UC DAVIS, APPLIED MATHEMATICS (MAT207C), SPRING 2016

Homework #2

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Problem 1

Compute the swimming speed of an undulating sheet moving at zero Reynolds number between two walls on which the velocity is zero (in lab frame) located at $y = \pm L$ in the limit of low amplitude. In the reference frame moving with the swimming, the shape of the swimmer is $y = A\sin(kx - \omega t)$.

Problem 2

Suppose the position of a mass on a damped linear spring obeys the following equation

$$m\ddot{x} + b\dot{x} + kx = 0,$$

where m, b, and k are constants representing the mass, damping coefficient, and spring constant, respectively.

(a) Each term in the above equation has dimensions of force. Identify the dimensions of b and k in terms of mass, length, and time.

(b) Identify the three time scales in the problem and discuss their physical meaning.

(c) Present two different nondimensionalizations: one appropriate for the limit of vanishing friction and the other appropriate for the limit of vanishing mass. Identify the small nondimensional parameter in each case.

(a) Since the dimensions of force are $\dfrac{\text{mass}\cdot\text{length}}{\text{time}^2}$, the dimensions of x are length, and the dimensions of $\dot{x}$ are $\dfrac{\text{length}}{\text{time}}$, then the dimensions of b are $\dfrac{\text{mass}}{\text{time}}$ and the dimensions of k are $\dfrac{\text{mass}}{\text{time}^2}$.

(b) Let $L(T) = \dfrac{x(t)}{X}$ and $T = \dfrac{t}{\tau}$. Then

$$\dot{L} = \frac{\mathrm{d}L}{\mathrm{d}T} = \frac{\mathrm{d}L}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}T} = \frac{\tau}{X}\dot{x} \qquad \text{and} \qquad \ddot{L} = \frac{\mathrm{d}}{\mathrm{d}T}\dot{L} = \frac{\mathrm{d}}{\mathrm{d}T}\left[\frac{\tau}{X}\dot{x}\right] = \frac{\tau}{X}\frac{\mathrm{d}}{\mathrm{d}T}\dot{x} = \frac{\tau}{X}\frac{\mathrm{d}t}{\mathrm{d}T}\frac{\mathrm{d}}{\mathrm{d}t}\dot{x} = \frac{\tau^2}{X}\ddot{x}$$

Thus,

$$m\ddot{L} + b\tau\dot{L} + k\tau^2 L = 0.$$

There are three timescales:

(i) Let $\tau = \frac{m}{b}$. Then

$$\ddot{L} + \dot{L} + \varepsilon L = 0$$

where $\varepsilon = \frac{km}{b^2}$.

(ii) Let $\tau = \sqrt{\frac{m}{k}}$. Then

$$\ddot{L} + \varepsilon\dot{L} + L = 0$$

where $\varepsilon = \frac{b}{\sqrt{km}}$.

(iii) Let $\tau = \frac{b}{k}$. Then

$$\varepsilon\ddot{L} + \dot{L} + L = 0$$

where $\varepsilon = \frac{km}{b^2}$.

(c) The nondimensionalization appropriate for the limit of vanishing friction is the second of the three given above:

$$\ddot{L} + \varepsilon \dot{L} + L = 0$$

where $\varepsilon = \frac{b}{\sqrt{km}}$.

(d) The nondimensionalization appropriate for the limit of vanishing mass is the third of the three given above:

$$\varepsilon \ddot{L} + \dot{L} + L = 0$$

where $\varepsilon = \frac{km}{b^2}$.