PBG 200A Notes

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1 Model Selection

Consider a time series $y_1, y_2, \ldots, y_T$ of log densities. Recall process error vs. observational error.

1.1 Process Error

$$x_{t+1} = g(x_t, a) + \underbrace{E^{\mathrm{p}}_{t+1}}_{\text{process error}} \tag{1}$$

$$y_t = x_t \qquad \text{(no observational error)} \tag{2}$$

We assume the random variables E^{p}_{t+1} are IID (independent, identically distributed) Normal with mean 0 and variance σ^2.

We run the model and ask..

- What value of a (parameter or vector of parameters) is most likely to generate the observed data y_t?
  - For example, for a given a, and given y_1, what is the likelihood, what is the probability that the model produce y_2?
  - This is the same as asking: what is the likelihood a normal with mean 0 and variance σ^2 yields a $y_2 - g(y_1, a)$?
  - The answer is

$$p(y_2 - g(y_1, a)) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{-(y_2 - g(y_1, a))^2}{2\sigma^2}\right] \tag{3}$$

    and we'd like to maximize this value with respect to a (optimization problem..)

Now we ask:

- What is the likelihood that the random variables $E^p_{t+1} = y_{t+1} - g(y_t, a)$ for $1 \leq t \leq T-1$?
- By independence, the likelihood a produces the data, denoted $L(a)$, is

$$L(a) = \prod_{t=1}^{T-1} \left[\frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(y_{t+1} - g(y_t, a))^2}{2\sigma^2}\right]\right] \tag{4}$$

$$\implies \log L(a) = \sum_{t=1}^{T-1} \left[\log \frac{1}{\sqrt{2\pi}\sigma} - \frac{(y_{t+1} - g(y_t, a))^2}{2\sigma^2}\right] \tag{5}$$

  So maximizing $\log L(a)$ is the same as minimizing the sum

$$\sum_{t=1}^{T-1} (y_{t+1} - g(y_t, a))^2 \tag{6}$$

1.1.1 Chickadee Data

We define the model $g(x, a)$ by the following:

$$N_{t+1} = N_t \exp[a_1 + a_2 N_t], \qquad \text{with} \qquad x = \log N \tag{7}$$

$$\implies x_{t+1} = x_t + a_1 + a_2 e^{x_t} =: g(x_t, [a_1, a_2]) \tag{8}$$

Off to RStudio... a negative value of a_2 corresponds to some negative density dependence.