PBG 200A Notes

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1 Bet Hedging

$$N_{t+1} = \underbrace{\Big(Y_{t+1}g + S_{t+1}(1-g)\Big)}_{\mathbb{E}[\log(\text{of this})]=:r} N_t$$

When is r maximized at $0 < g < 1$? Must have:

$$\mathbb{E}\left[\frac{Y_t}{S_t}\right] > 1$$
$$\mathbb{E}\left[\frac{S_t}{Y_t}\right] > 1$$

2 Discussion on Childs et al 2010 and Gremer and Venable 2014

2.1 ESS in G&V 2014

Two competing genotypes

$$N_{t+1} = \frac{gY_{t+1}N_t}{1 + a\Big(gN_t + \tilde{g}\tilde{N}_t\Big)} + (1-g)sN_t$$
$$\tilde{N}_{t+1} = \frac{\tilde{g}Y_{t+1}\tilde{N}_t}{1 + a\Big(gN_t + \tilde{g}\tilde{N}_t\Big)} + (1-\tilde{g})s\tilde{N}_t$$

An ESS is a strategy g which will exclude all other strategies $\tilde{g}$ at low densities. At low $\tilde{N}$, $\cancelto{0}{\tilde{g}\tilde{N}_t}$. Defining $r(g, \tilde{g})$ as the invasion rate of $\tilde{g}$ against g.

$$r(g, \tilde{g}) = \mathbb{E}\left[\log\left[\frac{\tilde{g}Y_{t+1}}{1 + agN_t} + (1-\tilde{g})s\right]\right] \tag{1}$$

Definition: the ESS is the value of g for which $r(g, \tilde{g}) < 0$ for all $\tilde{g} \neq g$. Side Note: As ESS must satisfy $\dfrac{\partial r}{\partial \tilde{g}}(g, g) = 0$.

3 Central Limit Theorem (CLT)

This is the whole basis of hypothesis testing.

Set $X_1, \ldots, X_t, \ldots$ are i.i.d. with mean μ and variance σ^2, then

$$\frac{X_1 + \cdots + X_t - t\mu}{\sqrt{t\sigma^2}} \to N(0, 1) \tag{2}$$

where $N(\mu, \sigma^2)$ is a normal distribution with mean μ and variance σ^2.

3.1 So What?

$$\log\frac{N_t}{N_0} = \log R_1 + \cdots + \log R_t, \tag{3}$$

so $\log N_t$ is approximately normally distributed with mean $rt + \log N_0$ and variance $\sigma^2 t$ where σ^2 is the variance of $\log R_1$.