UC DAVIS, NUMERICAL SOLUTIONS OF DIFFERENTIAL EQUATIONS (MAT 228A), FALL 2016

Homework #4

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Part I

White a multigrid V-cycle code to solve the Poisson equation in two dimensions on the unit square with Dirichlet boundary conditions. Use full weighting for restriction, bilinear interpolation for prolongation, and red-black Gauss-Seidel for smoothing.

Note: If you cannot get a V-cycle code working, write a simple code such as a 2-grid code. You can also experiment in one dimension (do not use GSRB in 1D). You may turn in one of these simplified coeds for reduced credit. You should state what your code does, and use your code for the assignment.

1. Use your V-cycle code to solve
$$\nabla^2 u = -\exp\left[-(x-0.25)^2 - (y-0.6)^2\right]$$
on the unit square $(0,1)\times(0,1)$ with homogeneous Dirichlet boundary conditions for different grid spaces. How many steps of pre and postsmoothing did you use? What tolerance did you use? How many cycles did it take to converge? Compare the amount of work needed to reach convergence with your solvers from Homework 3 taking into account how much work is involved in a V-cycle.

Part II

Choose **one** of the following problems.

1. Numerically estimate the average convergence factor,
$$\left(\frac{\|e^{(k)}\|_\infty}{\|e^{(0)}\|_\infty}\right)^{1/k},$$
for different numbers of presmoothing steps, ν_1, and postsmoothing steps, ν_2, for $\nu = \nu_1 + \nu_2 \leq 4$. Be sure to use a small value of k because convergence may be reached very quickly. What test problem did you use? Do your results depend on the grid spacing? Report the results in a table, and discuss which choices of ν_1 and ν_2 give the most efficient solver.

2. The multigrid V-cycle iteration is of the form
$$u^{k+1} = (I - BA)u^k + Bf,$$
where $M = I - BA$ is the multigrid iteration matrix. To compute the kth column of the multigrid iteration matrix, apply a single V-cycle to a problem with zero right-hand-side, f, and as an initial guess, u^0, that has a 1 in the kth entry and zeros everywhere else. For a small problem, e.g. $h = 2^{-5}$ or $h = 2^{-6}$, form the multigrid matrix, compute the eigenvalues, and plot them in the complex plane. Compute the spectral radius and 2-norm of the multigrid iteration matrix for different numbers of presmoothing steps ν_1, and postsmoothing steps, ν_2. Comment on your results.

Let $E_k = \left(\frac{\|e^{(k)}\|_\infty}{\|e^{(0)}\|_\infty}\right)^{1/k}$

$h = 2^{-5}$ $\quad$ $\mathrm{tol} = 10^{-6}$

ν_1	ν_2	E_1	$\frac{1}{2}\sum_{i=1}^{2} E_i$	$\frac{1}{3}\sum_{i=1}^{3} E_i$	$\frac{1}{5}\sum_{i=1}^{5} E_i$	$\frac{1}{\max}\sum_{i=1}^{\max} E_i$	iterations
0	0	0.396875	0.454525	0.50511	0.593587	0.767165	14
0	1	0.305393	0.288137	0.318529	0.422289	0.749344	23
1	0	0.240477	0.244411	0.316576	0.441658	0.745356	20
0	2	0.179352	0.185095	0.262303	0.390773	0.669029	16
1	1	0.0798504	0.181037	0.282042	0.420886	0.652608	13
2	0	0.0987041	0.212508	0.311878	0.447004	0.670126	13
0	3	0.121375	0.166698	0.251102	0.381308	0.617268	13
1	2	0.0556845	0.166925	0.267998	0.40584	0.622475	12
2	1	0.00870737	0.164355	0.276806	0.420276	0.596134	10
3	0	0.0844652	0.218674	0.3192	0.451548	0.616711	10
0	4	0.0914259	0.15609	0.24315	0.373443	0.592315	12
1	3	0.043239	0.158128	0.258661	0.395822	0.594257	11
2	2	0.0115739	0.158641	0.267094	0.407947	0.584198	10
3	1	0.0376952	0.186759	0.293044	0.429958	0.576603	9
4	0	0.117969	0.240874	0.334553	0.460074	0.571272	8