

# Nano Kiwi

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## Architecture

- We start from the **original GAN architecture** provided in the assignment.
- Latent prior:  $z \sim \mathcal{N}(0, I)$ .

## Classical GAN loss

$$\min_G \max_D \mathbb{E}_{x \sim P_{\text{data}}} [\log D(x)] + \mathbb{E}_{z \sim \mathcal{N}(0, I)} [\log(1 - D(G(z)))].$$

## Baseline performance

- FID: **108.1**
- Precision: **0.56**
- Recall: **0.09**

# Improving Recall with Static GM-GAN

## Motivation [1]

- The baseline GAN shows **limited recall**:
- The generator fails to cover all modes of the real data distribution.

## Static Gaussian Mixture GAN (Static GM-GAN)

- Replace the standard normal prior by a **mixture of  $K$  Gaussians**:

$$z \sim \frac{1}{K} \sum_{k=1}^K \mathcal{N}(\mu_k, \sigma_k^2 I).$$

- Means  $\mu_k$  are sampled once at initialization and kept fixed.
- The multi-modal latent prior encourages the generator to explore different regions of the data space.

## Static GM-GAN results (local)

- FID: **35.82**
- Precision: **0.28**
- Recall: **0.52** (improved recall compared to baseline)



# Modifying the Objective: Introducing WGAN

## Motivation

- So far, we modified the **latent distribution** (Static and Dynamic GM-GAN).
- The classical GAN loss still suffers from:
  - Vanishing gradients
- We now **modify the training objective itself** using Wasserstein GAN (WGAN).

## Wasserstein-1 distance

$$W(P_{\text{data}}, P_G) = \sup_{Lip(f) \leq 1} \mathbb{E}_{x \sim P_{\text{data}}} [f(x)] - \mathbb{E}_{z \sim P_Z} [f(G(z))].$$

# Motivation

## From GAN to Wasserstein GAN (WGAN)

The divergence used in the original GAN objective is replaced by the *(Wasserstein-1) distance*. The objective becomes:

$$\min_G \max_{D \in \text{Lip}(1)} \left( \mathbb{E}_{x \sim P_r} [D(x)] - \mathbb{E}_{z \sim P_z} [D(G(z))] \right),$$

where:

- $P_r$  is the real data distribution,
- $P_z$  is the prior distribution (e.g.,  $\mathcal{N}(0, I)$ ),
- $D$  is constrained to be 1-Lipschitz.

# WGAN-GP Objective

## Enforcing the 1-Lipschitz constraint

WGAN-GP [2] introduces a **gradient penalty** applied to random interpolations between real and generated samples.

$$\hat{x} = \epsilon x_{\text{real}} + (1 - \epsilon) x_{\text{fake}}, \quad \epsilon \sim \mathcal{U}(0, 1).$$

### Gradient Penalty term:

$$\text{GP} = \mathbb{E}_{\hat{x}} \left[ (\|\nabla_{\hat{x}} D(\hat{x})\|_2 - 1)^2 \right].$$

### Full WGAN-GP critic loss:

$$L_D = \mathbb{E}_{x_{\text{fake}}} [D(x_{\text{fake}})] - \mathbb{E}_{x_{\text{real}}} [D(x_{\text{real}})] + \lambda \text{GP}.$$

### Generator loss:

$$L_G = -\mathbb{E}_{x_{\text{fake}}} [D(x_{\text{fake}})].$$

## Training setup

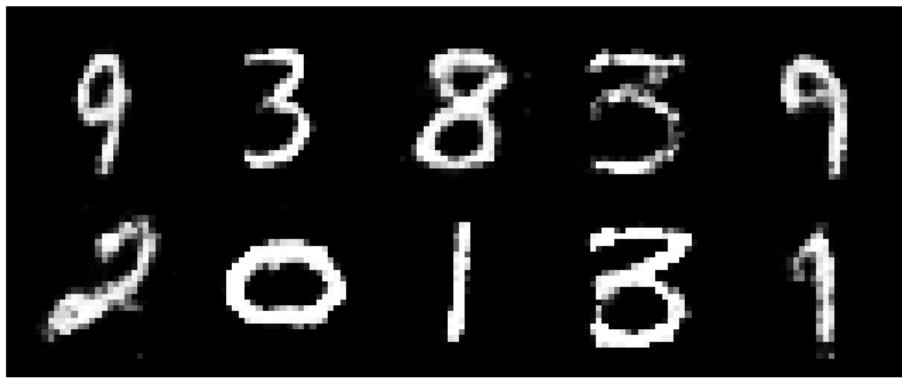
- Same architecture and latent prior as the baseline GAN.
- Loss replaced by the Wasserstein objective with gradient penalty.
- Gradient penalty weight:  $\lambda = 10$ .
- 5 discriminator updates per generator update.
- 100 epochs, batch size : 64

# WGAN-GP: Experimental Results

## Quantitative results

| Model                | FID   | Precision | Recall |
|----------------------|-------|-----------|--------|
| (local) WGAN(-GP)    | 26.90 | 0.42      | 0.59   |
| (platform) WGAN(-GP) | 35.48 | 0.48      | 0.32   |

## Observations



# Conditional WGAN-GP with Learnable Latent Distributions

**Goal:** Each digit has its own learnable Gaussian in the latent space.

**Conditional Discriminator:**

$$D(x, i) \rightarrow \text{score}$$

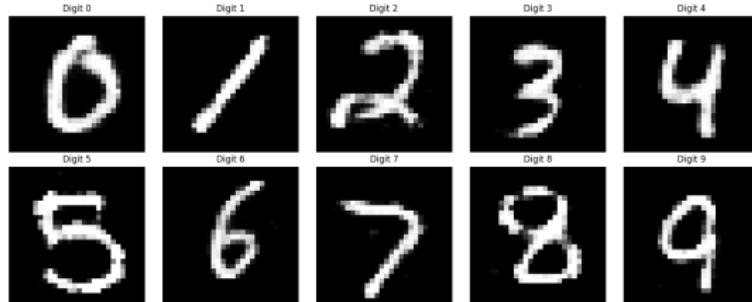
**Conditional Generator:**

$$G(z), \quad z \sim \mathcal{N}(a_i, b_i^2 I)$$

**Learnable parameters:**  $a_i, b_i \in \mathbb{R}^{100}$  for each digit  $i \in \{0, \dots, 9\}$

# Results

## Generated Samples (Epoch 600)



## Training Evolution

| Epoch      | FID ↓        | Precision   | Recall      |
|------------|--------------|-------------|-------------|
| 100        | 35.31        | 0.43        | 0.49        |
| 200        | 24.17        | 0.42        | 0.59        |
| 300        | 20.23        | 0.43        | 0.63        |
| 400        | 17.29        | 0.51        | 0.64        |
| 500        | 15.44        | 0.53        | 0.66        |
| <b>600</b> | <b>15.15</b> | <b>0.52</b> | <b>0.67</b> |

# Global Comparison of All Models

## Quantitative comparison

| Model                   | FID   | Precision | Recall |
|-------------------------|-------|-----------|--------|
| Baseline GAN            | 108.1 | 0.56      | 0.09   |
| Static GM-GAN           | 35.82 | 0.28      | 0.52   |
| WGAN-GP                 | 26.90 | 0.42      | 0.59   |
| WGAN-GP <sub>cond</sub> | 15.15 | 0.52      | 0.67   |

## Key observations

- **Recall** improves when introducing a multi-modal latent space (Static & Dynamic GM-GAN).
- Dynamic GM-GAN generally outperforms Static GM-GAN, confirming the benefit of learning mixture parameters.
- WGAN improves training stability and mode coverage, but does not explicitly target recall as a metric.



Matan Ben-Yosef and Daphna Weinshall.

Gaussian mixture generative adversarial networks for diverse datasets, and the unsupervised clustering of images.

*arXiv preprint arXiv:1808.10356*, 2018.



Ishaan Gulrajani, Faruk Ahmed, Martin Arjovsky, Vincent Dumoulin, and Aaron Courville.

Improved training of wasserstein GANs.

In *Advances in Neural Information Processing Systems (NeurIPS)*, 2017.