Lagrangian particles evolution through RRE refraction structure

M. Dutreuilh

Tsinghua University

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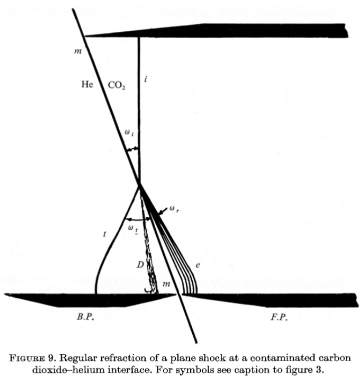
Outline

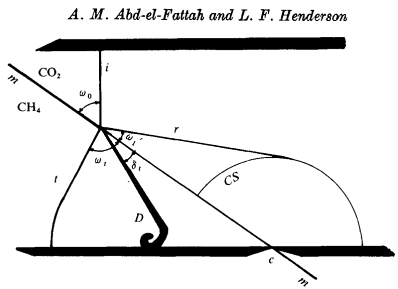
1. Reminder of the different refraction structures
2. Regular Refraction with reflected Expansion (RRE)
3. Theory and equations : gas dynamics
4. Results given by the inert gas dynamics theory
5. Use of CHEMKIN II to compute chemistry calculus
6. Results of the chemical calculus

Reminder of the different refraction structures

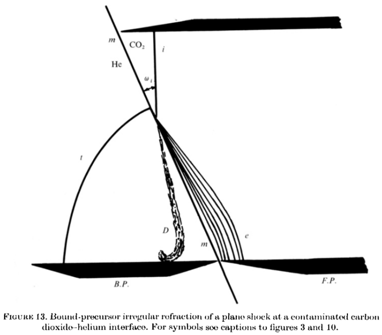
From Henderson 1976 and 1978



(a) RRE struture



(b) RRR struture



(c) BPR structure

Figure 1: Three of the already known refraction structures : RRE and BPR schemes are from Henderson, Abd-el-Fattah & Lozzi 1976; RRR scheme is from Henderson & Abd-el-Fattah 1978.

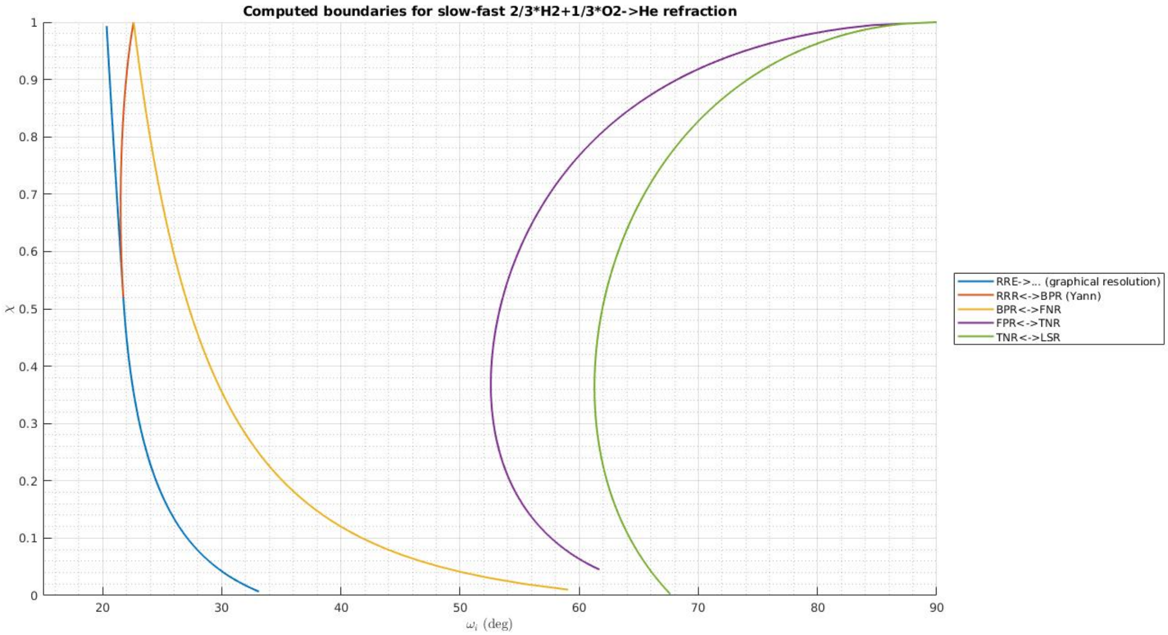


Figure 2: Boundaries of the differents structures in the $\chi - \omega$ plane, for a $H_2 - O_2//He$ system

Strength of the shock χ is related to Mach number of the shock M_{sh}.

Regular Refraction with reflected Expansion (RRE)

Relation between strength χ and Mach number M_{sh}

$$\chi = 1/\xi_i$$

$$\xi_i = \frac{1 - \gamma_I + 2\gamma_I M_{sh}}{\gamma_I + 1}$$

where γ_I is the heat ratio of phase I and M_{sh} is the normal component of the Mach number of the shock (see figure 5, slide 10). It is equal to M_{1n} the normal component of the Mach number of the incident flow. It is related to the Mach number of the flow by the following relation :

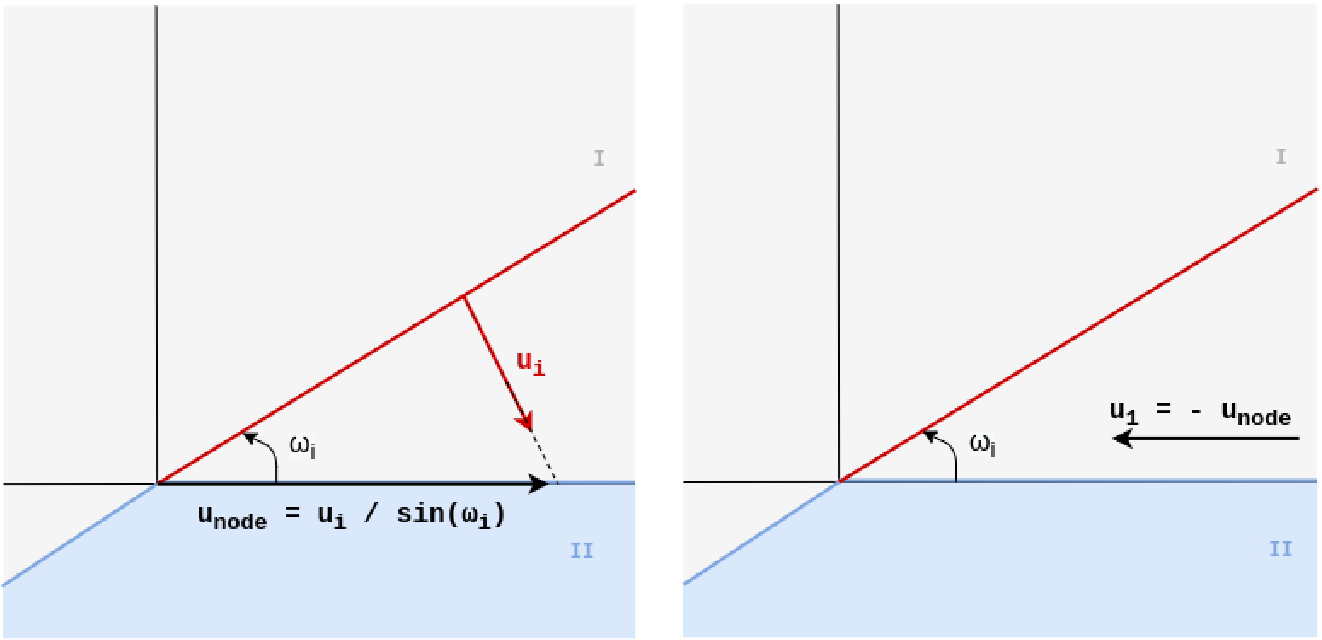
$$M_{sh} = \frac{M_1}{\sin(\omega_i)}$$

which finally leads to (with $\omega_i = 14.5°$):

$$\chi \in [0.01; 1] \iff M_{sh} \in [1; 2.95] \iff M_1 \in [4; 11.78] \qquad (1)$$

Theory and equations : gas dynamics

Change and rotation of frame of reference



(a) Frame of reference of the tube : shock is moving at u_i

(b) Frame of reference of the shock : flow is moving at u_1

Figure 3: Change of frame of reference (see legend on next slide)

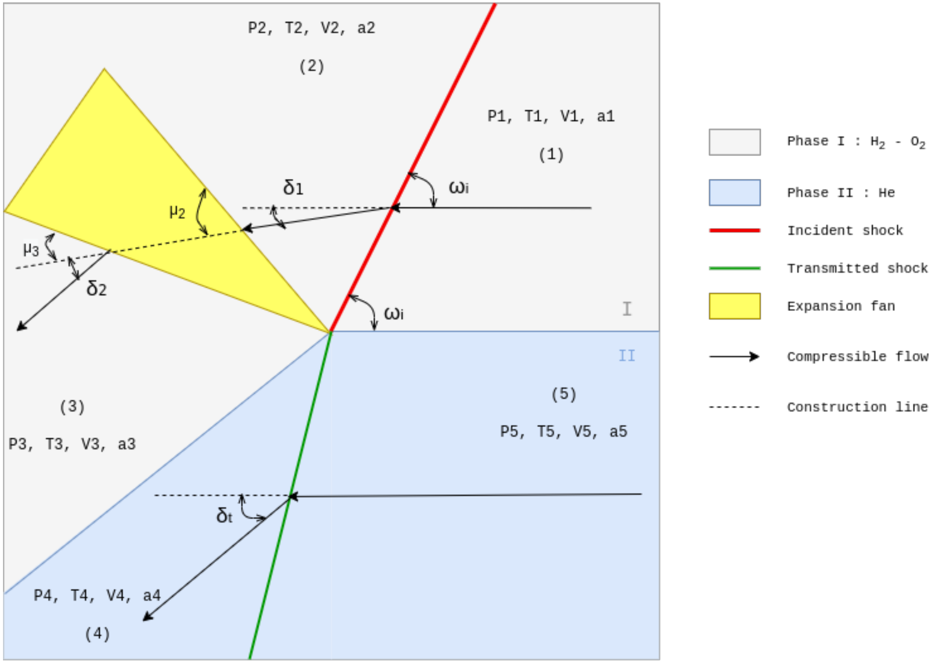


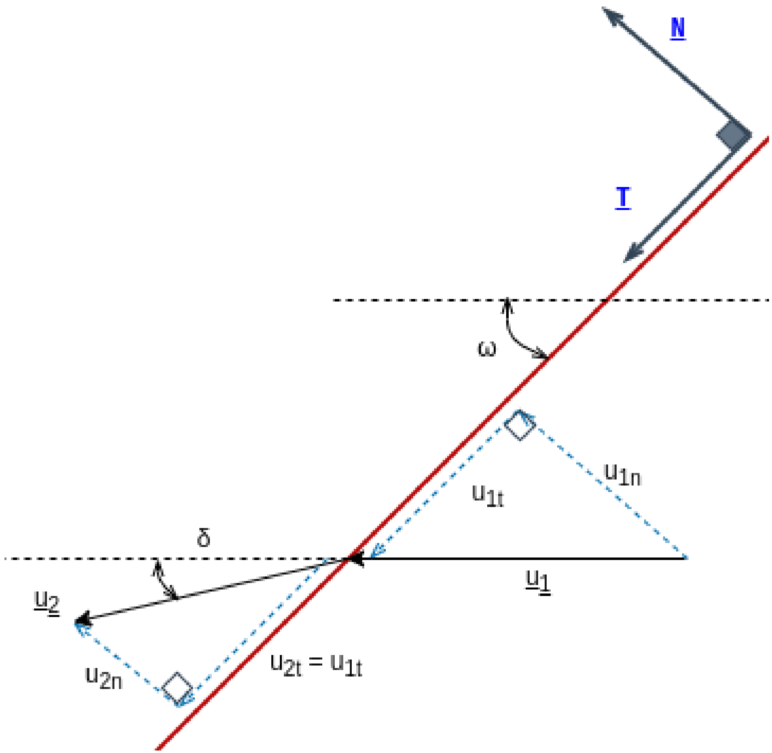
Figure 4: Symbols, zones and angles for computation

Theory and equations : gas dynamics

Initial conditions in zone (1)

Measure (unit)	Symbol	Value
Pressure (Pa)	$P1$	101 325
Temperature (K)	$T1$	600
Specific Volume	$V1$	$V1 = R_I * T1/P1$
Mach number	M1	6
Angle of incidence (deg)	ω_i	14.5
Normal mach number	$M1_n$	$M1_n = M1 \sin(\omega_i)$
Speed of sound (m/s)	a_1	$a_1 = \sqrt{\gamma_I R_I T1}$

Table 1: Initial conditions



$$M1_n = \frac{u1_n}{a_1} = \frac{u1 \sin(\omega_i)}{a_1} = M1 \sin(\omega_i)$$

$$\xi_i = \frac{1-\gamma_I+2\gamma_I M_{1n}}{\gamma_I+1}$$

Figure 5: Geometry associated with the oblique shock

Theory and equations : gas dynamics

From zone (1) to zone (2): oblique shock

$$M1_n = M1 \sin(\omega_i)$$

$$M2_n^2 = \frac{1+\frac{\gamma_I-1}{2}M1_n^2}{\gamma_I M1_n^2-\frac{\gamma_I-1}{2}}$$

$$\mathrm{M2} = \frac{M2_n}{\sin(\omega_i-\delta_1)}$$

$$\frac{V2}{V1} = \frac{(\gamma_I+1)M1_n^2}{2+(\gamma_I-1)M1_n^2}$$

$$\frac{P2}{P1} = 1+\frac{2\gamma_I}{\gamma_I+1}(M1_n^2-1)$$

$$\frac{T2}{T1} = \frac{P2}{P1}\frac{V2}{V1}$$

where δ_1 is the angle of deflection behind the incident shock.

$$\tan(\delta_1) = 2\cot(\omega_i)\frac{M1_n^2-1}{M1^2(\gamma_I+\cos 2\omega_i)+2}$$

Theory and equations : gas dynamics

From zone (5) to zone (4) : oblique shock of unknown angle

$$M5_n = M5\sin(\omega_t)$$

$$M4_n^2 = \frac{1+\frac{\gamma_{II}-1}{2}M5_n^2}{\gamma_{II}M5_n^2-\frac{\gamma_{II}-1}{2}}$$

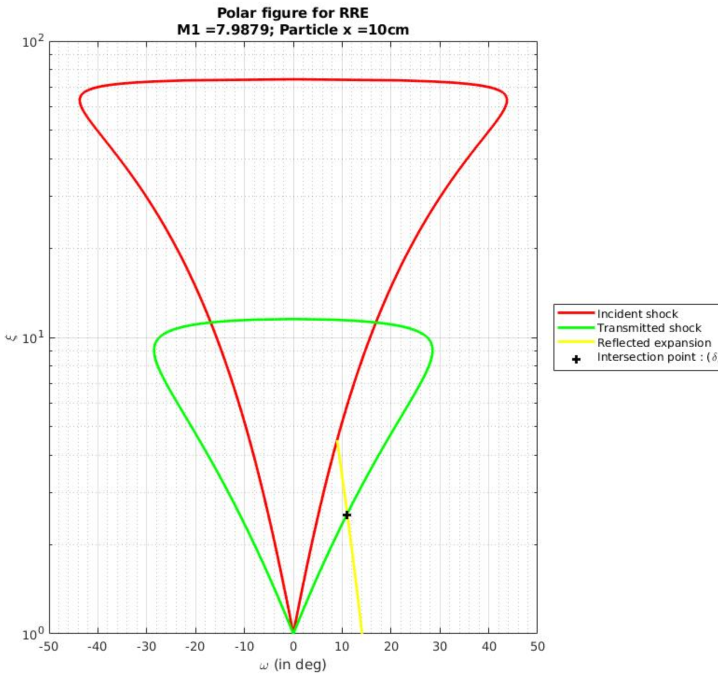
$$M4 = \frac{M4_n}{\sin(\omega_t-\delta_t)}$$

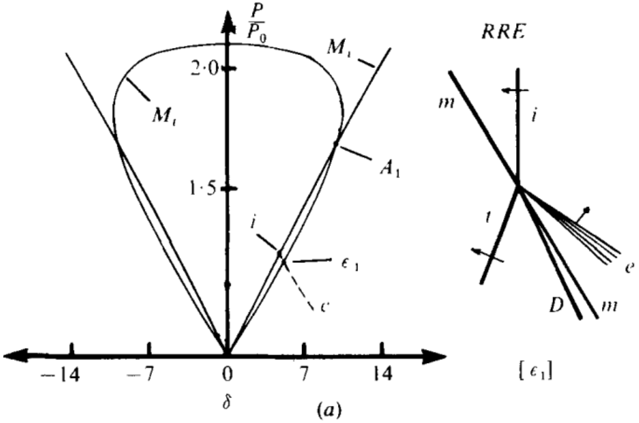
$$\frac{V4}{V5} = \frac{(\gamma_{II}+1)M5_n^2}{2+(\gamma_{II}-1)M5_n^2}$$

$$\frac{P4}{P5} = 1+\frac{2\gamma_{II}}{\gamma_{II}+1}(M5_n^2-1)$$

$$\frac{T4}{T5} = \frac{P4}{P5}\frac{V4}{V5}$$

where δ_t is the angle of deflection behind the transmitted shock. Unfortunately, ω_t, the angle between the transmitted shock and the flow in zone (5), remains unknown.





$$\delta_t = \delta_1 + \delta_2$$

$$P3 = P4$$

Figure 6: Polars for RRE structure, reference from Henderson et al. 1978 on the right

Thanks to the polars of the shock, δ_t and ξ_t can be determined (intersection of expansion and transmitted shock polar). $P4$ thus $P3$ are known and so $M3$ thanks to Prandtl-Meyer relations.
ν is the Prandtl-Meyer function, depending on the heat ratio of the gas at stake.

$$\nu(M3) = \delta_2 + \nu(M2)$$

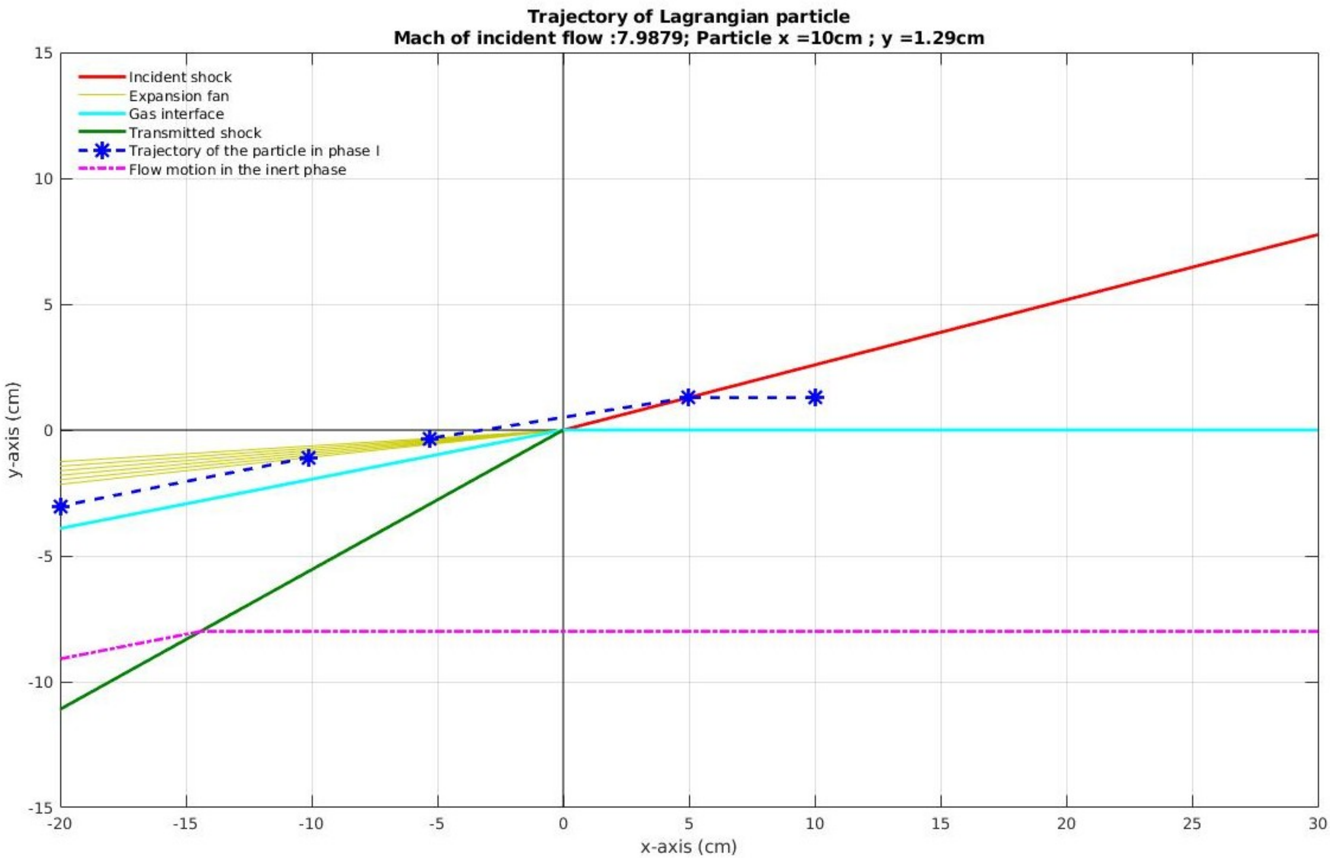
$$P3 = P4 = P5 \times \xi_t$$

$$T3 = T2\left(\frac{P3}{P2}\right)^{\frac{\gamma_I - 1}{\gamma_I}}$$

$$V3 = R_I \frac{T3}{P3}$$

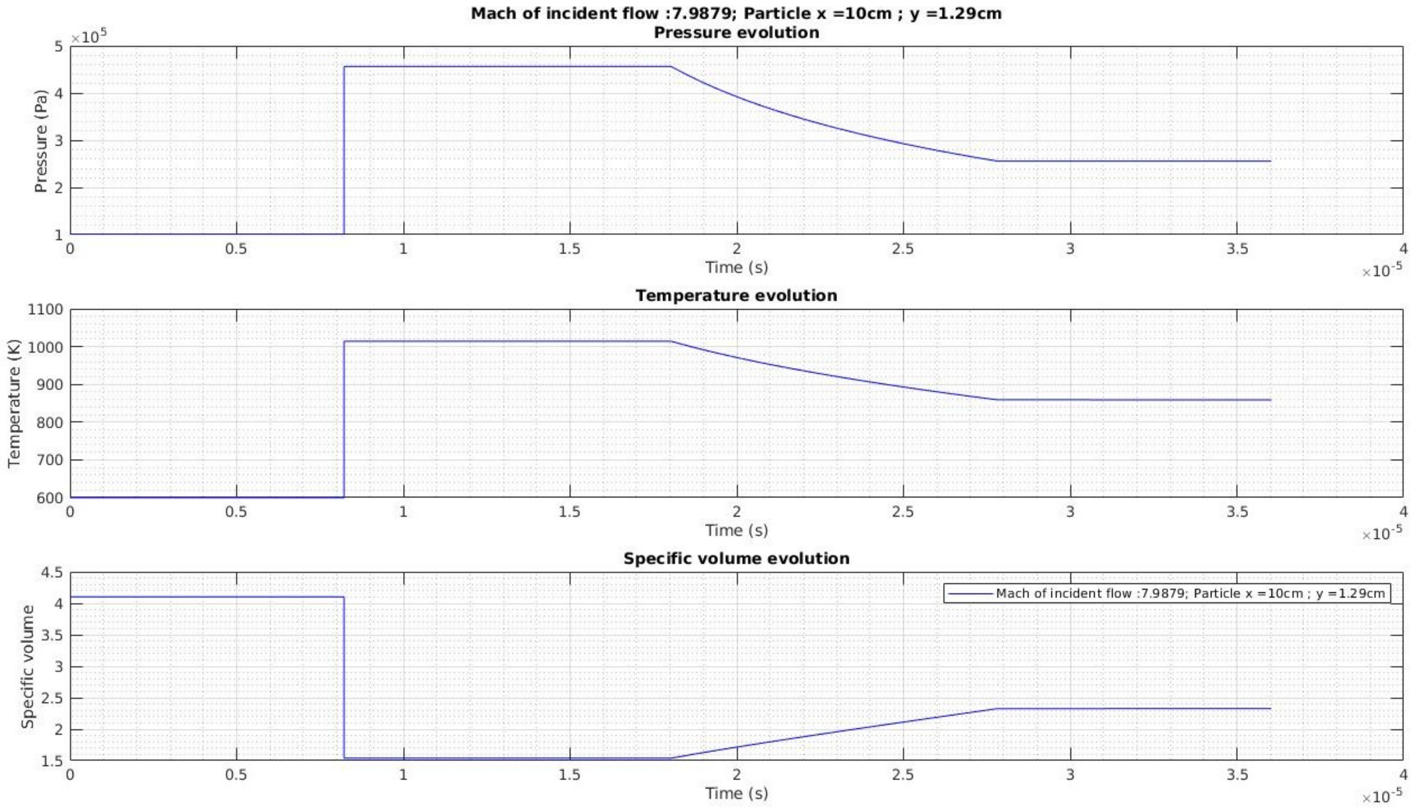
Results given by the inert gas dynamics theory

Evolution of a Lagrangian particle in the $H_2 - O_2$ phase



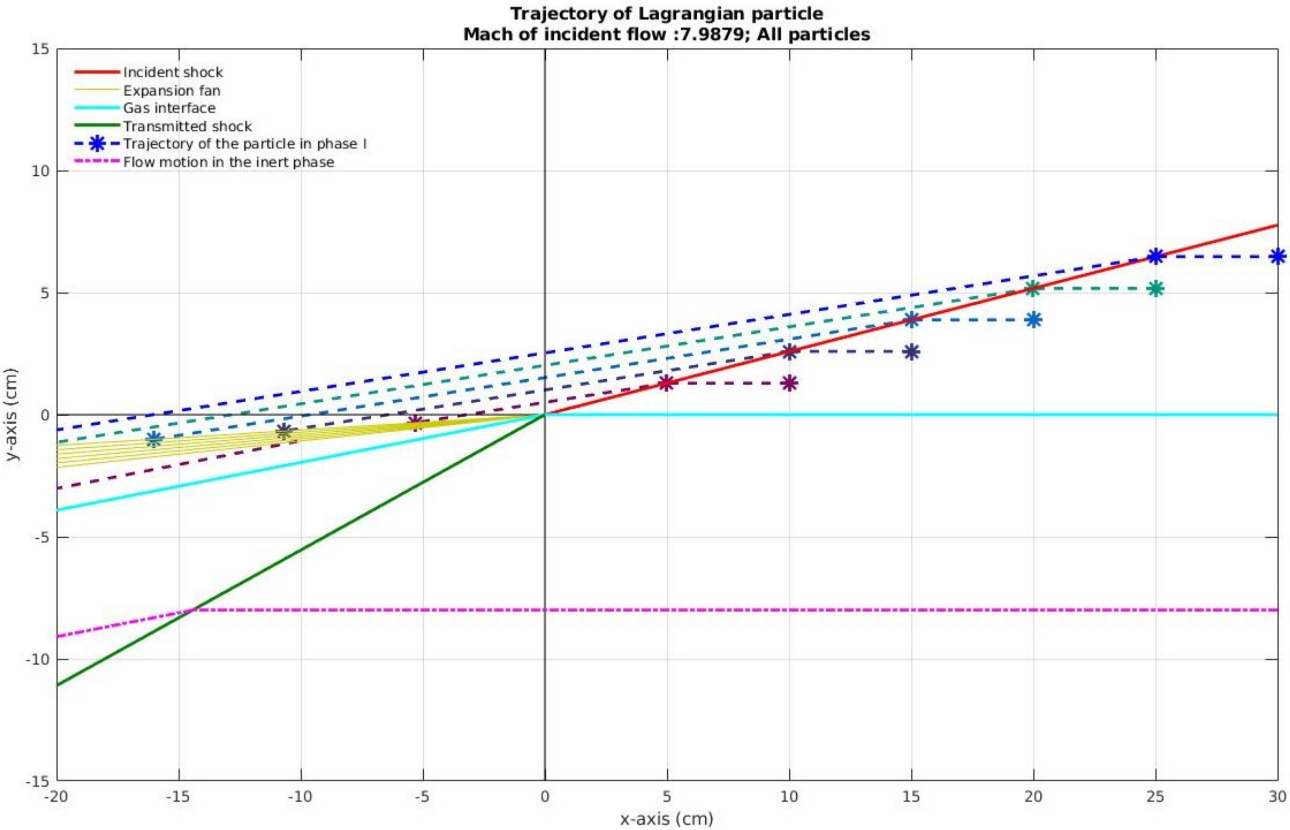
Results given by the inert gas dynamics theory

Evolution of a Lagrangian particle in the $H_2 - O_2$ phase



Results given by the inert gas dynamics theory

Evolution depends on position of the particle...



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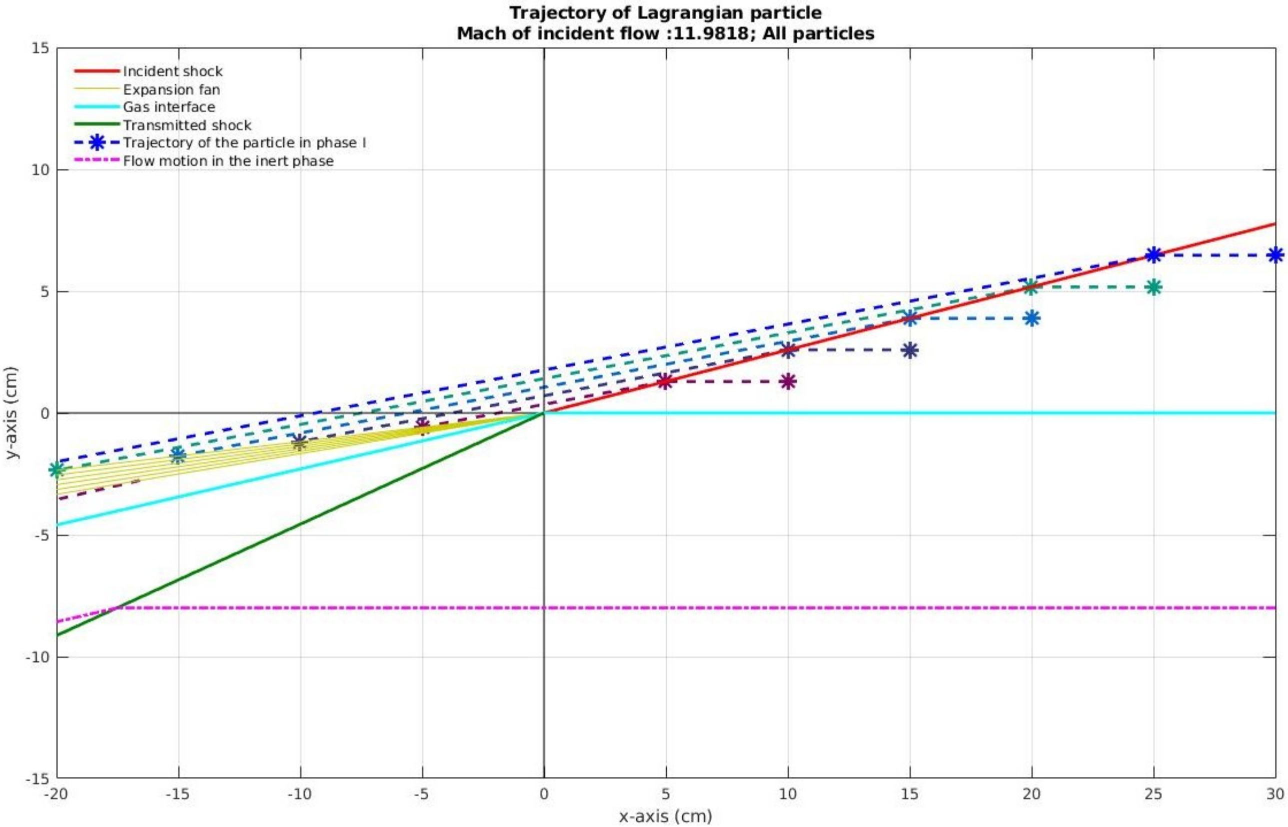

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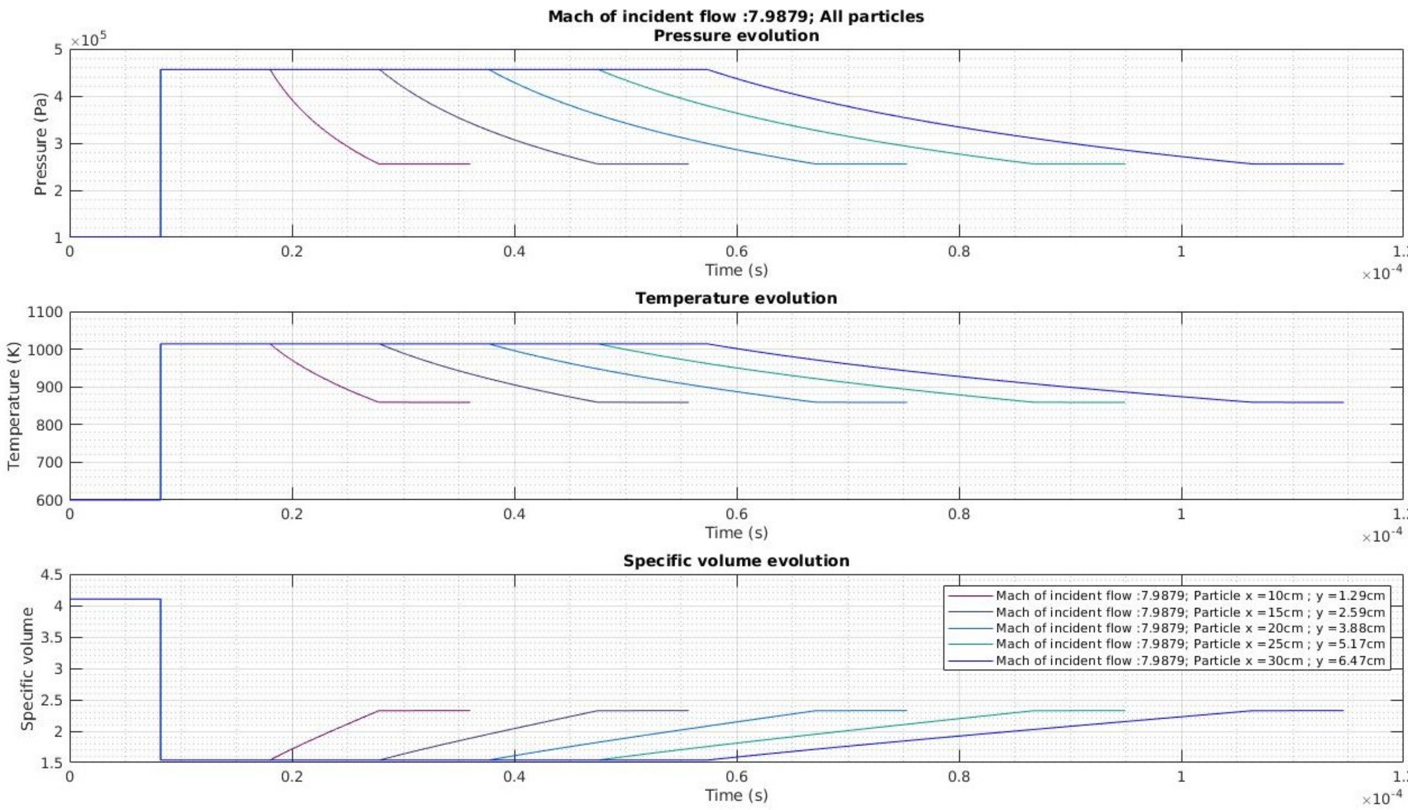

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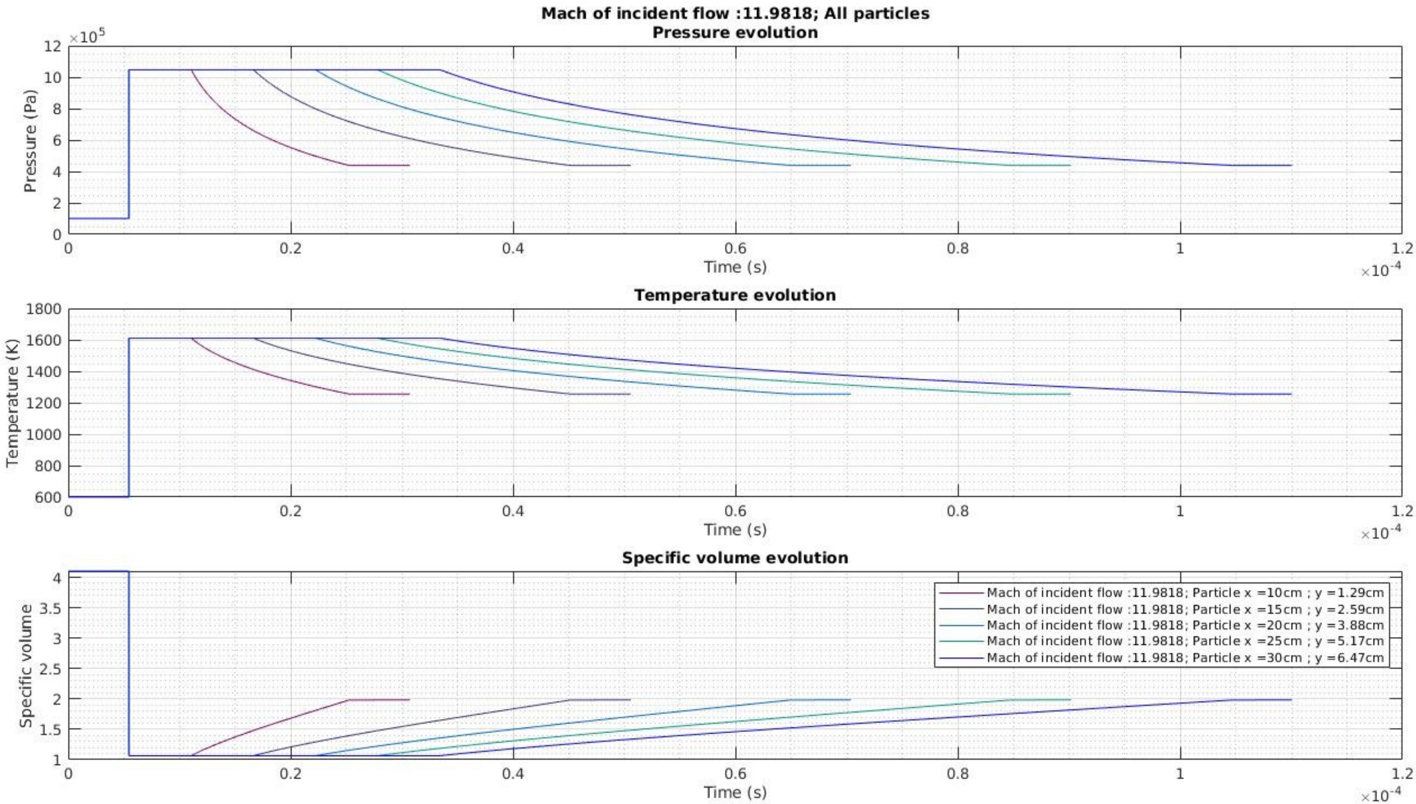
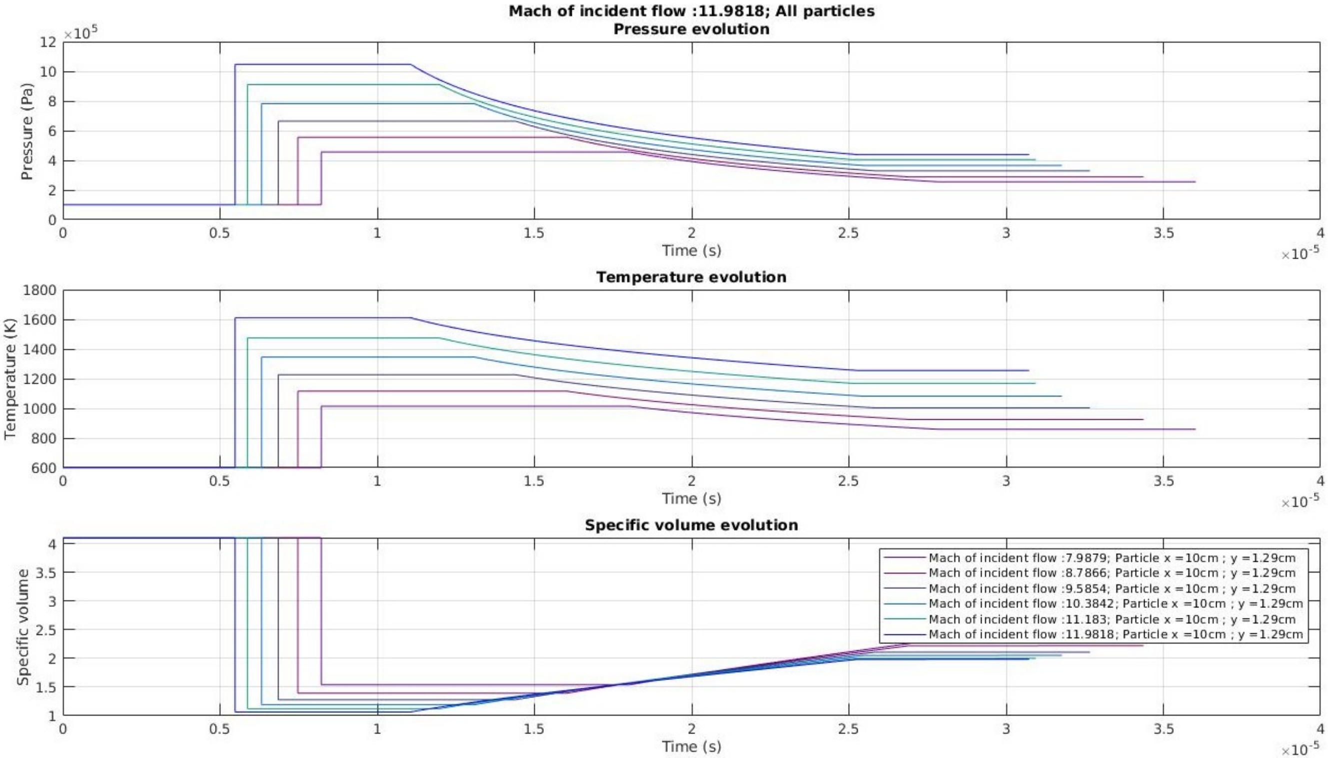


Results given by the inert gas dynamics theory

… and on Mach number of the shock



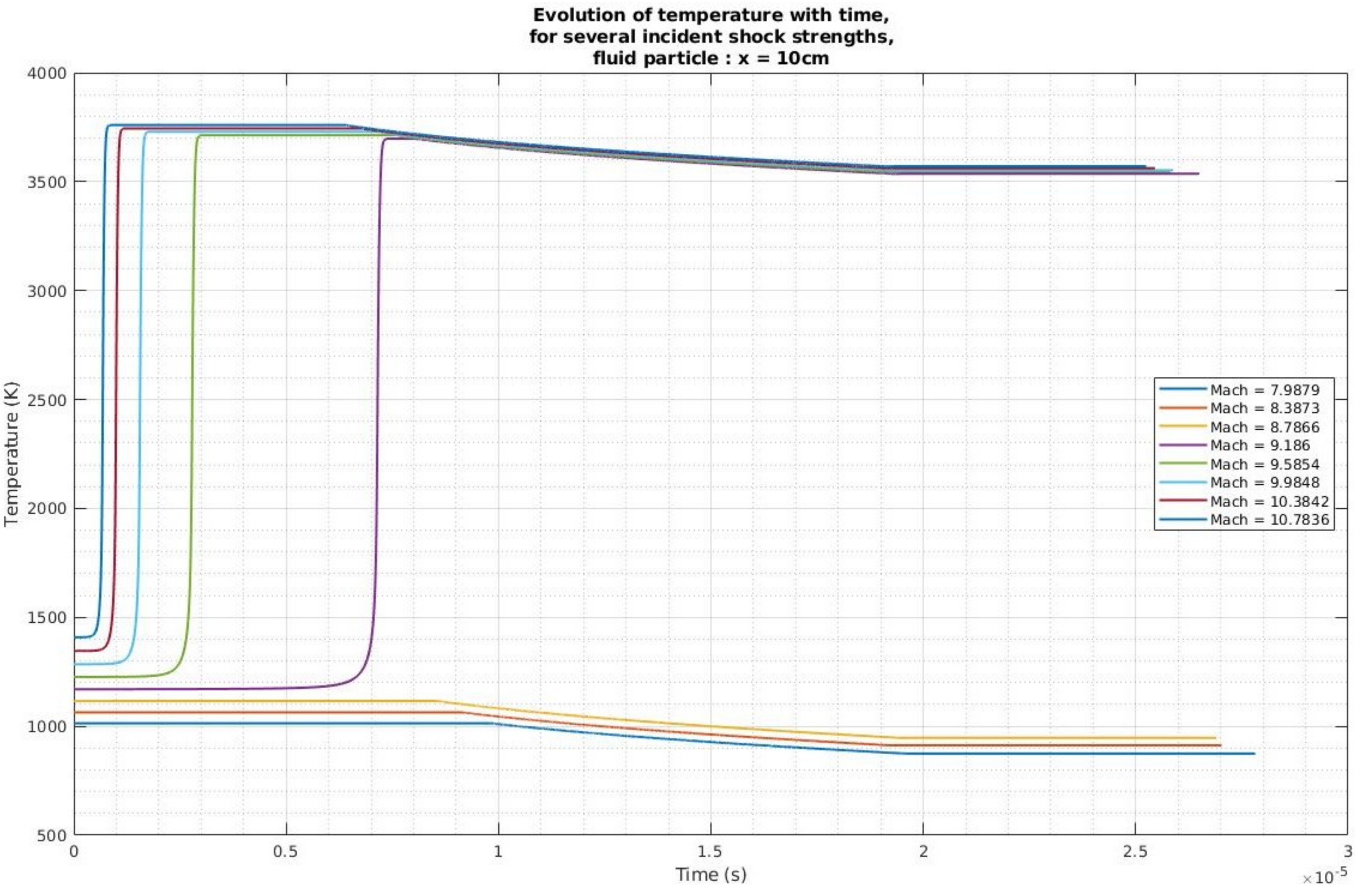
For a single particle, increasing M_{sh} leads to a rise in temperature and pressure : don't forget that $H_2 - O_2$ is a reactive phase. Is an ignition possible under certain conditions?

Main steps

- Calculate evolution of specific volume of different Lagrangian particles, for $\omega_i = 14.5°$, for different Mach numbers (see slide 6).
- Use of CHEMKIN II to calculate chemical reactions in the reactive phase
- Outputs of CHEMKIN II : evolution of pressure, temperature and ratios of chemical species
- Temperature jump in CHEMKIN II output = detonation

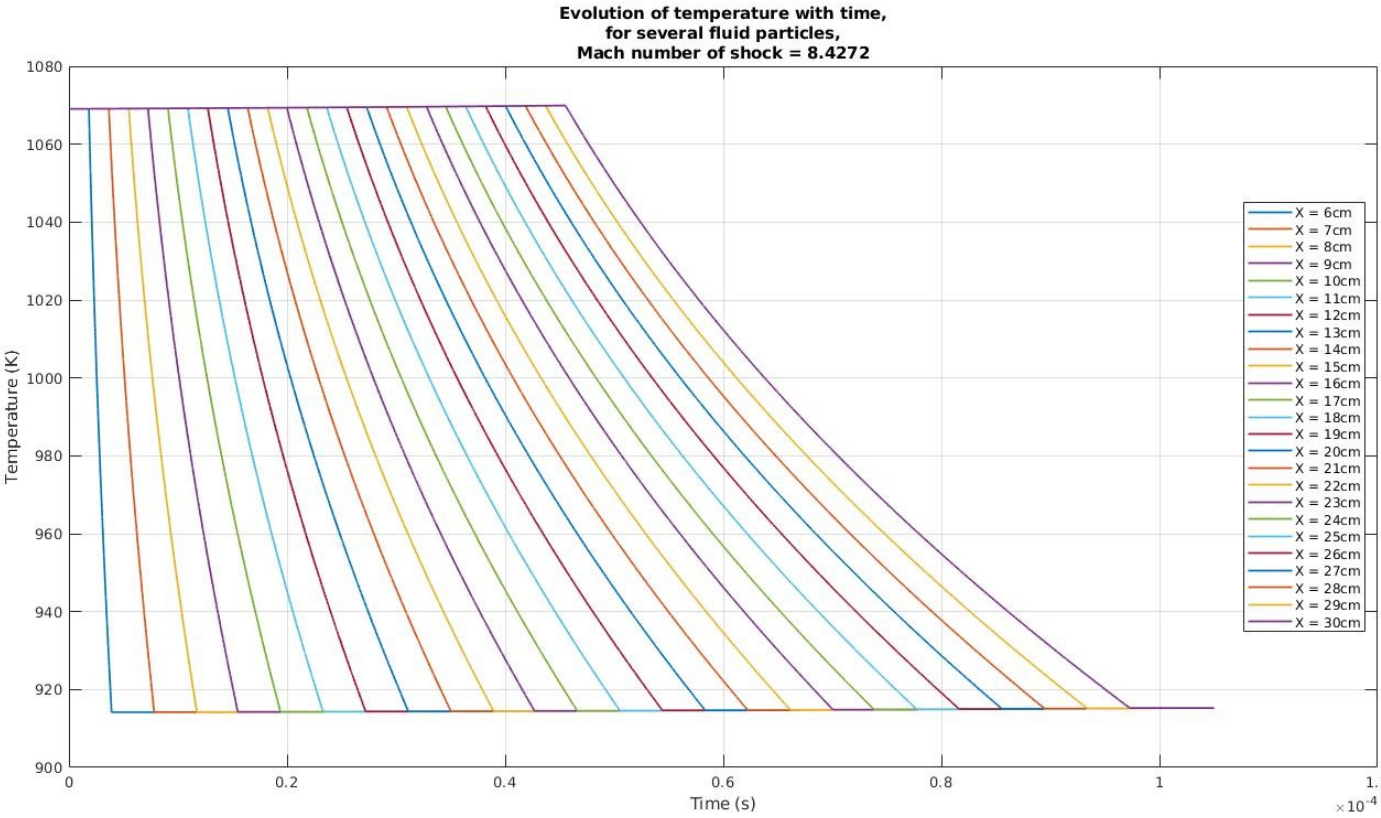
Results of the chemical calculus

Existence of a threshold : Mach number of ignition



Results of the chemical calculus

Each particle has its own M_{ignit}



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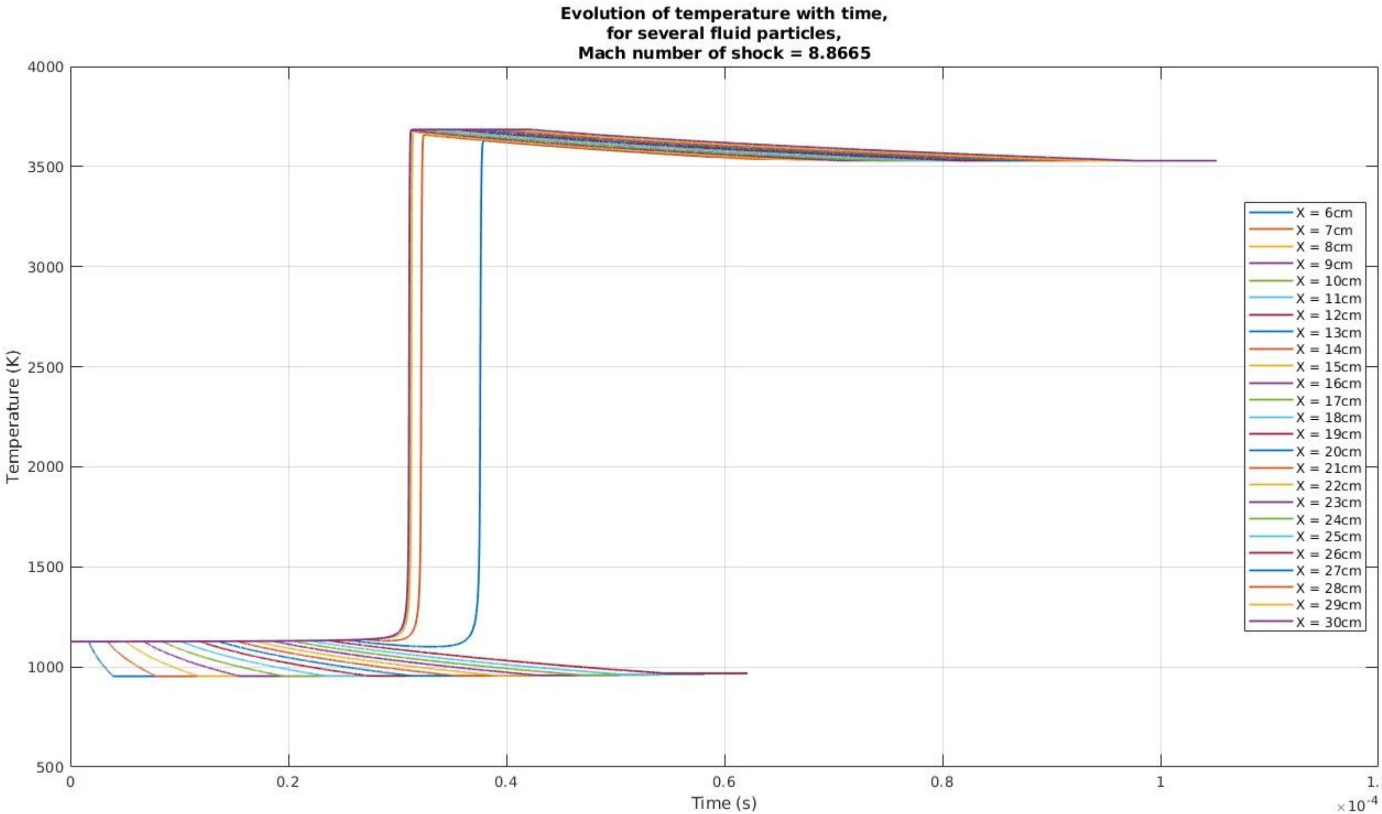

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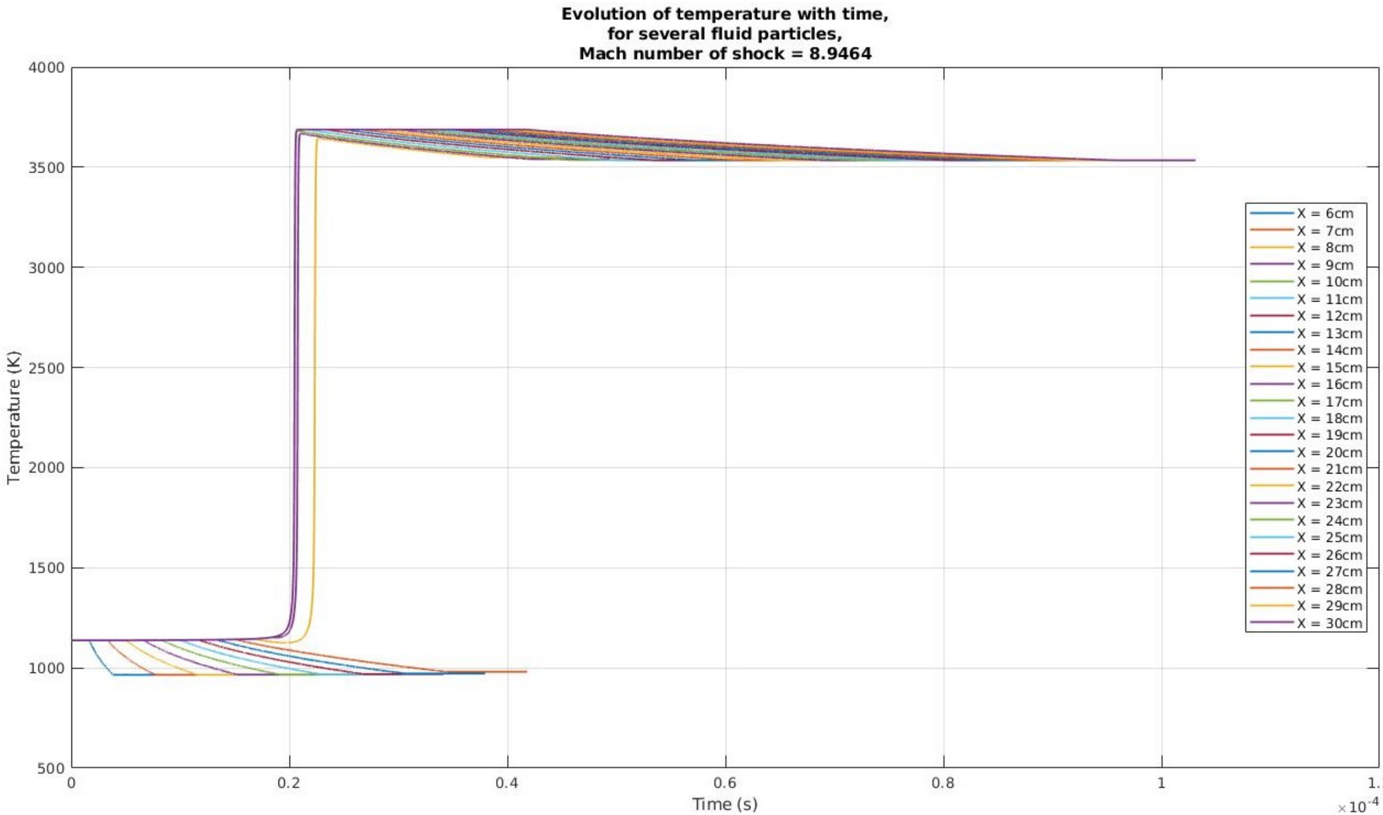
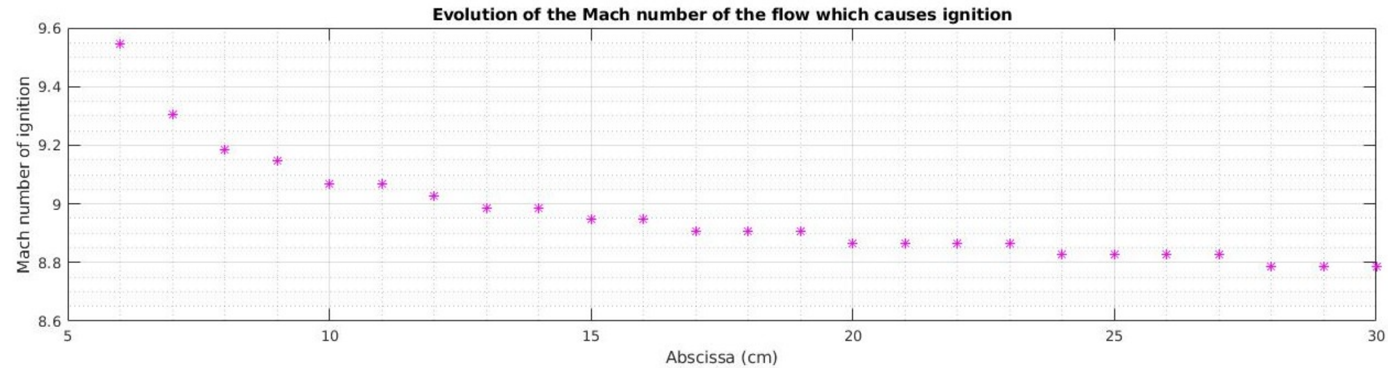
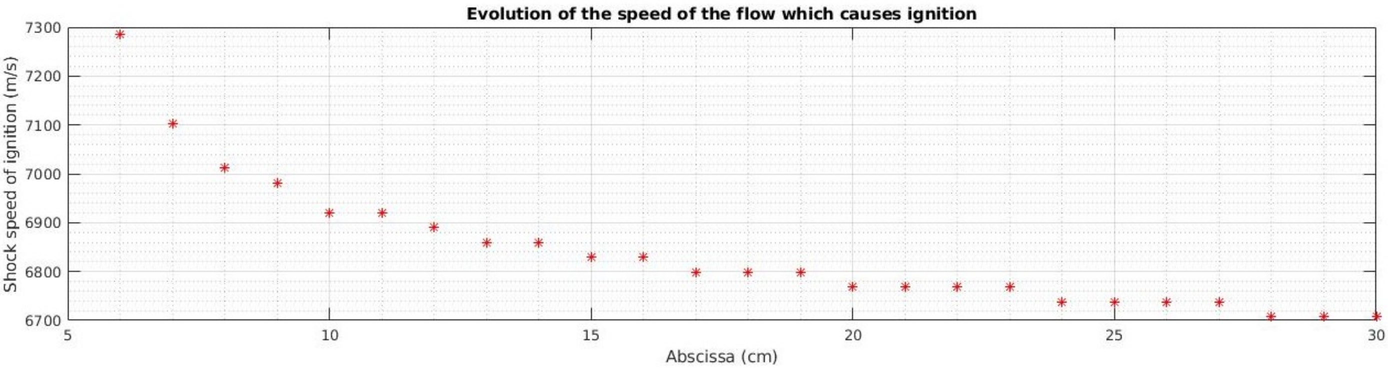


Results of the chemical calculus

M_{sh} plotted as a function of ordinate of the particle





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