Homework groups: You will complete each of seven homework assignment as part of a three- or four-person group. Group members are assigned randomly and will remain the same for the duration of the quarter. Each group turns in one homework, and each *participating* group member receives the same grade on the assignment. One member of the group is responsible for writing the homework (**the writer**), and this writer rotates for every assignment.

Homework groups work best if: Each member of the homework group finishes (or honestly attempts) the homework independently. At some appointed time, well before the due date, the group meets and everyone compares answers. Any discrepancies are discussed until a consensus is achieved. The writer notes the group consensus and makes sure she or he understands how to do the problem. After the meeting, but before class, the writer neatly and clearly writes the homework according to the **Homework guidelines** (described below).

Homework groups don't work if: One or more of the members skips meetings; each group member does not honestly attempt the homework prior to the meeting; a consensus in not reached for each assigned problem. *If a group member does not adequately participate in the homework, write a note on the homework and alert the TA. That person will not receive credit.*

Homework guidelines for writers: (Adapted from the website of Professor Andy Ruina). To get full credit, please do these things on each homework.

1. As a group writer, you must hand homework in by the end of class Monday, the day it is due. Homework is available via Smartsite Monday evenings, and is due the following week in class (unless stated otherwise). Late homework may or may not be accepted for reduced credit.

2. On the first page of your homework, please do the following to facilitate sorting:
   On the top left corner, please put the course information, homework number and date, e.g.:

   MAT207A
   HW 3
   Due October 21, 2015.

   On the top right corner, please put the names of your group members, with the writer at the top and clearly indicated. Non-participating group members should also be indicated, e.g.:

   Jaromir Jagr (writer)
   Sarah Jessica Parker
   Michelle Wie
   James Van der Beek (did not participate)

3. Please put a staple at the top left corner. Folded interlocked corners fall apart. Paperclips fall off.

4. **CITE YOUR HELP.** At the top of each problem, clearly acknowledge all help you got from TAs, faculty, students or any other source (with exceptions for lecture and the text, which need not be cited). You could write, for example: "Mary Jones pointed out to me that I had forgotten to divide by three in problem 2," or "Nadia Chow showed me how to do problem 3 from start to finish," or "I copied this solution word for word from Jane Lewenstein" or "I found a problem just like this one, number 9, at cheatonyourhomework.com, and copied it," etc. You will not lose credit for getting and citing such help. Don't violate academic integrity rules: be clear about which parts of your presentation you did not do on your own. Violations of this policy are violations of the UC Davis Code of Academic Conduct.

5. Your work should be laid out neatly enough to be read by someone who does not know how to do the problem. For most jobs, it is not sufficient to know how to do a problem, you must convince others that you know how to do it. Your job on the homework is to practice this. **Box your answers.**

6. Grading and regrading. We have a reasonable grading and regrading policy (see syllabus).

DUE: Wednesday, October 21, 2015. To be handed to me by the end of class.
The topics of this homework are 1. Transcritical bifurcations; 2. Pitchfork bifurcations (sub- and supercritical); 3. Hysteresis; 4. Imperfect bifurcations.

These topics are covered in §3.2–3.6 in Strogatz.

For your bifurcation diagrams:
i. Indicate stable fixed points with a solid line and unstable fixed points with a dashed line
ii. Show your calculations for how you determined the fixed points.
iii. Explain how you determined stability and/or show your calculations.
iv. Clearly indicate any bifurcation(s) (if they exist)
v. Clearly identify bifurcation(s) (if they exist - e.g. saddle-node, transcritical, sub/supercritical pitchfork)

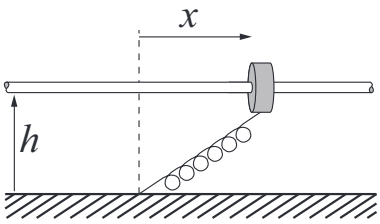

Figure 1:

1. In class, I've used a particle on a wire as a prototypical example of a system that exhibits a subcritical pitchfork bifurcation. I argued that this was true without any math. Now, you'll look at the equation.

Here is the differential equation

$$\dot{x} = -\frac{k}{b} \qquad \overline{x^2 + h^2} - \ _0 \quad \frac{x}{\overline{x^2 + h^2}}$$

If you want practice, it would be a good exercise to show that a non-dimensional form of this equation is

$$\frac{dX}{dT} = \frac{X}{\overline{X^2 + \ ^2}} - X \tag{1}$$

a. Draw a bifurcation diagram for this system.

b. Show that, near the bifurcation, Eq. 1 can be written in the normal form for a supercritical pitchfork bifurcation (i.e. $\dot{x} = -x^3 + ax$).

c. Suppose you add gravity to the system and that gravity acts along the wire.

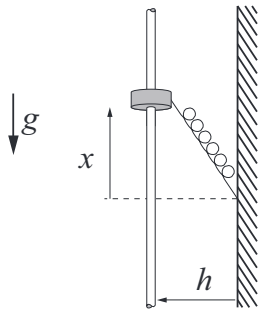

Figure 2:

The governing equation is then

$$\dot{x} = -\frac{k}{b} \quad \overline{x^2 + h^2} - \ _0 \quad \frac{x}{\overline{x^2 + h^2}} - \frac{mg}{b} \tag{2}$$

As in part a., If you want practice, it would be a good exercise to show that a non-dimensional form of this equation is

$$\frac{dX}{dT} = \frac{X}{\overline{X^2 + \ ^2}} - X - \tag{3}$$

d. Using Matlab, find the fixed points of this equation as a function of for $= 0$, $= 0.1$ and $= 0.2$. On your plots, indicate stable and unstable fixed points and label/identify any bifurcations. It might be useful to use the function **fzero**, which will numerically find the roots of a scalar function. To learn how to use it, type "help fzero" into Matlab's command line. I am also happy to help.

e. Again, using Matlab, plot the stability diagram for this system. It might be useful to use the function **fsolve**, which will numerically find the roots of a vector function. To learn how to use it, type "help fsolve" into Matlab's command line. I am also happy to help.

2.

$$\dot{x} = Ax - x(1 - x)^2 \tag{4}$$

a) Draw a bifurcation diagram for this equation as A varies.

b) In the neighborhood of all bifurcation(s) (if they exist), transform Equation 4 to the normal form.

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3. Consider the following bifurcation diagram (next page) showing fixed points x as a function of the parameter P. Note that there are three different parameter values indicated, P_1, P_2 and P_3.

a) Label and identify all bifurcations on the figure.

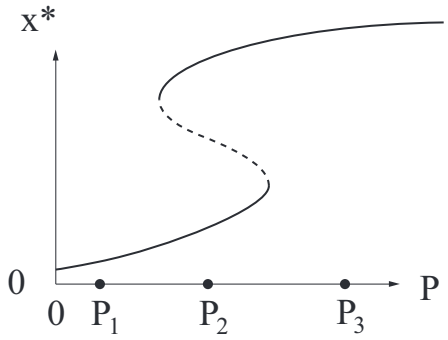

Figure 3: Bifurcation diagram; stable equilibria are drawn as a solid line, unstable as a dashed line.

b) Draw phase portraits consistent with the bifurcation curve at each of the parameter values P_1, P_2 and P_3. (You should draw one for each parameter value, a total of three phase portraits).

c) Can this system exhibit hysteresis (according to the definition used in class)?

d) How would your answer to c) change if the stability in Fig. 3 were flipped (i.e. each stable fixed point were unstable and each unstable fixed point were stable)? Explain.
