

# DrexelADS

*Tech talk*

# What's this about?

1. Cake cutting
2. Sorting/searching algorithms

# The cake cutting problem



**The cake** : Heterogeneous, divisible resource

## Fair division

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From Wikipedia, the free encyclopedia

**Fair division** is the problem of dividing a set of **resources** among several people who have an **entitlement** to them, such that each person receives their due share.



# Fairness

**Envy-free**: Each person feels that their piece is at *least as good as* the others' pieces

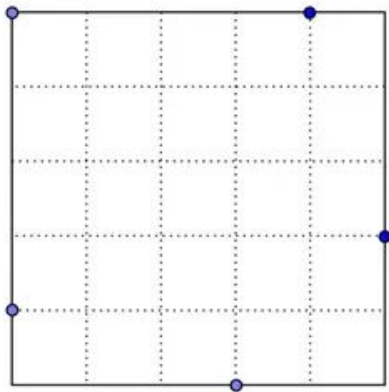
**Proportional**: Each person feels that their piece is  $1/N$  of the cake

**Equitable**: Each view their piece the same as everyone else views their piece

**Efficient**: There are no other division that dominates this one for all players

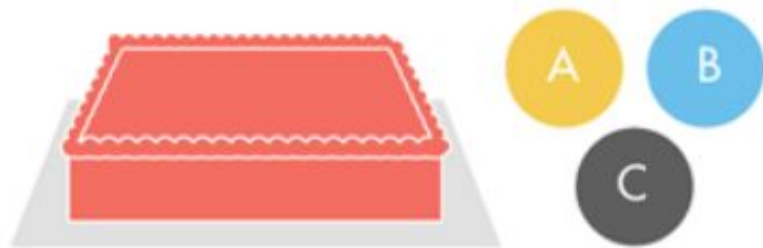


# Dividing a cake among 2 people

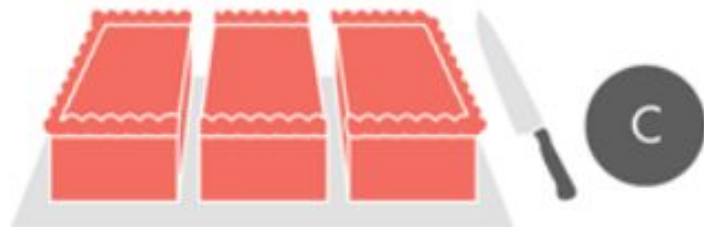


—— “I cut, you choose”

- ① Alice, Bob and Charlie want to share a cake so that none of them envies other pieces.

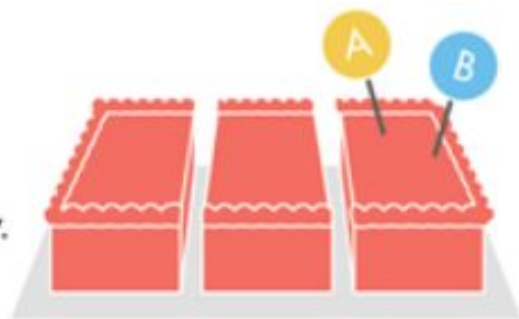


- ② Charlie cuts the cake into three pieces that are equally valuable from his perspective.

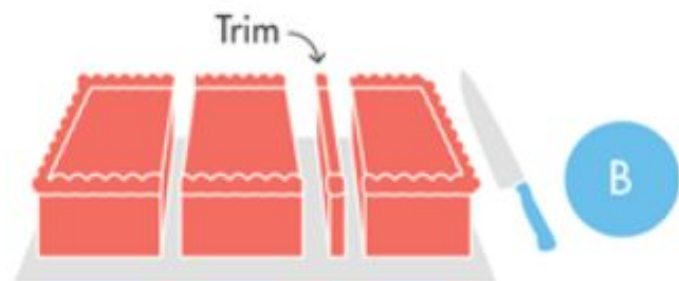




- 3 Alice and Bob identify their first choices.  
If they identify the same choice, things get tricky.



- 4 Bob trims his preferred piece to match his second most preferred piece.



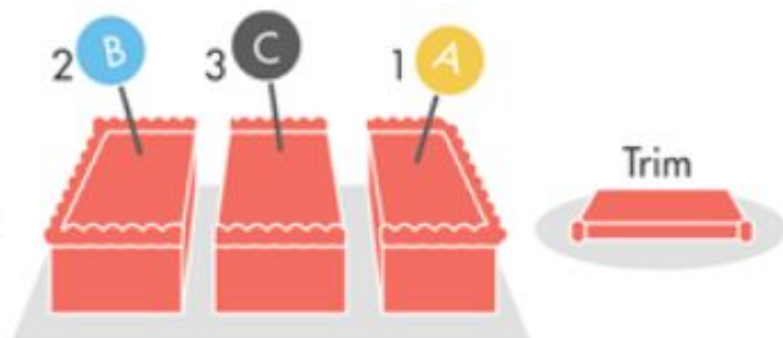
- 5 Putting the trim to one side they choose in this order: Alice first\*, Bob second and Charlie last.

It is envy free

...for Alice, because she got first choice.

...for Bob, because his second choice was equally valuable.

...for Charlie, because the three original slices were equal to him.



- 6 To divvy up the trimmed slice, first Bob cuts the trim into three pieces that are equally valuable from his perspective.



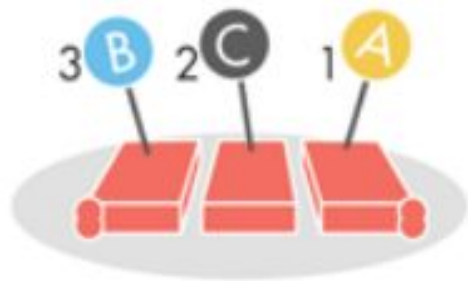
- ⑦ Now they choose a portion of trim in this order:  
Alice first, Charlie second and Bob last

It is envy free

...for Alice, because she got her first choice.

...for Charlie, because he got to choose before Bob.

...for Bob, because the three pieces of trim were equal to him.



# A bit history

Selfridge-Conway (1960)

Brams-Taylor (1995)

Aziz-Mackenzie (2016)

$$n^{n^{n^{n^n}}}$$



# Dubins-Spanier moving knife



## **Proportional:**

Each person  
feels that their  
piece is  $1/N$  of  
the cake



# Cutting a “bad” cake

House chore division

Rent harmony

# Cutting an “uncuttable” cake

