PBG 200A Notes

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1 Persistence, Boundedness, and Regulation

Suppose $r(N)$ is nonincreasing and

$$\frac{\mathrm{d}N}{\mathrm{d}t} = Nr(N) \tag{1}$$

If $r(N)$ is nonincreasing, the maximal per-capita growth rate is at $N = 0$. Then, if $r(0) < 0$, then there is always asymptotic extinction. In the real world, finite time. Or, supposing $r(0) > 0$. Then the population tends to increase at low densities, i.e. the population persists.

The minimal per-capito growth rate is at $N \to \infty$, i.e. $r(\infty) \coloneqq \lim_{N\to\infty} r(N)$ (probably $-\infty$ since it is nonincreasing). If $r(\infty) > 0$, then the population grows indefinitely, without bound. If $r(\infty) < 0$, then the population at high densities starts to decrease (population is bounded).

The population is "regulated" if it is peresistent and bounded. This requires $r(0) > 0$ and $r(\infty) < \infty$, i.e. there is negative density dependence.

2 Positive Density Dependence

Suppose $r(N)$ is increasing with density and

$$\frac{\mathrm{d}N}{\mathrm{d}t} = Nr(N) \tag{2}$$

- cooperative behavior
  - hunting (hyenas)
  - defense (schools of fish)
    - Considering a type two functional response of a generalist predator whose dynamics are relatively constant, or maybe there are time-scale differences, i.e. predators are much longer-lived, the individual risk of a prey individual decreases.
- mating chances - harder to find a mate at low densities
- inbreeding depression
- demographic stochasticity

A paper by Lamont, 1993 showed positive density dependence in *Banksia goodii*. Another: Levitan et al, 1992. Another: Bourbeau-Lemieux et al, 2011.

2.1 Example - Mate Limitation

Let $N \equiv$ density of females. Also assuming this is the density of wild, natural, non-sterilized males (1-1 sex ratio). Let $S \equiv$ density of sterile males. Let $b \equiv$ the per-capita birth rate of wild-fertilized females and $d \equiv$ the per-capita death rate of females. The model (assuming panmictic.. randomly choosing mates):

$$r(N) = b \cdot \underbrace{\frac{N}{N+S}}_{\text{prob. of randomly selecting a wild male}} - d \tag{3}$$

This is an increasing and saturating function.. $R(0) = -d$, $R(\infty) = b - d$, which is assumed to be positive. Two equilibria:

- $N = 0$ (stable)
- $N^* := \dfrac{S}{\frac{b}{d} - 1}$ (unstable)
  - Note: $\dfrac{b}{d}$ is the "R_0" of this population.

Initial conditions between 0 and N^* approach 0, and above N^* go off to infinity. So there is an Allee effect - a density where below there is a "spiral of doom," and above the population persists.

2.2 In general...

Supposing $r(N)$ is a strictly increasing function, the maximal per-capita growth rate is at $N = \infty$. So if $r(\infty) < 0$ (bad in the best of times), the population will go extinct for all initial conditions. The minimal per-capita growth rate is at $N = 0$. So if $r(0) > 0$ (good in the worst of times), the population will always persist and grow without bound.

If $r(0) < 0$ and $r(\infty) > 0$, then there is a critical density N_* such that $N(0) < N_*$ produces "spiral of doom" and $N(0) > N_*$ produces unbounded growth. This shows positive density dependence cannot produce regulated population - only negative density dependence can do that.

2.3 Example - Doomsday

$$\frac{\mathrm{d}N}{\mathrm{d}t} = N\big(aN^b\big) \qquad a, b > 0 \tag{4}$$

Turns out this model produces an infinite population in finite time (doomsday).

3 Negative and Positive Density Dependence

Big picture: anything can happen.

3.1 Example - Mate Limitation with Negative Density Dependence

$$\frac{\mathrm{d}N}{\mathrm{d}t} = N\left(\frac{bN}{N+S} - d_1 - d_2 N\right) \tag{5}$$

To stay interesting, assume $\dfrac{b}{S} > d_2$. So $r(0) = -d_2$ and increases initially, then when the linear term outweighs the saturating term, it decreases asymptotically linearly. Three equilibria (supposing the maximum is positive): 0, T (for "threshold"), and K (for "carrying capacity"). 0 and K are stable, and the threshold equilibrium is unstable. There are two alternative stable states. Increasing d_1 gives rise to a saddle-node bifurcation (sometimes called the "blue sky catasrophe").

Next time we will talk about hysteresis.