PBG 200A Notes

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1 Discrete-Time, Density-Independence

$$\begin{aligned} N_{t+1} - N_t &= (b-d)N_t \\ \implies N_{t+1} &= (1+b-d)N_t \coloneqq RN_t \end{aligned}$$

if N_0, then $N_1 = RN_0$, $N_2 = RN_1 = R^2N_0$, and so $N_t = R^tN_0$.

So from before, $e^r = R$. We will always use the variable r to denote intrinsic growth rate

2 Negative Density Dependence

This means r is decreasing with density. Examples:

- higher populations mean easier spread of pathogens
- predator has type 3 functional response
- cap of resources
- accumulation of waste producs

yeast model, T Carlson, 1913

picking functions to "graduate" a set of data.. the choice is, at its very best, only a combination of good judgement and good luck.

Simplest case of decrease function is a linear function, $r(N) = r_0\big(1 - \frac{N}{K}\big)$, and so we obtain the logistic growth model:

$$\dot{N} = Nr(N) = r_0N\left(1 - \frac{N}{K}\right)$$

This can be solved using separation of variables.. the solution is:

$$FILL\ \ IN\ \ LATER$$

Set $G(N) = Nr(N)$. Then we get a downward-facing parabola with $G(0) = G(K) = 0$. The vertex is at $\frac{K}{2}$. So 0 and K are equilibria, and there is an inflection point at $\frac{K}{2}$ on the N vs. t graph. All trajectories with $N(0) > 0$ approach K asymptotically.

Fitting linear decrease of growth rate to the 1913 data gives unbelievably accurate logistic growth.

Many populations exhibit (kind of) logistic growth. There are fluctuations...

2.1 Discrete-Time Version

$$N_{t+1} = \exp[r(N_t)]N_t$$

with $r(N) \coloneqq r_0\big(1 - \frac{N}{K}\big)$. This is called the "Ricker Equation." The equilbiria are still 0 and K.

(went to RStudio...)

2.1.1 Lyapunov Exponent

For Models of the form

$$N_{t+1} = F(N_t), \tag{1}$$

The Lyapunov Exponent for initial condition N_0 is

$$\chi = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \log |F'(N_t)| \tag{2}$$

3 Discussion of Tilman/Wedin and Bennica et. al.

4 How to Test for Density-Dependence?

Suppose we have some data $y_1, y_2, \ldots, y_T$, log-densities.

Which model parameter(s) a are best at describing the data?

$$x_{t+1} = g(x_t, a) \tag{3}$$

We assume there is some uncertainty in either data or descriptor..

Consider two types of "noise":

- Observational error, i.e. the model is perfect, but data is imperfect:

$$y_t = x_t + E_t^{\mathrm{o}} \tag{4}$$

  where E_t^{o} is observational error

- Process error, i.e. the observations are perfect, but descriptor is imperfect:

$$x_{t+1} = g(x_t, a) + E_{t+1}^{\mathrm{p}} \tag{5}$$

  where E_{t+1}^{p} is process noise.

For Monday, read Knape and Divalpine 2012.