

Homework groups: You will complete each of seven homework assignment as part of a three- or four-person group. Group members are assigned randomly and will remain the same for the duration of the quarter. Each group turns in one homework, and each *participating* group member receives the same grade on the assignment. One member of the group is responsible for writing the homework (**the writer**), and this writer rotates for every assignment.

Homework groups work best if: Each member of the homework group finishes (or honestly attempts) the homework independently. At some appointed time, well before the due date, the group meets and everyone compares answers. Any discrepancies are discussed until a consensus is achieved. The writer notes the group consensus and makes sure she or he understands how to do the problem. After the meeting, but before class, the writer neatly and clearly writes the homework according to the **Homework guidelines** (described below).

Homework groups don't work if: One or more of the members skips meetings; each group member does not honestly attempt the homework prior to the meeting; a consensus is not reached for each assigned problem. *If a group member does not adequately participate in the homework, write a note on the homework and alert the TA. That person will not receive credit.*

Homework guidelines for writers: (Adapted from the website of Professor Andy Ruina). To get full credit, please do these things on each homework.

1. As a group writer, you must hand homework in by the end of class Monday, the day it is due. Homework is available via Smartsite Monday evenings, and is due the following week in class (unless stated otherwise). Late homework may or may not be accepted for reduced credit.
2. On the first page of your homework, please do the following to facilitate sorting:
On the top left corner, please put the course information, homework number and date, e.g.:

MAT207A

HW 7

Due December 7, 2015.

On the top right corner, please put the names of your group members, with the writer at the top and clearly indicated. Non-participating group members should also be indicated, e.g.:

Jaromir Jagr (writer)

Sarah Jessica Parker

Michelle Wie

James Van der Beek (did not participate)

3. Please put a staple at the top left corner. Folded interlocked corners fall apart. Paperclips fall off.
4. **CITE YOUR HELP.** At the top of each problem, clearly acknowledge all help you got from TAs, faculty, students or any other source (with exceptions for lecture and the text, which need not be cited). You could write, for example: "Mary Jones pointed out to me that I had forgotten to divide by three in problem 2," or "Nadia Chow showed me how to do problem 3 from start to finish," or "I copied this solution word for word from Jane Lewenstein" or "I found a problem just like this one, number 9, at cheatonyourhomework.com, and copied it," etc. You will not lose credit for getting and citing such help. Don't violate academic integrity rules: be clear about which parts of your presentation you did not do on your own. Violations of this policy are violations of the UC Davis Code of Academic Conduct.
5. Your work should be laid out neatly enough to be read by someone who does not know how to do the problem. For most jobs, it is not sufficient to know how to do a problem, you must convince others that you know how to do it. Your job on the homework is to practice this. **Box your answers.**
6. Grading and regrading. We have a reasonable grading and regrading policy (see syllabus).

DUE: Monday, December 7, 2015. To be handed to me by the end of class.

The topics of this homework are 1. Poincarè-Bendixson, proving closed orbit(s) exist; 2. Hopf Bifurcations; 3. Global Bifurcations; 4. Maps and 1D Maps.

These topics are covered in §7, §8 and §10 in Strogatz.

1. (Problems 7.3.1 and 7.3.2 from Strogatz) Consider the following coupled differential equations:

$$\begin{aligned}\dot{x} &= x - y - x(x^2 + 5y^2) \\ \dot{y} &= x + y - y(x^2 + y^2)\end{aligned}$$

- Classify the fixed point at the origin.
 - Change to polar coordinates, using $r\dot{r} = x\dot{x} + y\dot{y}$ and $\dot{\theta} = (x\dot{y} - y\dot{x})/r^2$.
 - Find a minimum radius, r_{min} such that all trajectories have an outward directed radial component.
 - Find a maximum radius, r_{max} such that all trajectories have an inward directed radial component.
 - Prove that the system has a limit cycle somewhere in the trapping region, $r_{min} < r < r_{max}$.
 - Use Matlab to verify your results.
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2. The Fitzhugh-Nagumo model describes the generation of an action potential in a nerve cell (a neuron). These action potentials are an all-or-none signal that is used to transmit information to and from your brain.

$$\begin{aligned}\dot{V} &= V - \frac{V^3}{3} - W + I \\ \dot{W} &= 0.08(V + 0.7 - 0.8W)\end{aligned}\tag{1}$$

Where V is the voltage in the cell (the variable we care about) and W is a recovery variable. This system has one free parameter, I , the stimulus current.

- Sketch the null-clines for $I = 1$ and demonstrate that a limit cycle exists.
 - Explain why, for $I = 0$, you can't prove that a limit cycle exists (in fact, it does not).
 - Since a limit cycle does not exist for $I = 0$ and a limit cycle exists for $I = 1$, one might expect that there's a bifurcation for $0 < I < 1$. Classify this bifurcation and briefly explain your answer.
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3. Consider the following 1D map

$$x_{n+1} = x_n e^{-r(1-x_n)}$$

where $r > 0$ is a constant

- a. Find the period one motions and determine their stability.
- b. Find the period two motions and determine their stability (you may wish to do this numerically).
- c. Use Matlab and your answers to a. and b. to generate the beginning of a bifurcation diagram.
- d. Use Matlab to generate an orbit diagram.

4. I have uploaded some Matlab code onto Smartsite. Here is a brief description:

The first program is a function called **SimpleWalker**. As an input, it takes

1. an initial condition **Z**, a vector whose first entry is the angle of the leg and second entry is the angular velocity.
2. the slope, γ
3. a series of options: *animate*, *poincare*, *rootfind* and *per*. Set *animate* to 1 in order to animate the walker; *poincare* to 1 in order to plot a Poincaré section; *rootfind* to 1 in order to do a rootfind looking for period *per* motions.

It then outputs Z and t , the walker's legs' angles and angular velocities, and X the angle and angular velocity just after heel strike.

The second program is called **Walker_Demo**. This program shows a series of demonstrations to get you familiar with the **SimpleWalker** function.

- a. For a range of slopes, γ , find the period one motions and determine their stability
- b. For a range of slopes, find the period two motions and determine their stability.
- c. Use Matlab and your answers to a. and b. to generate the beginning of a “bifurcation diagram,” with axes θ^* (the initial angle, the first entry of **Z** in the function **SimpleWalker**) and γ (the slope).
(This is not a true bifurcation diagram, since you are not plotting the second entry of **Z**.)
- d. Use Matlab to generate an “orbit diagram,” with axes θ^* (the initial angle, the first entry of **Z** in the function **SimpleWalker**) and γ (the slope).
(This is not a true orbit diagram, since you are not plotting the second entry of **Z**.)
- e. (Optional) See if you can find a period-3 gait.
- f. (Optional) How well can you approximate Feigenbaum's number?