

We start with initial configuration

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

Let a clockwise rotation of the first square be a U move and a clockwise rotation of the second square be an R move.

We further write down the configuration in this format.

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \equiv \left\| \begin{array}{ccc} 1 & 4 & 2 \\ 3 & 6 & 5 \end{array} \right\|$$

Then an R move would take us to

$$\left\| \begin{array}{ccc} 1 & 4 & 5 \\ 2 & 3 & 6 \end{array} \right\|$$

and a U move would take us to

$$\left\| \begin{array}{ccc} 4 & 5 & 1 \\ 3 & 6 & 2 \end{array} \right\|$$

Define a flip move F_i as taking the elements of the i th column in our new representation and flipping them. Define a turn move T as either rotating the bottom three elements forward (T_d) or the top three elements backward T_u .

It can be shown that $R = F_3 T_d$ and $U = F_3 T_u$

It can be also shown that $F_1 F_3 = R' U R U'$, $F_2 F_3 = R U' R' U$ and $F_1 F_2 = (F_1 F_3)(F_2 F_3)$. Thus we can always do an even number of flips.

Now we color 1, 3 with color A , 4, 6 with B and 2, 5 with C

It can be shown that with any configuration, we can do an even number of flips so that all members of top row and bottom row are of distinct colors. Define this as it's group representation.

We divide group representations into four classes.

Class I is where each column consists of same elements. E.g.

$$\left\| \begin{array}{ccc} A & B & C \\ A & B & C \end{array} \right\|$$

Class II is where no column contains the same elements, and diagonal formed by the same elements goes top left to bottom right. E.g.

$$\left\| \begin{array}{ccc} A & B & C \\ C & A & B \end{array} \right\|$$

Class III is where no column contains the same elements, and diagonal formed by the same elements goes top right to bottom left. E.g.

$$\begin{vmatrix} A & B & C \\ B & C & A \end{vmatrix}$$

Class IV is where exactly one column contains the same elements. E.g.

$$\begin{vmatrix} A & B & C \\ A & C & B \end{vmatrix}$$

It can be shown that F moves take

$$I \rightarrow I \tag{1}$$

$$II \rightarrow III \tag{2}$$

$$III \rightarrow IV \tag{3}$$

$$IV \rightarrow IV \tag{4}$$

and T moves take

$$I \rightarrow II \tag{5}$$

$$II \rightarrow III \tag{6}$$

$$III \rightarrow I \tag{7}$$

$$IV \rightarrow IV \tag{8}$$

$$\tag{9}$$

Let I and II be good configurations. Then FT takes good configurations to good configurations and bad configurations to bad configurations. Since the original config is good, and the one we want is bad, it is not possible