Практика Мат. анализ
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ММИ

1) База $n=1$, верно (?)

2) Предп $n=k$, верно

3) Переход $n=k+1$, верно (?)

Арифм. прогр

$\{a_1, a_2, \dots, a_n\}$

$a_{n+1} = a_n + d$,

$a_n = a_1 + d\cdot(n-1)$

$a_{n+1} = a_1 + d\cdot n$;

$$S_n = \frac{(a_1+a_n)\cdot n}{2} = \frac{(2a_1+(n-1)\cdot d)\cdot n}{2} = a_1 + \frac{d\cdot n(n-1)}{2}$$

1) $n=1$, $S_1 = a_1$ верно

2) $n=k$, $S_k = a_1 + \frac{d\cdot k(k-1)}{2}$, верно

3) $n=k+1$ $\quad S_{k+1} = 2a_1 + \frac{d\cdot k(k+1)}{2}$ $\Big/ - a_{k+1}$

$a_{k+1} = a_1 + d\cdot k$

$$S_k = 2a_1 + \frac{d\cdot k(k+1)}{2} - a_{k+1}$$

$$a_1 + \frac{d\cdot k(k-1)}{2} = \cancel{a_1} + \frac{d\cdot k(k+1)}{2} - a_{k+1}$$

$$\frac{d\cdot k(k+1)}{2} - \frac{d\cdot k(k-1)}{2} = a_{k+1}$$

$$\frac{d\cdot k\cdot \cancel{2}}{\cancel{2}} = a_{k+1}$$

$(1+x)^n \geq 1+n\cdot x$ $\quad \begin{pmatrix} n\geq 2 \\ x > -1 \end{pmatrix}$

1) $(1+x)^2 \geq 1+2x$

$1+2x+x^2 \geq 1+2x$

$x^2 \geq 0$, ✓

2) $(1+x)^k \geq 1+k\cdot x$, ✓

3) $(1+x)^{k+1} \geq 1+(k+1)x$

$(1+x)(1+k\cdot x) \geq 1+(k+1)x$

$\cancel{1}+\cancel{x}+\cancel{kx}+k\cdot x^2 \geq \cancel{1}+\cancel{k\cdot x}+\cancel{x}$;

$k\cdot x^2 \geq 0$, ✓

Линейность суммирования:

$$\sum_{k=1}^{n}(\alpha a_k + \beta b_k) = \sum_{k=1}^{n}\alpha a_k + \sum_{k=1}^{n}\beta b_k = \alpha\sum_{k=1}^{n}a_k + \beta\sum_{k=1}^{n}b_k;$$

$$S_n = \sum_{k=1}^{n}(a_1 + (k-1)\cdot d) = n\cdot a_1 + d\sum_{k=1}^{n}(k-1) = n\cdot a_1 + \frac{n(n-1)d}{2}$$

$f(k),\ k\in N$

$S_n = S(n) = \sum\limits_{k=1}^{n} f(k)$

$a_1, a_2, \dots, a_n$

$f(k) = a_{k+1} - a_k$

$S_n = \sum\limits_{k=1}^{n} (a_{k+1} - a_k) = -a_1 + a_{n+1}$

$\sum\limits_{k=1}^{n} \frac{1}{k(k+1)} = \sum\limits_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{k+1}\right) =$

$= \sum\limits_{k=1}^{n} \frac{1}{k} - \sum\limits_{k=1}^{n} \frac{1}{k+1} = 1 - \frac{1}{n+1}$

$\sum\limits_{k=1}^{n} \frac{1}{k(k+1)(k+2)} = \frac{1}{2}\left(\sum\limits_{k=1}^{n} \frac{1}{k(k+1)} - \sum\limits_{k=1}^{n} \frac{1}{(k+1)(k+2)}\right) = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right)$

$= \frac{1}{4} - \frac{1}{2(n+1)(n+2)};$

$(a+b)^n = \sum\limits_{k=0}^{n} C_n^k \cdot a^{n-k} \cdot b^k$, $C_n^k = \frac{n!}{k!(n-k)!}$; ММИ

1) $n=1$

$a+b = \frac{1!}{0!\,1!} \cdot a \dots$

2) $n=k$, то

$(a+b)^m = \sum\limits_{k=0}^{m} (C_m^k \cdot a^{m-k} \cdot b^k)$, верно;

$n = m+1$

3) $(a+b)^{m+1} = (a+b)\sum\limits_{k=0}^{m} (C_m^k \cdot a^{m-k} \cdot b^k) =$

$= \sum\limits_{k=0}^{m+1} C_{m+1}^k\, a^{m+1-k} \cdot b^k.$

Д/З Шоригиев 1 том §4 №3(1), 6, 13(1,3)

$a_n = a_1 + d\cdot(n-1)$

$S_n = \frac{(a_1+a_n)\cdot n}{2}$, доказать $S_{n+3} = 3S_{n+2} - 3S_{n+1} + S_n$,

№3.(1)

Преобразую по формуле $S_{n+3} = \frac{(a_1 + a_{n+3})(n+3)}{2}$

$S_{n+2} = \frac{(a_1 + a_{n+2})}{2}\cdot(n+2)$

Имеем

$S_{n+1} = \frac{(a_1 + a_{n+1})}{2}\cdot(n+1)$

$\frac{(a_1 + a_1 + d\cdot(n+2))(n+3)}{2} = \frac{3\cdot(a_1 + a_1 + d(n+1))\cdot(n+2)}{2} -$ $\Big|\cdot 2$

$-\frac{3(a_1+a_1+d\cdot n)(n+1)}{2} + \frac{(a_1+a_1+d(n-1))\cdot n}{2}$

$(2a_1 + d(n+2))(n+3) = 3(2a_1+d(n+1))(n+2) - 3(2a_1+d\cdot n)(n+1) + (2a_1+d(n-1))n$

№13 (1, 3)

1) $\sum_{k=1}^{n} \frac{1}{(3k-2)(3k+1)} = \sum_{k=1}^{n} \left(\frac{1}{3} \cdot \left(\frac{1}{3k-2} - \frac{1}{3k+1} \right) \right) = \frac{1}{3}\left(\sum_{k=1}^{n} \frac{1}{3k-2} - \sum_{k=1}^{n} \frac{1}{3k+1} \right) =$

$= \frac{1}{3}\left(1 - \frac{1}{3n+1}\right) = \frac{1}{3} - \frac{1}{9n+3}$

3) $\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)(2k+3)} = \frac{1}{4}\sum_{k=1}^{n}\left(\frac{1}{(2k-1)(2k+1)} - \frac{1}{(2k+1)(2k+3)} \right) =$

$= \frac{1}{4}\left(\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} - \sum_{k=1}^{n} \frac{1}{(2k+1)(2k+3)} \right) = \frac{1}{4}\left(\frac{1}{3} - \frac{1}{(2n+1)(2n+3)} \right) =$

$= \frac{1}{12} - \frac{1}{4(2n+1)(2n+3)}$;

Разложение

$$\frac{P_m(X)}{Q_n(X)}$$

$$Q_n(X) = C_n \cdot (X-a_1)^{a_1} \dots (X-a_k)^{a_k} \cdot (X^2 + p_1 X + q_1)^{\beta_1} \dots (X^2 + p_s X + q_s)^{\beta_s}$$

$$\frac{P_m(X)}{(X-a)^k} = \frac{A_k}{(X-a)^k} + \frac{A_{k-1}}{(X-a)^{k-1}} + \dots + \frac{A_1}{X-a}, \ A_i \in R$$

$$\frac{1}{k(k+1)(k+2)} = \frac{A^{\backslash (k+1)(k+2)}}{k} + \frac{B^{\backslash k(k+2)}}{k+1} + \frac{C^{\backslash k(k+1)}}{k+2} ;$$

$1 = A(k+1)(k+2) + B \cdot k(k+2) + C \cdot k(k+1)$.

$k^2(A+B+C) + k(3A+2B+C) + 2A = 1 \iff$

$$\begin{cases} 2A = 1 \\ A+B+C = 0 \\ 3A+2B+C = 0 \end{cases} \quad \dots$$

Метод неопределенных коэф.

$$\frac{P_n(x)}{(x^2+px+q)^s} = \frac{B_sx+D_s}{(x^2+px+q)^s} + \dots + \frac{B_1x+D_1}{(x^2+px+q)^1}$$

$$\frac{1}{(x^2+1)^2} = \frac{B_1x+C_1}{x^2+1}^{|x^2+1} + \frac{B_2x+C_2}{(x^2+1)^2}$$

$$1 = x^3(B_1) + x^2\cdot C_1 + x(B_1+B_2) + x^0(C_1+C_2)$$

$$\begin{cases} B_1 = 0 \\ C_1 = 0 \\ B_1+B_2 = 0 \\ C_1+C_2 = 1 \end{cases}$$

Д/З: §6, №19(1, 2)
§4, №14(3)

№14 (3)

$\{a_n\}$ - арифм. прогр.

3) $$\sum_{k=1}^{n} \frac{1}{a_k\cdot a_{k+1}\cdot a_{k+2}\cdot a_{k+3}} = \frac{1}{3d}\left(\frac{1}{a_1a_2a_3} - \frac{1}{a_{n+1}\cdot a_{n+2}\cdot a_{n+3}}\right);$$

$$\sum_{k=1}^{n} \frac{1}{a_1^4\cdot d^{k-1+k+k+1+k+2}} = \frac{1}{3d}\left(\frac{1}{a_1^3\cdot d^3} - \frac{1}{a_1^3\cdot d^{n+n+1+n+2}}\right);$$

$$\sum_{k=1}^{n} \frac{1}{a_1^4\cdot d^{3k+2}} = \frac{1}{3d}\left(\frac{1}{a_1^3\cdot d^3} - \frac{1}{a_1^3\cdot d^{3n+3}}\right);$$

$$\sum_{k=1}^{n} \frac{1}{a_1^4\cdot d^{3k+2}} = \frac{1}{3a_1^3\cdot d^4}\left(1 - \frac{1}{d^{n}}\right) \Big| \cdot a_1^3\cdot d^2$$

$$\cancel{4} \sum_{k=1}^{n} \frac{1}{a_1\cdot d^{3k}} = \frac{1}{3d^2}\left(1 - \frac{1}{d^4}\right);$$

№5 (1,2)

1) $X_n = aX_{n-1} + bX_{n-2}$, $a=2, b=3$, то получим.

$X_n = 2X_{n-1} + 3X_{n-2}$; $X_2 = 2X_1 + 3X_0$; $X_3 = 2X_2 + 3X_0 = 2(2X_1 + 3X_0) + 3X_0$

$X_4 = 2X_3 + 3X_2 = 2(2(2X_1 + 3X_0) + 3X_0) + 3(2X_1 + 3X_0)$

$X_5 = 2X_4 + 3X_3 = 2((2(2X_1 + 3X_0) + 3X_0) + 3(2X_1 + 3X_0)) + 3(2(2X_1 + 3X_0) + 3X_0)$;

№7

1) $1\cdot 2 + 2\cdot 5 + \ldots + n(3n-1) = n^2(n+1)$,

1. $n=1$, $1\cdot 2 = 1^2\cdot 2$; верно

2. $n=k$,

$1\cdot 2 + 2\cdot 5 + \ldots + k(3k-1) = k^2(k+1)$; верно

3. $n=k+1$.

$1\cdot 2 + 2\cdot 5 + \ldots (k+1)(3k+2) = (k+1)^2(k+2)$

$k^2(k+1) + (k+1)(3k+2) = (k+1)^2(k+2)$;

$k^2 + 3k + 2 = (k+1)(k+2)$, верно.

2) $1^2 + 3^2 + \ldots (2n-1)^2 = \frac{n(4n^2-1)}{3}$

1. $n=2$ $1+3^2 = \frac{2\cdot 15}{3} = 10$, ✓

2. $n=k$, ✓

3. $1^2 + 3^2 + \ldots + (2k+1)^2 = \frac{(k+1)(4(k+1)^2-1)}{3}$

$\frac{k(4k^2-1)}{3} + (2k+1)^2 = \frac{(k+1)(4(k^2+2k+1)-1)}{3}$

$k(4k^2-1) + 3(2k^2+4k+1) = (k+1)(4k^2+8k+3)$ ✓

3) $1\cdot 2 + 2\cdot 3 + 3\cdot 4 + \ldots + (n-1)n = \frac{(n-1)\cdot n(n+1)}{3}$, 1. $n=6$

$\underset{2}{1\cdot 2} + \underset{6}{2\cdot 3} + \underset{12}{3\cdot 4} + \underset{20}{4\cdot 5} + \underset{30}{5\cdot 6} = \frac{5\cdot 6\cdot 7}{3} = 70$ ✓

2. $n=k$, ✓

3. $1\cdot 2 + 2\cdot 3 + 3\cdot 4 + \ldots + k(k+1) = \frac{k(k+1)(k+2)}{3}$

$\frac{(k-1)\cancel{k(k+1)}}{3} + \cancel{k(k+1)} = \frac{\cancel{k(k+1)}(k+2)}{3}$, ✓

4) $1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$;

1. $n=1$, ✓

2. $n=k$, ✓

3. $1^3 + \ldots + (k+1)^3 = \left(\frac{(k+1)(k+2)}{2}\right)^2$

$\left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3 = \left(\frac{(k+1)(k+2)}{2}\right)^2$

$k^2\cancel{(k+1)^2} + 4(k+1) = \cancel{(k+1)^2}(k+2)^2$

$k^2 + 4k + 4 = (k+2)^2$, ✓.

№8

8) $6^{2n-2}+3^{n+1}+3^{n-1}=11a$,

1. $n=1$. ✓

2. $n=k$, ✓

3. $n=k+1$. $6^{2k}+3^{k+2}+3^{k}=\frac{6^{2k-2}}{36}+3\cdot3^{k+1}+3^{k-1}\cdot3=3\left(\frac{6^{2k-2}}{12}+3^{k+1}+3^{k-1}\right)$

$=3\left(11a+\frac{6^{2k-2}}{12}-6^{2k-2}\right)=3\left(11a+\frac{6^{2k-2}}{11}\right)$.

№12

$x_1\cdot x_2\cdot\ldots\cdot x_n=1$ 1. $n=1$, ✓ 2. $n=k$, ✓ 3. $n=k+1$.

$x_1+x_2+\ldots+x_n\geq n$ $x_1+x_2+\ldots+x_{k+1}\geq k+1$, $k+x_{k+1}\geq k+1$.

$x_{k+1}\geq 1$.

№13

$\frac{x_1+x_2+\ldots+x_n}{n}\geq\sqrt{x_1\cdot x_2\cdots x_n}$ 1. $x=1$, ✓ 2. $n=k$, ✓ 3. $n=k+1$.

$\frac{x_1+x_2+\ldots+x_{k+1}}{k+1}\geq\sqrt{x_1\cdot x_2\cdot\ldots\cdot x_{k+1}}$;

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Верхняя и нижняя грани числового мн-ва

1. Промежутки в $\mathbb{R}$; $a,b\in\mathbb{R}$, $(a,b)\overset{def}{=}\{x\in\mathbb{R}: a<x<b\}$,

$[a,b]\overset{def}{=}\{x\in\mathbb{R}: a\leq x\leq b\}$. $(-\infty;b)\overset{def}{=}\{x\in\mathbb{R}: x<b\}$.

$(a;+\infty)\overset{def}{=}\{x\in\mathbb{R}: a<x\}$ $[a;+\infty)\overset{def}{=}\{x\in\mathbb{R}: a\leq x\}$.

$\mathbb{R}\equiv(-\infty;+\infty)$, $\forall x\in\mathbb{R}: -\infty<x<-\infty$;