Z prime matching

1 Z prime Model

The Z prime Lagrangian that we will consider is the following

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_{SM} - \frac{1}{4}X_{\mu\nu}^2 + \frac{1}{2}M^2X_\mu^2 - g_1\beta\mathcal{H}_\mu X^\mu - g_1\beta\Psi_\mu X^\mu + g_1^2Y_H^2\beta^2(H^\dagger H)X_\mu^2 \\
\mathcal{H} &= iY_H(H^\dagger \overleftrightarrow{D}_\mu H) \\
\Psi_\mu &= \sum_\psi Y_\psi(\bar{\psi}\gamma_\mu\psi)
\end{aligned}$$

I completely agree with Tyler's results for D6 matching.

$$\mathcal{L}_6 = -\frac{\beta^2}{2}Q_{2JB} + \frac{g_1^2}{2}Y_H^2\beta^2(2Q_{\phi,2} - 4Q_{\phi,1}) - ig_1^2\beta^2Y_HY_\psi(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{\psi}\gamma_\mu\psi) \tag{1}$$

Last operator needs to be traded for operators in our basis. It must be removed using field redefinitions to consistently include effects of $\mathcal{O}(1/\Lambda^4)$. Follow Tyler's notes, and after field redefinitions we get

$$\begin{aligned}
\mathcal{L}_{SM} + \mathcal{L}_6 \to &\mathcal{L}_{SM} - \frac{\beta^2}{2}Q_{2JB} + g_1^2Y_H^2\beta^2(2Q_{\phi,1} - Q_{\phi,2}) - g_1\beta^2Y_H\left[2Q_B + \frac{g_1}{2}Q_{BB} + \frac{g_2}{2}Q_{BW}\right] \\
&- g_1^4Y_H^4\beta^4(H^\dagger H)(H^\dagger \overleftrightarrow{D}_\mu H)^2 + \frac{g_1^2Y_H^2\beta^4}{2}\left[\partial_\mu(H^\dagger \overleftrightarrow{D}_\nu H) - \partial_\nu(H^\dagger \overleftrightarrow{D}_\mu H)\right]\partial_\mu(H^\dagger \overleftrightarrow{D}_\nu H) \\
&- 2ig_1^3Y_H^3\beta^4(H^\dagger H)(H^\dagger \overleftrightarrow{D}_\mu H)(\partial_\nu B_{\mu\nu})
\end{aligned}$$

The first line are all D6 terms, and the rest are D8 terms. Now, we perform tree-level matching at D8 using Matchete in Mathematica. We add the above dimension-eight terms to that result, and write everything in the bosonic basis.

2 Bosonic Basis

We get the complete matching result to dimension-eight

$$
\begin{aligned}
\mathcal{L} \to &\mathcal{L}_{SM} + [D6] - \tfrac{1}{2}\,\beta^2\, R^{(1)}_{B^2D^4} - 2\,g_1^2 Y_H^2\,\beta^4\, Q^{(1)}_{H^4} + 2\,g_1^2 Y_H^2\,\beta^4\, Q^{(2)}_{H^4} + g_1^2 Y_H^2\,\beta^4\, R^{(9)}_{B^2H^2D^2} \\
&+ 2i\,g_2 g_1^2 Y_H^2\,\beta^4\, Q^{(1)}_{WH^4D^2} + i\,g_2 g_1^2 Y_H^2\,\beta^4\, R^{(1)}_{WH^4D^2} - \tfrac{1}{2}\,g_2 g_1^2 Y_H^2\,\beta^4\, R'^{(2)}_{WH^4D^2} \\
&+ \tfrac{1}{4}\,g_2^2 g_1^2 Y_H^2\,\beta^4\, Q^{(1)}_{W^2H^4} - 2i\,g_1^3 Y_H^2\,\beta^4\, Q^{(1)}_{BH^4D^2} - \tfrac{1}{4}\,g_1^4 Y_H^2\,\beta^4\, Q^{(1)}_{B^2H^4} \\
&+ 2i\,g_1^3 Y_H^3\,\beta^4\, R^{(1)}_{BH^4D^2} + 3\,g_1^4 Y_H^4\,\beta^4\, Q^{(1)}_{H^6} + 2\,g_1^4 Y_H^4\,\beta^4\, Q^{(2)}_{H^6} \\
&+ \tfrac{1}{2}\,g_1^4 Y_H^4\,\beta^4\, R^{(1)}_{H^6D^2} + \tfrac{1}{2}\,g_1^4 Y_H^4\,\beta^4\, R^{(2)}_{H^6D^2}\,.
\end{aligned}
$$

Here we list the matching results for the Z prime model to our bosonic basis. Just to clarify, even in our "bosonic basis" we still use the same basis at dimension-six and it has fermionic operators. Note, that $R'^{(2)}_{WH^4D^2} = R^{(2)}_{WH^4D^2} + R^{(3)}_{WH^4D^2}$ is the CP even operator that we defined.

Wilson Coefficients			Operator		
c_{2JB}	$=$	$-\frac{\beta^2}{2}$	Q_{2JB}	$=$	$J_{B,\mu}J^\mu_B$
$c_{\phi,1}$	$=$	$2g_1^2Y_H^2\beta^2$	$Q_{\phi,1}$	$=$	$(D_\mu H^\dagger H)(H^\dagger D^\mu H)$
$c_{\phi,2}$	$=$	$-g_1^2Y_H^2$	$Q_{\phi,2}$	$=$	$\frac{1}{2}\partial^\mu(H^\dagger H)\partial_\mu(H^\dagger H)$
c_B	$=$	$2g_1\beta^2Y_H$	Q_B	$=$	$i(D_\mu H)^\dagger B^{\mu\nu}(D_\nu H)$
c_{BB}	$=$	$g_1^2\beta^2Y_H$	Q_{BB}	$=$	$H^\dagger B_{\mu\nu}B^{\mu\nu}H$
c_{BW}	$=$	$g_1g_2\beta^2Y_H$	Q_{BW}	$=$	$H^\dagger B_{\mu\nu}\tau^I W^{I\mu\nu}H$
$b^{(1)}_{H^4}$	$=$	$-2g_1^2Y_H^2\beta^4$	$Q^{(1)}_{H^4}$	$=$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$b^{(2)}_{H^4}$	$=$	$2g_1^2Y_H^2\beta^4$	$Q^{(2)}_{H^4}$	$=$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$b^{(1)}_{H^6}$	$=$	$3g_1^4Y_H^4\beta^4$	$Q^{(1)}_{H^6}$	$=$	$(H^\dagger H)^2(D_\mu H^\dagger D^\mu H)$
$b^{(2)}_{H^6}$	$=$	$2g_1^4Y_H^4\beta^4$	$Q^{(2)}_{H^6}$	$=$	$(H^\dagger H)(H^\dagger\tau^I H)(D_\mu H^\dagger\tau^I D^\mu H)$
$b^{(1)}_{B^2H^4}$	$=$	$-\frac{1}{4}g_1^4Y_H^2\beta^4$	$Q^{(1)}_{B^2H^4}$	$=$	$(H^\dagger H)^2B_{\mu\nu}B^{\mu\nu}$
$b^{(1)}_{W^2H^4}$	$=$	$\frac{1}{4}g_1^2g_2^2Y_H^2\beta^4$	$Q^{(1)}_{W^2H^4}$	$=$	$(H^\dagger H)^2W^I_{\mu\nu}W^{I,\mu\nu}$
$b^{(1)}_{BH^4D^2}$	$=$	$-2ig_1^3Y_H^2\beta^4$	$Q^{(1)}_{BH^4D^2}$	$=$	$(H^\dagger H)(D^\mu H^\dagger D^\nu H)B_{\mu\nu}$
$b^{(1)}_{WH^4D^2}$	$=$	$2ig_1^2g_2Y_H^2\beta^4$	$Q^{(1)}_{WH^4D^2}$	$=$	$(H^\dagger H)(D^\mu H^\dagger\tau^I D^\nu H)W^I_{\mu\nu}$
$r^{(1)}_{BH^4D^2}$	$=$	$2ig_1^3Y_H^3\beta^4$	$R^{(1)}_{BH^4D^2}$	$=$	$(D_\alpha B^{\alpha\mu})(H^\dagger\overleftrightarrow{D}_\mu H)(H^\dagger H)$
$r^{(1)}_{WH^4D^2}$	$=$	$ig_1^2g_2Y_H^2\beta^4$	$R^{(1)}_{WH^4D^2}$	$=$	$(D^\mu W^I_{\mu\nu})(H^\dagger\overleftrightarrow{D}^{I\nu}H)(H^\dagger H)$
$r'^{(2)}_{WH^4D^2}$	$=$	$-\frac{1}{2}ig_1^2g_2Y_H^2\beta^4$	$R'^{(2)}_{WH^4D^2}$	$=$	$\epsilon^{IJK}(H^\dagger\tau^I H)D^\nu(H^\dagger\tau^J H)(D^\mu W^K_{\mu\nu})$
$r^{(9)}_{B^2H^2D^2}$	$=$	$g_1^2Y_H^2\beta^4$	$R^{(9)}_{B^2H^2D^2}$	$=$	$(D^\mu B_{\mu\alpha})(D_\nu B^{\nu\alpha})(H^\dagger H)$
$r^{(1)}_{H^6D^2}$	$=$	$\frac{1}{2}g_1^4Y_H^4\beta^4$	$R^{(1)}_{H^6D^2}$	$=$	$(D^2H^\dagger H)(H^\dagger H)^2$
$r^{(2)}_{H^6D^2}$	$=$	$\frac{1}{2}g_1^4Y_H^4\beta^4$	$R^{(2)}_{H^6D^2}$	$=$	$(H^\dagger D^2H)(H^\dagger H)^2$
$r^{(1)}_{B^2D^4}$	$=$	$-\frac{\beta^2}{2}$	$R^{(1)}_{B^2D^4}$	$=$	$D^\rho D^\alpha B_{\alpha\mu}D_\rho D^\beta B^\mu_\beta$

3 Rotated Basis

Now, we rotate the operators from the *bosonic* basis to the *rotated* basis, using the relations obtained from the equations of motion in the appendix of arXiv:2404.03720. To simplify, I put $Y_H \to 1/2$. Here are the matching results that we get. We also get a new dimension six operator $Q_{\phi,6}$ from EOM, which we write down in the first line.

$$\begin{aligned}\mathcal{L} \to \mathcal{L}_{SM} &+ [D6] - \tfrac{1}{8}v^2 g_2^2 g_1^2 \beta^2(1+\beta^2)\lambda_h\, Q_{\phi,6} - \tfrac{1}{2}\beta^2\, Q^{(2)}_{\psi^4 D^2} + \tfrac{1}{4}g_1\beta^2\, Q^{(1)}_{\psi^2 H^2 D^3} \\ &- \tfrac{3}{8}g_1^2\beta^2\, Q^{(1)}_{\psi^4 H^2} + \tfrac{3}{8}g_1^2\beta^2\, Q^{(2)}_{\psi^4 H^2} + \tfrac{3}{8}g_1^2\beta^2\, Q^{\dagger(2)}_{\psi^4 H^2} - \tfrac{3}{4}g_1^2\beta^2\, Q^{(3)}_{\psi^4 H^2} + \tfrac{1}{4}g_1^2\beta^4\, Q^{(4)}_{\psi^4 H^2} \\ &- \tfrac{1}{2}g_1^2\beta^2(1+\beta^2)\, Q^{(1)}_{H^4} + \tfrac{1}{2}g_1^2\beta^2(1+\beta^2)\, Q^{(2)}_{H^4} + \tfrac{i}{2}g_2 g_1^2\beta^2(1+\beta^2)\, Q^{(1)}_{WH^4D^2} - \tfrac{1}{8}g_2 g_1^2\beta^2(1+\beta^2)\, Q^{(2)}_{\psi^2 H^4 D} \\ &- \tfrac{1}{2}g_2^2 g_1^2\beta^2(1+\beta^2)\, Q^{(1)}_{H^6} - \tfrac{1}{8}g_2^2 g_1^2\beta^2(1+\beta^2)\, Q^{(2)}_{H^6} + \tfrac{1}{16}g_2^2 g_1^2\beta^2(1+\beta^2)\,(Q^{(1)}_{\psi^2 H^5} + Q^{\dagger(1)}_{\psi^2 H^5}) \\ &+ \tfrac{1}{16}g_2^2 g_1^2\beta^2(1+\beta^2)\, Q^{(1)}_{W^2 H^4} - \tfrac{i}{2}g_1^3\beta^2(1+\beta^2)\, Q^{(1)}_{BH^4D^2} - \tfrac{1}{16}g_1^4\beta^2(1+\beta^2)\, Q^{(1)}_{B^2 H^4} + \tfrac{1}{4}g_2^2 g_1^2\beta^2(1+\beta^2)\lambda_h\, Q_{H^8}\end{aligned}$$

Now, we tabulate the matching results to the *rotated* basis.

Wilson Coefficients		
c_{2JB}	$=$	$-\frac{\beta^2}{2}$
$c_{\phi,1}$	$=$	$2g_1^2 Y_H^2 \beta^2$
$c_{\phi,2}$	$=$	$-g_1^2 Y_H^2$
$c_{\phi,6}$	$=$	$-\frac{1}{8}v^2\lambda_h\, g_2^2 g_1^2 \beta^2(1+\beta^2)$
c_B	$=$	$2g_1\beta^2 Y_H$
c_{BB}	$=$	$g_1^2\beta^2 Y_H$
c_{BW}	$=$	$g_1 g_2 \beta^2 Y_H$
c_{H^8}	$=$	$\frac{1}{4}g_2^2 g_1^2\beta^2(1+\beta^2)\lambda_h$
$c_{H^4}^{(1)}$	$=$	$-\frac{1}{2}g_1^2\beta^2(1+\beta^2)$
$c_{H^4}^{(2)}$	$=$	$\frac{1}{2}g_1^2\beta^2(1+\beta^2)$
$c_{H^6}^{(1)}$	$=$	$-\frac{1}{2}g_2^2 g_1^2\beta^2(1+\beta^2)$
$c_{H^6}^{(2)}$	$=$	$-\frac{1}{8}g_2^2 g_1^2\beta^2(1+\beta^2)$
$c_{B^2H^4}^{(1)}$	$=$	$-\frac{1}{16}g_1^4\beta^2(1+\beta^2)$
$c_{W^2H^4}^{(1)}$	$=$	$\frac{1}{16}g_2^2 g_1^2\beta^2(1+\beta^2)$
$c_{BH^4D^2}^{(1)}$	$=$	$-\frac{i}{2}g_1^3\beta^2(1+\beta^2)$
$c_{WH^4D^2}^{(1)}$	$=$	$\frac{i}{2}g_2 g_1^2\beta^2(1+\beta^2)$
$c_{\psi^2H^5}^{(1)}$	$=$	$\frac{1}{16}g_2^2 g_1^2\beta^2(1+\beta^2)$
$c_{\psi^2H^5}^{\dagger(1)}$	$=$	$\frac{1}{16}g_2^2 g_1^2\beta^2(1+\beta^2)$
$c_{\psi^2H^2D^3}^{(1)}$	$=$	$\frac{1}{4}g_1\beta^2$
$c_{\psi^2H^4D}^{(2)}$	$=$	$-\frac{1}{8}g_2 g_1^2\beta^2(1+\beta^2)$
$c_{\psi^4D^2}^{(2)}$	$=$	$-\frac{1}{2}\beta^2$
$c_{\psi^4H^2}^{(1)}$	$=$	$-\frac{3}{8}g_1^2\beta^2$
$c_{\psi^4H^2}^{(2)}$	$=$	$\frac{3}{8}g_1^2\beta^2$
$c_{\psi^4H^2}^{\dagger(2)}$	$=$	$\frac{3}{8}g_1^2\beta^2$
$c_{\psi^4H^2}^{(3)}$	$=$	$-\frac{3}{4}g_1^2\beta^2$
$c_{\psi^4H^2}^{(4)}$	$=$	$\frac{1}{4}g_1^2\beta^4$