$\boxed{7.}$ $w(x) = 1 + x^2$

$[a, b]$

$f_1(x) = 1 \qquad f_2(x) = x \qquad f_3(x) = x^2$

$L_1|$

$$\|f_1\| = \sqrt{\int_{-1}^{1} (f_1(x))^2 w(x)\, dx} = \sqrt{\frac{8}{3}}$$

$$= \sqrt{\int_{-1}^{1} 1 \cdot (x^2 + 1)\, dx}$$

$$= \sqrt{\left[\frac{x^3}{3} + x \right]_{-1}^{1}}$$

$$= \sqrt{\left(\frac{1}{3} + 1\right) - \left(-\frac{1}{3} - 1\right)}$$

$$L_1 = \frac{1}{\sqrt{\frac{8}{3}}} = \sqrt{\frac{3}{8}}$$

$L_2|$

$$\|f_2\| = \sqrt{\int_{-1}^{1} x^2 (x^2 + 1)\, dx} = \sqrt{\left[\frac{x^5}{5} + \frac{x^3}{3} \right]_{-1}^{1}}$$

$$= \sqrt{\int_{-1}^{1} x^4 + x^2\, dx} \qquad = \sqrt{\left(\frac{1}{5} + \frac{1}{3}\right) - \left(-\frac{1}{5} - \frac{1}{3}\right)}$$

$$= \sqrt{\frac{16}{15}}$$

$$L_2 = \frac{x}{\sqrt{\frac{16}{15}}} = \frac{\sqrt{15}}{4} x$$

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L_3 :

$$\|f_3\| = \sqrt{\int_{-1}^{1} x^4 (x^2+1)\,dx} = \sqrt{\frac{2}{7} + \frac{2}{5}}$$

$$= \sqrt{\int_{-1}^{1} x^6 + x^4 \,dx} = \sqrt{\frac{10}{35} + \frac{14}{35}}$$

$$= \sqrt{\left[\frac{x^7}{7} + \frac{x^5}{5} \right]_{-1}^{1}} = \sqrt{\frac{24}{35}}$$

$$= \sqrt{\left(\frac{1}{7} + \frac{1}{5}\right) - \left(-\frac{1}{7} - \frac{1}{5}\right)}$$

$$L_3 = \frac{x^2}{\sqrt{\frac{24}{35}}}$$

$$= \frac{\sqrt{35}\; x^2}{\sqrt{24}}$$

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