PBG 200A Notes

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1 From Last Time

$$N(t+1) = A(t+1)N(t)$$

where

$$\bar{A} = \mathbb{E}[A(t+1)], \qquad B(t+1) = A(t+1) - \mathbb{E}[A(t+1)],$$

and

$$A(t+1) = \bar{A} + B(t+1)$$

so

$$r \approx \log\lambda - \frac{1}{2\lambda^2}\sum_{i,j,k,\ell} S_{ij}S_{k\ell}\text{cov}[B_{ij}(t)B_{k\ell}(t)]$$

where $S_{ij} = \dfrac{\partial\lambda}{\partial\bar{a}_{ij}}$ and λ is the dominant eigenvalue for $\overline{A}$.

2 Demographic Stochasticity

Populations consist of a finite number of individuals whose fates are not perfectly correlated. The simplest model is a branching process.

2.1 Branching Process

$$N(t+1) = X_1 + \cdots + X_{N(t)}$$

where X_i are i.i.d. on $\mathbb{N}$. If the X_i are independent of $N(t)$ - this is a standard branching process.

There is a limit theorem for branching processes: Define $R \coloneqq \mathbb{E}[X_1]$. If $R \leq 1$, then $N_0 \to 0$ in finite time with probability 1. Note that R is defined this way since

$$\mathbb{E}[N(t+1)|N(t)] = RN(t)$$

If $R > 1$, then $N_t \to \infty$ with positive probability $p < 1$ and goes extinct with probability $1-p$. The probability of extinction when $R > 1$ is then $(1-p)^{N_0}$.

2.2 A Useful Approximation

We say $N(t+1) = X_1 + \cdots + X_{N(t)}$ is approximated by a normal curve with mean $RN(t)$ and variance $N(t)\text{var}[X_1]$ (denot $\sigma^2 \coloneqq \text{var}[X_1]$).

$$N(t+1) = X_1 + \cdots + X_{N(t)} \approx \underbrace{RN(t) + Z\sigma\sqrt{N(t)}}_{=N(t)\left(R + \frac{\sigma}{\sqrt{N(t)}}Z\right)}$$

This is called the diffusion approximation. This means smaller populations are at greater extinction risk. What happens to population size on the event of non-extinction?

$$\mathbb{E}[N(t+1)] = RN(t) \tag{1}$$

but

$$\mathbb{E}[N(t+1)] = \mathbb{E}[N(t+1)|N(t+1)>0]\overbrace{\mathbb{P}[N(t+1)>0]}^{<1} + \cancelto{0}{\mathbb{E}[N(t+1)|N(t+1)=0]}\mathbb{P}[N(t+1)=0] \tag{2}$$