Homework 2: due May 2, 2016

Problem 1: Let $i = \sqrt{-1}$ and set

$$A = \begin{bmatrix} i & 0 & -i \\ 0 & i & -i \end{bmatrix}.$$

Using the null space property, show that ℓ_1-minimization will recover any 1-sparse vector x, given $Ax = y$.

Problem 2: On the connection between (in)coherence parameter μ and restricted isometry constant δ_s: Show that $\delta_1 = 0, \delta_2 = \mu$, and $\delta_s \leq (s-1)\mu$.

Problem 3: Let $A \in \mathbb{R}^{k \times d}$ be a Gaussian random matrix. Give an estimate for the coherence μ of A.

Problem 4: Consider $y = Ax$, where A is a 100×400 Gaussian random matrix and x is an s-sparse vector of length 400. The locations of the non-zero entries of x are chosen uniformly at random and the non-zero coefficients of x are normal-distributed. For $s = 1, 2, \ldots,$ solve

$$\min_z \|z\|_1 \quad \text{subject to } Az = y,$$

(e.g. using the toolbox CVX). For each fixed s repeat the experiment 10 times. Create a graph plotting s versus the relative reconstruction error (averaged over the ten experiments for each s). Starting with which value of s (approximately) does ℓ_1-minimization fail to recover x?

Problem 5: Same setup as in Problem 4, but now the non-zero entries of x are non-negative. Taking this information into account, we now solve

$$\min_z \|z\|_1 \quad \text{subject to } Az = y \text{ and } z \geq 0$$

(here, $z \geq 0$ is meant entrywise, i.e., for each $k : z_k \geq 0$). (The positivity constraint is easy to include in CVX). Repeat the simulations as described in Problem 4. Compare your findings to the results from your experiments of Problem 4 and try to quantify the difference regarding the range for s for which recovery is still possible in this case.