

# Z prime matching

## 1 Z prime Model

The Z prime Lagrangian that we will consider is the following

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{SM} - \frac{1}{4}X_{\mu\nu}^2 + \frac{1}{2}M^2X_\mu^2 - g_1\beta\mathcal{H}_\mu X^\mu - g_1\beta\Psi_\mu X^\mu + g_1^2Y_H^2\beta^2(H^\dagger H)X_\mu^2 \\ \mathcal{H} &= iY_H(H^\dagger \overleftrightarrow{D}_\mu H) \\ \Psi_\mu &= \sum_\psi Y_\psi(\bar{\psi}\gamma_\mu\psi)\end{aligned}$$

I completely agree with Tyler's results for D6 matching.

$$\mathcal{L}_6 = -\frac{\beta^2}{2}Q_{2JB} + \frac{g_1^2}{2}Y_H^2\beta^2(2Q_{\phi,2} - 4Q_{\phi,1}) - ig_1^2\beta^2Y_HY_\psi(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{\psi}\gamma_\mu\psi) \quad (1)$$

Last operator needs to be traded for operators in our basis. It must be removed using field redefinitions to consistently include effects of  $\mathcal{O}(1/\Lambda^4)$ . Follow Tyler's notes, and after field redefinitions we get

$$\begin{aligned}\mathcal{L}_{SM} + \mathcal{L}_6 &\rightarrow \mathcal{L}_{SM} - \frac{\beta^2}{2}Q_{2JB} + g_1^2Y_H^2\beta^2(2Q_{\phi,1} - Q_{\phi,2}) - g_1\beta^2Y_H \left[ 2Q_B + \frac{g_1}{2}Q_{BB} + \frac{g_2}{2}Q_{BW} \right] \\ &\quad - g_1^4Y_H^4\beta^4(H^\dagger H)(H^\dagger \overleftrightarrow{D}_\mu H)^2 + \frac{g_1^2Y_H^2\beta^4}{2} \left[ \partial_\mu(H^\dagger \overleftrightarrow{D}_\nu H) - \partial_\nu(H^\dagger \overleftrightarrow{D}_\mu H) \right] \partial_\mu(H^\dagger \overleftrightarrow{D}_\nu H) \\ &\quad - 2ig_1^3Y_H^3\beta^4(H^\dagger H)(H^\dagger \overleftrightarrow{D}_\mu H)(\partial_\nu B_{\mu\nu})\end{aligned}$$

The first line are all D6 terms, and the rest are D8 terms. Now, we perform tree-level matching at D8 using Matchete in Mathematica. We add the above dimension-eight terms to that result, and write everything in the bosonic basis.

## 2 Bosonic Basis

We get the complete matching result to dimension-eight

$$\begin{aligned}
\mathcal{L} \rightarrow & \mathcal{L}_{SM} + [D6] - \frac{1}{2} \beta^2 R_{B^2 D^4}^{(1)} - 2 g_1^2 Y_H^2 \beta^4 Q_{H^4}^{(1)} + 2 g_1^2 Y_H^2 \beta^4 Q_{H^4}^{(2)} + g_1^2 Y_H^2 \beta^4 R_{B^2 H^2 D^2}^{(9)} \\
& + 2i g_2 g_1^2 Y_H^2 \beta^4 Q_{WH^4 D^2}^{(1)} + i g_2 g_1^2 Y_H^2 \beta^4 R_{WH^4 D^2}^{(1)} - \frac{1}{2} g_2 g_1^2 Y_H^2 \beta^4 R_{WH^4 D^2}'^{(2)} \\
& + \frac{1}{4} g_2^2 g_1^2 Y_H^2 \beta^4 Q_{W^2 H^4}^{(1)} - 2i g_1^3 Y_H^2 \beta^4 Q_{BH^4 D^2}^{(1)} - \frac{1}{4} g_1^4 Y_H^2 \beta^4 Q_{B^2 H^4}^{(1)} \\
& + 2i g_1^3 Y_H^3 \beta^4 R_{BH^4 D^2}^{(1)} + 3 g_1^4 Y_H^4 \beta^4 Q_{H^6}^{(1)} + 2 g_1^4 Y_H^4 \beta^4 Q_{H^6}^{(2)} \\
& + \frac{1}{2} g_1^4 Y_H^4 \beta^4 R_{H^6 D^2}^{(1)} + \frac{1}{2} g_1^4 Y_H^4 \beta^4 R_{H^6 D^2}^{(2)} .
\end{aligned}$$

Here we list the matching results for the Z prime model to our bosonic basis. Just to clarify, even in our “bosonic basis” we still use the same basis at dimension-six and it has fermionic operators. Note, that  $R_{WH^4D^2}^{(2)} = R_{WH^4D^2}^{(2)} + R_{WH^4D^2}^{(3)}$  is the CP even operator that we defined. The operators listed on the right column here are the ones defined in the basis paper.

Wilson Coefficients		Operator	
$c_{2JB}$	$= -\frac{\beta^2}{2}$	$Q_{2JB}$	$= J_{B,\mu} J_B^\mu$
$c_{\Phi,1}$	$= 2g_1^2 Y_H^2 \beta^2$	$Q_{\Phi,1}$	$= (D_\mu H^\dagger H)(H^\dagger D^\mu H)$
$c_{\Phi,2}$	$= -g_1^2 Y_H^2$	$Q_{\Phi,2}$	$= \frac{1}{2} \partial^\mu (H^\dagger H) \partial_\mu (H^\dagger H)$
$c_B$	$= 2g_1 \beta^2 Y_H$	$Q_B$	$= i(D_\mu H)^\dagger B^{\mu\nu} (D_\nu H)$
$c_{BB}$	$= g_1^2 \beta^2 Y_H$	$Q_{BB}$	$= H^\dagger B_{\mu\nu} B^{\mu\nu} H$
$c_{BW}$	$= g_1 g_2 \beta^2 Y_H$	$Q_{BW}$	$= H^\dagger B_{\mu\nu} \tau^I W^{I\mu\nu} H$
$b_{H^4}^{(1)}$	$= -2g_1^2 Y_H^2 \beta^4$	$Q_{H^4}^{(1)}$	$= (D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$b_{H^4}^{(2)}$	$= 2g_1^2 Y_H^2 \beta^4$	$Q_{H^4}^{(2)}$	$= (D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$b_{H^6}^{(1)}$	$= 3g_1^4 Y_H^4 \beta^4$	$Q_{H^6}^{(1)}$	$= (H^\dagger H)^2 (D_\mu H^\dagger D^\mu H)$
$b_{H^6}^{(2)}$	$= 2g_1^4 Y_H^4 \beta^4$	$Q_{H^6}^{(2)}$	$= (H^\dagger H)(H^\dagger \tau^I H)(D_\mu H^\dagger \tau^I D^\mu H)$
$b_{B^2 H^4}^{(1)}$	$= -\frac{1}{4} g_1^4 Y_H^2 \beta^4$	$Q_{B^2 H^4}^{(1)}$	$= (H^\dagger H)^2 B_{\mu\nu} B^{\mu\nu}$
$b_{W^2 H^4}^{(1)}$	$= \frac{1}{4} g_1^2 g_2^2 Y_H^2 \beta^4$	$Q_{W^2 H^4}^{(1)}$	$= (H^\dagger H)^2 W_{\mu\nu}^I W^{I,\mu\nu}$
$b_{BH^4 D^2}^{(1)}$	$= -2ig_1^3 Y_H^2 \beta^4$	$Q_{BH^4 D^2}^{(1)}$	$= (H^\dagger H)(D^\mu H^\dagger D^\nu H) B_{\mu\nu}$
$b_{WH^4 D^2}^{(1)}$	$= 2ig_1^2 g_2 Y_H^2 \beta^4$	$Q_{WH^4 D^2}^{(1)}$	$= (H^\dagger H)(D^\mu H^\dagger \tau^I D^\nu H) W_{\mu\nu}^I$
$r_{BH^4 D^2}^{(1)}$	$= 2ig_1^3 Y_H^3 \beta^4$	$R_{BH^4 D^2}^{(1)}$	$= (D_\alpha B^{\alpha\mu})(H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H)$
$r_{WH^4 D^2}^{(1)}$	$= ig_1^2 g_2 Y_H^2 \beta^4$	$R_{WH^4 D^2}^{(1)}$	$= (D^\mu W_{\mu\nu}^I)(H^\dagger \overleftrightarrow{D}^{I\nu} H)(H^\dagger H)$
$r_{WH^4 D^2}^{\prime(2)}$	$= -\frac{1}{2} g_1^2 g_2 Y_H^2 \beta^4$	$R_{WH^4 D^2}^{\prime(2)}$	$= \epsilon^{IJK} (H^\dagger \tau^I H) D^\nu (H^\dagger \tau^J H) (D^\mu W_{\mu\nu}^K)$
$r_{B^2 H^2 D^2}^{(9)}$	$= g_1^2 Y_H^2 \beta^4$	$R_{B^2 H^2 D^2}^{(9)}$	$= (D^\mu B_{\mu\alpha})(D_\nu B^{\nu\alpha})(H^\dagger H)$
$r_{H^6 D^2}^{(1)}$	$= \frac{1}{2} g_1^4 Y_H^4 \beta^4$	$R_{H^6 D^2}^{(1)}$	$= (D^2 H^\dagger H)(H^\dagger H)^2$
$r_{H^6 D^2}^{(2)}$	$= \frac{1}{2} g_1^4 Y_H^4 \beta^4$	$R_{H^6 D^2}^{(2)}$	$= (H^\dagger D^2 H)(H^\dagger H)^2$
$r_{B^2 D^4}^{(1)}$	$= -\frac{\beta^2}{2}$	$R_{B^2 D^4}^{(1)}$	$= D^\rho D^\alpha B_{\alpha\mu} D_\rho D^\beta B_\beta^\mu$

### 3 Rotated Basis

Now, we rotate the operators from the *bosonic* basis to the *rotated* basis, using the relations obtained from the equations of motion in the appendix of arXiv:2404.03720. To simplify, I put  $Y_H \rightarrow 1/2$ . Here are the matching results that we get. We also get a new dimension six operator  $Q_{\phi,6}$  from EOM, which we write down in the first line.

$$\begin{aligned}
\mathcal{L} \rightarrow \mathcal{L}_{SM} + [D6] &- \frac{1}{8} v^2 g_2^2 g_1^2 \beta^2 (1 + \beta^2) \lambda_h Q_{\phi,6} - \frac{1}{2} \beta^2 Q_{\psi^4 D^2}^{(2)} + \frac{1}{4} g_1 \beta^2 Q_{\psi^2 H^2 D^3}^{(1)} \\
&- \frac{3}{8} g_1^2 \beta^2 Q_{\psi^4 H^2}^{(1)} + \frac{3}{8} g_1^2 \beta^2 Q_{\psi^4 H^2}^{(2)} + \frac{3}{8} g_1^2 \beta^2 Q_{\psi^4 H^2}^{\dagger(2)} - \frac{3}{4} g_1^2 \beta^2 Q_{\psi^4 H^2}^{(3)} + \frac{1}{4} g_1^2 \beta^4 Q_{\psi^4 H^2}^{(4)} \\
&- \frac{1}{2} g_1^2 \beta^2 (1 + \beta^2) Q_{H^4}^{(1)} + \frac{1}{2} g_1^2 \beta^2 (1 + \beta^2) Q_{H^4}^{(2)} + \frac{i}{2} g_2 g_1^2 \beta^2 (1 + \beta^2) Q_{WH^4 D^2}^{(1)} - \frac{1}{8} g_2 g_1^2 \beta^2 (1 + \beta^2) Q_{\psi^2 H^4 D}^{(2)} \\
&- \frac{1}{2} g_2^2 g_1^2 \beta^2 (1 + \beta^2) Q_{H^6}^{(1)} - \frac{1}{8} g_2^2 g_1^2 \beta^2 (1 + \beta^2) Q_{H^6}^{(2)} + \frac{1}{16} g_2^2 g_1^2 \beta^2 (1 + \beta^2) (Q_{\psi^2 H^5}^{(1)} + Q_{\psi^2 H^5}^{\dagger(1)}) \\
&+ \frac{1}{16} g_2^2 g_1^2 \beta^2 (1 + \beta^2) Q_{W^2 H^4}^{(1)} - \frac{i}{2} g_1^3 \beta^2 (1 + \beta^2) Q_{BH^4 D^2}^{(1)} - \frac{1}{16} g_1^4 \beta^2 (1 + \beta^2) Q_{B^2 H^4}^{(1)} + \frac{1}{4} g_2^2 g_1^2 \beta^2 (1 + \beta^2) \lambda_h Q_{H^8}
\end{aligned}$$

Now, we tabulate the matching results to the *rotated* basis.

Wilson Coefficients		Operator	
$c_{2JB}$	$= -\frac{\beta^2}{2}$	$Q_{2JB}$	$= J_{B,\mu} J_B^\mu$
$c_{\Phi,1}$	$= 2g_1^2 Y_H^2 \beta^2$	$Q_{\Phi,1}$	$= (D_\mu H^\dagger H)(H^\dagger D^\mu H)$
$c_{\Phi,2}$	$= -g_1^2 Y_H^2$	$Q_{\Phi,2}$	$= \frac{1}{2} \partial^\mu (H^\dagger H) \partial_\mu (H^\dagger H)$
$c_{\Phi,6}$	$= -\frac{1}{8} v^2 \lambda_h g_2^2 g_1^2 \beta^2 (1 + \beta^2)$	$Q_{\Phi,6}$	$= (H^\dagger H)^3$
$c_B$	$= 2g_1 \beta^2 Y_H$	$Q_B$	$= i(D_\mu H)^\dagger B^{\mu\nu} (D_\nu H)$
$c_{BB}$	$= g_1^2 \beta^2 Y_H$	$Q_{BB}$	$= H^\dagger B_{\mu\nu} B^{\mu\nu} H$
$c_{BW}$	$= g_1 g_2 \beta^2 Y_H$	$Q_{BW}$	$= H^\dagger B_{\mu\nu} \tau^I W^{I\mu\nu} H$
$c_{H^8}$	$= \frac{1}{4} g_2^2 g_1^2 \beta^2 (1 + \beta^2) \lambda_h$	$Q_{H^8}$	$= (H^\dagger H)^4$
$c_{H^4}^{(1)}$	$= -\frac{1}{2} g_1^2 \beta^2 (1 + \beta^2)$	$Q_{H^4}^{(1)}$	$= (D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$c_{H^4}^{(2)}$	$= \frac{1}{2} g_1^2 \beta^2 (1 + \beta^2)$	$Q_{H^4}^{(2)}$	$= (D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$c_{H^6}^{(1)}$	$= -\frac{1}{2} g_2^2 g_1^2 \beta^2 (1 + \beta^2)$	$Q_{H^6}^{(1)}$	$= (H^\dagger H)^2 (D_\mu H^\dagger D^\mu H)$
$c_{H^6}^{(2)}$	$= -\frac{1}{8} g_2^2 g_1^2 \beta^2 (1 + \beta^2)$	$Q_{H^6}^{(2)}$	$= (H^\dagger H)(H^\dagger \tau^I H)(D_\mu H^\dagger \tau^I D^\mu H)$
$c_{B^2 H^4}^{(1)}$	$= -\frac{1}{16} g_1^4 \beta^2 (1 + \beta^2)$	$Q_{B^2 H^4}^{(1)}$	$= (H^\dagger H)^2 B_{\mu\nu} B^{\mu\nu}$
$c_{W^2 H^4}^{(1)}$	$= \frac{1}{16} g_2^2 g_1^2 \beta^2 (1 + \beta^2)$	$Q_{W^2 H^4}^{(1)}$	$= (H^\dagger H)^2 W_{\mu\nu}^I W^{I,\mu\nu}$
$c_{BH^4 D^2}^{(1)}$	$= -\frac{i}{2} g_1^3 \beta^2 (1 + \beta^2)$	$Q_{BH^4 D^2}^{(1)}$	$= (H^\dagger H)(D^\mu H^\dagger D^\nu H) B_{\mu\nu}$
$c_{WH^4 D^2}^{(1)}$	$= \frac{i}{2} g_2 g_1^2 \beta^2 (1 + \beta^2)$	$Q_{WH^4 D^2}^{(1)}$	$= (H^\dagger H)(D^\mu H^\dagger \tau^I D^\nu H) W_{\mu\nu}^I$
$c_{\psi^2 H^5}^{(1)}$	$= \frac{1}{16} g_2^2 g_1^2 \beta^2 (1 + \beta^2)$	$Q_{\psi^2 H^5}^{(1)}$	$= (H^\dagger H)^2 (J_H H)$
$c_{\psi^2 H^5}^{\dagger(1)}$	$= \frac{1}{16} g_2^2 g_1^2 \beta^2 (1 + \beta^2)$	$Q_{\psi^2 H^5}^{\dagger(1)}$	$= (H^\dagger H)^2 (H^\dagger J_H^\dagger)$
$c_{\psi^2 H^2 D^3}^{(1)}$	$= \frac{1}{4} g_1 \beta^2$	$Q_{\psi^2 H^2 D^3}^{(1)}$	$= i(D^\mu J_B^\nu + D^\nu J_B^\mu)(D_{(\mu} D_{\nu)} H^\dagger H$ $- H^\dagger D_{(\mu} D_{\nu)} H)$
$c_{\psi^2 H^4 D}^{(2)}$	$= -\frac{1}{8} g_2 g_1^2 \beta^2 (1 + \beta^2)$	$Q_{\psi^2 H^4 D}^{(2)}$	$= i J_W^{I\mu} \left[ (H^\dagger \overleftrightarrow{D}_\mu^I H)(H^\dagger H) \right.$ $\left. + (H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger \tau^I H) \right]$
$c_{\psi^4 D^2}^{(2)}$	$= -\frac{1}{2} \beta^2$	$Q_{\psi^4 D^2}^{(2)}$	$= D^\alpha J_B^\mu D_\alpha J_{B\mu}$
$c_{\psi^4 H^2}^{(1)}$	$= -\frac{3}{8} g_1^2 \beta^2$	$Q_{\psi^4 H^2}^{(1)}$	$= (J_H^j J_{Hj}^\dagger)(H^\dagger H)$
$c_{\psi^4 H^2}^{(2)}$	$= \frac{3}{8} g_1^2 \beta^2$	$Q_{\psi^4 H^2}^{(2)}$	$= J_H^j J_H^k H_j H_k$
$c_{\psi^4 H^2}^{\dagger(2)}$	$= \frac{3}{8} g_1^2 \beta^2$	$Q_{\psi^4 H^2}^{\dagger(2)}$	$= H^{\dagger j} H^{\dagger k} J_{Hj}^\dagger J_{Hk}^\dagger$
$c_{\psi^4 H^2}^{(3)}$	$= -\frac{3}{4} g_1^2 \beta^2$	$Q_{\psi^4 H^2}^{(3)}$	$= H^{\dagger k} J_{Hk}^\dagger J_H^j H_j - \frac{1}{2} Q_{\psi^4 H^2}^{(1)}$