TDT4171 EXERCISE 1

Due: 08.02.17
@ 08.00

Name: Mathilde Hotvedt
email: mathildh@stud.ntnu.no

# I) Counting and basic laws of probability

## 1) 5-card Poker Hands

INTRO: Consider the domain of dealing 5-card poker hands from a standard deck of 52 cards, under the assumption that the dealer is fair

a) Problem: How many atomic events are there in the joint probability distribution? i.e. how many 5-card hands are there?

Answer:

we want a sample size of $k=5$ and we may choose each card from the deck without replacement of the prev. card drawn.

The order of the cards does not matter such that $12345 = 54321$ etc..

Then all possible outcomes of 5-card hands will be $\binom{n}{k} = \binom{52}{5} = 2598960$

b) Problem: what is the probability of each atomic event?

Answer:

$$P(1 \text{ atomic event}) = \frac{\text{no. of outcomes favorable}}{\text{no. possible outcomes}}$$

$P(1 \text{ atomic event}) = \frac{1}{\binom{52}{5}} = 3.847 \times 10^{-7}$

c) Problem: what is the probability of being delt a royal straight flush? Four of a kind?

Answer:

- Royal straight Flush: is a card hand of ace, king, queen, knight, ten. in one suit. There will be 4 possible hands of this, one in each suit, Hence

$$P(\text{"Royal straight Flush}) = \frac{4}{\binom{52}{5}}$$

$$= 4 \times P(\text{"1 atomic event}) = 1.539 \times 10^{-6}$$

- Four of a kind: four cards of the same rank and 1 other card. for instance
5 clubs, 5 spades, 5 diamond, 5 hearts, 2 diamond.

we need to calculate with absolute probabilities. First we can draw any card of the 13 values in the deck $\binom{13}{1}$ but then we need 3 more of this → 4 in total → need all 4 cards of this value: $\binom{4}{4}$

$$P(\text{"Four equal"}) = \binom{13}{1}\binom{4}{4}$$

but then we need 1 last card which may be any of the values except the ones already drawn $13 - 1$ possible values, 1 card

$P(\text{"1 card not equal"}) = \binom{12}{1}\binom{4}{1}$

$P(\text{"Four of a kind"}) = \dfrac{\binom{13}{1}\binom{4}{4}\binom{12}{1}\binom{4}{1}}{\binom{52}{5}} = \dfrac{624}{\binom{52}{5}}$

$= 2.4 \times 10^{-4}$

2) Two cards in a deck

INTRO: Two cards are randomly selected from a deck of 52 playing cards

a) Problem:
what is the probability they constitute a pair - that they are of the same denomination ?

<u>Answer</u>:

First I can choose 1 cards randomly.
The next must be of the same value, and there are only $3/52$ cards I should draw.

So now, the number of possible 2-card hands are $\binom{52}{2} = 1326$

I can first choose any value $\binom{13}{1}$ but need $2/4$ cards of this value: $\binom{4}{2}$

$p(\text{"draw to equal valued cards"}) = \dfrac{\binom{13}{1}\binom{4}{2}}{\binom{52}{2}}$
$= 0.0588$

b) Problem: what is the conditional probability they constitute a pair given that they are of different suits?

Answer:

A: pair          B: different suits

$$P(A \mid B) = \frac{P(B \mid A)\,P(A)}{P(B)}$$

$P(B \mid A) = 1$

$P(B) = 1 - \frac{12}{51}$          1 st card drawn, then there will be 12 cards with same suit as first

$$P(A \mid B) = \frac{P(A)}{P(B)} = \frac{0.0588}{1 - \frac{12}{51}} = 0.07689$$

Scanned by CamScanner

3) Conditional Probability

1) Problem: if the occurrence of B makes A more likely, does the occurrence of A make B more likely? why?

Answer:

$P(A \mid B) > P(A)$ Given ⇑

$$P(B \mid A) = \frac{P(B,A)}{P(A)} = \frac{P(A,B)}{P(A)}$$

$$\Rightarrow P(A) = \frac{P(A,B)}{P(B \mid A)} < P(A \mid B) = \frac{P(A,B)}{P(B)}$$

$$\Rightarrow \frac{1}{P(B \mid A)} < \frac{1}{P(B)} \Rightarrow P(B) < P(B \mid A)$$

$$\Rightarrow P(B \mid A) > P(B)$$

Yes, occurrence of A makes B more likely

2) Problem: when coded msg are sent there are sometimes errors in the transmission. It is possible to use conditional probabilities to model this situation and reason about the sent and received msg. Suppose there is a binary code (0 or 1) with biased probabilities of occurrence in the sent msg of 0.6 for the code 0, $\Rightarrow P(\text{sent sym} = 0) = \frac{6}{10}$ $P(\text{sent sym} = 1) = \frac{4}{10}$. It is know that the conditional probability of mistakes in the transmission process by switching the symbols is $1/3$. $P(R=1 \mid S=0) = P(R=0 \mid S=1) = 1/3$ if you receive zero, what is the probability that zero was sent?

Answer :

we have :

$P(S=0) = \frac{6}{10} \qquad P(S=1) = \frac{4}{10}$

$P(R=1 \mid S=0) = P(R=0 \mid S=1) = 1/3$

$$P(S=0 \mid R=0) = \frac{P(R=0 \mid S=0)\, P(S=0)}{P(R=0)}$$

$$\begin{aligned} P(R=0) &= P(S=1) \cdot P(\text{"wrong received}) \\ &\quad + P(S=0) \cdot P(\text{"correct received"}) \\ &= \frac{4}{10} \cdot \frac{1}{3} + \frac{6}{10} \cdot \frac{2}{3} = \frac{8}{15} \end{aligned}$$

$$R(R=0 \mid S=0) = 1 - P(R=1 \mid S=0) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(S=0 \mid R=0) = \frac{\frac{2}{3} \cdot \frac{6}{10}}{\frac{8}{15}} = \frac{3}{4}$$

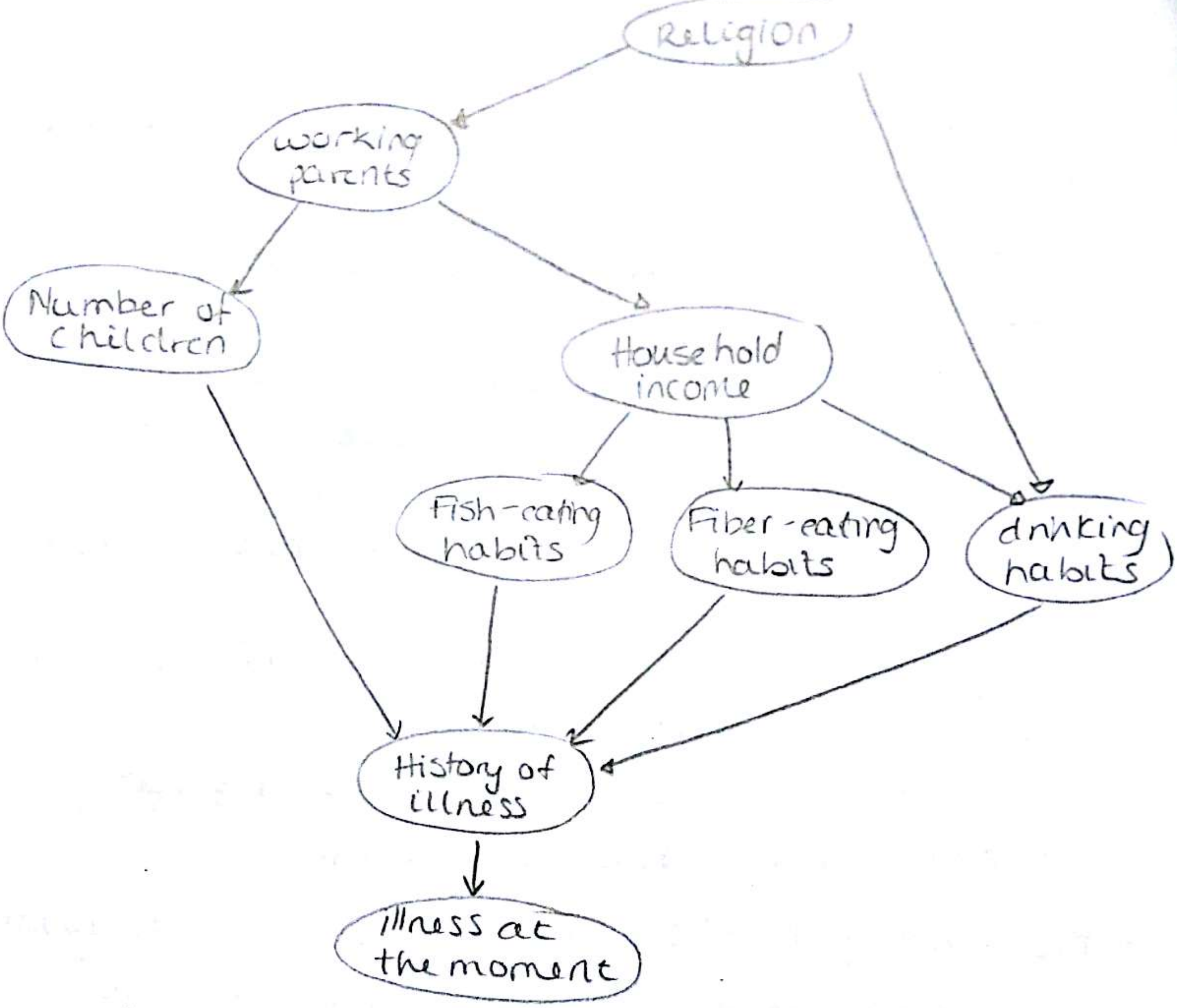
I) Bayesian Network Construction

INTRO: Consider the following variables relating to a single household consisting of a couple and possible some children.

- Illness at the moment, with states: severe illness, minor illness, no illness
- History of illness, with states: cases of severe illness, often minor illnessess, rarely minor illnessess
- Number of children, with states none, one, two, three, four and up
- Religion, with states Christianity, Judaism, Islam, Buddhism, Atheism, other
- Household income, with states \$0 – \$50 000, \$50 000 – \$100 000, \$100 000 and up
- Fish-eating habits, with states often fish, rarely fish
- Fiber-eating habits, with states lots of fiber, not much fiber
- Drinking habits, with states never alcohol, wine once in a while, often wine, wine every day.

Problem:
Try to construct a Bayesian netwok incorporating the above variables accurately according to your perception of the world. What are the conditional independence properties of the network you constructed? Are they reasonable?

Answer:




- This is my perception of the world.
for instance, Fish, fiber and alcohol affects your immune system and hence your ability to defeat illnesses before they break through. The number of children you have will also affect how often you have been ill, as small children often are carriers of bacteria and viruses.

- we can see here that illness at the moment is conditionally independent of all other variables given history of illness. That means, if history of illness is set, how much fish you eat will not affect whether or not you are ill <u>right now</u>

Scanned by CamScanner

- likewise we can see given working parents how many children you have will be conditionally independent of religion

- The general rule is: each node is conditionally independent of its nondescendants given its parents.

- Further: given household income, Fish eating habits and fiber-eating habits are conditionally independent of each other.

summing up: conditional independence:

- illness at the moment given history of illness
- history of illness given drinking, fish-eating fiber-eating habits and num. of children
- etc..

The only node without parent in my world perception is religion. This depends on how you look at it ofc. If you drink too much in a religion that doesnt allow it you may be forced to switch religion. However, we usually have a religion before starting to drink, and I therefore chose to set it this way.

In a real world there are ofcourse many other variables needed to add to the BN.

## III) Bayesian Network Application.

INTRO:

You are confronted with three doors A, B, C. Behind exactly one of the doors there is \$10000. The money is yours if you choose the correct door. After you have made your first choice of door but still not opened it, an official comes in. He is working according to some rules:

1) He starts by opening a door. He knows where the prize is, and he is not allowed to open that door. Furthermore, he cannot open the door you have chosen. Hence, he opens the door with nothing behind.

2) Now there are two closed doors, one of which contains the prize. The official will ask you if you want to alter your choice.

Problem:

Should you?

Answer:

First we need to create the different nodes with a probability table.

For MyChoice - I have 1/3 change at picking each door, and there is also 1/3 change that the price is behind one of the doors

| My choice | | Contains Price | |
|---|---|---|---|
| A | 1/3 | A | 1/3 |
| B | 1/3 | B | 1/3 |
| C | 1/3 | C | 1/3 |

Scanned by CamScanner

Now: My choice and where the price is will affect the officials choice so we will get a large table.

| My Choice | | A | | | B | | | C | | |
|---|---|---|---|---|---|---|---|---|---|---|
| contains price | | A | B | C | A | B | C | A | B | C |
| opened by official | A | 0 | 0 | 0 | 0 | 0.5 | 1 | 0 | 1 | 0.5 |
| | B | 0.5 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0.5 |
| | C | 0.5 | 1 | 0 | 1 | 0.5 | 0 | 0 | 0 | 0 |

- Hence, before anything is chosen, all states will have an equal oppertunity to happen in all three variable: $1/3$, see figure 1.

- Then, I choose door A. From figure 2 we see that this only effect opened By offical where there is a 50% change he opens B or C

- Then the offical choose to open door B, then suddenly Contains Prize is effected and we see that there is 67% probability that the prize is in C.

Result: we should change choice to door C

- The evidence behind this must be calculated using conditional probability

- Say that we have chosen door A and then the official chooses to open door B: we must then find:

$$P(\text{prize} \mid \text{opens } B) = \frac{P(\text{prize}, \text{opens } B)}{P(\text{opens } B)} \quad \text{Bayes regel}$$

official opens

P(prize, opens door)

| | | A | B | C |
|---|---|---|---|---|
| | A | 0 | 1/6 | 1/6 |
| prize | B | 0 | 0 | 1/3 |
| | C | 0 | 1/3 | 0 |

- The diagonal would be zero because official cannot open door with prize behind.
- official cannot open A because we have chosen it ⇒ zero on A column
- now there is 1/3 probability that the prize is behind any of the doors to begin with. if then the price is behind A (which we have chosen) then the official opens door B or door C with $p = 50\%$

therefore $p(\text{prize} = A, \text{opens } B) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$

$p(\text{prize} = A, \text{opens } C) = \frac{1}{6}$

- if price is behind B then official must open C

$p(\text{prize} = B, \text{opens } C) = \frac{1}{3} \cdot 1$

- same with last scenario

$p(\text{prize} = C, \text{opens } B) = \frac{1}{3} \cdot 1$

- now $p(\text{opens } B) = 1/6 + 1/3 = \frac{1}{2}$

$= p(\text{prize } A, \text{opens } B) + p(\text{prize } B, \text{opens } B)$
$+ p(\text{prize } C, \text{opens } B)$

$\Rightarrow \alpha = \frac{1}{p(\text{opens } B)} = 2$

- then: $P(\text{prize}, \text{opens } B) = [1/6, 0, 1/3]$
= distribution.

$p(\text{prize} \mid \text{opens } B) = \alpha [1/6, 0, 1/3]$
$= [1/3, 0, 2/3]$

↑ 66.6% chance that
price is behind door C
= should swap door!

Figure 1

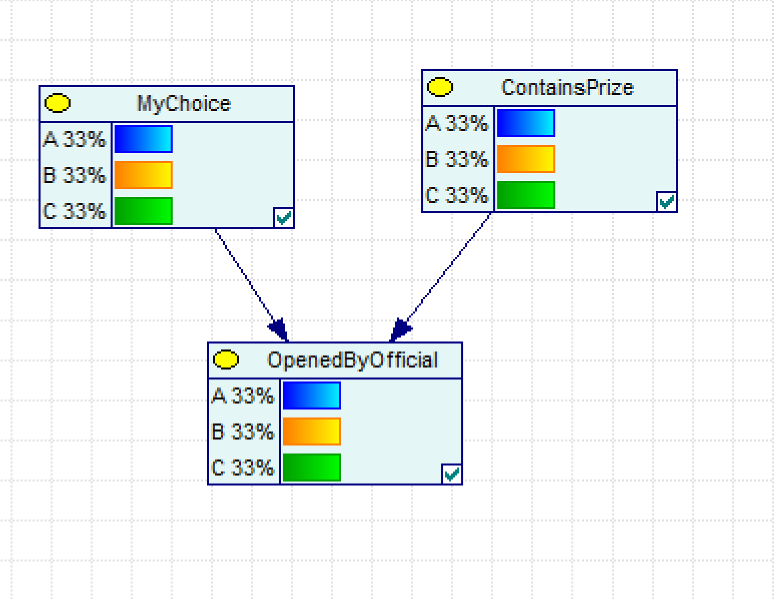



Figure 2

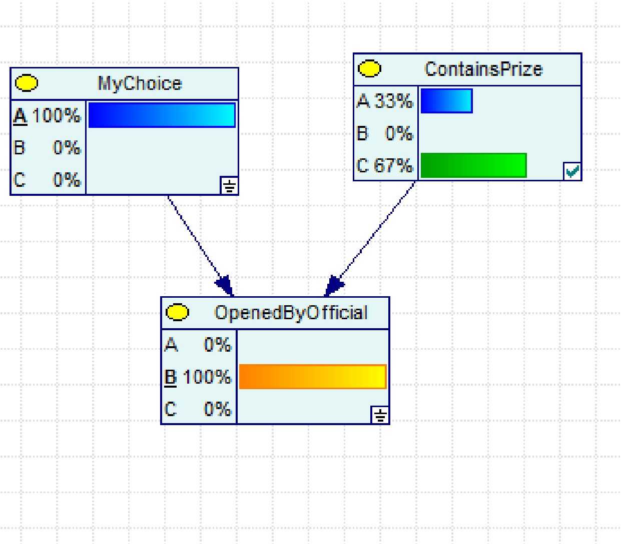



Figure 3