Calculus Cheat-Sheet

Trigonometry:

Definitions:

$\sin x = \sin\alpha \equiv x = \alpha + k2\pi \vee x = \pi - \alpha + k2\pi$
$\cos x = \cos\alpha \equiv x = \alpha \pm x + k2\pi$
$\tan\ = \tan\alpha \equiv x = \alpha + k\pi$

$\arcsin x = y \equiv x = \sin x \wedge y \in [\frac{-\pi}{2}, \frac{\pi}{2}]$
$\arccos x = y \equiv x = \cos x \wedge y \in [0, \pi]$
$\arctan x = y \equiv x = \tan x \wedge y \in]\frac{-\pi}{2}, \frac{\pi}{2}[$
$arccot\ x = y \equiv x = \cot x \wedge y \in]0, \pi[$

$$arccot\ a = \begin{cases} \arctan\frac{1}{a} & a \in \mathbb{R}_0^+ \\ \pi + \arctan\frac{1}{a} & a \in \mathbb{R}_0^- \\ \pi & a = 0 \end{cases}$$

Formulas:

$\sin^2 x + \cos^2 x = 1$	$\sin(a+b) = \sin a\cos b + \cos a\sin b$
$\tan^2 x + 1 = \frac{1}{\cos^2 x}$	$\sin(a-b) = \sin a\cos b - \cos a\sin b$
$\cot^2 x + 1 = \frac{1}{\sin^2 x}$	$\cos(a+b) = \cos a\cos b - \sin a\sin b$
$\sin 2x = 2\sin x\cos x$	$\cos(a-b) = \cos a\cos b + \sin a\sin b$
$\sin 2x = \frac{2\tan x}{1+\tan^2 x}$	$\tan(a+b) = \frac{\tan a+\tan b}{1-\tan a\tan b}$
$\cos 2x = \cos^2 x - \sin^2 x$	$\tan(a-b) = \frac{\tan a-\tan b}{1+\tan a\tan b}$
$\cos 2x = 1 - 2\sin^2 x$	$\sin^2 x = \frac{1-\cos 2x}{2}$
$\cos 2x = 2\cos^2 x - 1$	$\cos^2 x = \frac{1+\cos 2x}{2}$
$\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$	$\forall x \in [-1,1] : \arcsin x + \arccos x = \frac{\pi}{2}$
$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$	$\forall x \in \mathbb{R} : \arctan x + arccot x = \frac{\pi}{2}$

Formulas of Simpson:

$a = \frac{a+b}{2} + \frac{a-b}{2}$
$b = \frac{a+b}{2} - \frac{a-b}{2}$

$\sin a + \sin b = 2\sin\frac{a+b}{2}\cos\frac{a-b}{2}$
$\sin a - \sin b = 2\cos\frac{a+b}{2}\sin\frac{a-b}{2}$
$\cos a + \cos b = 2\cos\frac{a+b}{2}\cos\frac{a-b}{2}$
$\cos a - \cos b = -2\sin\frac{a+b}{2}\sin\frac{a-b}{2}$

Derivatives:

Basic Derivatives:

$D(c) = 0$	$D(\sin x) = \cos x$
$D(x) = 1$	$D(\cos x) = -\sin x$
$D(nx) = n$	$D(\tan x) = \frac{1}{\cos^2 x}$
$D(x^n) = nx^{n-1}$	$D(\cot x) = \frac{-1}{\sin^2 x}$
$D(\frac{1}{x}) = \frac{-1}{x^2}$	$D(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$
$D(\sqrt{x}) = \frac{1}{2\sqrt{x}}$	$D(\arccos x) = \frac{-1}{\sqrt{1-x^2}}$
$D(\sqrt[n]{x}) = D(x^{1/n}) = \frac{1}{n \cdot \sqrt[n]{x^{n-1}}}$	$D(arctan x) = \frac{1}{1+x^2}$
$D(e^x) = e^x$	$D(\sinh) = \cosh x$
$D(a^x) = a^x \ln a$	$D(\cosh x) = \sinh x$
$D(\log_a x) = \frac{1}{\ln a \cdot x}$	$D(\tanh x) = \frac{1}{\cosh^2 x}$
$D(x^x) = x^x(\ln x + 1)$	$D(\coth x) = \frac{-1}{\sinh^2 x}$

Rules of Calculation:

$D(f(x) + c) = D(f(x))$

$D(cf(x)) = c \cdot f'(x)$

$D(\lambda_1 f(x) + \lambda_2 g(x)) = \lambda_1 f'(x) + \lambda_2 g'(x)$

$(f \cdot g)'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$(\frac{f}{g})'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}$

$(f \circ g)'(x) = (f' \circ g)(x) \cdot g'(x)$

$$f(x) = \begin{cases} g(x) & \text{P(x)} \\ h(x) & \text{Q(x)} \end{cases} \implies f'(x) = \begin{cases} g'(x) & \text{P(x)} \\ h'(x) & \text{Q(x)} \end{cases}$$

$g(x) = f^{-1}(x) \implies g'(x) = \frac{1}{(f' \circ g)(x)}$

Integrals:

Fundamental Integrals:

$\int dx = x + c$	$\int \frac{1}{\cos^2 x} dx = \tan x + c$
$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad n \in \mathbb{N} \setminus \{-1\}$	$\int \frac{1}{\sin^2 x} dx = -\cot x + c$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + c$	$\int \frac{1}{1+x^2} dx = \arctan x + c$
$\int e^x dx = e^x + c$	$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \arctan \frac{x}{a} + c$
$\int a^x dx = \frac{a^x}{\ln a} + c$	$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + c$
$\int \frac{1}{\sqrt{x^2+k}} dx = \ln \lvert x + \sqrt{x^2+k} \rvert + c$	$\int \frac{1}{\sqrt{a^2-x^2}} dx = \arcsin \frac{x}{a} + c$
$\int \sin x \ dx = -\cos x + c$	$\int \sinh x \ dx = \cosh x + c$
$\int \cos x \ dx = \sin x + c$	$\int \cosh x \ dx = \sinh x + c$

Rules of Calculation:

$\int_a^b f + \int_a^b g = \int_a^b f + g$		
$\int_a^b rf = r \int_a^b f$	$r \in \mathbb{R}$	
$\int_a^c f + \int_c^b f = \int_a^b f$		
$\frac{d}{dx} \int_a^x f = f(x)$	$x \geq a$	
$\int f^n(x) f'(x) dx = \frac{f^{n+1}(x)}{n+1} + c$		
$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c$		
$\int f'(x) g(x) dx = (f \cdot g)(x) - \int f(x) g'(x) dx$		(partial integration)
$\int (f \circ g)(x) g'(x) dx = \int f(u) du$	$g(x) = u$	(substitution)