A Numerical Model of the Solar System

Project Proposal - Computational Physics

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Abstract

For my final project, I will attempt to simulate the solar system. A general model capable of representing the motion of N bodies in interaction with each other due to gravitational forces will be created. The first phase will consist of a 2-bodies system to describe the motion of Earth in orbit around the Sun. Then, another planet such as Jupiter will be added to the model, allowing us to study its effect on Earth's trajectory. Finally, the model will incorporate all 8 planets orbiting around the Sun. Although the 2-bodies system is stable and unchanging with time, the model will become chaotic when adding more planets. After simulating the system with different initial conditions, we will study the specific scenarios, calculating the system's properties and understand how the interactions change based on the initial position and velocity values. The model will be computed in a three-dimensional space and visualizations will be created. If time allows, an interactive visualization could be produced, allowing the user to play with different initial positions, velocities, distances between bodies, and masses. Finally, NASA's JPL (Jet Propulsion Laboratory) Horizons database provides online access to highly accurate positions and velocities for solar system objects. This data will be used to measure the accuracy of our simulation.

Background

This project is based on a classic problem in celestial mechanics called the N-bodies problem. It refers to understanding how N particles (or celestial bodies) interact with each other with the influence of gravity only. Although it has been solved analytically for 2 bodies (known as the Kepler problem), no closed-form solution exists for systems with 3 bodies or more. Nevertheless, numerical solutions have been used and have given approximations with reasonable accuracy.

Methods

The model will rely on two main laws of physics: the Newton second law of motion and the law of universal gravitation. Numerical methods will be used to integrate the differential equations of motion, including (in order):

- The fourth-order Runge-Kutta method
- The leapfrog method (most commonly used in the field)
- The Bulirsch-Stoer method

Astronomical units will be helpful to introduce a change of variables. For instance, the distance between the Sun and Earth will be the unit of length.

In addition, we will use the software Blender for visualization in three dimensions. This software has special packages for astrophysical visualizations, is free, and available on Mac, Linux, and Windows operating systems.

Finally, depending on arising problems during the computations, new techniques might be tried to improve the accuracy of the model. If one planet's acceleration is very high, smaller time steps will need to be used. However, this can be computationally expensive as other planets would not necessarily need such precision. Therefore, a method to explore is be to use different time steps for different orbits.

Timeline

Deadline	**Task**
March 5^{th}	First simulation for a 2-bodies system
March 10^{th} (before spring break)	First visualization with Blender
April 1^{st}	Simulation for a 3-bodies system and study of the system's properties
April 15^{th}	First Simulation of the solar system
May 1^{st}	Calculation of accuracy and improvements
May 12^{th}	Final paper and presentation

References

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Wikipedia, N-body problem, `https://en.wikipedia.org/wiki/N-body_problem`

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Astroblend: An astrophysical visualization package for Blender, `https://www.sciencedirect.com/science/article/pii/S2213133716300117`

Horizons Database, `https://ssd.jpl.nasa.gov/horizons/app.html#/`