From Eq. 1

$$\mathcal{L} = -\frac{\beta^2}{2} Q_{2JB} + g_1^2/_2\, Y_H^2 \beta^2 \left(2Q_{\phi,2} - 4Q_{\phi,1}\right) - \beta^2 Y_H\, 2\, Q_{HJB}$$

[Following the conv. from Wells]

Matching in the Warsaw basis:

$$C_{2JB} = -\beta^2/2$$

$$C_{\phi,1} = -2 g_1^2 Y_H^2 \beta^2 = -1/_2\, g_1^2 \beta^2$$

$$C_{HJB} = -2\beta^2 Y_H = -\beta^2$$

Oblique params: [Table 7 from Wells]

$$\hat{S} = g_2^2 \left[1/_4\, C_{HJB} - 1/_2\, C_{2JB} + \underbrace{\ldots}_{\text{not generated}} \right]$$

$$= 1/_2\, g_2^2 \left[1/_2 (-\beta^2) - (-\beta^2/_2) \right]$$

$$= 1/_2\, g_2^2 \left[-\beta^2/_2 + \beta^2/_2 \right] = 0$$

$$T = -1/_2\, C_{\phi,1} + g_1^2/_2 \left(C_{HJB} - C_{2JB} \right)$$

$$= -\frac{1}{2}\left(-\frac{1}{2} g_1^2 \beta^2\right) + g_1^2/_2 \left(-\beta^2 - \left(-\frac{\beta^2}{2}\right)\right)$$

$$= \frac{1}{4} g_1^2 \beta^2 - \frac{1}{4} g_1^2 \beta^2 = 0$$

$$Y = -g_2^2/_2\, C_{2JB}$$

$$= -g_2^2/_2 \left(-\beta^2/_2\right) = g_2^2 \beta^2/_4$$

In our basis, from Eq. 19 the matching reads

$$C_{2JB} = -\beta^2/_2$$

$$C_{\phi,1} = 2 g_1^2 Y_H^2 \beta^2 = 1/_2\, g_1^2 \beta^2$$

$$C_{BW} = -1/_2\, g_1 g_2 \beta^2 Y_H = -1/_4\, g_1 g_2 \beta^2$$

...

From our paper,

$$S = g_2/_{g_1}\, C_{BW} - g_2^2/_2\, C_{2JB} + \ldots$$

$$= g_2 \left[\frac{1}{g_1}\left(-\frac{1}{4} g_1 g_2 \beta^2\right) - g_2/_2 \left(-\beta^2/_2\right) \right]$$

$$= g_2 \left[-\frac{1}{4} g_2 \beta^2 + \frac{1}{4} g_2 \beta^2 \right] =$$

$$= 0 \checkmark$$

$$T = -\frac{1}{2}\left[C_{\phi,1} + g_1^2 C_{2JB} \right] + \ldots$$

$$= -\frac{1}{2}\left[\frac{1}{2} g_1^2 \beta^2 + g_1^2 \left(-\frac{\beta^2}{2}\right) \right]$$

$$= 0 \checkmark$$

$$Y = -g_2^2/_2\, C_{2JB} = -g_2^2/_2 \left(-\beta^2/_2\right)$$

$$= g_2^2/_4\, \beta^2 \checkmark$$