Math 207C
Homework 4
Due Friday, April 29th

1. Consider the Lagerstrom model for Low Reynolds number flow:

$$U'' + \frac{2}{R}U' + \epsilon U U' = 0$$
$$U(1) = 0$$
$$U(\infty) = 1.$$

   Compute the leading order expansion of $U'(1)$ in the limit of small ϵ. This is the analog to Stokes's original calculation for the force on a translating sphere in 3D.

2. Compute the expansion of $U'(1)$ up to order ϵ. You will encounter a problem similar to the "Whitehead paradox" which you will resolve using intermediate scale matching.

   You may find the following asymptotic expansion useful for small r:

$$\int_r^\infty \frac{\mathrm{e}^{-x}}{x^2}\,dx = \frac{1}{r} + \log(r) + \gamma - 1 + \mathcal{O}(r),$$

   where γ is Euler's constant.