Signals and Systems Notes

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1 Introduction

Alright boys. Here's the deal. I'm going to write all the important stuff in the past two weeks from S&S.

1.1 Fourier Series

So here's the thing. Fourier Series are a way to turn functions in the time domain (what we're used to – functions over time) to functions in the frequency domain (essentially, a sum of sines and cosines with different frequencies). The Fourier Series is a formula to transform a function from the time domain to the frequency domain. Don't worry about the proof for this; just know the formula and know how to integrate.

First, we need a definition for the fourier representation of any time-series x(t), as follows:

$$x(t) = \int_0^T a_k e^{jk\omega_0 t} dt \tag{1}$$

for every integral k from $-\infty$ to ∞. Now, how do we transform x(t) to the a_k's? That's the important thing. Intuitively, we know that any periodic function can be represented as a sum of a countably infinite number of sines and cosines; it's just a matter of finding what the scaling factors for these sines and cosines are.

So, the magic formula that converts the time domain to the frequency domain is the following:

$$a_k = \frac{1}{T} \int_0^T x(t) e^{-jk\omega_0 t} dt \tag{2}$$

Keep in mind, this only works for PERIODIC functions. In order to get it to work for NONPERIODIC functions, we essentially take the limit of the period to infinity (infinite period means it never repeats), and now instead of having a bunch of discrete a_k values for any integral k, we have a continuum of a(k) values for any real-valued k. But now, since we feel like it, we call it $x(jw_0)$. And now, the formula is:

$$x(jw_0) = \int_{-\infty}^{\infty} x(t)e^{-jk\omega_0 t}dt \tag{3}$$

Awesome! Now just one more thing. What happens if we want to go backwards? That is, given the function in the frequency domain (sum of a bunch of sines and cosines), how can we get the function over time back? Well, thanks for asking. Here's the formula.

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(jw_0)e^{j\omega_0 t}d\omega \tag{4}$$

1.1.1 Delta Train

Let's take a look at a specific example, the delta train. But first, let's look at a scaled delta function, defined as:

$$X(j\omega) = 2\pi\delta(\omega - \omega_0), \tag{5}$$

which basically means it's a single value at ω_0. Clearly, taking the integral in (4) gives you just a single value, which is $e^{j\omega_0 t}$. Now what if we stacked a bunch of delta's together in a periodic fashion, so now:

$$X(j\omega) = \sum_k 2\pi a_k \delta(\omega - k\omega_0). \tag{6}$$

This basically means that there's a scaled pulse after every period of time, and that's it. Now, clearly since the integral just adds all the values for which the function is nonzero, we get

$$X(j\omega) = \sum_k a_k e^{j\omega_0 t}. \tag{7}$$

1.1.2 What if $x(j\omega)$ is given in terms of magnitude and phase?

Since $x(j\omega)$ is often complex for real x(t), sometimes we will get it in terms of its magnitude and phase. But don't fret, just use this simple formula you probably learned in precalc to get the function in terms of magnitude and phase:

$$x(jw) = |x(jw)|\, e^{\angle x(j\omega)} \tag{8}$$

Literally just plug that in to the relevant formula, and you're good.

THINGS TO KNOW FOR TEST:

- Convolution (REMEMBER $-$inf to inf
- Fourier Series
- Fourier Transform