Mathimatika^2 Contest

Official website

Mathimatika Mathematics Competitions

August 14, 2021

1. DO NOT OPEN THIS BOOKLET UNTIL YOU ARE READY TO BEGIN.
2. This is a 25 question fill-in-the-blank test. For each question, there is only one correct answer. All answers are positive integers. There is no penalty for guessing, your score is the number of correct answers.
3. Submit your answers in the given blanks in the Google form that will be available when the test starts.
4. You will recieve 1 point for every correct answer, 0 points for each problem left unanswered, and 0 points for each incorrect answer.
5. The only materials allowed during this test are scrap paper, erasers, and pencils. Calculators and other electronic devices (other than to access the problems) or other forms of cheating are not permitted. This test does not require the use of a calculator or other aids. Do not open any other tabs other than the access to the problems, or you will be disqualified.
6. Figures are not necessarily drawn to scale.
7. Tie breaking: The student who solved the last problem will be ranked higher. If the tie persists, the second last problem will be used, then the third last problem, and so on. If two students solved the exact same set of problems correctly, a tie will not be broken.
8. You will have 45 minutes to complete this test, which includes the time for you to submit your answers on the google form.
9. The problems of the Mathimatika^2 Contest begin on the next page. Good luck!

The publication, reproduction or communication of the problems or solutions of the contest during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules.

1. Ramona baked cookies for herself and 7 of her friends. She makes multiple cookies, but when distributing them to everyone finds out that there are 5 cookies remaining. What is the smallest number of cookies she could have baked, if it is guaranteed that everyone got at least two cookies?

2. We define $\begin{bmatrix} a \\ d \end{bmatrix} \cdot \begin{bmatrix} b \\ c \end{bmatrix} = a \cdot c + b \cdot d$. Find $\begin{bmatrix} 48 \\ 75 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 25 \end{bmatrix}$.

3. Emma is thinking of a number. First, she adds three, multiplies the result by 10, and then adds 19. After taking the positive square root of the new number and subtracting 1, she finds that the result is the same as her initial number. What was the number she was thinking of?

4. Scott is trying to move all of his spare items into a cube-shaped storage room, which has 512 cubic yards of spare space. If he wants to fit as many 4ft $\times$ 4ft $\times$ 4ft boxes into the storage room as possible, how many boxes will he be able to put? (A yard is equal to 3 feet)

5. Let there be a 3$\times$3 grid with a caterpillar in the middle. Each day, the caterpillar randomly moves to another square diagonal from where it was. Assuming that the caterpiller does this for a random number of days between 1 and 99 inclusive, the probability that the caterpillar is on the outer 8 squares of the grid on the last day is $\frac{a}{b}$ where a and b are relatively prime. What is $b - a$?

6. Jack and Jill went up the hill at a speed of 4 and 5 miles per hour respectively. Then, immediately after reaching the top, they both travel back down the way they came. If Jill walks at a speed of 8 miles when jogging down the hill, at What speed should Jack descend so that he reaches the bottom with Jill at the same time, rounded to the nearest integer?

7. In the town of Cadosville, the dogs and cats are having an identity crisis. Twenty percent of dogs think they are cats, and twenty percent of cats think they are dogs. Luckily, the rest of the animals know what they are. One day, Old Macdonald found that amongst all of the dogs and cats, 32 percent of them think they are cats. If there are 180 more dogs than cats in this village, how many dogs are there?

8. Alex has a bag of candy. On the first day, he ate three eighths of all the candy. On the second day, he ate 50 pieces. On the third day, the amount of candy he has eaten is five sevenths of the candy remaining. How much candy is left?

9. Rosie is going to take 9 classes the coming quarter, but hasn't got her schedule. However her friend Donna had already seen Rosie's schedule, and told her that she had math as her 4th class and English sometime after. Rosie's sister Lucy hasn't gotten her schedule either, but really wants to be in the same English class as Rosie. If Lucy will also have 9 classes, the probability that Rosie and Lucy end up in the same English class is $\dfrac{x}{y}$ where $\gcd(x, y) = 1$. Find $y - x$.

10. Adria, Bill, Carl, and Daniel are having a conversation about a recent math test. Some of them are always telling the truth and the rest of them are always lying.

    Adria: "Well I totally screwed it. I only got 75%!"

    Bill: "You got a 90% liar, I know it."

    Carl: "Bill is obviously lying."

    Daniel: "Well, all I know is that there's only one honest student here."

    Lastly, Bill says: "I can vouch that Daniel is telling the truth."

    How many of the four are honest students?

11. If the roots of the equation $x(7x + 63) + 56 = 0$ are x_1 and x_2, what is the value of $|x_1(x_2 + x_1) \cdot x_2(x_1 + x_2)|$, where $|n|$ is equivalent to the non-negative value of n?

12. In the year of 2058, Bob bikes to school every 4th day (i.e. every 4 days), and Maya bikes to school every 7th day (every 7 days). Meanwhile, Donna only bikes to school every 16th day (every 16 days). If they all go to school on their bikes on February 24, on what day of the month will they next all bike to school together? (If your answer is January 1st, your answer should be 1.) For the sake of simplicity, let a leap year be any year divisible by 4.

13. Let $ABCD$ be a rectangle. Point E is on segment BC such that $\angle BAE = 30^\circ$ and $AE = 12$. In addition, let point F be on line DC so that $DF = CF$ and $\angle AEF = 75^\circ$. If the $BC = x + y\sqrt{z}$, what is $x + y \cdot z$?

14. In the span of 50 years, John and Jacob visit their grandmother's house on the first day of every 6 and 8 months, respectively. If they both initially visit their grandmother on the first day of January, on how many days will there be exactly one grandson at the house?

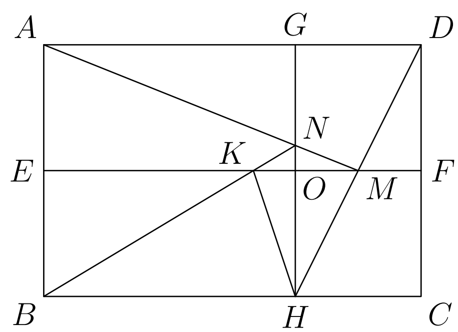
15. Let O be a point on the y-axis of the Cartesian plane. A circle centered at O is drawn to be tangent to the x-axis at a point A. Let B be a point on the circle such that the smaller angle formed by the x-axis and line AB is 50°, and let C be the point on the circle intersecting with the extension of $\overline{AO}$. What is the length of minor arc BC if the circumference of the circle is 126?

16. If the sum

$$\frac{1}{\binom{3}{3}} + \frac{1}{\binom{4}{3}} + \frac{1}{\binom{5}{3}} + \cdots$$

    can be written as $\frac{p}{q}$, where p and q are relatively prime integers, what is $p \cdot q$? Note that $\binom{n}{r} = \frac{n!}{r!(n-r)!}$.

17. As shown below, rectangle $ABCD$ has an area of 1260 and is divided into four smaller rectangles by segments $\overline{EF}$ and $\overline{GH}$. Let $\overline{DH}$ meet $\overline{EF}$ at M, $\overline{AM}$ meet $\overline{GH}$ at N, and $\overline{BN}$ meet $\overline{EF}$ at K. If E is the midpoint of $\overline{AB}$ and $AG = 2GD$, what is two the area of quadrilateral $MNKH$?



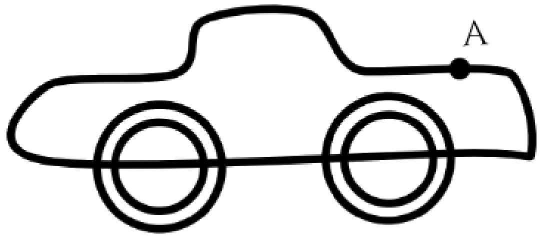
18. On a game board with a 6×6 grid of congruent squares, a game piece is placed in the square located in the top-left. Each turn, the piece can move to a square directly next to it, excluding diagonals. If there are 9 black squares in a 3×3 formation on the bottom-left of the grid, in how many ways can the game piece get to the square on the bottom-right of the board in 10 moves, without landing on the black squares?

19. In a bag, there are 10 balls numbered from 1 to 10. A fifth of these balls are red, five are blue, and the rest are white. Jack pulls out two balls simultaneously, and finds a white "10" and a red "4". If the probability of this happening is equal to $\frac{m}{n}$ when simplified, what is $m + n$? (We do not know which of the colored balls correspond to which number.)

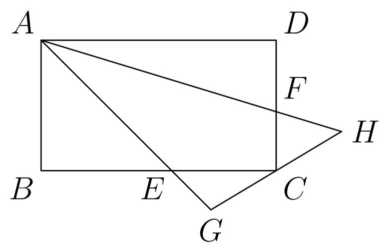
20. How many triples of integers (a, b, c) are there such that $-10 < a, b, c < 10$ and where the following expression is true?

$$\frac{\frac{a}{b}}{c} = \frac{a}{\frac{b}{c}}$$

21. In the diagram below, if one starts from point A, there are N ways to draw the entire car without lifting the pen nor repeating any previously drawn lines (you can, however, come across the same point twice). If $N = a^2 \cdot b$ where a and b are positive integers and b is as small as possible, what is $a + b$?



22. In rectangle $ABCD$, points E and F are the midpoints of segments BC and CD, respectively. Points G and H lie on the extensions of $\overline{AE}$ and $\overline{AF}$, respectively, such that G, C and H are colinear and the areas of both $\triangle ECG$ and $\triangle FCH$ are 12. Find the area of $ABCD$.



23. John takes an integer n_0, halves it, and then adds 1. He then takes the new number, n_1, and halves it again and adds 1 to get n_2. After doing this for 10 times, he notices that the new number, n_{10}, is just greater than $e \approx 2.718....$ He then realizes that the original number he started with was the smallest such that this was possible. What is n_0?

24. In a 4×4 checker board, 5 indistinguishable checkers are placed on different squares in such a way that one can find at least one checker in each row and column. How many different ways are there to place the checkers?

25. Consider the sequence $50 + n^2$ where $n = \{1, 2, 3, 4, ...\}$. If m is equal to the sequence of numbers constituting the greatest common divisor of consecutive terms in $50 + n^2$, find the sum of the distinct numbers in sequence m.