# Homework #8

## Sam Fleischer

June 3, 2016

---

### Problem 1

The whooping crane has been the target of protection efforts by the National Audubon Society and the federal governments of the US and Canada since 1941. There is one wild breeding population that winters at the Arkansas National Wildlife Refuge on the coast of Texas. The size of this population from 1941 to 1989 was studied by Dennis et al. (1991) in their Ecological Monographs paper Estimation of growth and extinction parameters for endangered species. The data set used by Dennis et al. is in the cranes.csv data file in the resources folder at SmartSite.

(a) Assuming the log population numbers can be modeled by

$$\log N(t) = \log N(0) + rt + \text{"observation error"} \tag{0.1}$$

where the "observation error" is normally distributed, use linear regression to estimate $\log N(0)$ and $r$. Use this model to estimate $N(2011)$.

(b) Assuming the log population numbers can be modeled by

$$\log \frac{N(t+1)}{N(t)} = r + \text{"process error"} \tag{0.2}$$

where the "process error" is normally distributed, use linear regression to estimate $r$. Use this model to estimate $N(2011)$.

(c) Do an online search to find an estimate for the *wild* whooping population size in August 2011. How do your model predictions compare?

(a) The following code snippet was used to load the CSV file and run a linear regression on the log population numbers.

```
1  # Load the file
2  cranes = read.csv("cranes-2013.csv")
3
4  # Set local variables to access the data
5  Count = cranes$Count
6  Year = cranes$Year
7
8  # Plot Count vs. Year
9  plot(Year, Count, pch=4)
10
11 # Take the logarithm of the population numbers.
12 logPop = log(Count)
13
14 # Perform a linear regression
15 regression = lm(logPop~Y)
16
17 # Add the exponential (line in log-space) to the plot
18 lines(Year,exp(regression$fitted.values))
19
20 # Print the summary of the regression
21 summary(regression)
```

The output from the summary command shows $\log N(0) \approx -78.86$ and $r \approx 0.04204$, so $\log N(t) \approx$

$-78.86 + 0.04204t$, which then implies

$$N(t) \approx N_0 \exp[0.04204t] \qquad \text{with} \qquad N_0 = \exp[-78.86].\tag{0.3}$$

Below is the plot generated from the code snippet. Using this linear regression, $N(2011) \approx 293.67$.
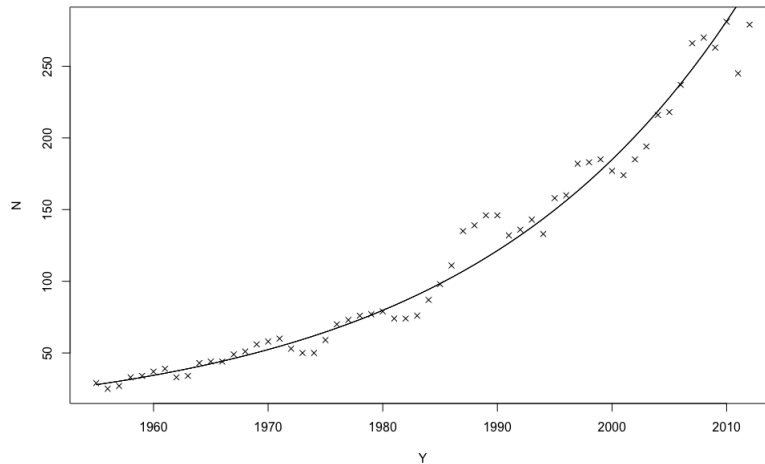


(b) The following code snippet was used to load the CSV file and run a linear regression on the ratio of successive log population numbers.

```
1  # Load the file
2  cranes = read.csv("cranes-2013.csv")
3
4  # Use the variable ''N'' as the population
5  N = cranes$Count
6
7  # Take the log of the population
8  logN = log(N)
9
10 # Get ''future'' and ''past'' arrays
11 logNtp1 = logN[-1]
12 logNt = logN[-length(logN)]
13
14 # Plot them against each other
15 plot(logNt,logNtp1)
16
17 # Linearly regress them and plot the result
18 regression = lm(logNtp1~logNt)
19 lines(logNt,regression$fitted.values)
20
21 # Print the result of the regression
22 summary(regression)
```

The output from the summary command shows $\log N_{t+1} \approx 0.99038 \log N_t + 0.08309$, which implies

$$N_{t+1} = e^{0.08309} N_t^{0.99038}\tag{0.4}$$

Using $N_{2010}$ from the data, ($N_{2010} = 281$), we get the approximation $N_{2011} = 289.22$. Using the initial condition $N_{1955} = 29$ and the above recursive rule, we get the approximation $N_{2011} = 262.5$.

(c) The website https://www.learner.org/jnorth/tm/crane/PopulTotals.html gives the wild Whooping Crane Population in the Western Flock as 278. It looks like the linear regression from (a) overestimated the most, while the average of the results from (b) is 275.86, which is very close to the actual value.

---

## Problem 2

**Monad's nightmare:** Starting with a single cell of *E. coli*, how long would it take to cover the Earth 1 foot deep with *E. coli*? Assume that *E. coli* are rectangular prisms of volume $1.1250 \times 10^{-18}$ meters$^3$ and that they reproduce every 20 minutes.

---

The radius of the Earth is approximately 20903520 feet, and thus the volume of the 1 foot film of *E. coli* around the Earth is

$$V_{\text{film}} = \frac{4}{3}\pi 20903521^3 - \frac{4}{3}\pi 2090352103^3 = 5490965741568000 \text{ feet}^3 \approx 155486834545908.5625 \text{ meters}^3$$

$$(0.5)$$

This means it would take approximately $\frac{155486834545908.5625}{1.1250 \times 10^{-18}} \approx 1.3821051959636316 \times 10^{32}$ *E. coli* cells to cover the earth with a 1 foot film. If we discretize time into twenty minute intervals and assume that the *E. coli* cells never die, then we just need to solve $1.3821051959636316 \times 10^{32} = 2^t$ for $t$, which has the solution

$$t = 106.7685664639302 \tag{0.6}$$

Since this is the number of twenty minute intervals, this corresponds to about 35.6 hours.

# Problem 4

In this problem, you will play around with the Bellows model using some of his data. To get things started you need to download the data file bellows1d.txt and the bifurcation file bellowsBif.R.

(a) For the data file bellows1d.txt, you are going to estimate the parameters $a$, $b$, and $c$ for the survivorship function

$$S(N) = \frac{aN}{1 + (bN)^c}$$

using nonlinear regression. To this end, load the data and rename it as follows

```
>temp=read.table("bellows1d.txt",header=FALSE)
>N=temp[,1]
>S=temp[,2]
```

To fit the model $S(N)$ to the data, use the nonlinear regression command

```
>reg=nls(S~a*N/(1+(b*N)^c),start=c(a=1,b=0.01,c=2))
```

where the second argument provides initial guesses for the parameter values. To see the parameter estimates, type

```
>summary(reg)
```

To see what the fit looks like, type

```
>plot(N,S,col="red")
>lines(N,fitted.values(reg),col="blue")
```

(b) With your fitted parameters in hand, simulate the Bellows model

$$N_{t+1} = FS(N_t)$$

with $N_0 = 1$ for the following fecundity values: $F = 1, 3, 4, 6, 7, 10$. Discuss what happens for each of these parameter values.

(c) Determine analytically the minimum value $F$ for which the population persists.

(d) To see how the behaviors change as you vary $F$, open the R-file bellowsBif.R and change the values of $a$, $b$, and $c$ in this file to what you hound in (a) and for the range of $F$ values used in (b). RUn the file. R will produce a *bifurcation diagram* that for each parameter value plots the abundances observed in the long-term in the vertical direction e.g. if there is only one point above a given $F$ value then the population tends toward that equilibrium value; if there are two values plotted either the population oscillates between the two values or there are two stable equilibria. Using this plot, do the following:

   (i) Estimate for what range of fecundities the population goes to a positive equilibrium.

   (ii) Estimate for what range of fecundities the population goes to a period two orbit.

   (iii) Estimate for what range of fecundities the population goes to a period four orbit.

   (iv) Discuss whether fecundity is always stabilizing or destabilizing.
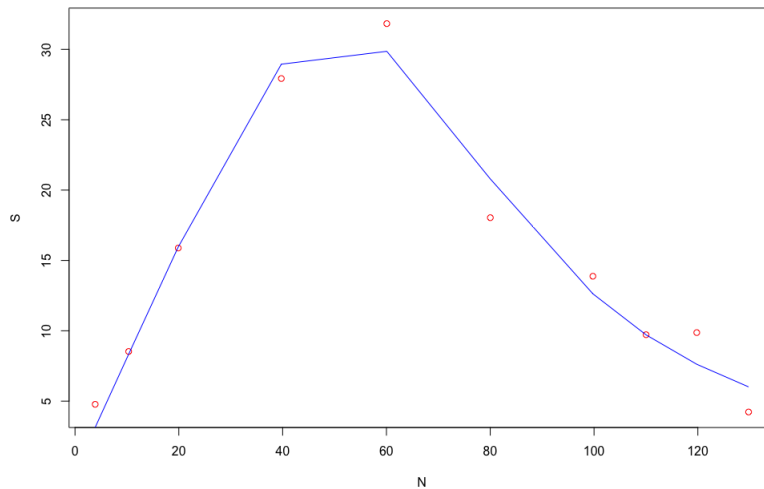
(a) Running the given code gives the following parameter values:

$$a \approx 0.8068723 \qquad b \approx 0.0148920 \qquad c \approx 4.2470522 \qquad (0.7)$$

so the model is

$$S(N) \approx \frac{0.8068723N}{1 + (0.0148920N)^{4.2470522}} \qquad (0.8)$$

and here is the plot with the data points in red dots and the model approximation as a blue line:



---

# Problem 5

You can find five more population data sets in the resources folder at Smartsite in the files data1.txt, data2.txt, data3.txt, data4.txt, data5.txt. For each of these data sets use Bayesian Information Criterion (BIC) to select one of the following three models where the "process error" is assumed to be normally distributed:

**Gompertz model:** is of the form

$$\log \frac{N(t+1)}{N(t)} = a + b \log N(t) + \text{"process error"}$$

**Ricker model:** is of the form

$$\log \frac{N(t+1)}{N(t)} = a + bN(t) + \text{"process error"}$$

**Random walk model:** is of the form

$$\log \frac{N(t+1)}{N(t)} = a + \text{"process error"}$$

## Problem 6

Consider a population who in the absence of predation exhibits exponential growth i.e. $G(N) = rN$ without predation. If this population is subject to a predator with a saturating function response, the predation would be of the form

$$\frac{aNP}{H + aN}$$

where $P$ is the predator abundance (assumed to be constant), $a$ is the searching efficiency of the predator and $H$ is the predator's "half-saturation constant". In the presence of predation, the population grwth rate is given by

$$G(N) = rN - \frac{aN}{H + aN}.$$

Let $R(N) = G(N)/N$ be the per-capita growth rate term.

(a) Find the maximum per-capita growth rate of the population. Discuss under what conditions the population is doomed to extincton for all initial conditions.

(b) Find the minimum per-capita growth rate of the population. Discuss under what conditions the population persists for all positive initial conditions. Is the population regulated?

(c) Assume that the maximum per-capita growth rate of the population is positive and the minimum per-capita growth rate is negative. Find the positive equilibrium for the model. What happens for populations whose initial abundance lies below this equilibrium? What happens if the initial abundance lies above this equilibrium? Discuss how the resilience of this population can be improved (by increasing the threshold value).