2012 Bonus Projects for MS&E 311

Yinyu Ye

Department of Management Science and Engineering

Stanford University

Stanford, CA 94305, U.S.A.

http://www.stanford.edu/~yyye

Project 1: Online Value-Function Optimization Algorithm I

Initialization of the online combinatorial auction for prediction markets discussed in the class:

$$\begin{array}{lll} \max & -z + U(\mathbf{s}) & \\ \text{s.t.} & -\mathbf{e} \cdot z + \mathbf{s} & = \mathbf{0}; \end{array}$$

and let z^0 and $\mathbf{s}^0$ be the optimal solution for the problem, and let $\mathbf{b}^0 = \mathbf{0}$ and initial price vector $\mathbf{p}^0 = \nabla U(\mathbf{s}^0)$.

Project 1: Online Value-Function Optimization Algorithm II

When the kth order arrives, solve a single order optimization problem

$$\begin{array}{lll} \max & \pi_k x_k - z + U(\mathbf{s}) & \\ \text{s.t.} & \mathbf{a}_k x_k - \mathbf{e} \cdot z + \mathbf{s} & = -\mathbf{b}^{k-1}, \\ & x_k & \leq q_k, \\ & x_k & \geq 0; \end{array}$$

and let $x_k^*, z^k, \mathbf{s}^k$ be the optimal solution for the problem, and let $\mathbf{b}^k = \mathbf{b}^{k-1} + \mathbf{a}_k x_k^*$ and price vector $\mathbf{p}^k = \nabla U(\mathbf{s}^k)$.

Project 1: What you need to do

- Implement the online algorithm and check the correctness of the implementation using the Logarithmic and Exponential value functions.
- Create data sets, and compare the online performance with the offline performance for each data set.
- Create a data set with a ground-truth probability distribution $\bar{\mathbf{p}}$, and run the online algorithm and compare the price vector of the algorithm with the true probability distribution order by order. More precisely, bid k is created as

$$\pi_k = \bar{\mathbf{p}}^T \mathbf{a}_k + \text{noise}, \text{ and } q_k = 10;$$

  where $\mathbf{a}_k$ is a random zero-one vector.
- Create a data set with a ground-truth probability distribution $\bar{\mathbf{p}}$ and the post price vector $\mathbf{p}^k$, and run the online algorithm and compare the price vector of the algorithm with the true probability distribution order by order. More

precisely, each bid is created as

$$\pi_k = \bar{\mathbf{p}}^T \mathbf{a}_k + \text{noise}, \text{ and } q_k = 10;$$

where $\mathbf{a}_k$ is zero-one vector: $(\mathbf{a}_k)_i = 1$ if $\bar{\mathbf{p}}_i \geq \mathbf{p}_i^{k+1}$ and $(\mathbf{a}_k)_i = 0$ otherwise.

- Any observation and conclusion you may draw from your simulation experiment.
- Electronic project report is due Sunday March 17.

Project 2: Concave L_p Minimization

Consider the two models

$$\begin{array}{ll} \min & p(\mathbf{x}) := \sum_{1\le j\le n} x_j^p \\ \text{s.t.} & A\mathbf{x} = \mathbf{b}, \\ & \mathbf{x} \ge \mathbf{0}; \end{array}$$

and

$$\begin{array}{ll} \min & q(\mathbf{x}, s_0) := s_0 + \lambda \cdot \left(\sum_{1\le j\le n} x_j^p \right) \\ \text{s.t.} & A\mathbf{x} + \mathbf{s} = \mathbf{b}, \\ & (s_0; \mathbf{s}) \in SOC, \end{array}$$

where $0 < p < 1$. We assume that the feasible region is bounded and has an interior.

The original real problem is to find the sparsest solution:

$$\begin{array}{rl} \min & \|\mathbf{x}\|_0^0 \\ \text{s.t.} & A\mathbf{x} = \mathbf{b}, \\ & \mathbf{x} \geq \mathbf{0}; \end{array}$$

and

$$\min \quad \|A\mathbf{x} - \mathbf{b}\| + \lambda \cdot \|\mathbf{x}\|_0^0$$

Project 2: What you need to do?

- Prove any possible theoretical result of the two minimization models.
- Implement the interior-point algorithm and check the correctness of the implementation for $p = 1/2$ for solving at least one of the two models.
- Create data sets each with a known sparse solution, and test the quality of the model to recover the sparse solution.
- Compare the quality of the models $p = 1/2$ with that of $p = 1$, i.e., the linear programming model.
- You may generate matrices A randomly with few different distributions.
- Electronic project report is due Sunday March 17.