Homework groups: You will complete each of seven homework assignment as part of a three- or four-person group. Group members are assigned randomly and will remain the same for the duration of the quarter. Each group turns in one homework, and each *participating* group member receives the same grade on the assignment. One member of the group is responsible for writing the homework (**the writer**), and this writer rotates for every assignment.

Homework groups work best if: Each member of the homework group finishes (or honestly attempts) the homework independently. At some appointed time, well before the due date, the group meets and everyone compares answers. Any discrepancies are discussed until a consensus is achieved. The writer notes the group consensus and makes sure she or he understands how to do the problem. After the meeting, but before class, the writer neatly and clearly writes the homework according to the **Homework guidelines** (described below).

Homework groups don't work if: One or more of the members skips meetings; each group member does not honestly attempt the homework prior to the meeting; a consensus in not reached for each assigned problem. *If a group member does not adequately participate in the homework, write a note on the homework and alert the TA. That person will not receive credit.*

Homework guidelines for writers: (Adapted from the website of Professor Andy Ruina). To get full credit, please do these things on each homework.

1. As a group writer, you must hand homework in by the end of class Monday, the day it is due. Homework is available via Smartsite Monday evenings, and is due the following week in class (unless stated otherwise). Late homework may or may not be accepted for reduced credit.

2. On the first page of your homework, please do the following to facilitate sorting:
   On the top left corner, please put the course information, homework number and date, e.g.:

   MAT207A
   HW 1
   Due October 5, 2015.

   On the top right corner, please put the names of your group members, with the writer at the top and clearly indicated. Non-participating group members should also be indicated, e.g.:

   Jaromir Jagr (writer)
   Sarah Jessica Parker
   Michelle Wie
   James Van der Beek (did not participate)

3. Please put a staple at the top left corner. Folded interlocked corners fall apart. Paperclips fall off.

4. **CITE YOUR HELP.** At the top of each problem, clearly acknowledge all help you got from TAs, faculty, students or any other source (with exceptions for lecture and the text, which need not be cited). You could write, for example: "Mary Jones pointed out to me that I had forgotten to divide by three in problem 2," or "Nadia Chow showed me how to do problem 3 from start to finish," or "I copied this solution word for word from Jane Lewenstein" or "I found a problem just like this one, number 9, at cheatonyourhomework.com, and copied it," etc. You will not lose credit for getting and citing such help. Don't violate academic integrity rules: be clear about which parts of your presentation you did not do on your own. Violations of this policy are violations of the UC Davis Code of Academic Conduct.

5. Your work should be laid out neatly enough to be read by someone who does not know how to do the problem. For most jobs, it is not sufficient to know how to do a problem, you must convince others that you know how to do it. Your job on the homework is to practice this. **Box your answers.**

6. Grading and regrading. We have a reasonable grading and regrading policy (see syllabus).

DUE: October 5, 2015. To be handed to me by the end of class.
The topics of this homework are 1. linear ODEs (review); 2. Getting familiar with Matlab; 3. flows on a line: 1-D phase plane analysis; and 4. linear stability analysis.

These topics are covered in §2.0–2.4 in Strogatz.

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1. Consider the following chemical reaction, where one chemical (A) turns into a different chemical (B) and vice versa. Suppose that the total amount of chemical is constant, that is $A(t) + B(t) = C$, where C is a positive constant. This reaction can be represented schematically in the following way:

$$A \underset{k^-}{\overset{k^+}{\rightleftharpoons}} B$$

where the two positive constants k^+ and k^- are called rate constants.

The following differential equation describes how A changes with time

$$\frac{dA}{dt} = -k^+A + k^-B \tag{1}$$

Recall that, in addition to this differential equation, we have the conservation constraint $A(t) + B(t) = C$.

a) Solve for $A(t)$, given $A(0) = A_0$, with A_0 being a positive constant such that $A_0 < C$.

b) Use Matlab to check your answer for a few choices of A_0, C, k^+ and k^- (I have provided code that will assist you).

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2. The position of an object moving in 1-D ($x(t)$) on a damped linear spring obeys the following differential equation

$$m\ddot{x} = -b\dot{x} - kx$$

where m, b and k are positive constants representing the mass of the object, the damping coefficient and the stiffness of the spring, respectively.

a) Solve for $x(t)$, given $x(0) = x_0$ and $\dot{x}(0) = v_0$.

b) Use Matlab to check your answer for a few choices of x_0, v_0, b, m and k (I have provided code that will assist you).

c) Assuming that both x_0 and v_0 are not zero, what inequality must m, b and k satisfy in order to give an oscillatory solution (i.e. a solution $x(t)$ the contains sine/cosine terms)?

3. The following equation describes the velocity $v(t)$ of a relatively large object falling through and inviscid medium (e.g. a baseball falling through the air)

$$m\dot{v} = -cv|v| + mg$$

where m, c and g are positive constants representing the mass of the object, the drag of the fluid and the pull of gravity, respectively.

a) Draw a plot of $\dot{v}$ vs. v. Label any fixed point(s) and indicate the stability of each. On the horizontal axis (v), indicate the flow direction.

b) Without solving the equation, sketch $v(t)$ as a function of t for several initial conditions.

c) Solve the equation for $v(t)$ given $v(0) = 0$. (It will simplify your life considerably to assume that $v \geq 0$ to get rid of the absolute value sign. Once you have a solution, you can determine whether this is a reasonable assumption).

d) Use Matlab to check your solution. I have not provided code, but you should be able to modify the code for problem 1.

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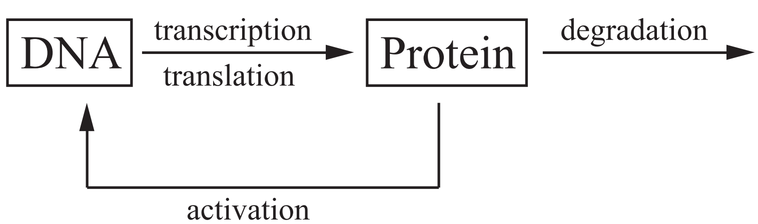
4. The DNA in your body codes for specific proteins. Proteins affect how cells in your body behave – even whether they live or die. In some cells, you might want a high concentration of a particular protein; in other cells, you might want only a small concentration of that protein.



Cells in your body typically destroy proteins; they have only a finite lifetime. Thus, the amount of a particular protein in one of your cells at a given time, $p(t)$, is determined by a balance between the rate of protein formation and destruction. Sometimes, the amount of protein affects how fast protein is made. Here we consider such a situation, as shown schematically in Fig. 1 above.

Mathematically, the system can be expressed with the following equation:

$$\dot{p} = -kp + A\frac{p^2}{p^2 + B^2}$$

where k, A and B are positive constants that determine how quickly the protein is destroyed, how quickly the protein is formed, and how strongly the protein affects its formation, respectively. For simplicity, assume that k = B = 1.

a) Assuming that k = B = 1, draw a phase portrait for the case where $A = 6$. (i.e. plot $\dot{p}$ vs. p, indicate any fixed point(s) and indicate the stability of each. On the horizontal axis (p) indicate the flow direction of phase points.)

b) Without solving the equation, sketch $p(t)$ as a function of t for several initial conditions for $A = 6$.

c) Assuming that $k = B = 1$, draw a phase portrait for the case where $A = 1$. (i.e. plot $\dot{p}$ vs. p, indicate any fixed point(s) and indicate the stability of each. On the horizontal axis (p) indicate the flow direction of phase points.)

d) Without solving the equation, sketch $p(t)$ as a function of t for several initial conditions for $A = 1$.

e) Suppose you wanted to maintain a non-zero steady-state concentration of protein in one of your cells. Would you choose $A = 1$ or $A = 6$? What else would you have to do to ensure a non-zero steady state?