PBG 200A Notes

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October 12, 2016

1 Continuous Structure in Populations

Canonical example: size or weight

1.1 Integral Projection Models (IPMs)

Individual states take values in an interval $[a, b]$. Write $N(t, x)$ as the density of size x individuals at time (or year) t. For a fixed t, we can look at $N(t, x)$ vs. x. Then the density of individuals in $[x, x + \Delta x]$ is approximately $N(t, x)\Delta x$.

Here is a classic

$$N(t+1, y) = \int_a^b s(x)[g(x, y) + p(x)f(x, y)]N(t, x)\mathrm{d}x \tag{1}$$

where

$$\begin{aligned} s(x) &\coloneqq \text{the percentage of size } x \text{ individuals which survive} && (2)\\ g(x, y) &\coloneqq \text{the infinitesimal probability of growing from } x \text{ to } y && (3)\\ p(x) &\coloneqq \text{the probability of a size } x \text{ individual reproducing} && (4)\\ f(x, y) &\coloneqq \text{the density of offspring of size } y \text{ produced by individuals of size } x && (5) \end{aligned}$$

2 Matrix Models with Temporal Heterogeneity

$$N(t) = \begin{pmatrix} N_1(t) \\ N_2(t) \\ \vdots \\ N_k(t) \end{pmatrix} \tag{6}$$

$$N(t+1) = A(t+1)N(t) \qquad \Longrightarrow \qquad N(t) = A(t)A(t-1)\dots A(1)N(0) \tag{7}$$

What is r?

2.1 Central Limit Theorem $^{++}$

Assume $A(i)$ are stationary, ergodic, and primitive for $i = 1, 2, \dots$. Then $\exists r$ and σ ($\sigma > 0$) such that

$$\log(N_1(t) + \dots + N_k(t)) \approx \log(N_1(0) + \dots + N_k(0))rt + \sigma\sqrt{t}Z \tag{8}$$

where Z is the standard normal.

2.2 Small Variance Approximation for r

For i.i.d. $A(t)$,

$$A(t) = \underbrace{\mathbb{E}[A(t)]}_{\overline{A}} + \underbrace{A(t) - \overline{A}}_{B(t)} \tag{9}$$

$$r \approx \log\lambda - \frac{1}{2\lambda^2}\sum_{i,j,r,s} S_{ij}S_{rs}\mathrm{cov}[B_{ij}(1), B_{rs}(1)] \tag{10}$$

where S are the sensitivities ($\partial\lambda/\partial\overline{a}_{ij}$).