MS&E 310 **Homework #4**
Linear Programming **2013**

HOMEWORK ASSIGNMENT 4: DUE November 29

Reading: Chapters 5 and 15.9 of (L&Y), *Linear and Nonlinear Programming* and Lecture Notes.

1. Exercise 7 (b) of Chapter 5 (L&Y)

2. Exercise 12 of Chapter 5 (L&Y)

3. Exercise 13 of Chapter 5 (L&Y)

4. Consider the LP feasible set:

$$\mathbf{a}_i \bullet \mathbf{x} = b_i, \quad i = 1, ..., m, \quad \mathbf{x} \geq \mathbf{0}$$

where $\mathbf{x}$ is an n-dimensional vector. Show that the polyhedral set

$$B = \{\mathbf{b} \in R^m : \; b_i = \mathbf{a}_i \bullet \mathbf{x}, \; i = 1, ..., m, \quad \mathbf{x} \geq \mathbf{0}\}$$

is a closed convex set.

Now consider the SDP feasible set:

$$A_i \bullet X = b_i, \quad i = 1, ..., m, \quad X \succeq 0$$

Suppose that there are scalars $\lambda_i > 0$ such that:

$$\sum_i^m \lambda_i A_i \succ 0.$$

Show that

- the set
$$B = \{\mathbf{b} \in R^m : \; b_i = A_i \bullet X, \; i = 1, ..., m, \quad X \succeq 0\}$$
is a closed convex set.
- The SDP feasible set has an alternative system:
$$\sum_i^m y_i A_i \preceq 0, \quad b^T y = 1.$$

5. When C is a cone, the dual of C is the closed convex cone

$$C^* = \{y : y \bullet x \geq 0 \quad \forall x \in C\},$$

where $\bullet$ denotes the inner product. In the following, let all cones be defined in n-dimensional symmetric matrices, and both C_1 and C_2 are convex cones. Show

(i) $C_1 \subset C_2 \Longrightarrow C_2^* \subset C_1^*$.

(ii) $(C_1 + C_2)^* = C_1^* \cap C_2^*$.

(iii) $C_1^* + C_2^* \subset (C_1 \cap C_2)^*$.

(iv) Let C be the cone of positive semidefinite matrices whose every entry is also nonnegative. What is the dual cone of C?

Note: If S and T are subsets of n-dimensional symmetric matrices, then by definition

$$S + T = \{s + t : s \in S,\ t \in T\}.$$

6. Exercise 13 of Chapter 15 (L&Y).

7. Extend the homogeneous and self-dual LP model to solving SDP and perform similar analyses. Can you guarantee that τ and κ are strictly complementary at optimality?