Lagrangian particles evolution through RRE, RRR and BPR refraction structures

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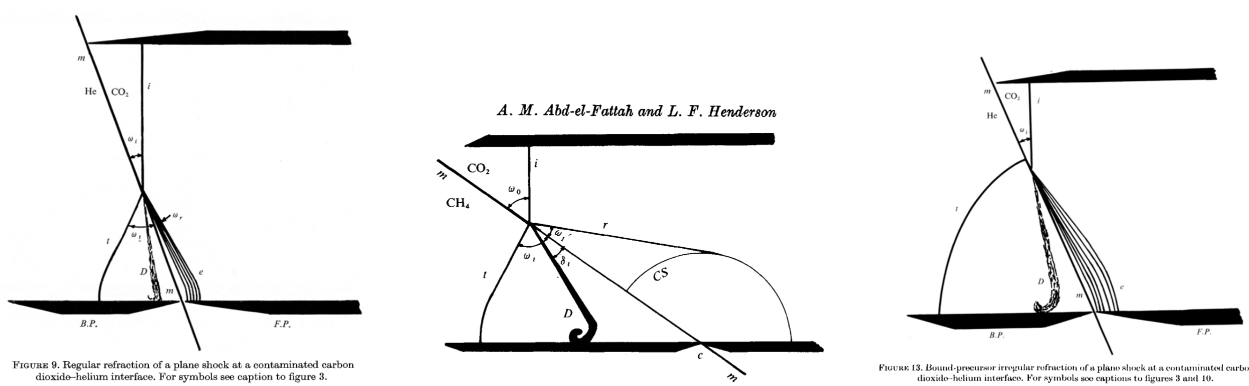
Outline

1. Reminder of the different refraction structures

2. Regular Refraction with reflected Expansion (RRE)
   - RRE - Theory and equations : gas dynamics
   - RRE - Results given by the inert gas dynamics theory
   - RRE - Use of CHEMKIN II to compute chemistry calculus
   - RRE - Results of the chemical calculus

3. Regular Refraction with Reflected Shock (RRR)
   - RRR - Theory and equations : gas dynamics
   - RRR - Results given by the inert gas dynamics theory
   - RRR - Results of the chemical calculus



(a) RRE struture (b) RRR struture (c) BPR structure

Figure 1: Three of the already known refraction structures : RRE and BPR schemes are from Henderson, Abd-el-Fattah & Lozzi 1976; RRR scheme is from Henderson & Abd-el-Fattah 1978.

Reminder of the different refraction structures

Refraction with reflected Expansion

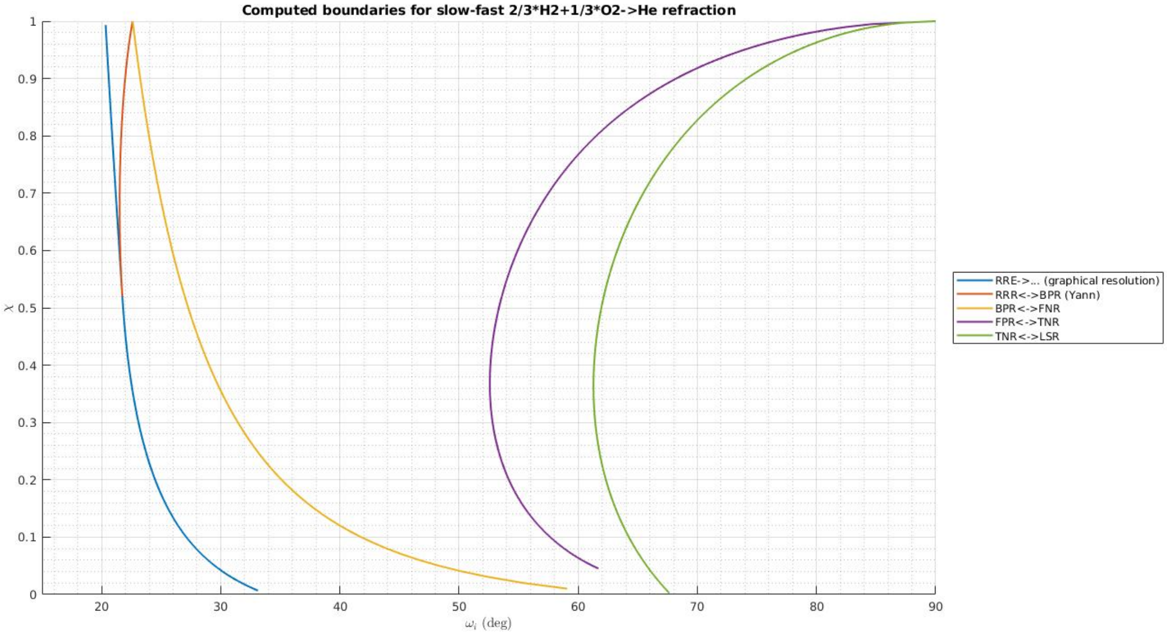


Figure 2: Boundaries of the differents structures in the $\chi - \omega$ plane, for a $H_2 - O_2//He$ system

Strength of the shock χ is related to Mach number of the shock M_{sh}.

Regular Refraction with reflected Expansion (RRE)

Relation between strength χ and Mach number M_{sh}

$$\chi = 1/\xi_i$$

$$\xi_i = \frac{1 - \gamma_I + 2\gamma_I M_{sh}}{\gamma_I + 1}$$

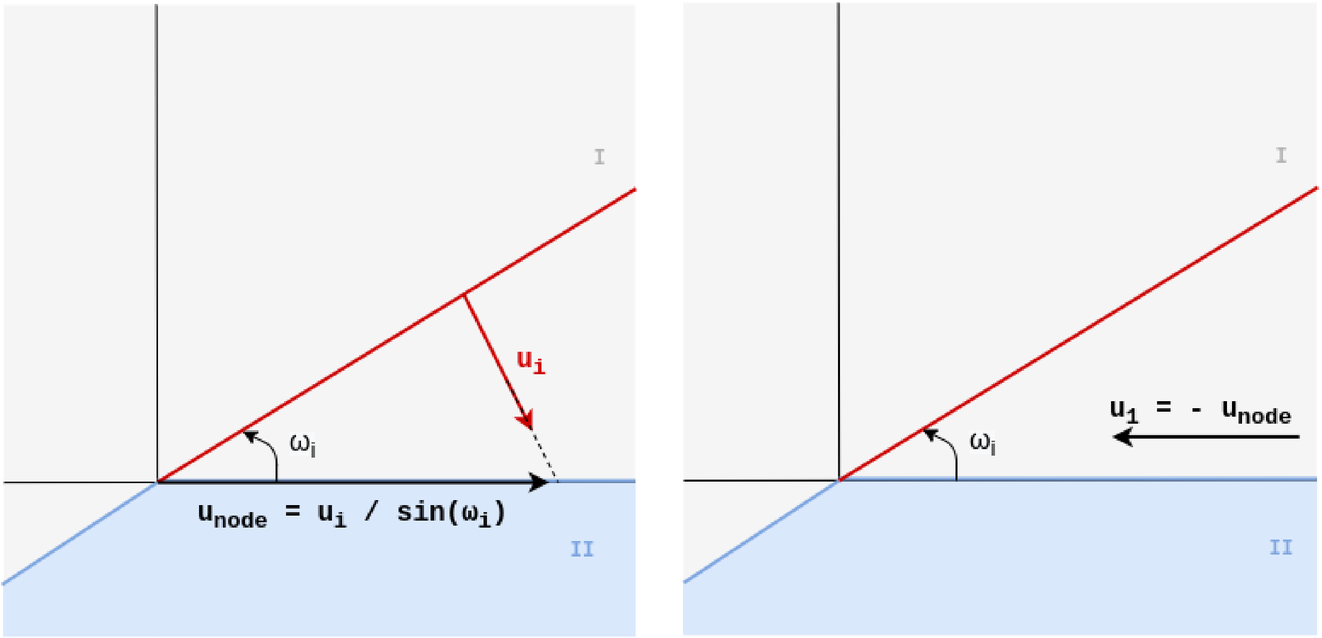
where γ_I is the heat ratio of phase I and M_{sh} is the normal component of the Mach number of the shock (see figure 5, slide 10). It is equal to M_{1n} the normal component of the Mach number of the incident flow. It is related to the Mach number of the flow by the following relation :

$$M_{sh} = \frac{M_1}{\sin(\omega_i)}$$

which finally leads to (with $\omega_i = 14.5°$):

$$\chi \in [0.01; 1] \iff M_{sh} \in [1; 2.95] \iff M_1 \in [4; 11.78] \tag{1}$$

Change and rotation of frame of reference



(a) Frame of reference of the tube : shock is moving at u_i

(b) Frame of reference of the shock : flow is moving at u_1

Figure 3: Change of frame of reference (see legend on next slide)

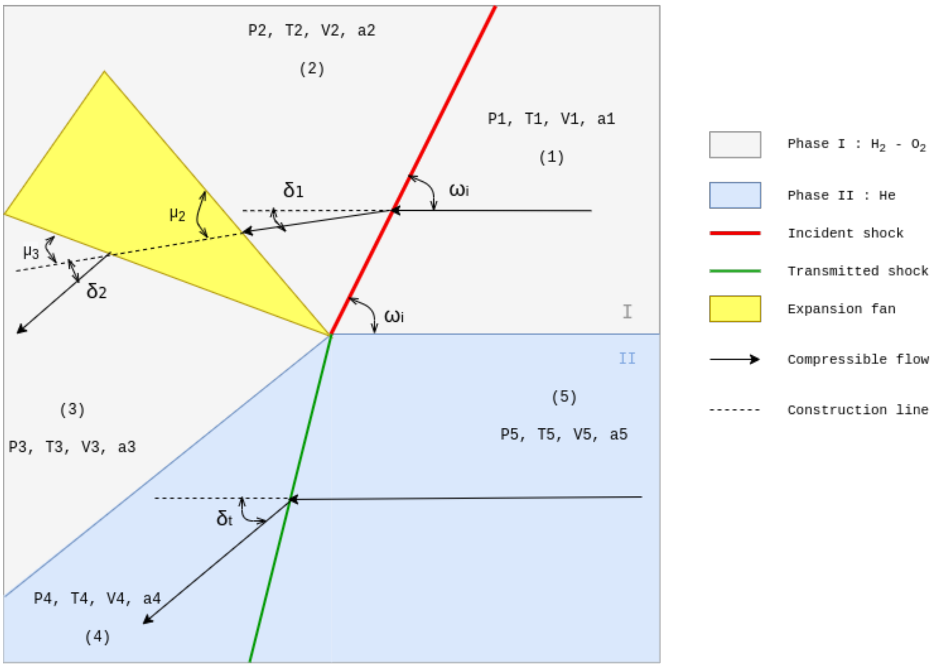


Figure 4: Symbols, zones and angles for computation

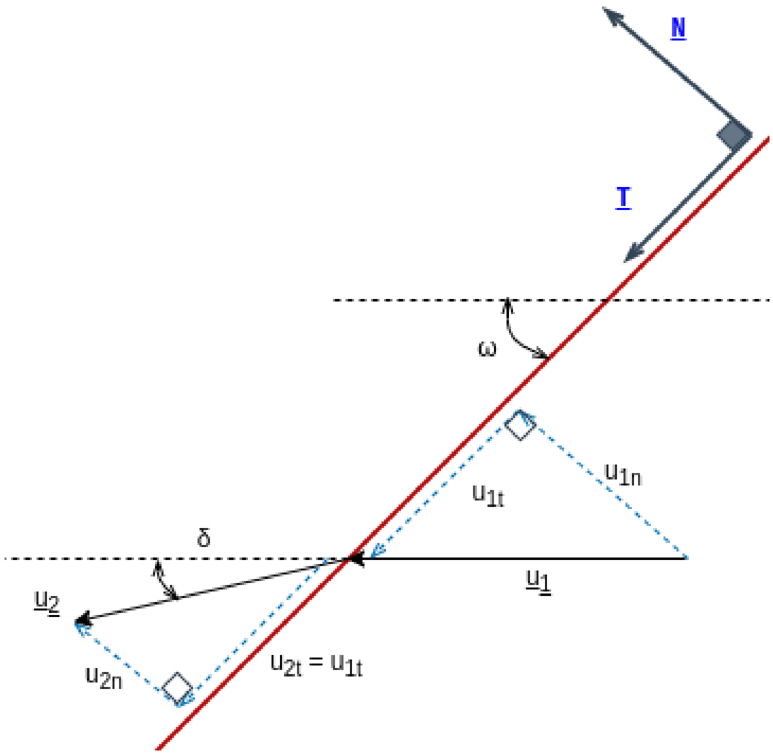
RRE - Theory and equations : gas dynamics

Initial conditions in zone (1)

Measure (unit)	Symbol	Value
Pressure (Pa)	$P1$	101 325
Temperature (K)	$T1$	600
Specific Volume	$V1$	$V1 = R_I * T1/P1$
Mach number	M1	6
Angle of incidence (deg)	ω_i	14.5
Normal mach number	$M1_n$	$M1_n = M1 \sin(\omega_i)$
Speed of sound (m/s)	a_1	$a_1 = \sqrt{\gamma_I R_I T1}$

Table 1: Initial conditions

Definition of normal Mach number



$$
\begin{aligned}
M1_n &= \frac{u1_n}{a_1} \\
&= \frac{u1\sin(\omega_i)}{a_1} \\
&= M1\sin(\omega_i)
\end{aligned}
$$

$$
\xi_i = \frac{1-\gamma_I+2\gamma_I M_{1n}}{\gamma_I+1}
$$

Figure 5: Geometry associated with the oblique shock

$$M1_n = M1\sin(\omega_i)$$

$$M2_n^2 = \frac{1+\frac{\gamma_I-1}{2}M1_n^2}{\gamma_I M1_n^2-\frac{\gamma_I-1}{2}}$$

$$\mathrm{M2} = \frac{M2_n}{\sin(\omega_i-\delta_1)}$$

$$\frac{V2}{V1} = \frac{(\gamma_I+1)M1_n^2}{2+(\gamma_I-1)M1_n^2}$$

$$\frac{P2}{P1} = 1+\frac{2\gamma_I}{\gamma_I+1}(M1_n^2-1)$$

$$\frac{T2}{T1} = \frac{P2}{P1}\frac{V2}{V1}$$

where δ_1 is the angle of deflection behind the incident shock.

$$\tan(\delta_1) = 2\cot(\omega_i)\frac{M1_n^2-1}{M1^2(\gamma_I+\cos 2\omega_i)+2}$$

RRE - Theory and equations : gas dynamics

From zone (5) to zone (4) : oblique shock of unknown angle

$$M5_n = M5\sin(\omega_t)$$

$$M4_n^2 = \frac{1+\frac{\gamma_{II}-1}{2}M5_n^2}{\gamma_{II}M5_n^2-\frac{\gamma_{II}-1}{2}}$$

$$M4 = \frac{M4_n}{\sin(\omega_t-\delta_t)}$$

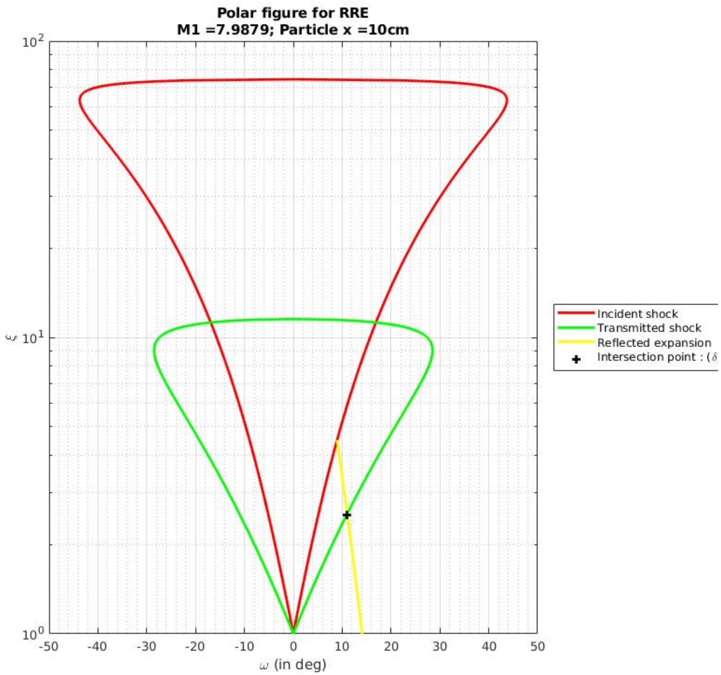
$$\frac{V4}{V5} = \frac{(\gamma_{II}+1)M5_n^2}{2+(\gamma_{II}-1)M5_n^2}$$

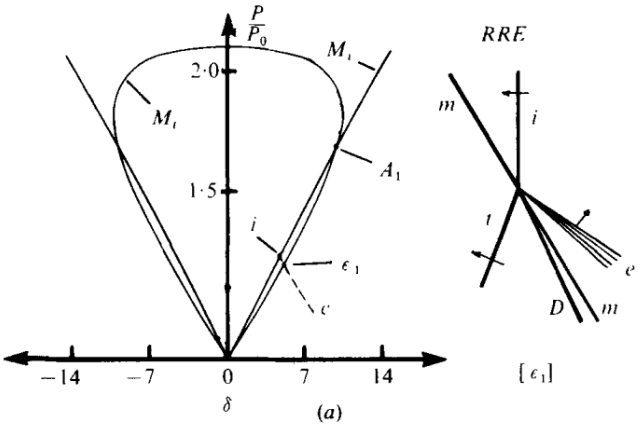
$$\frac{P4}{P5} = 1+\frac{2\gamma_{II}}{\gamma_{II}+1}(M5_n^2-1)$$

$$\frac{T4}{T5} = \frac{P4}{P5}\frac{V4}{V5}$$

where δ_t is the angle of deflection behind the transmitted shock. Unfortunately, ω_t, the angle between the transmitted shock and the flow in zone (5), remains unknown.

Membrane equilibrium between zone (4) and zone (3)





$$\delta_t = \delta_1 + \delta_2$$

$$P3 = P4$$

Figure 6: Polars for RRE structure, reference from Henderson et al. 1978 on the right

Thanks to the polars of the shock, δ_t and ξ_t can be determined (intersection of expansion and transmitted shock polar). $P4$ thus $P3$ are known and so $M3$ thanks to Prandtl-Meyer relations. ν is the Prandtl-Meyer function, depending on the heat ratio of the gas at stake.

$$\nu(M3) = \delta_2 + \nu(M2)$$

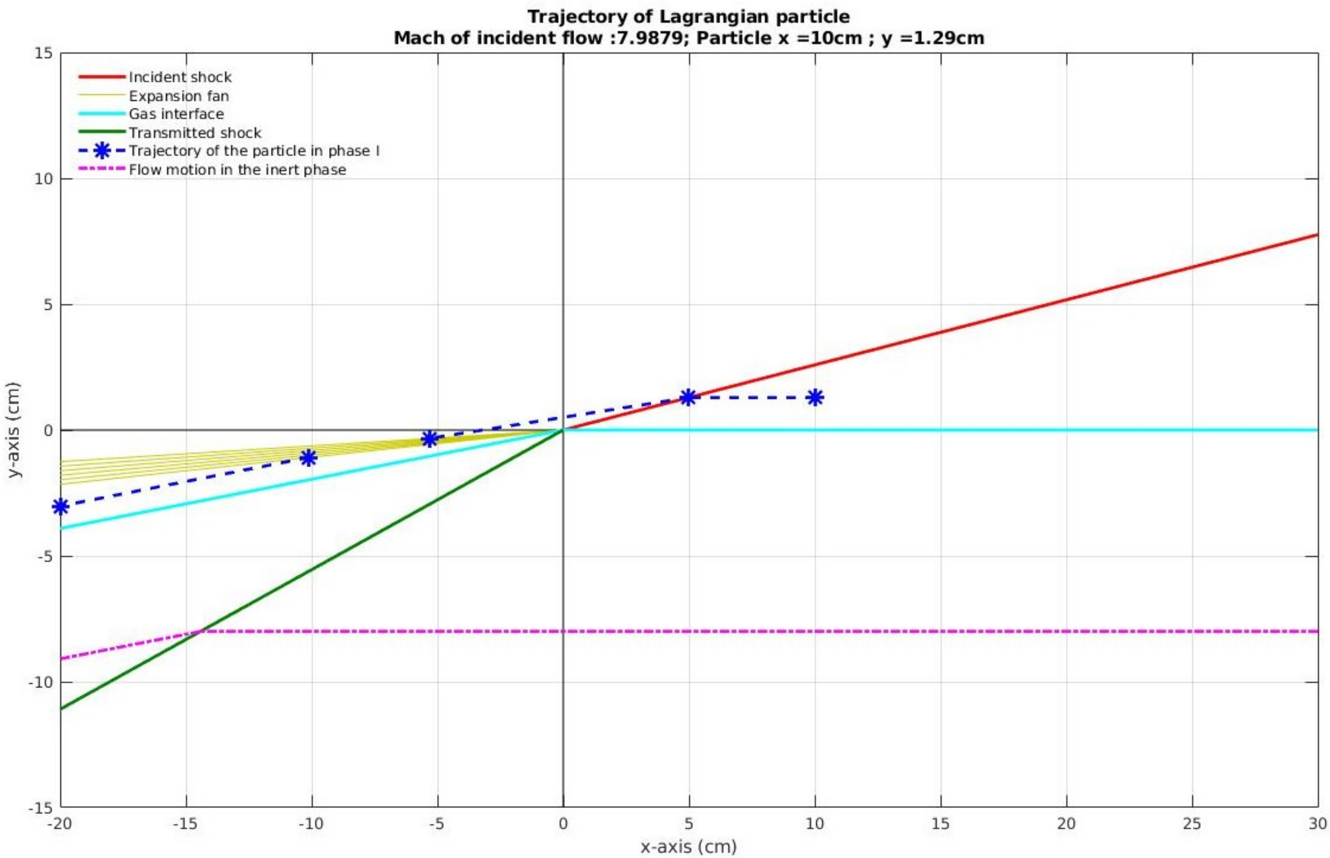
$$P3 = P4 = P5 \times \xi_t$$

$$T3 = T2\left(\frac{P3}{P2}\right)^{\frac{\gamma_I - 1}{\gamma_I}}$$

$$V3 = R_I \frac{T3}{P3}$$

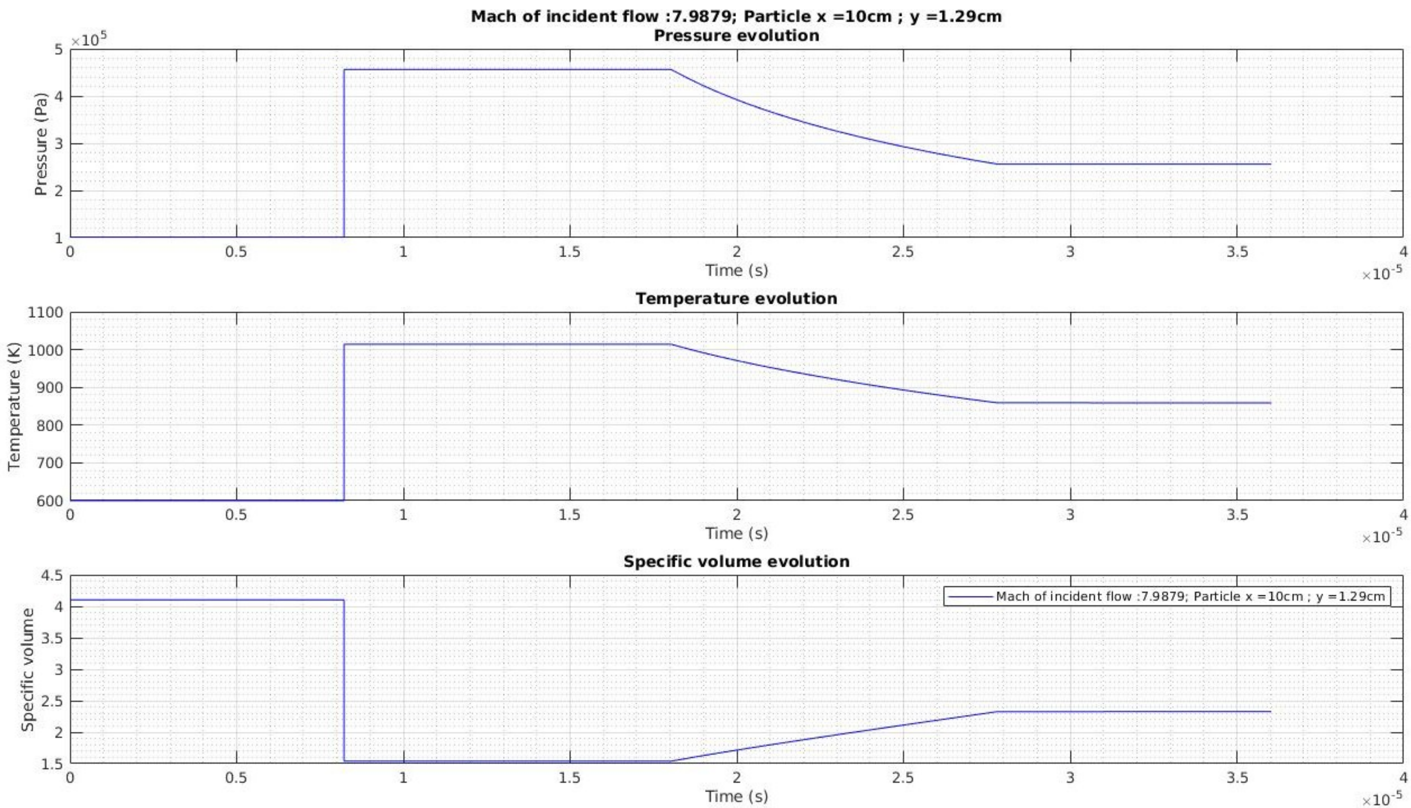
RRE - Results given by the inert gas dynamics theory

Evolution of a Lagrangian particle in the $H_2 - O_2$ phase



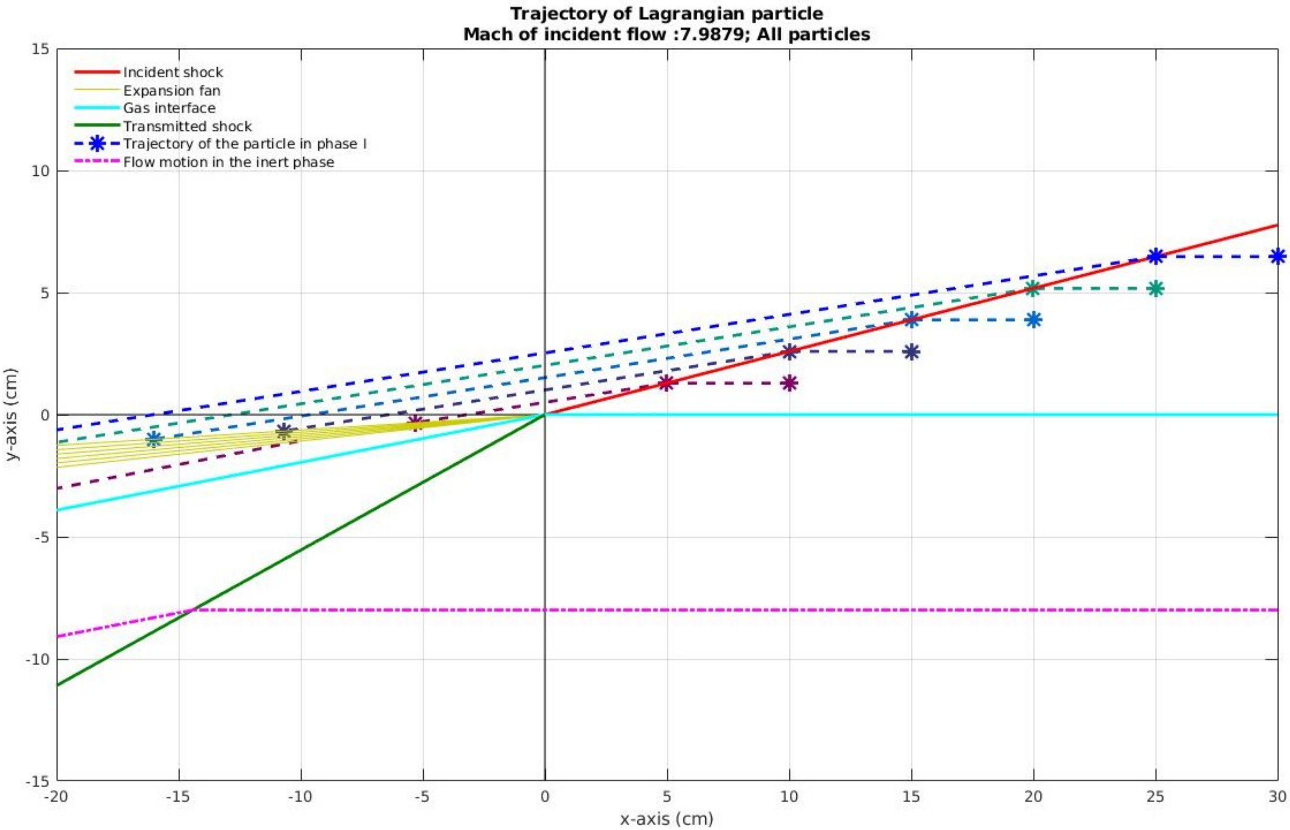
RRE - Results given by the inert gas dynamics theory

Evolution of a Lagrangian particle in the $H_2 - O_2$ phase



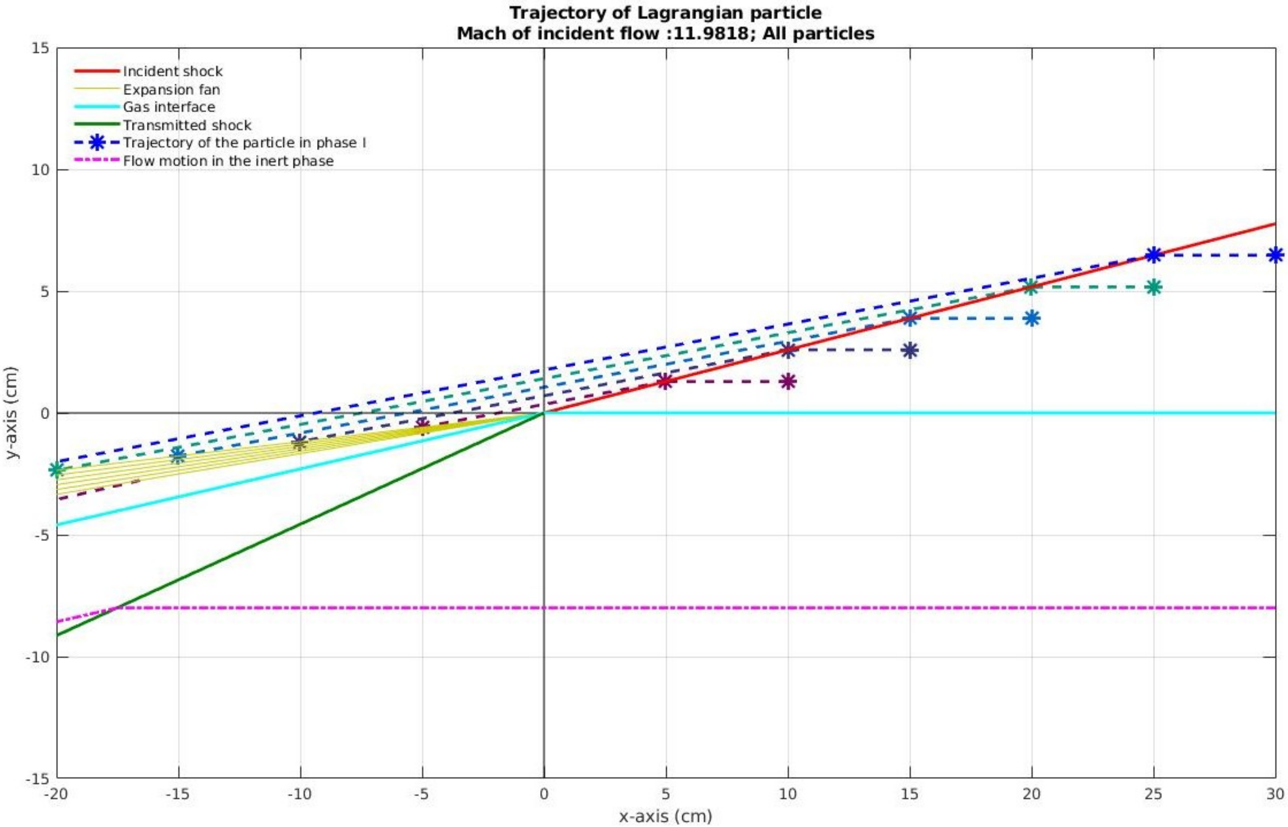
RRE - Results given by the inert gas dynamics theory

Evolution depends on position of the particle...



RRE - Results given by the inert gas dynamics theory

Evolution depends on position of the particle...



RRE - Results given by the inert gas dynamics theory

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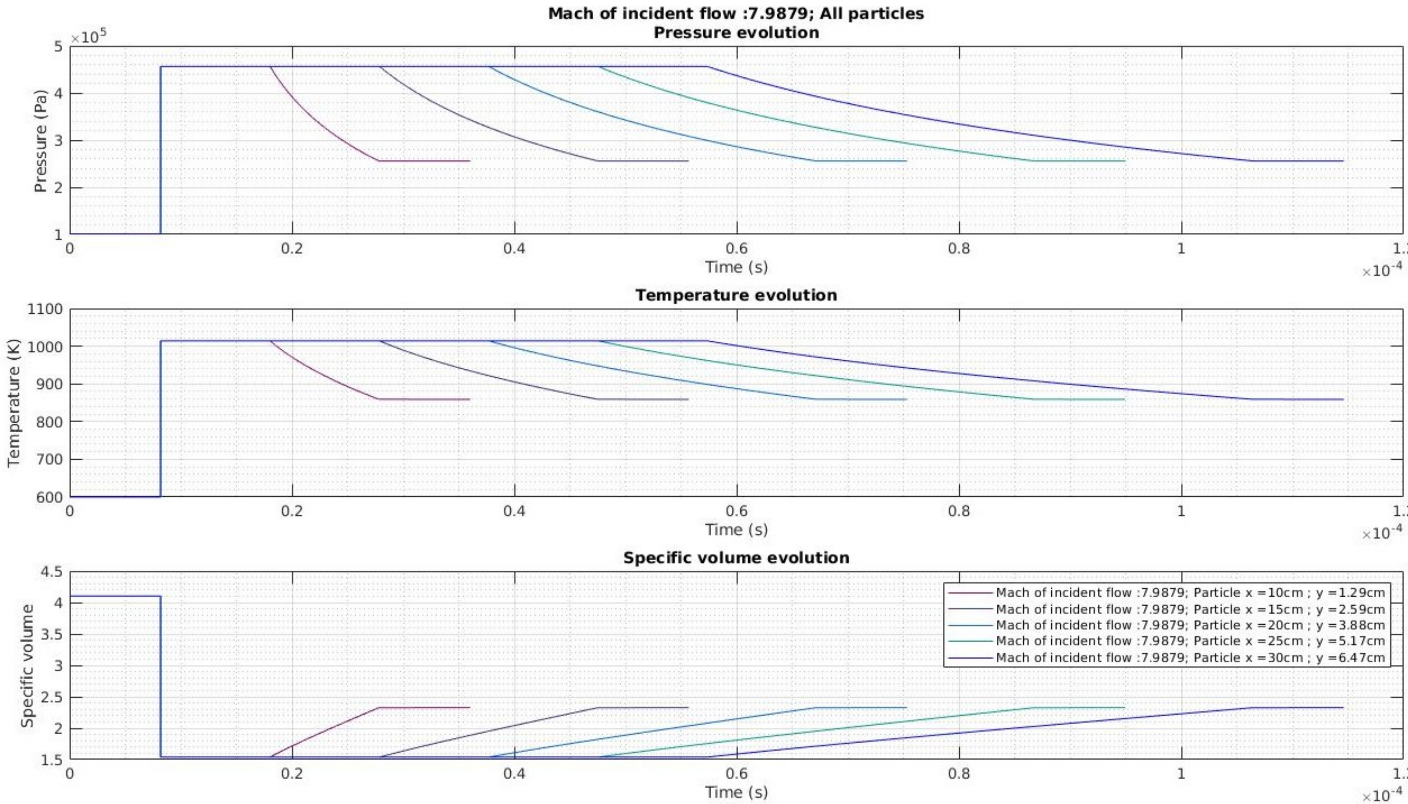

RRE - Results given by the inert gas dynamics theory

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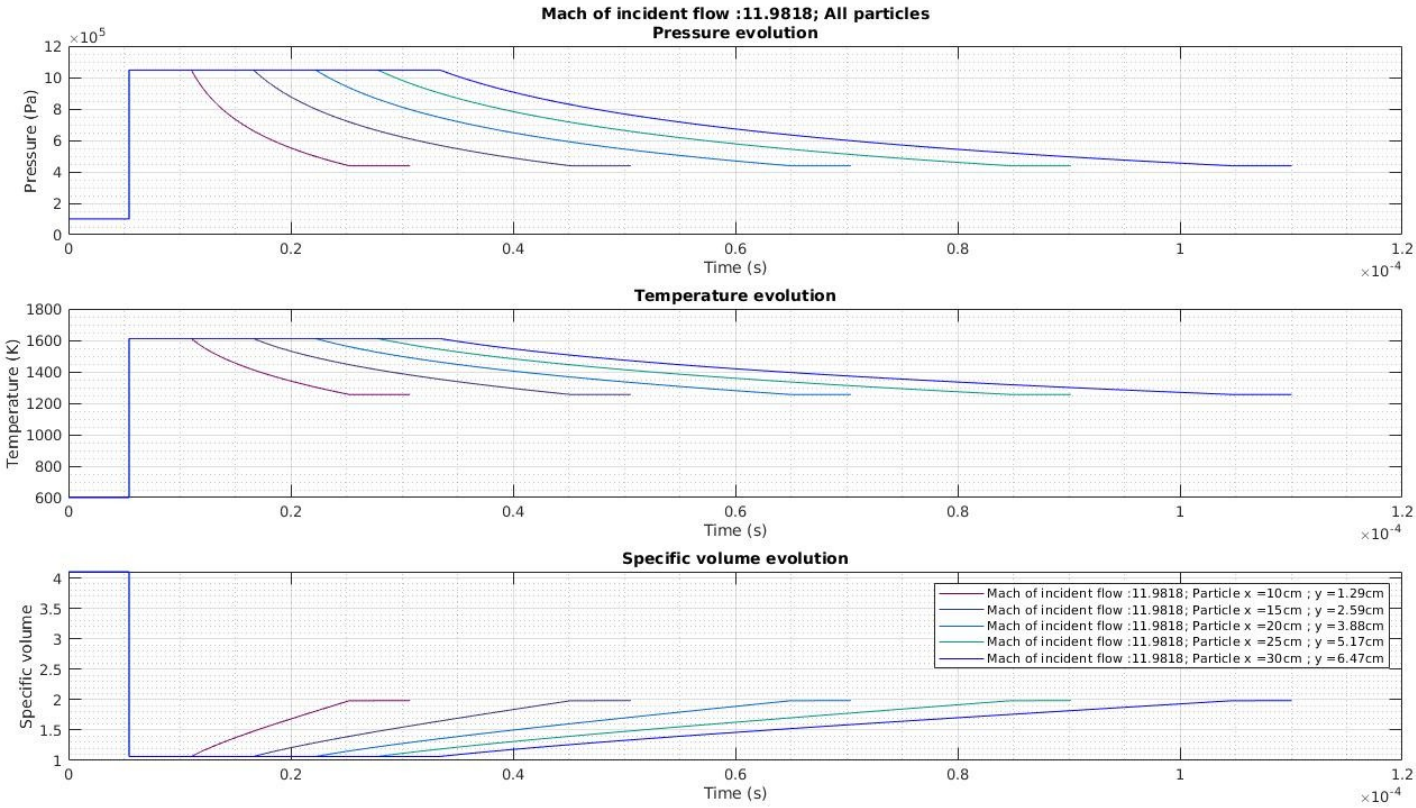
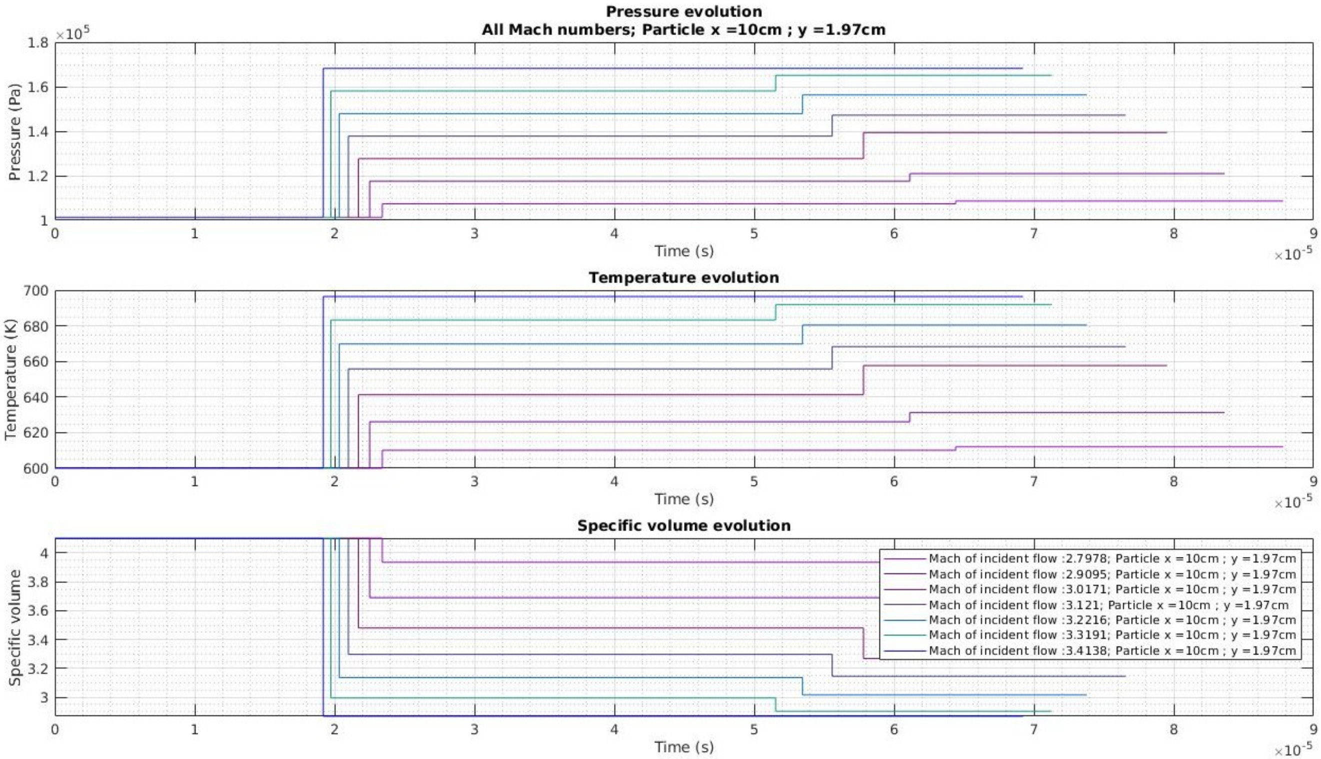


RRE - Results given by the inert gas dynamics theory

... and on Mach number of the shock



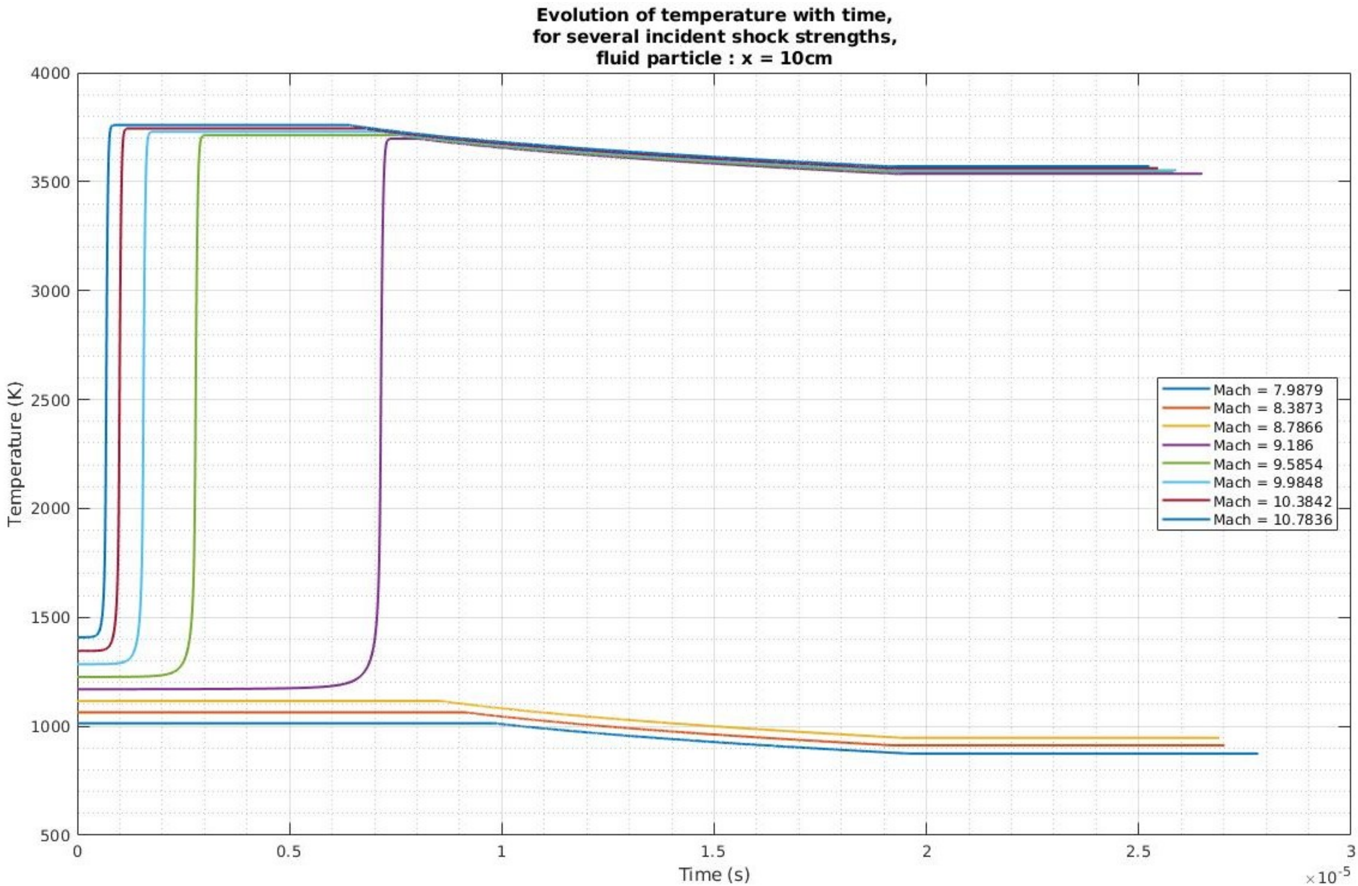
For a single particle, increasing M_{sh} leads to a rise in temperature and pressure : don't forget that $H_2 - O_2$ is a reactive phase. Is an ignition possible under certain conditions?

RRE - Main steps for chemistry calculus

- Calculate evolution of specific volume of different Lagrangian particles, for $\omega_i = 14.5°$, for different Mach numbers (see slide 29).
- Use of CHEMKIN II to calculate chemical reactions in the reactive phase
- Outputs of CHEMKIN II : evolution of pressure, temperature and ratios of chemical species
- Temperature jump in CHEMKIN II output = detonation

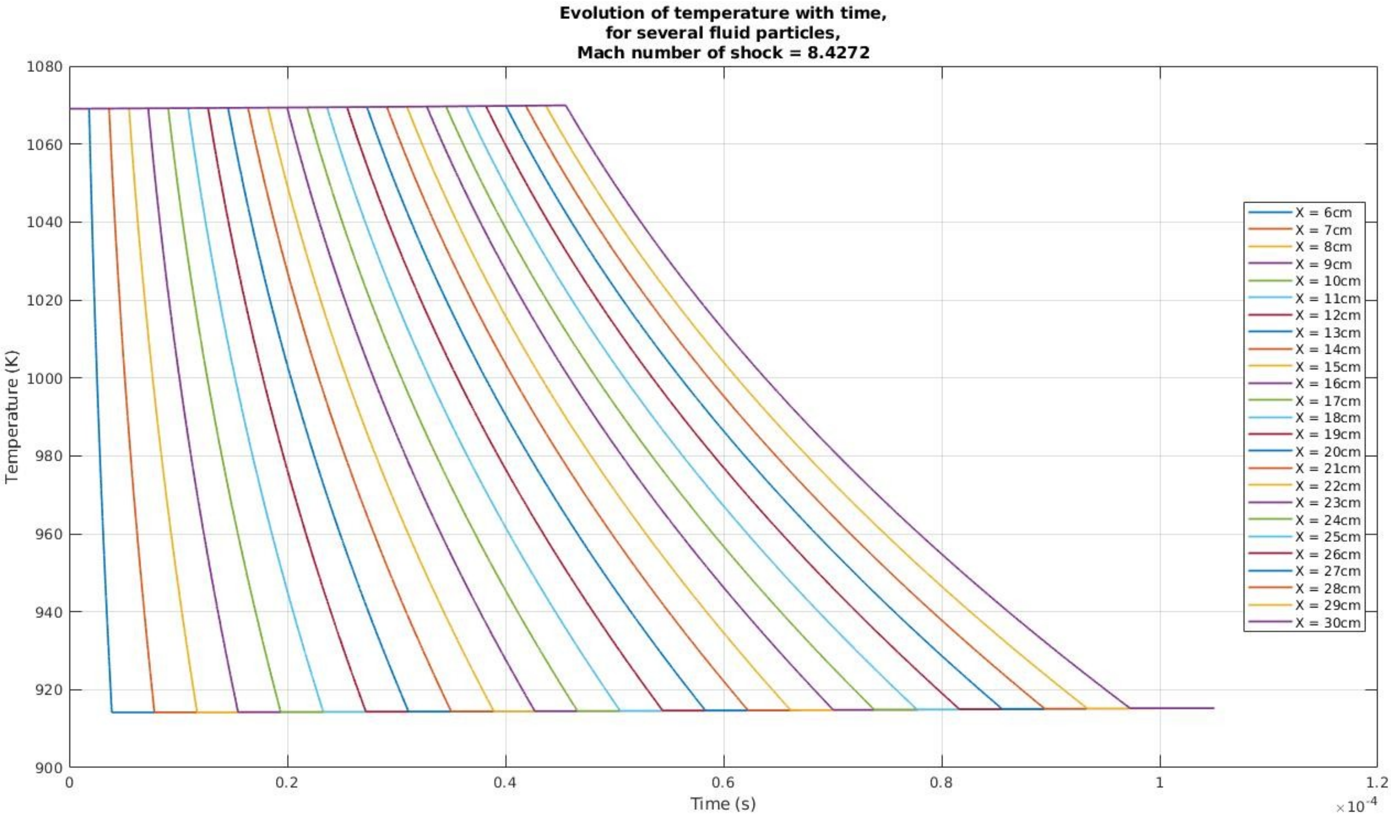
RRE - Results of the chemical calculus

Existence of a threshold : Mach number of ignition



RRE - Results of the chemical calculus

Each particle has its own M_{ignit}



RRE - Results of the chemical calculus

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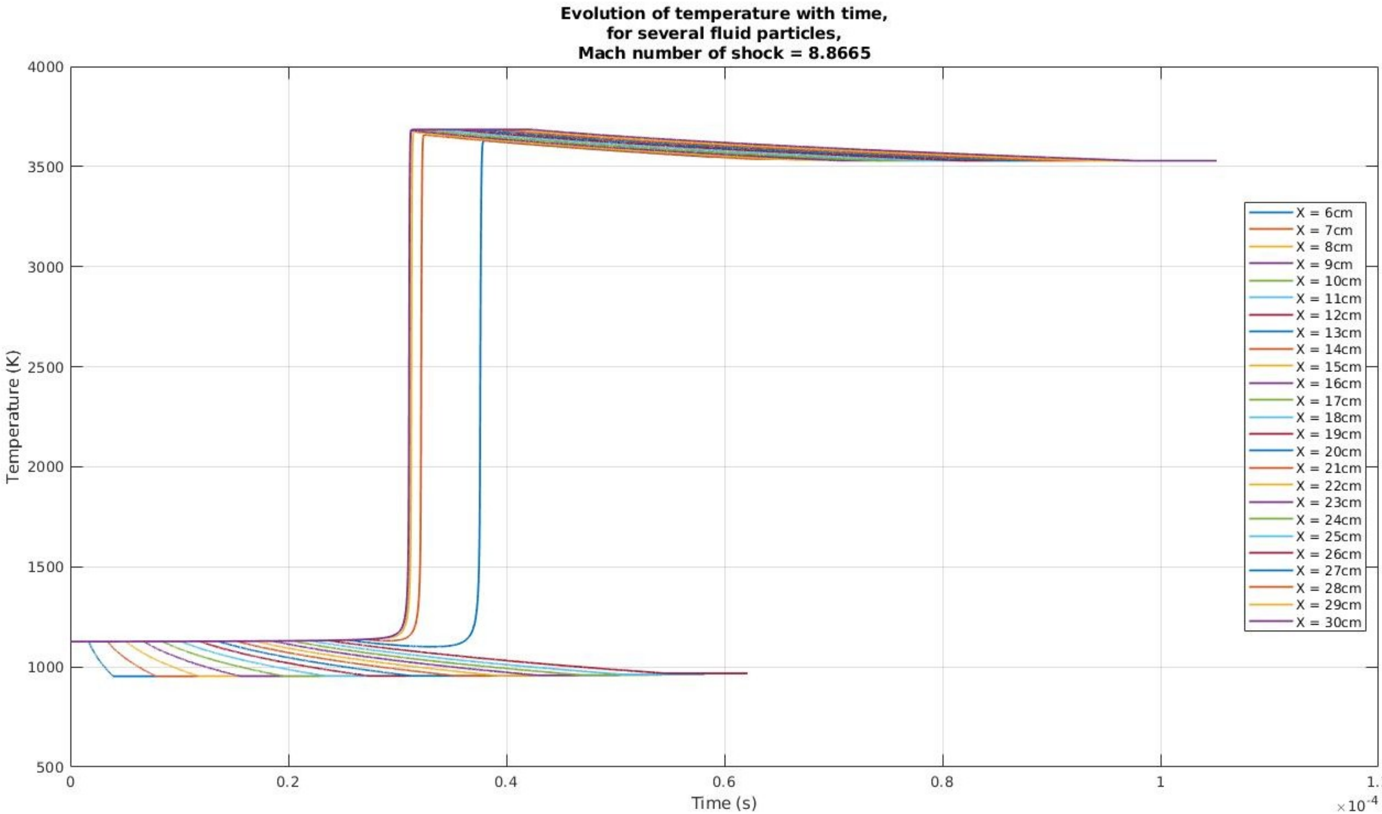
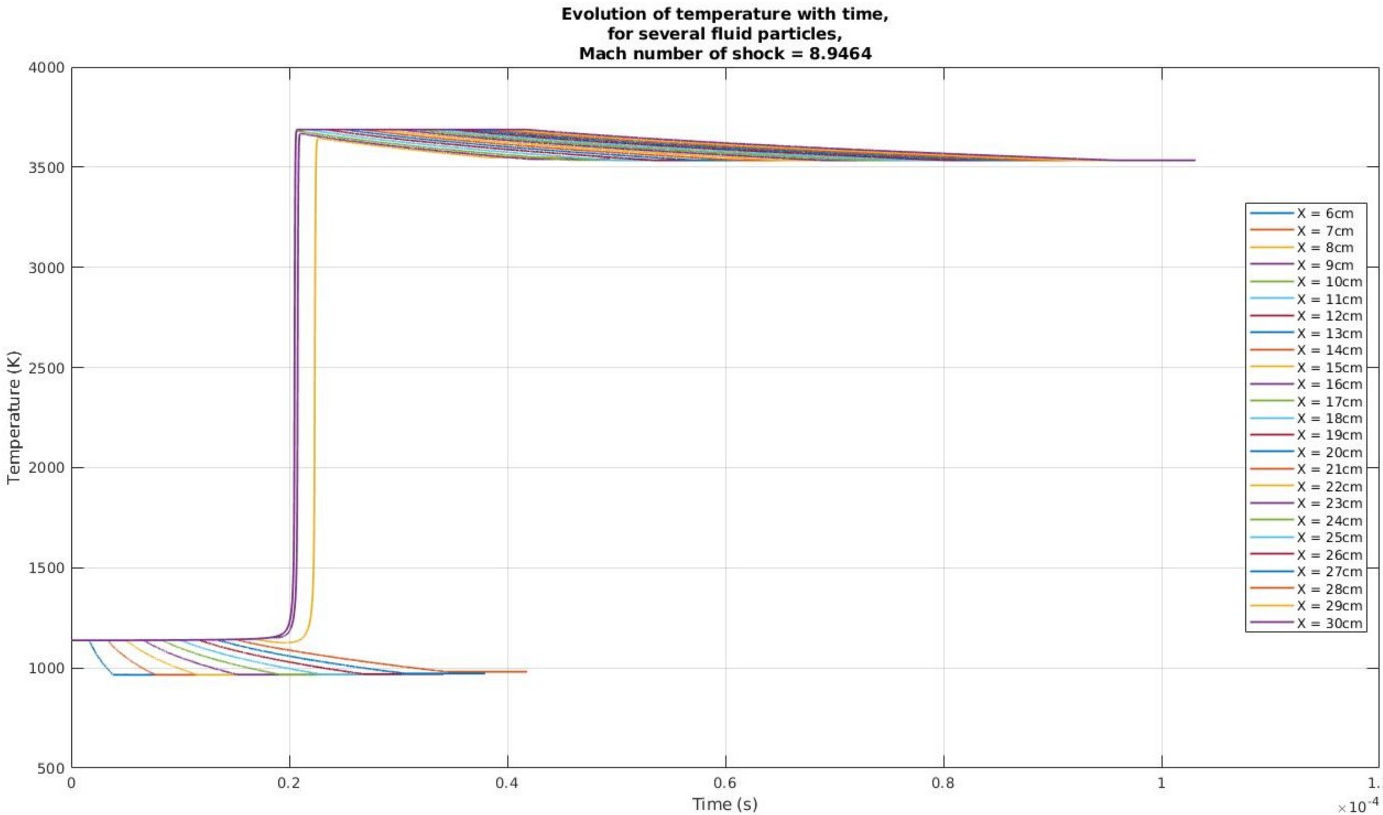


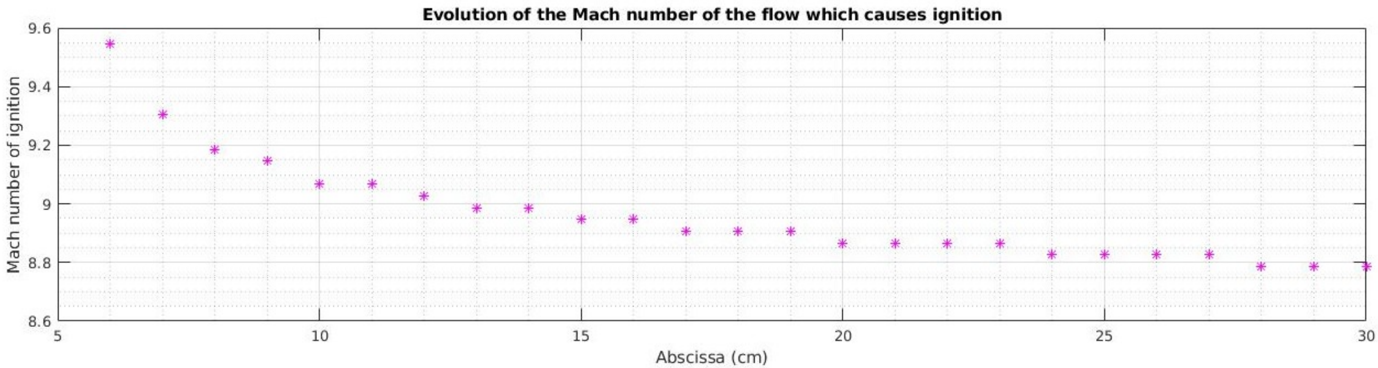
RRE - Results of the chemical calculus

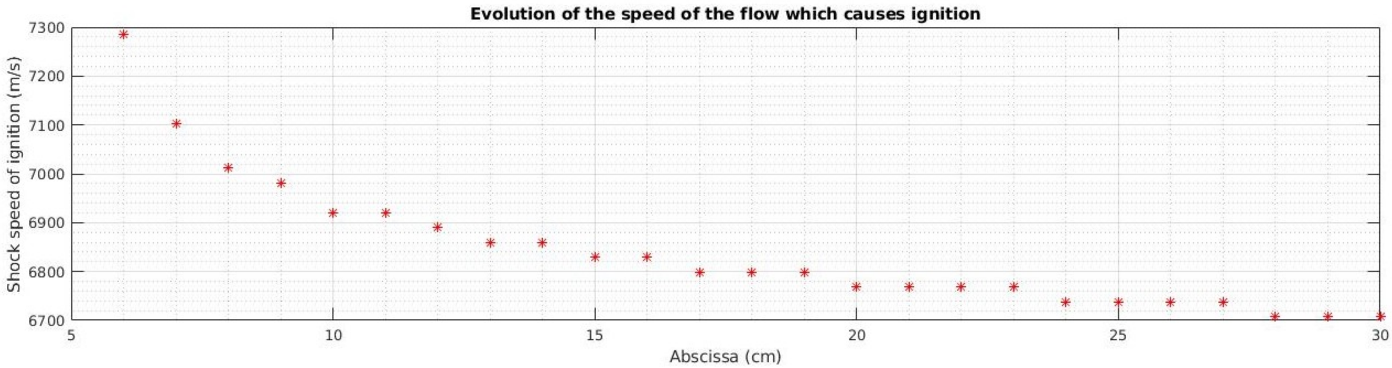
Each particle has its own M_{ignit}



RRE - Results of the chemical calculus

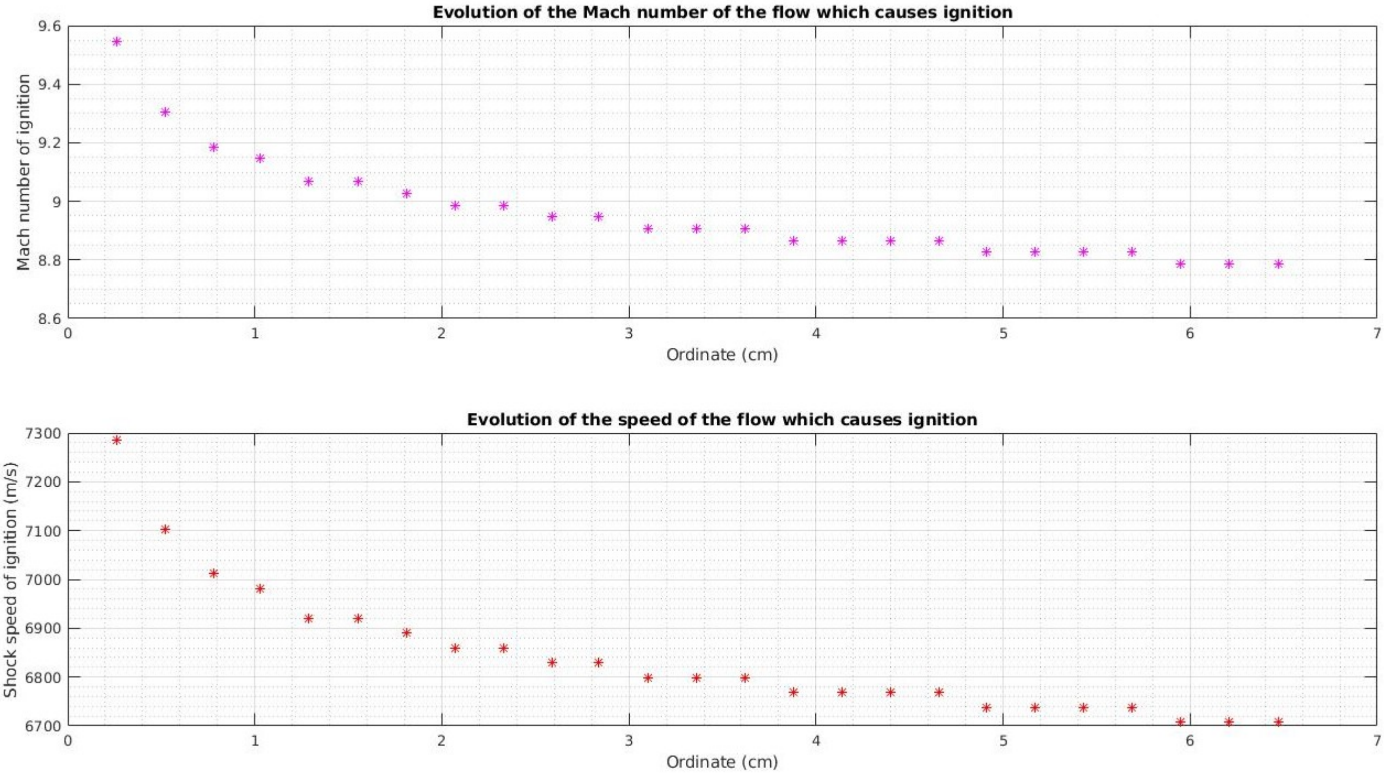
M_{sh} plotted as a function of ordinate of the particle





RRE - Results of the chemical calculus

M_{sh} plotted as a function of ordinate of the particle



Regular Refraction with Reflected Shock (RRR)

Strength χ and Mach number M_{sh} domains

For this structure, an angle of incidence of $\omega_i = 21.5°$ has been chosen so as to explore the largest range of Mach numbers which finally leads to :

$$\chi \in [0.6; 1] \iff M_{sh} \in [1; 1.26] \iff M_1 \in [2.76; 3.41] \tag{2}$$

The method employed for this structure will be highly similar to the method used for the RRE structure, and even simpler because of the reflected wave : a shock instead of an expansion fan.

Change and rotation of frame of reference

The change and rotation of frame of reference keeps the same and the symbols used for this structure are very similar with the previous ones : here is a little reminder

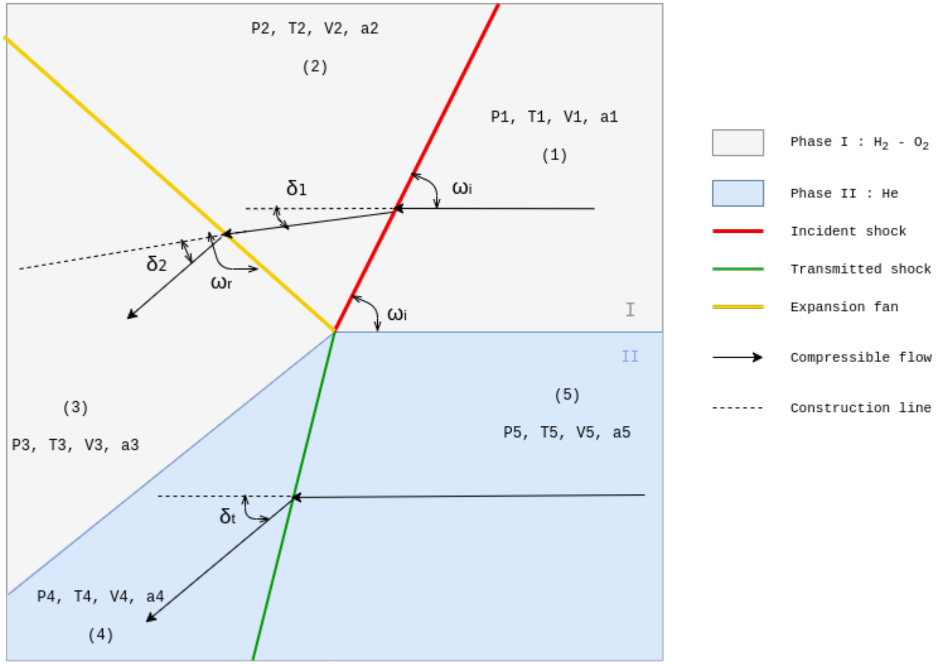


Figure 7: Symbols, zones and angles for computation

RRR - Theory and equations : gas dynamics

Initial conditions in zone (1)

Measure (unit)	Symbol	Value
Pressure (Pa)	$P1$	101 325
Temperature (K)	$T1$	600
Specific Volume	$V1$	$V1 = R_I * T1/P1$
Mach number	M1	6
Angle of incidence (deg)	ω_i	21.5
Normal mach number	$M1_n$	$M1_n = M1 \sin(\omega_i)$
Speed of sound (m/s)	a_1	$a_1 = \sqrt{\gamma_I R_I T1}$

Table 2: Initial conditions

Definition of the normal Mach number $M1_n$ is not changed.

RRR - Theory and equations : gas dynamics

From zone (1) to zone (2): oblique shock

$$
\begin{aligned}
M1_n &= M1 \sin(\omega_i) & \frac{V2}{V1} &= \frac{(\gamma_I+1)M1_n^2}{2+(\gamma_I-1)M1_n^2} \\
M2_n^2 &= \frac{1+\frac{\gamma_I-1}{2}M1_n^2}{\gamma_I M1_n^2-\frac{\gamma_I-1}{2}} & \frac{P2}{P1} &= 1+\frac{2\gamma_I}{\gamma_I+1}(M1_n^2-1) \\
\mathrm{M2} &= \frac{M2_n}{\sin(\omega_i-\delta_1)} & \frac{T2}{T1} &= \frac{P2}{P1}\frac{V2}{V1}
\end{aligned}
$$

where δ_1 is the angle of deflection behind the incident shock.

$$
\tan(\delta_1) = 2\cot(\omega_i)\frac{M1_n^2-1}{M1^2(\gamma_I+\cos 2\omega_i)+2}
$$

RRR - Theory and equations : gas dynamics

From zone (5) to zone (4) : oblique shock of unknown angle

$$M5_n = M5\sin(\omega_t)$$

$$M4_n^2 = \frac{1+\frac{\gamma_{II}-1}{2}M5_n^2}{\gamma_{II}M5_n^2-\frac{\gamma_{II}-1}{2}}$$

$$M4 = \frac{M4_n}{\sin(\omega_t-\delta_t)}$$

$$\frac{V4}{V5} = \frac{(\gamma_{II}+1)M5_n^2}{2+(\gamma_{II}-1)M5_n^2}$$

$$\frac{P4}{P5} = 1+\frac{2\gamma_{II}}{\gamma_{II}+1}(M5_n^2-1)$$

$$\frac{T4}{T5} = \frac{P4}{P5}\frac{V4}{V5}$$

where δ_t is the angle of deflection behind the transmitted shock. Unfortunately, ω_t, the angle between the transmitted shock and the flow in zone (5), remains unknown.

Membrane equilibrium between zone (4) and zone (3)

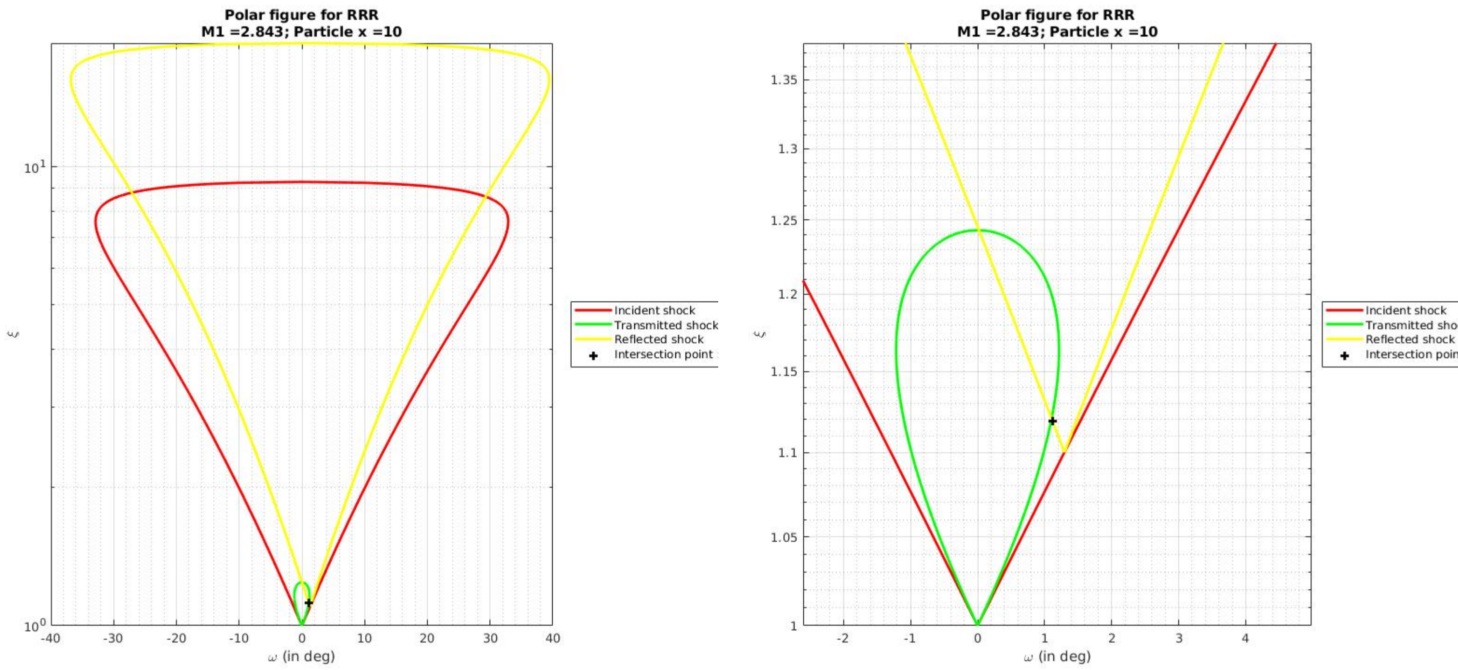
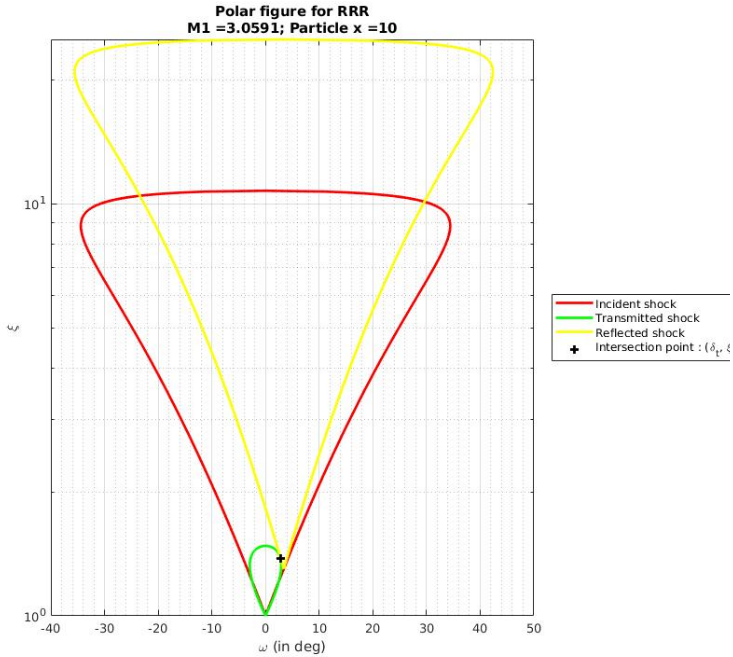
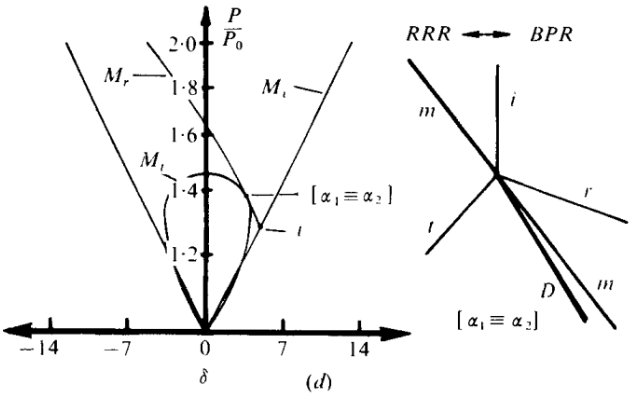


Figure 8: Polars for RRR structure, zoom on the intersection point on the right

Membrane equilibrium between zone (4) and zone (3)





$$\delta_t = \delta_1 + \delta_2$$

$$P3 = P4$$

Figure 9: Polars for RRR structure, reference from Henderson et al. 1978 on the right

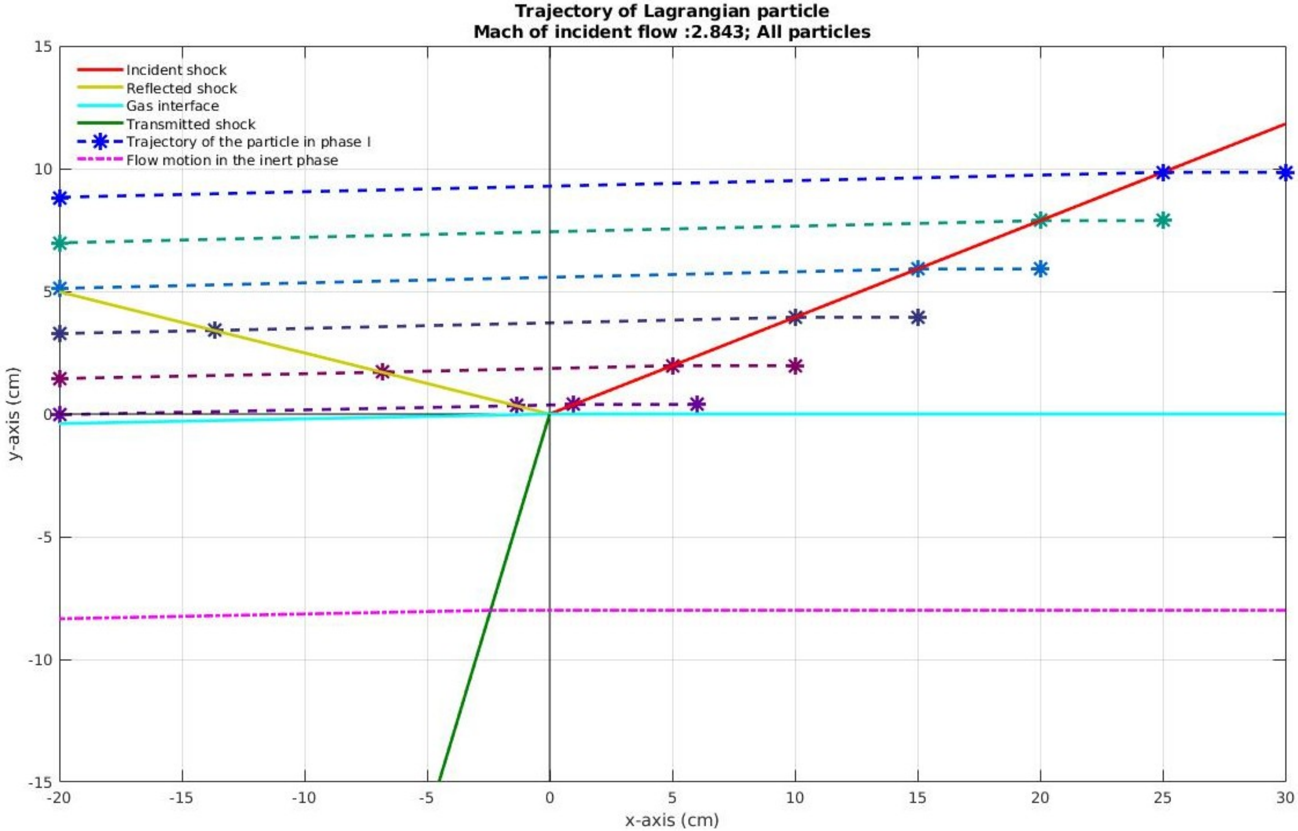
RRR - Theory and equations : gas dynamics

From zone `(2)` to zone `(3)` : oblique shock of unknown angle

Thanks to the polars of the shock, δ_t and ξ_t can be determined (intersection of expansion and transmitted shock polar) : $P4$ thus $P3$ are known. Then $\xi_r = \frac{P3}{P2}$ can be computed and all quantities in zone `(3)` can be easily deduced.

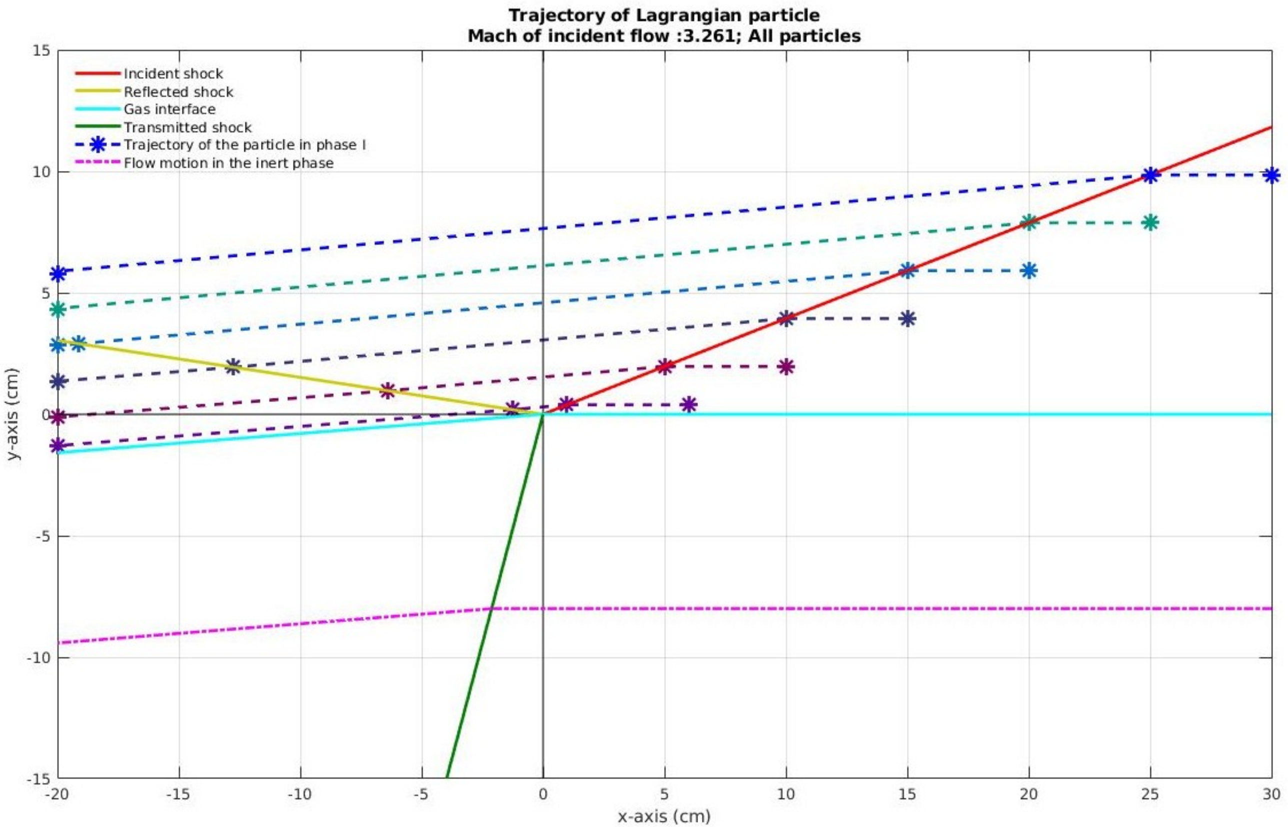
RRR - Results given by the inert gas dynamics theory

Evolution depends on position of the particle...



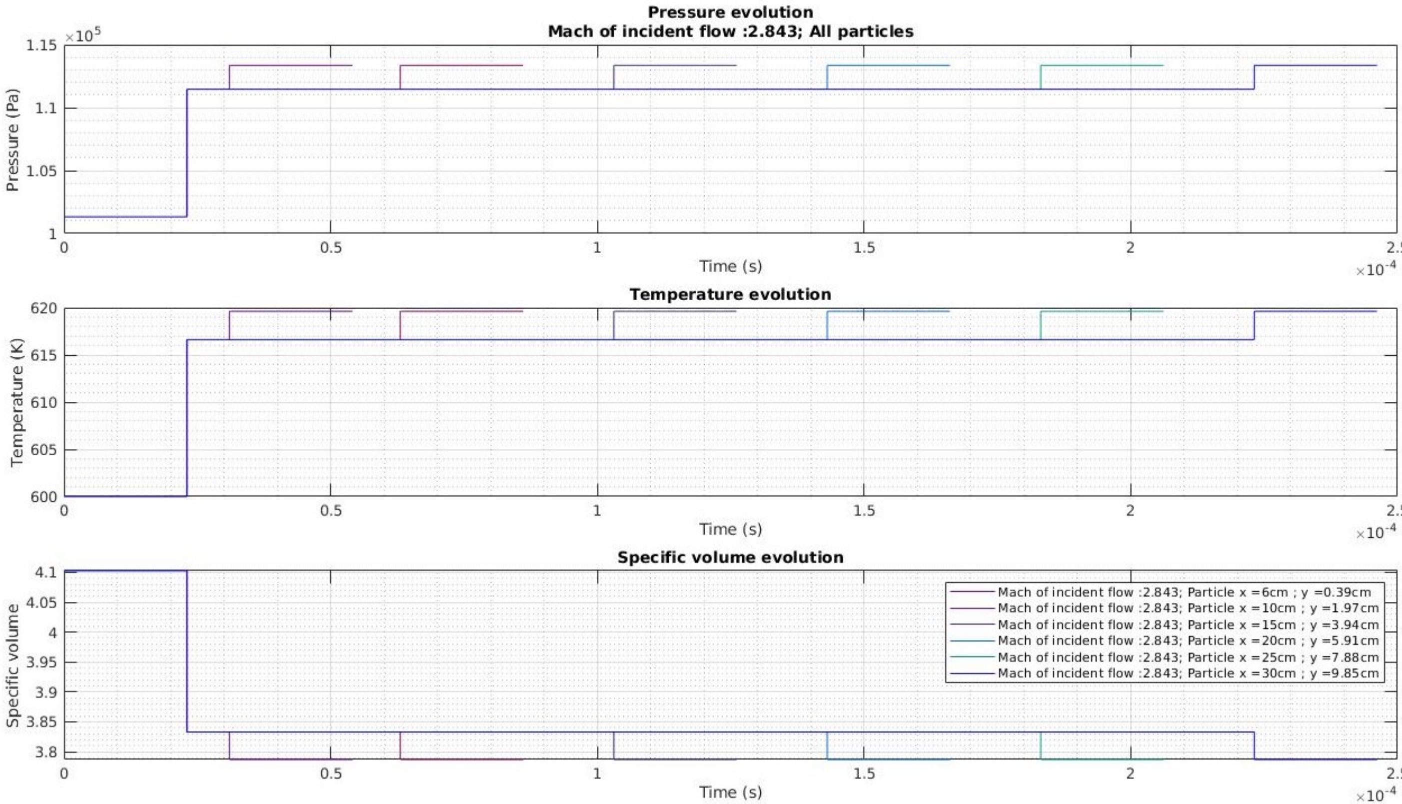
RRR - Results given by the inert gas dynamics theory

Evolution depends on position of the particle...



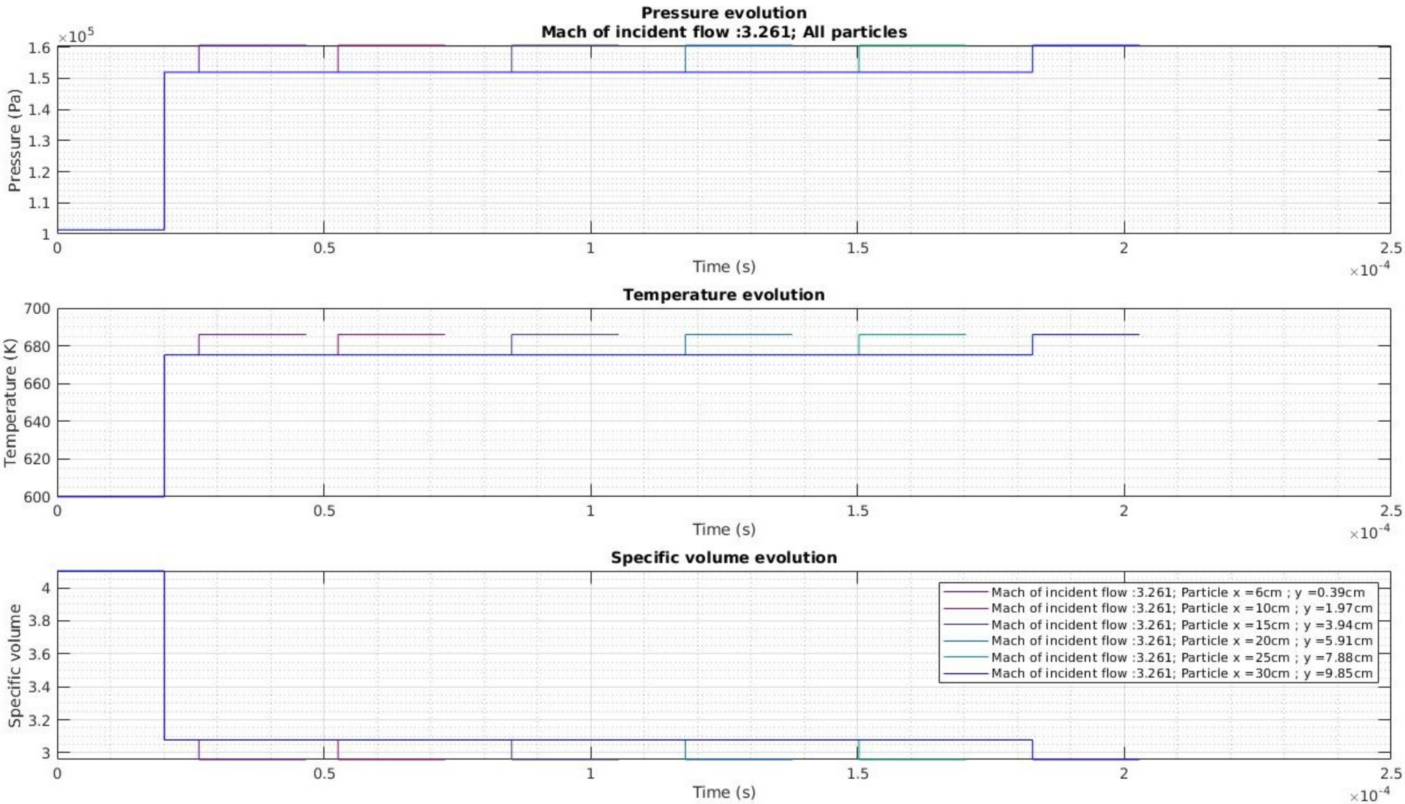
RRR - Results given by the inert gas dynamics theory

Evolution depends on position of the particle...



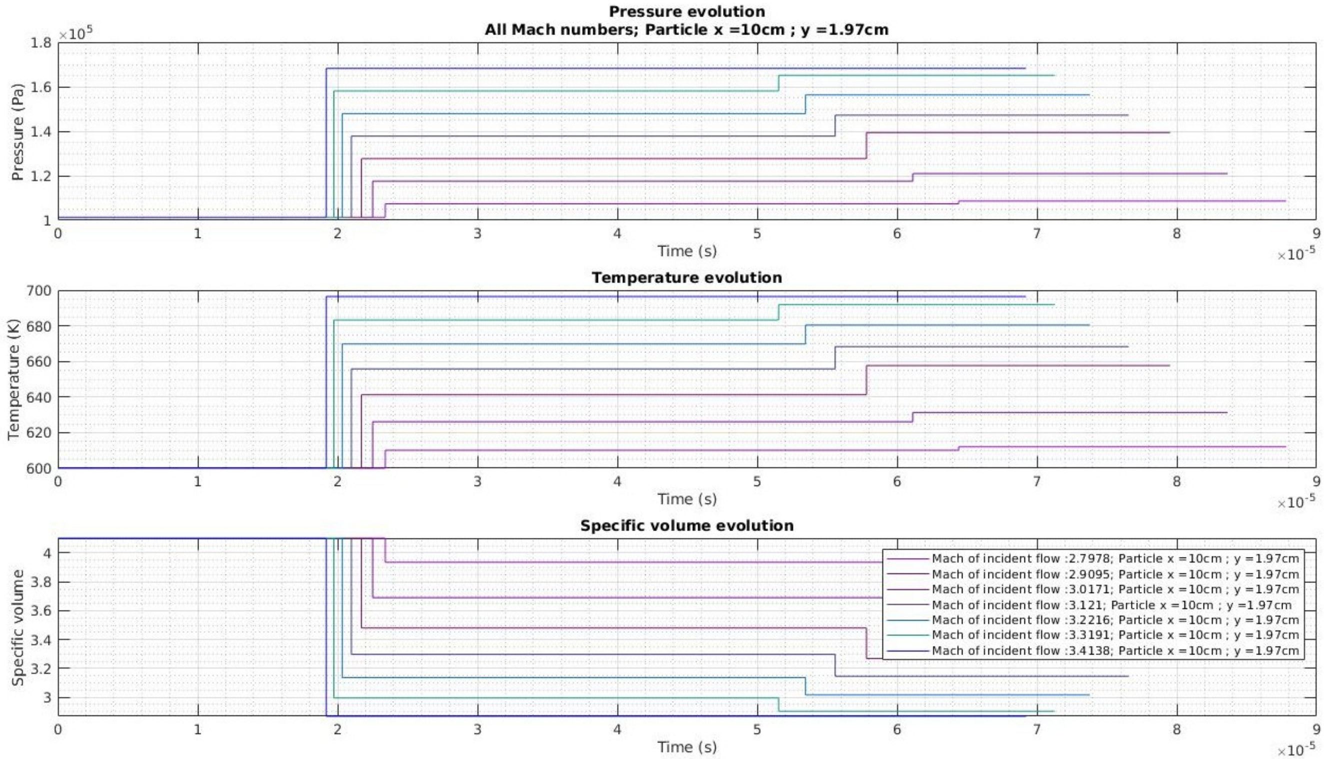
RRR - Results given by the inert gas dynamics theory

Evolution depends on position of the particle...



Results given by the inert gas dynamics theory

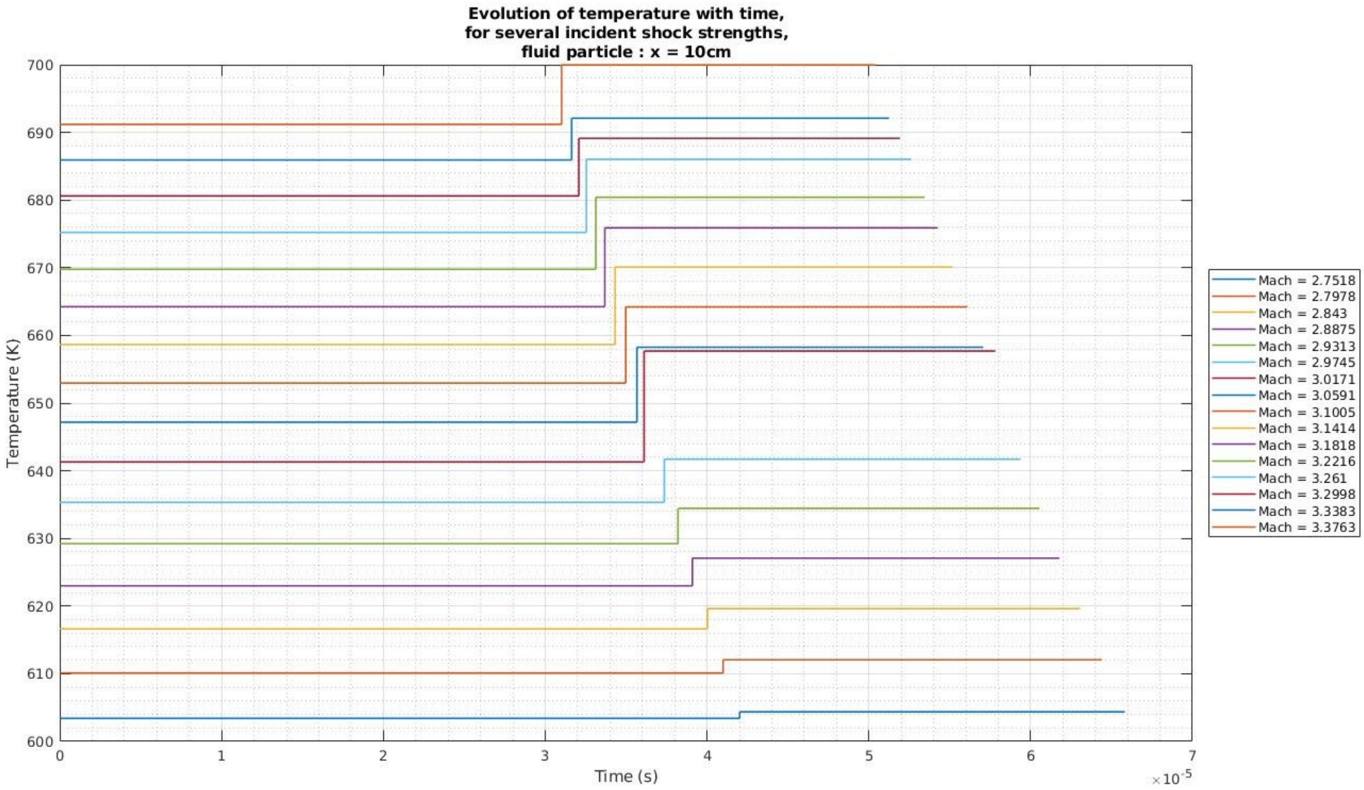
… and on Mach number of the shock



For a single particle, increasing M_{sh} leads to a rise in temperature and pressure : don't forget that $H_2 - O_2$ is a reactive phase. Is an ignition possible under certain conditions?

RRR - Results of the chemical calculus

Ignition doesn't occur for such low Mach numbers



RRR - Results of the chemical calculus

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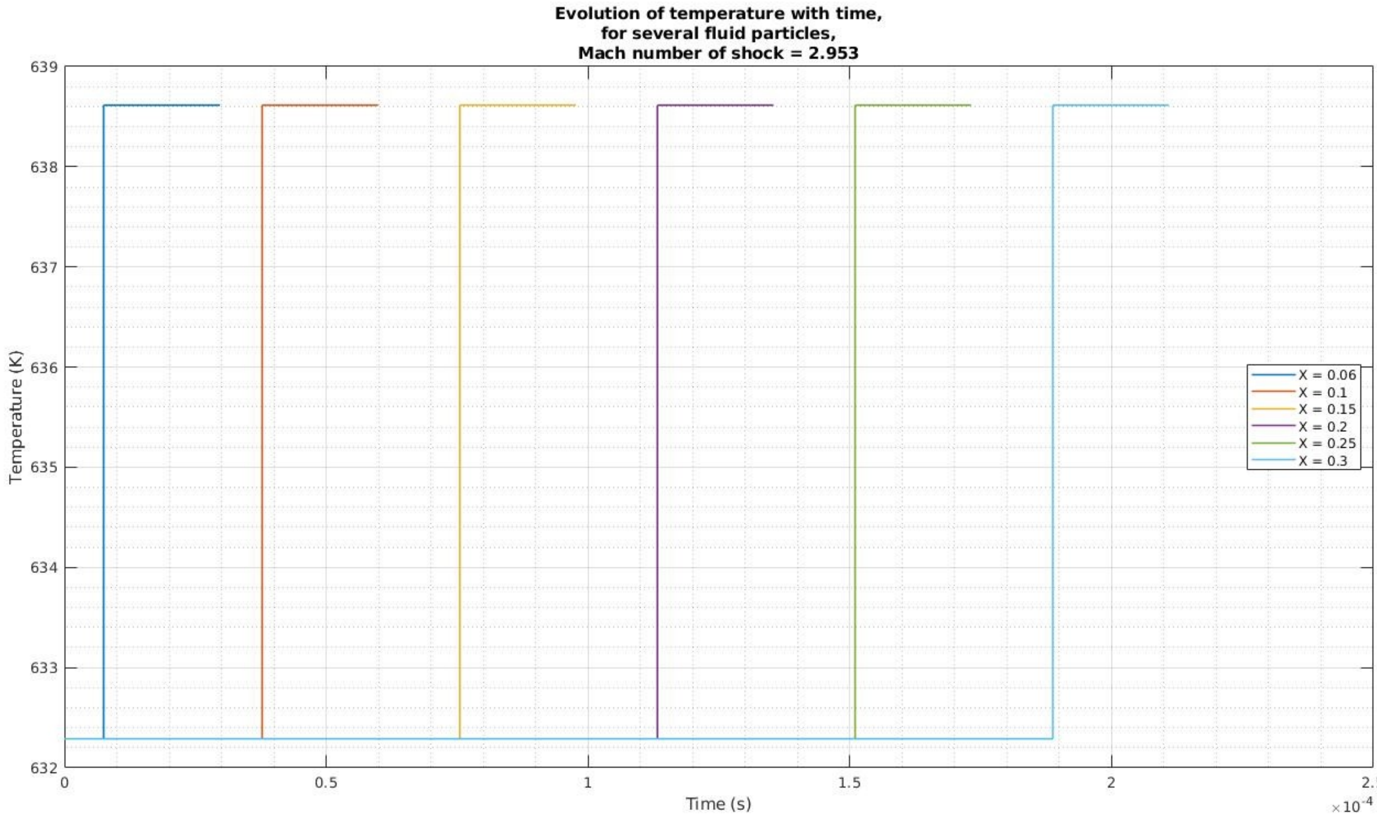

RRR - Results of the chemical calculus

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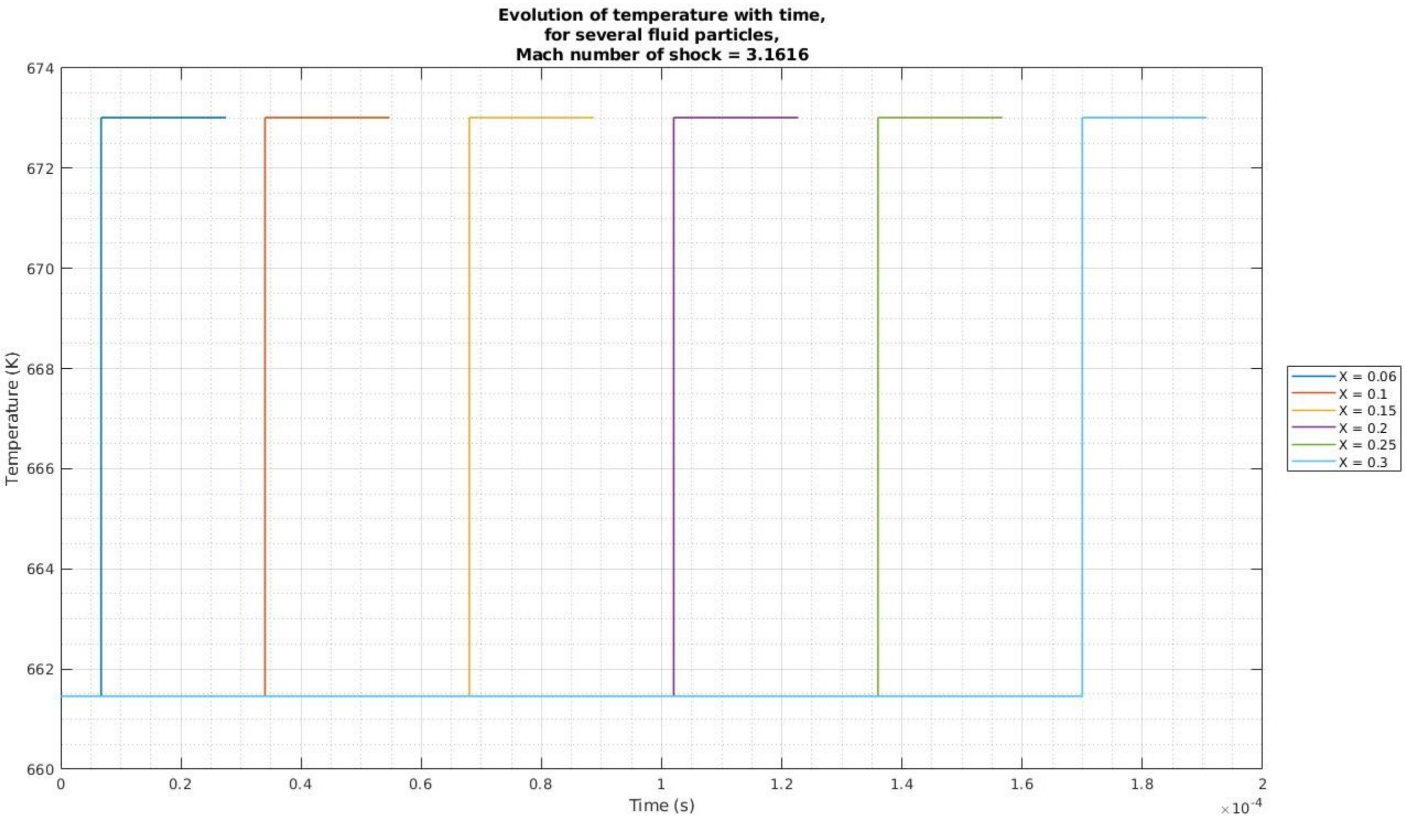

RRR - Results of the chemical calculus

Ignition doesn't occur for such low Mach numbers