignment $x = (x_1, \dots, x_n)$ as a consecutive sequence. Define the weight of an assignment x to be $\prod_{i=1}^{n-1} f_i(x) \prod_{p \in P} \gamma^{n_p}$. Intuitively, we multiply the weight by γ every time a notable pattern appears.

For example, suppose $n = 4$, $\gamma = 3$, $g(a, b) = 2[a = b] + 1[a \neq b]$ and $P = \{[1, 3, 3], [1, 2, 3]\}$. Then the assignment $x = [1, 3, 3, 2]$ has weight $2 \cdot 3 = 6$.

a. If we were to include the notable patterns as factors into the CSP, what would be the worst case treewidth? (You can assume each p has a maximum length of n.)