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HW2: Computational Physics

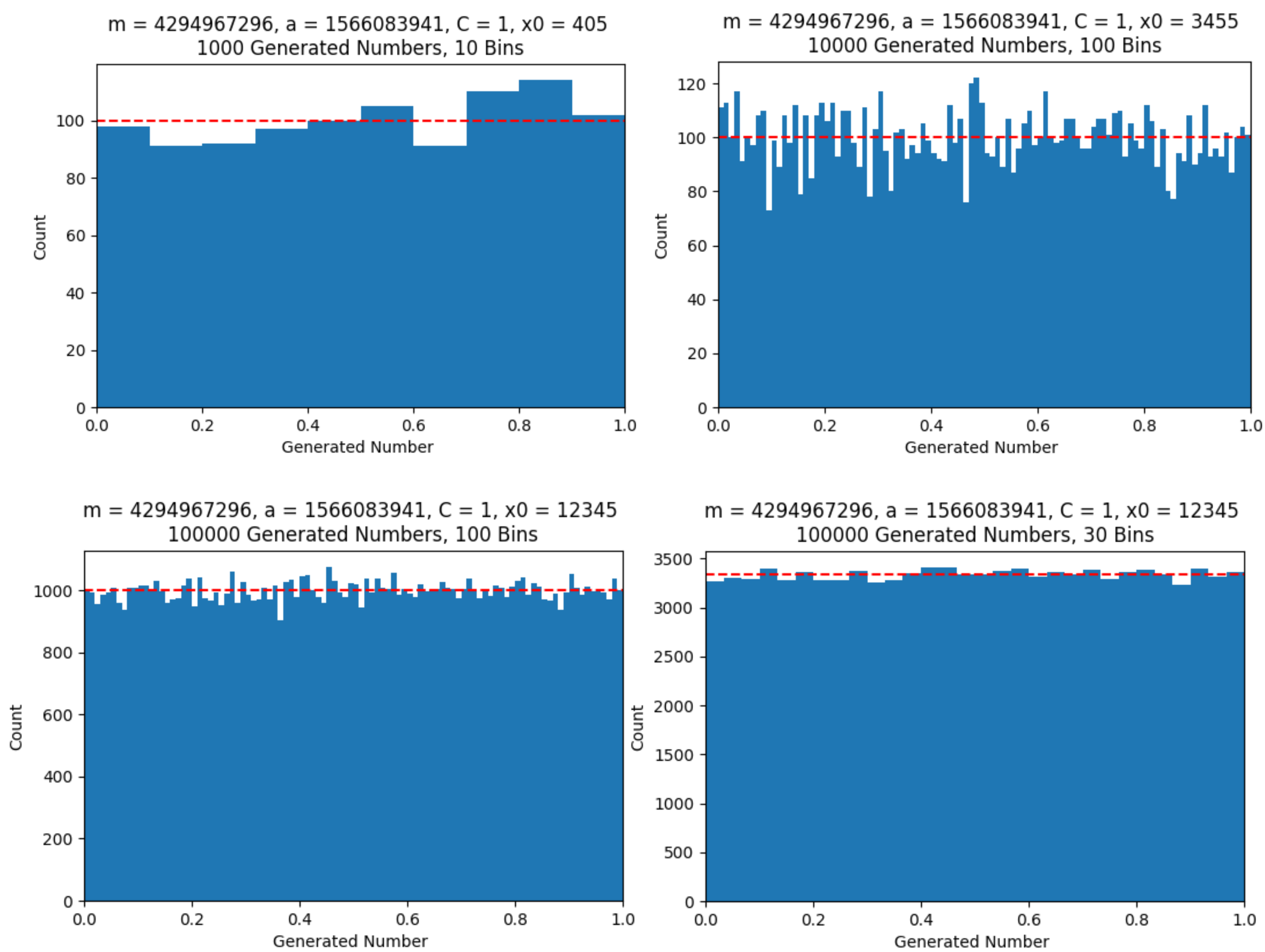
Question 1

For my random generator, I used the linear congruence method, creating a sequence:

$$X_{n+1} = (aX_n + C) \bmod m$$

As advised in the Appendix F1 of the book, I chose $m = 2^{32}$ as it is the machine word size for most computers: "integer computations modulo the machine word size amounts to truncating high order bits in integer overflows, which is typically performed automatically by the system in a very efficient manner". Moreover, a = 1566083941 and C = 1 are chosen to ensure that all numbers are produced in a period independent on the choice of the seed. (Appendix F1).

I tested my algorithm with 4 different initial conditions and display parameters:



Question 2:

First, I calculated the integral of x^2 from 0 to 10 using a fixed value N_slices = 100 (number of slices for Riemann sums and the trapezoidal method) and N_points = 100000 (number of points for the Sampling method) and got the following results:

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Real value: 333.33
Riemann left: 328.35
Riemann right: 338.35
Riemann center: 333.32
Trapezoidal: 333.35
Sampling: 332.77
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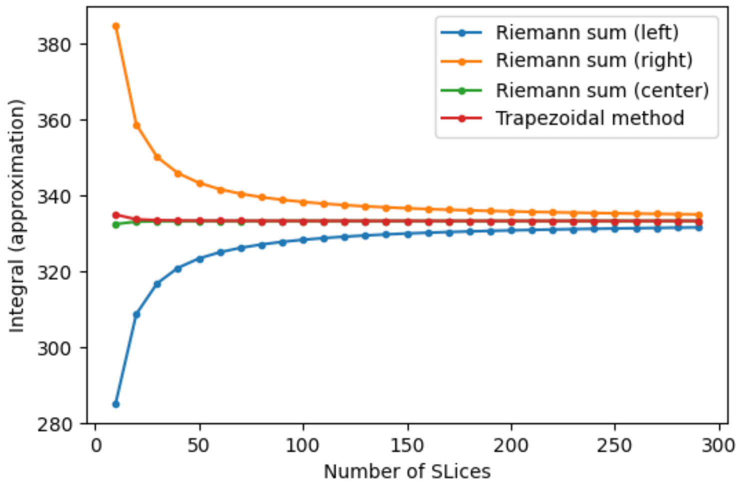
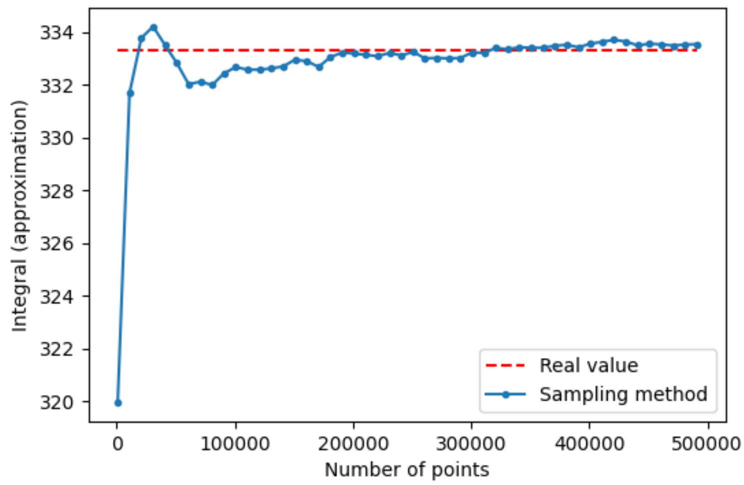
In addition, I calculated the integral of x^2 from 0 to 10 using different values for N_slices and N_points.

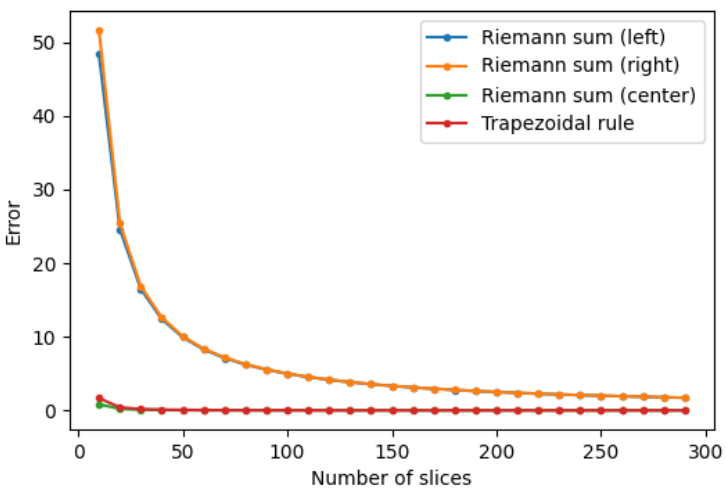


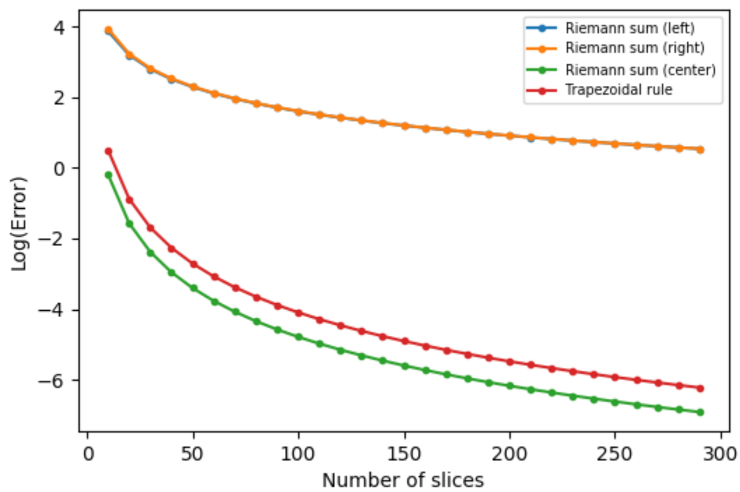


We observe that for all methods, the approximation seems to converge towards the real value (333.333…) when we increase the number of slices or the number of points.

Question 3

First, I plotted the Error and log of Error VS the number of slices for Riemann sums and the trapezoidal method.





We observe that the error is significantly lower for the centered Riemann sum and the trapezoidal method. In addition, the error for left and right Riemann sums is very similar. Finally, the error for all methods decreases with higher resolution (more slices).

The following are individual plots for each method. In each plot, the light blue line is a potential approximation and matches with the analysis we did in class.