Randomized Nyström Algorithm for Rank-*k* Approximation

Sequential Nyström algorithm

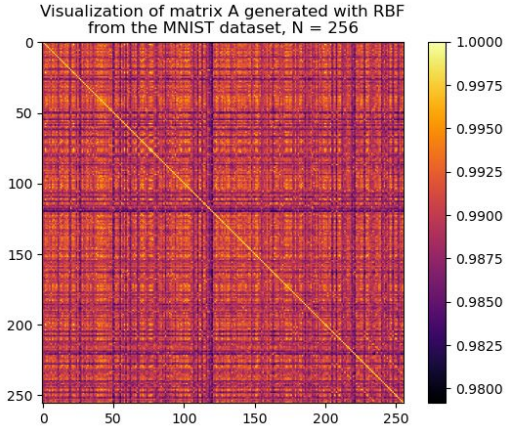
1 Compute $C = A\Omega \in \mathbb{R}^{n \times l}$
2 Compute $B = \Omega^T C \in \mathbb{R}^{l \times l}$ and its Cholesky factorization $B = LL^T$
3 Solve $Z = CL^{-T}$ with substitution
4 Compute the QR factorization $Z = QR$
5 Compute the truncated rank-k SVD $R = U_k \Sigma_k V_k^T$
6 Compute $\hat{U}_k = QU_k$
7 Output the factorization $\left[\left[A_{\text{Nyst}}\right]\right]_k = \hat{U}_k \Sigma_k^2 \hat{U}_k^T$

- C : Locally on every P + *sum_reduce* on rows
- B : Locally for P on the 1st row + *sum_reduce* on columns
  Cholesky: on root P + *broadcast* to 1st row
- Z and *TSQR* : Locally for P on 1st row
- SVD of R: On root P + *broadcast* to 1st row
- $\hat{U}$: Locally for P on 1st row

Choices for Ω:
- Gaussian
- SRHT

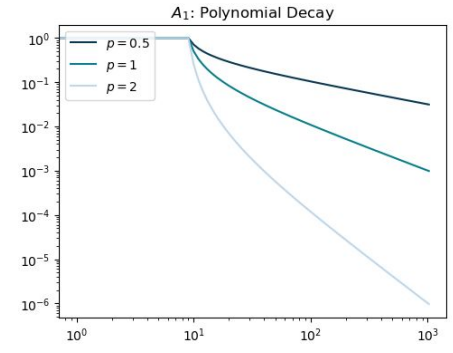
$$C = (\Omega A)^T = A\Omega^T$$
$$B = \Omega C = \Omega(\Omega A)^T = \Omega A \Omega^T$$

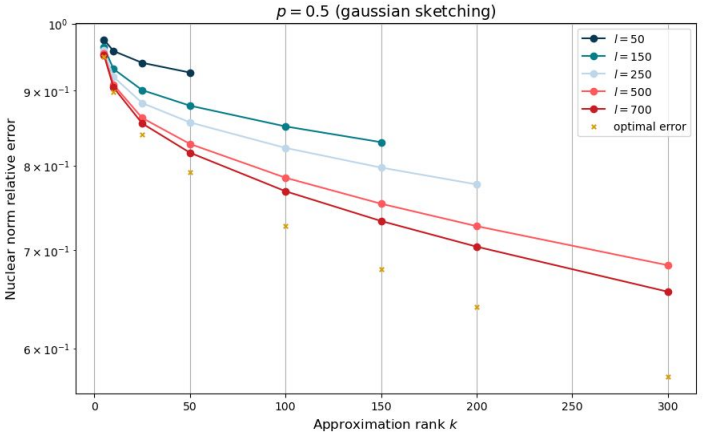
3 matrices:
- polynomial decay
- exponential decay
- RBF from MNIST dataset

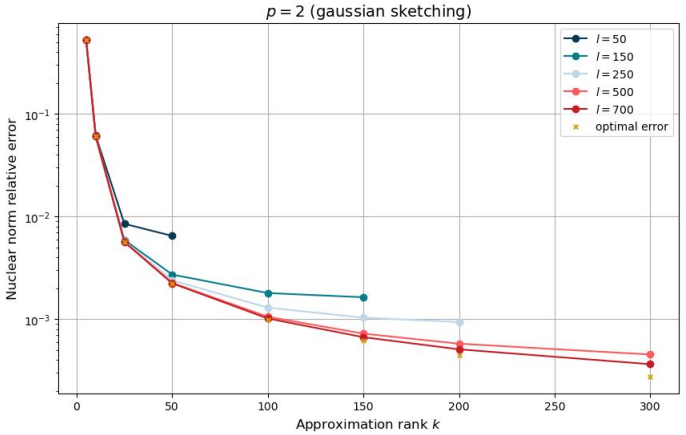


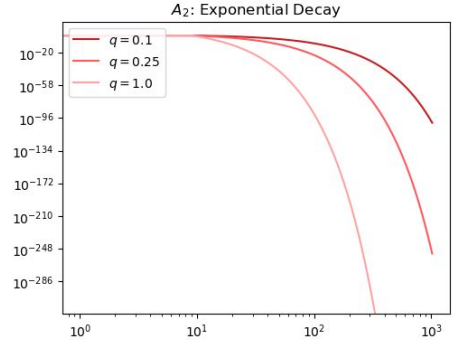
Stability Analysis

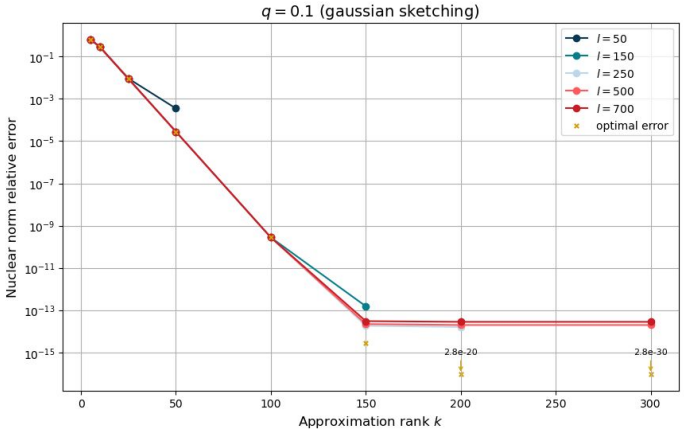
$$E = \frac{||A - [[A_{Nyst}]]_k||_*}{||A||_*} \qquad E_{opt} = \frac{||A - [[A]]_k||_*}{||A||_*} = \frac{\sum_{i=k+1}^{n} \sigma_i}{||A||_*}$$

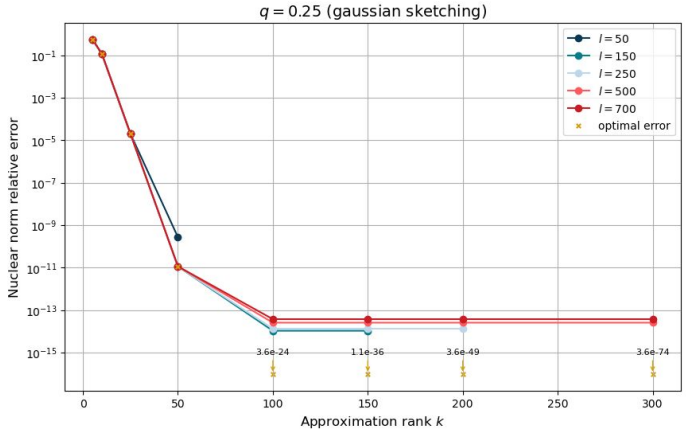












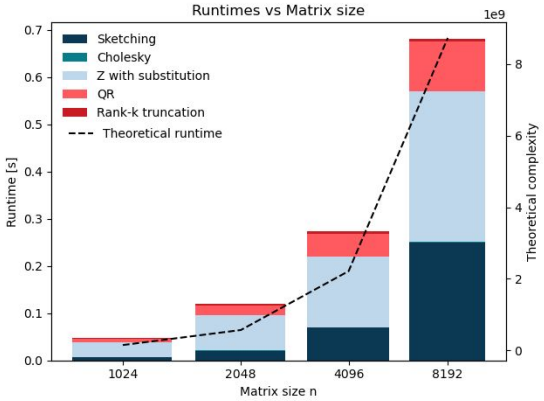
Sequential and Parallel Runtimes

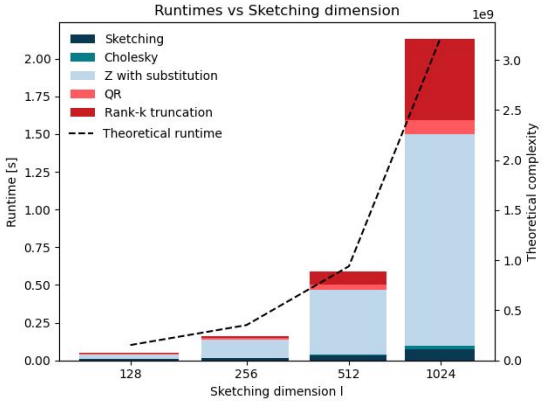
Sequential Algo | Runtime VS n

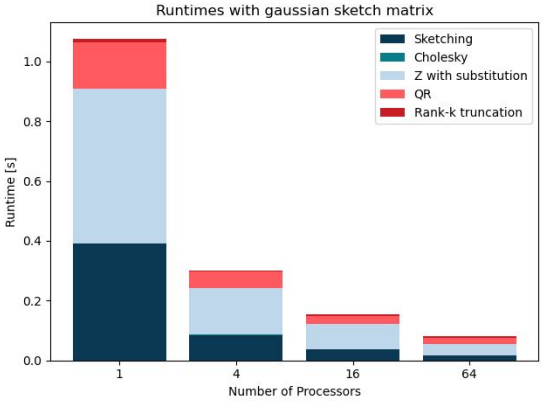
Sequential Algo | Runtime VS l

Parallel Algo | Runtime VS P

Gaussian sketching







SRHT sketching