Image Compression via Singular Value Decomposition

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Outline

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1. Singular Value Decomposition
2. Application to Image Compression
3. The Code
4. Examples

Images are Matrices

- An image can be viewed as a matrix where each entry in the matrix represents a pixel.
  - If we greyscale the image the entries are just numbers!
  - (otherwise they would be something like triples representing RGB values)
- So, we can apply techniques from linear algebra to manipluate images!

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Some Linear Algebra

- Given a matrix M we can decompose it as $M = U \Sigma V$ where U, Σ, and V have some nice properties.
- Σ is matrix whose non-zero entries are located on the main diagonal and these values contain information about how the columns of U relate to the matrix M.

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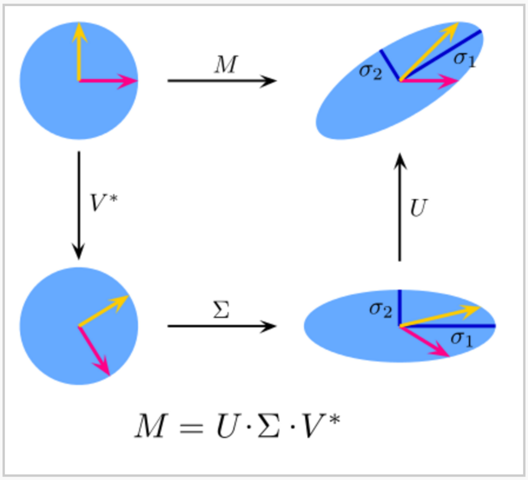
Schematic Example



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- The columns of U whose corresponding values in Σ (called singular values) are large can be interpreted as the "most important" in relation to A.
- In a sense we can approximate A by only paying attention to the first N "most important" columns of U.

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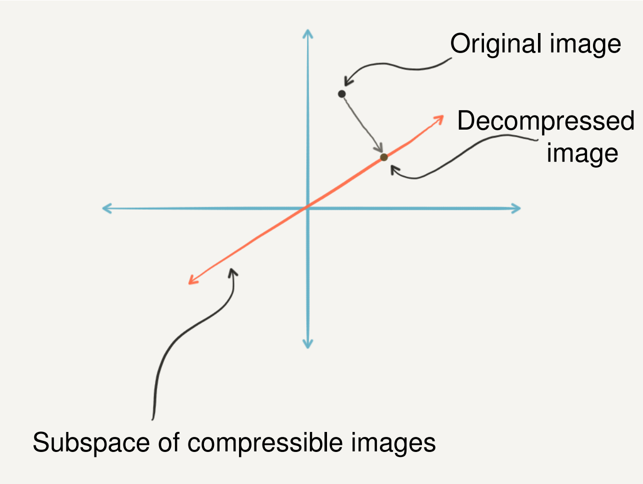
The Big Picture



Where's the Compression?

- The original image can be described with two coordinates.
- The compressed can be described with only one coordinate: the distance along the line from the origin!
- So, we have halved the amount of information needed to store the image.

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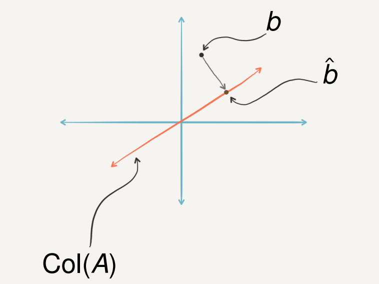
This is Really Least Squares (Some Math)



- If A is the matrix of "important" columns of the image matrix M, then the equation $Ax = b$ has a solution if b is in the span of the columns of A.

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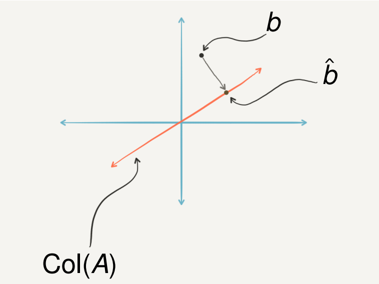

- But here, b is not in the column space of A, so we do the best we can: project b into $Col(A)$ and solve the equation $Ax = \hat{b}$ which DOES have a solution.

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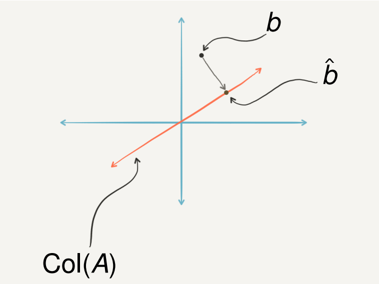

- A solution to $Ax = \hat{b}$ is a compressed representation of the image b and we recover the decompressed image $\hat{b}$ by multiplying by A.

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The following examples were generated using the same algorithm, but implemented in Mathematica.

I Like Dogs

Compressed 90%

Compressed 99%

I Like Dogs

Compressed 90%

Compressed 99%

I Like Dogs

Compressed 90%

Compressed 99%

I Like Dogs

Compressed 90%

Compressed 99%