# 2002 MATH OLYMPICS

LEVEL II

1. If the radius of a right circular cylinder is increased by 50%, what will the corresponding percent increase in the volume of the cylinder be?

   a. 25%   b. 50%   c. 100%   d. 125%   e. None of the above

2. For the two functions of x, $y = \log x^2$ and $y = 2\log x$, which of the following is true?

   a. The two functions are the same

   b. The two functions have different domains

   c. One function is a multiple of the other

   d. The two functions have the same domain

   e. None of the above are true

3. Tracy tried to compute the average of her five test scores and she mistakenly divided the correct total (T) of her scores by 6, and her result was 14 less than what the average should have been. Which of the following equations would determine the value of T.

   a. $5T+14=6T$ b. $\dfrac{T}{6} = \dfrac{T-14}{5}$ c. $\dfrac{T}{6} - 14 = \dfrac{T}{5}$

   d. $\dfrac{T}{6} + 14 = \dfrac{T}{5}$ e. None of the above

4. The floor of a room is a 10 m by 10 m square and the room is 4 m high. A spider is in one of the corners of the floor and sees a fly across the room at the diagonally opposite corner on the ceiling. If the fly does not move, what is the shortest distance (in meters) the spider must travel to catch the fly?

   a. $\sqrt{216}$ b. $4 + \sqrt{200}$ c. 24   d. $\sqrt{296}$ e. None of the above

5. When people play pool, they first arrange the balls inside a frame, as shown in the figure. The diameters of the balls are all the same. Given that AH=33 cm, find the diameter of a ball.

   a. $6\sqrt{3} - 3$ b. $\dfrac{22\sqrt{3}}{5}$ c. $\dfrac{11\sqrt{3}}{2}$

   d. $\dfrac{33\sqrt{3}}{8}$ e. None of the above
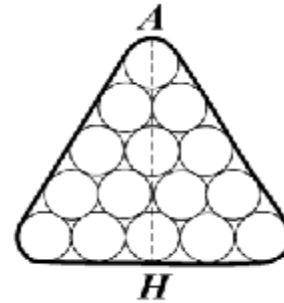
6. The first and seventh terms of a sequence are both 8. In addition, starting with the third term, each term is the sum of the previous two terms. What is the fifth term?

   a. -4 b. 0   c. 4   d. 12   e. None of the above

7. The sum of all different real values of x for which $\left(x^2 - 5x + 5\right)^{x^2 - 9x + 20} = 1$ is

   a. 9   b. 10   c. 12   d. 15   e. None of the above

8. Find the digit x such that the base three numeral $1x21$ represents the same number as the base five numeral 202.

   a. 0   b. 1   c. 2   d. 3   e. None of the above

9. Find the value of the product $(\log_2 3)(\log_3 4)(\log_4 5)(\log_5 6)(\log_6 7)(\log_7 8)$.

   a. 1   b. ½   c. 2   d. 3   e. None of the above

10. Let $f(x) = ax^3 + bx^2 + cx + 3$. If each of $(x-2)$, $(x-3)$, $(x-4)$ is a factor of f, find the value of a.

    a. -1 b. $-½$ c. $-⅛$ d. ½   e. None of the above

11. If $\sin a = \dfrac{3}{5}$ and $\cos a < 0$, what is $\sin(2a)$?

    a. $-12/25$ b. $-4/5$ c. $-24/25$ d. $-6/5$ e.   None of the above

12. A time trial for a car race consists of two laps around a race track. Driver 8 averages 180 miles per hour on the first lap, but, due to mechanical difficulties, only 120 miles per hour on the second lap. What is Driver 8's average speed for the time trial?

    a.   140 mph   b.   144 mph   c.   146 mph   d.   150 mph   e.   None of the above

13. If $\tan\theta = {-3}/{4}$ and $\sec\theta > 0$, then $\sin\left(\arctan\left({3}/{4}\right)\right)$ is equal to

    a.   3/5   b. $-3/5$ c.   4/5   d. $-4/5$ e.   None of the above

14. You ask a cashier for change for a half dollar and request no pennies. Assuming that the manner in which he makes change is purely arbitrary, determine the probability that the change he gives you will include just one quarter.

    a.   1/4   b.   1/5   c.   3/10   d.   7/10   e.   None of the above

15. In how many different orders can four children be arranged in a line if two children, John and Julie, refuse to stand next to each other?

    a.   6   b.   12   c.   16   d.   24   e.   None of the above

16. What is the first time, to the nearest minute, after 4:00 that the minute hand of a clock will overtake the hour hand?

    a.   4:20   b.   4:21   c.   4:22   d.   4:23   e.   None of the above

17. Call a positive integer *weird* if no two of its digits are equal and no digit is 0. Call a positive integer *goofy* if all its digits are even. How many four-digit positive integers are either *weird* or *goofy*?

    a.   3000   b.   3024   c.   3500   d.   3524   e.   None of the above

18. The terms a_n of a sequence of positive integers satisfy $a_{n+3} = a_{n+2}(a_{n+1} + a_n)$ for $n=1,2,3,\cdots$. If $a_6=144$, what is a_7?

    a.   256   b.   2304   c.   3248   d.   3456   e.   None of the above

19. Jim, in order to guess his grandfather's height and age, had him do the following:

    1) Write down his age. 2) Multiply it by 2. 3) Add 5. 4) Multiply this sum by 50. 5) Subtract 365. 6) Add his height (in inches). 7) Add 115. If the result was 6364. Guess Jim's grandfather's height and age.

    a.   height (5'3"); age (64)   b.   height (5'4"); age (64)   c.   height (5'3"); age (63)

    d.   height (5'4"); age (63)   e.   None of the above

20. The inverse function of $f(x) = x^2 - 1$, $x \le 0$, is

    a. $f^{-1}(x) = \sqrt{x+1}$ b. $f^{-1}(x) = -\sqrt{x+1}$ c. $f^{-1}(x) = \pm\sqrt{x+1}$

    d. $f^{-1}(x) = x^2 - 1$ e.   None of the above

21. Can the graph of $f(x) = \dfrac{x^3 + 2x + 2}{x^3 + x^2 + x}$ cross its horizontal asymptote? If yes, then where?

    a.   No   b.   Yes, at $x = -1, 2$ c.   Yes, at $x = 3$

    d.   Depends   e.   None of the above

22. The concentration of a mixture consisting of 10 gallons of 20%-acid and 40 gallons of 15%-acid is

    a.   12%-acid   b.   16%-acid   c.   17%-acid   d.   15%   e.   None of the above

23. Find the domain of the function $\log\left(4 + 3x - x^2\right)$.

    a. $(-\infty, +\infty)$ b. $(-1, 4)$ c. $(4, +\infty)$ d. $(-\infty, -1)$ e.   None of the above

24. Suppose that the quadratic expression $Ax^2 + Bx + C$ has no real zeros and that $A + B + C < 0$. One can conclude the following:

   a. $C < 0$ b. $C = 0$ c. $C > 0$

   d. $C \geq 0$ e. None of the above

25. When $\pi < t < \dfrac{3\pi}{2}$, the value of the expression $\dfrac{1}{1 - \cos t} - \dfrac{1}{1 + \cos t}$ in terms of $\sin t$ equals

   a. $\dfrac{-2\left(1 - \sin^2 t\right)}{\sin^2 t}$ b. $-\dfrac{2\sqrt{1 - \sin^2 t}}{\sin^2 t}$ c. $\dfrac{2\sqrt{1 - \sin^2 t}}{\sin^2 t}$

   d. $\dfrac{2\left(1 - \sin^2 t\right)}{\sin^2 t}$ e. None of the above