Homework groups: You will complete each of seven homework assignment as part of a three- or four-person group. Group members are assigned randomly and will remain the same for the duration of the quarter. Each group turns in one homework, and each *participating* group member receives the same grade on the assignment. One member of the group is responsible for writing the homework (**the writer**), and this writer rotates for every assignment.

Homework groups work best if: Each member of the homework group finishes (or honestly attempts) the homework independently. At some appointed time, well before the due date, the group meets and everyone compares answers. Any discrepancies are discussed until a consensus is achieved. The writer notes the group consensus and makes sure she or he understands how to do the problem. After the meeting, but before class, the writer neatly and clearly writes the homework according to the **Homework guidelines** (described below).

Homework groups don't work if: One or more of the members skips meetings; each group member does not honestly attempt the homework prior to the meeting; a consensus in not reached for each assigned problem. *If a group member does not adequately participate in the homework, write a note on the homework and alert the TA. That person will not receive credit.*

Homework guidelines for writers: (Adapted from the website of Professor Andy Ruina). To get full credit, please do these things on each homework.

1. As a group writer, you must hand homework in by the end of class Monday, the day it is due. Homework is available via Smartsite Monday evenings, and is due the following week in class (unless stated otherwise). Late homework may or may not be accepted for reduced credit.

2. On the first page of your homework, please do the following to facilitate sorting:
   On the top left corner, please put the course information, homework number and date, e.g.:

   MAT207A
   HW 4
   Due November 4, 2015.

   On the top right corner, please put the names of your group members, with the writer at the top and clearly indicated. Non-participating group members should also be indicated, e.g.:

   Jaromir Jagr (writer)
   Sarah Jessica Parker
   Michelle Wie
   James Van der Beek (did not participate)

3. Please put a staple at the top left corner. Folded interlocked corners fall apart. Paperclips fall off.

4. **CITE YOUR HELP.** At the top of each problem, clearly acknowledge all help you got from TAs, faculty, students or any other source (with exceptions for lecture and the text, which need not be cited). You could write, for example: "Mary Jones pointed out to me that I had forgotten to divide by three in problem 2," or "Nadia Chow showed me how to do problem 3 from start to finish," or "I copied this solution word for word from Jane Lewenstein" or "I found a problem just like this one, number 9, at cheatonyourhomework.com, and copied it," etc. You will not lose credit for getting and citing such help. Don't violate academic integrity rules: be clear about which parts of your presentation you did not do on your own. Violations of this policy are violations of the UC Davis Code of Academic Conduct.

5. Your work should be laid out neatly enough to be read by someone who does not know how to do the problem. For most jobs, it is not sufficient to know how to do a problem, you must convince others that you know how to do it. Your job on the homework is to practice this. **Box your answers.**

6. Grading and regrading. We have a reasonable grading and regrading policy (see syllabus).

DUE: Wednesday, November 4, 2015. To be handed to me by the end of class.
The topics of this homework are 1. Classification of fixed points (general); 2. Linearization (general); 3. 2-D systems; 4. 2-D phase portraits (phase planes).

These topics are covered in §5.0–6.3 in Strogatz.

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1. A damped linear oscillator is a classical mechanical system. One typically analyzes it to death in math, physics and engineering courses. Its importance lies in the fact that, near equilibrium, many systems behave like a damped linear oscillator. Here, you'll see how this works.

Here are three differential equations that govern non-linear oscillators of one sort or another:

1. A mass on a wire (like you saw on homework 3, but here it is not over damped so it obeys a second order differential equation).

$$m\ddot{x} = -b\dot{x} - k\left(\sqrt{x^2+h^2} - \ell_0\right)\frac{x}{\sqrt{x^2+h^2}}$$

A non-dimensional form of this equation is

$$\frac{d^2X}{dT^2} = \frac{X}{\sqrt{X^2+\alpha^2}} - X - \beta\frac{dX}{dT} \tag{1}$$

2. A pendulum on a torsional spring (like you saw on midterm 1, but not over-damped).

$$-m\ell^2\ddot{\theta} = \zeta\dot{\theta} + \kappa\theta - mg\ell\sin(\theta)$$

A non-dimensional form of this equation is

$$\frac{d^2X}{dT^2} = -\beta\frac{dX}{dT} - \alpha X + \sin(X) \tag{2}$$

3. The Duffing's oscillator (a model for a slender metal beam interacting with two magnets, which we will likely revisit), in non-dimensional form

$$\frac{d^2X}{dT^2} = -\frac{dX}{dT} + \beta X - \alpha X^3 \tag{3}$$

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a) Find the fixed points of each oscillator and classify them (i.e. stable node, unstable node, saddle, stable spiral, etc.). In all cases $\beta > 0$ and $\alpha > 0$.

b) For each oscillator, choose a fixed point that is stable in some parameter regime and write the linearized equations.

c) Compare your linearization to that of a linear oscillator $\ddot{x} = -(k/m)x - (b/m)\dot{x}$ and determine the effective spring constant k/m and effective damping b/m for each system.

d) Use Matlab to check your work. Pick values of α and β and run some simulations of the three non-linear oscillators. Compare these to the predictions of the linear system you found in part c, which can be solved analytically (as you did on HW 1).

Part of getting good at drawing phase portraits is experience. Here are four relatively simple systems to help you practice.

2. Problem 6.3.3:

a. Find all fixed points, classify them and fill in the rest of your phase portrait for the following system of equations.

$$\begin{aligned}\dot{x} &= 1 + y - e^{-x} \\ \dot{y} &= x^3 - y\end{aligned}$$

b. Check your answers by generating a phase portrait with Matlab. (Simulate several initial conditions, and plot x and y)

3. Problem 6.3.6:

a. Find all fixed points, classify them and fill in the rest of your phase portrait for the following system of equations.

$$\begin{aligned}\dot{x} &= xy - 1 \\ \dot{y} &= x - y^3\end{aligned}$$

b. Check your answers by generating a phase portrait with Matlab. (Simulate several initial conditions, and plot x and y)

4. Problem 6.4.1.

a. The following is a "rabbits vs. sheep" model of two species competing for resources (in this case, rabbits and sheep competing for grass). They are discussed in §6.4 in the book.

$$\begin{aligned}\dot{x} &= x(3 - x - y) \\ \dot{y} &= y(2 - x - y)\end{aligned}$$

Find the fixed points, investigate their stability and draw plausible phase portraits.

b. Check your answers by generating a phase portrait with Matlab. (Simulate several initial conditions, and plot x and y)